Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.2-Cosine/92-4.2.3.1-a+b-cos-^m-c+d-cos-^n-A+B-cos-

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [644]. This is test number [92].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (644)	0.00 (0)
Mathematica	98.60 (635)	1.40 (9)
Maple	98.45 (634)	1.55 (10)
Fricas	72.98 (470)	27.02 (174)
Mupad	35.87 (231)	64.13 (413)
Giac	32.30 (208)	67.70 (436)
Maxima	31.68 (204)	68.32 (440)
Sympy	10.09 (65)	89.91 (579)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

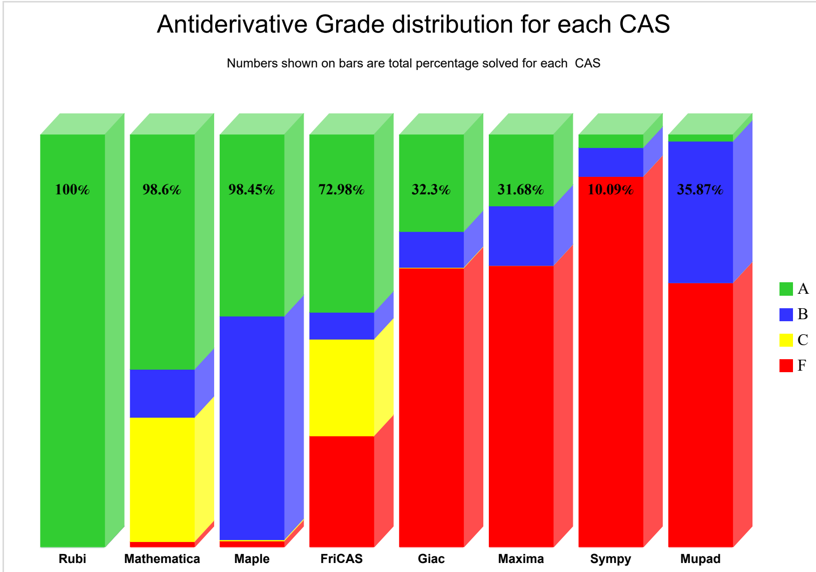
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

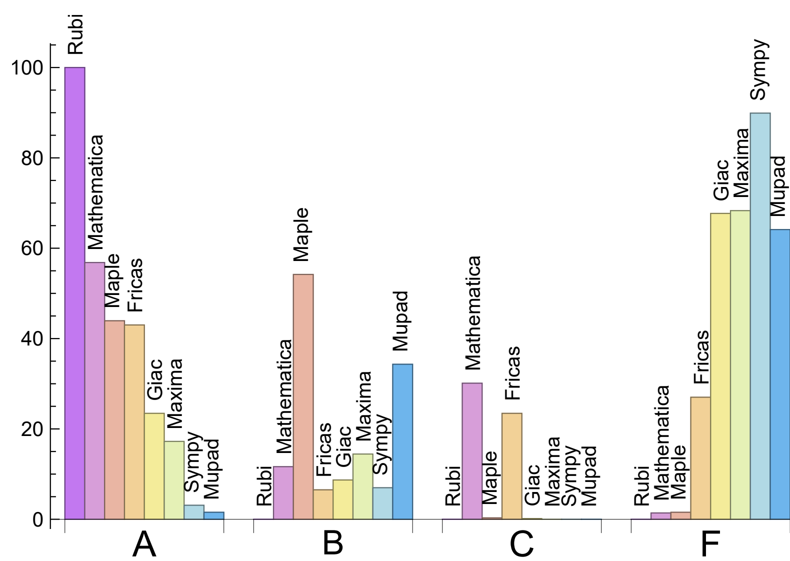
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	56.83	11.65	30.12	1.40
Maple	43.94	54.19	0.31	1.55
Fricas	43.01	6.52	23.45	27.02
Giac	23.45	8.70	0.16	67.70
Maxima	17.24	14.44	0.00	68.32
Sympy	3.11	6.99	0.00	89.91
Mupad	N/A	34.32	0.00	64.13

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	9	88.89 %	11.11 %	0.00 %
Maple	10	100.00 %	0.00 %	0.00 %
Fricas	174	60.34 %	39.66 %	0.00 %
Giac	436	78.21 %	19.50 %	2.29 %
Maxima	440	78.18 %	7.73 %	14.09 %
Sympy	579	33.33 %	41.45 %	25.22 %
Mupad	413	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

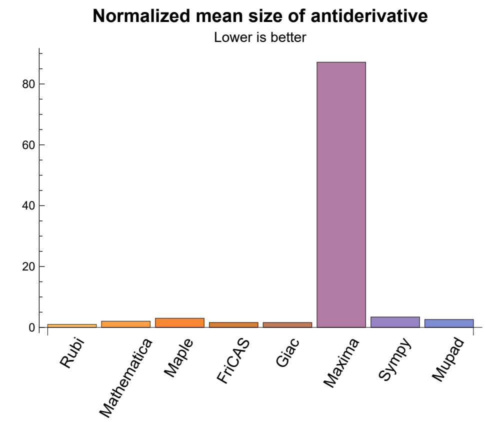
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

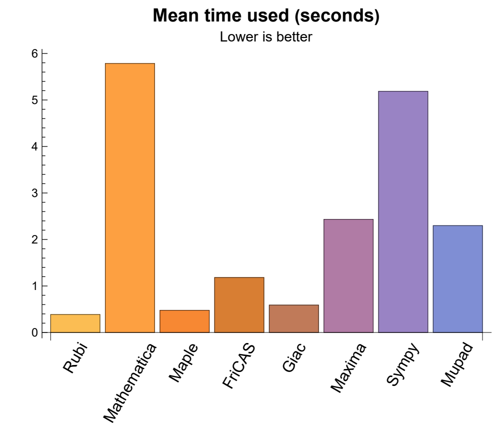
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.39	222.34	0.98	187.50	1.00
Mathematica	5.78	517.54	2.03	225.00	1.09
Maple	0.48	841.01	3.01	384.50	2.30
Maxima	2.43	15197.79	87.14	254.00	1.71
Fricas	1.18	289.15	1.60	197.50	1.23
Sympy	5.19	441.86	3.40	252.00	2.50
Giac	0.59	249.42	1.58	188.00	1.44
Mupad	2.30	453.45	2.59	202.00	1.43

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
20
40
60
80
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{449, 455, 456, 457, 458, 635, 641, 642, 643, 644\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {113, 114, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 157, 158, 159, 160, 161, 162, 164, 165, 194, 195, 196, 318, 332, 338, 339, 340, 376, 382, 395, 401, 402, 403, 404, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 426, 430, 431, 432, 434, 435, 436, 454, 522, 523, 524, 541, 548, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 590, 591, 592, 593, 595, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 612, 614, 615, 616, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 634, 640}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

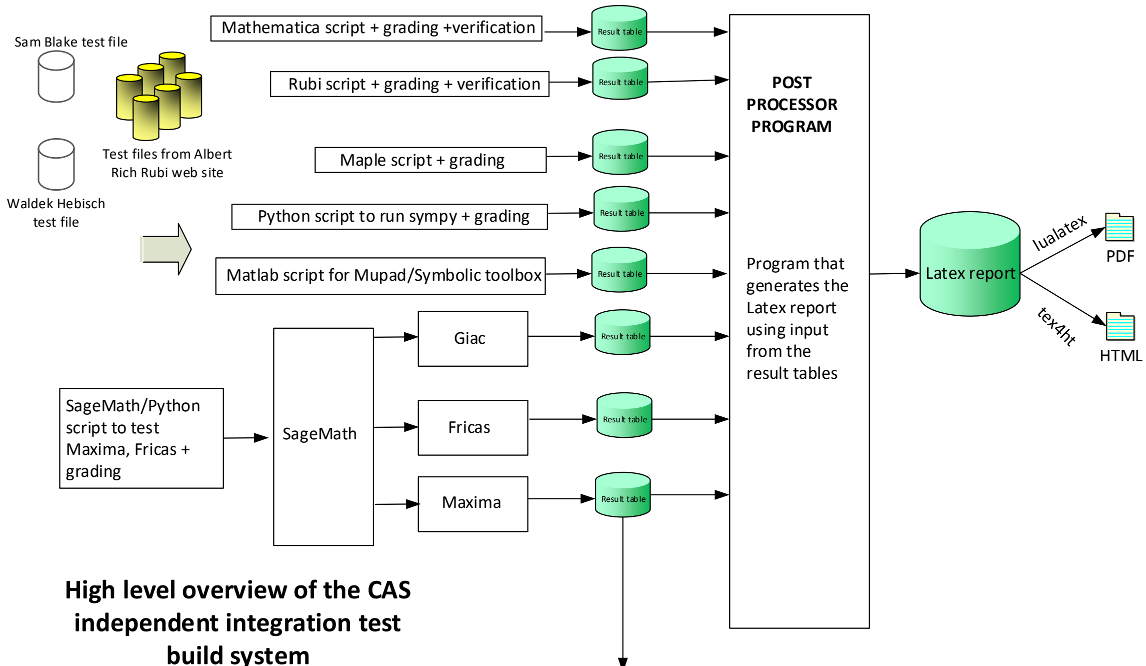
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 24, 26, 27, 28, 29, 30, 31, 35, 36, 37, 49, 51, 60, 61, 62, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 193, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 304, 305, 306, 311, 312, 313, 319, 320, 321, 322, 323, 326, 327, 328, 329, 333, 334, 335, 336, 337, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 397, 398, 399, 422, 423, 424, 438, 439, 440, 449, 450, 451, 452, 453, 455, 456, 457, 458, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 525, 528, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 572, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 591, 592, 593, 594, 599, 601, 613, 615, 616, 617, 621, 622, 628, 631, 632, 633, 635, 636, 637, 638, 639, 642, 643, 644 }

B grade: { 16, 17, 23, 25, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 65, 66, 67, 72, 73, 236, 246, 256, 257, 273, 274, 283, 369, 393, 454, 571, 573, 574, 575, 576, 590, 595, 596, 597, 598, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 614, 619, 620, 624, 625, 626, 627, 630, 640 }

C grade: { 113, 114, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 301, 302, 303, 307, 308, 309, 310, 314, 315, 316, 317, 318, 324, 325, 330, 331, 332, 338, 339, 340, 344, 395, 396, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 520, 521, 522, 523, 524, 526, 527, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 618, 623, 629, 634 }

F grade: { 441, 442, 443, 444, 445, 446, 447, 448, 641 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 82, 83, 84, 85, 91, 92, 93, 100, 101, 102, 107, 130, 131, 133, 137, 138, 139, 140, 145, 146, 147, 148, 149, 151, 158, 159, 170, 171, 172, 173, 178, 179, 180, 181, 188, 189, 190, 192, 193, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 300, 322, 323, 329, 330, 342, 343, 344, 367, 368, 370, 390, 391, 422, 437, 438, 439, 449, 455, 456, 457, 458, 461, 464, 467, 468, 469, 473, 474, 475, 477, 478, 479, 480, 481, 483, 484, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 504, 507, 508, 509, 510, 511, 512, 513, 517, 518, 519, 525, 526, 527, 528, 534, 535, 552, 555, 567, 568, 569, 570, 617, 632, 633, 634, 635, 641, 642, 643, 644 }

B grade: { 78, 79, 80, 81, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 134, 135, 136, 141, 142, 143, 144, 150, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 174, 175, 176, 177, 182, 183, 184, 185, 186, 187, 191, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 296, 297, 298, 299, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 440, 441, 442, 443, 444, 445, 446, 447, 448, 459, 460, 462, 463, 465, 466, 470, 471, 472, 476, 482, 487, 497, 505, 506, 514, 515, 516, 520, 521, 522, 523, 524, 529, 530, 531, 532, 533, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631 }

C grade: { 341, 386 }

F grade: { 450, 451, 452, 453, 454, 636, 637, 638, 639, 640 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 50, 51, 52, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 91, 92, 93, 94, 104, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 449, 455, 456, 457, 458, 635, 641, 642, 643, 644 }

B grade: { 6, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 56, 63, 64, 79, 80, 81, 87, 88, 89, 90, 95, 96, 97, 100, 101, 102, 103, 105, 106, 111, 112, 113, 120, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 219, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 519 }

C grade: { }

F grade: { 98, 99, 107, 108, 109, 110, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 516, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 107, 108, 109, 110, 114, 115, 116, 117, 122, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 257, 258, 261, 281, 282, 283, 284, 286, 287, 288, 289, 290, 291, 292, 293, 295, 449, 455, 456, 457, 458, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 635, 641, 642, 643, 644 }

B grade: { 6, 53, 72, 78, 79, 103, 104, 105, 106, 111, 112, 113, 118, 119, 120, 121, 219, 255, 256, 259, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 285, 294 }

C grade: { 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 296, 297, 298, 299, 304, 305, 306, 311, 312, 313, 319, 320, 321, 322, 326, 327, 328, 329, 333, 334, 335, 336, 337, 341, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 383, 384, 385, 386, 387, 388, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 584, 585, 586, 587, 588, 589 }

F grade: { 300, 301, 302, 303, 307, 308, 309, 310, 314, 315, 316, 317, 318, 323, 324, 325, 330, 331, 332, 338, 339, 340, 342, 344, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

2.1.6 Sympy

A grade: { 42, 50, 51, 59, 60, 61, 67, 68, 69, 70, 225, 284, 288, 449, 456, 457, 458, 642, 643, 644 }

B grade: { 1, 2, 3, 4, 10, 11, 12, 13, 19, 20, 21, 28, 29, 30, 38, 39, 40, 41, 47, 48, 49, 56, 57, 58, 65, 66, 215, 216, 217, 223, 224, 231, 232, 233, 240, 241, 242, 252, 253, 281, 282, 283, 285, 286, 292 }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 52, 53, 54, 55, 62, 63, 64, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 218, 219, 220, 221, 222, 226, 227, 228, 229, 230, 234, 235, 236, 237, 238, 239, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 287, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 455, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 114, 115, 116, 117, 118, 119, 120, 121, 122, 215, 216, 217, 223, 224, 225, 231, 232, 233, 235, 240, 241, 242, 244, 251, 252, 254, 255, 258, 260, 261, 262, 264, 281, 282, 283, 286, 287, 288, 289, 290, 292, 293, 294, 449, 455, 456, 457, 458, 635, 641, 642, 643, 644 }

B grade: { 5, 6, 7, 15, 79, 104, 105, 106, 111, 112, 218, 219, 220, 221, 222, 226, 227, 228, 229, 230, 234, 236, 237, 238, 239, 243, 245, 246, 247, 248, 249, 250, 253, 256, 257, 259, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 285, 291, 295 }

C grade: { 284 }

F grade: { 107, 108, 109, 110, 113, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

2.1.8 Mupad

A grade: { 449, 455, 456, 457, 458, 635, 641, 642, 643, 644 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 102, 103, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 171, 172, 173, 179, 180, 181, 188, 189, 190, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 321, 322, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 493, 494, 495, 496, 501, 502, 503, 504, 510, 511, 512, 513 }

C grade: { }

F grade: { 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 182, 183, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 497, 498, 499, 500, 505, 506, 507, 508, 509, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	110	128	124	88	333	112	236
N.S.	1	1.00	0.88	1.02	0.99	0.70	2.66	0.90	1.89
time (sec)	N/A	0.114	0.290	0.174	0.261	0.408	0.343	0.463	1.546

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	107	101	74	252	89	212
N.S.	1	1.00	0.77	1.10	1.04	0.76	2.60	0.92	2.19
time (sec)	N/A	0.101	0.285	0.134	0.262	0.373	0.209	0.481	1.228

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	65	85	79	56	168	68	84
N.S.	1	1.00	0.84	1.10	1.03	0.73	2.18	0.88	1.09
time (sec)	N/A	0.058	0.206	0.108	0.272	0.347	0.136	0.478	0.234

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	57	55	38	94	45	50
N.S.	1	1.00	0.94	1.21	1.17	0.81	2.00	0.96	1.06
time (sec)	N/A	0.015	0.122	0.082	0.262	0.359	0.081	0.420	0.194

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	46	48	47	51	0	79	100
N.S.	1	1.00	1.44	1.50	1.47	1.59	0.00	2.47	3.12
time (sec)	N/A	0.067	0.033	0.147	0.262	0.374	0.000	0.446	0.283

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	43	57	73	79	0	84	100
N.S.	1	1.00	1.34	1.78	2.28	2.47	0.00	2.62	3.12
time (sec)	N/A	0.072	0.029	0.173	0.270	0.355	0.000	0.476	0.307

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	75	95	89	0	124	94
N.S.	1	1.00	1.34	1.34	1.70	1.59	0.00	2.21	1.68
time (sec)	N/A	0.097	0.044	0.190	0.262	0.345	0.000	0.479	0.834

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	56	105	127	105	0	154	126
N.S.	1	1.00	0.65	1.22	1.48	1.22	0.00	1.79	1.47
time (sec)	N/A	0.107	0.379	0.234	0.258	0.355	0.000	0.432	2.067

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	77	131	163	127	0	188	166
N.S.	1	1.00	0.73	1.24	1.54	1.20	0.00	1.77	1.57
time (sec)	N/A	0.114	0.437	0.271	0.266	0.369	0.000	0.480	2.665

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	134	217	216	130	600	166	315
N.S.	1	1.00	0.70	1.14	1.13	0.68	3.14	0.87	1.65
time (sec)	N/A	0.211	0.694	0.210	0.260	0.353	0.508	0.450	1.585

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	108	186	178	110	459	137	277
N.S.	1	1.00	0.68	1.16	1.11	0.69	2.87	0.86	1.73
time (sec)	N/A	0.197	0.470	0.168	0.268	0.358	0.351	0.443	1.500

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	86	154	144	90	338	110	134
N.S.	1	1.00	0.67	1.19	1.12	0.70	2.62	0.85	1.04
time (sec)	N/A	0.124	0.387	0.137	0.266	0.355	0.240	0.437	0.291

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	61	116	110	70	199	85	98
N.S.	1	1.00	0.65	1.23	1.17	0.74	2.12	0.90	1.04
time (sec)	N/A	0.041	0.209	0.102	0.258	0.348	0.166	0.426	0.233

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	96	96	94	79	0	145	141
N.S.	1	1.00	1.17	1.17	1.15	0.96	0.00	1.77	1.72
time (sec)	N/A	0.128	0.206	0.171	0.264	0.382	0.000	0.449	0.337

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	143	88	105	108	0	155	161
N.S.	1	1.00	1.93	1.19	1.42	1.46	0.00	2.09	2.18
time (sec)	N/A	0.140	0.389	0.199	0.276	0.359	0.000	0.443	0.320

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	277	114	142	119	0	154	162
N.S.	1	1.00	3.15	1.30	1.61	1.35	0.00	1.75	1.84
time (sec)	N/A	0.152	1.356	0.227	0.264	0.383	0.000	0.495	0.299

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	451	145	174	125	0	178	145
N.S.	1	1.00	3.99	1.28	1.54	1.11	0.00	1.58	1.28
time (sec)	N/A	0.181	5.879	0.247	0.269	0.363	0.000	0.491	2.082

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	262	187	230	145	0	212	183
N.S.	1	1.00	1.82	1.30	1.60	1.01	0.00	1.47	1.27
time (sec)	N/A	0.202	1.251	0.300	0.266	0.367	0.000	0.466	2.682

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	134	266	262	130	695	166	315
N.S.	1	1.00	0.67	1.32	1.30	0.65	3.46	0.83	1.57
time (sec)	N/A	0.291	0.601	0.220	0.268	0.351	0.514	0.510	1.606

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	108	223	213	110	530	136	277
N.S.	1	1.00	0.70	1.45	1.38	0.71	3.44	0.88	1.80
time (sec)	N/A	0.157	0.467	0.167	0.268	0.343	0.364	0.434	1.505

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	86	176	167	90	371	112	134
N.S.	1	1.00	0.74	1.52	1.44	0.78	3.20	0.97	1.16
time (sec)	N/A	0.076	0.341	0.133	0.269	0.347	0.224	0.495	0.272

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	147	141	102	0	180	178
N.S.	1	1.00	1.02	1.32	1.27	0.92	0.00	1.62	1.60
time (sec)	N/A	0.199	0.289	0.195	0.269	0.359	0.000	0.455	0.421

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	272	128	140	127	0	192	197
N.S.	1	1.00	2.47	1.16	1.27	1.15	0.00	1.75	1.79
time (sec)	N/A	0.209	1.833	0.237	0.268	0.353	0.000	0.457	0.371

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	208	137	165	137	0	192	207
N.S.	1	1.00	1.82	1.20	1.45	1.20	0.00	1.68	1.82
time (sec)	N/A	0.223	1.943	0.362	0.274	0.358	0.000	0.511	0.372

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	786	176	212	141	0	189	209
N.S.	1	1.00	6.29	1.41	1.70	1.13	0.00	1.51	1.67
time (sec)	N/A	0.227	6.416	0.282	0.272	0.368	0.000	0.550	0.332

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	273	219	269	145	0	212	185
N.S.	1	1.00	1.77	1.42	1.75	0.94	0.00	1.38	1.20
time (sec)	N/A	0.270	1.435	0.309	0.282	0.361	0.000	0.480	2.708

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	294	271	337	165	0	246	224
N.S.	1	1.00	1.59	1.46	1.82	0.89	0.00	1.33	1.21
time (sec)	N/A	0.290	1.541	0.257	0.297	0.364	0.000	0.501	2.816

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	156	358	356	150	960	193	353
N.S.	1	1.00	0.65	1.49	1.48	0.62	3.98	0.80	1.46
time (sec)	N/A	0.379	0.877	0.278	0.272	0.365	0.777	0.469	1.637

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	134	306	297	130	765	166	316
N.S.	1	1.00	0.72	1.65	1.61	0.70	4.14	0.90	1.71
time (sec)	N/A	0.181	0.560	0.214	0.269	0.344	0.549	0.510	1.616

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	108	248	236	110	544	139	278
N.S.	1	1.00	0.72	1.65	1.57	0.73	3.63	0.93	1.85
time (sec)	N/A	0.089	0.372	0.160	0.271	0.347	0.377	0.435	1.558

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	138	208	198	118	0	214	188
N.S.	1	1.00	0.91	1.38	1.31	0.78	0.00	1.42	1.25
time (sec)	N/A	0.270	0.419	0.225	0.266	0.362	0.000	0.483	0.673

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	312	179	187	150	0	226	242
N.S.	1	1.00	2.08	1.19	1.25	1.00	0.00	1.51	1.61
time (sec)	N/A	0.298	1.682	0.266	0.272	0.362	0.000	0.475	0.421

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	343	177	199	156	0	230	243
N.S.	1	1.00	2.12	1.09	1.23	0.96	0.00	1.42	1.50
time (sec)	N/A	0.309	4.433	0.300	0.271	0.367	0.000	0.475	0.402

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	380	199	235	159	0	227	254
N.S.	1	1.00	2.30	1.21	1.42	0.96	0.00	1.38	1.54
time (sec)	N/A	0.340	6.227	0.336	0.275	0.377	0.000	0.512	0.407

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	326	250	307	157	0	223	255
N.S.	1	1.00	1.88	1.45	1.77	0.91	0.00	1.29	1.47
time (sec)	N/A	0.310	1.980	0.260	0.264	0.363	0.000	0.542	0.378

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	306	303	376	165	0	246	224
N.S.	1	1.00	1.55	1.53	1.90	0.83	0.00	1.24	1.13
time (sec)	N/A	0.368	1.690	0.273	0.267	0.358	0.000	0.496	2.791

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	358	365	464	185	0	280	262
N.S.	1	1.00	1.56	1.59	2.03	0.81	0.00	1.22	1.14
time (sec)	N/A	0.405	2.214	0.314	0.278	0.354	0.000	0.511	2.839

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	311	143	394	120	1794	181	170
N.S.	1	1.00	2.03	0.93	2.58	0.78	11.73	1.18	1.11
time (sec)	N/A	0.130	0.670	0.181	0.478	0.344	2.203	0.415	0.379

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	249	122	310	98	1161	151	138
N.S.	1	1.00	2.04	1.00	2.54	0.80	9.52	1.24	1.13
time (sec)	N/A	0.111	0.603	0.153	0.491	0.350	1.415	0.414	1.357

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	99	197	105	225	83	665	124	107
N.S.	1	1.10	2.19	1.17	2.50	0.92	7.39	1.38	1.19
time (sec)	N/A	0.081	0.492	0.136	0.478	0.347	0.919	0.468	0.490

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	126	76	143	61	264	78	65
N.S.	1	1.00	2.33	1.41	2.65	1.13	4.89	1.44	1.20
time (sec)	N/A	0.088	0.275	0.120	0.520	0.344	0.628	0.422	0.265

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	72	45	73	43	49	43	30
N.S.	1	1.00	2.12	1.32	2.15	1.26	1.44	1.26	0.88
time (sec)	N/A	0.032	0.141	0.094	0.495	0.352	0.385	0.429	0.198

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	109	61	99	74	0	71	42
N.S.	1	1.00	2.48	1.39	2.25	1.68	0.00	1.61	0.95
time (sec)	N/A	0.051	0.280	0.164	0.276	0.360	0.000	0.435	0.216

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	201	100	196	127	0	110	78
N.S.	1	1.00	2.91	1.45	2.84	1.84	0.00	1.59	1.13
time (sec)	N/A	0.096	1.312	0.196	0.280	0.379	0.000	0.444	0.290

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	289	142	282	156	0	157	119
N.S.	1	1.00	2.70	1.33	2.64	1.46	0.00	1.47	1.11
time (sec)	N/A	0.112	3.485	0.224	0.284	0.349	0.000	0.457	0.373

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	490	190	368	168	0	182	152
N.S.	1	1.00	3.74	1.45	2.81	1.28	0.00	1.39	1.16
time (sec)	N/A	0.119	4.476	0.259	0.280	0.352	0.000	0.451	0.639

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	369	154	372	154	1425	192	189
N.S.	1	1.00	2.17	0.91	2.19	0.91	8.38	1.13	1.11
time (sec)	N/A	0.215	0.658	0.178	0.492	0.359	3.274	0.454	0.335

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	315	135	283	138	843	164	152
N.S.	1	1.00	2.14	0.92	1.93	0.94	5.73	1.12	1.03
time (sec)	N/A	0.185	0.831	0.163	0.488	0.350	2.073	0.444	0.288

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	137	106	191	117	411	119	105
N.S.	1	1.00	1.38	1.07	1.93	1.18	4.15	1.20	1.06
time (sec)	N/A	0.182	0.765	0.143	0.480	0.341	1.258	0.454	0.259

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	153	74	120	91	105	86	65
N.S.	1	1.00	2.19	1.06	1.71	1.30	1.50	1.23	0.93
time (sec)	N/A	0.105	0.383	0.123	0.496	0.340	0.759	0.483	0.218

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	76	60	93	58	94	60	45
N.S.	1	1.00	1.17	0.92	1.43	0.89	1.45	0.92	0.69
time (sec)	N/A	0.036	0.198	0.100	0.271	0.332	0.555	0.430	0.189

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	170	91	145	131	0	113	74
N.S.	1	1.00	2.15	1.15	1.84	1.66	0.00	1.43	0.94
time (sec)	N/A	0.120	0.546	0.187	0.273	0.349	0.000	0.452	0.229

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	264	134	244	207	0	155	123
N.S.	1	1.00	2.47	1.25	2.28	1.93	0.00	1.45	1.15
time (sec)	N/A	0.194	1.799	0.214	0.271	0.346	0.000	0.460	0.280

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	496	177	336	228	0	198	165
N.S.	1	1.00	3.26	1.16	2.21	1.50	0.00	1.30	1.09
time (sec)	N/A	0.209	3.339	0.266	0.272	0.360	0.000	0.510	0.301

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	609	222	425	247	0	226	203
N.S.	1	1.00	3.40	1.24	2.37	1.38	0.00	1.26	1.13
time (sec)	N/A	0.224	4.988	0.284	0.281	0.358	0.000	0.476	0.341

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	491	182	412	205	1584	228	238
N.S.	1	1.00	2.25	0.83	1.89	0.94	7.27	1.05	1.09
time (sec)	N/A	0.338	0.964	0.220	0.498	0.346	6.798	0.470	0.333

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	435	163	322	190	966	200	203
N.S.	1	1.00	2.25	0.84	1.67	0.98	5.01	1.04	1.05
time (sec)	N/A	0.298	0.852	0.197	0.509	0.345	4.416	0.454	0.267

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	361	134	231	165	496	155	152
N.S.	1	1.00	2.46	0.91	1.57	1.12	3.37	1.05	1.03
time (sec)	N/A	0.293	0.982	0.179	0.482	0.341	2.723	0.434	0.263

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	241	102	160	137	148	120	134
N.S.	1	1.00	2.08	0.88	1.38	1.18	1.28	1.03	1.16
time (sec)	N/A	0.186	0.599	0.154	0.478	0.339	1.581	0.484	0.384

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	135	64	115	93	117	75	66
N.S.	1	1.00	1.32	0.63	1.13	0.91	1.15	0.74	0.65
time (sec)	N/A	0.123	0.366	0.168	0.273	0.342	1.169	0.450	0.208

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	96	64	115	93	114	75	66
N.S.	1	1.00	0.94	0.63	1.13	0.91	1.12	0.74	0.65
time (sec)	N/A	0.053	0.293	0.143	0.271	0.337	0.885	0.427	0.195

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	197	119	187	185	0	148	130
N.S.	1	1.00	1.68	1.02	1.60	1.58	0.00	1.26	1.11
time (sec)	N/A	0.202	0.990	0.210	0.275	0.356	0.000	0.457	0.246

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	482	162	286	272	0	190	168
N.S.	1	1.00	3.32	1.12	1.97	1.88	0.00	1.31	1.16
time (sec)	N/A	0.307	3.177	0.244	0.272	0.355	0.000	0.470	0.280

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	610	206	377	295	0	233	216
N.S.	1	1.00	3.11	1.05	1.92	1.51	0.00	1.19	1.10
time (sec)	N/A	0.327	4.988	0.283	0.274	0.378	0.000	0.493	0.277

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	555	191	364	238	1085	233	259
N.S.	1	1.00	2.42	0.83	1.59	1.04	4.74	1.02	1.13
time (sec)	N/A	0.414	1.314	0.163	0.483	0.340	9.283	0.440	0.307

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	481	162	271	213	578	188	201
N.S.	1	1.00	2.60	0.88	1.46	1.15	3.12	1.02	1.09
time (sec)	N/A	0.401	0.918	0.212	0.485	0.343	5.908	0.478	0.388

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	329	130	201	180	192	155	162
N.S.	1	1.00	2.14	0.84	1.31	1.17	1.25	1.01	1.05
time (sec)	N/A	0.281	0.797	0.189	0.477	0.346	3.619	0.433	0.345

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	193	90	175	124	182	117	86
N.S.	1	1.00	1.42	0.66	1.29	0.91	1.34	0.86	0.63
time (sec)	N/A	0.225	0.496	0.194	0.273	0.355	2.541	0.440	0.247

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	163	88	174	124	178	117	84
N.S.	1	1.00	1.18	0.64	1.26	0.90	1.29	0.85	0.61
time (sec)	N/A	0.139	0.405	0.198	0.277	0.381	1.965	0.427	0.248

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	109	88	175	125	177	117	87
N.S.	1	1.00	0.79	0.64	1.27	0.91	1.28	0.85	0.63
time (sec)	N/A	0.071	0.365	0.153	0.270	0.342	1.661	0.451	0.243

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	239	146	228	236	0	182	199
N.S.	1	1.00	1.63	0.99	1.55	1.61	0.00	1.24	1.35
time (sec)	N/A	0.288	1.555	0.232	0.278	0.382	0.000	0.470	0.361

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	595	190	326	337	0	224	236
N.S.	1	1.00	3.40	1.09	1.86	1.93	0.00	1.28	1.35
time (sec)	N/A	0.417	5.421	0.262	0.274	0.370	0.000	0.466	0.280

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	798	234	419	360	0	267	273
N.S.	1	1.00	3.44	1.01	1.81	1.55	0.00	1.15	1.18
time (sec)	N/A	0.433	6.503	0.315	0.280	0.362	0.000	0.496	0.287

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	103	121	145	99	0	182	-1
N.S.	1	1.00	0.55	0.65	0.78	0.53	0.00	0.97	-0.01
time (sec)	N/A	0.195	0.702	0.224	0.569	0.344	0.000	0.814	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	80	102	118	82	0	147	-1
N.S.	1	1.00	0.56	0.71	0.82	0.57	0.00	1.02	-0.01
time (sec)	N/A	0.152	0.386	0.217	0.565	0.334	0.000	0.576	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	64	83	88	64	0	107	-1
N.S.	1	1.00	0.63	0.82	0.87	0.63	0.00	1.06	-0.01
time (sec)	N/A	0.131	0.225	0.175	0.548	0.346	0.000	0.472	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	46	62	57	47	0	70	-1
N.S.	1	1.00	0.74	1.00	0.92	0.76	0.00	1.13	-0.02
time (sec)	N/A	0.040	0.089	0.179	0.537	0.335	0.000	0.455	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	212	21	127	0	89	-1
N.S.	1	1.00	1.00	3.21	0.32	1.92	0.00	1.35	-0.02
time (sec)	N/A	0.089	0.103	0.329	0.516	0.356	0.000	0.455	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	85	650	710	153	0	121	-1
N.S.	1	1.00	1.25	9.56	10.44	2.25	0.00	1.78	-0.01
time (sec)	N/A	0.102	0.228	0.373	0.554	0.380	0.000	0.451	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	101	1003	3352	178	0	195	-1
N.S.	1	1.00	0.86	8.57	28.65	1.52	0.00	1.67	-0.01
time (sec)	N/A	0.144	0.875	0.392	3.462	0.383	0.000	0.470	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	129	1327	5021	197	0	244	-1
N.S.	1	1.00	0.81	8.29	31.38	1.23	0.00	1.52	-0.01
time (sec)	N/A	0.181	1.939	0.457	3.548	0.393	0.000	0.509	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	125	142	185	125	0	235	-1
N.S.	1	1.00	0.53	0.61	0.79	0.53	0.00	1.00	-0.00
time (sec)	N/A	0.334	1.026	0.239	0.587	0.349	0.000	2.226	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	103	123	154	107	0	191	-1
N.S.	1	1.00	0.54	0.65	0.81	0.57	0.00	1.01	-0.01
time (sec)	N/A	0.282	0.607	0.201	0.575	0.344	0.000	1.053	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	81	104	123	88	0	155	-1
N.S.	1	1.00	0.59	0.75	0.89	0.64	0.00	1.12	-0.01
time (sec)	N/A	0.159	0.409	0.196	0.549	0.345	0.000	0.625	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	65	85	93	69	0	115	-1
N.S.	1	1.00	0.64	0.84	0.92	0.68	0.00	1.14	-0.01
time (sec)	N/A	0.059	0.206	0.161	0.541	0.358	0.000	0.503	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	85	274	39	149	0	140	-1
N.S.	1	1.00	0.81	2.61	0.37	1.42	0.00	1.33	-0.01
time (sec)	N/A	0.169	0.222	0.328	0.513	0.395	0.000	0.534	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	98	704	1315	172	0	149	-1
N.S.	1	1.00	0.95	6.83	12.77	1.67	0.00	1.45	-0.01
time (sec)	N/A	0.184	0.334	0.362	0.557	0.382	0.000	0.534	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	109	1003	3339	182	0	201	-1
N.S.	1	1.00	0.92	8.43	28.06	1.53	0.00	1.69	-0.01
time (sec)	N/A	0.221	0.561	0.405	0.712	0.377	0.000	0.527	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	132	1326	7567	202	0	252	-1
N.S.	1	1.00	0.80	8.09	46.14	1.23	0.00	1.54	-0.01
time (sec)	N/A	0.265	0.992	0.414	153.572	0.401	0.000	0.580	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	151	1651	10504	220	0	302	-1
N.S.	1	1.00	0.72	7.90	50.26	1.05	0.00	1.44	-0.00
time (sec)	N/A	0.316	1.587	0.476	154.601	0.391	0.000	0.637	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	127	142	207	137	0	257	-1
N.S.	1	1.00	0.54	0.60	0.87	0.58	0.00	1.08	-0.00
time (sec)	N/A	0.406	1.095	0.187	0.600	0.343	0.000	3.044	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	105	123	172	116	0	213	-1
N.S.	1	1.00	0.60	0.70	0.98	0.66	0.00	1.22	-0.01
time (sec)	N/A	0.185	0.748	0.175	0.583	0.352	0.000	1.171	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	83	104	139	95	0	169	-1
N.S.	1	1.00	0.60	0.75	1.01	0.69	0.00	1.22	-0.01
time (sec)	N/A	0.076	0.401	0.160	0.562	0.342	0.000	0.732	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	104	313	61	177	0	202	-1
N.S.	1	1.00	0.73	2.20	0.43	1.25	0.00	1.42	-0.01
time (sec)	N/A	0.267	0.432	0.333	0.533	0.358	0.000	1.089	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	120	764	8114	202	0	209	-1
N.S.	1	1.00	0.83	5.31	56.35	1.40	0.00	1.45	-0.01
time (sec)	N/A	0.294	0.558	0.379	0.836	0.380	0.000	0.785	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	126	1028	11782	204	0	239	-1
N.S.	1	1.00	0.81	6.59	75.53	1.31	0.00	1.53	-0.01
time (sec)	N/A	0.308	0.669	0.408	3.729	0.378	0.000	0.796	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	131	1326	7994	212	0	268	-1
N.S.	1	1.00	0.80	8.09	48.74	1.29	0.00	1.63	-0.01
time (sec)	N/A	0.328	1.130	0.417	3.870	0.380	0.000	0.781	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	152	1650	0	232	0	322	-1
N.S.	1	1.00	0.73	7.89	0.00	1.11	0.00	1.54	-0.00
time (sec)	N/A	0.392	1.770	0.480	0.000	0.413	0.000	0.905	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	176	1975	0	252	0	376	-1
N.S.	1	1.00	0.69	7.78	0.00	0.99	0.00	1.48	-0.00
time (sec)	N/A	0.441	2.155	0.540	0.000	0.417	0.000	0.983	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	111	281	1604723	184	0	217	-1
N.S.	1	1.00	0.55	1.39	7944.17	0.91	0.00	1.07	-0.00
time (sec)	N/A	0.373	0.706	0.306	37.376	0.365	0.000	0.813	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	94	240	927957	166	0	183	-1
N.S.	1	1.00	0.59	1.51	5836.21	1.04	0.00	1.15	-0.01
time (sec)	N/A	0.251	0.359	0.283	18.749	0.373	0.000	0.571	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	78	194	38386	149	0	149	160
N.S.	1	1.00	0.66	1.64	325.31	1.26	0.00	1.26	1.36
time (sec)	N/A	0.139	0.177	0.277	1.061	0.368	0.000	0.485	0.379

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	60	160	19040	135	0	125	112
N.S.	1	1.00	0.77	2.05	244.10	1.73	0.00	1.60	1.44
time (sec)	N/A	0.046	0.078	0.228	0.776	0.362	0.000	0.448	0.350

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	72	270	91	171	0	165	-1
N.S.	1	1.00	0.79	2.97	1.00	1.88	0.00	1.81	-0.01
time (sec)	N/A	0.109	0.088	0.344	0.529	0.363	0.000	0.507	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	95	820	18436	259	0	233	-1
N.S.	1	1.00	0.80	6.89	154.92	2.18	0.00	1.96	-0.01
time (sec)	N/A	0.204	0.376	0.391	0.715	0.407	0.000	0.528	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	114	1252	76209	284	0	299	-1
N.S.	1	1.00	0.69	7.59	461.87	1.72	0.00	1.81	-0.01
time (sec)	N/A	0.317	0.857	0.473	3.880	0.411	0.000	0.491	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	167	448	0	241	0	0	-1
N.S.	1	1.00	0.64	1.72	0.00	0.92	0.00	0.00	-0.00
time (sec)	N/A	0.517	1.133	0.399	0.000	0.375	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	142	407	0	224	0	0	-1
N.S.	1	1.00	0.66	1.88	0.00	1.04	0.00	0.00	-0.00
time (sec)	N/A	0.390	0.940	0.328	0.000	0.364	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	97	327	0	205	0	0	-1
N.S.	1	1.00	0.57	1.91	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.271	0.767	0.325	0.000	0.350	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	104	256	0	189	0	0	-1
N.S.	1	1.00	0.88	2.17	0.00	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.441	0.285	0.000	0.358	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	63	220	62254	172	0	160	-1
N.S.	1	1.00	0.72	2.53	715.56	1.98	0.00	1.84	-0.01
time (sec)	N/A	0.052	0.210	0.257	3.353	0.341	0.000	0.490	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	131	374	15722	281	0	220	-1
N.S.	1	1.00	1.03	2.94	123.80	2.21	0.00	1.73	-0.01
time (sec)	N/A	0.201	0.681	0.387	1.156	0.378	0.000	0.835	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	608	1051	47933	339	0	0	-1
N.S.	1	1.00	3.58	6.18	281.96	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.327	6.392	0.437	1.989	0.403	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	1402	1540	0	361	0	366	-1
N.S.	1	1.00	6.34	6.97	0.00	1.63	0.00	1.66	-0.00
time (sec)	N/A	0.457	8.591	0.526	0.000	0.417	0.000	0.609	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	139	467	0	270	0	309	-1
N.S.	1	1.00	0.53	1.79	0.00	1.03	0.00	1.18	-0.00
time (sec)	N/A	0.538	1.525	0.346	0.000	0.363	0.000	8.395	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	117	397	0	254	0	263	-1
N.S.	1	1.00	0.54	1.84	0.00	1.18	0.00	1.22	-0.00
time (sec)	N/A	0.409	1.053	0.410	0.000	0.431	0.000	4.815	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	100	327	0	237	0	224	-1
N.S.	1	1.00	0.59	1.93	0.00	1.40	0.00	1.33	-0.01
time (sec)	N/A	0.282	0.717	0.312	0.000	0.372	0.000	2.689	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	87	292	0	223	0	194	-1
N.S.	1	1.00	0.69	2.32	0.00	1.77	0.00	1.54	-0.01
time (sec)	N/A	0.160	0.585	0.322	0.000	0.347	0.000	1.310	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	80	292	0	223	0	194	-1
N.S.	1	1.00	0.63	2.32	0.00	1.77	0.00	1.54	-0.01
time (sec)	N/A	0.071	0.494	0.302	0.000	0.358	0.000	0.788	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	126	445	84333	339	0	264	-1
N.S.	1	1.00	0.77	2.71	514.23	2.07	0.00	1.61	-0.01
time (sec)	N/A	0.310	1.449	0.424	23.660	0.415	0.000	2.271	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	632	1122	0	404	0	347	-1
N.S.	1	1.00	3.05	5.42	0.00	1.95	0.00	1.68	-0.00
time (sec)	N/A	0.473	4.982	0.526	0.000	0.458	0.000	0.882	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	656	1610	0	428	0	374	-1
N.S.	1	1.00	2.48	6.10	0.00	1.62	0.00	1.42	-0.00
time (sec)	N/A	0.608	6.260	0.574	0.000	0.522	0.000	0.855	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	914	411	0	193	0	0	177
N.S.	1	1.00	5.75	2.58	0.00	1.21	0.00	0.00	1.11
time (sec)	N/A	0.142	6.327	0.306	0.000	0.147	0.000	0.000	1.085

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	872	383	0	179	0	0	166
N.S.	1	1.00	6.61	2.90	0.00	1.36	0.00	0.00	1.26
time (sec)	N/A	0.125	6.274	0.327	0.000	0.130	0.000	0.000	0.609

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	830	355	0	161	0	0	128
N.S.	1	1.00	8.22	3.51	0.00	1.59	0.00	0.00	1.27
time (sec)	N/A	0.111	6.236	0.264	0.000	0.124	0.000	0.000	0.522

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	309	321	0	142	0	0	79
N.S.	1	1.00	4.41	4.59	0.00	2.03	0.00	0.00	1.13
time (sec)	N/A	0.102	6.190	0.266	0.000	0.112	0.000	0.000	0.526

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	256	242	0	173	0	0	90
N.S.	1	1.00	3.88	3.67	0.00	2.62	0.00	0.00	1.36
time (sec)	N/A	0.103	6.087	0.306	0.000	0.115	0.000	0.000	0.964

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	813	399	0	196	0	0	150
N.S.	1	1.00	8.56	4.20	0.00	2.06	0.00	0.00	1.58
time (sec)	N/A	0.115	6.370	0.470	0.000	0.113	0.000	0.000	1.300

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	865	634	0	219	0	0	177
N.S.	1	1.00	6.55	4.80	0.00	1.66	0.00	0.00	1.34
time (sec)	N/A	0.127	6.434	0.659	0.000	0.135	0.000	0.000	1.606

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	944	413	0	223	0	0	266
N.S.	1	1.00	4.87	2.13	0.00	1.15	0.00	0.00	1.37
time (sec)	N/A	0.212	6.298	0.283	0.000	0.147	0.000	0.000	1.071

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	898	385	0	203	0	0	231
N.S.	1	1.00	5.58	2.39	0.00	1.26	0.00	0.00	1.43
time (sec)	N/A	0.196	6.260	0.305	0.000	0.128	0.000	0.000	1.012

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	852	357	0	179	0	0	153
N.S.	1	1.00	6.76	2.83	0.00	1.42	0.00	0.00	1.21
time (sec)	N/A	0.187	6.310	0.334	0.000	0.117	0.000	0.000	1.004

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	623	244	0	198	0	0	134
N.S.	1	1.00	5.28	2.07	0.00	1.68	0.00	0.00	1.14
time (sec)	N/A	0.180	6.389	0.373	0.000	0.108	0.000	0.000	1.136

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	624	513	0	210	0	0	196
N.S.	1	1.00	5.20	4.28	0.00	1.75	0.00	0.00	1.63
time (sec)	N/A	0.190	6.431	0.367	0.000	0.107	0.000	0.000	1.692

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	883	714	0	239	0	0	229
N.S.	1	1.00	5.55	4.49	0.00	1.50	0.00	0.00	1.44
time (sec)	N/A	0.208	6.536	0.731	0.000	0.107	0.000	0.000	1.974

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	925	824	0	263	0	0	235
N.S.	1	1.00	4.77	4.25	0.00	1.36	0.00	0.00	1.21
time (sec)	N/A	0.221	6.630	0.803	0.000	0.146	0.000	0.000	2.299

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	990	441	0	243	0	0	360
N.S.	1	1.00	4.18	1.86	0.00	1.03	0.00	0.00	1.52
time (sec)	N/A	0.322	6.345	0.296	0.000	0.139	0.000	0.000	1.310

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	944	413	0	223	0	0	323
N.S.	1	1.00	4.63	2.02	0.00	1.09	0.00	0.00	1.58
time (sec)	N/A	0.299	6.295	0.286	0.000	0.131	0.000	0.000	1.074

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	898	385	0	203	0	0	255
N.S.	1	1.00	5.25	2.25	0.00	1.19	0.00	0.00	1.49
time (sec)	N/A	0.286	6.364	0.285	0.000	0.125	0.000	0.000	0.998

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	888	337	0	229	0	0	229
N.S.	1	1.00	5.25	1.99	0.00	1.36	0.00	0.00	1.36
time (sec)	N/A	0.285	6.475	0.344	0.000	0.132	0.000	0.000	1.043

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	879	654	0	223	0	0	251
N.S.	1	1.00	5.46	4.06	0.00	1.39	0.00	0.00	1.56
time (sec)	N/A	0.281	6.559	0.371	0.000	0.127	0.000	0.000	1.628

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	890	916	0	243	0	0	287
N.S.	1	1.00	5.20	5.36	0.00	1.42	0.00	0.00	1.68
time (sec)	N/A	0.298	6.624	0.693	0.000	0.118	0.000	0.000	2.503

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	925	902	0	263	0	0	307
N.S.	1	1.00	4.53	4.42	0.00	1.29	0.00	0.00	1.50
time (sec)	N/A	0.311	6.689	0.837	0.000	0.121	0.000	0.000	2.666

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	967	1151	0	283	0	0	552
N.S.	1	1.00	4.08	4.86	0.00	1.19	0.00	0.00	2.33
time (sec)	N/A	0.331	6.783	1.112	0.000	0.132	0.000	0.000	3.031

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	1182	281	0	269	0	0	-1
N.S.	1	1.00	7.58	1.80	0.00	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.131	6.667	0.309	0.000	0.129	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	1129	262	0	250	0	0	-1
N.S.	1	1.00	9.18	2.13	0.00	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.114	6.580	0.296	0.000	0.109	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	1098	244	0	237	0	0	-1
N.S.	1	1.00	12.92	2.87	0.00	2.79	0.00	0.00	-0.01
time (sec)	N/A	0.095	6.487	0.291	0.000	0.111	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1094	243	0	241	0	0	-1
N.S.	1	1.00	13.18	2.93	0.00	2.90	0.00	0.00	-0.01
time (sec)	N/A	0.097	6.522	0.283	0.000	0.110	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	1130	319	0	290	0	0	-1
N.S.	1	1.00	9.50	2.68	0.00	2.44	0.00	0.00	-0.01
time (sec)	N/A	0.110	6.742	0.392	0.000	0.107	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	1167	466	0	320	0	0	-1
N.S.	1	1.00	7.63	3.05	0.00	2.09	0.00	0.00	-0.01
time (sec)	N/A	0.125	7.156	0.605	0.000	0.121	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	1262	465	0	383	0	0	-1
N.S.	1	1.00	6.22	2.29	0.00	1.89	0.00	0.00	-0.00
time (sec)	N/A	0.224	6.933	0.356	0.000	0.155	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	1218	435	0	367	0	0	-1
N.S.	1	1.00	7.34	2.62	0.00	2.21	0.00	0.00	-0.01
time (sec)	N/A	0.213	6.823	0.320	0.000	0.134	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	1184	421	0	352	0	0	-1
N.S.	1	1.00	8.71	3.10	0.00	2.59	0.00	0.00	-0.01
time (sec)	N/A	0.194	6.703	0.315	0.000	0.121	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	815	350	0	314	0	0	-1
N.S.	1	1.00	6.74	2.89	0.00	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.178	6.559	0.331	0.000	0.109	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	815	350	0	318	0	0	-1
N.S.	1	1.00	6.74	2.89	0.00	2.63	0.00	0.00	-0.01
time (sec)	N/A	0.183	6.593	0.349	0.000	0.120	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	1217	494	0	407	0	0	-1
N.S.	1	1.00	7.24	2.94	0.00	2.42	0.00	0.00	-0.01
time (sec)	N/A	0.220	6.853	0.428	0.000	0.126	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	1258	723	0	436	0	0	-1
N.S.	1	1.00	6.26	3.60	0.00	2.17	0.00	0.00	-0.00
time (sec)	N/A	0.229	7.492	0.721	0.000	0.129	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	1346	493	0	495	0	0	-1
N.S.	1	1.00	5.30	1.94	0.00	1.95	0.00	0.00	-0.00
time (sec)	N/A	0.355	7.228	0.374	0.000	0.165	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	1306	465	0	478	0	0	-1
N.S.	1	1.00	5.96	2.12	0.00	2.18	0.00	0.00	-0.00
time (sec)	N/A	0.328	7.055	0.332	0.000	0.145	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	1273	451	0	467	0	0	-1
N.S.	1	1.00	6.77	2.40	0.00	2.48	0.00	0.00	-0.01
time (sec)	N/A	0.313	6.949	0.350	0.000	0.125	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	1265	451	0	465	0	0	-1
N.S.	1	1.00	7.03	2.51	0.00	2.58	0.00	0.00	-0.01
time (sec)	N/A	0.303	6.886	0.332	0.000	0.125	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	1264	451	0	465	0	0	-1
N.S.	1	1.00	7.10	2.53	0.00	2.61	0.00	0.00	-0.01
time (sec)	N/A	0.292	6.772	0.319	0.000	0.117	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	1265	451	0	465	0	0	-1
N.S.	1	1.00	6.95	2.48	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.304	6.848	0.369	0.000	0.134	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	1305	685	0	521	0	0	-1
N.S.	1	1.00	5.90	3.10	0.00	2.36	0.00	0.00	-0.00
time (sec)	N/A	0.336	7.177	0.447	0.000	0.131	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	1346	876	0	548	0	0	-1
N.S.	1	1.00	5.30	3.45	0.00	2.16	0.00	0.00	-0.00
time (sec)	N/A	0.347	7.950	0.629	0.000	0.165	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	135	428	8220	151	0	0	-1
N.S.	1	1.00	0.61	1.94	37.19	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.231	1.071	3.003	1.080	0.453	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	118	356	2981	134	0	0	-1
N.S.	1	1.00	0.67	2.02	16.94	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.578	0.372	0.826	0.410	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	100	284	1851	117	0	0	-1
N.S.	1	1.00	0.76	2.17	14.13	0.89	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.316	0.322	0.710	0.412	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	83	164	939	97	0	0	-1
N.S.	1	1.00	1.06	2.10	12.04	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.174	0.391	0.660	0.416	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	86	109	245	109	0	0	-1
N.S.	1	1.00	1.13	1.43	3.22	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.187	0.381	0.613	0.380	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	57	62	289	67	0	0	112
N.S.	1	1.00	0.67	0.73	3.40	0.79	0.00	0.00	1.32
time (sec)	N/A	0.105	0.168	0.505	0.541	0.382	0.000	0.000	1.557

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	78	86	428	86	0	0	194
N.S.	1	1.00	0.60	0.66	3.29	0.66	0.00	0.00	1.49
time (sec)	N/A	0.145	0.271	0.366	0.540	0.369	0.000	0.000	3.253

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	102	108	522	104	0	0	479
N.S.	1	1.00	0.58	0.62	2.98	0.59	0.00	0.00	2.74
time (sec)	N/A	0.189	0.418	0.378	0.541	0.387	0.000	0.000	6.226

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	136	429	8904	162	0	0	-1
N.S.	1	1.00	0.60	1.89	39.22	0.71	0.00	0.00	-0.00
time (sec)	N/A	0.330	1.196	0.279	1.092	0.475	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	119	357	3023	144	0	0	-1
N.S.	1	1.00	0.66	1.98	16.79	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.262	0.706	0.364	0.854	0.416	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	101	283	1884	125	0	0	-1
N.S.	1	1.00	0.76	2.13	14.17	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.408	0.345	0.742	0.420	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	107	300	1801	135	0	0	-1
N.S.	1	1.00	0.85	2.38	14.29	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.353	0.330	0.715	0.413	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	106	211	1124	133	0	0	-1
N.S.	1	1.00	0.85	1.69	8.99	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.408	0.329	0.649	0.389	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	80	87	344	88	0	0	195
N.S.	1	1.00	0.60	0.65	2.57	0.66	0.00	0.00	1.46
time (sec)	N/A	0.224	0.346	0.264	0.549	0.364	0.000	0.000	3.094

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	102	109	481	107	0	0	236
N.S.	1	1.00	0.56	0.60	2.66	0.59	0.00	0.00	1.30
time (sec)	N/A	0.279	0.570	0.274	0.569	0.360	0.000	0.000	6.720

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	124	131	573	126	0	0	289
N.S.	1	1.00	0.54	0.57	2.51	0.55	0.00	0.00	1.27
time (sec)	N/A	0.324	0.721	0.396	0.584	0.379	0.000	0.000	7.023

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	159	503	10042	194	0	0	-1
N.S.	1	1.00	0.58	1.84	36.65	0.71	0.00	0.00	-0.00
time (sec)	N/A	0.451	2.039	0.431	1.188	0.466	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	137	431	9415	174	0	0	-1
N.S.	1	1.00	0.60	1.90	41.48	0.77	0.00	0.00	-0.00
time (sec)	N/A	0.388	1.284	0.256	1.132	0.451	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	121	357	3071	154	0	0	-1
N.S.	1	1.00	0.67	1.98	17.06	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.337	0.815	0.381	0.846	0.429	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	126	336	2080	164	0	0	-1
N.S.	1	1.00	0.71	1.89	11.69	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.334	0.725	0.353	0.736	0.411	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	130	484	2370	169	0	0	-1
N.S.	1	1.00	0.75	2.80	13.70	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.329	0.753	0.384	0.761	0.422	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	130	306	1548	161	0	0	-1
N.S.	1	1.00	0.76	1.78	9.00	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.301	0.813	0.397	0.667	0.427	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	104	111	396	114	0	0	551
N.S.	1	1.00	0.57	0.61	2.19	0.63	0.00	0.00	3.04
time (sec)	N/A	0.342	0.648	0.276	0.562	0.378	0.000	0.000	6.864

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	126	133	533	135	0	0	647
N.S.	1	1.00	0.55	0.58	2.34	0.59	0.00	0.00	2.84
time (sec)	N/A	0.405	0.909	0.337	0.558	0.381	0.000	0.000	8.236

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	147	155	626	156	0	0	773
N.S.	1	1.00	0.53	0.56	2.28	0.57	0.00	0.00	2.81
time (sec)	N/A	0.449	1.059	0.501	0.565	0.391	0.000	0.000	7.322

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	348	347	0	184	0	0	-1
N.S.	1	1.00	1.83	1.83	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.375	2.100	0.405	0.000	2.361	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	333	216	0	168	0	0	-1
N.S.	1	1.00	2.36	1.53	0.00	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.240	1.761	0.291	0.000	1.400	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	82	149	0	96	0	0	-1
N.S.	1	1.00	0.82	1.49	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.155	0.275	0.000	1.141	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	203	230	0	143	0	0	-1
N.S.	1	1.00	2.05	2.32	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.119	1.703	0.303	0.000	0.395	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	627	383	0	163	0	0	-1
N.S.	1	1.00	4.42	2.70	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.213	6.818	0.349	0.000	0.414	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	1728	519	0	180	0	0	-1
N.S.	1	1.00	9.24	2.78	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.340	7.941	0.379	0.000	0.424	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	362	379	0	237	0	0	-1
N.S.	1	1.00	1.84	1.92	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.395	2.253	0.343	0.000	3.985	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	313	298	0	203	0	0	-1
N.S.	1	1.00	2.16	2.06	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.249	2.650	0.303	0.000	3.125	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	212	246	0	164	0	0	-1
N.S.	1	1.00	1.98	2.30	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.131	1.237	0.296	0.000	0.404	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	206	299	0	201	0	0	-1
N.S.	1	1.00	1.32	1.92	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.242	3.563	0.329	0.000	0.412	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	213	443	0	221	0	0	-1
N.S.	1	1.00	1.05	2.18	0.00	1.09	0.00	0.00	-0.00
time (sec)	N/A	0.358	3.247	0.392	0.000	0.411	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	376	647	0	302	0	0	-1
N.S.	1	1.00	1.53	2.63	0.00	1.23	0.00	0.00	-0.00
time (sec)	N/A	0.527	3.486	0.385	0.000	8.389	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	329	515	0	267	0	0	-1
N.S.	1	1.00	1.70	2.65	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.366	2.761	0.339	0.000	6.422	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	198	413	0	215	0	0	-1
N.S.	1	1.00	1.29	2.68	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.238	1.516	0.322	0.000	0.382	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	200	413	0	217	0	0	-1
N.S.	1	1.00	1.28	2.65	0.00	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.246	1.473	0.311	0.000	0.382	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	217	443	0	248	0	0	-1
N.S.	1	1.00	1.07	2.18	0.00	1.22	0.00	0.00	-0.00
time (sec)	N/A	0.372	2.619	0.350	0.000	0.404	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	239	571	0	270	0	0	-1
N.S.	1	1.00	0.96	2.28	0.00	1.08	0.00	0.00	-0.00
time (sec)	N/A	0.499	2.859	0.286	0.000	0.419	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	396	887	0	368	0	0	-1
N.S.	1	1.00	1.35	3.03	0.00	1.26	0.00	0.00	-0.00
time (sec)	N/A	0.666	5.659	0.408	0.000	14.420	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	350	703	0	327	0	0	-1
N.S.	1	1.00	1.45	2.92	0.00	1.36	0.00	0.00	-0.00
time (sec)	N/A	0.493	4.168	0.371	0.000	8.273	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	217	549	0	266	0	0	-1
N.S.	1	1.00	1.08	2.73	0.00	1.32	0.00	0.00	-0.00
time (sec)	N/A	0.360	2.303	0.351	0.000	0.387	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	215	549	0	264	0	0	-1
N.S.	1	1.00	1.07	2.73	0.00	1.31	0.00	0.00	-0.00
time (sec)	N/A	0.372	2.140	0.356	0.000	0.409	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	216	549	0	266	0	0	-1
N.S.	1	1.00	1.06	2.70	0.00	1.31	0.00	0.00	-0.00
time (sec)	N/A	0.368	2.185	0.339	0.000	0.432	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	240	581	0	298	0	0	-1
N.S.	1	1.00	0.96	2.32	0.00	1.19	0.00	0.00	-0.00
time (sec)	N/A	0.509	2.768	0.378	0.000	0.415	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	262	715	0	319	0	0	-1
N.S.	1	1.00	0.88	2.41	0.00	1.07	0.00	0.00	-0.00
time (sec)	N/A	0.642	4.690	0.286	0.000	0.409	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	91	107	101	81	252	89	117
N.S.	1	1.00	0.87	1.02	0.96	0.77	2.40	0.85	1.11
time (sec)	N/A	0.116	0.252	0.137	0.264	0.417	0.198	0.429	0.474

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	75	85	79	60	168	68	84
N.S.	1	1.00	0.89	1.01	0.94	0.71	2.00	0.81	1.00
time (sec)	N/A	0.060	0.188	0.107	0.267	0.350	0.132	0.414	0.397

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	51	57	55	42	94	45	50
N.S.	1	1.00	0.98	1.10	1.06	0.81	1.81	0.87	0.96
time (sec)	N/A	0.015	0.108	0.081	0.265	0.381	0.082	0.444	0.356

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	46	48	47	54	0	79	100
N.S.	1	1.00	1.31	1.37	1.34	1.54	0.00	2.26	2.86
time (sec)	N/A	0.072	0.034	0.135	0.280	0.382	0.000	0.530	0.479

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	57	73	85	0	84	114
N.S.	1	1.00	1.23	1.63	2.09	2.43	0.00	2.40	3.26
time (sec)	N/A	0.077	0.021	0.161	0.291	0.383	0.000	0.436	0.484

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	75	75	95	96	0	151	104
N.S.	1	1.00	1.23	1.23	1.56	1.57	0.00	2.48	1.70
time (sec)	N/A	0.102	0.036	0.180	0.279	0.393	0.000	0.465	1.273

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	67	105	127	115	0	210	145
N.S.	1	1.00	0.72	1.13	1.37	1.24	0.00	2.26	1.56
time (sec)	N/A	0.111	0.324	0.216	0.273	0.372	0.000	0.498	2.574

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	85	131	163	136	0	304	194
N.S.	1	1.00	0.75	1.15	1.43	1.19	0.00	2.67	1.70
time (sec)	N/A	0.121	0.685	0.250	0.273	0.364	0.000	0.453	3.863

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	146	184	176	142	459	156	307
N.S.	1	1.00	0.77	0.97	0.93	0.75	2.43	0.83	1.62
time (sec)	N/A	0.204	0.517	0.168	0.286	0.367	0.358	0.458	3.933

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	118	152	142	114	338	124	169
N.S.	1	1.00	0.69	0.89	0.84	0.67	1.99	0.73	0.99
time (sec)	N/A	0.153	0.506	0.129	0.270	0.369	0.223	0.425	0.512

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	90	114	108	85	199	93	115
N.S.	1	1.00	0.84	1.07	1.01	0.79	1.86	0.87	1.07
time (sec)	N/A	0.063	0.254	0.100	0.263	0.399	0.144	0.428	0.453

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	120	94	92	87	0	178	169
N.S.	1	1.00	1.40	1.09	1.07	1.01	0.00	2.07	1.97
time (sec)	N/A	0.115	0.260	0.166	0.281	0.372	0.000	0.454	0.693

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	109	86	103	117	0	152	169
N.S.	1	1.00	1.82	1.43	1.72	1.95	0.00	2.53	2.82
time (sec)	N/A	0.109	0.547	0.186	0.273	0.366	0.000	0.462	0.876

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	67	112	140	136	0	190	176
N.S.	1	1.00	0.84	1.40	1.75	1.70	0.00	2.38	2.20
time (sec)	N/A	0.136	0.311	0.216	0.271	0.389	0.000	0.525	0.978

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	92	143	172	150	0	294	227
N.S.	1	1.00	0.79	1.23	1.48	1.29	0.00	2.53	1.96
time (sec)	N/A	0.181	0.523	0.237	0.269	0.360	0.000	0.454	3.660

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	120	185	228	180	0	478	314
N.S.	1	1.00	0.77	1.19	1.46	1.15	0.00	3.06	2.01
time (sec)	N/A	0.197	0.784	0.295	0.284	0.368	0.000	0.483	3.868

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	289	270	266	211	721	230	352
N.S.	1	1.00	1.07	1.00	0.99	0.78	2.68	0.86	1.31
time (sec)	N/A	0.336	0.714	0.215	0.274	0.366	0.540	0.447	1.108

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	176	227	217	174	551	188	277
N.S.	1	1.00	0.72	0.93	0.89	0.72	2.27	0.77	1.14
time (sec)	N/A	0.227	0.763	0.166	0.286	0.369	0.376	0.453	0.778

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	140	180	171	136	386	148	202
N.S.	1	1.00	0.82	1.05	1.00	0.80	2.26	0.87	1.18
time (sec)	N/A	0.133	0.452	0.127	0.282	0.351	0.230	0.461	0.569

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	159	151	145	131	0	314	1924
N.S.	1	1.00	1.16	1.10	1.06	0.96	0.00	2.29	14.04
time (sec)	N/A	0.216	0.441	0.186	0.278	0.380	0.000	0.475	1.910

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	217	132	144	152	0	234	236
N.S.	1	1.00	1.66	1.01	1.10	1.16	0.00	1.79	1.80
time (sec)	N/A	0.218	0.729	0.214	0.297	0.375	0.000	0.487	1.351

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	277	141	169	167	0	239	249
N.S.	1	1.00	2.23	1.14	1.36	1.35	0.00	1.93	2.01
time (sec)	N/A	0.224	2.196	0.246	0.274	0.368	0.000	0.490	1.556

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	108	180	216	189	0	336	526
N.S.	1	1.00	0.74	1.24	1.49	1.30	0.00	2.32	3.63
time (sec)	N/A	0.226	0.637	0.279	0.279	0.376	0.000	0.611	1.948

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	140	223	273	211	0	586	395
N.S.	1	1.00	0.74	1.19	1.45	1.12	0.00	3.12	2.10
time (sec)	N/A	0.300	0.875	0.302	0.279	0.397	0.000	0.481	3.946

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	181	275	341	249	0	722	470
N.S.	1	1.00	0.77	1.17	1.44	1.06	0.00	3.06	1.99
time (sec)	N/A	0.323	2.863	0.240	0.276	0.378	0.000	0.513	3.894

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	408	368	366	289	1017	313	436
N.S.	1	1.00	1.11	1.01	1.00	0.79	2.78	0.86	1.19
time (sec)	N/A	0.539	0.576	0.275	0.283	0.416	0.786	0.507	2.635

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	333	316	307	243	811	263	403
N.S.	1	1.00	1.02	0.97	0.94	0.75	2.50	0.81	1.24
time (sec)	N/A	0.335	0.750	0.215	0.291	0.390	0.555	0.470	1.371

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	263	258	246	197	580	212	307
N.S.	1	1.00	1.09	1.07	1.02	0.82	2.41	0.88	1.27
time (sec)	N/A	0.220	0.434	0.166	0.276	0.373	0.371	0.452	0.878

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	210	218	208	183	0	603	369
N.S.	1	1.00	1.05	1.09	1.04	0.92	0.00	3.02	1.84
time (sec)	N/A	0.359	0.413	0.215	0.274	0.397	0.000	0.512	1.419

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	257	189	197	196	0	371	2522
N.S.	1	1.00	1.32	0.97	1.01	1.01	0.00	1.90	12.93
time (sec)	N/A	0.369	0.714	0.253	0.287	0.409	0.000	0.532	2.269

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	310	187	209	202	0	526	330
N.S.	1	1.00	1.48	0.89	1.00	0.97	0.00	2.52	1.58
time (sec)	N/A	0.403	1.600	0.290	0.281	0.378	0.000	0.536	2.314

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	415	209	245	219	0	387	636
N.S.	1	1.00	2.10	1.06	1.24	1.11	0.00	1.95	3.21
time (sec)	N/A	0.383	3.770	0.331	0.273	0.389	0.000	0.497	2.831

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	160	260	317	250	0	635	1969
N.S.	1	1.00	0.74	1.20	1.47	1.16	0.00	2.94	9.12
time (sec)	N/A	0.376	0.721	0.239	0.278	0.407	0.000	0.503	2.976

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	198	313	386	281	0	850	555
N.S.	1	1.00	0.74	1.17	1.45	1.05	0.00	3.18	2.08
time (sec)	N/A	0.467	4.379	0.258	0.296	0.370	0.000	0.524	3.879

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	244	375	474	327	0	1186	706
N.S.	1	1.00	0.75	1.16	1.46	1.01	0.00	3.66	2.18
time (sec)	N/A	0.508	2.844	0.292	0.288	0.406	0.000	0.550	3.751

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	152	240	0	541	0	360	2500
N.S.	1	1.00	0.85	1.35	0.00	3.04	0.00	2.02	14.04
time (sec)	N/A	0.335	0.520	0.286	0.000	0.407	0.000	0.456	5.088

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	121	169	0	426	0	227	2500
N.S.	1	1.00	0.90	1.26	0.00	3.18	0.00	1.69	18.66
time (sec)	N/A	0.195	0.350	0.233	0.000	0.408	0.000	0.459	3.998

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	85	110	0	322	3225	142	541
N.S.	1	1.00	0.96	1.24	0.00	3.62	36.24	1.60	6.08
time (sec)	N/A	0.114	0.233	0.196	0.000	0.416	63.543	0.479	1.117

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	73	0	242	524	296	344
N.S.	1	1.00	1.01	1.09	0.00	3.61	7.82	4.42	5.13
time (sec)	N/A	0.045	0.135	0.159	0.000	0.383	12.766	0.467	1.720

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	112	92	0	304	0	127	342
N.S.	1	1.00	1.47	1.21	0.00	4.00	0.00	1.67	4.50
time (sec)	N/A	0.074	0.179	0.377	0.000	0.743	0.000	0.478	1.600

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	129	144	0	460	0	175	675
N.S.	1	1.00	1.30	1.45	0.00	4.65	0.00	1.77	6.82
time (sec)	N/A	0.131	0.611	0.440	0.000	0.477	0.000	0.451	1.988

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	300	229	0	589	0	269	2500
N.S.	1	1.00	2.10	1.60	0.00	4.12	0.00	1.88	17.48
time (sec)	N/A	0.315	1.762	0.539	0.000	2.808	0.000	0.472	4.207

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	422	335	0	729	0	412	2500
N.S.	1	1.00	2.26	1.79	0.00	3.90	0.00	2.20	13.37
time (sec)	N/A	0.505	2.339	0.623	0.000	0.817	0.000	0.475	4.895

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	184	267	0	965	0	338	2500
N.S.	1	1.00	0.70	1.02	0.00	3.67	0.00	1.29	9.51
time (sec)	N/A	0.445	1.127	0.411	0.000	0.433	0.000	0.442	9.209

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	147	205	0	788	0	1116	2500
N.S.	1	1.00	0.95	1.32	0.00	5.08	0.00	7.20	16.13
time (sec)	N/A	0.302	0.882	0.345	0.000	0.439	0.000	0.527	5.170

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	119	161	0	552	0	199	2500
N.S.	1	1.00	0.98	1.32	0.00	4.52	0.00	1.63	20.49
time (sec)	N/A	0.162	0.585	0.265	0.000	0.425	0.000	0.441	7.786

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	97	128	0	379	0	159	113
N.S.	1	1.00	0.97	1.28	0.00	3.79	0.00	1.59	1.13
time (sec)	N/A	0.061	0.387	0.202	0.000	0.371	0.000	0.483	0.732

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	191	182	0	684	0	223	2500
N.S.	1	1.00	1.44	1.37	0.00	5.14	0.00	1.68	18.80
time (sec)	N/A	0.189	0.666	0.571	0.000	2.961	0.000	0.483	7.811

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	240	241	0	1088	0	404	2500
N.S.	1	1.00	1.27	1.28	0.00	5.76	0.00	2.14	13.23
time (sec)	N/A	0.445	2.064	0.762	0.000	7.073	0.000	0.488	8.516

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	438	326	0	1329	0	378	2500
N.S.	1	1.00	1.62	1.21	0.00	4.92	0.00	1.40	9.26
time (sec)	N/A	0.650	6.292	0.873	0.000	12.301	0.000	0.482	9.279

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	734	402	0	1812	0	2712	2500
N.S.	1	1.00	1.84	1.01	0.00	4.55	0.00	6.81	6.28
time (sec)	N/A	1.141	3.592	0.761	0.000	0.504	0.000	0.702	12.006

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	232	341	0	1561	0	543	2500
N.S.	1	1.00	0.83	1.22	0.00	5.58	0.00	1.94	8.93
time (sec)	N/A	0.808	2.224	0.665	0.000	0.459	0.000	0.537	7.663

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	204	282	0	1152	0	455	2500
N.S.	1	1.00	0.97	1.34	0.00	5.46	0.00	2.16	11.85
time (sec)	N/A	0.383	1.448	0.547	0.000	0.424	0.000	0.476	9.949

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	172	234	0	740	0	391	248
N.S.	1	1.00	0.96	1.30	0.00	4.11	0.00	2.17	1.38
time (sec)	N/A	0.198	0.899	0.413	0.000	0.406	0.000	0.476	3.742

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	157	232	0	742	0	390	248
N.S.	1	1.00	0.96	1.41	0.00	4.52	0.00	2.38	1.51
time (sec)	N/A	0.132	0.700	0.356	0.000	0.400	0.000	0.457	3.544

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	269	302	0	1400	0	481	2500
N.S.	1	1.00	1.26	1.41	0.00	6.54	0.00	2.25	11.68
time (sec)	N/A	0.482	1.392	0.986	0.000	12.444	0.000	0.485	9.627

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	352	376	0	2100	0	574	2500
N.S.	1	1.00	1.18	1.26	0.00	7.02	0.00	1.92	8.36
time (sec)	N/A	1.143	6.064	1.193	0.000	26.824	0.000	0.515	12.905

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	507	460	0	2416	0	1395	2500
N.S.	1	1.00	1.26	1.14	0.00	6.01	0.00	3.47	6.22
time (sec)	N/A	1.446	3.076	1.422	0.000	40.781	0.000	0.535	12.558

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	1155	552	0	2567	0	966	2500
N.S.	1	1.00	2.82	1.35	0.00	6.28	0.00	2.36	6.11
time (sec)	N/A	3.291	6.563	1.101	0.000	0.597	0.000	0.497	12.515

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	717	459	0	1857	0	813	2500
N.S.	1	1.00	2.38	1.52	0.00	6.17	0.00	2.70	8.31
time (sec)	N/A	0.784	3.323	0.902	0.000	0.530	0.000	0.493	12.575

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	251	371	0	1220	0	689	440
N.S.	1	1.00	0.92	1.35	0.00	4.45	0.00	2.51	1.61
time (sec)	N/A	0.415	1.352	0.706	0.000	0.467	0.000	0.495	4.147

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	252	384	0	1232	0	722	451
N.S.	1	1.00	0.96	1.46	0.00	4.68	0.00	2.75	1.71
time (sec)	N/A	0.359	1.186	0.633	0.000	0.446	0.000	0.553	4.032

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	227	372	0	1228	0	691	440
N.S.	1	1.00	0.96	1.57	0.00	5.18	0.00	2.92	1.86
time (sec)	N/A	0.280	2.308	0.553	0.000	0.462	0.000	0.458	4.002

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	368	479	0	2269	0	837	2500
N.S.	1	1.00	1.22	1.59	0.00	7.54	0.00	2.78	8.31
time (sec)	N/A	0.949	1.736	1.404	0.000	35.533	0.000	0.523	12.810

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	549	587	0	3393	0	996	2500
N.S.	1	1.00	1.31	1.40	0.00	8.08	0.00	2.37	5.95
time (sec)	N/A	3.866	3.274	1.990	0.000	72.795	0.000	0.512	18.114

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	547	547	781	672	0	3819	0	1090	2500
N.S.	1	1.00	1.43	1.23	0.00	6.98	0.00	1.99	4.57
time (sec)	N/A	4.507	5.322	2.253	0.000	97.305	0.000	0.537	13.936

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	0	25	56	25	24
N.S.	1	1.00	1.00	0.82	0.00	0.89	2.00	0.89	0.86
time (sec)	N/A	0.010	0.011	0.157	0.000	0.372	0.460	0.434	0.479

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	28	0	24	68	33	50
N.S.	1	1.00	0.89	1.04	0.00	0.89	2.52	1.22	1.85
time (sec)	N/A	0.008	0.027	0.127	0.000	0.433	0.387	0.503	0.867

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	12	0	11	31	11	11
N.S.	1	1.00	2.09	1.09	0.00	1.00	2.82	1.00	1.00
time (sec)	N/A	0.004	0.010	0.105	0.000	0.393	0.323	0.425	0.472

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	3	2	10	3
N.S.	1	1.00	1.00	1.33	0.00	1.00	0.67	3.33	1.00
time (sec)	N/A	0.001	0.000	0.047	0.000	0.365	0.051	0.436	0.446

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	20	0	31	39	47	16
N.S.	1	1.00	1.00	1.67	0.00	2.58	3.25	3.92	1.33
time (sec)	N/A	0.004	0.005	0.121	0.000	0.386	2.263	0.472	0.487

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	19	32	11	30
N.S.	1	1.00	1.00	1.09	0.00	1.73	2.91	1.00	2.73
time (sec)	N/A	0.008	0.007	0.134	0.000	0.384	1.824	0.490	0.474

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	0	64	0	52	73
N.S.	1	1.00	1.00	1.03	0.00	1.78	0.00	1.44	2.03
time (sec)	N/A	0.012	0.011	0.178	0.000	0.405	0.000	0.479	0.855

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	25	0	32	42	25	39
N.S.	1	1.00	0.86	0.89	0.00	1.14	1.50	0.89	1.39
time (sec)	N/A	0.010	0.046	0.204	0.000	0.396	10.706	0.457	0.520

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	98	139	0	350	0	185	173
N.S.	1	1.00	0.86	1.22	0.00	3.07	0.00	1.62	1.52
time (sec)	N/A	0.140	0.283	0.303	0.000	0.443	0.000	0.437	1.167

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	98	0	281	0	128	193
N.S.	1	1.00	0.92	1.24	0.00	3.56	0.00	1.62	2.44
time (sec)	N/A	0.085	0.153	0.227	0.000	0.404	0.000	0.507	0.894

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	66	0	231	0	245	101
N.S.	1	1.00	0.97	1.08	0.00	3.79	0.00	4.02	1.66
time (sec)	N/A	0.043	0.085	0.177	0.000	0.433	0.000	0.461	0.799

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	45	0	177	190	78	44
N.S.	1	1.00	0.98	0.90	0.00	3.54	3.80	1.56	0.88
time (sec)	N/A	0.024	0.044	0.123	0.000	0.394	179.957	0.454	0.504

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	103	86	0	292	0	122	101
N.S.	1	1.00	1.47	1.23	0.00	4.17	0.00	1.74	1.44
time (sec)	N/A	0.052	0.094	0.320	0.000	0.443	0.000	0.520	0.758

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	116	125	0	398	0	155	326
N.S.	1	1.00	1.32	1.42	0.00	4.52	0.00	1.76	3.70
time (sec)	N/A	0.095	0.414	0.352	0.000	0.494	0.000	0.489	1.058

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	239	194	0	487	0	221	1099
N.S.	1	1.00	1.94	1.58	0.00	3.96	0.00	1.80	8.93
time (sec)	N/A	0.230	1.131	0.443	0.000	0.520	0.000	0.508	1.826

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	292	1635	0	639	0	0	-1
N.S.	1	1.00	0.76	4.24	0.00	1.66	0.00	0.00	-0.00
time (sec)	N/A	0.522	1.697	0.436	0.000	0.199	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	232	1305	0	561	0	0	-1
N.S.	1	1.00	0.77	4.31	0.00	1.85	0.00	0.00	-0.00
time (sec)	N/A	0.347	1.062	0.399	0.000	0.169	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	179	993	0	492	0	0	-1
N.S.	1	1.00	0.77	4.30	0.00	2.13	0.00	0.00	-0.00
time (sec)	N/A	0.257	0.930	0.346	0.000	0.153	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	146	600	0	435	0	0	-1
N.S.	1	1.00	0.85	3.51	0.00	2.54	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.629	0.324	0.000	0.143	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	107	247	0	0	0	0	-1
N.S.	1	1.00	0.60	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	2.386	0.301	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	372	746	0	0	0	0	-1
N.S.	1	1.00	1.75	3.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	18.382	0.481	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	420	1290	0	0	0	0	-1
N.S.	1	1.00	1.44	4.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.606	14.384	0.661	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	635	2213	0	0	0	0	-1
N.S.	1	1.00	1.68	5.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.846	16.556	1.028	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	291	1635	0	639	0	0	-1
N.S.	1	1.00	0.77	4.33	0.00	1.69	0.00	0.00	-0.00
time (sec)	N/A	0.482	1.606	0.437	0.000	0.190	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	233	1305	0	562	0	0	-1
N.S.	1	1.00	0.78	4.39	0.00	1.89	0.00	0.00	-0.00
time (sec)	N/A	0.344	1.126	0.383	0.000	0.165	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	203	993	0	493	0	0	-1
N.S.	1	1.00	0.90	4.41	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.231	0.831	0.365	0.000	0.216	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	406	738	0	0	0	0	-1
N.S.	1	1.00	1.72	3.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	12.685	0.356	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	398	1167	0	0	0	0	-1
N.S.	1	1.00	1.72	5.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.444	12.643	0.407	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	422	1403	0	0	0	0	-1
N.S.	1	1.00	1.43	4.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.676	15.018	0.672	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	634	2327	0	0	0	0	-1
N.S.	1	1.00	1.69	6.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.923	16.721	1.062	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	357	1983	0	726	0	0	-1
N.S.	1	1.00	0.77	4.29	0.00	1.57	0.00	0.00	-0.00
time (sec)	N/A	0.616	2.219	0.475	0.000	0.198	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	291	1635	0	639	0	0	-1
N.S.	1	1.00	0.78	4.40	0.00	1.72	0.00	0.00	-0.00
time (sec)	N/A	0.461	1.665	0.455	0.000	0.180	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	254	1305	0	562	0	0	-1
N.S.	1	1.00	0.88	4.53	0.00	1.95	0.00	0.00	-0.00
time (sec)	N/A	0.310	1.140	0.375	0.000	0.151	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	453	1067	0	0	0	0	-1
N.S.	1	1.00	1.55	3.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.638	13.055	0.419	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	442	1563	0	0	0	0	-1
N.S.	1	1.00	1.49	5.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.655	14.026	0.490	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	451	1742	0	0	0	0	-1
N.S.	1	1.00	1.43	5.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.678	16.010	0.739	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	486	2438	0	0	0	0	-1
N.S.	1	1.00	1.29	6.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.924	16.224	1.095	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	465	465	729	3548	0	0	0	0	-1
N.S.	1	1.00	1.57	7.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.198	16.774	1.648	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	230	1305	0	562	0	0	-1
N.S.	1	1.00	0.72	4.08	0.00	1.76	0.00	0.00	-0.00
time (sec)	N/A	0.394	1.124	0.409	0.000	0.158	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	180	993	0	493	0	0	-1
N.S.	1	1.00	0.73	4.04	0.00	2.00	0.00	0.00	-0.00
time (sec)	N/A	0.273	0.953	0.363	0.000	0.143	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	154	671	0	435	0	0	199
N.S.	1	1.00	0.84	3.67	0.00	2.38	0.00	0.00	1.09
time (sec)	N/A	0.181	0.729	0.333	0.000	0.115	0.000	0.000	0.800

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	93	249	0	371	0	0	135
N.S.	1	1.00	0.72	1.92	0.00	2.85	0.00	0.00	1.04
time (sec)	N/A	0.084	3.430	0.306	0.000	0.114	0.000	0.000	0.885

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	81	194	0	0	0	0	-1
N.S.	1	1.00	0.69	1.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.213	0.281	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	320	639	0	0	0	0	-1
N.S.	1	1.00	1.48	2.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.393	14.781	0.455	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	420	1182	0	0	0	0	-1
N.S.	1	1.00	1.40	3.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.610	15.977	0.648	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	304	1312	0	910	0	0	-1
N.S.	1	1.00	0.79	3.39	0.00	2.35	0.00	0.00	-0.00
time (sec)	N/A	0.463	1.916	0.740	0.000	0.347	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	189	1336	0	789	0	0	-1
N.S.	1	1.00	0.72	5.10	0.00	3.01	0.00	0.00	-0.00
time (sec)	N/A	0.312	1.562	0.631	0.000	0.321	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	170	519	0	683	0	0	-1
N.S.	1	1.00	0.83	2.54	0.00	3.35	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.877	0.460	0.000	0.168	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	151	432	0	610	0	0	-1
N.S.	1	1.00	0.82	2.34	0.00	3.30	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.600	0.450	0.000	0.262	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	460	433	0	0	0	0	-1
N.S.	1	1.00	2.42	2.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.322	14.086	0.451	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	482	912	0	0	0	0	-1
N.S.	1	1.00	1.59	3.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.638	15.838	0.763	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	678	1568	0	0	0	0	-1
N.S.	1	1.00	1.70	3.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.901	16.865	1.007	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	372	1750	0	1538	0	0	-1
N.S.	1	1.00	0.68	3.18	0.00	2.80	0.00	0.00	-0.00
time (sec)	N/A	0.778	4.022	1.739	0.000	0.677	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	334	1412	0	1348	0	0	-1
N.S.	1	1.00	0.81	3.42	0.00	3.26	0.00	0.00	-0.00
time (sec)	N/A	0.523	3.035	1.421	0.000	0.408	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	274	954	0	1193	0	0	-1
N.S.	1	1.00	0.83	2.88	0.00	3.60	0.00	0.00	-0.00
time (sec)	N/A	0.356	2.432	1.232	0.000	0.261	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	224	864	0	1076	0	0	-1
N.S.	1	1.00	0.73	2.81	0.00	3.50	0.00	0.00	-0.00
time (sec)	N/A	0.310	2.108	1.077	0.000	0.189	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	193	754	0	956	0	0	-1
N.S.	1	1.00	0.70	2.74	0.00	3.48	0.00	0.00	-0.00
time (sec)	N/A	0.250	1.742	0.987	0.000	0.177	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	743	858	0	0	0	0	-1
N.S.	1	1.00	2.13	2.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.695	16.855	1.258	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	750	1345	0	0	0	0	-1
N.S.	1	1.00	1.72	3.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.952	17.308	1.783	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	820	2004	0	0	0	0	-1
N.S.	1	1.00	1.54	3.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.245	17.419	2.665	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	76	0	148	0	0	-1
N.S.	1	1.00	1.00	1.31	0.00	2.55	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.063	0.167	0.000	0.111	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	167	0	0	0	0	-1
N.S.	1	1.00	1.00	2.83	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.089	0.091	0.258	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	84	218	0	491	0	0	-1
N.S.	1	1.00	0.78	2.02	0.00	4.55	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.224	0.325	0.000	0.134	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	403	377	0	0	0	0	-1
N.S.	1	1.00	2.25	2.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	5.564	0.377	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	125	451	0	211	0	0	177
N.S.	1	1.00	0.74	2.65	0.00	1.24	0.00	0.00	1.04
time (sec)	N/A	0.139	1.349	0.308	0.000	0.151	0.000	0.000	1.349

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	103	413	0	192	0	0	166
N.S.	1	1.00	0.74	2.95	0.00	1.37	0.00	0.00	1.19
time (sec)	N/A	0.126	0.903	0.286	0.000	0.163	0.000	0.000	1.156

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	86	371	0	175	0	0	128
N.S.	1	1.00	0.80	3.44	0.00	1.62	0.00	0.00	1.19
time (sec)	N/A	0.109	0.444	0.287	0.000	0.144	0.000	0.000	1.011

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	326	0	156	0	0	85
N.S.	1	1.00	0.89	4.35	0.00	2.08	0.00	0.00	1.13
time (sec)	N/A	0.097	0.244	0.277	0.000	0.118	0.000	0.000	0.993

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	246	0	185	0	0	96
N.S.	1	1.00	0.90	3.46	0.00	2.61	0.00	0.00	1.35
time (sec)	N/A	0.104	0.369	0.326	0.000	0.115	0.000	0.000	1.440

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	107	401	0	213	0	0	150
N.S.	1	1.00	1.04	3.89	0.00	2.07	0.00	0.00	1.46
time (sec)	N/A	0.120	0.498	0.544	0.000	0.161	0.000	0.000	1.965

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	134	636	0	235	0	0	177
N.S.	1	1.00	0.96	4.54	0.00	1.68	0.00	0.00	1.26
time (sec)	N/A	0.132	0.860	0.720	0.000	0.126	0.000	0.000	2.394

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	196	666	0	299	0	0	275
N.S.	1	1.00	0.74	2.52	0.00	1.13	0.00	0.00	1.04
time (sec)	N/A	0.240	1.841	0.327	0.000	0.146	0.000	0.000	1.535

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	167	610	0	271	0	0	264
N.S.	1	1.00	0.75	2.74	0.00	1.22	0.00	0.00	1.18
time (sec)	N/A	0.217	1.500	0.342	0.000	0.140	0.000	0.000	1.350

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	139	548	0	243	0	0	229
N.S.	1	1.00	0.76	3.01	0.00	1.34	0.00	0.00	1.26
time (sec)	N/A	0.198	1.191	0.325	0.000	0.143	0.000	0.000	1.342

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	106	487	0	216	0	0	177
N.S.	1	1.00	0.76	3.48	0.00	1.54	0.00	0.00	1.26
time (sec)	N/A	0.177	0.635	0.310	0.000	0.138	0.000	0.000	1.340

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	405	0	240	0	0	158
N.S.	1	1.00	0.84	3.35	0.00	1.98	0.00	0.00	1.31
time (sec)	N/A	0.163	0.665	0.349	0.000	0.116	0.000	0.000	1.570

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	105	650	0	255	0	0	194
N.S.	1	1.00	0.83	5.16	0.00	2.02	0.00	0.00	1.54
time (sec)	N/A	0.177	1.228	0.562	0.000	0.115	0.000	0.000	2.288

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	175	723	0	286	0	0	227
N.S.	1	1.00	1.02	4.20	0.00	1.66	0.00	0.00	1.32
time (sec)	N/A	0.203	1.162	0.830	0.000	0.130	0.000	0.000	2.616

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	235	825	0	358	0	0	364
N.S.	1	1.00	0.77	2.70	0.00	1.17	0.00	0.00	1.19
time (sec)	N/A	0.349	2.040	0.367	0.000	0.162	0.000	0.000	1.738

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	197	745	0	321	0	0	328
N.S.	1	1.00	0.77	2.92	0.00	1.26	0.00	0.00	1.29
time (sec)	N/A	0.324	1.268	0.348	0.000	0.153	0.000	0.000	1.539

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	158	664	0	284	0	0	275
N.S.	1	1.00	0.77	3.24	0.00	1.39	0.00	0.00	1.34
time (sec)	N/A	0.311	1.392	0.357	0.000	0.153	0.000	0.000	1.432

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	150	641	0	302	0	0	248
N.S.	1	1.00	0.74	3.17	0.00	1.50	0.00	0.00	1.23
time (sec)	N/A	0.299	1.195	0.409	0.000	0.178	0.000	0.000	1.458

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	165	1210	0	306	0	0	255
N.S.	1	1.00	0.86	6.30	0.00	1.59	0.00	0.00	1.33
time (sec)	N/A	0.287	1.168	0.677	0.000	0.138	0.000	0.000	2.339

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	176	970	0	326	0	0	291
N.S.	1	1.00	0.86	4.75	0.00	1.60	0.00	0.00	1.43
time (sec)	N/A	0.297	2.304	0.834	0.000	0.143	0.000	0.000	3.587

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	260	1074	0	0	0	0	-1
N.S.	1	1.00	1.43	5.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.504	12.531	0.364	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	207	822	0	0	0	0	-1
N.S.	1	1.00	1.51	6.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.320	11.474	0.471	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	128	295	0	0	0	0	-1
N.S.	1	1.00	1.44	3.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	10.942	0.278	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	58	217	0	0	0	0	-1
N.S.	1	1.00	0.95	3.56	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.097	0.224	0.291	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	206	300	0	0	0	0	-1
N.S.	1	1.00	2.40	3.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	12.586	0.386	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	260	441	0	0	0	0	-1
N.S.	1	1.00	1.73	2.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.485	12.294	0.705	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	318	1066	0	0	0	0	-1
N.S.	1	1.00	1.05	3.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.596	13.275	0.817	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	280	849	0	0	0	0	-1
N.S.	1	1.00	1.25	3.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.398	12.771	0.676	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	260	808	0	0	0	0	-1
N.S.	1	1.00	1.31	4.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.347	12.403	0.632	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	274	721	0	0	0	0	-1
N.S.	1	1.00	1.37	3.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	12.712	0.779	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	316	856	0	0	0	0	-1
N.S.	1	1.00	1.23	3.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.596	14.280	0.740	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	427	1004	0	0	0	0	-1
N.S.	1	1.00	1.24	2.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.829	16.965	1.386	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	390	1977	0	0	0	0	-1
N.S.	1	1.00	1.06	5.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.664	15.113	1.394	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	360	1937	0	0	0	0	-1
N.S.	1	1.00	1.05	5.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.636	13.794	1.284	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	365	1850	0	0	0	0	-1
N.S.	1	1.00	1.08	5.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.608	14.517	1.223	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	383	1744	0	0	0	0	-1
N.S.	1	1.00	1.11	5.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.695	14.901	1.240	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	458	1975	0	0	0	0	-1
N.S.	1	1.00	1.09	4.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.957	15.561	1.547	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	523	523	570	2131	0	0	0	0	-1
N.S.	1	1.00	1.09	4.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.265	17.325	2.869	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	203	0	77	0	0	-1
N.S.	1	1.00	0.93	4.61	0.00	1.75	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.057	0.237	0.000	0.121	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	37	180	0	71	0	0	-1
N.S.	1	1.00	0.84	4.09	0.00	1.61	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.045	0.257	0.000	0.246	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	134	0	59	0	0	-1
N.S.	1	1.00	1.00	7.88	0.00	3.47	0.00	0.00	-0.06
time (sec)	N/A	0.008	0.019	0.236	0.000	0.190	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	0	53	0	0	-1
N.S.	1	1.00	1.00	1.12	0.00	3.12	0.00	0.00	-0.06
time (sec)	N/A	0.008	0.024	0.075	0.000	0.153	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	183	0	96	0	0	-1
N.S.	1	1.00	1.00	4.58	0.00	2.40	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.068	0.251	0.000	0.117	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	37	214	0	95	0	0	-1
N.S.	1	1.00	0.84	4.86	0.00	2.16	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.083	0.224	0.000	0.114	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	159	553	0	0	0	0	-1
N.S.	1	1.00	1.37	4.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.253	1.820	0.331	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	82	228	0	0	0	0	-1
N.S.	1	1.00	1.05	2.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.113	0.258	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	189	0	0	0	0	-1
N.S.	1	1.00	0.89	3.44	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.062	0.288	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	151	0	0	0	0	-1
N.S.	1	1.00	1.00	5.03	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.033	0.075	0.209	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	196	355	0	0	0	0	-1
N.S.	1	1.00	2.45	4.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	2.749	0.308	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	211	425	0	0	0	0	-1
N.S.	1	1.00	1.59	3.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.360	4.121	0.568	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	1224	2949	0	0	0	0	-1
N.S.	1	1.00	2.19	5.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.984	6.336	1.953	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	1175	2052	0	0	0	0	-1
N.S.	1	1.00	2.48	4.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.674	20.686	0.322	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	408	1693	0	0	0	0	-1
N.S.	1	1.00	1.06	4.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.463	10.964	0.530	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	273	1687	0	0	0	0	-1
N.S.	1	1.00	0.78	4.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.325	12.774	0.869	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	407	1729	0	0	0	0	-1
N.S.	1	1.00	1.43	6.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	13.521	0.311	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	1315	2481	0	0	0	0	-1
N.S.	1	1.00	3.76	7.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.537	6.400	0.368	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	1408	3427	0	0	0	0	-1
N.S.	1	1.00	3.25	7.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.773	6.479	0.493	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	670	670	1284	4048	0	0	0	0	-1
N.S.	1	1.00	1.92	6.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.361	6.446	0.641	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	566	566	1227	3139	0	0	0	0	-1
N.S.	1	1.00	2.17	5.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.086	6.334	0.441	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	472	472	1198	2430	0	0	0	0	-1
N.S.	1	1.00	2.54	5.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.757	6.383	0.366	0.000	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	1196	2185	0	0	0	0	-1
N.S.	1	1.00	2.66	4.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.757	6.366	0.307	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	1236	2321	0	0	0	0	-1
N.S.	1	1.00	2.95	5.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.547	6.399	0.315	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	1314	2666	0	0	0	0	-1
N.S.	1	1.00	3.72	7.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.573	6.473	0.366	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	1407	3413	0	0	0	0	-1
N.S.	1	1.00	3.25	7.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.822	6.540	0.478	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	1515	4392	0	0	0	0	-1
N.S.	1	1.00	2.90	8.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.171	6.665	0.653	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	779	779	1353	5164	0	0	0	0	-1
N.S.	1	1.00	1.74	6.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.057	6.541	0.946	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	664	664	1287	4238	0	0	0	0	-1
N.S.	1	1.00	1.94	6.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.429	6.439	0.607	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	564	564	1251	3512	0	0	0	0	-1
N.S.	1	1.00	2.22	6.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.084	6.533	0.485	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	547	547	1241	3270	0	0	0	0	-1
N.S.	1	1.00	2.27	5.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.053	6.543	0.390	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	1269	3204	0	0	0	0	-1
N.S.	1	1.00	2.37	5.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.048	6.524	0.349	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	1319	3274	0	0	0	0	-1
N.S.	1	1.00	2.68	6.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.781	6.567	0.378	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	1409	3628	0	0	0	0	-1
N.S.	1	1.00	3.25	8.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.827	6.643	0.446	0.000	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	1517	4392	0	0	0	0	-1
N.S.	1	1.00	2.91	8.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.168	6.757	0.645	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	622	622	1640	5373	0	0	0	0	-1
N.S.	1	1.00	2.64	8.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.657	6.872	0.937	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	1236	2346	0	0	0	0	-1
N.S.	1	1.00	2.96	5.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.597	19.451	0.375	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	1175	1871	0	0	0	0	-1
N.S.	1	1.00	2.45	3.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.676	12.121	0.354	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	4017	1005	0	0	0	0	-1
N.S.	1	1.00	9.41	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.685	17.284	0.452	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	144	197	0	0	0	0	-1
N.S.	1	1.00	0.63	0.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.168	1.630	0.302	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	299	935	0	0	0	0	-1
N.S.	1	1.00	1.30	4.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.198	13.123	0.304	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	416	1536	0	0	0	0	-1
N.S.	1	1.00	1.43	5.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.322	16.024	0.309	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	1319	2480	0	0	0	0	-1
N.S.	1	1.00	3.63	6.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.539	6.441	0.370	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	500	500	1234	2881	0	0	0	0	-1
N.S.	1	1.00	2.47	5.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.827	6.430	0.365	0.000	0.000	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	1012	2013	0	0	0	0	-1
N.S.	1	1.00	2.43	4.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	18.109	0.319	0.000	0.000	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	1223	1633	0	0	0	0	-1
N.S.	1	1.00	4.31	5.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.326	6.387	0.308	0.000	0.000	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	1281	2282	0	0	0	0	-1
N.S.	1	1.00	4.20	7.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	6.521	0.326	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	1357	3334	0	0	0	0	-1
N.S.	1	1.00	3.45	8.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.610	6.705	0.361	0.000	0.000	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	674	674	1396	8611	0	0	0	0	-1
N.S.	1	1.00	2.07	12.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.420	6.714	0.574	0.000	0.000	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	1342	5749	0	0	0	0	-1
N.S.	1	1.00	2.46	10.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.888	6.522	0.434	0.000	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	1335	4241	0	0	0	0	-1
N.S.	1	1.00	3.41	10.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.557	6.462	0.422	0.000	0.000	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	1384	5203	0	0	0	0	-1
N.S.	1	1.00	3.23	12.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.638	6.593	0.446	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	456	456	1431	6500	0	0	0	0	-1
N.S.	1	1.00	3.14	14.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.737	6.724	0.510	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	567	567	1499	8093	0	0	0	0	-1
N.S.	1	1.00	2.64	14.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.205	6.962	0.531	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	480	623	0	0	0	0	-1
N.S.	1	1.00	1.15	1.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.510	1.570	0.345	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	131	160	0	0	0	0	-1
N.S.	1	1.00	1.12	1.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.152	0.301	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	171	124	0	0	0	0	-1
N.S.	1	1.00	1.55	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.979	0.280	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	212	613	0	0	0	0	-1
N.S.	1	1.00	0.94	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.168	2.207	0.289	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	0	658	0	0	0	0	-1
N.S.	1	1.00	0.00	9.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	45.895	2.444	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	0	600	0	0	0	0	-1
N.S.	1	1.00	0.00	8.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	49.128	1.805	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	611	0	0	0	0	-1
N.S.	1	1.00	0.00	6.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	44.421	0.892	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	0	705	0	0	0	0	-1
N.S.	1	1.00	0.00	7.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	38.576	0.896	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	0	665	0	0	0	0	-1
N.S.	1	1.00	0.00	9.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	48.754	2.000	0.000	0.000	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	0	663	0	0	0	0	-1
N.S.	1	1.00	0.00	8.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	48.609	2.434	0.000	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	0	714	0	0	0	0	-1
N.S.	1	1.00	0.00	7.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	51.968	0.658	0.000	0.000	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	0	740	0	0	0	0	-1
N.S.	1	1.00	0.00	7.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	38.610	0.557	0.000	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.054	7.246	0.210	0.000	0.000	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	595	595	487	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.306	6.217	0.585	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	269	0	0	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.700	2.872	0.490	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	217	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.361	1.773	0.674	0.000	0.000	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	151	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.351	0.591	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	10482	0	0	0	0	0	-1
N.S.	1	1.00	36.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	27.018	0.243	0.000	0.000	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.341	75.977	0.153	0.000	0.000	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.070	8.890	0.145	0.000	0.000	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.073	9.950	0.207	0.000	0.000	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.315	11.541	0.212	0.000	0.000	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	292	634	0	219	0	0	-1
N.S.	1	1.00	1.70	3.69	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.143	2.081	0.878	0.000	0.119	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	225	399	0	188	0	0	-1
N.S.	1	1.00	1.67	2.96	0.00	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.129	1.334	0.627	0.000	0.112	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	157	242	0	141	0	0	-1
N.S.	1	1.00	1.48	2.28	0.00	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.120	1.147	0.396	0.000	0.114	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	148	321	0	142	0	0	-1
N.S.	1	1.00	1.35	2.92	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.122	1.370	0.361	0.000	0.111	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	148	355	0	169	0	0	-1
N.S.	1	1.00	1.05	2.52	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.132	1.704	0.379	0.000	0.126	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	182	383	0	187	0	0	-1
N.S.	1	1.00	1.06	2.23	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.147	2.361	0.364	0.000	0.121	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	299	714	0	239	0	0	-1
N.S.	1	1.00	1.50	3.59	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.224	3.142	0.842	0.000	0.143	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	279	513	0	202	0	0	-1
N.S.	1	1.00	1.74	3.21	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.208	2.464	0.473	0.000	0.114	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	302	244	0	166	0	0	-1
N.S.	1	1.00	1.89	1.52	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.210	1.984	0.426	0.000	0.109	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	153	357	0	187	0	0	-1
N.S.	1	1.00	0.92	2.15	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.221	1.766	0.369	0.000	0.111	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	193	385	0	211	0	0	-1
N.S.	1	1.00	0.96	1.92	0.00	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.240	2.473	0.375	0.000	0.127	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	435	902	0	263	0	0	-1
N.S.	1	1.00	1.78	3.70	0.00	1.08	0.00	0.00	-0.00
time (sec)	N/A	0.322	4.489	1.039	0.000	0.131	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	268	916	0	243	0	0	-1
N.S.	1	1.00	1.27	4.34	0.00	1.15	0.00	0.00	-0.00
time (sec)	N/A	0.312	3.377	0.934	0.000	0.125	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	202	654	0	215	0	0	-1
N.S.	1	1.00	1.02	3.29	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.305	2.067	0.487	0.000	0.116	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	207	337	0	197	0	0	-1
N.S.	1	1.00	0.98	1.60	0.00	0.93	0.00	0.00	-0.00
time (sec)	N/A	0.313	1.738	0.478	0.000	0.135	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	194	385	0	211	0	0	-1
N.S.	1	1.00	0.92	1.82	0.00	1.00	0.00	0.00	-0.00
time (sec)	N/A	0.321	2.563	0.440	0.000	0.121	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	196	413	0	231	0	0	-1
N.S.	1	1.00	0.80	1.69	0.00	0.95	0.00	0.00	-0.00
time (sec)	N/A	0.338	2.945	0.396	0.000	0.144	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	650	466	0	308	0	0	-1
N.S.	1	1.00	3.37	2.41	0.00	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.187	7.388	0.767	0.000	0.120	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	400	319	0	250	0	0	-1
N.S.	1	1.00	2.52	2.01	0.00	1.57	0.00	0.00	-0.01
time (sec)	N/A	0.167	4.752	0.493	0.000	0.108	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	200	243	0	241	0	0	-1
N.S.	1	1.00	1.63	1.98	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.154	1.197	0.369	0.000	0.104	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	422	244	0	237	0	0	-1
N.S.	1	1.00	3.38	1.95	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.159	2.756	0.432	0.000	0.114	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	444	262	0	261	0	0	-1
N.S.	1	1.00	2.72	1.61	0.00	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.180	5.354	0.382	0.000	0.114	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	518	281	0	278	0	0	-1
N.S.	1	1.00	2.64	1.43	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.186	3.439	0.386	0.000	0.115	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	303	494	0	367	0	0	-1
N.S.	1	1.00	1.46	2.38	0.00	1.76	0.00	0.00	-0.00
time (sec)	N/A	0.274	3.415	0.524	0.000	0.123	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	256	350	0	326	0	0	-1
N.S.	1	1.00	1.59	2.17	0.00	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.254	2.130	0.424	0.000	0.123	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	256	350	0	324	0	0	-1
N.S.	1	1.00	1.52	2.08	0.00	1.93	0.00	0.00	-0.01
time (sec)	N/A	0.252	2.567	0.443	0.000	0.130	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	732	421	0	362	0	0	-1
N.S.	1	1.00	4.16	2.39	0.00	2.06	0.00	0.00	-0.01
time (sec)	N/A	0.256	6.829	0.490	0.000	0.142	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	777	435	0	376	0	0	-1
N.S.	1	1.00	3.77	2.11	0.00	1.83	0.00	0.00	-0.00
time (sec)	N/A	0.275	6.956	0.495	0.000	0.130	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	358	685	0	481	0	0	-1
N.S.	1	1.00	1.37	2.62	0.00	1.84	0.00	0.00	-0.00
time (sec)	N/A	0.389	5.774	0.612	0.000	0.115	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	793	451	0	476	0	0	-1
N.S.	1	1.00	3.57	2.03	0.00	2.14	0.00	0.00	-0.00
time (sec)	N/A	0.370	7.028	0.471	0.000	0.111	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	792	451	0	472	0	0	-1
N.S.	1	1.00	3.67	2.09	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.357	7.002	0.476	0.000	0.133	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	793	451	0	474	0	0	-1
N.S.	1	1.00	3.57	2.03	0.00	2.14	0.00	0.00	-0.00
time (sec)	N/A	0.366	7.130	0.486	0.000	0.129	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	817	451	0	478	0	0	-1
N.S.	1	1.00	3.58	1.98	0.00	2.10	0.00	0.00	-0.00
time (sec)	N/A	0.379	7.220	0.492	0.000	0.140	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	589	465	0	489	0	0	-1
N.S.	1	1.00	2.27	1.80	0.00	1.89	0.00	0.00	-0.00
time (sec)	N/A	0.403	4.886	0.459	0.000	0.145	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	124	138	659	121	0	0	479
N.S.	1	1.00	0.56	0.63	3.00	0.55	0.00	0.00	2.18
time (sec)	N/A	0.299	0.624	2.278	0.563	0.358	0.000	0.000	5.605

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	102	116	568	104	0	0	441
N.S.	1	1.00	0.58	0.66	3.25	0.59	0.00	0.00	2.52
time (sec)	N/A	0.256	0.499	0.387	0.600	0.368	0.000	0.000	4.552

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	78	94	475	86	0	0	196
N.S.	1	1.00	0.60	0.72	3.65	0.66	0.00	0.00	1.51
time (sec)	N/A	0.206	0.313	0.376	0.554	0.363	0.000	0.000	2.689

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	57	70	380	65	0	0	114
N.S.	1	1.00	0.67	0.82	4.47	0.76	0.00	0.00	1.34
time (sec)	N/A	0.164	0.205	0.359	0.572	0.344	0.000	0.000	1.074

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	86	171	906	91	0	0	-1
N.S.	1	1.00	0.90	1.78	9.44	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.242	0.391	0.714	0.370	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	103	168	939	97	0	0	-1
N.S.	1	1.00	1.05	1.71	9.58	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.235	0.398	0.684	0.400	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	120	238	1851	127	0	0	-1
N.S.	1	1.00	0.79	1.58	12.26	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.435	0.440	0.734	0.411	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	138	308	2981	146	0	0	-1
N.S.	1	1.00	0.70	1.57	15.21	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.260	0.772	1.627	0.839	0.431	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	146	161	712	144	0	0	348
N.S.	1	1.00	0.53	0.59	2.59	0.52	0.00	0.00	1.27
time (sec)	N/A	0.456	0.851	0.528	0.601	0.358	0.000	0.000	5.142

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	124	139	619	126	0	0	316
N.S.	1	1.00	0.54	0.61	2.71	0.55	0.00	0.00	1.39
time (sec)	N/A	0.404	0.765	0.393	0.578	0.373	0.000	0.000	4.915

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	102	117	527	107	0	0	259
N.S.	1	1.00	0.56	0.65	2.91	0.59	0.00	0.00	1.43
time (sec)	N/A	0.349	0.615	0.382	0.550	0.370	0.000	0.000	4.804

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	80	95	436	88	0	0	197
N.S.	1	1.00	0.60	0.71	3.25	0.66	0.00	0.00	1.47
time (sec)	N/A	0.299	0.366	0.363	0.594	0.354	0.000	0.000	2.444

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	106	287	1462	130	0	0	-1
N.S.	1	1.00	0.73	1.98	10.08	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.284	0.435	0.419	0.691	0.399	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	107	308	1801	119	0	0	-1
N.S.	1	1.00	0.73	2.11	12.34	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.294	0.355	0.421	0.748	0.401	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	121	233	1884	133	0	0	-1
N.S.	1	1.00	0.79	1.52	12.31	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.293	0.485	0.411	0.733	0.403	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	141	309	3023	153	0	0	-1
N.S.	1	1.00	0.70	1.54	15.12	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.346	0.548	0.446	0.820	0.401	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	158	381	8901	171	0	0	-1
N.S.	1	1.00	0.64	1.54	36.04	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.404	0.815	0.373	1.128	0.449	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	171	185	763	176	0	0	789
N.S.	1	1.00	0.53	0.57	2.37	0.55	0.00	0.00	2.45
time (sec)	N/A	0.601	0.968	0.569	0.617	0.402	0.000	0.000	6.192

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	147	163	672	156	0	0	751
N.S.	1	1.00	0.53	0.59	2.44	0.57	0.00	0.00	2.73
time (sec)	N/A	0.542	1.350	0.420	0.567	0.393	0.000	0.000	5.810

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	126	141	579	135	0	0	617
N.S.	1	1.00	0.55	0.62	2.54	0.59	0.00	0.00	2.71
time (sec)	N/A	0.476	0.990	0.396	0.576	0.381	0.000	0.000	5.725

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	104	119	488	114	0	0	579
N.S.	1	1.00	0.57	0.66	2.70	0.63	0.00	0.00	3.20
time (sec)	N/A	0.425	0.740	0.380	0.557	0.368	0.000	0.000	4.995

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	130	389	1713	162	0	0	-1
N.S.	1	1.00	0.68	2.03	8.92	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.396	0.888	0.430	0.676	0.398	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	130	492	2780	166	0	0	-1
N.S.	1	1.00	0.67	2.55	14.40	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.428	0.780	0.447	0.752	0.466	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	126	344	0	147	0	0	-1
N.S.	1	1.00	0.64	1.74	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.425	0.838	0.459	0.000	0.400	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	141	305	3071	163	0	0	-1
N.S.	1	1.00	0.70	1.52	15.36	0.82	0.00	0.00	-0.00
time (sec)	N/A	0.417	1.044	0.439	0.865	0.448	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	159	383	9390	183	0	0	-1
N.S.	1	1.00	0.64	1.55	38.02	0.74	0.00	0.00	-0.00
time (sec)	N/A	0.469	1.021	0.392	1.106	0.439	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	181	455	10042	203	0	0	-1
N.S.	1	1.00	0.62	1.55	34.16	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.540	1.499	0.381	1.147	0.444	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	272	793	0	198	0	0	-1
N.S.	1	1.00	0.92	2.69	0.00	0.67	0.00	0.00	-0.00
time (sec)	N/A	0.665	9.599	0.398	0.000	0.430	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	250	657	0	181	0	0	-1
N.S.	1	1.00	1.00	2.63	0.00	0.72	0.00	0.00	-0.00
time (sec)	N/A	0.535	6.980	0.498	0.000	0.425	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	1718	521	0	164	0	0	-1
N.S.	1	1.00	8.30	2.52	0.00	0.79	0.00	0.00	-0.00
time (sec)	N/A	0.408	8.013	0.462	0.000	0.390	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	617	384	0	143	0	0	-1
N.S.	1	1.00	3.81	2.37	0.00	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.285	6.842	0.441	0.000	0.381	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	203	231	0	110	0	0	-1
N.S.	1	1.00	1.71	1.94	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.194	1.682	0.411	0.000	0.420	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	102	153	0	96	0	0	-1
N.S.	1	1.00	0.73	1.09	0.00	0.69	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.241	0.401	0.000	1.128	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	467	232	0	168	0	0	-1
N.S.	1	1.00	2.58	1.28	0.00	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.325	1.476	0.423	0.000	1.412	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	412	301	0	194	0	0	-1
N.S.	1	1.00	1.79	1.31	0.00	0.84	0.00	0.00	-0.00
time (sec)	N/A	0.438	1.555	0.459	0.000	2.362	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	143	317	0	208	0	0	-1
N.S.	1	1.00	0.74	1.65	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.415	0.491	0.511	0.000	14.318	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	262	731	0	237	0	0	-1
N.S.	1	1.00	0.83	2.31	0.00	0.75	0.00	0.00	-0.00
time (sec)	N/A	0.709	3.116	0.517	0.000	0.401	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	236	595	0	220	0	0	-1
N.S.	1	1.00	0.87	2.20	0.00	0.81	0.00	0.00	-0.00
time (sec)	N/A	0.561	2.819	0.466	0.000	0.393	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	215	457	0	197	0	0	-1
N.S.	1	1.00	0.96	2.05	0.00	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.436	2.463	0.444	0.000	0.379	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	191	311	0	163	0	0	-1
N.S.	1	1.00	1.09	1.77	0.00	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.322	2.007	0.413	0.000	0.394	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	196	235	0	144	0	0	-1
N.S.	1	1.00	1.54	1.85	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.209	1.669	0.391	0.000	0.373	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	326	288	0	203	0	0	-1
N.S.	1	1.00	1.76	1.56	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.340	3.433	0.414	0.000	3.120	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	836	370	0	246	0	0	-1
N.S.	1	1.00	3.53	1.56	0.00	1.04	0.00	0.00	-0.00
time (sec)	N/A	0.475	6.683	0.420	0.000	4.106	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	261	729	0	266	0	0	-1
N.S.	1	1.00	0.82	2.30	0.00	0.84	0.00	0.00	-0.00
time (sec)	N/A	0.710	6.853	0.494	0.000	0.432	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	243	585	0	246	0	0	-1
N.S.	1	1.00	0.90	2.17	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.584	3.095	0.477	0.000	0.420	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	219	457	0	210	0	0	-1
N.S.	1	1.00	0.98	2.05	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.452	2.203	0.430	0.000	0.419	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	216	375	0	207	0	0	-1
N.S.	1	1.00	1.23	2.13	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.330	1.813	0.403	0.000	0.407	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	213	375	0	205	0	0	-1
N.S.	1	1.00	1.22	2.16	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.325	1.858	0.410	0.000	0.402	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	347	476	0	277	0	0	-1
N.S.	1	1.00	1.48	2.03	0.00	1.18	0.00	0.00	-0.00
time (sec)	N/A	0.460	3.580	0.411	0.000	6.504	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	929	609	0	313	0	0	-1
N.S.	1	1.00	3.25	2.13	0.00	1.09	0.00	0.00	-0.00
time (sec)	N/A	0.617	7.025	0.454	0.000	8.447	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	267	729	0	295	0	0	-1
N.S.	1	1.00	0.84	2.30	0.00	0.93	0.00	0.00	-0.00
time (sec)	N/A	0.724	5.027	0.510	0.000	0.448	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	242	595	0	260	0	0	-1
N.S.	1	1.00	0.90	2.20	0.00	0.96	0.00	0.00	-0.00
time (sec)	N/A	0.591	3.195	0.460	0.000	0.404	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	228	512	0	257	0	0	-1
N.S.	1	1.00	1.02	2.30	0.00	1.15	0.00	0.00	-0.00
time (sec)	N/A	0.460	3.187	0.421	0.000	0.428	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	233	512	0	255	0	0	-1
N.S.	1	1.00	1.05	2.32	0.00	1.15	0.00	0.00	-0.00
time (sec)	N/A	0.456	3.024	0.425	0.000	0.448	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	488	512	0	257	0	0	-1
N.S.	1	1.00	2.21	2.32	0.00	1.16	0.00	0.00	-0.00
time (sec)	N/A	0.458	7.089	0.416	0.000	0.418	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	365	667	0	338	0	0	-1
N.S.	1	1.00	1.30	2.37	0.00	1.20	0.00	0.00	-0.00
time (sec)	N/A	0.584	6.172	0.436	0.000	8.512	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	1017	855	0	379	0	0	-1
N.S.	1	1.00	3.05	2.57	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	0.768	7.374	0.487	0.000	14.490	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	132	636	0	235	0	0	-1
N.S.	1	1.00	0.73	3.53	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.149	2.023	0.885	0.000	0.127	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	104	401	0	205	0	0	-1
N.S.	1	1.00	0.73	2.80	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.924	0.641	0.000	0.113	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	85	246	0	153	0	0	-1
N.S.	1	1.00	0.77	2.22	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.299	0.389	0.000	0.104	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	90	326	0	156	0	0	-1
N.S.	1	1.00	0.78	2.83	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.268	0.368	0.000	0.112	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	108	371	0	184	0	0	-1
N.S.	1	1.00	0.73	2.51	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.606	0.373	0.000	0.125	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	125	413	0	203	0	0	-1
N.S.	1	1.00	0.69	2.29	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.158	1.079	0.348	0.000	0.134	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	171	723	0	286	0	0	-1
N.S.	1	1.00	0.77	3.27	0.00	1.29	0.00	0.00	-0.00
time (sec)	N/A	0.247	2.627	0.987	0.000	0.121	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	125	650	0	247	0	0	-1
N.S.	1	1.00	0.71	3.67	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.222	1.240	0.675	0.000	0.114	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	124	405	0	208	0	0	-1
N.S.	1	1.00	0.77	2.52	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.820	0.463	0.000	0.132	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	128	487	0	226	0	0	-1
N.S.	1	1.00	0.75	2.85	0.00	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.999	0.389	0.000	0.116	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	161	548	0	254	0	0	-1
N.S.	1	1.00	0.76	2.57	0.00	1.19	0.00	0.00	-0.00
time (sec)	N/A	0.234	1.468	0.433	0.000	0.148	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	225	917	0	364	0	0	-1
N.S.	1	1.00	0.76	3.11	0.00	1.23	0.00	0.00	-0.00
time (sec)	N/A	0.381	3.905	1.326	0.000	0.146	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	192	970	0	326	0	0	-1
N.S.	1	1.00	0.79	3.98	0.00	1.34	0.00	0.00	-0.00
time (sec)	N/A	0.366	1.726	1.003	0.000	0.135	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	166	1210	0	298	0	0	-1
N.S.	1	1.00	0.69	5.06	0.00	1.25	0.00	0.00	-0.00
time (sec)	N/A	0.355	2.068	0.817	0.000	0.146	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	172	641	0	270	0	0	-1
N.S.	1	1.00	0.73	2.70	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	0.347	1.544	0.499	0.000	0.152	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	180	664	0	295	0	0	-1
N.S.	1	1.00	0.73	2.71	0.00	1.20	0.00	0.00	-0.00
time (sec)	N/A	0.345	1.404	0.443	0.000	0.149	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	219	745	0	332	0	0	-1
N.S.	1	1.00	0.74	2.53	0.00	1.13	0.00	0.00	-0.00
time (sec)	N/A	0.375	1.981	0.428	0.000	0.169	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	225	441	0	0	0	0	-1
N.S.	1	1.00	1.07	2.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.510	33.680	0.797	0.000	0.000	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	125	300	0	0	0	0	-1
N.S.	1	1.00	0.99	2.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.279	31.371	0.465	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	76	217	0	0	0	0	-1
N.S.	1	1.00	0.75	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	20.606	0.342	0.000	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	220	295	0	0	0	0	-1
N.S.	1	1.00	1.48	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	36.910	0.367	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	542	822	0	0	0	0	-1
N.S.	1	1.00	2.75	4.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.349	36.817	0.455	0.000	0.000	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	735	1004	0	0	0	0	-1
N.S.	1	1.00	1.81	2.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.827	37.165	1.683	0.000	0.000	0.000	0.000	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	681	856	0	0	0	0	-1
N.S.	1	1.00	2.16	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.597	36.983	0.914	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	639	721	0	0	0	0	-1
N.S.	1	1.00	2.46	2.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.388	36.894	0.724	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	626	808	0	0	0	0	-1
N.S.	1	1.00	2.43	3.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.378	36.872	0.754	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	655	849	0	0	0	0	-1
N.S.	1	1.00	2.31	2.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.419	36.892	0.893	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	701	1066	0	0	0	0	-1
N.S.	1	1.00	1.93	2.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.604	37.106	1.034	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	844	1975	0	0	0	0	-1
N.S.	1	1.00	1.76	4.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.920	37.360	1.839	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	797	1744	0	0	0	0	-1
N.S.	1	1.00	1.97	4.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.666	37.079	1.507	0.000	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	784	1850	0	0	0	0	-1
N.S.	1	1.00	1.95	4.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.669	37.031	1.520	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	786	1937	0	0	0	0	-1
N.S.	1	1.00	1.96	4.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.624	37.027	1.552	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	820	1977	0	0	0	0	-1
N.S.	1	1.00	1.92	4.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.674	37.157	1.661	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	521	521	865	2195	0	0	0	0	-1
N.S.	1	1.00	1.66	4.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.965	37.386	1.906	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	47	214	0	91	0	0	-1
N.S.	1	1.00	0.73	3.34	0.00	1.42	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.082	0.290	0.000	0.136	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	46	183	0	76	0	0	-1
N.S.	1	1.00	0.77	3.05	0.00	1.27	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.056	0.297	0.000	0.104	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	134	0	53	0	0	-1
N.S.	1	1.00	1.00	3.62	0.00	1.43	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.037	0.286	0.000	0.095	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	134	0	59	0	0	-1
N.S.	1	1.00	1.00	3.62	0.00	1.59	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.043	0.238	0.000	0.106	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	50	180	0	71	0	0	-1
N.S.	1	1.00	0.78	2.81	0.00	1.11	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.054	0.300	0.000	0.112	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	203	0	77	0	0	-1
N.S.	1	1.00	0.88	3.17	0.00	1.20	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.086	0.340	0.000	0.114	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	3321	3436	0	0	0	0	-1
N.S.	1	1.00	7.02	7.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.898	24.034	2.176	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	423	2487	0	0	0	0	-1
N.S.	1	1.00	1.08	6.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.610	17.670	0.554	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	346	1735	0	0	0	0	-1
N.S.	1	1.00	1.07	5.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.403	14.691	0.417	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	635	1361	0	0	0	0	-1
N.S.	1	1.00	1.55	3.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	17.246	0.462	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	445	787	1369	0	0	0	0	-1
N.S.	1	1.00	1.77	3.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.551	17.291	0.587	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	1121	2054	0	0	0	0	-1
N.S.	1	1.00	2.10	3.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.792	18.065	0.506	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	620	620	1533	2956	0	0	0	0	-1
N.S.	1	1.00	2.47	4.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.071	14.611	0.532	0.000	0.000	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	3739	4399	0	0	0	0	-1
N.S.	1	1.00	6.65	7.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.302	25.756	0.769	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	3318	3421	0	0	0	0	-1
N.S.	1	1.00	7.01	7.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.930	24.136	0.583	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	427	2674	0	0	0	0	-1
N.S.	1	1.00	1.09	6.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.690	18.123	0.480	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	5981	2329	0	0	0	0	-1
N.S.	1	1.00	12.49	4.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.670	24.447	0.422	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	927	2193	0	0	0	0	-1
N.S.	1	1.00	1.82	4.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.859	16.232	0.426	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	1134	2432	0	0	0	0	-1
N.S.	1	1.00	2.13	4.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.865	18.475	0.444	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	626	626	1489	3141	0	0	0	0	-1
N.S.	1	1.00	2.38	5.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.248	19.110	0.543	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	1888	4056	0	0	0	0	-1
N.S.	1	1.00	2.59	5.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.522	21.017	0.681	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	662	662	4198	5381	0	0	0	0	-1
N.S.	1	1.00	6.34	8.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.849	27.048	0.997	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	3755	4400	0	0	0	0	-1
N.S.	1	1.00	6.68	7.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.288	25.900	0.733	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	3348	3636	0	0	0	0	-1
N.S.	1	1.00	7.06	7.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.930	24.587	0.581	0.000	0.000	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	7032	3282	0	0	0	0	-1
N.S.	1	1.00	12.72	5.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.911	25.519	0.498	0.000	0.000	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	7700	3212	0	0	0	0	-1
N.S.	1	1.00	12.92	5.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.186	25.759	0.476	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	607	607	1278	3278	0	0	0	0	-1
N.S.	1	1.00	2.11	5.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.193	19.079	0.509	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	1504	3514	0	0	0	0	-1
N.S.	1	1.00	2.41	5.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.225	19.624	0.544	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	724	724	1857	4240	0	0	0	0	-1
N.S.	1	1.00	2.56	5.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.584	20.229	0.701	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	839	839	703	5172	0	0	0	0	-1
N.S.	1	1.00	0.84	6.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.244	16.369	0.918	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	2987	2488	0	0	0	0	-1
N.S.	1	1.00	7.41	6.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.633	22.290	0.481	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	355	1544	0	0	0	0	-1
N.S.	1	1.00	1.08	4.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.411	15.610	0.416	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	279	812	0	0	0	0	-1
N.S.	1	1.00	1.03	3.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.292	14.158	0.457	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	157	199	0	0	0	0	-1
N.S.	1	1.00	0.59	0.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	2.635	0.441	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	1091	1007	0	0	0	0	-1
N.S.	1	1.00	2.24	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.788	18.062	0.546	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	539	539	1157	1879	0	0	0	0	-1
N.S.	1	1.00	2.15	3.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.784	19.087	0.441	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	3433	3343	0	0	0	0	-1
N.S.	1	1.00	7.93	7.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.731	24.330	0.462	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	433	2291	0	0	0	0	-1
N.S.	1	1.00	1.26	6.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.490	18.181	0.514	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	305	1636	0	0	0	0	-1
N.S.	1	1.00	0.94	5.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.418	13.610	0.498	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	1403	2016	0	0	0	0	-1
N.S.	1	1.00	2.95	4.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	13.898	0.487	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	1551	2890	0	0	0	0	-1
N.S.	1	1.00	2.77	5.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.939	19.390	0.441	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	607	607	4316	8101	0	0	0	0	-1
N.S.	1	1.00	7.11	13.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.405	27.038	0.661	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	3891	6506	0	0	0	0	-1
N.S.	1	1.00	7.84	13.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.889	25.849	0.550	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	3493	5205	0	0	0	0	-1
N.S.	1	1.00	7.45	11.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.759	24.311	0.460	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	528	4243	0	0	0	0	-1
N.S.	1	1.00	1.23	9.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.678	19.103	0.498	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	602	602	1994	5759	0	0	0	0	-1
N.S.	1	1.00	3.31	9.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.051	16.074	0.478	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	733	733	2318	8621	0	0	0	0	-1
N.S.	1	1.00	3.16	11.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.609	22.281	0.619	0.000	0.000	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	298	621	0	0	0	0	-1
N.S.	1	1.00	1.12	2.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.230	6.118	0.378	0.000	0.000	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	104	126	0	0	0	0	-1
N.S.	1	1.00	0.80	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.162	0.404	0.000	0.000	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	147	144	0	0	0	0	-1
N.S.	1	1.00	1.07	1.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.200	0.372	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	508	631	0	0	0	0	-1
N.S.	1	1.00	1.06	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.576	3.147	0.451	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.114	8.538	0.300	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	644	644	317	0	0	0	0	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.321	4.408	0.866	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	455	259	0	0	0	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.738	2.109	0.710	0.000	0.000	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	205	0	0	0	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.413	0.996	0.586	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	163	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	0.398	0.513	0.000	0.000	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	10630	0	0	0	0	0	-1
N.S.	1	1.00	35.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.368	26.343	0.396	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.445	180.002	0.245	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.143	116.798	0.238	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.163	13.877	0.363	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	213	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	14.275	0.345	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [528] had the largest ratio of [54]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	6	1.00	29	0.207
2	A	7	6	1.00	29	0.207
3	A	3	3	1.00	27	0.111
4	A	1	1	1.00	21	0.048
5	A	4	4	1.00	27	0.148
6	A	4	4	1.00	29	0.138
7	A	6	6	1.00	29	0.207
8	A	7	7	1.00	29	0.241
9	A	7	6	1.00	29	0.207
10	A	9	7	1.00	31	0.226
11	A	8	7	1.00	31	0.226
12	A	4	4	1.00	29	0.138
13	A	2	2	1.00	23	0.087
14	A	5	5	1.00	29	0.172
15	A	5	5	1.00	31	0.161
16	A	5	5	1.00	31	0.161
17	A	7	7	1.00	31	0.226
18	A	8	8	1.00	31	0.258
19	A	9	7	1.00	31	0.226
20	A	10	8	1.00	29	0.276
21	A	8	6	1.00	23	0.261
22	A	6	5	1.00	29	0.172
23	A	6	6	1.00	31	0.194
24	A	6	5	1.00	31	0.161
25	A	6	5	1.00	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	8	7	1.00	31	0.226
27	A	9	8	1.00	31	0.258
28	A	10	7	1.00	31	0.226
29	A	13	8	1.00	29	0.276
30	A	11	6	1.00	23	0.261
31	A	7	5	1.00	29	0.172
32	A	7	6	1.00	31	0.194
33	A	7	6	1.00	31	0.194
34	A	7	5	1.00	31	0.161
35	A	7	5	1.00	31	0.161
36	A	9	7	1.00	31	0.226
37	A	10	8	1.00	31	0.258
38	A	7	5	1.00	31	0.161
39	A	6	5	1.00	31	0.161
40	A	2	2	1.10	31	0.065
41	A	5	5	1.00	29	0.172
42	A	2	2	1.00	23	0.087
43	A	3	3	1.00	29	0.103
44	A	5	5	1.00	31	0.161
45	A	6	6	1.00	31	0.194
46	A	6	5	1.00	31	0.161
47	A	7	5	1.00	31	0.161
48	A	3	2	1.00	31	0.065
49	A	6	6	1.00	31	0.194
50	A	4	4	1.00	29	0.138
51	A	2	2	1.00	23	0.087
52	A	4	3	1.00	29	0.103
53	A	6	5	1.00	31	0.161
54	A	7	6	1.00	31	0.194
55	A	7	5	1.00	31	0.161
56	A	8	5	1.00	31	0.161
57	A	4	2	1.00	31	0.065
58	A	7	6	1.00	31	0.194
59	A	5	5	1.00	31	0.161
60	A	4	4	1.00	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	3	1.00	23	0.130
62	A	5	3	1.00	29	0.103
63	A	7	5	1.00	31	0.161
64	A	8	6	1.00	31	0.194
65	A	5	2	1.00	31	0.065
66	A	8	6	1.00	31	0.194
67	A	6	5	1.00	31	0.161
68	A	5	5	1.00	31	0.161
69	A	5	5	1.00	29	0.172
70	A	4	3	1.00	23	0.130
71	A	6	3	1.00	29	0.103
72	A	8	5	1.00	31	0.161
73	A	9	6	1.00	31	0.194
74	A	5	5	1.00	33	0.152
75	A	4	4	1.00	33	0.121
76	A	4	4	1.00	31	0.129
77	A	2	2	1.00	25	0.080
78	A	3	3	1.00	31	0.097
79	A	3	3	1.00	33	0.091
80	A	4	4	1.00	33	0.121
81	A	5	4	1.00	33	0.121
82	A	6	6	1.00	33	0.182
83	A	5	5	1.00	33	0.152
84	A	5	5	1.00	31	0.161
85	A	3	3	1.00	25	0.120
86	A	4	4	1.00	31	0.129
87	A	4	4	1.00	33	0.121
88	A	4	4	1.00	33	0.121
89	A	5	5	1.00	33	0.152
90	A	6	5	1.00	33	0.152
91	A	6	5	1.00	33	0.152
92	A	6	5	1.00	31	0.161
93	A	4	3	1.00	25	0.120
94	A	5	4	1.00	31	0.129
95	A	5	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	4	1.00	33	0.121
97	A	5	4	1.00	33	0.121
98	A	6	5	1.00	33	0.152
99	A	7	5	1.00	33	0.152
100	A	7	6	1.00	33	0.182
101	A	6	6	1.00	33	0.182
102	A	5	5	1.00	31	0.161
103	A	3	3	1.00	25	0.120
104	A	5	4	1.00	31	0.129
105	A	6	5	1.00	33	0.152
106	A	7	5	1.00	33	0.152
107	A	8	7	1.00	33	0.212
108	A	7	7	1.00	33	0.212
109	A	6	6	1.00	33	0.182
110	A	5	5	1.00	31	0.161
111	A	3	3	1.00	25	0.120
112	A	6	5	1.00	31	0.161
113	A	7	6	1.00	33	0.182
114	A	8	6	1.00	33	0.182
115	A	8	7	1.00	33	0.212
116	A	7	6	1.00	33	0.182
117	A	6	6	1.00	33	0.182
118	A	5	5	1.00	31	0.161
119	A	4	4	1.00	25	0.160
120	A	7	5	1.00	31	0.161
121	A	8	6	1.00	33	0.182
122	A	9	6	1.00	33	0.182
123	A	8	6	1.00	31	0.194
124	A	7	6	1.00	31	0.194
125	A	6	6	1.00	31	0.194
126	A	5	5	1.00	31	0.161
127	A	5	5	1.00	31	0.161
128	A	6	6	1.00	31	0.194
129	A	7	6	1.00	31	0.194
130	A	8	7	1.00	33	0.212

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	7	7	1.00	33	0.212
132	A	6	6	1.00	33	0.182
133	A	6	6	1.00	33	0.182
134	A	6	6	1.00	33	0.182
135	A	7	7	1.00	33	0.212
136	A	8	7	1.00	33	0.212
137	A	9	7	1.00	33	0.212
138	A	8	7	1.00	33	0.212
139	A	7	6	1.00	33	0.182
140	A	7	7	1.00	33	0.212
141	A	7	6	1.00	33	0.182
142	A	7	6	1.00	33	0.182
143	A	8	7	1.00	33	0.212
144	A	9	7	1.00	33	0.212
145	A	6	5	1.00	33	0.152
146	A	5	5	1.00	33	0.152
147	A	4	4	1.00	33	0.121
148	A	4	4	1.00	33	0.121
149	A	5	5	1.00	33	0.152
150	A	6	5	1.00	33	0.152
151	A	7	5	1.00	33	0.152
152	A	6	5	1.00	33	0.152
153	A	5	4	1.00	33	0.121
154	A	5	5	1.00	33	0.152
155	A	5	4	1.00	33	0.121
156	A	6	5	1.00	33	0.152
157	A	7	5	1.00	33	0.152
158	A	8	5	1.00	33	0.152
159	A	7	5	1.00	33	0.152
160	A	6	4	1.00	33	0.121
161	A	6	5	1.00	33	0.152
162	A	6	5	1.00	33	0.152
163	A	6	4	1.00	33	0.121
164	A	7	5	1.00	33	0.152
165	A	8	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	6	4	1.00	35	0.114
167	A	5	4	1.00	35	0.114
168	A	4	4	1.00	35	0.114
169	A	3	3	1.00	35	0.086
170	A	3	3	1.00	35	0.086
171	A	2	2	1.00	35	0.057
172	A	3	3	1.00	35	0.086
173	A	4	3	1.00	35	0.086
174	A	6	5	1.00	35	0.143
175	A	5	5	1.00	35	0.143
176	A	4	4	1.00	35	0.114
177	A	4	4	1.00	35	0.114
178	A	4	4	1.00	35	0.114
179	A	3	3	1.00	35	0.086
180	A	4	4	1.00	35	0.114
181	A	5	4	1.00	35	0.114
182	A	7	5	1.00	35	0.143
183	A	6	5	1.00	35	0.143
184	A	5	4	1.00	35	0.114
185	A	5	5	1.00	35	0.143
186	A	5	4	1.00	35	0.114
187	A	5	4	1.00	35	0.114
188	A	4	3	1.00	35	0.086
189	A	5	4	1.00	35	0.114
190	A	6	4	1.00	35	0.114
191	A	7	6	1.00	35	0.171
192	A	6	6	1.00	35	0.171
193	A	5	5	1.00	35	0.143
194	A	4	4	1.00	35	0.114
195	A	5	4	1.00	35	0.114
196	A	6	4	1.00	35	0.114
197	A	7	7	1.00	35	0.200
198	A	6	6	1.00	35	0.171
199	A	4	4	1.00	35	0.114
200	A	5	5	1.00	35	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	6	5	1.00	35	0.143
202	A	8	7	1.00	35	0.200
203	A	7	6	1.00	35	0.171
204	A	5	5	1.00	35	0.143
205	A	5	4	1.00	35	0.114
206	A	6	5	1.00	35	0.143
207	A	7	5	1.00	35	0.143
208	A	9	7	1.00	35	0.200
209	A	8	6	1.00	35	0.171
210	A	6	5	1.00	35	0.143
211	A	6	5	1.00	35	0.143
212	A	6	4	1.00	35	0.114
213	A	7	5	1.00	35	0.143
214	A	8	5	1.00	35	0.143
215	A	7	6	1.00	29	0.207
216	A	3	3	1.00	27	0.111
217	A	1	1	1.00	21	0.048
218	A	4	4	1.00	27	0.148
219	A	4	4	1.00	29	0.138
220	A	6	6	1.00	29	0.207
221	A	7	7	1.00	29	0.241
222	A	7	6	1.00	29	0.207
223	A	7	6	1.00	31	0.194
224	A	4	4	1.00	29	0.138
225	A	2	2	1.00	23	0.087
226	A	4	4	1.00	29	0.138
227	A	4	4	1.00	31	0.129
228	A	4	4	1.00	31	0.129
229	A	6	6	1.00	31	0.194
230	A	7	7	1.00	31	0.226
231	A	8	7	1.00	31	0.226
232	A	5	4	1.00	29	0.138
233	A	3	2	1.00	23	0.087
234	A	5	5	1.00	29	0.172
235	A	5	5	1.00	31	0.161

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	5	5	1.00	31	0.161
237	A	5	5	1.00	31	0.161
238	A	7	7	1.00	31	0.226
239	A	8	8	1.00	31	0.258
240	A	9	8	1.00	31	0.258
241	A	6	4	1.00	29	0.138
242	A	4	2	1.00	23	0.087
243	A	6	6	1.00	29	0.207
244	A	6	6	1.00	31	0.194
245	A	6	6	1.00	31	0.194
246	A	6	6	1.00	31	0.194
247	A	6	6	1.00	31	0.194
248	A	8	8	1.00	31	0.258
249	A	9	9	1.00	31	0.290
250	A	6	6	1.00	31	0.194
251	A	5	5	1.00	31	0.161
252	A	6	6	1.00	29	0.207
253	A	3	3	1.00	23	0.130
254	A	4	4	1.00	29	0.138
255	A	6	6	1.00	31	0.194
256	A	6	6	1.00	31	0.194
257	A	7	6	1.00	31	0.194
258	A	6	6	1.00	31	0.194
259	A	5	5	1.00	31	0.161
260	A	5	5	1.00	29	0.172
261	A	4	4	1.00	23	0.174
262	A	5	5	1.00	29	0.172
263	A	6	6	1.00	31	0.194
264	A	7	6	1.00	31	0.194
265	A	7	7	1.00	31	0.226
266	A	6	6	1.00	31	0.194
267	A	5	5	1.00	31	0.161
268	A	6	6	1.00	29	0.207
269	A	5	4	1.00	23	0.174
270	A	6	6	1.00	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	7	6	1.00	31	0.194
272	A	8	6	1.00	31	0.194
273	A	7	7	1.00	31	0.226
274	A	6	6	1.00	31	0.194
275	A	6	6	1.00	31	0.194
276	A	7	6	1.00	29	0.207
277	A	6	4	1.00	23	0.174
278	A	7	6	1.00	29	0.207
279	A	8	6	1.00	31	0.194
280	A	9	6	1.00	31	0.194
281	A	3	2	1.00	34	0.059
282	A	3	3	1.00	34	0.088
283	A	2	2	1.00	32	0.062
284	A	2	2	1.00	26	0.077
285	A	2	2	1.00	32	0.062
286	A	3	3	1.00	34	0.088
287	A	3	3	1.00	34	0.088
288	A	3	2	1.00	34	0.059
289	A	6	6	1.00	34	0.176
290	A	6	6	1.00	34	0.176
291	A	4	4	1.00	32	0.125
292	A	3	3	1.00	26	0.115
293	A	5	5	1.00	32	0.156
294	A	7	7	1.00	34	0.206
295	A	7	7	1.00	34	0.206
296	A	9	9	1.00	33	0.273
297	A	8	8	1.00	33	0.242
298	A	8	8	1.00	31	0.258
299	A	6	6	1.00	25	0.240
300	A	8	8	1.00	31	0.258
301	A	9	9	1.00	33	0.273
302	A	10	10	1.00	33	0.303
303	A	11	10	1.00	33	0.303
304	A	9	8	1.00	33	0.242
305	A	9	8	1.00	31	0.258

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	7	6	1.00	25	0.240
307	A	9	9	1.00	31	0.290
308	A	9	9	1.00	33	0.273
309	A	10	10	1.00	33	0.303
310	A	11	10	1.00	33	0.303
311	A	10	8	1.00	33	0.242
312	A	10	8	1.00	31	0.258
313	A	8	6	1.00	25	0.240
314	A	10	10	1.00	31	0.323
315	A	10	10	1.00	33	0.303
316	A	10	10	1.00	33	0.303
317	A	11	11	1.00	33	0.333
318	A	12	11	1.00	33	0.333
319	A	8	8	1.00	33	0.242
320	A	7	7	1.00	33	0.212
321	A	7	7	1.00	31	0.226
322	A	5	5	1.00	25	0.200
323	A	5	5	1.00	31	0.161
324	A	9	9	1.00	33	0.273
325	A	10	10	1.00	33	0.303
326	A	8	8	1.00	33	0.242
327	A	7	7	1.00	33	0.212
328	A	7	7	1.00	31	0.226
329	A	6	6	1.00	25	0.240
330	A	7	7	1.00	31	0.226
331	A	10	10	1.00	33	0.303
332	A	11	10	1.00	33	0.303
333	A	9	9	1.00	33	0.273
334	A	8	8	1.00	33	0.242
335	A	7	7	1.00	33	0.212
336	A	8	8	1.00	31	0.258
337	A	7	6	1.00	25	0.240
338	A	10	10	1.00	31	0.323
339	A	11	11	1.00	33	0.333
340	A	12	10	1.00	33	0.303

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	3	3	1.00	28	0.107
342	A	3	3	1.00	34	0.088
343	A	5	4	1.00	28	0.143
344	A	8	8	1.00	34	0.235
345	A	8	6	1.00	31	0.194
346	A	7	6	1.00	31	0.194
347	A	6	6	1.00	31	0.194
348	A	5	5	1.00	31	0.161
349	A	5	5	1.00	31	0.161
350	A	6	6	1.00	31	0.194
351	A	7	6	1.00	31	0.194
352	A	8	6	1.00	33	0.182
353	A	7	6	1.00	33	0.182
354	A	6	6	1.00	33	0.182
355	A	5	5	1.00	33	0.152
356	A	5	5	1.00	33	0.152
357	A	5	5	1.00	33	0.152
358	A	6	6	1.00	33	0.182
359	A	8	7	1.00	33	0.212
360	A	7	7	1.00	33	0.212
361	A	6	6	1.00	33	0.182
362	A	6	6	1.00	33	0.182
363	A	6	6	1.00	33	0.182
364	A	6	6	1.00	33	0.182
365	A	7	7	1.00	33	0.212
366	A	6	6	1.00	33	0.182
367	A	5	5	1.00	33	0.152
368	A	3	3	1.00	33	0.091
369	A	5	5	1.00	33	0.152
370	A	7	7	1.00	33	0.212
371	A	7	7	1.00	33	0.212
372	A	6	6	1.00	33	0.182
373	A	6	6	1.00	33	0.182
374	A	6	6	1.00	33	0.182
375	A	7	7	1.00	33	0.212

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	8	7	1.00	33	0.212
377	A	7	7	1.00	33	0.212
378	A	7	7	1.00	33	0.212
379	A	7	7	1.00	33	0.212
380	A	7	7	1.00	33	0.212
381	A	8	7	1.00	33	0.212
382	A	9	7	1.00	33	0.212
383	A	3	3	1.00	36	0.083
384	A	3	3	1.00	36	0.083
385	A	2	2	1.00	36	0.056
386	A	2	2	1.00	36	0.056
387	A	3	3	1.00	36	0.083
388	A	3	3	1.00	36	0.083
389	A	7	7	1.00	36	0.194
390	A	6	6	1.00	36	0.167
391	A	4	4	1.00	36	0.111
392	A	2	2	1.00	36	0.056
393	A	6	6	1.00	36	0.167
394	A	8	8	1.00	36	0.222
395	A	8	8	1.00	35	0.229
396	A	7	7	1.00	35	0.200
397	A	6	6	1.00	35	0.171
398	A	5	5	1.00	35	0.143
399	A	4	4	1.00	35	0.114
400	A	5	5	1.00	35	0.143
401	A	6	5	1.00	35	0.143
402	A	9	8	1.00	35	0.229
403	A	8	8	1.00	35	0.229
404	A	7	7	1.00	35	0.200
405	A	7	7	1.00	35	0.200
406	A	6	6	1.00	35	0.171
407	A	5	5	1.00	35	0.143
408	A	6	5	1.00	35	0.143
409	A	7	5	1.00	35	0.143
410	A	10	8	1.00	35	0.229

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	9	8	1.00	35	0.229
412	A	8	8	1.00	35	0.229
413	A	8	8	1.00	35	0.229
414	A	8	8	1.00	35	0.229
415	A	7	7	1.00	35	0.200
416	A	6	6	1.00	35	0.171
417	A	7	6	1.00	35	0.171
418	A	8	6	1.00	35	0.171
419	A	6	6	1.00	43	0.140
420	A	7	7	1.00	35	0.200
421	A	7	7	1.00	35	0.200
422	A	3	3	1.00	35	0.086
423	A	3	3	1.00	35	0.086
424	A	4	4	1.00	35	0.114
425	A	5	5	1.00	35	0.143
426	A	7	7	1.00	35	0.200
427	A	6	6	1.00	35	0.171
428	A	4	4	1.00	35	0.114
429	A	4	4	1.00	35	0.114
430	A	5	5	1.00	35	0.143
431	A	8	8	1.00	35	0.229
432	A	7	7	1.00	35	0.200
433	A	5	5	1.00	35	0.143
434	A	5	5	1.00	35	0.143
435	A	5	5	1.00	35	0.143
436	A	6	5	1.00	35	0.143
437	A	9	9	1.00	38	0.237
438	A	2	2	1.00	38	0.053
439	A	2	2	1.00	38	0.053
440	A	4	4	1.00	38	0.105
441	A	1	1	1.00	33	0.030
442	A	1	1	1.00	33	0.030
443	A	2	2	1.00	33	0.061
444	A	2	2	1.00	33	0.061
445	A	1	1	1.00	33	0.030

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	1	1	1.00	33	0.030
447	A	2	2	1.00	33	0.061
448	A	2	2	1.00	33	0.061
449	A	0	0	0.00	0	0.000
450	A	7	6	1.00	33	0.182
451	A	6	5	1.00	33	0.152
452	A	5	4	1.00	33	0.121
453	A	5	4	1.00	31	0.129
454	A	7	5	1.00	33	0.152
455	A	0	0	0.00	0	0.000
456	A	0	0	0.00	0	0.000
457	A	0	0	0.00	0	0.000
458	A	0	0	0.00	0	0.000
459	A	9	7	1.00	31	0.226
460	A	8	7	1.00	31	0.226
461	A	7	6	1.00	31	0.194
462	A	7	6	1.00	31	0.194
463	A	8	7	1.00	31	0.226
464	A	9	7	1.00	31	0.226
465	A	9	8	1.00	33	0.242
466	A	8	7	1.00	33	0.212
467	A	8	7	1.00	33	0.212
468	A	8	7	1.00	33	0.212
469	A	9	8	1.00	33	0.242
470	A	10	8	1.00	33	0.242
471	A	9	7	1.00	33	0.212
472	A	9	8	1.00	33	0.242
473	A	9	7	1.00	33	0.212
474	A	9	7	1.00	33	0.212
475	A	10	8	1.00	33	0.242
476	A	9	7	1.00	33	0.212
477	A	8	7	1.00	33	0.212
478	A	7	6	1.00	33	0.182
479	A	7	6	1.00	33	0.182
480	A	8	7	1.00	33	0.212

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	9	7	1.00	33	0.212
482	A	9	7	1.00	33	0.212
483	A	8	6	1.00	33	0.182
484	A	8	7	1.00	33	0.212
485	A	8	6	1.00	33	0.182
486	A	9	7	1.00	33	0.212
487	A	10	7	1.00	33	0.212
488	A	9	6	1.00	33	0.182
489	A	9	7	1.00	33	0.212
490	A	9	7	1.00	33	0.212
491	A	9	6	1.00	33	0.182
492	A	10	7	1.00	33	0.212
493	A	6	4	1.00	35	0.114
494	A	5	4	1.00	35	0.114
495	A	4	4	1.00	35	0.114
496	A	3	3	1.00	35	0.086
497	A	4	4	1.00	35	0.114
498	A	4	4	1.00	35	0.114
499	A	5	5	1.00	35	0.143
500	A	6	5	1.00	35	0.143
501	A	7	5	1.00	35	0.143
502	A	6	5	1.00	35	0.143
503	A	5	5	1.00	35	0.143
504	A	4	4	1.00	35	0.114
505	A	5	5	1.00	35	0.143
506	A	5	5	1.00	35	0.143
507	A	5	5	1.00	35	0.143
508	A	6	6	1.00	35	0.171
509	A	7	6	1.00	35	0.171
510	A	8	5	1.00	35	0.143
511	A	7	5	1.00	35	0.143
512	A	6	5	1.00	35	0.143
513	A	5	4	1.00	35	0.114
514	A	6	5	1.00	35	0.143
515	A	6	5	1.00	35	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	6	6	1.00	35	0.171
517	A	6	5	1.00	35	0.143
518	A	7	6	1.00	35	0.171
519	A	8	6	1.00	35	0.171
520	A	9	5	1.00	35	0.143
521	A	8	5	1.00	35	0.143
522	A	7	5	1.00	35	0.143
523	A	6	5	1.00	35	0.143
524	A	5	5	1.00	35	0.143
525	A	6	6	1.00	35	0.171
526	A	7	7	1.00	35	0.200
527	A	8	7	1.00	35	0.200
528	A	7	7	1.00	54	0.130
529	A	9	6	1.00	35	0.171
530	A	8	6	1.00	35	0.171
531	A	7	6	1.00	35	0.171
532	A	6	6	1.00	35	0.171
533	A	5	5	1.00	35	0.143
534	A	7	7	1.00	35	0.200
535	A	8	8	1.00	35	0.229
536	A	9	6	1.00	35	0.171
537	A	8	6	1.00	35	0.171
538	A	7	6	1.00	35	0.171
539	A	6	5	1.00	35	0.143
540	A	6	6	1.00	35	0.171
541	A	8	7	1.00	35	0.200
542	A	9	8	1.00	35	0.229
543	A	9	6	1.00	35	0.171
544	A	8	6	1.00	35	0.171
545	A	7	5	1.00	35	0.143
546	A	7	6	1.00	35	0.171
547	A	7	6	1.00	35	0.171
548	A	9	7	1.00	35	0.200
549	A	10	8	1.00	35	0.229
550	A	9	7	1.00	31	0.226

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	8	7	1.00	31	0.226
552	A	7	6	1.00	31	0.194
553	A	7	6	1.00	31	0.194
554	A	8	7	1.00	31	0.226
555	A	9	7	1.00	31	0.226
556	A	9	8	1.00	33	0.242
557	A	8	7	1.00	33	0.212
558	A	8	7	1.00	33	0.212
559	A	8	7	1.00	33	0.212
560	A	9	8	1.00	33	0.242
561	A	10	9	1.00	33	0.273
562	A	9	8	1.00	33	0.242
563	A	9	8	1.00	33	0.242
564	A	9	8	1.00	33	0.242
565	A	9	8	1.00	33	0.242
566	A	10	9	1.00	33	0.273
567	A	11	10	1.00	33	0.303
568	A	8	8	1.00	33	0.242
569	A	6	6	1.00	33	0.182
570	A	8	8	1.00	33	0.242
571	A	10	9	1.00	33	0.273
572	A	12	10	1.00	33	0.303
573	A	11	10	1.00	33	0.303
574	A	10	9	1.00	33	0.273
575	A	10	9	1.00	33	0.273
576	A	10	9	1.00	33	0.273
577	A	11	10	1.00	33	0.303
578	A	12	11	1.00	33	0.333
579	A	11	10	1.00	33	0.303
580	A	11	10	1.00	33	0.303
581	A	11	10	1.00	33	0.303
582	A	11	10	1.00	33	0.303
583	A	12	11	1.00	33	0.333
584	A	4	4	1.00	36	0.111
585	A	4	4	1.00	36	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
586	A	3	3	1.00	36	0.083
587	A	3	3	1.00	36	0.083
588	A	4	4	1.00	36	0.111
589	A	4	4	1.00	36	0.111
590	A	7	6	1.00	35	0.171
591	A	6	6	1.00	35	0.171
592	A	5	5	1.00	35	0.143
593	A	6	6	1.00	35	0.171
594	A	7	7	1.00	35	0.200
595	A	8	8	1.00	35	0.229
596	A	9	9	1.00	35	0.257
597	A	8	6	1.00	35	0.171
598	A	7	6	1.00	35	0.171
599	A	6	6	1.00	35	0.171
600	A	7	7	1.00	35	0.200
601	A	8	8	1.00	35	0.229
602	A	8	8	1.00	35	0.229
603	A	9	9	1.00	35	0.257
604	A	10	9	1.00	35	0.257
605	A	9	7	1.00	35	0.200
606	A	8	7	1.00	35	0.200
607	A	7	7	1.00	35	0.200
608	A	8	8	1.00	35	0.229
609	A	9	9	1.00	35	0.257
610	A	9	9	1.00	35	0.257
611	A	9	9	1.00	35	0.257
612	A	10	9	1.00	35	0.257
613	A	11	9	1.00	35	0.257
614	A	6	6	1.00	35	0.171
615	A	5	5	1.00	35	0.143
616	A	4	4	1.00	35	0.114
617	A	4	4	1.00	35	0.114
618	A	8	8	1.00	35	0.229
619	A	8	8	1.00	35	0.229
620	A	6	6	1.00	35	0.171

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	5	5	1.00	35	0.143
622	A	5	5	1.00	35	0.143
623	A	7	7	1.00	35	0.200
624	A	8	8	1.00	35	0.229
625	A	7	6	1.00	35	0.171
626	A	6	6	1.00	35	0.171
627	A	6	6	1.00	35	0.171
628	A	6	6	1.00	35	0.171
629	A	8	8	1.00	35	0.229
630	A	9	9	1.00	35	0.257
631	A	5	5	1.00	38	0.132
632	A	3	3	1.00	38	0.079
633	A	3	3	1.00	38	0.079
634	A	10	10	1.00	38	0.263
635	A	0	0	0.00	0	0.000
636	A	10	8	1.00	33	0.242
637	A	9	7	1.00	33	0.212
638	A	8	6	1.00	33	0.182
639	A	7	5	1.00	31	0.161
640	A	10	8	1.00	33	0.242
641	A	0	0	0.00	0	0.000
642	A	0	0	0.00	0	0.000
643	A	0	0	0.00	0	0.000
644	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=125

$$\frac{3}{8}a(A+B)x+\frac{a(5A+4B)\sin(c+dx)}{5d}+\frac{3a(A+B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a(A+B)\cos^3(c+dx)\sin(c+dx)}{4d}$$

[Out] 3/8*a*(A+B)*x+1/5*a*(5*A+4*B)*sin(d*x+c)/d+3/8*a*(A+B)*cos(d*x+c)*sin(d*x+c)/d+1/4*a*(A+B)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*a*B*cos(d*x+c)^4*sin(d*x+c)/d-1/15*a*(5*A+4*B)*sin(d*x+c)^3/d

Rubi [A]
time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3047, 3102, 2827, 2713, 2715, 8}

$$-\frac{a(5A+4B)\sin^3(c+dx)}{15d}+\frac{a(5A+4B)\sin(c+dx)}{5d}+\frac{a(A+B)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a(A+B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}ax(A+B)+\frac{aB\sin(c+dx)\cos^4(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]

[Out] (3*a*(A + B)*x)/8 + (a*(5*A + 4*B)*Sin[c + d*x])/(5*d) + (3*a*(A + B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(A + B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (a*B*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(5*A + 4*B)*Sin[c + d*x]^3)/(15*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx &= \int \cos^3(c+dx)\left(aA+(aA+aB)\cos(c+dx)\right. \\
&= \frac{aB\cos^4(c+dx)\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^3(c+d \\
&= \frac{aB\cos^4(c+dx)\sin(c+dx)}{5d}+(a(A+B))\int \mathrm{c} \\
&= \frac{a(A+B)\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{aB\cos^4(\\
&= \frac{a(5A+4B)\sin(c+dx)}{5d}+\frac{3a(A+B)\cos(c+}{8d} \\
&= \frac{3}{8}a(A+B)x+\frac{a(5A+4B)\sin(c+dx)}{5d}+\frac{3a(}{}
\end{aligned}$$

Mathematica [A]
time = 0.29, size = 110, normalized size = 0.88

$$\frac{a(180Ac+180Bc+180Adx+180Bdx+60(8A+5B)\sin(c+dx)-160A\sin^3(c+dx)+120(A+B)\sin(2(c+dx))+50B\sin(3(c+dx))+15A\sin(4(c+dx))+15B\sin(4(c+dx))+6B\sin(5(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(180*A*c + 180*B*c + 180*A*d*x + 180*B*d*x + 60*(8*A + 5*B)*Sin[c + d*x]
 - 160*A*Sin[c + d*x]^3 + 120*(A + B)*Sin[2*(c + d*x)] + 50*B*Sin[3*(c + d*
x)] + 15*A*Sin[4*(c + d*x)] + 15*B*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)])
)/(480*d)
```

Maple [A]

time = 0.17, size = 128, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{aB\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+aA\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+aB\left(\frac{\left(\cos^3(dx+c)+\right.}{}\right.}{d}$
default	$\frac{\frac{aB\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+aA\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+aB\left(\frac{\left(\cos^3(dx+c)+\right.}{}\right.}{d}$
risch	$\frac{3axA}{8}+\frac{3aBx}{8}+\frac{3\sin(dx+c)aA}{4d}+\frac{5aB\sin(dx+c)}{8d}+\frac{aB\sin(5dx+5c)}{80d}+\frac{\sin(4dx+4c)aA}{32d}+\frac{\sin(4dx+4c)aB}{32d}+$
norman	$\frac{\frac{3a(A+B)x}{8}+\frac{15a(A+B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{15a(A+B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{15a(A+B)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{15a(A+B)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*a*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a*B*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*A*(cos(d*x+c)^2+2)*sin(d*x+c)
)
```

Maxima [A]

time = 0.26, size = 124, normalized size = 0.99

$$-\frac{160\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa-15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)Aa-32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Ba-15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)Ba}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/480*(160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 15*(12*d*x + 12*c + sin
(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 32*(3*sin(d*x + c)^5 - 10*sin(d*x
 + c)^3 + 15*sin(d*x + c))*B*a - 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*s
in(2*d*x + 2*c))*B*a)/d
```

Fricas [A]
time = 0.41, size = 88, normalized size = 0.70

$$\frac{45\,(A+B)adx+\left(24\,Ba\cos\left(dx+c\right)^{4}+30\,(A+B)a\cos\left(dx+c\right)^{3}+8\,(5\,A+4\,B)a\cos\left(dx+c\right)^{2}+45\,(A+B)a\cos\left(dx+c\right)+16\,(5\,A+4\,B)a\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/120*(45*(A + B)*a*d*x + (24*B*a*cos(d*x + c)^4 + 30*(A + B)*a*cos(d*x + c
)^3 + 8*(5*A + 4*B)*a*cos(d*x + c)^2 + 45*(A + B)*a*cos(d*x + c) + 16*(5*A
+ 4*B)*a)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(117) = 234$.
time = 0.34, size = 333, normalized size = 2.66

$$\begin{cases} \frac{3Aax\sin^{4}(c+dx)}{8}+\frac{3Aax\sin^{2}(c+dx)\cos^{2}(c+dx)}{4}+\frac{3Aax\cos^{4}(c+dx)}{8}+\frac{3Aa\sin^{3}(c+dx)\cos(c+dx)}{8d}+\frac{2Aa\sin^{3}(c+dx)}{3d}+\frac{5Aa\sin(c+dx)\cos^{3}(c+dx)}{8d}+\frac{Aa\sin(c+dx)\cos^{2}(c+dx)}{d}+\frac{3Bax\sin^{4}(c+dx)}{8}+\frac{3Bax\sin^{2}(c+dx)\cos^{2}(c+dx)}{4}+\frac{3Bax\cos^{4}(c+dx)}{8}+\frac{8Ba\sin^{5}(c+dx)}{15d}+\frac{4Ba\sin^{3}(c+dx)\cos^{2}(c+dx)}{3d}+\frac{3Ba\sin^{3}(c+dx)\cos(c+dx)}{8d}+\frac{Ba\sin(c+dx)\cos^{4}(c+dx)}{d}+\frac{5Ba\sin(c+dx)\cos^{3}(c+dx)}{8d} & \text{for } d\neq 0 \\ x(A+B\cos(c))\,(a\cos(c)+a)\cos^{3}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a*x*sin(c + d*x)**4/8 + 3*A*a*x*sin(c + d*x)**2*cos(c + d*x)
**2/4 + 3*A*a*x*cos(c + d*x)**4/8 + 3*A*a*sin(c + d*x)**3*cos(c + d*x)/(8*d
) + 2*A*a*sin(c + d*x)**3/(3*d) + 5*A*a*sin(c + d*x)*cos(c + d*x)**3/(8*d)
+ A*a*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a*x*sin(c + d*x)**4/8 + 3*B*a*x*
sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B*a*x*cos(c + d*x)**4/8 + 8*B*a*sin(c
 + d*x)**5/(15*d) + 4*B*a*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a*sin
(c + d*x)**3*cos(c + d*x)/(8*d) + B*a*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*
a*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c
) + a)*cos(c)**3, True))
```

Giac [A]
time = 0.46, size = 112, normalized size = 0.90

$$\frac{3}{8}\,(Aa+Ba)x+\frac{Ba\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{(Aa+Ba)\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{(4\,Aa+5\,Ba)\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{(Aa+Ba)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{(6\,Aa+5\,Ba)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 3/8*(A*a + B*a)*x + 1/80*B*a*sin(5*d*x + 5*c)/d + 1/32*(A*a + B*a)*sin(4*d*
x + 4*c)/d + 1/48*(4*A*a + 5*B*a)*sin(3*d*x + 3*c)/d + 1/4*(A*a + B*a)*sin(
2*d*x + 2*c)/d + 1/8*(6*A*a + 5*B*a)*sin(d*x + c)/d
```

Mupad [B]
time = 1.55, size = 236, normalized size = 1.89

$$\frac{\left(\frac{3\,A\,a}{4}+\frac{3\,B\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{29\,A\,a}{6}+\frac{13\,B\,a}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{20\,A\,a}{3}+\frac{116\,B\,a}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{35\,A\,a}{6}+\frac{19\,B\,a}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{13\,A\,a}{4}+\frac{13\,B\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{3\,a\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)\,\left(A+B\right)}{4\,d}+\frac{3\,a\,\mathrm{atan}\left(\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A+B\right)}{4\,\left(\frac{3\,A\,a}{4}+\frac{3\,B\,a}{4}\right)}\right)\,\left(A+B\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((13*A*a)/4 + (13*B*a)/4) + tan(c/2 + (d*x)/2)^9*((3*A*
a)/4 + (3*B*a)/4) + tan(c/2 + (d*x)/2)^7*((29*A*a)/6 + (13*B*a)/6) + tan(c/
2 + (d*x)/2)^3*((35*A*a)/6 + (19*B*a)/6) + tan(c/2 + (d*x)/2)^5*((20*A*a)/3
 + (116*B*a)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10
*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)
) - (3*a*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(A + B))/(4*d) + (3*a*atan((3
*a*tan(c/2 + (d*x)/2)*(A + B))/(4*((3*A*a)/4 + (3*B*a)/4)))*(A + B))/(4*d)
```

3.2 $\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{1}{8}a(4A+3B)x+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(4A+3B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{aB\cos^3(c+dx)\sin(c+dx)}{4d}-$$

```
[Out] 1/8*a*(4*A+3*B)*x+a*(A+B)*sin(d*x+c)/d+1/8*a*(4*A+3*B)*cos(d*x+c)*sin(d*x+c
)/d+1/4*a*B*cos(d*x+c)^3*sin(d*x+c)/d-1/3*a*(A+B)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3047, 3102, 2827, 2715, 8, 2713}

$$-\frac{a(A+B)\sin^3(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(4A+3B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+3B)+\frac{aB\sin(c+dx)\cos^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(4*A + 3*B)*x)/8 + (a*(A + B)*Sin[c + d*x])/d + (a*(4*A + 3*B)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(A +
 B)*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx &= \int \cos^2(c+dx)\left(aA+(aA+aB)\cos(c+dx)+\right. \\ &= \frac{aB\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^2(c+dx) \\ &= \frac{aB\cos^3(c+dx)\sin(c+dx)}{4d}+(a(A+B))\int \mathrm{co} \\ &= \frac{a(4A+3B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{aB\cos^3(}{} \\ &= \frac{1}{8}a(4A+3B)x+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(4A}{}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 75, normalized size = 0.77

$$\frac{a(48Ac+36Bc+48Adx+36Bdx+96(A+B)\sin(c+dx)-32(A+B)\sin^3(c+dx)+24(A+B)\sin(2(c+dx))+3B\sin(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
[Out] (a*(48*A*c + 36*B*c + 48*A*d*x + 36*B*d*x + 96*(A + B)*Sin[c + d*x] - 32*(A
 + B)*Sin[c + d*x]^3 + 24*(A + B)*Sin[2*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))
/(96*d)
```

Maple [A]
time = 0.13, size = 107, normalized size = 1.10

method	result
derivativedivides	$\frac{aB\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{aA\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+\frac{aB\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+aA\left(\right.}{d}$
default	$\frac{aB\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{aA\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+\frac{aB\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+aA\left(\right.}{d}$
risch	$\frac{axA}{2}+\frac{3aBx}{8}+\frac{3\sin(dx+c)aA}{4d}+\frac{3aB\sin(dx+c)}{4d}+\frac{\sin(4dx+4c)aB}{32d}+\frac{\sin(3dx+3c)aA}{12d}+\frac{\sin(3dx+3c)aB}{12d}+$
norman	$\frac{\frac{a(4A+3B)x}{8}+\frac{a(4A+3B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a(4A+3B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3a(4A+3B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{a(4A+3B)x\left(\tan^6\left(\frac{dx}{2}+\right.\right.}{2}}{\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a
*A*(cos(d*x+c)^2+2)*sin(d*x+c)+1/3*a*B*(cos(d*x+c)^2+2)*sin(d*x+c)+a*A*(1/2
*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.26, size = 101, normalized size = 1.04

$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Aa-24\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa+32\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Ba-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)Ba}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 24*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a - 3*(12*d*x + 12*
c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a)/d
```

Fricas [A]
time = 0.37, size = 74, normalized size = 0.76

$$\frac{3\left(4\,A+3\,B\right)adx+\left(6\,Ba\cos\left(dx+c\right)^{3}+8\left(A+B\right)a\cos\left(dx+c\right)^{2}+3\left(4\,A+3\,B\right)a\cos\left(dx+c\right)+16\left(A+B\right)a\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(4*A + 3*B)*a*d*x + (6*B*a*cos(d*x + c)^3 + 8*(A + B)*a*cos(d*x + c
)^2 + 3*(4*A + 3*B)*a*cos(d*x + c) + 16*(A + B)*a)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(88) = 176$.
time = 0.21, size = 252, normalized size = 2.60

$$\begin{cases} \frac{Aax\sin^2(c+dx)}{2} + \frac{Aax\cos^2(c+dx)}{2} + \frac{2Aa\sin^3(c+dx)}{3d} + \frac{Aa\sin(c+dx)\cos^2(c+dx)}{d} + \frac{Aa\sin(c+dx)\cos(c+dx)}{2d} + \frac{3Bax\sin^4(c+dx)}{8} + \frac{3Bax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Bax\cos^4(c+dx)}{8} + \frac{3Ba\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2Ba\sin^3(c+dx)}{3d} + \frac{5Ba\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{Ba\sin(c+dx)\cos^2(c+dx)}{d} & \text{for } d \neq 0 \\ x(A + B\cos(c))(a\cos(c) + a)\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + 2*A*a*sin(c
+ d*x)**3/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**2/d + A*a*sin(c + d*x)*cos
(c + d*x)/(2*d) + 3*B*a*x*sin(c + d*x)**4/8 + 3*B*a*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 + 3*B*a*x*cos(c + d*x)**4/8 + 3*B*a*sin(c + d*x)**3*cos(c + d*
x)/(8*d) + 2*B*a*sin(c + d*x)**3/(3*d) + 5*B*a*sin(c + d*x)*cos(c + d*x)**3
/(8*d) + B*a*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(
a*cos(c) + a)*cos(c)**2, True))
```

Giac [A]
time = 0.48, size = 89, normalized size = 0.92

$$\frac{1}{8}(4\,Aa + 3\,Ba)x + \frac{Ba\sin(4\,dx + 4\,c)}{32\,d} + \frac{(Aa + Ba)\sin(3\,dx + 3\,c)}{12\,d} + \frac{(Aa + Ba)\sin(2\,dx + 2\,c)}{4\,d} + \frac{3\,(Aa + Ba)\sin(dx + c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/8*(4*A*a + 3*B*a)*x + 1/32*B*a*sin(4*d*x + 4*c)/d + 1/12*(A*a + B*a)*sin(
3*d*x + 3*c)/d + 1/4*(A*a + B*a)*sin(2*d*x + 2*c)/d + 3/4*(A*a + B*a)*sin(d
*x + c)/d
```

Mupad [B]
time = 1.23, size = 212, normalized size = 2.19

$$\frac{\left(A\,a + \frac{3\,B\,a}{4}\right)\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^7 + \left(\frac{7\,A\,a}{3} + \frac{49\,B\,a}{12}\right)\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^5 + \left(\frac{13\,A\,a}{3} + \frac{31\,B\,a}{12}\right)\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3 + \left(3\,A\,a + \frac{13\,B\,a}{4}\right)\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^8 + 4\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^6 + 6\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4 + 4\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)} + \frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\,(4\,A + 3\,B)}{4\,\left(A\,a + \frac{3\,B\,a}{4}\right)}\right)\,(4\,A + 3\,B)}{4\,d} - \frac{a\,(4\,A + 3\,B)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right) - \frac{d\,x}{2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(3*A*a + (13*B*a)/4) + tan(c/2 + (d*x)/2)^7*(A*a + (3*B
*a)/4) + tan(c/2 + (d*x)/2)^3*((13*A*a)/3 + (31*B*a)/12) + tan(c/2 + (d*x)/
2)^5*((7*A*a)/3 + (49*B*a)/12))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d
*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (a*atan((a
*tan(c/2 + (d*x)/2)*(4*A + 3*B))/(4*(A*a + (3*B*a)/4)))*(4*A + 3*B))/(4*d)
 - (a*(4*A + 3*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4*d)
```

3.3 $\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=77

$$\frac{1}{2}a(A+B)x+\frac{a(3A+2B)\sin(c+dx)}{3d}+\frac{a(A+B)\cos(c+dx)\sin(c+dx)}{2d}+\frac{aB\cos^2(c+dx)\sin(c+dx)}{3d}$$

```
[Out] 1/2*a*(A+B)*x+1/3*a*(3*A+2*B)*sin(d*x+c)/d+1/2*a*(A+B)*cos(d*x+c)*sin(d*x+c
)/d+1/3*a*B*cos(d*x+c)^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{3047, 3102, 2813\}$

$$\frac{a(3A+2B)\sin(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+B)+\frac{aB\sin(c+dx)\cos^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(A + B)*x)/2 + (a*(3*A + 2*B)*Sin[c + d*x])/(3*d) + (a*(A + B)*Cos[c + d
*x]*Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos(c+dx)\left(aA+(aA+aB)\cos(c+dx)+a\right.$$
$$= \frac{aB\cos^2(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}\int \cos(c+dx)(a$$
$$= \frac{1}{2}a(A+B)x+\frac{a(3A+2B)\sin(c+dx)}{3d}+\frac{a(A+}{}$$

Mathematica [A]
time = 0.21, size = 65, normalized size = 0.84

$$\frac{a(6Ac+6Bc+6Adx+6Bdx+3(4A+3B)\sin(c+dx)+3(A+B)\sin(2(c+dx))+B\sin(3(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(6*A*c + 6*B*c + 6*A*d*x + 6*B*d*x + 3*(4*A + 3*B)*Sin[c + d*x] + 3*(A +
 B)*Sin[2*(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A]
time = 0.11, size = 85, normalized size = 1.10

method	result
derivativedivides	$\frac{\frac{aB\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+aA\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aB\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aA\sin(dx+c)}{d}$
default	$\frac{\frac{aB\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+aA\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aB\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aA\sin(dx+c)}{d}$
risch	$\frac{axA}{2}+\frac{aBx}{2}+\frac{\sin(dx+c)aA}{d}+\frac{3aB\sin(dx+c)}{4d}+\frac{\sin(3dx+3c)aB}{12d}+\frac{\sin(2dx+2c)aA}{4d}+\frac{aB\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{(A+B)a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a(A+B)x}{2}+\frac{3a(A+B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3a(A+B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{a(A+B)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{4a(3A}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a*B*(cos(d*x+c)^2+2)*sin(d*x+c)+a*A*(1/2*sin(d*x+c)*cos(d*x+c)+1/2
*d*x+1/2*c)+a*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+a*A*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 79, normalized size = 1.03

$$\frac{3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa-4\,\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ba+3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba+12\,Aa\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a - 4*(sin(d*x + c)^3 - 3*sin(d*
x + c))*B*a + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 12*A*a*sin(d*x + c))
/d
```

Fricas [A]
time = 0.35, size = 56, normalized size = 0.73

$$\frac{3\,(A+B)adx+\left(2\,Ba\cos\left(dx+c\right)^{2}+3\,(A+B)a\cos\left(dx+c\right)+2\,(3\,A+2\,B)a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/6*(3*(A + B)*a*d*x + (2*B*a*cos(d*x + c)^2 + 3*(A + B)*a*cos(d*x + c) + 2
*(3*A + 2*B)*a)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(70) = 140$.
time = 0.14, size = 168, normalized size = 2.18

$$\begin{cases}\frac{Aax\sin^{2}(c+dx)}{2}+\frac{Aax\cos^{2}(c+dx)}{2}+\frac{Aa\sin(c+dx)\cos(c+dx)}{2d}+\frac{Aa\sin(c+dx)}{d}+\frac{Bax\sin^{2}(c+dx)}{2}+\frac{Bax\cos^{2}(c+dx)}{2}+\frac{2Ba\sin^{3}(c+dx)}{3d}+\frac{Ba\sin(c+dx)\cos^{2}(c+dx)}{d}+\frac{Ba\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d\neq 0\\ x(A+B\cos(c))\,(a\cos(c)+a)\cos(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
d*x)*cos(c + d*x)/(2*d) + A*a*sin(c + d*x)/d + B*a*x*sin(c + d*x)**2/2 + B*
a*x*cos(c + d*x)**2/2 + 2*B*a*sin(c + d*x)**3/(3*d) + B*a*sin(c + d*x)*cos(
c + d*x)**2/d + B*a*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A + B*c
os(c))*(a*cos(c) + a)*cos(c), True))
```

Giac [A]
time = 0.48, size = 68, normalized size = 0.88

$$\frac{1}{2}\,(Aa+Ba)x+\frac{Ba\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{(Aa+Ba)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{(4\,Aa+3\,Ba)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(A*a + B*a)*x + 1/12*B*a*sin(3*d*x + 3*c)/d + 1/4*(A*a + B*a)*sin(2*d*x
 + 2*c)/d + 1/4*(4*A*a + 3*B*a)*sin(d*x + c)/d
```

Mupad [B]
time = 0.23, size = 84, normalized size = 1.09

$$\frac{A\,a\,x}{2}+\frac{B\,a\,x}{2}+\frac{A\,a\,\sin\left(c+d\,x\right)}{d}+\frac{3\,B\,a\,\sin\left(c+d\,x\right)}{4\,d}+\frac{A\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] (A*a*x)/2 + (B*a*x)/2 + (A*a*sin(c + d*x))/d + (3*B*a*sin(c + d*x))/(4*d) +
 (A*a*sin(2*c + 2*d*x))/(4*d) + (B*a*sin(2*c + 2*d*x))/(4*d) + (B*a*sin(3*c
 + 3*d*x))/(12*d)
```

3.4 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=47

$$\frac{1}{2}a(2A+B)x+\frac{a(A+B)\sin(c+dx)}{d}+\frac{aB\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*(2*A+B)*x+a*(A+B)*sin(d*x+c)/d+1/2*a*B*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2813}

$$\frac{a(A+B)\sin(c+dx)}{d}+\frac{1}{2}ax(2A+B)+\frac{aB\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(2*A + B)*x)/2 + (a*(A + B)*Sin[c + d*x])/d + (a*B*Cos[c + d*x]*Sin[c +
d*x])/(2*d)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\,dx=\frac{1}{2}a(2A+B)x+\frac{a(A+B)\sin(c+dx)}{d}+\frac{aB\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A]
time = 0.12, size = 44, normalized size = 0.94

$$\frac{a(2Bc+4Adx+2Bdx+4(A+B)\sin(c+dx)+B\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(2*B*c + 4*A*d*x + 2*B*d*x + 4*(A + B)*Sin[c + d*x] + B*Sin[2*(c + d*x)]
))/(4*d)
```

Maple [A]
time = 0.08, size = 57, normalized size = 1.21

method	result	s
risch	$axA+\frac{aBx}{2}+\frac{\sin(dx+c)aA}{d}+\frac{aB\sin(dx+c)}{d}+\frac{aB\sin(2dx+2c)}{4d}$	5
derivativedivides	$\frac{aB\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aA\sin(dx+c)+aB\sin(dx+c)+aA(dx+c)}{d}$	5
default	$\frac{aB\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aA\sin(dx+c)+aB\sin(dx+c)+aA(dx+c)}{d}$	5
norman	$\frac{\frac{a(2A+B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+a(2A+B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{a(2A+3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a(2A+B)x}{2}+\frac{a(2A+B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+a*A*sin(d*x+c)+a*B*sin(d
*x+c)+a*A*(d*x+c))
```

Maxima [A]
time = 0.26, size = 55, normalized size = 1.17

$$\frac{4\,(dx+c)Aa+(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba+4\,Aa\sin\left(dx+c\right)+4\,Ba\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 4*A*a*sin(d*x
 + c) + 4*B*a*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 38, normalized size = 0.81

$$\frac{(2\,A+B)adx+(Ba\cos\left(dx+c\right)+2\,(A+B)a)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((2*A + B)*a*d*x + (B*a*cos(d*x + c) + 2*(A + B)*a)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.
time = 0.08, size = 94, normalized size = 2.00

$$\begin{cases} Aax + \frac{Aa\sin(c+dx)}{d} + \frac{Bax\sin^2(c+dx)}{2} + \frac{Bax\cos^2(c+dx)}{2} + \frac{Ba\sin(c+dx)\cos(c+dx)}{2d} + \frac{Ba\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x(A + B\cos(c))(a\cos(c) + a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x + A*a*sin(c + d*x)/d + B*a*x*sin(c + d*x)**2/2 + B*a*x*cos
(c + d*x)**2/2 + B*a*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*sin(c + d*x)/d,
Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a), True))
```

Giac [A]
time = 0.42, size = 45, normalized size = 0.96

$$\frac{1}{2}(2Aa + Ba)x + \frac{Ba\sin(2dx + 2c)}{4d} + \frac{(Aa + Ba)\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a + B*a)*x + 1/4*B*a*sin(2*d*x + 2*c)/d + (A*a + B*a)*sin(d*x + c)
/d
```

Mupad [B]
time = 0.19, size = 50, normalized size = 1.06

$$A\,a\,x + \frac{B\,a\,x}{2} + \frac{A\,a\,\sin(c + d\,x)}{d} + \frac{B\,a\,\sin(c + d\,x)}{d} + \frac{B\,a\,\sin(2\,c + 2\,d\,x)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] A*a*x + (B*a*x)/2 + (A*a*sin(c + d*x))/d + (B*a*sin(c + d*x))/d + (B*a*sin(
2*c + 2*d*x))/(4*d)
```

3.5 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=32

$$a(A+B)x+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\sin(c+dx)}{d}$$

[Out] a*(A+B)*x+a*A*arctanh(sin(d*x+c))/d+a*B*sin(d*x+c)/d

Rubi [A]
time = 0.07, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.148$, Rules used = {3047, 3102, 2814, 3855}

$$ax(A+B)+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]

[Out] a*(A + B)*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*Sin[c + d*x])/d

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))(A + B\cos(c+dx))\sec(c+dx)\,dx = \int (aA + (aA+aB)\cos(c+dx) + aB\cos^2(c+$$

$$= \frac{aB\sin(c+dx)}{d} + \int (aA + a(A+B)\cos(c+dx$$

$$= a(A+B)x + \frac{aB\sin(c+dx)}{d} + (aA)\int \sec(c+$$

$$= a(A+B)x + \frac{aA\tanh^{-1}(\sin(c+dx))}{d} + \frac{aB\sin}{}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 1.44

$$aAx + aBx + \frac{aA\tanh^{-1}(\sin(c+dx))}{d} + \frac{aB\cos(dx)\sin(c)}{d} + \frac{aB\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
[Out] a*A*x + a*B*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*Cos[d*x]*Sin[c])/d + (
a*B*Cos[c]*Sin[d*x])/d
```

Maple [A]
time = 0.15, size = 48, normalized size = 1.50

method	result
derivativedivides	$\frac{aA(dx+c)+aB\sin(dx+c)+aA\ln(\sec(dx+c)+\tan(dx+c))+aB(dx+c)}{d}$
default	$\frac{aA(dx+c)+aB\sin(dx+c)+aA\ln(\sec(dx+c)+\tan(dx+c))+aB(dx+c)}{d}$
risch	$axA + aBx - \frac{iaB\,e^{i(dx+c)}}{2d} + \frac{iaB\,e^{-i(dx+c)}}{2d} + \frac{aA\ln\left(e^{i(dx+c)}+i\right)}{d} - \frac{aA\ln\left(e^{i(dx+c)}-i\right)}{d}$
norman	$\frac{(aA+aB)x+(aA+aB)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2aA+2aB)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2aB\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2aB\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} + \frac{aA}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*A*(d*x+c)+a*B*sin(d*x+c)+a*A*ln(sec(d*x+c)+tan(d*x+c))+a*B*(d*x+c))
```

Maxima [A]
time = 0.26, size = 47, normalized size = 1.47

$$\frac{(dx+c)Aa+(dx+c)Ba+Aa\log(\sec(dx+c)+\tan(dx+c))+Ba\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima
")
```

```
[Out] ((d*x + c)*A*a + (d*x + c)*B*a + A*a*log(sec(d*x + c) + tan(d*x + c)) + B*a
*sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 51, normalized size = 1.59

$$\frac{2(A+B)adx+Aa\log(\sin(dx+c)+1)-Aa\log(-\sin(dx+c)+1)+2Ba\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(A + B)*a*d*x + A*a*log(sin(d*x + c) + 1) - A*a*log(-sin(d*x + c) +
1) + 2*B*a*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec(c+dx)\,dx+\int A\cos(c+dx)\sec(c+dx)\,dx+\int B\cos(c+dx)\sec(c+dx)\,dx+\int B\cos^2(c+dx)\sec(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a*(Integral(A*sec(c + d*x), x) + Integral(A*cos(c + d*x)*sec(c + d*x), x) +
 Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(B*cos(c + d*x)**2*sec(
c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(32) = 64.
time = 0.45, size = 79, normalized size = 2.47

$$\frac{Aa\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-Aa\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+(Aa+Ba)(dx+c)+\frac{2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

```
[Out] (A*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - A*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (A*a + B*a)*(d*x + c) + 2*B*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

Mupad [B]
time = 0.28, size = 100, normalized size = 3.12

$$\frac{B\,a\,\sin\left(c+d\,x\right)}{d}+\frac{2\,A\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x),x)
```

```
[Out] (B*a*sin(c + d*x))/d + (2*A*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d + (2*A*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.6 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=32

$$aBx+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}$$

```
[Out] a*B*x+a*(A+B)*arctanh(sin(d*x+c))/d+a*A*tan(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.138$, Rules used = {3047, 3100, 2814, 3855}

$$\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+aBx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] a*B*x + (a*(A + B)*ArcTanh[Sin[c + d*x]])/d + (a*A*Tan[c + d*x])/d
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int (aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+$$
$$= \frac{aA\tan(c+dx)}{d} + \int (a(A+B)+aB\cos(c+$$
$$= aBx + \frac{aA\tan(c+dx)}{d} + (a(A+B))\int \sec(c$$
$$= aBx + \frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d} + \frac{aA\,\mathrm{ta}}{}$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 1.34

$$aBx + \frac{aA\tanh^{-1}(\sin(c+dx))}{d} + \frac{aB\tanh^{-1}(\sin(c+dx))}{d} + \frac{aA\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
[Out] a*B*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*
A*Tan[c + d*x])/d
```

Maple [A]
time = 0.17, size = 57, normalized size = 1.78

method	result
derivativedivides	$\frac{aA\ln(\sec(dx+c)+\tan(dx+c))+aB(dx+c)+aA\tan(dx+c)+aB\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{aA\ln(\sec(dx+c)+\tan(dx+c))+aB(dx+c)+aA\tan(dx+c)+aB\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$aBx+\frac{2iaA}{d(\mathrm{e}^{2i(dx+c)}+1)}+\frac{aA\ln(\mathrm{e}^{i(dx+c)}+i)}{d}+\frac{a\ln(\mathrm{e}^{i(dx+c)}+i)B}{d}-\frac{aA\ln(\mathrm{e}^{i(dx+c)}-i)}{d}-\frac{a\ln(\mathrm{e}^{i(dx+c)}-i)B}{d}$
norman	$\frac{aBx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+aBx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-aBx-\frac{2aA\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4aA\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2aA\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-aBx(\mathrm{t}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*A*ln(sec(d*x+c)+tan(d*x+c))+a*B*(d*x+c)+a*A*tan(d*x+c)+a*B*ln(sec(d*
x+c)+tan(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.
time = 0.27, size = 73, normalized size = 2.28

$$\frac{2\,(dx+c)Ba + Aa(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + Ba(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2\,Aa\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(d*x + c)*B*a + A*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*A*a*tan(d*x + c))
/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.
time = 0.36, size = 79, normalized size = 2.47

$$\frac{2\,Badx\cos(dx+c) + (A+B)a\cos(dx+c)\log(\sin(dx+c)+1) - (A+B)a\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,Aa\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fric
as")
```

```
[Out] 1/2*(2*B*a*d*x*cos(d*x + c) + (A + B)*a*cos(d*x + c)*log(sin(d*x + c) + 1)
- (A + B)*a*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*A*a*sin(d*x + c))/(d*co
s(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^2(c+dx)\,dx + \int A\cos(c+dx)\sec^2(c+dx)\,dx + \int B\cos(c+dx)\sec^2(c+dx)\,dx + \int B\cos^2(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**2, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**2
, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x
)**2*sec(c + d*x)**2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(32) = 64$.
time = 0.48, size = 84, normalized size = 2.62

$$\frac{(dx+c)Ba + (Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - (Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac
")
```

```
[Out] ((d*x + c)*B*a + (A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + B*
a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B]
time = 0.31, size = 100, normalized size = 3.12

$$\frac{A\,a\,\tan(c+d\,x)}{d}+\frac{2\,A\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] (A*a*tan(c + d*x))/d + (2*A*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d + (2*B*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a*atanh(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.7 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=56

$$\frac{a(A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{aA\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*a*(A+2*B)*arctanh(sin(d*x+c))/d+a*(A+B)*tan(d*x+c)/d+1/2*a*A*sec(d*x+c)
*tan(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3047, 3100, 2827, 3852, 8, 3855}

$$\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*(A + 2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(A + B)*Tan[c + d*x])/d + (a
*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx &= \int (aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+\\ &= \frac{aA\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (2a(A+B)\\ &= \frac{aA\sec(c+dx)\tan(c+dx)}{2d}+(a(A+B))\int \mathrm{se}\\ &= \frac{a(A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\sec(c+\\ &= \frac{a(A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\,\mathrm{ta}}{d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 1.34

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+\frac{aB\tan(c+dx)}{d}+\frac{aA\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/(2*d) + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Ta
n[c + d*x])/d + (a*B*Tan[c + d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Maple [A]
time = 0.19, size = 75, normalized size = 1.34

method	result
derivativedivides	$\frac{aA\tan(dx+c)+aB\ln(\sec(dx+c)+\tan(dx+c))+aA\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+aB\tan(dx+c)}{d}$
default	$\frac{aA\tan(dx+c)+aB\ln(\sec(dx+c)+\tan(dx+c))+aA\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+aB\tan(dx+c)}{d}$
risch	$-\frac{ia\left(A\,e^{3i(dx+c)}-2A\,e^{2i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+c)}-2A-2B\right)}{d\left(e^{2i(dx+c)}+1\right)^{2}}+\frac{aA\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)B}{d}-$
norman	$\frac{\frac{a(A-2B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a(3A+2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a(5A+2B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a(A+2B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{a(A+2B)\ln\left(\tan\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*A*tan(d*x+c)+a*B*ln(sec(d*x+c)+tan(d*x+c))+a*A*(1/2*sec(d*x+c)*tan(d
*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a*B*tan(d*x+c))
```

Maxima [A]
time = 0.26, size = 95, normalized size = 1.70

$$-\frac{Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-2\,Ba(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))-4\,Aa\tan\left(dx+c\right)-4\,Ba\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/4*(A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 2*B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 4*A*a*tan(d*x + c) - 4*B*a*tan(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 89, normalized size = 1.59

$$\frac{(A+2\,B)a\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-(A+2\,B)a\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\left(A+B\right)a\cos\left(dx+c\right)+Aa\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fric
as")
```

```
[Out] 1/4*((A + 2*B)*a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A + 2*B)*a*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 2*(2*(A + B)*a*cos(d*x + c) + A*a)*sin(d*x
 + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^3{(c+dx)}\,dx+\int A\cos{(c+dx)}\sec^3{(c+dx)}\,dx+\int B\cos{(c+dx)}\sec^3{(c+dx)}\,dx+\int B\cos^2{(c+dx)}\sec^3{(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**3, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**3
, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(B*cos(c + d*x
)**2*sec(c + d*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(52) = 104.
time = 0.48, size = 124, normalized size = 2.21

$$\frac{(Aa+2Ba)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(Aa+2Ba)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac
")
```

```
[Out] 1/2*((A*a + 2*B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + 2*B*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*tan(1
/2*d*x + 1/2*c)^3 - 3*A*a*tan(1/2*d*x + 1/2*c) - 2*B*a*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 0.83, size = 94, normalized size = 1.68

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,A\,a+2\,B\,a\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(A\,a+2\,B\,a\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A+2\,B\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(3*A*a + 2*B*a) - tan(c/2 + (d*x)/2)^3*(A*a + 2*B*a))/(
d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (a*atanh(tan(c/2 +
 (d*x)/2))*(A + 2*B))/d
```

3.8 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=86

$$\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(2A+3B)\tan(c+dx)}{3d}+\frac{a(A+B)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}$$

```
[Out] 1/2*a*(A+B)*arctanh(sin(d*x+c))/d+1/3*a*(2*A+3*B)*tan(d*x+c)/d+1/2*a*(A+B)*
sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3047, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{a(2A+3B)\tan(c+dx)}{3d}+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a*(A + B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*A + 3*B)*Tan[c + d*x])/(3*d
) + (a*(A + B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= \int \left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+\right.\\
&= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (3a(A+B\\
&= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}+(a(A+B))\int \mathrm{s}\\
&= \frac{a(A+B)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aA\sec^2(c}{}\\
&= \frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(2A+3B)\,\mathrm{t}}{3a}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 56, normalized size = 0.65

$$\frac{a\left(3(A+B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(6(A+B)+3(A+B)\sec(c+dx)+2A\tan^2(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a*(3*(A + B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*(A + B) + 3*(A + B)*S
ec[c + d*x] + 2*A*Tan[c + d*x]^2)))/(6*d)
```

Maple [A]
time = 0.23, size = 105, normalized size = 1.22

method	result
derivativedivides	$\frac{aA\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+aB\tan(dx+c)-aA\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+aB\left(\frac{\sec(dx+c)\ \text{t}}{2}\right.}{d}$
default	$\frac{aA\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+aB\tan(dx+c)-aA\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+aB\left(\frac{\sec(dx+c)\ \text{t}}{2}\right.}{d}$
norman	$\frac{-\frac{4a(A-3B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a(A-3B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a(7A+3B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{3(A+B)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{(A+B)a\left(\tan^9\left(\frac{c}{}\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$
risch	$-\frac{ia\left(3A\,\mathrm{e}^{5i(dx+c)}+3B\,\mathrm{e}^{5i(dx+c)}-6B\,\mathrm{e}^{4i(dx+c)}-12A\,\mathrm{e}^{2i(dx+c)}-12B\,\mathrm{e}^{2i(dx+c)}-3A\,\mathrm{e}^{i(dx+c)}-3B\,\mathrm{e}^{i(dx+c)}-4A-6B\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a*B*tan(
d*x+c)-a*A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a*B*(1/2*sec(d*x+c)*tan(d*x+c
)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.26, size = 127, normalized size = 1.48

$$\frac{4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa-3\,Aa\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ba\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,Ba\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a - 3*A*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*a*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*B*a*tan(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 105, normalized size = 1.22

$$\frac{3\left(A+B\right)a\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(A+B\right)a\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\left(2\,A+3\,B\right)a\cos\left(dx+c\right)^2+3\left(A+B\right)a\cos\left(dx+c\right)+2\,Aa\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(A + B)*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(A + B)*a*cos(d*
x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(2*A + 3*B)*a*cos(d*x + c)^2 + 3*(A
+ B)*a*cos(d*x + c) + 2*A*a)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^4(c+dx)\,dx+\int A\cos(c+dx)\sec^4(c+dx)\,dx+\int B\cos(c+dx)\sec^4(c+dx)\,dx+\int B\cos^2(c+dx)\sec^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**4, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**4
, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**4, x) + Integral(B*cos(c + d*x
)**2*sec(c + d*x)**4, x))
```

Giac [A]
time = 0.43, size = 154, normalized size = 1.79

$$\frac{3\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-4\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-12\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac
")
```

```
[Out] 1/6*(3*(A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(A*a + B*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a*tan(1/2*d*x + 1/2*c)^5 + 3*B*a*tan
(1/2*d*x + 1/2*c)^5 - 4*A*a*tan(1/2*d*x + 1/2*c)^3 - 12*B*a*tan(1/2*d*x + 1
/2*c)^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 9*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2
*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 2.07, size = 126, normalized size = 1.47

$$\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A+B\right)}{d}-\frac{\left(A\,a+B\,a\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(-\frac{4\,A\,a}{3}-4\,B\,a\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(3\,A\,a+3\,B\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] (a*atanh(tan(c/2 + (d*x)/2))*(A + B))/d - (tan(c/2 + (d*x)/2)*(3*A*a + 3*B*
a) + tan(c/2 + (d*x)/2)^5*(A*a + B*a) - tan(c/2 + (d*x)/2)^3*((4*A*a)/3 + 4
*B*a))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x
)/2)^6 - 1))
```

3.9 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=106

$$\frac{a(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(3A+4B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aA\sec^3(c+$$

```
[Out] 1/8*a*(3*A+4*B)*arctanh(sin(d*x+c))/d+a*(A+B)*tan(d*x+c)/d+1/8*a*(3*A+4*B)*
sec(d*x+c)*tan(d*x+c)/d+1/4*a*A*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*(A+B)*tan(d
*x+c)^3/d
```

Rubi [A]
time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3047, 3100, 2827, 3852, 3853, 3855}

$$\frac{a(A+B)\tan^3(c+dx)}{3d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3A+4B)\tan(c+dx)\sec(c+dx)}{8d}+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a*(3*A + 4*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(A + B)*Tan[c + d*x])/d +
(a*(3*A + 4*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*A*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (a*(A + B)*Tan[c + d*x]^3)/(3*d)
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
```

```
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx &= \int (aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+\\ &= \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (4a(A+B\\ &= \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+(a(A+B))\int \mathrm{s}\\ &= \frac{a(3A+4B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aA\sec^3}{}\\ &= \frac{a(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(A+B)\,\mathrm{t}}{}\end{aligned}$$

Mathematica [A]
time = 0.44, size = 77, normalized size = 0.73

$$\frac{a\left(3(3A+4B)\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\left(9A+12B+8(A+B)(2+\cos(2(c+dx)))\sec(c+dx)+6A\sec^2(c+dx)\right)\tan(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a*(3*(3*A + 4*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(9*A + 12*B + 8*(A +
 B)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*A*Sec[c + d*x]^2)*Tan[c + d*x])
)/(24*d)
```

Maple [A]
time = 0.27, size = 131, normalized size = 1.24

method	result
derivativedivides	$\frac{-aA\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+aB\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+aA\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec}{}\right.\right.}{d}$
default	$\frac{-aA\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+aB\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+aA\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec}{}\right.\right.}{d}$
norman	$\frac{-\frac{a(3A+4B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a(13A-20B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{a(13A+12B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{a(29A-4B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{a(31A+4}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$
risch	$-\frac{ia\left(9A\,e^{7i(dx+c)}+12B\,e^{7i(dx+c)}+33A\,e^{5i(dx+c)}+12B\,e^{5i(dx+c)}-48A\,e^{4i(dx+c)}-48B\,e^{4i(dx+c)}-33A\,e^{3i(dx+c)}-12B\,e^{3}\right.}{12d\left(e^{2i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a*B*(1/2*sec(d*x+c)*tan(d*x+c)
+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a*A*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*ta
n(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-a*B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x
+c))
```

Maxima [A]
time = 0.27, size = 163, normalized size = 1.54

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba-3Aa\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-12Ba\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*a - 3*A*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.37, size = 127, normalized size = 1.20

$$\frac{3(3A+4B)a\cos(dx+c)^4\log(\sin(dx+c)+1)-3(3A+4B)a\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(16(A+B)a\cos(dx+c)^3+3(3A+4B)a\cos(dx+c)^2+8(A+B)a\cos(dx+c)+6Aa\right)\sin(dx+c)}{48d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(3*A + 4*B)*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A + 4*B)*
a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(A + B)*a*cos(d*x + c)^3 +
3*(3*A + 4*B)*a*cos(d*x + c)^2 + 8*(A + B)*a*cos(d*x + c) + 6*A*a)*sin(d*x
+ c))/(d*cos(d*x + c)^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^{5}\left(c+dx\right)dx+\int A\cos\left(c+dx\right)\sec^{5}\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec^{5}\left(c+dx\right)dx+\int B\cos^{2}\left(c+dx\right)\sec^{5}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**5, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**5
, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**5, x) + Integral(B*cos(c + d*x
)**2*sec(c + d*x)**5, x))
```

Giac [**A**]
time = 0.48, size = 188, normalized size = 1.77

$$\frac{3\left(3\,Aa+4\,Ba\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(3\,Aa+4\,Ba\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-49\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-28\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+31\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+52\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-39\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-36\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac
")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a + 4*B
*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9*A*a*tan(1/2*d*x + 1/2*c)^7 +
12*B*a*tan(1/2*d*x + 1/2*c)^7 - 49*A*a*tan(1/2*d*x + 1/2*c)^5 - 28*B*a*tan(
1/2*d*x + 1/2*c)^5 + 31*A*a*tan(1/2*d*x + 1/2*c)^3 + 52*B*a*tan(1/2*d*x + 1
/2*c)^3 - 39*A*a*tan(1/2*d*x + 1/2*c) - 36*B*a*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [**B**]
time = 2.66, size = 166, normalized size = 1.57

$$\frac{\left(-\frac{3\,A\,a}{4}-B\,a\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{49\,A\,a}{12}+\frac{7\,B\,a}{3}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{31\,A\,a}{12}-\frac{13\,B\,a}{3}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{13\,A\,a}{4}+3\,B\,a\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{a\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(3\,A+4\,B\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((13*A*a)/4 + 3*B*a) - tan(c/2 + (d*x)/2)^7*((3*A*a)/4
+ B*a) - tan(c/2 + (d*x)/2)^3*((31*A*a)/12 + (13*B*a)/3) + tan(c/2 + (d*x)/
2)^5*((49*A*a)/12 + (7*B*a)/3))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d
*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (a*atanh(t
an(c/2 + (d*x)/2))*(3*A + 4*B))/(4*d)
```

3.10 $\int \cos^3(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=191

$$\frac{1}{16}a^2(12A+11B)x+\frac{a^2(9A+8B)\sin(c+dx)}{5d}+\frac{a^2(12A+11B)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^2(12A+11B)\cos}{2}$$

```
[Out] 1/16*a^2*(12*A+11*B)*x+1/5*a^2*(9*A+8*B)*sin(d*x+c)/d+1/16*a^2*(12*A+11*B)*
cos(d*x+c)*sin(d*x+c)/d+1/24*a^2*(12*A+11*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/30
*a^2*(6*A+7*B)*cos(d*x+c)^4*sin(d*x+c)/d+1/6*B*cos(d*x+c)^4*(a^2+a^2*cos(d*
x+c))*sin(d*x+c)/d-1/15*a^2*(9*A+8*B)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.21, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3055, 3047, 3102, 2827, 2713, 2715, 8}

$$-\frac{a^2(9A+8B)\sin^3(c+dx)}{15d}+\frac{a^2(9A+8B)\sin(c+dx)}{5d}+\frac{a^2(6A+7B)\sin(c+dx)\cos^4(c+dx)}{30d}+\frac{a^2(12A+11B)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a^2(12A+11B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}a^2x(12A+11B)+\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(12*A + 11*B)*x)/16 + (a^2*(9*A + 8*B)*Sin[c + d*x])/(5*d) + (a^2*(12*
A + 11*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(12*A + 11*B)*Cos[c + d*
x]^3*Sin[c + d*x])/(24*d) + (a^2*(6*A + 7*B)*Cos[c + d*x]^4*Sin[c + d*x])/(
30*d) + (B*Cos[c + d*x]^4*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(6*d) - (a
^2*(9*A + 8*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx &= \frac{B\cos^4(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+d}{6d} \\
&= \frac{B\cos^4(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+d}{6d} \\
&= \frac{a^2(6A+7B)\cos^4(c+dx)\sin(c+dx)}{30d}+\frac{B\,\mathrm{co}}{} \\
&= \frac{a^2(6A+7B)\cos^4(c+dx)\sin(c+dx)}{30d}+\frac{B\,\mathrm{co}}{} \\
&= \frac{a^2(12A+11B)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a^2}{} \\
&= \frac{a^2(9A+8B)\sin(c+dx)}{5d}+\frac{a^2(12A+11B)\,\mathrm{co}}{} \\
&= \frac{1}{16}a^2(12A+11B)x+\frac{a^2(9A+8B)\sin(c+dx}{5d}
\end{aligned}
$$

Mathematica [A]
time = 0.69, size = 134, normalized size = 0.70

$$
\frac{a^2(660Bc+720Adx+660Bdx+120(11A+10B)\sin(c+dx)+15(32A+31B)\sin(2(c+dx))+180A\sin(3(c+dx))+200B\sin(3(c+dx))+60A\sin(4(c+dx))+75B\sin(4(c+dx))+12A\sin(5(c+dx))+24B\sin(5(c+dx))+5B\sin(6(c+dx)))}{960d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(660*B*c + 720*A*d*x + 660*B*d*x + 120*(11*A + 10*B)*Sin[c + d*x] + 15
*(32*A + 31*B)*Sin[2*(c + d*x)] + 180*A*Sin[3*(c + d*x)] + 200*B*Sin[3*(c +
 d*x)] + 60*A*Sin[4*(c + d*x)] + 75*B*Sin[4*(c + d*x)] + 12*A*Sin[5*(c + d*
x)] + 24*B*Sin[5*(c + d*x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A]
time = 0.21, size = 217, normalized size = 1.14

method	result
risch	$\frac{3a^2xA}{4}+\frac{11a^2Bx}{16}+\frac{11\sin(dx+c)a^2A}{8d}+\frac{5\sin(dx+c)B\,a^2}{4d}+\frac{B\,a^2\sin(6dx+6c)}{192d}+\frac{\sin(5dx+5c)a^2A}{80d}+\frac{\sin(5dx+}{40}$
derivativedivides	$\frac{\frac{a^2A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+B\,a^2\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{}$

default	$\frac{\frac{a^2 A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+B\,a^2\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+}{}$
norman	$\frac{a^2(12A+11B)x}{16}+\frac{17a^2(12A+11B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{a^2(12A+11B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{3a^2(12A+11B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{15a^2(12A}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*a^2*(1/6*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+2
*a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/5*B*a
^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+1/3*a^2*A*(cos(d*x+c)^2+2
)*sin(d*x+c)+B*a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/
8*c))
```

Maxima [A]
time = 0.26, size = 216, normalized size = 1.13

$$\frac{64\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^2-320\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2+60\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Aa^2+128\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Ba^2-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-48\sin(2dx+2c)\right)Ba^2+30\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Ba^2}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 -
320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 + 60*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 + 128*(3*sin(d*x + c)^5 - 10*sin(d*x +
c)^3 + 15*sin(d*x + c))*B*a^2 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*a^2 + 30*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2)/d
```

Fricas [A]
time = 0.35, size = 130, normalized size = 0.68

$$\frac{15(12A+11B)a^2dx+\left(40Ba^2\cos(dx+c)^5+48(A+2B)a^2\cos(dx+c)^4+10(12A+11B)a^2\cos(dx+c)^3+16(9A+8B)a^2\cos(dx+c)^2+15(12A+11B)a^2\cos(dx+c)+32(9A+8B)a^2\right)\sin(dx+c)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(12*A + 11*B)*a^2*d*x + (40*B*a^2*cos(d*x + c)^5 + 48*(A + 2*B)*a
^2*cos(d*x + c)^4 + 10*(12*A + 11*B)*a^2*cos(d*x + c)^3 + 16*(9*A + 8*B)*a^
```

```
2*cos(d*x + c)^2 + 15*(12*A + 11*B)*a^2*cos(d*x + c) + 32*(9*A + 8*B)*a^2)*
sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(173) = 346$.
time = 0.51, size = 600, normalized size = 3.14

$$\begin{cases} \frac{3Aa^2x\sin^4(c+dx)}{4} + \frac{3Aa^2x\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{3Aa^2x\cos^4(c+dx)}{4} + \frac{8Aa^2\sin^5(c+dx)}{15d} + \frac{4Aa^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{3Aa^2\sin^3(c+dx)\cos(c+dx)}{4d} + \frac{2Aa^2\sin^3(c+dx)}{3d} + \frac{Aa^2\sin(c+dx)\cos^4(c+dx)}{d} + \frac{5Aa^2\sin(c+dx)\cos^3(c+dx)}{4d} + \frac{Aa^2\sin(c+dx)\cos^2(c+dx)}{d} + \frac{5Ba^2x\sin^6(c+dx)}{16} + \frac{15Ba^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3Ba^2x\sin^4(c+dx)}{8} + \frac{15Ba^2x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3Ba^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{5Ba^2x\cos^6(c+dx)}{16} + \frac{3Ba^2x\cos^4(c+dx)}{8} + \frac{5Ba^2\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{16Ba^2\sin^5(c+dx)}{15d} + \frac{5Ba^2\sin^3(c+dx)\cos^3(c+dx)}{6d} + \frac{8Ba^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{3Ba^2\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{11Ba^2\sin(c+dx)\cos^5(c+dx)}{16d} + \frac{2Ba^2\sin(c+dx)\cos^4(c+dx)}{d} + \frac{5Ba^2\sin(c+dx)\cos^3(c+dx)}{8d} & \text{for } d \neq 0 \\ x(A+B\cos(c))(a\cos(c)+a)^2\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**2*x*sin(c + d*x)**4/4 + 3*A*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/2 + 3*A*a**2*x*cos(c + d*x)**4/4 + 8*A*a**2*sin(c + d*x)**5/(15*d
) + 4*A*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*A*a**2*sin(c + d*x)*
*3*cos(c + d*x)/(4*d) + 2*A*a**2*sin(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x
)*cos(c + d*x)**4/d + 5*A*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + A*a**2*
sin(c + d*x)*cos(c + d*x)**2/d + 5*B*a**2*x*sin(c + d*x)**6/16 + 15*B*a**2*
x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*B*a**2*x*sin(c + d*x)**4/8 + 15*B*
a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*B*a**2*x*sin(c + d*x)**2*cos(
c + d*x)**2/4 + 5*B*a**2*x*cos(c + d*x)**6/16 + 3*B*a**2*x*cos(c + d*x)**4/
8 + 5*B*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 16*B*a**2*sin(c + d*x)**
5/(15*d) + 5*B*a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 8*B*a**2*sin(c
+ d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*
d) + 11*B*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 2*B*a**2*sin(c + d*x)*
cos(c + d*x)**4/d + 5*B*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)),
 (x*(A + B*cos(c))*(a*cos(c) + a)**2*cos(c)**3, True))
```

Giac [A]
time = 0.45, size = 166, normalized size = 0.87

$$\frac{Ba^2\sin(6\,dx+6\,c)}{192\,d} + \frac{1}{16}\left(12\,Aa^2+11\,Ba^2\right)x + \frac{\left(Aa^2+2\,Ba^2\right)\sin(5\,dx+5\,c)}{80\,d} + \frac{\left(4\,Aa^2+5\,Ba^2\right)\sin(4\,dx+4\,c)}{64\,d} + \frac{\left(9\,Aa^2+10\,Ba^2\right)\sin(3\,dx+3\,c)}{48\,d} + \frac{\left(32\,Aa^2+31\,Ba^2\right)\sin(2\,dx+2\,c)}{64\,d} + \frac{\left(11\,Aa^2+10\,Ba^2\right)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*B*a^2*sin(6*d*x + 6*c)/d + 1/16*(12*A*a^2 + 11*B*a^2)*x + 1/80*(A*a^2
 + 2*B*a^2)*sin(5*d*x + 5*c)/d + 1/64*(4*A*a^2 + 5*B*a^2)*sin(4*d*x + 4*c)/
d + 1/48*(9*A*a^2 + 10*B*a^2)*sin(3*d*x + 3*c)/d + 1/64*(32*A*a^2 + 31*B*a^
2)*sin(2*d*x + 2*c)/d + 1/8*(11*A*a^2 + 10*B*a^2)*sin(d*x + c)/d
```

Mupad [B]
time = 1.59, size = 315, normalized size = 1.65

$$\frac{\left(\frac{3\,A\,a^2}{2}+\frac{11\,B\,a^2}{8}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{17\,A\,a^2}{2}+\frac{187\,B\,a^2}{24}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{107\,A\,a^2}{5}+\frac{331\,B\,a^2}{20}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{117\,A\,a^2}{5}+\frac{501\,B\,a^2}{20}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{31\,A\,a^2}{2}+\frac{87\,B\,a^2}{8}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{13\,A\,a^2}{2}+\frac{53\,B\,a^2}{8}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+6\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+20\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+6\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{a^2\,\left(12\,A+11\,B\right)\,\left(\mathrm{atan}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{8\,d}+\frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(12\,A+11\,B\right)}{8\,\left(\frac{3\,A\,a^2}{2}+\frac{11\,B\,a^2}{8}\right)}\right)\,\left(12\,A+11\,B\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((13*A*a^2)/2 + (53*B*a^2)/8) + tan(c/2 + (d*x)/2)^11*(
(3*A*a^2)/2 + (11*B*a^2)/8) + tan(c/2 + (d*x)/2)^3*((31*A*a^2)/2 + (87*B*a^
2)/8) + tan(c/2 + (d*x)/2)^9*((17*A*a^2)/2 + (187*B*a^2)/24) + tan(c/2 + (d
*x)/2)^7*((107*A*a^2)/5 + (331*B*a^2)/20) + tan(c/2 + (d*x)/2)^5*((117*A*a^
2)/5 + (501*B*a^2)/20))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^
4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2
)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - (a^2*(12*A + 11*B)*(atan(tan(c/2 + (d*
x)/2)) - (d*x)/2))/(8*d) + (a^2*atan((a^2*tan(c/2 + (d*x)/2)*(12*A + 11*B))
/(8*((3*A*a^2)/2 + (11*B*a^2)/8)))*(12*A + 11*B))/(8*d)
```

3.11 $\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=160

$$\frac{1}{8}a^2(7A+6B)x+\frac{a^2(10A+9B)\sin(c+dx)}{5d}+\frac{a^2(7A+6B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2(5A+6B)\cos^3(c+d}{20d}$$

```
[Out] 1/8*a^2*(7*A+6*B)*x+1/5*a^2*(10*A+9*B)*sin(d*x+c)/d+1/8*a^2*(7*A+6*B)*cos(d
*x+c)*sin(d*x+c)/d+1/20*a^2*(5*A+6*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*B*cos(d
*x+c)^3*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d-1/15*a^2*(10*A+9*B)*sin(d*x+c)^3/
d
```

Rubi [A]
time = 0.20, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3055, 3047, 3102, 2827, 2715, 8, 2713}

$$-\frac{a^2(10A+9B)\sin^3(c+dx)}{15d}+\frac{a^2(10A+9B)\sin(c+dx)}{5d}+\frac{a^2(5A+6B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{a^2(7A+6B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}a^2x(7A+6B)+\frac{B\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(7*A + 6*B)*x)/8 + (a^2*(10*A + 9*B)*Sin[c + d*x])/(5*d) + (a^2*(7*A +
 6*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(5*A + 6*B)*Cos[c + d*x]^3*Si
n[c + d*x])/(20*d) + (B*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x
])/(5*d) - (a^2*(10*A + 9*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{B\cos^3(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+d}{5d}$$
$$= \frac{B\cos^3(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+d}{5d}$$
$$= \frac{a^2(5A+6B)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{B\,\mathrm{co}}{}$$
$$= \frac{a^2(5A+6B)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{B\,\mathrm{co}}{}$$
$$= \frac{a^2(7A+6B)\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2(5A}{}$$
$$= \frac{1}{8}a^2(7A+6B)x + \frac{a^2(10A+9B)\sin(c+dx)}{5d}$$

Mathematica [A]
time = 0.47, size = 108, normalized size = 0.68

$$\frac{a^2(360Bc+420Adx+360Bdx+60(12A+11B)\sin(c+dx)+240(A+B)\sin(2(c+dx))+80A\sin(3(c+dx))+90B\sin(3(c+dx))+15A\sin(4(c+dx))+30B\sin(4(c+dx))+6B\sin(5(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
[Out] (a^2*(360*B*c + 420*A*d*x + 360*B*d*x + 60*(12*A + 11*B)*Sin[c + d*x] + 240
*(A + B)*Sin[2*(c + d*x)] + 80*A*Sin[3*(c + d*x)] + 90*B*Sin[3*(c + d*x)] +
 15*A*Sin[4*(c + d*x)] + 30*B*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)]))/(48
0*d)
```

Maple [A]
time = 0.17, size = 186, normalized size = 1.16

method	result
risch	$\frac{7a^2xA}{8}+\frac{3a^2Bx}{4}+\frac{3\sin(dx+c)a^2A}{2d}+\frac{11\sin(dx+c)B\,a^2}{8d}+\frac{\sin(5dx+5c)B\,a^2}{80d}+\frac{\sin(4dx+4c)a^2A}{32d}+\frac{\sin(4dx+4}{16d}$
derivativedivides	$\frac{a^2A\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{B\,a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+\frac{2a^2A\left(\cos^2(dx+\right.}{3}}{}$
default	$\frac{a^2A\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{B\,a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+\frac{2a^2A\left(\cos^2(dx+\right.}{3}}{}$

norman	$\frac{a^2(7A+6B)x}{8}+\frac{7a^2(7A+6B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{a^2(7A+6B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a^2(7A+6B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{5a^2(7A+6B)x\left(\tan^4\left(\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5
*B*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2/3*a^2*A*(cos(d*x+c)
^2+2)*sin(d*x+c)+2*B*a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*
d*x+3/8*c)+a^2*A*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+1/3*B*a^2*(cos(d
*x+c)^2+2)*sin(d*x+c))
```

Maxima [**A**]
time = 0.27, size = 178, normalized size = 1.11

$$-\frac{320\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Aa^{2}-15\left(12dx+12c+\sin\left(4dx+4c\right)+8\sin\left(2dx+2c\right)\right)Aa^{2}-120\left(2dx+2c+\sin\left(2dx+2c\right)\right)Aa^{2}-32\left(3\sin\left(dx+c\right)^{5}-10\sin\left(dx+c\right)^{3}+15\sin\left(dx+c\right)\right)Ba^{2}+160\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Ba^{2}-30\left(12dx+12c+\sin\left(4dx+4c\right)+8\sin\left(2dx+2c\right)\right)Ba^{2}}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/480*(320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 120*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*a^2 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*B*a^2 + 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 30*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2)/d
```

Fricas [**A**]
time = 0.36, size = 110, normalized size = 0.69

$$\frac{15\,(7\,A+6\,B)a^{2}dx+\left(24\,Ba^{2}\cos\left(dx+c\right)^{4}+30\,(A+2\,B)a^{2}\cos\left(dx+c\right)^{3}+8\,(10\,A+9\,B)a^{2}\cos\left(dx+c\right)^{2}+15\,(7\,A+6\,B)a^{2}\cos\left(dx+c\right)+16\,(10\,A+9\,B)a^{2}\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(7*A + 6*B)*a^2*d*x + (24*B*a^2*cos(d*x + c)^4 + 30*(A + 2*B)*a^2
*cos(d*x + c)^3 + 8*(10*A + 9*B)*a^2*cos(d*x + c)^2 + 15*(7*A + 6*B)*a^2*co
s(d*x + c) + 16*(10*A + 9*B)*a^2)*sin(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(144) = 288$.
time = 0.35, size = 459, normalized size = 2.87

$$\begin{cases}\frac{3Aa^{2}x\sin^{4}{\left(c+dx\right)}}{8}+\frac{3Aa^{2}x\sin^{2}{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{4}+\frac{Aa^{2}x\sin^{2}{\left(c+dx\right)}}{2}+\frac{3Aa^{2}x\cos^{4}{\left(c+dx\right)}}{8}+\frac{Aa^{2}x\cos^{2}{\left(c+dx\right)}}{2}+\frac{3Aa^{2}\sin^{3}{\left(c+dx\right)}\cos{\left(c+dx\right)}}{8d}+\frac{4Aa^{2}\sin^{3}{\left(c+dx\right)}}{3d}+\frac{5Aa^{2}\sin{\left(c+dx\right)}\cos^{3}{\left(c+dx\right)}}{8d}+\frac{2Aa^{2}\sin{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{d}+\frac{Aa^{2}\sin{\left(c+dx\right)}\cos{\left(c+dx\right)}}{2d}+\frac{3Ba^{2}x\sin^{4}{\left(c+dx\right)}}{4}+\frac{3Ba^{2}x\sin^{2}{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{2}+\frac{3Ba^{2}x\cos^{4}{\left(c+dx\right)}}{4}+\frac{8Ba^{2}\sin^{5}{\left(c+dx\right)}}{15d}+\frac{4Ba^{2}\sin^{3}{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{3d}+\frac{3Ba^{2}\sin^{3}{\left(c+dx\right)}\cos{\left(c+dx\right)}}{4d}+\frac{2Ba^{2}\sin^{3}{\left(c+dx\right)}}{3d}+\frac{Ba^{2}\sin{\left(c+dx\right)}\cos^{4}{\left(c+dx\right)}}{d}+\frac{5Ba^{2}\sin{\left(c+dx\right)}\cos^{3}{\left(c+dx\right)}}{4d}+\frac{Ba^{2}\sin{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{d} & \text{for}\: d \neq 0 \\x\left(A+B\cos{\left(c\right)}\right)\left(a\cos{\left(c\right)}+a\right)^{2}\cos^{2}{\left(c\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**2*x*sin(c + d*x)**4/8 + 3*A*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + A*a**2*x*sin(c + d*x)**2/2 + 3*A*a**2*x*cos(c + d*x)**4/8 + A
*a**2*x*cos(c + d*x)**2/2 + 3*A*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4
*A*a**2*sin(c + d*x)**3/(3*d) + 5*A*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d)
 + 2*A*a**2*sin(c + d*x)*cos(c + d*x)**2/d + A*a**2*sin(c + d*x)*cos(c + d*
x)/(2*d) + 3*B*a**2*x*sin(c + d*x)**4/4 + 3*B*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/2 + 3*B*a**2*x*cos(c + d*x)**4/4 + 8*B*a**2*sin(c + d*x)**5/(15*d
) + 4*B*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a**2*sin(c + d*x)*
*3*cos(c + d*x)/(4*d) + 2*B*a**2*sin(c + d*x)**3/(3*d) + B*a**2*sin(c + d*x
)*cos(c + d*x)**4/d + 5*B*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + B*a**2*
sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)
**2*cos(c)**2, True))
```

Giac [A]
time = 0.44, size = 137, normalized size = 0.86

$$\frac{Ba^2 \sin(5dx+5c)}{80d} + \frac{1}{8}\left(7Aa^2+6Ba^2\right)x + \frac{\left(Aa^2+2Ba^2\right)\sin(4dx+4c)}{32d} + \frac{\left(8Aa^2+9Ba^2\right)\sin(3dx+3c)}{48d} + \frac{\left(Aa^2+Ba^2\right)\sin(2dx+2c)}{2d} + \frac{\left(12Aa^2+11Ba^2\right)\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/80*B*a^2*sin(5*d*x + 5*c)/d + 1/8*(7*A*a^2 + 6*B*a^2)*x + 1/32*(A*a^2 + 2
*B*a^2)*sin(4*d*x + 4*c)/d + 1/48*(8*A*a^2 + 9*B*a^2)*sin(3*d*x + 3*c)/d +
1/2*(A*a^2 + B*a^2)*sin(2*d*x + 2*c)/d + 1/8*(12*A*a^2 + 11*B*a^2)*sin(d*x
+ c)/d
```

Mupad [B]
time = 1.50, size = 277, normalized size = 1.73

$$\frac{\left(\frac{7Aa^2}{4}+\frac{3Ba^2}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(\frac{49Aa^2}{6}+7Ba^2\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{40Aa^2}{3}+\frac{72Ba^2}{5}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{79Aa^2}{6}+9Ba^2\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(\frac{25Aa^2}{4}+\frac{13Ba^2}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)} - \frac{a^2(7A+6B)\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{4d} + \frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(7A+6B)}{4\left(\frac{7Aa^2}{4}+\frac{3Ba^2}{2}\right)}\right)(7A+6B)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((25*A*a^2)/4 + (13*B*a^2)/2) + tan(c/2 + (d*x)/2)^9*((
7*A*a^2)/4 + (3*B*a^2)/2) + tan(c/2 + (d*x)/2)^7*((49*A*a^2)/6 + 7*B*a^2) +
 tan(c/2 + (d*x)/2)^3*((79*A*a^2)/6 + 9*B*a^2) + tan(c/2 + (d*x)/2)^5*((40*
A*a^2)/3 + (72*B*a^2)/5))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2
)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)
^10 + 1)) - (a^2*(7*A + 6*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4*d) +
(a^2*atan((a^2*tan(c/2 + (d*x)/2)*(7*A + 6*B))/(4*((7*A*a^2)/4 + (3*B*a^2)/
2)))*(7*A + 6*B))/(4*d)
```

3.12 $\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{1}{8}a^2(8A+7B)x+\frac{a^2(8A+7B)\sin(c+dx)}{6d}+\frac{a^2(8A+7B)\cos(c+dx)\sin(c+dx)}{24d}+\frac{(4A-B)(a+a\cos(c+dx$$

```
[Out] 1/8*a^2*(8*A+7*B)*x+1/6*a^2*(8*A+7*B)*sin(d*x+c)/d+1/24*a^2*(8*A+7*B)*cos(d
*x+c)*sin(d*x+c)/d+1/12*(4*A-B)*(a+a*cos(d*x+c))^2*sin(d*x+c)/d+1/4*B*(a+a*
cos(d*x+c))^3*sin(d*x+c)/a/d
```

Rubi [A]

time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3047, 3102, 2830, 2723}

$$\frac{a^2(8A+7B)\sin(c+dx)}{6d}+\frac{a^2(8A+7B)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}a^2x(8A+7B)+\frac{(4A-B)\sin(c+dx)(a\cos(c+dx)+a)^2}{12d}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^3}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(8*A + 7*B)*x)/8 + (a^2*(8*A + 7*B)*Sin[c + d*x])/(6*d) + (a^2*(8*A +
7*B)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*A - B)*(a + a*Cos[c + d*x])^2*
Sin[c + d*x])/(12*d) + (B*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^2\left(A\cos(c+dx)+B\cos^2\right.$$

$$= \frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4ad} + \frac{\int(a+a\,c}{}$$

$$= \frac{(4A-B)(a+a\cos(c+dx))^2\sin(c+dx)}{12d} + \frac{B}{}$$

$$= \frac{1}{8}a^2(8A+7B)x + \frac{a^2(8A+7B)\sin(c+dx)}{6d} +$$

Mathematica [A]
time = 0.39, size = 86, normalized size = 0.67

$$\frac{a^2(84Bc+96Adx+84Bdx+24(7A+6B)\sin(c+dx)+48(A+B)\sin(2(c+dx))+8A\sin(3(c+dx))+16B\sin(3(c+dx))+3B\sin(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
[Out] (a^2*(84*B*c + 96*A*d*x + 84*B*d*x + 24*(7*A + 6*B)*Sin[c + d*x] + 48*(A +
B)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 16*B*Sin[3*(c + d*x)] + 3*B*Si
n[4*(c + d*x)]))/(96*d)
```

Maple [A]
time = 0.14, size = 154, normalized size = 1.19

method	result
risch	$a^2xA+\frac{7a^2Bx}{8}+\frac{7\sin(dx+c)a^2A}{4d}+\frac{3\sin(dx+c)B\,a^2}{2d}+\frac{\sin(4dx+4c)B\,a^2}{32d}+\frac{\sin(3dx+3c)a^2A}{12d}+\frac{\sin(3dx+3c}{6d}$
derivativedivides	$\frac{\frac{a^2A(\cos^2(dx+c)+2)\sin(dx+c)}{3}+B\,a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^2A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\right.}{d}$

default	$\frac{\frac{a^{2} A\left(\cos^{2}(dx+c)+2\right)\sin(dx+c)}{3}+B\,a^{2}\left(\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^{2} A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right.}{d}$
norman	$\frac{\frac{a^{2}(8A+7B)x}{8}+\frac{11a^{2}(8A+7B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{a^{2}(8A+7B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a^{2}(8A+7B)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3a^{2}(8A+7B)x\left(\tan^{4}\left(\right.\right.}{4}}{\left(1+\tan^{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^2*A*(cos(d*x+c)^2+2)*sin(d*x+c)+B*a^2*(1/4*(cos(d*x+c)^3+3/2*cos
(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^2*A*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d
*x+1/2*c)+2/3*B*a^2*(cos(d*x+c)^2+2)*sin(d*x+c)+a^2*A*sin(d*x+c)+B*a^2*(1/2
*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 144, normalized size = 1.12

$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Aa^{2}-48\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^{2}+64\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Ba^{2}-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)Ba^{2}-24\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba^{2}-96\,Aa^{2}\sin\left(dx+c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 48*(2*d*x + 2*c + sin(2
*d*x + 2*c))*A*a^2 + 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 3*(12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 - 24*(2*d*x + 2*c +
sin(2*d*x + 2*c))*B*a^2 - 96*A*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 90, normalized size = 0.70

$$\frac{3\,(8\,A+7\,B)a^{2}dx+\left(6\,Ba^{2}\cos\left(dx+c\right)^{3}+8\,(A+2\,B)a^{2}\cos\left(dx+c\right)^{2}+3\,(8\,A+7\,B)a^{2}\cos\left(dx+c\right)+8\,(5\,A+4\,B)a^{2}\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(8*A + 7*B)*a^2*d*x + (6*B*a^2*cos(d*x + c)^3 + 8*(A + 2*B)*a^2*cos
(d*x + c)^2 + 3*(8*A + 7*B)*a^2*cos(d*x + c) + 8*(5*A + 4*B)*a^2)*sin(d*x +
 c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(112) = 224$.
time = 0.24, size = 338, normalized size = 2.62

$$\begin{cases} Aa^{2}x\sin^{2}(c+dx)+Aa^{2}x\cos^{2}(c+dx)+\frac{2Aa^{2}\sin^{3}(c+dx)}{3d}+\frac{Aa^{2}\sin(c+dx)\cos^{2}(c+dx)}{d}+\frac{Aa^{2}\sin(c+dx)\cos(c+dx)}{d}+\frac{Aa^{2}\sin(c+dx)}{d}+\frac{3Ba^{2}x\sin^{4}(c+dx)}{8}+\frac{3Ba^{2}x\sin^{2}(c+dx)\cos^{2}(c+dx)}{4}+\frac{Ba^{2}x\sin^{2}(c+dx)}{2}+\frac{3Ba^{2}x\cos^{4}(c+dx)}{8}+\frac{Ba^{2}x\cos^{2}(c+dx)}{2}+\frac{3Ba^{2}\sin^{3}(c+dx)\cos(c+dx)}{8d}+\frac{4Ba^{2}\sin^{3}(c+dx)}{3d}+\frac{5Ba^{2}\sin(c+dx)\cos^{3}(c+dx)}{8d}+\frac{2Ba^{2}\sin(c+dx)\cos^{2}(c+dx)}{d}+\frac{Ba^{2}\sin(c+dx)\cos(c+dx)}{2d} & \text{for}\ d\neq 0 \\ x(A+B\cos(c))\,(a\cos(c)+a)^{2}\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2 + A*a**2*x*cos(c + d*x)**2 + 2*A*a**2*s
in(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)*cos(c + d*x)**2/d + A*a**2*sin(c
 + d*x)*cos(c + d*x)/d + A*a**2*sin(c + d*x)/d + 3*B*a**2*x*sin(c + d*x)**4
/8 + 3*B*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + B*a**2*x*sin(c + d*x)**
2/2 + 3*B*a**2*x*cos(c + d*x)**4/8 + B*a**2*x*cos(c + d*x)**2/2 + 3*B*a**2*
sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4*B*a**2*sin(c + d*x)**3/(3*d) + 5*B*a
**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 2*B*a**2*sin(c + d*x)*cos(c + d*x)
**2/d + B*a**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A + B*cos(c)
)*(a*cos(c) + a)**2*cos(c), True))
```

Giac [A]
time = 0.44, size = 110, normalized size = 0.85

$$\frac{Ba^2\sin(4dx+4c)}{32d}+\frac{1}{8}\left(8Aa^2+7Ba^2\right)x+\frac{(Aa^2+2Ba^2)\sin(3dx+3c)}{12d}+\frac{(Aa^2+Ba^2)\sin(2dx+2c)}{2d}+\frac{(7Aa^2+6Ba^2)\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/32*B*a^2*sin(4*d*x + 4*c)/d + 1/8*(8*A*a^2 + 7*B*a^2)*x + 1/12*(A*a^2 + 2
*B*a^2)*sin(3*d*x + 3*c)/d + 1/2*(A*a^2 + B*a^2)*sin(2*d*x + 2*c)/d + 1/4*(
7*A*a^2 + 6*B*a^2)*sin(d*x + c)/d
```

Mupad [B]
time = 0.29, size = 134, normalized size = 1.04

$$A\,a^2\,x+\frac{7\,B\,a^2\,x}{8}+\frac{7\,A\,a^2\,\sin(c+d\,x)}{4\,d}+\frac{3\,B\,a^2\,\sin(c+d\,x)}{2\,d}+\frac{A\,a^2\,\sin(2\,c+2\,d\,x)}{2\,d}+\frac{A\,a^2\,\sin(3\,c+3\,d\,x)}{12\,d}+\frac{B\,a^2\,\sin(2\,c+2\,d\,x)}{2\,d}+\frac{B\,a^2\,\sin(3\,c+3\,d\,x)}{6\,d}+\frac{B\,a^2\,\sin(4\,c+4\,d\,x)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

```
[Out] A*a^2*x + (7*B*a^2*x)/8 + (7*A*a^2*sin(c + d*x))/(4*d) + (3*B*a^2*sin(c + d
*x))/(2*d) + (A*a^2*sin(2*c + 2*d*x))/(2*d) + (A*a^2*sin(3*c + 3*d*x))/(12*
d) + (B*a^2*sin(2*c + 2*d*x))/(2*d) + (B*a^2*sin(3*c + 3*d*x))/(6*d) + (B*a
^2*sin(4*c + 4*d*x))/(32*d)
```

3.13 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=94

$$\frac{1}{2}a^2(3A+2B)x+\frac{2a^2(3A+2B)\sin(c+dx)}{3d}+\frac{a^2(3A+2B)\cos(c+dx)\sin(c+dx)}{6d}+\frac{B(a+a\cos(c+dx))^2\sin}{3d}$$

```
[Out] 1/2*a^2*(3*A+2*B)*x+2/3*a^2*(3*A+2*B)*sin(d*x+c)/d+1/6*a^2*(3*A+2*B)*cos(d*
x+c)*sin(d*x+c)/d+1/3*B*(a+a*cos(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2830, 2723}

$$\frac{2a^2(3A+2B)\sin(c+dx)}{3d}+\frac{a^2(3A+2B)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}a^2x(3A+2B)+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(3*A + 2*B)*x)/2 + (2*a^2*(3*A + 2*B)*Sin[c + d*x])/(3*d) + (a^2*(3*A
+ 2*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (B*(a + a*Cos[c + d*x])^2*Sin[c +
 d*x])/(3*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx=\frac{B(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}(3A+2B)\int (a+a\,c$$

$$=\frac{1}{2}a^2(3A+2B)x+\frac{2a^2(3A+2B)\sin(c+dx)}{3d}+\frac{a^2(3A+2B}{}$$

Mathematica [A]
time = 0.21, size = 61, normalized size = 0.65

$$\frac{a^2(18Adx+12Bdx+3(8A+7B)\sin(c+dx)+3(A+2B)\sin(2(c+dx))+B\sin(3(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(18*A*d*x + 12*B*d*x + 3*(8*A + 7*B)*Sin[c + d*x] + 3*(A + 2*B)*Sin[2*
(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A]
time = 0.10, size = 116, normalized size = 1.23

method	result
risch	$\frac{3a^2xA}{2}+a^2Bx+\frac{2\sin(dx+c)a^2A}{d}+\frac{7\sin(dx+c)B\,a^2}{4d}+\frac{\sin(3dx+3c)B\,a^2}{12d}+\frac{\sin(2dx+2c)a^2A}{4d}+\frac{\sin(2dx+2c}{2d}$
derivativedivides	$\frac{\frac{B\,a^2\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+a^2A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2B\,a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2A\sin(dx+c}{d}$
default	$\frac{\frac{B\,a^2\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+a^2A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2B\,a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2A\sin(dx+c}{d}$
norman	$\frac{\frac{a^2(3A+2B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2(5A+6B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a^2(3A+2B)x}{2}+\frac{8a^2(3A+2B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{3a^2(3A+2B)x\left(\tan^2\left(\frac{dx}{2}\right.\right.}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*B*a^2*(cos(d*x+c)^2+2)*sin(d*x+c)+a^2*A*(1/2*sin(d*x+c)*cos(d*x+c)
+1/2*d*x+1/2*c)+2*B*a^2*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+2*a^2*A*s
in(d*x+c)+B*a^2*sin(d*x+c)+a^2*A*(d*x+c))
```

Maxima [A]
time = 0.26, size = 110, normalized size = 1.17

$$\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2+12\,(dx+c)Aa^2-4\,(\sin(dx+c)^3-3\,\sin(dx+c))Ba^2+6\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2+24\,Aa^2\sin(dx+c)+12\,Ba^2\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 12*(d*x + c)*A*a^2 - 4*(si
n(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 + 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*B
*a^2 + 24*A*a^2*sin(d*x + c) + 12*B*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 70, normalized size = 0.74

$$\frac{3\,(3\,A+2\,B)a^2dx+\left(2\,Ba^2\cos\left(dx+c\right)^2+3\,(A+2\,B)a^2\cos\left(dx+c\right)+2\,(6\,A+5\,B)a^2\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(3*A + 2*B)*a^2*d*x + (2*B*a^2*cos(d*x + c)^2 + 3*(A + 2*B)*a^2*cos(
d*x + c) + 2*(6*A + 5*B)*a^2)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(85) = 170.
time = 0.17, size = 199, normalized size = 2.12

$$\begin{cases} \frac{Aa^2x\sin^2\left(c+dx\right)}{2}+\frac{Aa^2x\cos^2\left(c+dx\right)}{2}+Aa^2x+\frac{Aa^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{2Aa^2\sin\left(c+dx\right)}{d}+Ba^2x\sin^2\left(c+dx\right)+Ba^2x\cos^2\left(c+dx\right)+\frac{2Ba^2\sin^3\left(c+dx\right)}{3d}+\frac{Ba^2\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{Ba^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{Ba^2\sin\left(c+dx\right)}{d} & \text{for}\; d\neq 0 \\ x(A+B\cos\left(c\right))\left(a\cos\left(c\right)+a\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**2/2 + A*a**2
*x + A*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*a**2*sin(c + d*x)/d + B*a
**2*x*sin(c + d*x)**2 + B*a**2*x*cos(c + d*x)**2 + 2*B*a**2*sin(c + d*x)**3
/(3*d) + B*a**2*sin(c + d*x)*cos(c + d*x)**2/d + B*a**2*sin(c + d*x)*cos(c
+ d*x)/d + B*a**2*sin(c + d*x)/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) +
a)**2, True))
```

Giac [A]
time = 0.43, size = 85, normalized size = 0.90

$$\frac{Ba^2\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{1}{2}\left(3\,Aa^2+2\,Ba^2\right)x+\frac{\left(Aa^2+2\,Ba^2\right)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{\left(8\,Aa^2+7\,Ba^2\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*B*a^2*sin(3*d*x + 3*c)/d + 1/2*(3*A*a^2 + 2*B*a^2)*x + 1/4*(A*a^2 + 2*
B*a^2)*sin(2*d*x + 2*c)/d + 1/4*(8*A*a^2 + 7*B*a^2)*sin(d*x + c)/d
```

Mupad [B]
time = 0.23, size = 98, normalized size = 1.04

$$\frac{3\,A\,a^2\,x}{2}+B\,a^2\,x+\frac{2\,A\,a^2\,\sin\left(c+d\,x\right)}{d}+\frac{7\,B\,a^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{A\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{B\,a^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (3*A*a^2*x)/2 + B*a^2*x + (2*A*a^2*sin(c + d*x))/d + (7*B*a^2*sin(c + d*x))
/(4*d) + (A*a^2*sin(2*c + 2*d*x))/(4*d) + (B*a^2*sin(2*c + 2*d*x))/(2*d) +
(B*a^2*sin(3*c + 3*d*x))/(12*d)
```

3.14 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}a^2(4A+3B)x+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2(2A+3B)\sin(c+dx)}{2d}+\frac{B(a^2+a^2\cos(c+dx))\sin(c+dx)}{2d}$$

```
[Out] 1/2*a^2*(4*A+3*B)*x+a^2*A*arctanh(sin(d*x+c))/d+1/2*a^2*(2*A+3*B)*sin(d*x+c
)/d+1/2*B*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3055, 3047, 3102, 2814, 3855}

$$\frac{a^2(2A+3B)\sin(c+dx)}{2d}+\frac{1}{2}a^2x(4A+3B)+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^2*(4*A + 3*B)*x)/2 + (a^2*A*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*A + 3*B)*
Sin[c + d*x])/(2*d) + (B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
```

```
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx &= \frac{B(a^2+a^2\cos(c+dx))\sin(c+dx)}{2d}+\frac{1}{2}\int (a+c\\ &= \frac{B(a^2+a^2\cos(c+dx))\sin(c+dx)}{2d}+\frac{1}{2}\int (2a^2A\\ &= \frac{a^2(2A+3B)\sin(c+dx)}{2d}+\frac{B(a^2+a^2\cos(c+d}{2d}\\ &= \frac{1}{2}a^2(4A+3B)x+\frac{a^2(2A+3B)\sin(c+dx)}{2d}+\frac{B}{}\\ &= \frac{1}{2}a^2(4A+3B)x+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{a}{}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 96, normalized size = 1.17

$$\frac{a^2\left(8Adx+6Bdx-4A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+4(A+2B)\sin(c+dx)+B\sin(2(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^2*(8*A*d*x + 6*B*d*x - 4*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*
A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*(A + 2*B)*Sin[c + d*x] + B*S
in[2*(c + d*x)]))/(4*d)
```

Maple [A]
time = 0.17, size = 96, normalized size = 1.17

method	result
derivativedivides	$\frac{a^2 A \sin(dx+c)+B a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2 A(dx+c)+2B a^2 \sin(dx+c)+a^2 A \ln(\sec(dx+c)+\tan(dx+c))+}{d}$
default	$\frac{a^2 A \sin(dx+c)+B a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2 A(dx+c)+2B a^2 \sin(dx+c)+a^2 A \ln(\sec(dx+c)+\tan(dx+c))+}{d}$
risch	$2a^2xA+\frac{3a^2Bx}{2}-\frac{i e^{i(dx+c)}a^2 A}{2d}-\frac{i e^{i(dx+c)}B a^2}{d}+\frac{i e^{-i(dx+c)}a^2 A}{2d}+\frac{i e^{-i(dx+c)}B a^2}{d}+\frac{a^2 A \ln(e^{i(dx+c)}+i}{d}$
norman	$\frac{(2a^2A+\frac{3}{2}B a^2)x+(2a^2A+\frac{3}{2}B a^2)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(6a^2A+\frac{9}{2}B a^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(6a^2A+\frac{9}{2}B a^2)x\left(\tan^4\left(\frac{dx}{2}+\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*A*sin(d*x+c)+B*a^2*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+2*a^2
*A*(d*x+c)+2*B*a^2*sin(d*x+c)+a^2*A*ln(sec(d*x+c)+tan(d*x+c))+B*a^2*(d*x+c)
)
```

Maxima [A]
time = 0.26, size = 94, normalized size = 1.15

$$\frac{8\,(dx+c)Aa^2+(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2+4\,(dx+c)Ba^2+4\,Aa^2\log(\sec(dx+c)+\tan(dx+c))+4\,Aa^2\sin(dx+c)+8\,Ba^2\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(8*(d*x + c)*A*a^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 + 4*(d*x +
c)*B*a^2 + 4*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 4*A*a^2*sin(d*x + c)
+ 8*B*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 79, normalized size = 0.96

$$\frac{(4\,A+3\,B)a^2dx+Aa^2\log(\sin(dx+c)+1)-Aa^2\log(-\sin(dx+c)+1)+(Ba^2\cos(dx+c)+2\,(A+2\,B)a^2)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/2*((4*A + 3*B)*a^2*d*x + A*a^2*log(sin(d*x + c) + 1) - A*a^2*log(-sin(d*x
 + c) + 1) + (B*a^2*cos(d*x + c) + 2*(A + 2*B)*a^2)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int A\sec\left(c+dx\right)dx+\int 2A\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int A\cos^{2}\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int 2B\cos^{2}\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos^{3}\left(c+dx\right)\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x), x) + Integral(2*A*cos(c + d*x)*sec(c + d*x),
 x) + Integral(A*cos(c + d*x)**2*sec(c + d*x), x) + Integral(B*cos(c + d*x)
*sec(c + d*x), x) + Integral(2*B*cos(c + d*x)**2*sec(c + d*x), x) + Integra
l(B*cos(c + d*x)**3*sec(c + d*x), x))
```

Giac [A]
time = 0.45, size = 145, normalized size = 1.77

$$\frac{2\,Aa^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Aa^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\left(4\,Aa^{2}+3\,Ba^{2}\right)\left(dx+c\right)+\frac{2\left(2\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/2*(2*A*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*A*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (4*A*a^2 + 3*B*a^2)*(d*x + c) + 2*(2*A*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 3*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*a^2*tan(1/2*d*x + 1/2*c)
+ 5*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 0.34, size = 141, normalized size = 1.72

$$\frac{A\,a^{2}\,\sin\left(c+d\,x\right)}{d}+\frac{2\,B\,a^{2}\,\sin\left(c+d\,x\right)}{d}+\frac{4\,A\,a^{2}\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,a^{2}\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,B\,a^{2}\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{B\,a^{2}\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x),x)
```

```
[Out] (A*a^2*sin(c + d*x))/d + (2*B*a^2*sin(c + d*x))/d + (4*A*a^2*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)))/d + (3*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
+ (B*a^2*sin(2*c + 2*d*x))/(4*d)
```

3.15 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=74

$$a^2(A+2B)x+\frac{a^2(2A+B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{A(a^2+a^2\cos(c+dx))\tan(c+dx)}{d}$$

```
[Out] a^2*(A+2*B)*x+a^2*(2*A+B)*arctanh(sin(d*x+c))/d-a^2*(A-B)*sin(d*x+c)/d+A*(a
^2+a^2*cos(d*x+c))*tan(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3054, 3047, 3102, 2814, 3855}

$$-\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{a^2(2A+B)\tanh^{-1}(\sin(c+dx))}{d}+a^2x(A+2B)+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] a^2*(A + 2*B)*x + (a^2*(2*A + B)*ArcTanh[Sin[c + d*x]])/d - (a^2*(A - B)*Si
n[c + d*x])/d + (A*(a^2 + a^2*Cos[c + d*x])*Tan[c + d*x])/d
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx &= \frac{A(a^2+a^2\cos(c+dx))\tan(c+dx)}{d}+\int (a+a \\ &= \frac{A(a^2+a^2\cos(c+dx))\tan(c+dx)}{d}+\int (a^2(2A \\ &= -\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{A(a^2+a^2\cos(c+d}{d} \\ &= a^2(A+2B)x-\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{A(a^2}{} \\ &= a^2(A+2B)x+\frac{a^2(2A+B)\tanh^{-1}(\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.39, size = 143, normalized size = 1.93

$$\frac{a^2\left(Ac+2Bc+Adx+2Bdx-2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+B\sin(c+dx)+A\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^2*(A*c + 2*B*c + A*d*x + 2*B*d*x - 2*A*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*A*Log[Cos[(c + d*x
```

```
)/2] + Sin[(c + d*x)/2]] + B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + B*S
in[c + d*x] + A*Tan[c + d*x]))/d
```

Maple [A]
time = 0.20, size = 88, normalized size = 1.19

method	result
derivativedivides	$\frac{a^2 A(dx+c)+B a^2 \sin(dx+c)+2a^2 A \ln(\sec(dx+c)+\tan(dx+c))+2B a^2(dx+c)+a^2 A \tan(dx+c)+B a^2 \ln(\sec(dx+c)+\tan}{d}$
default	$\frac{a^2 A(dx+c)+B a^2 \sin(dx+c)+2a^2 A \ln(\sec(dx+c)+\tan(dx+c))+2B a^2(dx+c)+a^2 A \tan(dx+c)+B a^2 \ln(\sec(dx+c)+\tan}{d}$
risch	$a^2xA+2a^2Bx-\frac{i e^{i(dx+c)} B a^2}{2d}+\frac{i e^{-i(dx+c)} B a^2}{2d}+\frac{2ia^2 A}{d(e^{2i(dx+c)}+1)}-\frac{2a^2 A \ln(e^{i(dx+c)}-i)}{d}-\frac{a^2 \ln(e^{i(dx}}{d}$
norman	$\frac{(-a^2A-2B a^2)x+(-2a^2A-4B a^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(a^2A+2B a^2)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2a^2A+4B a^2)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(d*x+c)+B*a^2*sin(d*x+c)+2*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+2*B*a
^2*(d*x+c)+a^2*A*tan(d*x+c)+B*a^2*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.28, size = 105, normalized size = 1.42

$$\frac{2\,(dx+c)Aa^2+4\,(dx+c)Ba^2+2\,Aa^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+Ba^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+2\,Ba^2\sin\left(dx+c\right)+2\,Aa^2\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*(2*(d*x + c)*A*a^2 + 4*(d*x + c)*B*a^2 + 2*A*a^2*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*B*a^2*sin(d*x + c) + 2*A*a^2*tan(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 108, normalized size = 1.46

$$\frac{2\,(A+2\,B)a^2dx\cos\left(dx+c\right)+(2\,A+B)a^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-(2\,A+B)a^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,(Ba^2\cos\left(dx+c\right)+Aa^2)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*(2*(A + 2*B)*a^2*d*x*cos(d*x + c) + (2*A + B)*a^2*cos(d*x + c)*log(sin(
d*x + c) + 1) - (2*A + B)*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(B*a^
2*cos(d*x + c) + A*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sec^2(c+dx)\,dx+\int 2A\cos(c+dx)\sec^2(c+dx)\,dx+\int A\cos^2(c+dx)\sec^2(c+dx)\,dx+\int B\cos(c+dx)\sec^2(c+dx)\,dx+\int 2B\cos^2(c+dx)\sec^2(c+dx)\,dx+\int B\cos^3(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**2, x) + Integral(2*A*cos(c + d*x)*sec(c + d*
x)**2, x) + Integral(A*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(B*cos
(c + d*x)*sec(c + d*x)**2, x) + Integral(2*B*cos(c + d*x)**2*sec(c + d*x)**
2, x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)**2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(74) = 148.
time = 0.44, size = 155, normalized size = 2.09

$$\frac{(Aa^2+2Ba^2)(dx+c)+(2Aa^2+Ba^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(2Aa^2+Ba^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] ((A*a^2 + 2*B*a^2)*(d*x + c) + (2*A*a^2 + B*a^2)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (2*A*a^2 + B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a^2*
tan(1/2*d*x + 1/2*c)^3 - B*a^2*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*x +
 1/2*c) + B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

Mupad [B]
time = 0.32, size = 161, normalized size = 2.18

$$\frac{B\,a^2\,\sin\left(c+d\,x\right)}{d}+\frac{2\,A\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] (B*a^2*sin(c + d*x))/d + (2*A*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d + (4*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*B*a^2
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^2*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^2*sin(c + d*x))/(d*cos(c + d*x))
```

3.16 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=88

$$a^2Bx+\frac{a^2(3A+4B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(3A+2B)\tan(c+dx)}{2d}+\frac{A(a^2+a^2\cos(c+dx))\sec(c+dx)\,\mathrm{ta}}{2d}$$

```
[Out] a^2*B*x+1/2*a^2*(3*A+4*B)*arctanh(sin(d*x+c))/d+1/2*a^2*(3*A+2*B)*tan(d*x+c
)/d+1/2*A*(a^2+a^2*cos(d*x+c))*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3054, 3047, 3100, 2814, 3855}

$$\frac{a^2(3A+2B)\tan(c+dx)}{2d}+\frac{a^2(3A+4B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}+a^2Bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] a^2*B*x + (a^2*(3*A + 4*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(3*A + 2*B)*
Tan[c + d*x])/(2*d) + (A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x]
)/(2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
```

```
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx &= \frac{A(a^2+a^2\cos(c+dx))\sec(c+dx)\tan(c+dx)}{2d}\\ &= \frac{A(a^2+a^2\cos(c+dx))\sec(c+dx)\tan(c+dx)}{2d}\\ &= \frac{a^2(3A+2B)\tan(c+dx)}{2d}+\frac{A(a^2+a^2\cos(c+\end{aligned}$$
$$\begin{aligned}&= a^2Bx+\frac{a^2(3A+2B)\tan(c+dx)}{2d}+\frac{A(a^2+a^2\end{aligned}$$
$$\begin{aligned}&= a^2Bx+\frac{a^2(3A+4B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 277 vs. 2(88) = 176.
time = 1.36, size = 277, normalized size = 3.15

$$\frac{1}{16}a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(4Bx-\frac{2(3A+4B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{2(3A+4B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{A}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(2A+B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{A}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(2A+B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(4*B*x - (2*(3*A + 4*B)*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(3*A + 4*B)*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]])/d + A/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
4*(2*A + B)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])) - A/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(2*A +
B)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]))))/16
```

Maple [A]
time = 0.23, size = 114, normalized size = 1.30

method	result
derivativedivides	$\frac{a^2 A \ln(\sec(dx+c)+\tan(dx+c))+B\,a^2(dx+c)+2a^2 A\tan(dx+c)+2B\,a^2\ln(\sec(dx+c)+\tan(dx+c))+a^2 A\left(\frac{\sec(dx+c)\tan(d}{2}\right.}{d}$
default	$\frac{a^2 A \ln(\sec(dx+c)+\tan(dx+c))+B\,a^2(dx+c)+2a^2 A\tan(dx+c)+2B\,a^2\ln(\sec(dx+c)+\tan(dx+c))+a^2 A\left(\frac{\sec(dx+c)\tan(d}{2}\right.}{d}$
risch	$a^2 Bx-\frac{ia^2\left(A\,e^{3i(dx+c)}-4A\,e^{2i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+c)}-4A-2B\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{3a^2 A\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{2a^2\ln(e^{i(}}{}$
norman	$\frac{a^2Bx+a^2Bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^2Bx\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^2Bx\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{a^2(5A+2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{6a^2A\left(\tan^5\left(\frac{dx}{2}\right.\right.}{d}}{\left(1+\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*ln(sec(d*x+c)+tan(d*x+c))+B*a^2*(d*x+c)+2*a^2*A*tan(d*x+c)+2*B*a
^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*
x+c)+tan(d*x+c)))+B*a^2*tan(d*x+c))
```

Maxima [A]
time = 0.26, size = 142, normalized size = 1.61

$$\frac{4\,(dx+c)Ba^2-Aa^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+2\,Aa^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+4\,Ba^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+8\,Aa^2\tan\left(dx+c\right)+4\,Ba^2\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*(d*x + c)*B*a^2 - A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*A*a^2*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 4*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 8*A*a^2*tan(d*x + c) + 4*B*a^2*tan(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 119, normalized size = 1.35

$$\frac{4\,Ba^2dx\cos\left(dx+c\right)^2+(3\,A+4\,B)a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(3\,A+4\,B)a^2\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,(2\,(2\,A+B)a^2\cos\left(dx+c\right)+Aa^2)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*B*a^2*d*x*cos(d*x + c)^2 + (3*A + 4*B)*a^2*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - (3*A + 4*B)*a^2*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
(2*A + B)*a^2*cos(d*x + c) + A*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sec^3(c+dx)\,dx+\int 2A\cos(c+dx)\sec^3(c+dx)\,dx+\int A\cos^2(c+dx)\sec^3(c+dx)\,dx+\int B\cos(c+dx)\sec^3(c+dx)\,dx+\int 2B\cos^2(c+dx)\sec^3(c+dx)\,dx+\int B\cos^3(c+dx)\sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**3, x) + Integral(2*A*cos(c + d*x)*sec(c + d*
x)**3, x) + Integral(A*cos(c + d*x)**2*sec(c + d*x)**3, x) + Integral(B*cos
(c + d*x)*sec(c + d*x)**3, x) + Integral(2*B*cos(c + d*x)**2*sec(c + d*x)**
3, x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)**3, x))
```

Giac [A]
time = 0.50, size = 154, normalized size = 1.75

$$\frac{2(dx+c)Ba^2+(3Aa^2+4Ba^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(3Aa^2+4Ba^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(3Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(d*x + c)*B*a^2 + (3*A*a^2 + 4*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - (3*A*a^2 + 4*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a^2*
tan(1/2*d*x + 1/2*c)^3 + 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*A*a^2*tan(1/2*d
*x + 1/2*c) - 2*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)
/d
```

Mupad [B]
time = 0.30, size = 162, normalized size = 1.84

$$\frac{3Aa^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{2Ba^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{4Ba^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{2Aa^2\sin(c+dx)}{d\cos(c+dx)}+\frac{Aa^2\sin(c+dx)}{2d\cos(c+dx)^2}+\frac{Ba^2\sin(c+dx)}{d\cos(c+dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^3,x)
```

```
[Out] (3*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^2*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*B*a^2*atanh(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)))/d + (2*A*a^2*sin(c + d*x))/(d*cos(c + d*x)) + (A*a^2*
sin(c + d*x))/(2*d*cos(c + d*x)^2) + (B*a^2*sin(c + d*x))/(d*cos(c + d*x))
```

3.17 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=113

$$\frac{a^2(2A+3B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(5A+6B)\tan(c+dx)}{3d}+\frac{a^2(4A+3B)\sec(c+dx)\tan(c+dx)}{6d}+\frac{A(a^2$$

[Out] 1/2*a^2*(2*A+3*B)*arctanh(sin(d*x+c))/d+1/3*a^2*(5*A+6*B)*tan(d*x+c)/d+1/6*a^2*(4*A+3*B)*sec(d*x+c)*tan(d*x+c)/d+1/3*A*(a^2+a^2*cos(d*x+c))*sec(d*x+c)^2*tan(d*x+c)/d

Rubi [A]
time = 0.18, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3054, 3047, 3100, 2827, 3852, 8, 3855}

$$\frac{a^2(5A+6B)\tan(c+dx)}{3d}+\frac{a^2(2A+3B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(4A+3B)\tan(c+dx)\sec(c+dx)}{6d}+\frac{A\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]

[Out] (a^2*(2*A + 3*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(5*A + 6*B)*Tan[c + d*x])/(3*d) + (a^2*(4*A + 3*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^2 (A + B\cos(c+dx)) \sec^4(c+dx)\,dx = \frac{A(a^2 + a^2\cos(c+dx))\sec^2(c+dx)\tan(c+d}{3d}$$
$$= \frac{A(a^2 + a^2\cos(c+dx))\sec^2(c+dx)\tan(c+d}{3d}$$
$$= \frac{a^2(4A+3B)\sec(c+dx)\tan(c+dx)}{6d} + \frac{A(a^2}{}$$
$$= \frac{a^2(4A+3B)\sec(c+dx)\tan(c+dx)}{6d} + \frac{A(a^2}{}$$
$$= \frac{a^2(2A+3B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2(4A+}{}$$
$$= \frac{a^2(2A+3B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2(5A+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 451 vs. $2(113) = 226$.
time = 5.88, size = 451, normalized size = 3.99

$$\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-6(2A+3B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+6(2A+3B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2A\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{(7A+3B)\cos\left(\frac{c}{2}\right)-(5A+3B)\sin\left(\frac{c}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(5A+6B)\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{2A\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{(7A+3B)\cos\left(\frac{c}{2}\right)+(5A+3B)\sin\left(\frac{c}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(5A+6B)\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-6*(2*A + 3*B)*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] + 6*(2*A + 3*B)*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + (2*A*Sin[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^3) + ((7*A + 3*B)*Cos[c/2] - (5*A + 3*B)*Sin[c/2])/((Cos[c/
2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(5*A + 6*B)*Si
n[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) +
 (2*A*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^3) - ((7*A + 3*B)*Cos[c/2] + (5*A + 3*B)*Sin[c/2])/((Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(5*A + 6*B)*Sin[(d*x)/2]
)/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(48*d)
```

Maple [A]
time = 0.25, size = 145, normalized size = 1.28

method	result
derivativedivides	$\frac{a^2 A\tan(dx+c)+B\,a^2\ln(\sec(dx+c)+\tan(dx+c))+2a^2 A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2B\,a^2\tan(dx}{d}$

default	$\frac{a^2 A\tan(dx+c)+B\,a^2\ln(\sec(dx+c)+\tan(dx+c))+2a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2B\,a^2\tan(dx+c}{d}$
risch	$-\frac{ia^2\left(6A\,e^{5i(dx+c)}+3B\,e^{5i(dx+c)}-6A\,e^{4i(dx+c)}-12B\,e^{4i(dx+c)}-24A\,e^{2i(dx+c)}-24B\,e^{2i(dx+c)}-6A\,e^{i(dx+c)}-3B\,e^{i(dx+c}\right.}{3d\left(e^{2i(dx+c)}+1\right)^3}$
norman	$\frac{-\frac{2a^2(2A-3B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2(2A+3B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{a^2(2A+3B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2(2A+5B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2(6}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*tan(d*x+c)+B*a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*A*(1/2*sec(d*x+
c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+2*B*a^2*tan(d*x+c)-a^2*A*(-2/3
-1/3*sec(d*x+c)^2)*tan(d*x+c)+B*a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d
*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 174, normalized size = 1.54

$$\frac{4\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Aa^2-6\,Aa^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ba^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,Ba^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+12\,Aa^2\tan\left(dx+c\right)+24\,Ba^2\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 - 6*A*a^2*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*
a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 6*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
12*A*a^2*tan(d*x + c) + 24*B*a^2*tan(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 125, normalized size = 1.11

$$\frac{3\left(2\,A+3\,B\right)a^2\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,A+3\,B\right)a^2\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\left(5\,A+6\,B\right)a^2\cos\left(dx+c\right)^2+3\left(2\,A+B\right)a^2\cos\left(dx+c\right)+2\,Aa^2\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*(2*A + 3*B)*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*A + 3*B
)*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(5*A + 6*B)*a^2*cos(d*x
+ c)^2 + 3*(2*A + B)*a^2*cos(d*x + c) + 2*A*a^2)*sin(d*x + c))/(d*cos(d*x +
 c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 178, normalized size = 1.58

$$\frac{3\,(2\,Aa^2+3\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(2\,Aa^2+3\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-16\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(2*A*a^2 + 3*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*A*a^2
+ 3*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^2*tan(1/2*d*x + 1/
2*c)^5 + 9*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 16*A*a^2*tan(1/2*d*x + 1/2*c)^3 -
 24*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 18*A*a^2*tan(1/2*d*x + 1/2*c) + 15*B*a^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [**B**]
time = 2.08, size = 145, normalized size = 1.28

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A+\frac{3\,B}{2}\right)}{d}-\frac{\left(2\,A\,a^2+3\,B\,a^2\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(-\frac{16\,A\,a^2}{3}-8\,B\,a^2\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(6\,A\,a^2+5\,B\,a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^4,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2))*(A + (3*B)/2))/d - (tan(c/2 + (d*x)/2)*(6*
A*a^2 + 5*B*a^2) + tan(c/2 + (d*x)/2)^5*(2*A*a^2 + 3*B*a^2) - tan(c/2 + (d*
x)/2)^3*((16*A*a^2)/3 + 8*B*a^2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 +
(d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.18 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{a^2(7A+8B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(4A+5B)\tan(c+dx)}{3d}+\frac{a^2(7A+8B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2(5A$$

```
[Out] 1/8*a^2*(7*A+8*B)*arctanh(sin(d*x+c))/d+1/3*a^2*(4*A+5*B)*tan(d*x+c)/d+1/8*
a^2*(7*A+8*B)*sec(d*x+c)*tan(d*x+c)/d+1/12*a^2*(5*A+4*B)*sec(d*x+c)^2*tan(d
*x+c)/d+1/4*A*(a^2+a^2*cos(d*x+c))*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.20, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3054, 3047, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{a^2(4A+5B)\tan(c+dx)}{3d}+\frac{a^2(7A+8B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(5A+4B)\tan(c+dx)\sec^2(c+dx)}{12d}+\frac{a^2(7A+8B)\tan(c+dx)\sec(c+dx)}{8d}+\frac{A\tan(c+dx)\sec^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a^2*(7*A + 8*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(4*A + 5*B)*Tan[c + d*
x])/(3*d) + (a^2*(7*A + 8*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(5*A +
 4*B)*Sec[c + d*x]^2*Tan[c + d*x])/(12*d) + (A*(a^2 + a^2*Cos[c + d*x])*Sec
[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{A(a^2+a^2\cos(c+dx))\sec^3(c+dx)\tan(c+dx)}{4d}$$
$$= \frac{A(a^2+a^2\cos(c+dx))\sec^3(c+dx)\tan(c+dx}{4d}$$
$$= \frac{a^2(5A+4B)\sec^2(c+dx)\tan(c+dx)}{12d} + \frac{A(a^2}{}$$
$$= \frac{a^2(5A+4B)\sec^2(c+dx)\tan(c+dx)}{12d} + \frac{A(a^2}{}$$
$$= \frac{a^2(7A+8B)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^2(5A}{}$$
$$= \frac{a^2(7A+8B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^2(4A+5}{}$$

Mathematica [A]
time = 1.25, size = 262, normalized size = 1.82

$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(24(7A+8B)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(-24(4A+5B)\sin(c)+3(15A+8B)\sin(dx)+45A\sin(2c+dx)+24B\sin(2c+dx)+128A\sin(c+2dx)+136B\sin(c+2dx)-24B\sin(3c+2dx)+21A\sin(2c+3dx)+24B\sin(2c+3dx)+21A\sin(4c+3dx)+24B\sin(4c+3dx)+32A\sin(3c+4dx)+40B\sin(3c+4dx))\right)}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] -1/768*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^4*(24*(7*A
 + 8*B)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-24*(4*A + 5*B)*Sin[c] + 3*(15*
A + 8*B)*Sin[d*x] + 45*A*Sin[2*c + d*x] + 24*B*Sin[2*c + d*x] + 128*A*Sin[c
 + 2*d*x] + 136*B*Sin[c + 2*d*x] - 24*B*Sin[3*c + 2*d*x] + 21*A*Sin[2*c + 3
*d*x] + 24*B*Sin[2*c + 3*d*x] + 21*A*Sin[4*c + 3*d*x] + 24*B*Sin[4*c + 3*d*
x] + 32*A*Sin[3*c + 4*d*x] + 40*B*Sin[3*c + 4*d*x]))))/d
```

Maple [A]
time = 0.30, size = 187, normalized size = 1.30

method	result
derivativedivides	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+B\,a^2\tan(dx+c)-2a^2A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+2B\,a^2\left(\mathrm{sec}\right.}{}$
default	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+B\,a^2\tan(dx+c)-2a^2A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+2B\,a^2\left(\mathrm{sec}\right.}{}$
norman	$\frac{\frac{a^2(3A-8B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2(7A+8B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{a^2(7A+8B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a^2(25A+24B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{a^2(53A-}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

risch	$-\frac{ia^2(21A\,\mathrm{e}^{7i(dx+c)}+24B\,\mathrm{e}^{7i(dx+c)}-24B\,\mathrm{e}^{6i(dx+c)}+45A\,\mathrm{e}^{5i(dx+c)}+24B\,\mathrm{e}^{5i(dx+c)}-96A\,\mathrm{e}^{4i(dx+c)}-120B\,\mathrm{e}^{4i(dx+c)}-4}{12d(\mathrm{e}^{2i(dx+c)}+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+B*a^2*
tan(d*x+c)-2*a^2*A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2*B*a^2*(1/2*sec(d*x+
c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*A*(-(-1/4*sec(d*x+c)^3-3/8
*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-B*a^2*(-2/3-1/3*sec(
d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 230, normalized size = 1.60

$$\frac{32\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^2+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2-3\,Aa^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-12\,Aa^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-24\,Ba^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+48\,Ba^2\tan(dx+c)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^2 - 3*A*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 12*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 24*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*B*a^2*tan(d*x +
 c))/d
```

Fricas [A]
time = 0.37, size = 145, normalized size = 1.01

$$\frac{3(7A+8B)a^2\cos(dx+c)^4\log(\sin(dx+c)+1)-3(7A+8B)a^2\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(8(4A+5B)a^2\cos(dx+c)^3+3(7A+8B)a^2\cos(dx+c)^2+8(2A+B)a^2\cos(dx+c)+6\,Aa^2\right)\sin(dx+c)}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(7*A + 8*B)*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(7*A + 8*B
)*a^2*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(4*A + 5*B)*a^2*cos(d*x
+ c)^3 + 3*(7*A + 8*B)*a^2*cos(d*x + c)^2 + 8*(2*A + B)*a^2*cos(d*x + c) +
6*A*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 212, normalized size = 1.47

$$\frac{3\,(7\,Aa^2+8\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(7\,Aa^2+8\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+24\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-77\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-88\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+83\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+136\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-75\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-72\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(7*A*a^2 + 8*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(7*A*a^2
 + 8*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(21*A*a^2*tan(1/2*d*x +
1/2*c)^7 + 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*A*a^2*tan(1/2*d*x + 1/2*c)^
5 - 88*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 83*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 136
*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*A*a^2*tan(1/2*d*x + 1/2*c) - 72*B*a^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 2.68, size = 183, normalized size = 1.27

$$\frac{\left(-\frac{7\,A\,a^2}{4}-2\,B\,a^2\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{77\,A\,a^2}{12}+\frac{22\,B\,a^2}{3}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{83\,A\,a^2}{12}-\frac{34\,B\,a^2}{3}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{25\,A\,a^2}{4}+6\,B\,a^2\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{2\,a^2\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{7\,A}{8}+B\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((25*A*a^2)/4 + 6*B*a^2) - tan(c/2 + (d*x)/2)^7*((7*A*a
^2)/4 + 2*B*a^2) + tan(c/2 + (d*x)/2)^5*((77*A*a^2)/12 + (22*B*a^2)/3) - ta
n(c/2 + (d*x)/2)^3*((83*A*a^2)/12 + (34*B*a^2)/3))/(d*(6*tan(c/2 + (d*x)/2)
^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8
 + 1)) + (2*a^2*atanh(tan(c/2 + (d*x)/2))*((7*A)/8 + B))/d
```

3.19 $\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=201

$$\frac{1}{16}a^3(26A+23B)x+\frac{a^3(19A+17B)\sin(c+dx)}{5d}+\frac{a^3(26A+23B)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^3(22A+21B)\,c$$

```
[Out] 1/16*a^3*(26*A+23*B)*x+1/5*a^3*(19*A+17*B)*sin(d*x+c)/d+1/16*a^3*(26*A+23*B
)*cos(d*x+c)*sin(d*x+c)/d+1/40*a^3*(22*A+21*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/
6*a*B*cos(d*x+c)^3*(a+a*cos(d*x+c))^2*sin(d*x+c)/d+1/15*(3*A+4*B)*cos(d*x+c
)^3*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d-1/15*a^3*(19*A+17*B)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.29, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3055, 3047, 3102, 2827, 2715, 8, 2713}

$$-\frac{a^3(19A+17B)\sin^3(c+dx)}{15d}+\frac{a^3(19A+17B)\sin(c+dx)}{5d}+\frac{a^3(22A+21B)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{(3A+4B)\sin(c+dx)\cos^3(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d}+\frac{a^3(26A+23B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}a^3x(26A+23B)+\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^3*(26*A + 23*B)*x)/16 + (a^3*(19*A + 17*B)*Sin[c + d*x])/(5*d) + (a^3*(2
6*A + 23*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(22*A + 21*B)*Cos[c +
d*x]^3*Sin[c + d*x])/(40*d) + (a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*Si
n[c + d*x])/(6*d) + ((3*A + 4*B)*Cos[c + d*x]^3*(a^3 + a^3*Cos[c + d*x])*Si
n[c + d*x])/(15*d) - (a^3*(19*A + 17*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+d}{6d}$$
$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+d}{6d}$$
$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+d}{6d}$$
$$= \frac{a^3(22A+21B)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{aI}{}$$
$$= \frac{a^3(22A+21B)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{aI}{}$$
$$= \frac{a^3(26A+23B)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^3(}{}$$
$$= \frac{1}{16}a^3(26A+23B)x + \frac{a^3(19A+17B)\sin(c+}{5d}$$

Mathematica [A]
time = 0.60, size = 134, normalized size = 0.67

$$\frac{a^3(1380Bc+1560Adx+1380Bdx+120(23A+21B)\sin(c+dx)+15(64A+63B)\sin(2(c+dx))+340A\sin(3(c+dx))+380B\sin(3(c+dx))+90A\sin(4(c+dx))+135B\sin(4(c+dx))+12A\sin(5(c+dx))+36B\sin(5(c+dx))+5B\sin(6(c+dx)))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^3*(1380*B*c + 1560*A*d*x + 1380*B*d*x + 120*(23*A + 21*B)*Sin[c + d*x] +
 15*(64*A + 63*B)*Sin[2*(c + d*x)] + 340*A*Sin[3*(c + d*x)] + 380*B*Sin[3*(
c + d*x)] + 90*A*Sin[4*(c + d*x)] + 135*B*Sin[4*(c + d*x)] + 12*A*Sin[5*(c
+ d*x)] + 36*B*Sin[5*(c + d*x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A]
time = 0.22, size = 266, normalized size = 1.32

method	result
risch	$\frac{13a^3xA}{8}+\frac{23a^3Bx}{16}+\frac{23a^3A\sin(dx+c)}{8d}+\frac{21a^3B\sin(dx+c)}{8d}+\frac{a^3B\sin(6dx+6c)}{192d}+\frac{\sin(5dx+5c)Aa^3}{80d}+\frac{3\sin(5}{}$
derivativedivides	$\frac{\frac{Aa^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a^3B\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{}$

default	$\frac{\frac{A a^3 \left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a^3 B\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+}{}$
norman	$\frac{a^3(26A+23B)x}{16}+\frac{33a^3(26A+23B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}+\frac{17a^3(26A+23B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{a^3(26A+23B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(1/6*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+3
*A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3/5*a^3
*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^3*(cos(d*x+c)^2+2)*si
n(d*x+c)+3*a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*
c)+A*a^3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+1/3*a^3*B*(cos(d*x+c)^2+
2)*sin(d*x+c))
```

<u>Maxima</u> [A]
time = 0.27, size = 262, normalized size = 1.30

$$\frac{64\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)Aa^3-960\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa^3+90\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)Aa^3+240\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^3+192\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)Ba^3-5\left(4\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\sin\left(4\,dx+4\,c\right)-48\sin\left(2\,dx+2\,c\right)\right)Ba^3-320\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Ba^3+90\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)Ba^3}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 -
960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 90*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*
A*a^3 + 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^3
- 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d
*x + 2*c))*B*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 90*(12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3)/d
```

<u>Fricas</u> [A]
time = 0.35, size = 130, normalized size = 0.65

$$\frac{15\left(26\,A+23\,B\right)a^3dx+\left(40\,Ba^3\cos\left(dx+c\right)^5+48\left(A+3\,B\right)a^3\cos\left(dx+c\right)^4+10\left(18\,A+23\,B\right)a^3\cos\left(dx+c\right)^3+16\left(19\,A+17\,B\right)a^3\cos\left(dx+c\right)^2+15\left(26\,A+23\,B\right)a^3\cos\left(dx+c\right)+32\left(19\,A+17\,B\right)a^3\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(26*A + 23*B)*a^3*d*x + (40*B*a^3*cos(d*x + c)^5 + 48*(A + 3*B)*a
^3*cos(d*x + c)^4 + 10*(18*A + 23*B)*a^3*cos(d*x + c)^3 + 16*(19*A + 17*B)*
a^3*cos(d*x + c)^2 + 15*(26*A + 23*B)*a^3*cos(d*x + c) + 32*(19*A + 17*B)*a
^3)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(184) = 368$.
time = 0.51, size = 695, normalized size = 3.46

$$\begin{cases} \frac{9Aa^3x\sin^4(c+dx)}{8} + \frac{9Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{Aa^3x\sin^2(c+dx)}{2} + \frac{9Aa^3x\cos^4(c+dx)}{8} + \frac{Aa^3x\cos^2(c+dx)}{2} + \frac{8Aa^3\sin^5(c+dx)}{15d} + \frac{4Aa^3\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{9Aa^3\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2Aa^3\sin^3(c+dx)}{d} + \frac{Aa^3\sin(c+dx)\cos^4(c+dx)}{d} + \frac{15Aa^3\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{3Aa^3\sin(c+dx)\cos^2(c+dx)}{d} + \frac{Aa^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{5Ba^3x\sin^6(c+dx)}{16} + \frac{15Ba^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{9Ba^3x\sin^4(c+dx)}{8} + \frac{15Ba^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{9Ba^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{5Ba^3x\cos^6(c+dx)}{16} + \frac{9Ba^3x\cos^4(c+dx)}{8} + \frac{5Ba^3\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{8Ba^3\sin^5(c+dx)}{5d} + \frac{5Ba^3\sin^3(c+dx)\cos^3(c+dx)}{6d} + \frac{4Ba^3\sin^3(c+dx)\cos^2(c+dx)}{d} + \frac{9Ba^3\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2Ba^3\sin^3(c+dx)}{3d} + \frac{11Ba^3\sin(c+dx)\cos^5(c+dx)}{16d} + \frac{3Ba^3\sin(c+dx)\cos^4(c+dx)}{d} + \frac{15Ba^3\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{Ba^3\sin(c+dx)\cos^2(c+dx)}{d} & \text{for}\ d \neq 0 \\ x(A + B\cos(c))(a\cos(c) + a)^3\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((9*A*a**3*x*sin(c + d*x)**4/8 + 9*A*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + A*a**3*x*sin(c + d*x)**2/2 + 9*A*a**3*x*cos(c + d*x)**4/8 + A
*a**3*x*cos(c + d*x)**2/2 + 8*A*a**3*sin(c + d*x)**5/(15*d) + 4*A*a**3*sin(
c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(
8*d) + 2*A*a**3*sin(c + d*x)**3/d + A*a**3*sin(c + d*x)*cos(c + d*x)**4/d +
 15*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*A*a**3*sin(c + d*x)*cos(c
 + d*x)**2/d + A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 5*B*a**3*x*sin(c +
d*x)**6/16 + 15*B*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*B*a**3*x*si
n(c + d*x)**4/8 + 15*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 9*B*a**3
*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*B*a**3*x*cos(c + d*x)**6/16 + 9*B*
a**3*x*cos(c + d*x)**4/8 + 5*B*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 8
*B*a**3*sin(c + d*x)**5/(5*d) + 5*B*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6
*d) + 4*B*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d + 9*B*a**3*sin(c + d*x)**3
*cos(c + d*x)/(8*d) + 2*B*a**3*sin(c + d*x)**3/(3*d) + 11*B*a**3*sin(c + d*
x)*cos(c + d*x)**5/(16*d) + 3*B*a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*B*
a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**3*sin(c + d*x)*cos(c + d*x)*
*2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**3*cos(c)**2, True))
```

Giac [A]
time = 0.51, size = 166, normalized size = 0.83

$$\frac{Ba^3\sin(6dx+6c)}{192d} + \frac{1}{16}(26Aa^3 + 23Ba^3)x + \frac{(Aa^3 + 3Ba^3)\sin(5dx+5c)}{80d} + \frac{3(2Aa^3 + 3Ba^3)\sin(4dx+4c)}{64d} + \frac{(17Aa^3 + 19Ba^3)\sin(3dx+3c)}{48d} + \frac{(64Aa^3 + 63Ba^3)\sin(2dx+2c)}{64d} + \frac{(23Aa^3 + 21Ba^3)\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*B*a^3*sin(6*d*x + 6*c)/d + 1/16*(26*A*a^3 + 23*B*a^3)*x + 1/80*(A*a^3
 + 3*B*a^3)*sin(5*d*x + 5*c)/d + 3/64*(2*A*a^3 + 3*B*a^3)*sin(4*d*x + 4*c)/
d + 1/48*(17*A*a^3 + 19*B*a^3)*sin(3*d*x + 3*c)/d + 1/64*(64*A*a^3 + 63*B*a
^3)*sin(2*d*x + 2*c)/d + 1/8*(23*A*a^3 + 21*B*a^3)*sin(d*x + c)/d
```

Mupad [B]

time = 1.61, size = 315, normalized size = 1.57

$$\frac{\left(\frac{13\,A\,a^3}{4}+\frac{23\,B\,a^3}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{221\,A\,a^3}{12}+\frac{391\,B\,a^3}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{429\,A\,a^3}{10}+\frac{759\,B\,a^3}{20}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{499\,A\,a^3}{10}+\frac{969\,B\,a^3}{20}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{419\,A\,a^3}{12}+\frac{211\,B\,a^3}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{51\,A\,a^3}{4}+\frac{105\,B\,a^3}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{a^3\,\left(26\,A+23\,B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{8\,d}+\frac{a^3\,\mathrm{atan}\left(\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(26\,A+23\,B\right)}{8\,\left(\frac{13\,A\,a^3}{4}+\frac{23\,B\,a^3}{8}\right)}\right)\,\left(26\,A+23\,B\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((51*A*a^3)/4 + (105*B*a^3)/8) + tan(c/2 + (d*x)/2)^11*
((13*A*a^3)/4 + (23*B*a^3)/8) + tan(c/2 + (d*x)/2)^3*((419*A*a^3)/12 + (211
*B*a^3)/8) + tan(c/2 + (d*x)/2)^9*((221*A*a^3)/12 + (391*B*a^3)/24) + tan(c
/2 + (d*x)/2)^7*((429*A*a^3)/10 + (759*B*a^3)/20) + tan(c/2 + (d*x)/2)^5*((
499*A*a^3)/10 + (969*B*a^3)/20))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 +
(d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2
+ (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - (a^3*(26*A + 23*B)*(atan(tan(
c/2 + (d*x)/2)) - (d*x)/2))/(8*d) + (a^3*atan((a^3*tan(c/2 + (d*x)/2)*(26*A
 + 23*B))/(8*((13*A*a^3)/4 + (23*B*a^3)/8)))*(26*A + 23*B))/(8*d)
```

3.20 $\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=154

$$\frac{1}{8}a^3(15A+13B)x+\frac{a^3(15A+13B)\sin(c+dx)}{5d}+\frac{3a^3(15A+13B)\cos(c+dx)\sin(c+dx)}{40d}+\frac{(5A-B)(a+a$$

```
[Out] 1/8*a^3*(15*A+13*B)*x+1/5*a^3*(15*A+13*B)*sin(d*x+c)/d+3/40*a^3*(15*A+13*B)
*cos(d*x+c)*sin(d*x+c)/d+1/20*(5*A-B)*(a+a*cos(d*x+c))^3*sin(d*x+c)/d+1/5*B
*(a+a*cos(d*x+c))^4*sin(d*x+c)/a/d-1/60*a^3*(15*A+13*B)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.16, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3047, 3102, 2830, 2724, 2717, 2715, 8, 2713}

$$-\frac{a^3(15A+13B)\sin^3(c+dx)}{60d}+\frac{a^3(15A+13B)\sin(c+dx)}{5d}+\frac{3a^3(15A+13B)\sin(c+dx)\cos(c+dx)}{40d}+\frac{1}{8}a^3x(15A+13B)+\frac{(5A-B)\sin(c+dx)(a\cos(c+dx)+a)^3}{20d}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^3*(15*A + 13*B)*x)/8 + (a^3*(15*A + 13*B)*Sin[c + d*x])/(5*d) + (3*a^3*(
15*A + 13*B)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + ((5*A - B)*(a + a*Cos[c +
d*x])^3*Sin[c + d*x])/(20*d) + (B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*a
*d) - (a^3*(15*A + 13*B)*Sin[c + d*x]^3)/(60*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^3\left(A\cos(c+dx)+B\cos^2\right.$$
$$= \frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5ad} + \frac{\int(a+a\,c}{}$$
$$= \frac{(5A-B)(a+a\cos(c+dx))^3\sin(c+dx)}{20d} + \frac{B}{}$$
$$= \frac{(5A-B)(a+a\cos(c+dx))^3\sin(c+dx)}{20d} + \frac{B}{}$$
$$= \frac{1}{20}a^3(15A+13B)x + \frac{(5A-B)(a+a\cos(c+}{20d}$$
$$= \frac{1}{20}a^3(15A+13B)x + \frac{3a^3(15A+13B)\sin(c+}{20d}$$
$$= \frac{1}{8}a^3(15A+13B)x + \frac{a^3(15A+13B)\sin(c+dx}{5d}$$

Mathematica [A]

time = 0.47, size = 108, normalized size = 0.70

$$\frac{a^3(780Bc+900Adx+780Bdx+60(26A+23B)\sin(c+dx)+480(A+B)\sin(2(c+dx))+120A\sin(3(c+dx))+170B\sin(3(c+dx))+15A\sin(4(c+dx))+45B\sin(4(c+dx))+6B\sin(5(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
[Out] (a^3*(780*B*c + 900*A*d*x + 780*B*d*x + 60*(26*A + 23*B)*Sin[c + d*x] + 480
*(A + B)*Sin[2*(c + d*x)] + 120*A*Sin[3*(c + d*x)] + 170*B*Sin[3*(c + d*x)]
 + 15*A*Sin[4*(c + d*x)] + 45*B*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)]))/(
480*d)
```

Maple [A]

time = 0.17, size = 223, normalized size = 1.45

method	result
risch	$\frac{15a^3xA}{8} + \frac{13a^3Bx}{8} + \frac{13a^3A\sin(dx+c)}{4d} + \frac{23a^3B\sin(dx+c)}{8d} + \frac{\sin(5dx+5c)a^3B}{80d} + \frac{\sin(4dx+4c)A\,a^3}{32d} + \frac{3\sin(4c}{}$
derivativedivides	$\frac{A\,a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^3B\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+A\,a^3\left(\cos^2(dx+\right.}{}$
default	$\frac{A\,a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^3B\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+A\,a^3\left(\cos^2(dx+\right.}{}$

norman	$\frac{a^3(15A+13B)x}{8}+\frac{32a^3(15A+13B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{7a^3(15A+13B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{a^3(15A+13B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a^3(15A+13$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5
*a^3*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^3*(cos(d*x+c)^2+2
)*sin(d*x+c)+3*a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+
3/8*c)+3*A*a^3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+a^3*B*(cos(d*x+c)^
2+2)*sin(d*x+c)+A*a^3*sin(d*x+c)+a^3*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1
/2*c))
```

Maxima [A]
time = 0.27, size = 213, normalized size = 1.38

$$-\frac{480\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa^3-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^3-360\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^3-32\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)Ba^3+480\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Ba^3-45\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Ba^3-120\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba^3-480\,Aa^3\sin\left(dx+c\right)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/480*(480*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 360*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*a^3 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*B*a^3 + 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 45*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 - 120*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*B*a^3 - 480*A*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 0.34, size = 110, normalized size = 0.71

$$\frac{15\,(15\,A+13\,B)a^3dx+\left(24\,Ba^3\cos\left(dx+c\right)^4+30\,(A+3\,B)a^3\cos\left(dx+c\right)^3+8\,(15\,A+19\,B)a^3\cos\left(dx+c\right)^2+15\,(15\,A+13\,B)a^3\cos\left(dx+c\right)+8\,(45\,A+38\,B)a^3\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/120*(15*(15*A + 13*B)*a^3*d*x + (24*B*a^3*cos(d*x + c)^4 + 30*(A + 3*B)*a
^3*cos(d*x + c)^3 + 8*(15*A + 19*B)*a^3*cos(d*x + c)^2 + 15*(15*A + 13*B)*a
^3*cos(d*x + c) + 8*(45*A + 38*B)*a^3)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(136) = 272$.
time = 0.36, size = 530, normalized size = 3.44

$$\begin{cases} \frac{3Aa^3x\sin^4(c+dx)}{8}+\frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3Aa^3x\sin^2(c+dx)}{2}+\frac{3Aa^3x\cos^4(c+dx)}{8}+\frac{3Aa^3x\cos^2(c+dx)}{2}+\frac{3Aa^3\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{2Aa^3\sin^3(c+dx)}{d}+\frac{5Aa^3\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3Aa^3\sin(c+dx)\cos^2(c+dx)}{d}+\frac{3Aa^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{Aa^3\sin(c+dx)}{d}+\frac{9Ba^3x\sin^4(c+dx)}{8}+\frac{9Ba^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{Ba^3x\sin^2(c+dx)}{2}+\frac{9Ba^3x\cos^4(c+dx)}{8}+\frac{Ba^3x\cos^2(c+dx)}{2}+\frac{8Ba^3\sin^5(c+dx)}{15d}+\frac{4Ba^3\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{9Ba^3\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{2Ba^3\sin^3(c+dx)}{d}+\frac{Ba^3\sin(c+dx)\cos^4(c+dx)}{d}+\frac{15Ba^3\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3Ba^3\sin(c+dx)\cos^2(c+dx)}{d}+\frac{Ba^3\sin(c+dx)\cos(c+dx)}{2d} & \text{for}\: d \neq 0 \\ x(A+B\cos(c))(a\cos(c)+a)^3\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + 3*A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**4/8 +
 3*A*a**3*x*cos(c + d*x)**2/2 + 3*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d)
 + 2*A*a**3*sin(c + d*x)**3/d + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d)
 + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a**3*sin(c + d*x)*cos(c +
d*x)/(2*d) + A*a**3*sin(c + d*x)/d + 9*B*a**3*x*sin(c + d*x)**4/8 + 9*B*a**
3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + B*a**3*x*sin(c + d*x)**2/2 + 9*B*a*
*3*x*cos(c + d*x)**4/8 + B*a**3*x*cos(c + d*x)**2/2 + 8*B*a**3*sin(c + d*x)
**5/(15*d) + 4*B*a**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*B*a**3*sin(
c + d*x)**3*cos(c + d*x)/(8*d) + 2*B*a**3*sin(c + d*x)**3/d + B*a**3*sin(c
+ d*x)*cos(c + d*x)**4/d + 15*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3
*B*a**3*sin(c + d*x)*cos(c + d*x)**2/d + B*a**3*sin(c + d*x)*cos(c + d*x)/(
2*d), Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**3*cos(c), True))
```

Giac [A]
time = 0.43, size = 136, normalized size = 0.88

$$\frac{Ba^3\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{8}\,(15\,Aa^3+13\,Ba^3)x+\frac{(Aa^3+3\,Ba^3)\sin(4\,dx+4\,c)}{32\,d}+\frac{(12\,Aa^3+17\,Ba^3)\sin(3\,dx+3\,c)}{48\,d}+\frac{(Aa^3+Ba^3)\sin(2\,dx+2\,c)}{d}+\frac{(26\,Aa^3+23\,Ba^3)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/80*B*a^3*sin(5*d*x + 5*c)/d + 1/8*(15*A*a^3 + 13*B*a^3)*x + 1/32*(A*a^3 +
 3*B*a^3)*sin(4*d*x + 4*c)/d + 1/48*(12*A*a^3 + 17*B*a^3)*sin(3*d*x + 3*c)/
d + (A*a^3 + B*a^3)*sin(2*d*x + 2*c)/d + 1/8*(26*A*a^3 + 23*B*a^3)*sin(d*x
+ c)/d
```

Mupad [B]
time = 1.50, size = 277, normalized size = 1.80

$$\frac{\left(\frac{15\,A\,a^3}{4}+\frac{13\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{35\,A\,a^3}{2}+\frac{91\,B\,a^3}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(32\,A\,a^3+\frac{416\,B\,a^3}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{61\,A\,a^3}{2}+\frac{133\,B\,a^3}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{49\,A\,a^3}{4}+\frac{51\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{a^3\,\left(15\,A+13\,B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{4\,d}+\frac{a^3\,\mathrm{atan}\left(\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(15\,A+13\,B\right)}{4\,\left(\frac{15\,A\,a^3}{4}+\frac{13\,B\,a^3}{4}\right)}\right)\,\left(15\,A+13\,B\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((49*A*a^3)/4 + (51*B*a^3)/4) + tan(c/2 + (d*x)/2)^9*((
15*A*a^3)/4 + (13*B*a^3)/4) + tan(c/2 + (d*x)/2)^7*((35*A*a^3)/2 + (91*B*a^
3)/6) + tan(c/2 + (d*x)/2)^3*((61*A*a^3)/2 + (133*B*a^3)/6) + tan(c/2 + (d*
x)/2)^5*(32*A*a^3 + (416*B*a^3)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/
2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2
 + (d*x)/2)^10 + 1)) - (a^3*(15*A + 13*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)
/2))/(4*d) + (a^3*atan((a^3*tan(c/2 + (d*x)/2)*(15*A + 13*B))/(4*((15*A*a^3
)/4 + (13*B*a^3)/4)))*(15*A + 13*B))/(4*d)
```

3.21 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=116

$$\frac{5}{8}a^3(4A+3B)x+\frac{a^3(4A+3B)\sin(c+dx)}{d}+\frac{3a^3(4A+3B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{B(a+a\cos(c+dx))^3\sin}{4d}$$

```
[Out] 5/8*a^3*(4*A+3*B)*x+a^3*(4*A+3*B)*sin(d*x+c)/d+3/8*a^3*(4*A+3*B)*cos(d*x+c)
*sin(d*x+c)/d+1/4*B*(a+a*cos(d*x+c))^3*sin(d*x+c)/d-1/12*a^3*(4*A+3*B)*sin(
d*x+c)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2830, 2724, 2717, 2715, 8, 2713}

$$-\frac{a^3(4A+3B)\sin^3(c+dx)}{12d}+\frac{a^3(4A+3B)\sin(c+dx)}{d}+\frac{3a^3(4A+3B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3x(4A+3B)+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (5*a^3*(4*A + 3*B)*x)/8 + (a^3*(4*A + 3*B)*Sin[c + d*x])/d + (3*a^3*(4*A +
3*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (B*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(4*d) - (a^3*(4*A + 3*B)*Sin[c + d*x]^3)/(12*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}(4A+3B)\int (a+a$$

$$= \frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}(4A+3B)\int \left(a^3+\right.$$

$$= \frac{1}{4}a^3(4A+3B)x+\frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}($$

$$= \frac{1}{4}a^3(4A+3B)x+\frac{3a^3(4A+3B)\sin(c+dx)}{4d}+\frac{3a^3(4A+}{}$$

$$= \frac{5}{8}a^3(4A+3B)x+\frac{a^3(4A+3B)\sin(c+dx)}{d}+\frac{3a^3(4A+3}{}$$

Mathematica [A]
time = 0.34, size = 86, normalized size = 0.74

$$\frac{a^3(240Adx+180Bdx+24(15A+13B)\sin(c+dx)+24(3A+4B)\sin(2(c+dx))+8A\sin(3(c+dx))+24B\sin(3(c+dx))+3B\sin(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^3*(240*A*d*x + 180*B*d*x + 24*(15*A + 13*B)*Sin[c + d*x] + 24*(3*A + 4*B
)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 24*B*Sin[3*(c + d*x)] + 3*B*Sin
[4*(c + d*x)]))/(96*d)
```

Maple [A]
time = 0.13, size = 176, normalized size = 1.52

method	result
risch	$\frac{5a^3xA}{2}+\frac{15a^3Bx}{8}+\frac{15a^3A\sin(dx+c)}{4d}+\frac{13a^3B\sin(dx+c)}{4d}+\frac{\sin(4dx+4c)a^3B}{32d}+\frac{\sin(3dx+3c)A\,a^3}{12d}+\frac{\sin(3dx+3}{4d}$
derivativedivides	$\frac{a^3B\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{A\,a^3\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+a^3B\left(\cos^2(dx+c)+2\right)\sin(dx+c)+3}{a}$
default	$\frac{a^3B\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{A\,a^3\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+a^3B\left(\cos^2(dx+c)+2\right)\sin(dx+c)+3}{a}$
norman	$\frac{\frac{5a^3(4A+3B)x}{8}+\frac{73a^3(4A+3B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{55a^3(4A+3B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{5a^3(4A+3B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a^3(4A+3B)x(\tan}{2}}{(1+\mathrm{ta}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*A*a^3*(cos(d*x+c)^2+2)*sin(d*x+c)+a^3*B*(cos(d*x+c)^2+2)*sin(d*x+c)+3*A*a^
3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+3*a^3*B*(1/2*sin(d*x+c)*cos(d*x
+c)+1/2*d*x+1/2*c)+3*A*a^3*sin(d*x+c)+a^3*B*sin(d*x+c)+A*a^3*(d*x+c))
```

Maxima [A]

time = 0.27, size = 167, normalized size = 1.44

$$-\frac{32\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa^3-72\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^3-96\left(dx+c\right)Aa^3+96\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Ba^3-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)Ba^3-72\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba^3-288\,Aa^3\sin\left(dx+c\right)-96\,Ba^3\sin\left(dx+c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 72*(2*d*x + 2*c + sin(2
*d*x + 2*c))*A*a^3 - 96*(d*x + c)*A*a^3 + 96*(sin(d*x + c)^3 - 3*sin(d*x +
c))*B*a^3 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3
 - 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 - 288*A*a^3*sin(d*x + c) - 96*
B*a^3*sin(d*x + c))/d
```

Fricas [A]

time = 0.35, size = 90, normalized size = 0.78

$$\frac{15\,(4\,A+3\,B)a^3dx+\left(6\,Ba^3\cos\left(dx+c\right)^3+8\,(A+3\,B)a^3\cos\left(dx+c\right)^2+9\,(4\,A+5\,B)a^3\cos\left(dx+c\right)+8\,(11\,A+9\,B)a^3\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(15*(4*A + 3*B)*a^3*d*x + (6*B*a^3*cos(d*x + c)^3 + 8*(A + 3*B)*a^3*co
s(d*x + c)^2 + 9*(4*A + 5*B)*a^3*cos(d*x + c) + 8*(11*A + 9*B)*a^3)*sin(d*x
 + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(107) = 214$.
time = 0.22, size = 371, normalized size = 3.20

$$\begin{cases} \frac{3Aa^3x\sin^2(c+dx)}{2} + \frac{3Aa^3x\cos^2(c+dx)}{2} + Aa^3x + \frac{2Aa^3\sin^3(c+dx)}{3d} + \frac{Aa^3\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Aa^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{3Aa^3\sin(c+dx)}{d} + \frac{3Ba^3x\sin^4(c+dx)}{8} + \frac{3Ba^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Ba^3x\sin^2(c+dx)}{2} + \frac{3Ba^3x\cos^4(c+dx)}{8} + \frac{3Ba^3x\cos^2(c+dx)}{2} + \frac{3Ba^3\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2Ba^3\sin^3(c+dx)}{d} + \frac{5Ba^3\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{3Ba^3\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Ba^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{Ba^3\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x(A + B\cos(c))(a\cos(c) + a)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**2/2 + A*
a**3*x + 2*A*a**3*sin(c + d*x)**3/(3*d) + A*a**3*sin(c + d*x)*cos(c + d*x)*
*2/d + 3*A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*A*a**3*sin(c + d*x)/d +
 3*B*a**3*x*sin(c + d*x)**4/8 + 3*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*B*a**3*x*sin(c + d*x)**2/2 + 3*B*a**3*x*cos(c + d*x)**4/8 + 3*B*a**3*
x*cos(c + d*x)**2/2 + 3*B*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*B*a**
3*sin(c + d*x)**3/d + 5*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*B*a**
3*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a**3*sin(c + d*x)*cos(c + d*x)/(2*d)
 + B*a**3*sin(c + d*x)/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**3, T
rue))
```

Giac [A]
time = 0.49, size = 112, normalized size = 0.97

$$\frac{Ba^3\sin(4\,dx + 4\,c)}{32\,d} + \frac{5}{8}\left(4\,Aa^3 + 3\,Ba^3\right)x + \frac{\left(Aa^3 + 3\,Ba^3\right)\sin(3\,dx + 3\,c)}{12\,d} + \frac{\left(3\,Aa^3 + 4\,Ba^3\right)\sin(2\,dx + 2\,c)}{4\,d} + \frac{\left(15\,Aa^3 + 13\,Ba^3\right)\sin(dx + c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*B*a^3*sin(4*d*x + 4*c)/d + 5/8*(4*A*a^3 + 3*B*a^3)*x + 1/12*(A*a^3 + 3
*B*a^3)*sin(3*d*x + 3*c)/d + 1/4*(3*A*a^3 + 4*B*a^3)*sin(2*d*x + 2*c)/d + 1
/4*(15*A*a^3 + 13*B*a^3)*sin(d*x + c)/d
```

Mupad [B]
time = 0.27, size = 134, normalized size = 1.16

$$\frac{5\,A\,a^3\,x}{2} + \frac{15\,B\,a^3\,x}{8} + \frac{15\,A\,a^3\,\sin(c + d\,x)}{4\,d} + \frac{13\,B\,a^3\,\sin(c + d\,x)}{4\,d} + \frac{3\,A\,a^3\,\sin(2\,c + 2\,d\,x)}{4\,d} + \frac{A\,a^3\,\sin(3\,c + 3\,d\,x)}{12\,d} + \frac{B\,a^3\,\sin(2\,c + 2\,d\,x)}{d} + \frac{B\,a^3\,\sin(3\,c + 3\,d\,x)}{4\,d} + \frac{B\,a^3\,\sin(4\,c + 4\,d\,x)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (5*A*a^3*x)/2 + (15*B*a^3*x)/8 + (15*A*a^3*sin(c + d*x))/(4*d) + (13*B*a^3*
sin(c + d*x))/(4*d) + (3*A*a^3*sin(2*c + 2*d*x))/(4*d) + (A*a^3*sin(3*c + 3
*d*x))/(12*d) + (B*a^3*sin(2*c + 2*d*x))/d + (B*a^3*sin(3*c + 3*d*x))/(4*d)
 + (B*a^3*sin(4*c + 4*d*x))/(32*d)
```

3.22 $\int (a+a\cos (c+dx))^3 (A+B\cos (c+dx)) \sec (c+dx) \, dx$

Optimal. Leaf size=111

$$\frac{1}{2}a^3(7A+5B)x+\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{aB(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{(3$$

```
[Out] 1/2*a^3*(7*A+5*B)*x+a^3*A*arctanh(sin(d*x+c))/d+5/2*a^3*(A+B)*sin(d*x+c)/d+
1/3*a*B*(a+a*cos(d*x+c))^2*sin(d*x+c)/d+1/6*(3*A+5*B)*(a^3+a^3*cos(d*x+c))*
sin(d*x+c)/d
```

Rubi [A]
time = 0.20, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3055, 3047, 3102, 2814, 3855}

$$\frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{(3A+5B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}+\frac{1}{2}a^3x(7A+5B)+\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^3*(7*A + 5*B)*x)/2 + (a^3*A*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(A + B)*Si
n[c + d*x])/(2*d) + (a*B*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((3*A
 + 5*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(6*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
```

```
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx &= \frac{aB(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (a+\\ &= \frac{aB(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{(3A+5\\ &= \frac{aB(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{(3A+5\\ &= \frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{aB(a+a\cos(c+d}{3d}\\ &= \frac{1}{2}a^3(7A+5B)x+\frac{5a^3(A+B)\sin(c+dx)}{2d}+\\ &= \frac{1}{2}a^3(7A+5B)x+\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+\end{aligned}$$

Mathematica [A]
time = 0.29, size = 113, normalized size = 1.02

$$\frac{a^3\left(42Adx+30Bdx-12A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+9(4A+5B)\sin(c+dx)+3(A+3B)\sin(2(c+dx))+B\sin(3(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^3*(42*A*d*x + 30*B*d*x - 12*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 12*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*(4*A + 5*B)*Sin[c + d*x]
 + 3*(A + 3*B)*Sin[2*(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A]
time = 0.20, size = 147, normalized size = 1.32

method	result
derivativedivides	$\frac{A a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^3 B\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+3A a^3\sin(dx+c)+3a^3 B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right) \cdots}{d}$
default	$\frac{A a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^3 B\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+3A a^3\sin(dx+c)+3a^3 B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right) \cdots}{d}$
risch	$\frac{7a^3xA}{2}+\frac{5a^3Bx}{2}-\frac{3i\mathrm{e}^{i(dx+c)}A a^3}{2d}-\frac{15i\mathrm{e}^{i(dx+c)}a^3 B}{8d}+\frac{3i\mathrm{e}^{-i(dx+c)}A a^3}{2d}+\frac{15i\mathrm{e}^{-i(dx+c)}a^3 B}{8d}+\frac{A a^3\ln(\mathrm{e}^{i(dx+c} \cdots}{d}$
norman	$\left(\frac{7}{2}A a^3+\frac{5}{2}a^3 B\right)x+\left(\frac{7}{2}A a^3+\frac{5}{2}a^3 B\right)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(14A a^3+10a^3 B\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(14A a^3+10a^3 B\right)x\left(\tan^6\left(\frac{dx}{2} \cdots\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+1/3*a^3*B*(cos(d*x+c)^
2+2)*sin(d*x+c)+3*A*a^3*sin(d*x+c)+3*a^3*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d
*x+1/2*c)+3*A*a^3*(d*x+c)+3*a^3*B*sin(d*x+c)+A*a^3*ln(sec(d*x+c)+tan(d*x+c)
)+a^3*B*(d*x+c))
```

Maxima [A]
time = 0.27, size = 141, normalized size = 1.27

$$\frac{3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^3+36\left(dx+c\right)Aa^3-4\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Ba^3+9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba^3+12\left(dx+c\right)Ba^3+12\,Aa^3\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+36\,Aa^3\sin\left(dx+c\right)+36\,Ba^3\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 36*(d*x + c)*A*a^3 - 4*(si
n(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*B
*a^3 + 12*(d*x + c)*B*a^3 + 12*A*a^3*log(sec(d*x + c) + tan(d*x + c)) + 36*
A*a^3*sin(d*x + c) + 36*B*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 102, normalized size = 0.92

$$\frac{3\left(7\,A+5\,B\right)a^3dx+3\,Aa^3\log\left(\sin\left(dx+c\right)+1\right)-3\,Aa^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,Ba^3\cos\left(dx+c\right)^2+3\left(A+3\,B\right)a^3\cos\left(dx+c\right)+2\left(9\,A+11\,B\right)a^3\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(7*A + 5*B)*a^3*d*x + 3*A*a^3*log(sin(d*x + c) + 1) - 3*A*a^3*log(-s
in(d*x + c) + 1) + (2*B*a^3*cos(d*x + c)^2 + 3*(A + 3*B)*a^3*cos(d*x + c) +
 2*(9*A + 11*B)*a^3)*sin(d*x + c))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int A\sec{\left(c+dx\right)}\,dx+\int 3A\cos{\left(c+dx\right)}\sec{\left(c+dx\right)}\,dx+\int 3A\cos^{2}{\left(c+dx\right)}\sec{\left(c+dx\right)}\,dx+\int A\cos^{3}{\left(c+dx\right)}\sec{\left(c+dx\right)}\,dx+\int B\cos{\left(c+dx\right)}\sec{\left(c+dx\right)}\,dx+\int 3B\cos^{2}{\left(c+dx\right)}\sec{\left(c+dx\right)}\,dx+\int 3B\cos^{3}{\left(c+dx\right)}\sec{\left(c+dx\right)}\,dx+\int B\cos^{4}{\left(c+dx\right)}\sec{\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x), x) + Integral(3*A*cos(c + d*x)*sec(c + d*x),
 x) + Integral(3*A*cos(c + d*x)**2*sec(c + d*x), x) + Integral(A*cos(c + d*
x)**3*sec(c + d*x), x) + Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integra
l(3*B*cos(c + d*x)**2*sec(c + d*x), x) + Integral(3*B*cos(c + d*x)**3*sec(c
 + d*x), x) + Integral(B*cos(c + d*x)**4*sec(c + d*x), x))
```

Giac [**A**]
time = 0.45, size = 180, normalized size = 1.62

$$\frac{6\,Aa^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Aa^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\,(7\,Aa^{3}+5\,Ba^{3})(dx+c)+\frac{2\left(15\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+36\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+40\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+21\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+33\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/6*(6*A*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*A*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(7*A*a^3 + 5*B*a^3)*(d*x + c) + 2*(15*A*a^3*tan(1/2*d
*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^3*tan(1/2*d*x + 1/
2*c)^3 + 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*A*a^3*tan(1/2*d*x + 1/2*c) +
33*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [**B**]
time = 0.42, size = 178, normalized size = 1.60

$$\frac{3\,A\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{15\,B\,a^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{7\,A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{5\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,B\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a^3\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x),x)
```

```
[Out] (3*A*a^3*sin(c + d*x))/d + (15*B*a^3*sin(c + d*x))/(4*d) + (7*A*a^3*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*A*a^3*atanh(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)))/d + (5*B*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/d + (A*a^3*sin(2*c + 2*d*x))/(4*d) + (3*B*a^3*sin(2*c + 2*d*x))/(4*d)
+ (B*a^3*sin(3*c + 3*d*x))/(12*d)
```

3.23 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=110

$$\frac{1}{2}a^3(6A+7B)x+\frac{a^3(3A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^3B\sin(c+dx)}{2d}-\frac{(2A-B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+}{2d}$$

```
[Out] 1/2*a^3*(6*A+7*B)*x+a^3*(3*A+B)*arctanh(sin(d*x+c))/d+5/2*a^3*B*sin(d*x+c)/
d-1/2*(2*A-B)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d+a*A*(a+a*cos(d*x+c))^2*tan(
d*x+c)/d
```

Rubi [A]
time = 0.21, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3054, 3055, 3047, 3102, 2814, 3855}

$$\frac{a^3(3A+B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{1}{2}a^3x(6A+7B)+\frac{5a^3B\sin(c+dx)}{2d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^3*(6*A + 7*B)*x)/2 + (a^3*(3*A + B)*ArcTanh[Sin[c + d*x]])/d + (5*a^3*B*
Sin[c + d*x])/(2*d) - ((2*A - B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(2*
d) + (a*A*(a + a*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
```

```
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx &= \frac{aA(a+a\cos(c+dx))^2\tan(c+dx)}{d} + \int (a+a \\
&= -\frac{(2A-B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{2d} + \\
&= -\frac{(2A-B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{2d} + \\
&= \frac{5a^3B\sin(c+dx)}{2d} - \frac{(2A-B)\left(a^3+a^3\cos(c+\right.}{2d} \\
&= \frac{1}{2}a^3(6A+7B)x + \frac{5a^3B\sin(c+dx)}{2d} - \frac{(2A-\right.}{} \\
&= \frac{1}{2}a^3(6A+7B)x + \frac{a^3(3A+B)\tanh^{-1}(\sin(c+}{d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 272 vs. $2(110) = 220$.
time = 1.83, size = 272, normalized size = 2.47

$$
\frac{1}{32}a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(2(6A+7B)x - \frac{4(3A+B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{4(3A+B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{4(A+3B)\cos(dx)\sin(c)}{d} + \frac{B\cos(2dx)\sin(2c)}{d} + \frac{4(A+3B)\cos(c)\sin(dx)}{d} + \frac{B\cos(2c)\sin(2dx)}{d} + \frac{4A\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{4A\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(2*(6*A + 7*B)*x - (4*(3*A + B
)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (4*(3*A + B)*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]])/d + (4*(A + 3*B)*Cos[d*x]*Sin[c])/d + (B*Cos[2*
d*x]*Sin[2*c])/d + (4*(A + 3*B)*Cos[c]*Sin[d*x])/d + (B*Cos[2*c]*Sin[2*d*x]
)/d + (4*A*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])) + (4*A*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]))))/32
```

Maple [A]
time = 0.24, size = 128, normalized size = 1.16

method	result
derivativedivides	$\frac{A\,a^3\sin(dx+c)+a^3B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3A\,a^3(dx+c)+3a^3B\sin(dx+c)+3A\,a^3\ln(\sec(dx+c)+\tan(dx+c))+3}{d}$
default	$\frac{A\,a^3\sin(dx+c)+a^3B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3A\,a^3(dx+c)+3a^3B\sin(dx+c)+3A\,a^3\ln(\sec(dx+c)+\tan(dx+c))+3}{d}$
risch	$3a^3xA+\frac{7a^3Bx}{2}-\frac{ia^3B\,\mathrm{e}^{2i(dx+c)}}{8d}-\frac{i\mathrm{e}^{i(dx+c)}A\,a^3}{2d}-\frac{3i\mathrm{e}^{i(dx+c)}a^3B}{2d}+\frac{i\mathrm{e}^{-i(dx+c)}A\,a^3}{2d}+\frac{3i\mathrm{e}^{-i(dx+c)}a^3B}{2d}+$

norman	$\frac{\left(-\frac{7}{2}a^3B-3Aa^3\right)x+\left(-\frac{21}{2}a^3B-9Aa^3\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{7}{2}a^3B+3Aa^3\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{21}{2}a^3B+9Aa^3\right)x\left(\tan^8\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*sin(d*x+c)+a^3*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+3*A*a
^3*(d*x+c)+3*a^3*B*sin(d*x+c)+3*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*B*(d*
x+c)+A*a^3*tan(d*x+c)+a^3*B*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.27, size = 140, normalized size = 1.27

$$\frac{12(dx+c)Aa^3+(2dx+2c+\sin(2dx+2c))Ba^3+12(dx+c)Ba^3+6Aa^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2Ba^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+4Aa^3\sin(dx+c)+12Ba^3\sin(dx+c)+4Aa^3\tan(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/4*(12*(d*x + c)*A*a^3 + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 + 12*(d*x
+ c)*B*a^3 + 6*A*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*B*
a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*A*a^3*sin(d*x + c)
+ 12*B*a^3*sin(d*x + c) + 4*A*a^3*tan(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 127, normalized size = 1.15

$$\frac{(6A+7B)a^3dx\cos(dx+c)+(3A+B)a^3\cos(dx+c)\log(\sin(dx+c)+1)-(3A+B)a^3\cos(dx+c)\log(-\sin(dx+c)+1)+\left(Ba^3\cos(dx+c)^2+2(A+3B)a^3\cos(dx+c)+2Aa^3\right)\sin(dx+c)}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*((6*A + 7*B)*a^3*d*x*cos(d*x + c) + (3*A + B)*a^3*cos(d*x + c)*log(sin(
d*x + c) + 1) - (3*A + B)*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + (B*a^3*
cos(d*x + c)^2 + 2*(A + 3*B)*a^3*cos(d*x + c) + 2*A*a^3)*sin(d*x + c))/(d*c
os(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\sec^2(c+dx)\,dx+\int 3A\cos(c+dx)\sec^2(c+dx)\,dx+\int 3A\cos^2(c+dx)\sec^2(c+dx)\,dx+\int A\cos^3(c+dx)\sec^2(c+dx)\,dx+\int B\cos(c+dx)\sec^2(c+dx)\,dx+\int 3B\cos^2(c+dx)\sec^2(c+dx)\,dx+\int 3B\cos^3(c+dx)\sec^2(c+dx)\,dx+\int B\cos^4(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x)**2, x) + Integral(3*A*cos(c + d*x)*sec(c + d*
x)**2, x) + Integral(3*A*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(A*c
os(c + d*x)**3*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**
2, x) + Integral(3*B*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(3*B*cos
(c + d*x)**3*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x)**4*sec(c + d*x)*
*2, x))
```

Giac [A]
time = 0.46, size = 192, normalized size = 1.75

$$-\frac{\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-(6\,Aa^3+7\,Ba^3)(dx+c)-2\,(3\,Aa^3+Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+2\,(3\,Aa^3+Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(2\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (6*A*a^3
+ 7*B*a^3)*(d*x + c) - 2*(3*A*a^3 + B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) + 2*(3*A*a^3 + B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*A*a^3*ta
n(1/2*d*x + 1/2*c)^3 + 5*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 2*A*a^3*tan(1/2*d*x
 + 1/2*c) + 7*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 0.37, size = 197, normalized size = 1.79

$$\frac{A\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{3\,B\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{6\,A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{7\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{B\,a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] (A*a^3*sin(c + d*x))/d + (3*B*a^3*sin(c + d*x))/d + (6*A*a^3*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*A*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)))/d + (7*B*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
+ (2*B*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^3*sin(c +
 d*x))/(d*cos(c + d*x)) + (B*a^3*cos(c + d*x)*sin(c + d*x))/(2*d)
```

3.24 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=114

$$a^3(A+3B)x+\frac{a^3(7A+6B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5a^3A\sin(c+dx)}{2d}+\frac{(2A+B)\left(a^3+a^3\cos(c+dx)\right)\tan(c+dx)}{d}$$

```
[Out] a^3*(A+3*B)*x+1/2*a^3*(7*A+6*B)*arctanh(sin(d*x+c))/d-5/2*a^3*A*sin(d*x+c)/
d+(2*A+B)*(a^3+a^3*cos(d*x+c))*tan(d*x+c)/d+1/2*a*A*(a+a*cos(d*x+c))^2*sec(
d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.22, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3054, 3047, 3102, 2814, 3855}

$$\frac{a^3(7A+6B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2A+B)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{d}+a^3x(A+3B)-\frac{5a^3A\sin(c+dx)}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] a^3*(A + 3*B)*x + (a^3*(7*A + 6*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*A*
Sin[c + d*x])/(2*d) + ((2*A + B)*(a^3 + a^3*Cos[c + d*x])*Tan[c + d*x])/d +
 (a*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
```

```
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}$$
$$= \frac{(2A+B)\left(a^3+a^3\cos(c+dx)\right)\tan(c+dx)}{d} +$$
$$= \frac{(2A+B)\left(a^3+a^3\cos(c+dx)\right)\tan(c+dx)}{d} +$$
$$= -\frac{5a^3A\sin(c+dx)}{2d} + \frac{(2A+B)\left(a^3+a^3\cos(c\right.}{d}$$
$$= a^3(A+3B)x - \frac{5a^3A\sin(c+dx)}{2d} + \frac{(2A+B)(}{}$$
$$= a^3(A+3B)x + \frac{a^3(7A+6B)\tanh^{-1}(\sin(c+d}{2d}$$

Mathematica [A]
time = 1.94, size = 208, normalized size = 1.82

$$\frac{a^3\left(4Ac+12Bc+4Adx+12Bdx-14A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-12B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+14A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+12B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+4B\sin(c+dx)+4(3A+B)\tan(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^3*(4*A*c + 12*B*c + 4*A*d*x + 12*B*d*x - 14*A*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - 12*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 14*A*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*B*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + A/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - A/(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^2 + 4*B*Sin[c + d*x] + 4*(3*A + B)*Tan[c + d*x]))/(4*d)
```

Maple [A]
time = 0.36, size = 137, normalized size = 1.20

method	result
derivativedivides	$\frac{A\,a^3(dx+c)+a^3B\sin(dx+c)+3A\,a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3B(dx+c)+3A\,a^3\tan(dx+c)+3a^3B\ln(\sec(dx+c)+\mathrm{t}}{d}$
default	$\frac{A\,a^3(dx+c)+a^3B\sin(dx+c)+3A\,a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3B(dx+c)+3A\,a^3\tan(dx+c)+3a^3B\ln(\sec(dx+c)+\mathrm{t}}{d}$
risch	$a^3xA+3a^3Bx-\frac{i e^{i(dx+c)}a^3B}{2d}+\frac{i e^{-i(dx+c)}a^3B}{2d}-\frac{i a^3\left(A\,e^{3i(dx+c)}-6A\,e^{2i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)^2}$
norman	$\frac{(A\,a^3+3a^3B)x+(-4A\,a^3-12a^3B)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-A\,a^3-3a^3B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-A\,a^3-3a^3B)x\left(\tan^8\left(\frac{dx}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*(d*x+c)+a^3*B*sin(d*x+c)+3*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3
*B*(d*x+c)+3*A*a^3*tan(d*x+c)+3*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+A*a^3*(1/2*
sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^3*B*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 165, normalized size = 1.45

$$\frac{4\,(dx+c)Aa^3+12\,(dx+c)Ba^3-Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,Aa^3(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+6\,Ba^3(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+4\,Ba^3\sin\left(dx+c\right)+12\,Aa^3\tan\left(dx+c\right)+4\,Ba^3\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a^3 + 12*(d*x + c)*B*a^3 - A*a^3*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*A*a^3*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*B*a^3*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 4*B*a^3*sin(d*x + c) + 12*A*a^3*tan(d*x + c)
 + 4*B*a^3*tan(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 137, normalized size = 1.20

$$\frac{4\,(A+3\,B)a^3dx\cos\left(dx+c\right)^2+(7\,A+6\,B)a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(7\,A+6\,B)a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(2\,Ba^3\cos\left(dx+c\right)^2+2\,(3\,A+B)a^3\cos\left(dx+c\right)+Aa^3\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*(A + 3*B)*a^3*d*x*cos(d*x + c)^2 + (7*A + 6*B)*a^3*cos(d*x + c)^2*lo
g(sin(d*x + c) + 1) - (7*A + 6*B)*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
 + 2*(2*B*a^3*cos(d*x + c)^2 + 2*(3*A + B)*a^3*cos(d*x + c) + A*a^3)*sin(d*
x + c))/(d*cos(d*x + c)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int A\sec^{3}(c+dx)\,dx+\int 3A\cos(c+dx)\sec^{3}(c+dx)\,dx+\int 3A\cos^{2}(c+dx)\sec^{3}(c+dx)\,dx+\int A\cos^{3}(c+dx)\sec^{3}(c+dx)\,dx+\int B\cos(c+dx)\sec^{3}(c+dx)\,dx+\int 3B\cos^{2}(c+dx)\sec^{3}(c+dx)\,dx+\int 3B\cos^{3}(c+dx)\sec^{3}(c+dx)\,dx+\int B\cos^{4}(c+dx)\sec^{3}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x)**3, x) + Integral(3*A*cos(c + d*x)*sec(c + d*
x)**3, x) + Integral(3*A*cos(c + d*x)**2*sec(c + d*x)**3, x) + Integral(A*c
os(c + d*x)**3*sec(c + d*x)**3, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**
3, x) + Integral(3*B*cos(c + d*x)**2*sec(c + d*x)**3, x) + Integral(3*B*cos
(c + d*x)**3*sec(c + d*x)**3, x) + Integral(B*cos(c + d*x)**4*sec(c + d*x)*
*3, x))
```

Giac [**A**]
time = 0.51, size = 192, normalized size = 1.68

$$\frac{\frac{4\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+2\,(Aa^{3}+3\,Ba^{3})(dx+c)+(7\,Aa^{3}+6\,Ba^{3})\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(7\,Aa^{3}+6\,Ba^{3})\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(5\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-7\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*B*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(A*a^3 +
 3*B*a^3)*(d*x + c) + (7*A*a^3 + 6*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - (7*A*a^3 + 6*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*A*a^3*tan
(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*A*a^3*tan(1/2*d*x
+ 1/2*c) - 2*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [**B**]
time = 0.37, size = 207, normalized size = 1.82

$$\frac{B\,a^{3}\,\sin\left(c+d\,x\right)}{d}+\frac{2\,A\,a^{3}\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{7\,A\,a^{3}\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,B\,a^{3}\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,B\,a^{3}\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,A\,a^{3}\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{A\,a^{3}\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^{2}}+\frac{B\,a^{3}\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^3,x)
```

```
[Out] (B*a^3*sin(c + d*x))/d + (2*A*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d + (7*A*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*B*a^3
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*B*a^3*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*A*a^3*sin(c + d*x))/(d*cos(c + d*x)) +
 (A*a^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (B*a^3*sin(c + d*x))/(d*cos(c
+ d*x))
```

3.25 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=125

$$a^3Bx+\frac{a^3(5A+7B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^3(A+B)\tan(c+dx)}{2d}+\frac{(5A+3B)\left(a^3+a^3\cos(c+dx)\right)\sec(c+}{6d}$$

```
[Out] a^3*B*x+1/2*a^3*(5*A+7*B)*arctanh(sin(d*x+c))/d+5/2*a^3*(A+B)*tan(d*x+c)/d+
1/6*(5*A+3*B)*(a^3+a^3*cos(d*x+c))*sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*(a+a*cos
(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.23, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3054, 3047, 3100, 2814, 3855}

$$\frac{5a^3(A+B)\tan(c+dx)}{2d}+\frac{a^3(5A+7B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(5A+3B)\tan(c+dx)\sec(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}+a^3Bx+\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] a^3*B*x + (a^3*(5*A + 7*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*(A + B)*Ta
n[c + d*x])/(2*d) + ((5*A + 3*B)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]*Tan[
c + d*x])/(6*d) + (a*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/
(3*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
```

```
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= \frac{aA(a+a\cos(c+dx))^2\sec^2(c+dx)\tan(c+d \\
&= \frac{(5A+3B)\left(a^3+a^3\cos(c+dx)\right)\sec(c+dx)\,\mathrm{t} \\
&= \frac{(5A+3B)\left(a^3+a^3\cos(c+dx)\right)\sec(c+dx)\,\mathrm{t} \\
&= \frac{5a^3(A+B)\tan(c+dx)}{2d}+\frac{(5A+3B)\left(a^3+a \\
&= a^3Bx+\frac{5a^3(A+B)\tan(c+dx)}{2d}+\frac{(5A+3B \\
&= a^3Bx+\frac{a^3(5A+7B)\tanh^{-1}(\sin(c+dx))}{2d}+
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 786 vs. 2(125) = 250.
time = 6.42, size = 786, normalized size = 6.29

$$
\frac{1}{8}Bx(a+a\cos(c+dx))^3\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{(-5A-7B)(a+a\cos(c+dx))^3\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{16d}+\frac{(5A+7B)(a+a\cos(c+dx))^3\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{16d}+\frac{A(a+a\cos(c+dx))^3\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{48d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}+\frac{(a+a\cos(c+dx))^3\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)\left(10A\cos\left(\frac{c}{2}\right)+3B\cos\left(\frac{c}{2}\right)-8A\sin\left(\frac{c}{2}\right)-3B\sin\left(\frac{c}{2}\right)\right)}{96d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}+\frac{(a+a\cos(c+dx))^3\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)\left(11A\sin\left(\frac{dx}{2}\right)+9B\sin\left(\frac{dx}{2}\right)\right)}{24d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{A(a+a\cos(c+dx))^3\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{48d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}+\frac{(a+a\cos(c+dx))^3\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-10A\cos\left(\frac{c}{2}\right)-3B\cos\left(\frac{c}{2}\right)-8A\sin\left(\frac{c}{2}\right)-3B\sin\left(\frac{c}{2}\right)\right)}{96d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}+\frac{(a+a\cos(c+dx))^3\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)\left(11A\sin\left(\frac{dx}{2}\right)+9B\sin\left(\frac{dx}{2}\right)\right)}{24d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (B*x*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6)/8 + ((-5*A - 7*B)*(a + a*
Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x
)/2]^6)/(16*d) + ((5*A + 7*B)*(a + a*Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6)/(16*d) + (A*(a + a*Cos[c + d*x
])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(48*d*(Cos[c/2] - Sin[c/2])*(Cos[c/
2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (
d*x)/2]^6*(10*A*Cos[c/2] + 3*B*Cos[c/2] - 8*A*Sin[c/2] - 3*B*Sin[c/2]))/(96
*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + ((a
 + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(11*A*Sin[(d*x)/2] + 9*B*Sin[(d*x
)/2]))/(24*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]
)) + (A*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(48*d*(Co
s[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + ((a + a*C
os[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-10*A*Cos[c/2] - 3*B*Cos[c/2] - 8*A*Si
n[c/2] - 3*B*Sin[c/2]))/(96*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + S
in[c/2 + (d*x)/2])^2) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(11*A*
Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/(24*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d
*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A]
time = 0.28, size = 176, normalized size = 1.41

method	result
derivativedivides	$\frac{A a^3 \ln(\sec(dx+c)+\tan(dx+c))+a^3 B(dx+c)+3A a^3 \tan(dx+c)+3a^3 B \ln(\sec(dx+c)+\tan(dx+c))+3A a^3 \left(\frac{\sec(dx+c)\tan(d}{2}\right.}{}$
default	$\frac{A a^3 \ln(\sec(dx+c)+\tan(dx+c))+a^3 B(dx+c)+3A a^3 \tan(dx+c)+3a^3 B \ln(\sec(dx+c)+\tan(dx+c))+3A a^3 \left(\frac{\sec(dx+c)\tan(d}{2}\right.}{}$
risch	$a^3 Bx - \frac{i a^3 (9A\,e^{5i(dx+c)}+3B\,e^{5i(dx+c)}-18A\,e^{4i(dx+c)}-18B\,e^{4i(dx+c)}-48A\,e^{2i(dx+c)}-36B\,e^{2i(dx+c)}-9A\,e^{i(dx+c)}-}{3d\left(e^{2i(dx+c)}+1\right)^3}$
norman	$\frac{a^3 Bx\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^3 Bx\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^3 Bx-a^3 Bx\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a^3 Bx\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a^3 Bx\left(\tan^{6}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*B*(d*x+c)+3*A*a^3*tan(d*x+c)+3*a^3
*B*ln(sec(d*x+c)+tan(d*x+c))+3*A*a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(
d*x+c)+tan(d*x+c)))+3*a^3*B*tan(d*x+c)-A*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*
x+c)+a^3*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 212, normalized size = 1.70

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Aa^{3}+12\left(dx+c\right)Ba^{3}-9\,Aa^{3}\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ba^{3}\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,Aa^{3}(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+18\,Ba^{3}(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+36\,Aa^{3}\tan\left(dx+c\right)+36\,Ba^{3}\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 12*(d*x + c)*B*a^3 - 9*A*
a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 3*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 6*A*a^3*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1)) + 18*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)) + 36*A*a^3*tan(d*x + c) + 36*B*a^3*tan(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 141, normalized size = 1.13

$$\frac{12\,Ba^{3}dx\cos\left(dx+c\right)^{3}+3\left(5\,A+7\,B\right)a^{3}\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\left(5\,A+7\,B\right)a^{3}\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\left(11\,A+9\,B\right)a^{3}\cos\left(dx+c\right)^{2}+3\left(3\,A+B\right)a^{3}\cos\left(dx+c\right)+2\,Aa^{3}\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(12*B*a^3*d*x*cos(d*x + c)^3 + 3*(5*A + 7*B)*a^3*cos(d*x + c)^3*log(si
n(d*x + c) + 1) - 3*(5*A + 7*B)*a^3*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
 2*(2*(11*A + 9*B)*a^3*cos(d*x + c)^2 + 3*(3*A + B)*a^3*cos(d*x + c) + 2*A*
a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 189, normalized size = 1.51

$$\frac{6\left(dx+c\right)Ba^{3}+3\left(5\,Aa^{3}+7\,Ba^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(5\,Aa^{3}+7\,Ba^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-40\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-36\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+33\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(d*x + c)*B*a^3 + 3*(5*A*a^3 + 7*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3*(5*A*a^3 + 7*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*A
*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 36*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 33*A*a^3*tan(1/2*d
*x + 1/2*c) + 21*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3
)/d
```

Mupad [B]
time = 0.33, size = 209, normalized size = 1.67

$$\frac{5\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{7\,B\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{11\,A\,a^3\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)}+\frac{3\,A\,a^3\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}+\frac{A\,a^3\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3}+\frac{3\,B\,a^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{B\,a^3\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] (5*A*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^3*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (7*B*a^3*atanh(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)))/d + (11*A*a^3*sin(c + d*x))/(3*d*cos(c + d*x)) + (3*A
*a^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (A*a^3*sin(c + d*x))/(3*d*cos(c +
 d*x)^3) + (3*B*a^3*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^3*sin(c + d*x))/(
2*d*cos(c + d*x)^2)
```

3.26 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=154

$$\frac{5a^3(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(9A+11B)\tan(c+dx)}{3d}+\frac{a^3(27A+28B)\sec(c+dx)\tan(c+dx)}{24d}+$$

```
[Out] 5/8*a^3*(3*A+4*B)*arctanh(sin(d*x+c))/d+1/3*a^3*(9*A+11*B)*tan(d*x+c)/d+1/2
4*a^3*(27*A+28*B)*sec(d*x+c)*tan(d*x+c)/d+1/6*(3*A+2*B)*(a^3+a^3*cos(d*x+c)
)*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a*A*(a+a*cos(d*x+c))^2*sec(d*x+c)^3*tan(d*x
+c)/d
```

Rubi [A]
time = 0.27, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3054, 3047, 3100, 2827, 3852, 8, 3855}

$$\frac{a^3(9A+11B)\tan(c+dx)}{3d}+\frac{5a^3(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(27A+28B)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(3A+2B)\tan(c+dx)\sec^2(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}+\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (5*a^3*(3*A + 4*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(9*A + 11*B)*Tan[c +
 d*x])/(3*d) + (a^3*(27*A + 28*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((3*A
 + 2*B)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a*A*
(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^2\sec^3(c+dx)\tan(c+d}{4d}$$
$$= \frac{(3A+2B)\left(a^3+a^3\cos(c+dx)\right)\sec^2(c+dx)}{6d}$$
$$= \frac{(3A+2B)\left(a^3+a^3\cos(c+dx)\right)\sec^2(c+dx)}{6d}$$
$$= \frac{a^3(27A+28B)\sec(c+dx)\tan(c+dx)}{24d} + \frac{(3A}{}$$
$$= \frac{a^3(27A+28B)\sec(c+dx)\tan(c+dx)}{24d} + \frac{(3A}{}$$
$$= \frac{5a^3(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(27A}{}$$
$$= \frac{5a^3(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(9A+}{}$$

Mathematica [A]
time = 1.43, size = 273, normalized size = 1.77

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(120(3A+4B)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(-24(9A+11B)\sin(c)+(69A+36B)\sin(dx)+69A\sin(2c+dx)+36B\sin(2c+dx)+264A\sin(c+2dx)+280B\sin(c+2dx)-24A\sin(3c+2dx)-72B\sin(3c+2dx)+45A\sin(2c+3dx)+36B\sin(2c+3dx)+45A\sin(4c+3dx)+36B\sin(4c+3dx)+72A\sin(3c+4dx)+88B\sin(3c+4dx))\right)}{1536d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] -1/1536*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^4*(120*(3
*A + 4*B)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-24*(9*A + 11*B)*Sin[c] + (69
*A + 36*B)*Sin[d*x] + 69*A*Sin[2*c + d*x] + 36*B*Sin[2*c + d*x] + 264*A*Sin
[c + 2*d*x] + 280*B*Sin[c + 2*d*x] - 24*A*Sin[3*c + 2*d*x] - 72*B*Sin[3*c +
 2*d*x] + 45*A*Sin[2*c + 3*d*x] + 36*B*Sin[2*c + 3*d*x] + 45*A*Sin[4*c + 3*
d*x] + 36*B*Sin[4*c + 3*d*x] + 72*A*Sin[3*c + 4*d*x] + 88*B*Sin[3*c + 4*d*x
])))/d
```

Maple [A]
time = 0.31, size = 219, normalized size = 1.42

method	result
derivativedivides	$\frac{A\,a^3\tan(dx+c)+a^3B\ln(\sec(dx+c)+\tan(dx+c))+3A\,a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3B\tan(dx}{}$
default	$\frac{A\,a^3\tan(dx+c)+a^3B\ln(\sec(dx+c)+\tan(dx+c))+3A\,a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3B\tan(dx}{}$

risch	$-\frac{ia^3(45A\,e^{7i(dx+c)}+36B\,e^{7i(dx+c)}-24A\,e^{6i(dx+c)}-72B\,e^{6i(dx+c)}+69A\,e^{5i(dx+c)}+36B\,e^{5i(dx+c)}-216A\,e^{4i(dx+c)}-264}{12d(e^{2i(d}}$
norman	$\frac{\frac{a^3(49A+44B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{a^3(81A+44B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{a^3(111A+404B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{a^3(369A+236B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*tan(d*x+c)+a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3*A*a^3*(1/2*sec(d*x+
c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*B*tan(d*x+c)-3*A*a^3*(-2
/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a^3*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(s
ec(d*x+c)+tan(d*x+c)))+A*a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c
)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-a^3*B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 269, normalized size = 1.75

$$\frac{48\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa^3+16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ba^3-3\,Aa^3\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-36\,Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-36\,Ba^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+24\,Ba^3\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+48\,Aa^3\tan\left(dx+c\right)+144\,Ba^3\tan\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(48*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^3 - 3*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 36*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 36*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*B*a^3*(log(sin(
d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*A*a^3*tan(d*x + c) + 144*B*a^3*
tan(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 145, normalized size = 0.94

$$\frac{15\left(3\,A+4\,B\right)a^3\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-15\left(3\,A+4\,B\right)a^3\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\left(9\,A+11\,B\right)a^3\cos\left(dx+c\right)^3+9\left(5\,A+4\,B\right)a^3\cos\left(dx+c\right)^2+8\left(3\,A+B\right)a^3\cos\left(dx+c\right)+6\,Aa^3\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(15*(3*A + 4*B)*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*(3*A + 4
*B)*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(9*A + 11*B)*a^3*cos(d
```

```
*x + c)^3 + 9*(5*A + 4*B)*a^3*cos(d*x + c)^2 + 8*(3*A + B)*a^3*cos(d*x + c)
 + 6*A*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.48, size = 212, normalized size = 1.38

$$\frac{15\left(3Aa^3+4Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(3Aa^3+4Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(45Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+60Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-165Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-220Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+219Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+292Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-147Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-132Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(15*(3*A*a^3 + 4*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*A*a
^3 + 4*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*A*a^3*tan(1/2*d*x
+ 1/2*c)^7 + 60*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 165*A*a^3*tan(1/2*d*x + 1/2*
c)^5 - 220*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 219*A*a^3*tan(1/2*d*x + 1/2*c)^3
+ 292*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 147*A*a^3*tan(1/2*d*x + 1/2*c) - 132*B
*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 2.71, size = 185, normalized size = 1.20

$$\frac{\left(-\frac{15Aa^3}{4}-5Ba^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{55Aa^3}{4}+\frac{55Ba^3}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\frac{73Aa^3}{4}-\frac{73Ba^3}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(\frac{49Aa^3}{4}+11Ba^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{5a^3\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(3A+4B\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((49*A*a^3)/4 + 11*B*a^3) - tan(c/2 + (d*x)/2)^7*((15*A
*a^3)/4 + 5*B*a^3) + tan(c/2 + (d*x)/2)^5*((55*A*a^3)/4 + (55*B*a^3)/3) - t
an(c/2 + (d*x)/2)^3*((73*A*a^3)/4 + (73*B*a^3)/3))/(d*(6*tan(c/2 + (d*x)/2)
^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8
 + 1)) + (5*a^3*atanh(tan(c/2 + (d*x)/2))*(3*A + 4*B))/(4*d)
```

3.27 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=185

$$\frac{a^3(13A+15B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(38A+45B)\tan(c+dx)}{15d}+\frac{a^3(13A+15B)\sec(c+dx)\tan(c+dx)}{8d}+$$

```
[Out] 1/8*a^3*(13*A+15*B)*arctanh(sin(d*x+c))/d+1/15*a^3*(38*A+45*B)*tan(d*x+c)/d
+1/8*a^3*(13*A+15*B)*sec(d*x+c)*tan(d*x+c)/d+1/60*a^3*(43*A+45*B)*sec(d*x+c
)^2*tan(d*x+c)/d+1/20*(7*A+5*B)*(a^3+a^3*cos(d*x+c))*sec(d*x+c)^3*tan(d*x+c
)/d+1/5*a*A*(a+a*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d
```

Rubi [A]
time = 0.29, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3054, 3047, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{a^3(38A+45B)\tan(c+dx)}{15d}+\frac{a^3(13A+15B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(43A+45B)\tan(c+dx)\sec^2(c+dx)}{60d}+\frac{a^3(13A+15B)\tan(c+dx)\sec(c+dx)}{8d}+\frac{(7A+5B)\tan(c+dx)\sec^3(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{20d}+\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (a^3*(13*A + 15*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(38*A + 45*B)*Tan[c
+ d*x])/(15*d) + (a^3*(13*A + 15*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3
*(43*A + 45*B)*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) + ((7*A + 5*B)*(a^3 + a^
3*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*A*(a + a*Cos[c + d
*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx}{5d}$$

$$= \frac{(7A+5B)\left(a^3+a^3\cos(c+dx)\right)\sec^3(c+dx)\,\mathrm{ta}}{20d}$$

$$= \frac{(7A+5B)\left(a^3+a^3\cos(c+dx)\right)\sec^3(c+dx)\,\mathrm{ta}}{20d}$$

$$= \frac{a^3(43A+45B)\sec^2(c+dx)\tan(c+dx)}{60d} + \frac{(7A}{}$$

$$= \frac{a^3(43A+45B)\sec^2(c+dx)\tan(c+dx)}{60d} + \frac{(7A}{}$$

$$= \frac{a^3(13A+15B)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^3(4}{}$$

$$= \frac{a^3(13A+15B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(38A}{}$$

Mathematica [A]
time = 1.54, size = 294, normalized size = 1.59

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(240(13A+15B)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(80(29A+30B)\sin(dx)-240(3A+5B)\sin(2c+dx)+750A\sin(c+2dx)+570B\sin(c+2dx)+750A\sin(3c+2dx)+570B\sin(3c+2dx)+1520A\sin(2c+3dx)+1680B\sin(2c+3dx)-120B\sin(4c+3dx)+195A\sin(3c+4dx)+225B\sin(3c+4dx)+195A\sin(5c+4dx)+225B\sin(5c+4dx)+304A\sin(4c+5dx)+360B\sin(4c+5dx))\right)}{15360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] -1/15360*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^5*(240*(
13*A + 15*B)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(29*A + 30*B)*Sin[d*x]
- 240*(3*A + 5*B)*Sin[2*c + d*x] + 750*A*Sin[c + 2*d*x] + 570*B*Sin[c + 2*d
*x] + 750*A*Sin[3*c + 2*d*x] + 570*B*Sin[3*c + 2*d*x] + 1520*A*Sin[2*c + 3*
d*x] + 1680*B*Sin[2*c + 3*d*x] - 120*B*Sin[4*c + 3*d*x] + 195*A*Sin[3*c + 4
*d*x] + 225*B*Sin[3*c + 4*d*x] + 195*A*Sin[5*c + 4*d*x] + 225*B*Sin[5*c + 4
*d*x] + 304*A*Sin[4*c + 5*d*x] + 360*B*Sin[4*c + 5*d*x]))/d
```

Maple [A]
time = 0.26, size = 271, normalized size = 1.46

method	result
derivativedivides	$\frac{A\,a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^3B\tan(dx+c)-3A\,a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+3a^3B\left(\mathrm{sec}\right.}{}$
default	$\frac{A\,a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^3B\tan(dx+c)-3A\,a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+3a^3B\left(\mathrm{sec}\right.}{}$

risch	$-\frac{i a^3 (195 A e^{9i(dx+c)}+225 B e^{9i(dx+c)}-120 B e^{8i(dx+c)}+750 A e^{7i(dx+c)}+570 B e^{7i(dx+c)}-720 A e^{6i(dx+c)}-1200 B e^{6i(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^3*B*
tan(d*x+c)-3*A*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a^3*B*(1/2*sec(d*x+
c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*A*a^3*(-(-1/4*sec(d*x+c)^3-3
/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-3*a^3*B*(-2/3-1/3*
sec(d*x+c)^2)*tan(d*x+c)-A*a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*t
an(d*x+c)+a^3*B*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(
d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.30, size = 337, normalized size = 1.82

$$\frac{16 \left(3 \tan (dx+c)^5+10 \tan (dx+c)^3+15 \tan (dx+c)\right)Aa^3+240 \left(\tan (dx+c)^3+3 \tan (dx+c)\right)Aa^3+240 \left(\tan (dx+c)^3+3 \tan (dx+c)\right)Ba^3-45\,Aa^3\left(\frac{2 \left(3 \sin(dx+c)^3-5 \sin(dx+c)\right)}{\sin(dx+c)^4-2 \sin(dx+c)^2+1}-3 \log (\sin (dx+c)+1)+3 \log (\sin (dx+c)-1)\right)-15\,Ba^3\left(\frac{2 \left(3 \sin(dx+c)^3-5 \sin(dx+c)\right)}{\sin(dx+c)^4-2 \sin(dx+c)^2+1}-3 \log (\sin (dx+c)+1)+3 \log (\sin (dx+c)-1)\right)-60\,Aa^3\left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1}-\log (\sin (dx+c)+1)+\log (\sin (dx+c)-1)\right)-180\,Ba^3\left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1}-\log (\sin (dx+c)+1)+\log (\sin (dx+c)-1)\right)+240\,Ba^3 \tan (dx+c)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
240*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^3 - 45*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 15*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 60*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) - 180*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - l
og(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*B*a^3*tan(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 165, normalized size = 0.89

$$\frac{15\,(13\,A+15\,B)a^3 \cos (dx+c)^5 \log (\sin (dx+c)+1)-15\,(13\,A+15\,B)a^3 \cos (dx+c)^5 \log (-\sin (dx+c)+1)+2\left(8\,(38\,A+45\,B)a^3 \cos (dx+c)^4+15\,(13\,A+15\,B)a^3 \cos (dx+c)^3+8\,(19\,A+15\,B)a^3 \cos (dx+c)^2+30\,(3\,A+B)a^3 \cos (dx+c)+24\,Aa^3\right) \sin (dx+c)}{240\,d \cos (dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(13*A + 15*B)*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(13*A
 + 15*B)*a^3*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(38*A + 45*B)*a^3
```

```
*cos(d*x + c)^4 + 15*(13*A + 15*B)*a^3*cos(d*x + c)^3 + 8*(19*A + 15*B)*a^3
*cos(d*x + c)^2 + 30*(3*A + B)*a^3*cos(d*x + c) + 24*A*a^3)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.50, size = 246, normalized size = 1.33

$$\frac{15\,(13\,Aa^3+15\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(13\,Aa^3+15\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(195\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+225\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-910\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1050\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1664\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1920\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1330\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1830\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+765\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+735\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(13*A*a^3 + 15*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(13
*A*a^3 + 15*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(195*A*a^3*tan(1/
2*d*x + 1/2*c)^9 + 225*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*A*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 1050*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*A*a^3*tan(1/2*d*x +
1/2*c)^5 + 1920*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*A*a^3*tan(1/2*d*x + 1/2
*c)^3 - 1830*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*A*a^3*tan(1/2*d*x + 1/2*c)
+ 735*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 2.82, size = 224, normalized size = 1.21

$$\frac{a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(13\,A+15\,B\right)}{4\,d}-\frac{\left(\frac{13\,A\,a^3}{4}+\frac{15\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{91\,A\,a^3}{6}-\frac{35\,B\,a^3}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{416\,A\,a^3}{15}+32\,B\,a^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{133\,A\,a^3}{6}-\frac{61\,B\,a^3}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{51\,A\,a^3}{4}+\frac{49\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^6,x)
```

```
[Out] (a^3*atanh(tan(c/2 + (d*x)/2))*(13*A + 15*B))/(4*d) - (tan(c/2 + (d*x)/2)*(
(51*A*a^3)/4 + (49*B*a^3)/4) + tan(c/2 + (d*x)/2)^9*((13*A*a^3)/4 + (15*B*a
^3)/4) - tan(c/2 + (d*x)/2)^7*((91*A*a^3)/6 + (35*B*a^3)/2) - tan(c/2 + (d*
x)/2)^3*((133*A*a^3)/6 + (61*B*a^3)/2) + tan(c/2 + (d*x)/2)^5*((416*A*a^3)/
15 + 32*B*a^3))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*t
an(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.28 $\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=241

$$\frac{1}{16}a^4(49A+44B)x+\frac{a^4(252A+227B)\sin(c+dx)}{35d}+\frac{a^4(49A+44B)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^4(301A+276B)}{}$$

```
[Out] 1/16*a^4*(49*A+44*B)*x+1/35*a^4*(252*A+227*B)*sin(d*x+c)/d+1/16*a^4*(49*A+4
4*B)*cos(d*x+c)*sin(d*x+c)/d+1/280*a^4*(301*A+276*B)*cos(d*x+c)^3*sin(d*x+c
)/d+1/7*a*B*cos(d*x+c)^3*(a+a*cos(d*x+c))^3*sin(d*x+c)/d+1/42*(7*A+10*B)*co
s(d*x+c)^3*(a^2+a^2*cos(d*x+c))^2*sin(d*x+c)/d+7/15*(A+B)*cos(d*x+c)^3*(a^4
+a^4*cos(d*x+c))*sin(d*x+c)/d-1/105*a^4*(252*A+227*B)*sin(d*x+c)^3/d
```

Rubi [A]

time = 0.38, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3055, 3047, 3102, 2827, 2715, 8, 2713}

$$-\frac{a^4(252A+227B)\sin^3(c+dx)}{105d}+\frac{a^4(252A+227B)\sin(c+dx)}{35d}+\frac{a^4(301A+276B)\sin(c+dx)\cos^3(c+dx)}{280d}+\frac{7(A+B)\sin(c+dx)\cos^3(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{15d}+\frac{a^4(49A+44B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}a^4x(49A+44B)+\frac{(7A+10B)\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{42d}+\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^4*(49*A + 44*B)*x)/16 + (a^4*(252*A + 227*B)*Sin[c + d*x])/(35*d) + (a^4
*(49*A + 44*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^4*(301*A + 276*B)*Cos
[c + d*x]^3*Sin[c + d*x])/(280*d) + (a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x]
)^3*Sin[c + d*x])/(7*d) + ((7*A + 10*B)*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d
*x])^2*Sin[c + d*x])/(42*d) + (7*(A + B)*Cos[c + d*x]^3*(a^4 + a^4*Cos[c +
d*x])*Sin[c + d*x])/(15*d) - (a^4*(252*A + 227*B)*Sin[c + d*x]^3)/(105*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+d}{7d}$$
$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+d}{7d}$$
$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+d}{7d}$$
$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+d}{7d}$$
$$= \frac{a^4(301A+276B)\cos^3(c+dx)\sin(c+dx)}{280d} +$$
$$= \frac{a^4(301A+276B)\cos^3(c+dx)\sin(c+dx)}{280d} +$$
$$= \frac{a^4(49A+44B)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^4(}{}$$
$$= \frac{1}{16}a^4(49A+44B)x + \frac{a^4(252A+227B)\sin(c}{35d}$$

Mathematica [A]
time = 0.88, size = 156, normalized size = 0.65

$$\frac{a^4(18480Bc+20580Adx+18480Bdx+105(352A+323B)\sin(c+dx)+105(127A+124B)\sin(2(c+dx))+5040A\sin(3(c+dx))+5495B\sin(3(c+dx))+1575A\sin(4(c+dx))+2100B\sin(4(c+dx))+336A\sin(5(c+dx))+651B\sin(5(c+dx))+35A\sin(6(c+dx))+140B\sin(6(c+dx))+15B\sin(7(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^4*(18480*B*c + 20580*A*d*x + 18480*B*d*x + 105*(352*A + 323*B)*Sin[c + d
*x] + 105*(127*A + 124*B)*Sin[2*(c + d*x)] + 5040*A*Sin[3*(c + d*x)] + 5495
*B*Sin[3*(c + d*x)] + 1575*A*Sin[4*(c + d*x)] + 2100*B*Sin[4*(c + d*x)] + 3
36*A*Sin[5*(c + d*x)] + 651*B*Sin[5*(c + d*x)] + 35*A*Sin[6*(c + d*x)] + 14
0*B*Sin[6*(c + d*x)] + 15*B*Sin[7*(c + d*x)]))/(6720*d)
```

Maple [A]
time = 0.28, size = 358, normalized size = 1.49 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/7*a^4*B*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+
```

```
c)^2)*sin(d*x+c)+4/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4
*a^4*B*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)+6*A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+
3/8*c)+6/5*a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4/3*A*a^4*(
cos(d*x+c)^2+2)*sin(d*x+c)+4*a^4*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d
*x+c)+3/8*d*x+3/8*c)+A*a^4*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+1/3*a^
4*B*(cos(d*x+c)^2+2)*sin(d*x+c))
```

Maxima [**A**]
time = 0.27, size = 356, normalized size = 1.48

$$\frac{1792\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))Aa^4-35\,(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c))Aa^4-8960\,(\sin(dx+c)^3-3\,\sin(dx+c))Aa^4+1260\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^4+1680\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^4-192\,(5\,\sin(dx+c)^7-21\,\sin(dx+c)^5+35\,\sin(dx+c)^3-35\,\sin(dx+c))Ba^4+2688\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))Ba^4-140\,(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c))Ba^4-2240\,(\sin(dx+c)^3-3\,\sin(dx+c))Ba^4+840\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Ba^4}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/6720*(1792*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4
 - 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2
*d*x + 2*c))*A*a^4 - 8960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 1260*(1
2*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 1680*(2*d*x +
 2*c + sin(2*d*x + 2*c))*A*a^4 - 192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5
+ 35*sin(d*x + c)^3 - 35*sin(d*x + c))*B*a^4 + 2688*(3*sin(d*x + c)^5 - 10*
sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^4 - 140*(4*sin(2*d*x + 2*c)^3 - 60*d*
x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*a^4 - 2240*(sin(d*x
+ c)^3 - 3*sin(d*x + c))*B*a^4 + 840*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*
sin(2*d*x + 2*c))*B*a^4)/d
```

Fricas [**A**]
time = 0.36, size = 150, normalized size = 0.62

$$\frac{105\,(49\,A+44\,B)a^4dx+\left(240\,Ba^4\cos(dx+c)^6+280\,(A+4\,B)a^4\cos(dx+c)^5+192\,(7\,A+12\,B)a^4\cos(dx+c)^4+70\,(41\,A+44\,B)a^4\cos(dx+c)^3+16\,(252\,A+227\,B)a^4\cos(dx+c)^2+105\,(49\,A+44\,B)a^4\cos(dx+c)+32\,(252\,A+227\,B)a^4\right)\sin(dx+c)}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1680*(105*(49*A + 44*B)*a^4*d*x + (240*B*a^4*cos(d*x + c)^6 + 280*(A + 4*
B)*a^4*cos(d*x + c)^5 + 192*(7*A + 12*B)*a^4*cos(d*x + c)^4 + 70*(41*A + 44
*B)*a^4*cos(d*x + c)^3 + 16*(252*A + 227*B)*a^4*cos(d*x + c)^2 + 105*(49*A
+ 44*B)*a^4*cos(d*x + c) + 32*(252*A + 227*B)*a^4)*sin(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(226) = 452$.
time = 0.78, size = 960, normalized size = 3.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((5*A*a**4*x*sin(c + d*x)**6/16 + 15*A*a**4*x*sin(c + d*x)**4*cos(
c + d*x)**2/16 + 9*A*a**4*x*sin(c + d*x)**4/4 + 15*A*a**4*x*sin(c + d*x)**2
*cos(c + d*x)**4/16 + 9*A*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + A*a**4
*x*sin(c + d*x)**2/2 + 5*A*a**4*x*cos(c + d*x)**6/16 + 9*A*a**4*x*cos(c + d
*x)**4/4 + A*a**4*x*cos(c + d*x)**2/2 + 5*A*a**4*sin(c + d*x)**5*cos(c + d*
x)/(16*d) + 32*A*a**4*sin(c + d*x)**5/(15*d) + 5*A*a**4*sin(c + d*x)**3*cos
(c + d*x)**3/(6*d) + 16*A*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*A*
a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*A*a**4*sin(c + d*x)**3/(3*d) +
11*A*a**4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 4*A*a**4*sin(c + d*x)*cos(c
 + d*x)**4/d + 15*A*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 4*A*a**4*sin(
c + d*x)*cos(c + d*x)**2/d + A*a**4*sin(c + d*x)*cos(c + d*x)/(2*d) + 5*B*a
**4*x*sin(c + d*x)**6/4 + 15*B*a**4*x*sin(c + d*x)**4*cos(c + d*x)**2/4 + 3
*B*a**4*x*sin(c + d*x)**4/2 + 15*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/4
 + 3*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2 + 5*B*a**4*x*cos(c + d*x)**6/
4 + 3*B*a**4*x*cos(c + d*x)**4/2 + 16*B*a**4*sin(c + d*x)**7/(35*d) + 8*B*a
**4*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 5*B*a**4*sin(c + d*x)**5*cos(c
+ d*x)/(4*d) + 16*B*a**4*sin(c + d*x)**5/(5*d) + 2*B*a**4*sin(c + d*x)**3*c
os(c + d*x)**4/d + 10*B*a**4*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) + 8*B*a*
*4*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*B*a**4*sin(c + d*x)**3*cos(c + d*x
)/(2*d) + 2*B*a**4*sin(c + d*x)**3/(3*d) + B*a**4*sin(c + d*x)*cos(c + d*x)
**6/d + 11*B*a**4*sin(c + d*x)*cos(c + d*x)**5/(4*d) + 6*B*a**4*sin(c + d*x
)*cos(c + d*x)**4/d + 5*B*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*d) + B*a**4*
sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)
**4*cos(c)**2, True))
```

Giac [A]
time = 0.47, size = 193, normalized size = 0.80

$$\frac{Ba^4\sin(7dx+7c)}{448d}+\frac{1}{16}(49Aa^4+44Ba^4)x+\frac{(Aa^4+4Ba^4)\sin(6dx+6c)}{192d}+\frac{(16Aa^4+31Ba^4)\sin(5dx+5c)}{320d}+\frac{5(3Aa^4+4Ba^4)\sin(4dx+4c)}{64d}+\frac{(144Aa^4+157Ba^4)\sin(3dx+3c)}{192d}+\frac{(127Aa^4+124Ba^4)\sin(2dx+2c)}{64d}+\frac{(352Aa^4+323Ba^4)\sin(dx+c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/448*B*a^4*sin(7*d*x + 7*c)/d + 1/16*(49*A*a^4 + 44*B*a^4)*x + 1/192*(A*a^
4 + 4*B*a^4)*sin(6*d*x + 6*c)/d + 1/320*(16*A*a^4 + 31*B*a^4)*sin(5*d*x + 5
*c)/d + 5/64*(3*A*a^4 + 4*B*a^4)*sin(4*d*x + 4*c)/d + 1/192*(144*A*a^4 + 15
7*B*a^4)*sin(3*d*x + 3*c)/d + 1/64*(127*A*a^4 + 124*B*a^4)*sin(2*d*x + 2*c)
/d + 1/64*(352*A*a^4 + 323*B*a^4)*sin(d*x + c)/d
```

Mupad [B]
time = 1.64, size = 353, normalized size = 1.46

$$\frac{\left(\frac{49Aa^4}{8}+\frac{11Ba^4}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+\left(\frac{245Aa^4}{6}+\frac{110Ba^4}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+\left(\frac{13867Aa^4}{120}+\frac{3113Ba^4}{30}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+\left(\frac{896Aa^4}{5}+\frac{5632Ba^4}{35}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+\left(\frac{19157Aa^4}{120}+\frac{1501Ba^4}{10}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+\left(\frac{523Aa^4}{6}+70Ba^4\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+\left(\frac{207Aa^4}{8}+\frac{53Ba^4}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}-\frac{a^4(49A+44B)\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{8d}+\frac{a^4\,\mathrm{atan}\left(\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(49A+44B)}{8\left(\frac{49Aa^4}{8}+\frac{11Ba^4}{2}\right)}\right)(49A+44B)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((207*A*a^4)/8 + (53*B*a^4)/2) + tan(c/2 + (d*x)/2)^13*
((49*A*a^4)/8 + (11*B*a^4)/2) + tan(c/2 + (d*x)/2)^11*((245*A*a^4)/6 + (110
*B*a^4)/3) + tan(c/2 + (d*x)/2)^3*((523*A*a^4)/6 + 70*B*a^4) + tan(c/2 + (d
*x)/2)^7*((896*A*a^4)/5 + (5632*B*a^4)/35) + tan(c/2 + (d*x)/2)^9*((13867*A
*a^4)/120 + (3113*B*a^4)/30) + tan(c/2 + (d*x)/2)^5*((19157*A*a^4)/120 + (1
501*B*a^4)/10))/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*t
an(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 +
7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1)) - (a^4*(49*A + 44*B)*
(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(8*d) + (a^4*atan((a^4*tan(c/2 + (d*x
)/2)*(49*A + 44*B))/(8*((49*A*a^4)/8 + (11*B*a^4)/2)))*(49*A + 44*B))/(8*d)
```

3.29 $\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=185

$$\frac{7}{16}a^4(8A+7B)x+\frac{4a^4(8A+7B)\sin(c+dx)}{5d}+\frac{27a^4(8A+7B)\cos(c+dx)\sin(c+dx)}{80d}+\frac{a^4(8A+7B)\cos^3(c}{40d}$$

```
[Out] 7/16*a^4*(8*A+7*B)*x+4/5*a^4*(8*A+7*B)*sin(d*x+c)/d+27/80*a^4*(8*A+7*B)*cos
(d*x+c)*sin(d*x+c)/d+1/40*a^4*(8*A+7*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/30*(6*A
-B)*(a+a*cos(d*x+c))^4*sin(d*x+c)/d+1/6*B*(a+a*cos(d*x+c))^5*sin(d*x+c)/a/d
-2/15*a^4*(8*A+7*B)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.18, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3047, 3102, 2830, 2724, 2717, 2715, 8, 2713}

$$-\frac{2a^4(8A+7B)\sin^3(c+dx)}{15d}+\frac{4a^4(8A+7B)\sin(c+dx)}{5d}+\frac{a^4(8A+7B)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{27a^4(8A+7B)\sin(c+dx)\cos(c+dx)}{80d}+\frac{7}{16}a^4x(8A+7B)+\frac{(6A-B)\sin(c+dx)(a\cos(c+dx)+a)^4}{30d}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^5}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (7*a^4*(8*A + 7*B)*x)/16 + (4*a^4*(8*A + 7*B)*Sin[c + d*x])/(5*d) + (27*a^4
*(8*A + 7*B)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + (a^4*(8*A + 7*B)*Cos[c + d
*x]^3*Sin[c + d*x])/(40*d) + ((6*A - B)*(a + a*Cos[c + d*x])^4*Sin[c + d*x]
)/(30*d) + (B*(a + a*Cos[c + d*x])^5*Sin[c + d*x])/(6*a*d) - (2*a^4*(8*A +
7*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^4\left(A\cos(c+dx)+B\cos^2\right.$$

$$= \frac{B(a+a\cos(c+dx))^5\sin(c+dx)}{6ad} + \frac{\int(a+a\,\mathrm{c}}{}$$

$$= \frac{(6A-B)(a+a\cos(c+dx))^4\sin(c+dx)}{30d} + \frac{B}{}$$

$$= \frac{(6A-B)(a+a\cos(c+dx))^4\sin(c+dx)}{30d} + \frac{B}{}$$

$$= \frac{1}{10}a^4(8A+7B)x + \frac{(6A-B)(a+a\cos(c+dx}{30d}$$

$$= \frac{1}{10}a^4(8A+7B)x + \frac{2a^4(8A+7B)\sin(c+dx)}{5d}$$

$$= \frac{2}{5}a^4(8A+7B)x + \frac{4a^4(8A+7B)\sin(c+dx)}{5d} +$$

$$= \frac{7}{16}a^4(8A+7B)x + \frac{4a^4(8A+7B)\sin(c+dx)}{5d}$$

Mathematica [A]
time = 0.56, size = 134, normalized size = 0.72

$$\frac{a^4(2940Bc+3360Adx+2940Bdx+120(49A+44B)\sin(c+dx)+15(128A+127B)\sin(2(c+dx))+580A\sin(3(c+dx))+720B\sin(3(c+dx))+120A\sin(4(c+dx))+225B\sin(4(c+dx))+12A\sin(5(c+dx))+48B\sin(5(c+dx))+5B\sin(6(c+dx)))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^4*(2940*B*c + 3360*A*d*x + 2940*B*d*x + 120*(49*A + 44*B)*Sin[c + d*x] +
 15*(128*A + 127*B)*Sin[2*(c + d*x)] + 580*A*Sin[3*(c + d*x)] + 720*B*Sin[3
*(c + d*x)] + 120*A*Sin[4*(c + d*x)] + 225*B*Sin[4*(c + d*x)] + 12*A*Sin[5*
(c + d*x)] + 48*B*Sin[5*(c + d*x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A]
time = 0.21, size = 306, normalized size = 1.65

method	result
risch	$\frac{7a^4xA}{2}+\frac{49a^4Bx}{16}+\frac{49\sin(dx+c)Aa^4}{8d}+\frac{11a^4B\sin(dx+c)}{2d}+\frac{\sin(6dx+6c)a^4B}{192d}+\frac{\sin(5dx+5c)Aa^4}{80d}+\frac{a^4B\sin}{2}$
derivativedivides	$\frac{\frac{Aa^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a^4B\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{}$

default	$\frac{\frac{A a^4 \left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a^4 B\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+}{}$
norman	$\frac{7a^4(8A+7B)x}{16}+\frac{281a^4(8A+7B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}+\frac{231a^4(8A+7B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}+\frac{119a^4(8A+7B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{7a^4(8A+7B)(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^4*B*(1/6*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+4
*A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/5*a^4
*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*A*a^4*(cos(d*x+c)^2+2)*
sin(d*x+c)+6*a^4*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/
8*c)+4*A*a^4*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+4/3*a^4*B*(cos(d*x+c
)^2+2)*sin(d*x+c)+A*a^4*sin(d*x+c)+a^4*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x
+1/2*c))
```

Maxima [A]
time = 0.27, size = 297, normalized size = 1.61

$$\frac{64\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^4-1920\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4+120\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Aa^4+960\left(2dx+2c+\sin(2dx+2c)\right)Aa^4+256\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Ba^4-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-48\sin(2dx+2c)\right)Ba^4-1280\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^4+180\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Ba^4+240\left(2dx+2c+\sin(2dx+2c)\right)Ba^4+960Aa^4\sin(dx+c)}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 -
1920*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 120*(12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 960*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*A*a^4 + 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^
4 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2
*d*x + 2*c))*B*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 180*(12
*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 240*(2*d*x + 2
*c + sin(2*d*x + 2*c))*B*a^4 + 960*A*a^4*sin(d*x + c))/d
```

Fricas [A]
time = 0.34, size = 130, normalized size = 0.70

$$\frac{105(8A+7B)a^4dx+\left(40Ba^4\cos(dx+c)^5+48(A+4B)a^4\cos(dx+c)^4+10(24A+41B)a^4\cos(dx+c)^3+32(17A+18B)a^4\cos(dx+c)^2+105(8A+7B)a^4\cos(dx+c)+16(83A+72B)a^4\right)\sin(dx+c)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

[Out] 1/240*(105*(8*A + 7*B)*a^4*d*x + (40*B*a^4*cos(d*x + c)^5 + 48*(A + 4*B)*a^4*cos(d*x + c)^4 + 10*(24*A + 41*B)*a^4*cos(d*x + c)^3 + 32*(17*A + 18*B)*a^4*cos(d*x + c)^2 + 105*(8*A + 7*B)*a^4*cos(d*x + c) + 16*(83*A + 72*B)*a^4)*sin(d*x + c))/d

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(170) = 340$.
time = 0.55, size = 765, normalized size = 4.14

$$\begin{cases} \frac{3Aa^4x\sin^4(c+dx)}{2} + 3Aa^4x\sin^2(c+dx)\cos^2(c+dx) + 2Aa^4x\sin^2(c+dx) + \frac{3Aa^4x\cos^4(c+dx)}{2} + 2Aa^4x\cos^2(c+dx) + \frac{8Aa^4\sin^5(c+dx)}{15d} + \frac{4Aa^4\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{3Aa^4\sin^3(c+dx)\cos(c+dx)}{2d} + \frac{4Aa^4\sin^3(c+dx)}{d} + \frac{Aa^4\sin(c+dx)\cos^4(c+dx)}{d} + \frac{5Aa^4\sin(c+dx)\cos^3(c+dx)}{2d} + \frac{6Aa^4\sin(c+dx)\cos^2(c+dx)}{d} + \frac{2Aa^4\sin(c+dx)\cos(c+dx)}{d} + \frac{Aa^4\sin(c+dx)}{d} + \frac{5Ba^4x\sin^6(c+dx)}{16} + \frac{15Ba^4x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{9Ba^4x\sin^4(c+dx)}{4} + \frac{15Ba^4x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{9Ba^4x\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{Ba^4x\sin^2(c+dx)}{2} + \frac{5Ba^4x\cos^6(c+dx)}{16} + \frac{9Ba^4x\cos^4(c+dx)}{4} + \frac{Ba^4x\cos^2(c+dx)}{2} + \frac{5Ba^4\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{32Ba^4\sin^5(c+dx)}{15d} + \frac{5Ba^4\sin^3(c+dx)\cos^3(c+dx)}{6d} + \frac{16Ba^4\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{9Ba^4\sin^3(c+dx)\cos(c+dx)}{4d} + \frac{8Ba^4\sin^3(c+dx)}{3d} + \frac{11Ba^4\sin(c+dx)\cos^5(c+dx)}{16d} + \frac{4Ba^4\sin(c+dx)\cos^4(c+dx)}{d} + \frac{15Ba^4\sin(c+dx)\cos^3(c+dx)}{4d} + \frac{4Ba^4\sin(c+dx)\cos^2(c+dx)}{d} + \frac{Ba^4\sin(c+dx)\cos(c+dx)}{2d} & \text{for}\: d \neq 0 \\ x(A+B\cos(c))(a\cos(c)+a)^4\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)

[Out] Piecewise((3*A*a**4*x*sin(c + d*x)**4/2 + 3*A*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2 + 2*A*a**4*x*sin(c + d*x)**2 + 3*A*a**4*x*cos(c + d*x)**4/2 + 2*A*a**4*x*cos(c + d*x)**2 + 8*A*a**4*sin(c + d*x)**5/(15*d) + 4*A*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*A*a**4*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 4*A*a**4*sin(c + d*x)**3/d + A*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 5*A*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 6*A*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 2*A*a**4*sin(c + d*x)*cos(c + d*x)/d + A*a**4*sin(c + d*x)/d + 5*B*a**4*x*sin(c + d*x)**6/16 + 15*B*a**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*B*a**4*x*sin(c + d*x)**4/4 + 15*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 9*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + B*a**4*x*sin(c + d*x)**2/2 + 5*B*a**4*x*cos(c + d*x)**6/16 + 9*B*a**4*x*cos(c + d*x)**4/4 + B*a**4*x*cos(c + d*x)**2/2 + 5*B*a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 32*B*a**4*sin(c + d*x)**5/(15*d) + 5*B*a**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 16*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*B*a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*B*a**4*sin(c + d*x)**3/(3*d) + 11*B*a**4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 4*B*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 15*B*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 4*B*a**4*sin(c + d*x)*cos(c + d*x)**2/d + B*a**4*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**4*cos(c), True))

Giac [A]
time = 0.51, size = 166, normalized size = 0.90

$$\frac{Ba^4\sin(6\,dx+6\,c)}{192\,d} + \frac{7}{16}\left(8\,Aa^4+7\,Ba^4\right)x + \frac{(Aa^4+4\,Ba^4)\sin(5\,dx+5\,c)}{80\,d} + \frac{(8\,Aa^4+15\,Ba^4)\sin(4\,dx+4\,c)}{64\,d} + \frac{(29\,Aa^4+36\,Ba^4)\sin(3\,dx+3\,c)}{48\,d} + \frac{(128\,Aa^4+127\,Ba^4)\sin(2\,dx+2\,c)}{64\,d} + \frac{(49\,Aa^4+44\,Ba^4)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] 1/192*B*a^4*sin(6*d*x + 6*c)/d + 7/16*(8*A*a^4 + 7*B*a^4)*x + 1/80*(A*a^4 + 4*B*a^4)*sin(5*d*x + 5*c)/d + 1/64*(8*A*a^4 + 15*B*a^4)*sin(4*d*x + 4*c)/d

```
 + 1/48*(29*A*a^4 + 36*B*a^4)*sin(3*d*x + 3*c)/d + 1/64*(128*A*a^4 + 127*B*
a^4)*sin(2*d*x + 2*c)/d + 1/8*(49*A*a^4 + 44*B*a^4)*sin(d*x + c)/d
```

Mupad [B]
time = 1.62, size = 316, normalized size = 1.71

$$\frac{\left(7\,A\,a^4+\frac{49\,B\,a^4}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{119\,A\,a^4}{3}+\frac{833\,B\,a^4}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{462\,A\,a^4}{5}+\frac{1617\,B\,a^4}{20}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{562\,A\,a^4}{5}+\frac{1967\,B\,a^4}{20}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{233\,A\,a^4}{3}+\frac{1471\,B\,a^4}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(25\,A\,a^4+\frac{207\,B\,a^4}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{7\,a^4\,\left(8\,A+7\,B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{8\,d}+\frac{7\,a^4\,\mathrm{atan}\left(\frac{7\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(8\,A+7\,B\right)}{8\,\left(7\,A\,a^4+\frac{49\,B\,a^4}{8}\right)}\right)\,\left(8\,A+7\,B\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(25*A*a^4 + (207*B*a^4)/8) + tan(c/2 + (d*x)/2)^11*(7*A
*a^4 + (49*B*a^4)/8) + tan(c/2 + (d*x)/2)^9*((119*A*a^4)/3 + (833*B*a^4)/24
) + tan(c/2 + (d*x)/2)^3*((233*A*a^4)/3 + (1471*B*a^4)/24) + tan(c/2 + (d*x
)/2)^7*((462*A*a^4)/5 + (1617*B*a^4)/20) + tan(c/2 + (d*x)/2)^5*((562*A*a^4
)/5 + (1967*B*a^4)/20))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^
4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2
)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - (7*a^4*(8*A + 7*B)*(atan(tan(c/2 + (d*
x)/2)) - (d*x)/2))/(8*d) + (7*a^4*atan((7*a^4*tan(c/2 + (d*x)/2)*(8*A + 7*B
))/(8*(7*A*a^4 + (49*B*a^4)/8)))*(8*A + 7*B))/(8*d)
```

3.30 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=150

$$\frac{7}{8}a^4(5A+4B)x+\frac{8a^4(5A+4B)\sin(c+dx)}{5d}+\frac{27a^4(5A+4B)\cos(c+dx)\sin(c+dx)}{40d}+\frac{a^4(5A+4B)\cos^3(c+dx)\sin(c+dx)}{20d}$$

```
[Out] 7/8*a^4*(5*A+4*B)*x+8/5*a^4*(5*A+4*B)*sin(d*x+c)/d+27/40*a^4*(5*A+4*B)*cos(
d*x+c)*sin(d*x+c)/d+1/20*a^4*(5*A+4*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*B*(a+a
*cos(d*x+c))^4*sin(d*x+c)/d-4/15*a^4*(5*A+4*B)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2830, 2724, 2717, 2715, 8, 2713}

$$-\frac{4a^4(5A+4B)\sin^3(c+dx)}{15d}+\frac{8a^4(5A+4B)\sin(c+dx)}{5d}+\frac{a^4(5A+4B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{27a^4(5A+4B)\sin(c+dx)\cos(c+dx)}{40d}+\frac{7}{8}a^4x(5A+4B)+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (7*a^4*(5*A + 4*B)*x)/8 + (8*a^4*(5*A + 4*B)*Sin[c + d*x])/(5*d) + (27*a^4*
(5*A + 4*B)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + (a^4*(5*A + 4*B)*Cos[c + d*
x]^3*Sin[c + d*x])/(20*d) + (B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d) -
 (4*a^4*(5*A + 4*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5d}+\frac{1}{5}(5A+4B)\int (a+a\,\text{c}$$

$$= \frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5d}+\frac{1}{5}(5A+4B)\int \left(a^4+4\right.$$

$$= \frac{1}{5}a^4(5A+4B)x+\frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5d}+\frac{1}{5}(a$$

$$= \frac{1}{5}a^4(5A+4B)x+\frac{4a^4(5A+4B)\sin(c+dx)}{5d}+\frac{3a^4(5A+4.}{}$$

$$= \frac{4}{5}a^4(5A+4B)x+\frac{8a^4(5A+4B)\sin(c+dx)}{5d}+\frac{27a^4(5A+}{}$$

$$= \frac{7}{8}a^4(5A+4B)x+\frac{8a^4(5A+4B)\sin(c+dx)}{5d}+\frac{27a^4(5A+}{}$$

Mathematica [A]
time = 0.37, size = 108, normalized size = 0.72

$$\frac{a^4(2100Adx+1680Bdx+420(8A+7B)\sin(c+dx)+120(7A+8B)\sin(2(c+dx))+160A\sin(3(c+dx))+290B\sin(3(c+dx))+15A\sin(4(c+dx))+60B\sin(4(c+dx))+6B\sin(5(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^4*(2100*A*d*x + 1680*B*d*x + 420*(8*A + 7*B)*Sin[c + d*x] + 120*(7*A + 8
*B)*Sin[2*(c + d*x)] + 160*A*Sin[3*(c + d*x)] + 290*B*Sin[3*(c + d*x)] + 15
*A*Sin[4*(c + d*x)] + 60*B*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)]))/(480*d
)
```

Maple [A]

time = 0.16, size = 248, normalized size = 1.65

method	result
risch	$\frac{35a^4xA}{8}+\frac{7a^4Bx}{2}+\frac{7\sin(dx+c)A\,a^4}{d}+\frac{49a^4B\sin(dx+c)}{8d}+\frac{a^4B\sin(5dx+5c)}{80d}+\frac{\sin(4dx+4c)A\,a^4}{32d}+\frac{a^4B\sin(\cdots}{8\cdots}$
derivativedivides	$\frac{\frac{a^4B\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+A\,a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+4a^4B\left(\frac{\left(\cos^3(\cdots\right.}{}\right.}{}$
default	$\frac{\frac{a^4B\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+A\,a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+4a^4B\left(\frac{\left(\cos^3(\cdots\right.}{}\right.}{}$
norman	$\frac{7a^4(5A+4B)x}{8}+\frac{79a^4(5A+4B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{224a^4(5A+4B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{49a^4(5A+4B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{7a^4(5A+4B)(\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^4*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*a^4*B*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*A*a^4*(cos(d*x+c)^2+2)*
sin(d*x+c)+2*a^4*B*(cos(d*x+c)^2+2)*sin(d*x+c)+6*A*a^4*(1/2*sin(d*x+c)*cos(
d*x+c)+1/2*d*x+1/2*c)+4*a^4*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+4*A
*a^4*sin(d*x+c)+a^4*B*sin(d*x+c)+A*a^4*(d*x+c))
```

Maxima [A]

time = 0.27, size = 236, normalized size = 1.57

$$-\frac{640\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4-15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c)\right)Aa^4-720\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)Aa^4-480\,(dx+c)Aa^4-32\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c)\right)Ba^4+960\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^4-60\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c)\right)Ba^4-480\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)Ba^4-1920\,Aa^4\sin(dx+c)-480\,Ba^4\sin(dx+c)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/480*(640*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 720*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*a^4 - 480*(d*x + c)*A*a^4 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c
)^3 + 15*sin(d*x + c))*B*a^4 + 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4
- 60*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 480*(2
*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 1920*A*a^4*sin(d*x + c) - 480*B*a^4*
sin(d*x + c))/d
```

Fricas [A]

time = 0.35, size = 110, normalized size = 0.73

$$\frac{105\,(5\,A+4\,B)a^4dx+\left(24\,Ba^4\cos(dx+c)^4+30\,(A+4\,B)a^4\cos(dx+c)^3+16\,(10\,A+17\,B)a^4\cos(dx+c)^2+15\,(27\,A+28\,B)a^4\cos(dx+c)+8\,(100\,A+83\,B)a^4\right)\sin(dx+c)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(105*(5*A + 4*B)*a^4*d*x + (24*B*a^4*cos(d*x + c)^4 + 30*(A + 4*B)*a^
4*cos(d*x + c)^3 + 16*(10*A + 17*B)*a^4*cos(d*x + c)^2 + 15*(27*A + 28*B)*a
^4*cos(d*x + c) + 8*(100*A + 83*B)*a^4)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(141) = 282$.
time = 0.38, size = 544, normalized size = 3.63

$$\begin{cases} \frac{3Aa^4x\sin^4(c+dx)}{8} + \frac{3Aa^4x\sin^2(c+dx)\cos^2(c+dx)}{4} + 3Aa^4x\sin^2(c+dx) + \frac{3Aa^4x\cos^4(c+dx)}{8} + 3Aa^4x\cos^2(c+dx) + Aa^4x + \frac{3Aa^4\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{8Aa^4\sin^3(c+dx)}{3d} + \frac{5Aa^4\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{4Aa^4\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Aa^4\sin(c+dx)\cos(c+dx)}{d} + \frac{4Aa^4\sin(c+dx)}{d} + \frac{3Ba^4x\sin^4(c+dx)}{2} + 3Ba^4x\sin^2(c+dx)\cos^2(c+dx) + 2Ba^4x\sin^2(c+dx) + \frac{3Ba^4x\cos^4(c+dx)}{2} + 2Ba^4x\cos^2(c+dx) + \frac{8Ba^4\sin^5(c+dx)}{15d} + \frac{4Ba^4\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{3Ba^4\sin^3(c+dx)\cos(c+dx)}{2d} + \frac{4Ba^4\sin^3(c+dx)}{d} + \frac{Ba^4\sin(c+dx)\cos^4(c+dx)}{d} + \frac{5Ba^4\sin(c+dx)\cos^3(c+dx)}{2d} + \frac{6Ba^4\sin(c+dx)\cos^2(c+dx)}{d} + \frac{2Ba^4\sin(c+dx)\cos(c+dx)}{d} + \frac{Ba^4\sin(c+dx)}{d} & \text{for}\ d \neq 0 \\ x(A+B\cos(c))(a\cos(c)+a)^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**4*x*sin(c + d*x)**4/8 + 3*A*a**4*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + 3*A*a**4*x*sin(c + d*x)**2 + 3*A*a**4*x*cos(c + d*x)**4/8 + 3
*A*a**4*x*cos(c + d*x)**2 + A*a**4*x + 3*A*a**4*sin(c + d*x)**3*cos(c + d*x
)/(8*d) + 8*A*a**4*sin(c + d*x)**3/(3*d) + 5*A*a**4*sin(c + d*x)*cos(c + d*
x)**3/(8*d) + 4*A*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a**4*sin(c + d*
x)*cos(c + d*x)/d + 4*A*a**4*sin(c + d*x)/d + 3*B*a**4*x*sin(c + d*x)**4/2
+ 3*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2 + 2*B*a**4*x*sin(c + d*x)**2 +
 3*B*a**4*x*cos(c + d*x)**4/2 + 2*B*a**4*x*cos(c + d*x)**2 + 8*B*a**4*sin(c
 + d*x)**5/(15*d) + 4*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a*
*4*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 4*B*a**4*sin(c + d*x)**3/d + B*a**4
*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*
d) + 6*B*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 2*B*a**4*sin(c + d*x)*cos(c
+ d*x)/d + B*a**4*sin(c + d*x)/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) +
a)**4, True))
```

Giac [A]
time = 0.43, size = 139, normalized size = 0.93

$$\frac{Ba^4\sin(5\,dx+5\,c)}{80\,d} + \frac{7}{8}\left(5\,Aa^4+4\,Ba^4\right)x + \frac{\left(Aa^4+4\,Ba^4\right)\sin(4\,dx+4\,c)}{32\,d} + \frac{\left(16\,Aa^4+29\,Ba^4\right)\sin(3\,dx+3\,c)}{48\,d} + \frac{\left(7\,Aa^4+8\,Ba^4\right)\sin(2\,dx+2\,c)}{4\,d} + \frac{7\left(8\,Aa^4+7\,Ba^4\right)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/80*B*a^4*sin(5*d*x + 5*c)/d + 7/8*(5*A*a^4 + 4*B*a^4)*x + 1/32*(A*a^4 + 4
*B*a^4)*sin(4*d*x + 4*c)/d + 1/48*(16*A*a^4 + 29*B*a^4)*sin(3*d*x + 3*c)/d
+ 1/4*(7*A*a^4 + 8*B*a^4)*sin(2*d*x + 2*c)/d + 7/8*(8*A*a^4 + 7*B*a^4)*sin(
d*x + c)/d
```

Mupad [B]
time = 1.56, size = 278, normalized size = 1.85

$$\frac{\left(\frac{35\,A\,a^4}{4}+7\,B\,a^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{245\,A\,a^4}{6}+\frac{98\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{224\,A\,a^4}{3}+\frac{896\,B\,a^4}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{395\,A\,a^4}{6}+\frac{158\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{93\,A\,a^4}{4}+25\,B\,a^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{7\,a^4\,\left(5\,A+4\,B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{4\,d}+\frac{7\,a^4\,\mathrm{atan}\left(\frac{7\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,A+4\,B\right)}{4\,\left(\frac{35\,A\,a^4}{4}+7\,B\,a^4\right)}\right)\,\left(5\,A+4\,B\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((93*A*a^4)/4 + 25*B*a^4) + tan(c/2 + (d*x)/2)^9*((35*A
*a^4)/4 + 7*B*a^4) + tan(c/2 + (d*x)/2)^7*((245*A*a^4)/6 + (98*B*a^4)/3) +
tan(c/2 + (d*x)/2)^3*((395*A*a^4)/6 + (158*B*a^4)/3) + tan(c/2 + (d*x)/2)^5
*((224*A*a^4)/3 + (896*B*a^4)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2
+ (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 +
 (d*x)/2)^10 + 1)) - (7*a^4*(5*A + 4*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2
))/(4*d) + (7*a^4*atan((7*a^4*tan(c/2 + (d*x)/2)*(5*A + 4*B))/(4*((35*A*a^4
)/4 + 7*B*a^4)))*(5*A + 4*B))/(4*d)
```

3.31 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=151

$$\frac{1}{8}a^4(48A+35B)x+\frac{a^4A\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^4(8A+7B)\sin(c+dx)}{8d}+\frac{aB(a+a\cos(c+dx))^3\sin(c+dx)}{4d}$$

```
[Out] 1/8*a^4*(48*A+35*B)*x+a^4*A*arctanh(sin(d*x+c))/d+5/8*a^4*(8*A+7*B)*sin(d*x
+c)/d+1/4*a*B*(a+a*cos(d*x+c))^3*sin(d*x+c)/d+1/12*(4*A+7*B)*(a^2+a^2*cos(d
*x+c))^2*sin(d*x+c)/d+1/24*(32*A+35*B)*(a^4+a^4*cos(d*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.27, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3055, 3047, 3102, 2814, 3855}

$$\frac{5a^4(8A+7B)\sin(c+dx)}{8d}+\frac{(32A+35B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{24d}+\frac{1}{8}a^4x(48A+35B)+\frac{a^4A\tanh^{-1}(\sin(c+dx))}{d}+\frac{(4A+7B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{12d}+\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^4*(48*A + 35*B)*x)/8 + (a^4*A*ArcTanh[Sin[c + d*x]])/d + (5*a^4*(8*A + 7
*B)*Sin[c + d*x])/(8*d) + (a*B*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) +
 ((4*A + 7*B)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(12*d) + ((32*A + 35
*B)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(24*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
```

```
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx &= \frac{aB(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}\int (a+\\ &= \frac{aB(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{(4A+7\\ &= \frac{aB(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{(4A+7\\ &= \frac{aB(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{(4A+7\\ &= \frac{5a^4(8A+7B)\sin(c+dx)}{8d}+\frac{aB(a+a\cos(c+\\ &= \frac{1}{8}a^4(48A+35B)x+\frac{5a^4(8A+7B)\sin(c+dx)}{8d}\\ &= \frac{1}{8}a^4(48A+35B)x+\frac{a^4A\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.42, size = 138, normalized size = 0.91

$$\frac{a^4\left(576Adx+420Bdx-96A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+96A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+24(27A+28B)\sin(c+dx)+24(4A+7B)\sin(2(c+dx))+8A\sin(3(c+dx))+32B\sin(3(c+dx))+3B\sin(4(c+dx))\right)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^4*(576*A*d*x + 420*B*d*x - 96*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 96*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24*(27*A + 28*B)*Sin[c +
 d*x] + 24*(4*A + 7*B)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 32*B*Sin[3
*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A]
time = 0.22, size = 208, normalized size = 1.38

method	result
derivativedivides	$\frac{\frac{A a^4\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+a^4 B\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+4A a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right.}{}$
default	$\frac{\frac{A a^4\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+a^4 B\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+4A a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right.}{}$
risch	$6a^4xA+\frac{35a^4Bx}{8}-\frac{27ie^{i(dx+c)}A a^4}{8d}-\frac{7ie^{i(dx+c)}a^4B}{2d}+\frac{27ie^{-i(dx+c)}A a^4}{8d}+\frac{7ie^{-i(dx+c)}a^4B}{2d}+\frac{A a^4\ln(e^{i(dx}}{d}$
norman	$\frac{(6A a^4+\frac{35}{8}a^4B)x+(6A a^4+\frac{35}{8}a^4B)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(30A a^4+\frac{175}{8}a^4B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(30A a^4+\frac{175}{8}a^4B)x\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*A*a^4*(cos(d*x+c)^2+2)*sin(d*x+c)+a^4*B*(1/4*(cos(d*x+c)^3+3/2*cos
(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*A*a^4*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d
*x+1/2*c)+4/3*a^4*B*(cos(d*x+c)^2+2)*sin(d*x+c)+6*A*a^4*sin(d*x+c)+6*a^4*B*
(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+4*A*a^4*(d*x+c)+4*a^4*B*sin(d*x+c
)+A*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*B*(d*x+c))
```

Maxima [A]
time = 0.27, size = 198, normalized size = 1.31

$$-\frac{32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4-96\left(2dx+2c+\sin(2dx+2c)\right)Aa^4-384(dx+c)Aa^4+128\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^4-3\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Ba^4-144\left(2dx+2c+\sin(2dx+2c)\right)Ba^4-96(dx+c)Ba^4-96Aa^4\log\left(\sec(dx+c)+\tan(dx+c)\right)-576Aa^4\sin(dx+c)-384Ba^4\sin(dx+c)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 96*(2*d*x + 2*c + sin(2
*d*x + 2*c))*A*a^4 - 384*(d*x + c)*A*a^4 + 128*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*B*a^4 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a
^4 - 144*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 96*(d*x + c)*B*a^4 - 96*A
```

```
*a^4*log(sec(d*x + c) + tan(d*x + c)) - 576*A*a^4*sin(d*x + c) - 384*B*a^4*
sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 118, normalized size = 0.78

$$\frac{3\,(48\,A+35\,B)a^4dx+12\,Aa^4\log\left(\sin\left(dx+c\right)+1\right)-12\,Aa^4\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,Ba^4\cos\left(dx+c\right)^3+8\,(A+4\,B)a^4\cos\left(dx+c\right)^2+3\,(16\,A+27\,B)a^4\cos\left(dx+c\right)+160\,(A+B)a^4\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(48*A + 35*B)*a^4*d*x + 12*A*a^4*log(sin(d*x + c) + 1) - 12*A*a^4*l
og(-sin(d*x + c) + 1) + (6*B*a^4*cos(d*x + c)^3 + 8*(A + 4*B)*a^4*cos(d*x +
 c)^2 + 3*(16*A + 27*B)*a^4*cos(d*x + c) + 160*(A + B)*a^4)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int A\sec\left(c+dx\right)dx+\int 4A\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int 6A\cos^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int 4A\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int A\cos^4\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int 4B\cos^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int 6B\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int 4B\cos^4\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos^5\left(c+dx\right)\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a**4*(Integral(A*sec(c + d*x), x) + Integral(4*A*cos(c + d*x)*sec(c + d*x),
 x) + Integral(6*A*cos(c + d*x)**2*sec(c + d*x), x) + Integral(4*A*cos(c +
d*x)**3*sec(c + d*x), x) + Integral(A*cos(c + d*x)**4*sec(c + d*x), x) + In
tegral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(4*B*cos(c + d*x)**2*sec(c
 + d*x), x) + Integral(6*B*cos(c + d*x)**3*sec(c + d*x), x) + Integral(4*B*
cos(c + d*x)**4*sec(c + d*x), x) + Integral(B*cos(c + d*x)**5*sec(c + d*x),
 x))
```

Giac [A]
time = 0.48, size = 214, normalized size = 1.42

$$\frac{24\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\,(48\,Aa^4+35\,Ba^4)(dx+c)+\frac{2\left(120\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+105\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+424\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+385\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+520\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+511\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+216\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+279\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/24*(24*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*A*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(48*A*a^4 + 35*B*a^4)*(d*x + c) + 2*(120*A*a^4*tan
(1/2*d*x + 1/2*c)^7 + 105*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 424*A*a^4*tan(1/2*
d*x + 1/2*c)^5 + 385*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 520*A*a^4*tan(1/2*d*x +
```

```
1/2*c)^3 + 511*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 216*A*a^4*tan(1/2*d*x + 1/2*
c) + 279*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 0.67, size = 188, normalized size = 1.25

$$\frac{144\,A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+24\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+105\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+12\,A\,a^4\,\sin\left(2\,c+2\,d\,x\right)+A\,a^4\,\sin\left(3\,c+3\,d\,x\right)+21\,B\,a^4\,\sin\left(2\,c+2\,d\,x\right)+4\,B\,a^4\,\sin\left(3\,c+3\,d\,x\right)+\frac{3\,B\,a^4\,\sin(4\,c+4\,d\,x)}{8}+81\,A\,a^4\,\sin\left(c+d\,x\right)+84\,B\,a^4\,\sin\left(c+d\,x\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x),x)
```

```
[Out] (144*A*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 24*A*a^4*atanh(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 105*B*a^4*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)) + 12*A*a^4*sin(2*c + 2*d*x) + A*a^4*sin(3*c + 3*d*x) + 21*
B*a^4*sin(2*c + 2*d*x) + 4*B*a^4*sin(3*c + 3*d*x) + (3*B*a^4*sin(4*c + 4*d*
x))/8 + 81*A*a^4*sin(c + d*x) + 84*B*a^4*sin(c + d*x))/(12*d)
```

3.32 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=150

$$\frac{1}{2}a^4(13A+12B)x+\frac{a^4(4A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^4(A+2B)\sin(c+dx)}{2d}-\frac{(3A-B)\left(a^2+a^2\cos(c+\right.}{3d}$$

```
[Out] 1/2*a^4*(13*A+12*B)*x+a^4*(4*A+B)*arctanh(sin(d*x+c))/d+5/2*a^4*(A+2*B)*sin
(d*x+c)/d-1/3*(3*A-B)*(a^2+a^2*cos(d*x+c))^2*sin(d*x+c)/d-1/6*(3*A-8*B)*(a^
4+a^4*cos(d*x+c))*sin(d*x+c)/d+a*A*(a+a*cos(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.30, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3054, 3055, 3047, 3102, 2814, 3855}

$$\frac{5a^4(A+2B)\sin(c+dx)}{2d}+\frac{a^4(4A+B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(3A-8B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{6d}+\frac{1}{2}a^4x(13A+12B)-\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^4*(13*A + 12*B)*x)/2 + (a^4*(4*A + B)*ArcTanh[Sin[c + d*x]])/d + (5*a^4*
(A + 2*B)*Sin[c + d*x])/(2*d) - ((3*A - B)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c
 + d*x])/(3*d) - ((3*A - 8*B)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(6*d)
+ (a*A*(a + a*Cos[c + d*x])^3*Tan[c + d*x])/d
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
```

```
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\tan(c+dx)}{d} + \int (a+$$

$$= -\frac{(3A-B)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{3d}$$

$$= -\frac{(3A-B)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{3d}$$

$$= -\frac{(3A-B)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{3d}$$

$$= \frac{5a^4(A+2B)\sin(c+dx)}{2d} - \frac{(3A-B)\left(a^2+a\right.}{}$$

$$= \frac{1}{2}a^4(13A+12B)x + \frac{5a^4(A+2B)\sin(c+dx)}{2d}$$

$$= \frac{1}{2}a^4(13A+12B)x + \frac{a^4(4A+B)\tanh^{-1}(\sin(}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 312 vs. 2(150) = 300.
time = 1.68, size = 312, normalized size = 2.08

$$\frac{1}{192}a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(78Ax+72Bx-\frac{12(4A+B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{12(4A+B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{3(16A+27B)\cos(dx)\sin(c)}{d}+\frac{3(A+4B)\cos(2dx)\sin(2c)}{d}+\frac{B\cos(3dx)\sin(3c)}{d}+\frac{3(16A+27B)\cos(c)\sin(dx)}{d}+\frac{3(A+4B)\cos(2c)\sin(2dx)}{d}+\frac{B\cos(3c)\sin(3dx)}{d}+\frac{12A\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{12A\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(78*A*x + 72*B*x - (12*(4*A +
B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (12*(4*A + B)*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]])/d + (3*(16*A + 27*B)*Cos[d*x]*Sin[c])/d + (3*
(A + 4*B)*Cos[2*d*x]*Sin[2*c])/d + (B*Cos[3*d*x]*Sin[3*c])/d + (3*(16*A + 2
7*B)*Cos[c]*Sin[d*x])/d + (3*(A + 4*B)*Cos[2*c]*Sin[2*d*x])/d + (B*Cos[3*c]
*Sin[3*d*x])/d + (12*A*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])) + (12*A*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))))/192
```

Maple [A]
time = 0.27, size = 179, normalized size = 1.19

method	result
derivativedivides	$\frac{A\,a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^4B\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+4A\,a^4\sin(dx+c)+4a^4B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right.}{}$

default	$\frac{A\,a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^4B\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+4A\,a^4\sin(dx+c)+4a^4B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{}$
risch	$\frac{13a^4xA}{2}+6a^4Bx+\frac{ie^{-2i(dx+c)}A\,a^4}{8d}+\frac{ie^{-2i(dx+c)}a^4B}{2d}+\frac{2iA\,a^4}{d\left(e^{2i(dx+c)}+1\right)}-\frac{27ie^{i(dx+c)}a^4B}{8d}-\frac{ie^{2i(dx+c)}a^4}{2d}$
norman	$\frac{\left(-\frac{13}{2}A\,a^4-6a^4B\right)x+\left(-\frac{65}{2}A\,a^4-30a^4B\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{13}{2}A\,a^4+6a^4B\right)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{65}{2}A\,a^4+30a^4B\right)x\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+1/3*a^4*B*(cos(d*x+c)^
2+2)*sin(d*x+c)+4*A*a^4*sin(d*x+c)+4*a^4*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d
*x+1/2*c)+6*A*a^4*(d*x+c)+6*a^4*B*sin(d*x+c)+4*A*a^4*ln(sec(d*x+c)+tan(d*x+
c))+4*a^4*B*(d*x+c)+A*a^4*tan(d*x+c)+a^4*B*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.27, size = 187, normalized size = 1.25

$$\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^4+72\,(dx+c)Aa^4-4\,(\sin(dx+c)^3-3\,\sin(dx+c))Ba^4+12\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^4+48\,(dx+c)Ba^4+24\,Aa^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+6\,Ba^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+48\,Aa^4\sin(dx+c)+72\,Ba^4\sin(dx+c)+12\,Aa^4\tan(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 72*(d*x + c)*A*a^4 - 4*(si
n(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 12*(2*d*x + 2*c + sin(2*d*x + 2*c))*
B*a^4 + 48*(d*x + c)*B*a^4 + 24*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x
+ c) - 1)) + 6*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*A
*a^4*sin(d*x + c) + 72*B*a^4*sin(d*x + c) + 12*A*a^4*tan(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 150, normalized size = 1.00

$$\frac{3\,(13\,A+12\,B)a^4dx\cos(dx+c)+3\,(4\,A+B)a^4\cos(dx+c)\log(\sin(dx+c)+1)-3\,(4\,A+B)a^4\cos(dx+c)\log(-\sin(dx+c)+1)+\left(2\,Ba^4\cos(dx+c)^3+3\,(A+4\,B)a^4\cos(dx+c)^2+8\,(3\,A+5\,B)a^4\cos(dx+c)+6\,Aa^4\right)\sin(dx+c)}{6\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(13*A + 12*B)*a^4*d*x*cos(d*x + c) + 3*(4*A + B)*a^4*cos(d*x + c)*lo
g(sin(d*x + c) + 1) - 3*(4*A + B)*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) +
 (2*B*a^4*cos(d*x + c)^3 + 3*(A + 4*B)*a^4*cos(d*x + c)^2 + 8*(3*A + 5*B)*a
^4*cos(d*x + c) + 6*A*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 226, normalized size = 1.51

$$-\frac{\frac{12\,A a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - 3\,(13\,A a^4 + 12\,B a^4)(dx+c) - 6\,(4\,A a^4 + B a^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 6\,(4\,A a^4 + B a^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(21\,A a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 30\,B a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 48\,A a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 76\,B a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 27\,A a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 54\,B a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(13*A*
a^4 + 12*B*a^4)*(d*x + c) - 6*(4*A*a^4 + B*a^4)*log(abs(tan(1/2*d*x + 1/2*c
) + 1)) + 6*(4*A*a^4 + B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(21*A*
a^4*tan(1/2*d*x + 1/2*c)^5 + 30*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^4*tan
(1/2*d*x + 1/2*c)^3 + 76*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 27*A*a^4*tan(1/2*d*
x + 1/2*c) + 54*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)
/d
```

Mupad [**B**]
time = 0.42, size = 242, normalized size = 1.61

$$\frac{4\,A\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{20\,B\,a^4\,\sin\left(c+d\,x\right)}{3\,d}+\frac{13\,A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{8\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{12\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{B\,a^4\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{A\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}+\frac{2\,B\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^2,x)
```

```
[Out] (4*A*a^4*sin(c + d*x))/d + (20*B*a^4*sin(c + d*x))/(3*d) + (13*A*a^4*atan(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*A*a^4*atanh(sin(c/2 + (d*x)/2
)/cos(c/2 + (d*x)/2)))/d + (12*B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/d + (2*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^
4*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^4*cos(c + d*x)^2*sin(c + d*x))/(3*d
) + (A*a^4*cos(c + d*x)*sin(c + d*x))/(2*d) + (2*B*a^4*cos(c + d*x)*sin(c +
 d*x))/d
```

3.33 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=162

$$\frac{1}{2}a^4(8A+13B)x+\frac{a^4(13A+8B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5a^4(A-B)\sin(c+dx)}{2d}-\frac{(6A+B)\left(a^4+a^4\cos(c+d\right.}{2d}$$

```
[Out] 1/2*a^4*(8*A+13*B)*x+1/2*a^4*(13*A+8*B)*arctanh(sin(d*x+c))/d-5/2*a^4*(A-B)
*sin(d*x+c)/d-1/2*(6*A+B)*(a^4+a^4*cos(d*x+c))*sin(d*x+c)/d+1/2*(5*A+2*B)*(
a^2+a^2*cos(d*x+c))^2*tan(d*x+c)/d+1/2*a*A*(a+a*cos(d*x+c))^3*sec(d*x+c)*ta
n(d*x+c)/d
```

Rubi [A]
time = 0.31, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3054, 3055, 3047, 3102, 2814, 3855}

$$-\frac{5a^4(A-B)\sin(c+dx)}{2d}+\frac{a^4(13A+8B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(6A+B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{2d}+\frac{1}{2}a^4x(8A+13B)+\frac{(5A+2B)\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^4*(8*A + 13*B)*x)/2 + (a^4*(13*A + 8*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (
5*a^4*(A - B)*Sin[c + d*x])/(2*d) - ((6*A + B)*(a^4 + a^4*Cos[c + d*x])*Sin
[c + d*x])/(2*d) + ((5*A + 2*B)*(a^2 + a^2*Cos[c + d*x])^2*Tan[c + d*x])/(2
*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
```

```
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d}$$
$$= \frac{(5A+2B)\left(a^2+a^2\cos(c+dx)\right)^2\tan(c+dx)}{2d} +$$
$$= -\frac{(6A+B)\left(a^4+a^4\cos(c+dx)\right)\sin(c+dx)}{2d} +$$
$$= -\frac{(6A+B)\left(a^4+a^4\cos(c+dx)\right)\sin(c+dx)}{2d} +$$
$$= -\frac{5a^4(A-B)\sin(c+dx)}{2d} - \frac{(6A+B)\left(a^4+a^4\right.}{}$$
$$= \frac{1}{2}a^4(8A+13B)x - \frac{5a^4(A-B)\sin(c+dx)}{2d} -$$
$$= \frac{1}{2}a^4(8A+13B)x + \frac{a^4(13A+8B)\tanh^{-1}(\sin(c}{2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 343 vs. $2(162) = 324$.
time = 4.43, size = 343, normalized size = 2.12

$$\frac{1}{64}a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(2(8A+13B)x-\frac{2(13A+8B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{2(13A+8B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{4(A+4B)\cos(dx)\sin(c)}{d}+\frac{B\cos(2dx)\sin(2c)}{d}+\frac{4(A+4B)\cos(c)\sin(dx)}{d}+\frac{B\cos(2c)\sin(2dx)}{d}+\frac{A}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(4A+B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{A}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(4A+B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(2*(8*A + 13*B)*x - (2*(13*A +
 8*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(13*A + 8*B)*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (4*(A + 4*B)*Cos[d*x]*Sin[c])/d + (B
*Cos[2*d*x]*Sin[2*c])/d + (4*(A + 4*B)*Cos[c]*Sin[d*x])/d + (B*Cos[2*c]*Sin
[2*d*x])/d + A/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(4*A + B)*S
in[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
) - A/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(4*A + B)*Sin[(d*x)/
2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/64
```

Maple [A]
time = 0.30, size = 177, normalized size = 1.09

method	result
derivativedivides	$\frac{A\,a^4\sin(dx+c)+a^4B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4A\,a^4(dx+c)+4a^4B\sin(dx+c)+6A\,a^4\ln(\sec(dx+c)+\tan(dx+c))+6}{}$

default	$\frac{A a^4 \sin(dx+c)+a^4 B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4A a^4(dx+c)+4a^4 B\sin(dx+c)+6A a^4\ln(\sec(dx+c)+\tan(dx+c))-}{}$
risch	$4a^4xA+\frac{13a^4Bx}{2}-\frac{ie^{2i(dx+c)}a^4B}{8d}-\frac{ie^{i(dx+c)}A a^4}{2d}-\frac{2ie^{i(dx+c)}a^4B}{d}+\frac{ie^{-i(dx+c)}A a^4}{2d}+\frac{2ie^{-i(dx+c)}a^4B}{d}$
norman	$\frac{(4A a^4+\frac{13}{2}a^4B)x+(-20A a^4-\frac{65}{2}a^4B)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-20A a^4-\frac{65}{2}a^4B)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(4A a^4+\frac{13}{2}a^4B)x\left(\mathrm{t}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*sin(d*x+c)+a^4*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+4*A*a
^4*(d*x+c)+4*a^4*B*sin(d*x+c)+6*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+6*a^4*B*(d*
x+c)+4*A*a^4*tan(d*x+c)+4*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+A*a^4*(1/2*sec(d*
x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^4*B*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 199, normalized size = 1.23

$$\frac{16\,(dx+c)Aa^4+(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^4+24\,(dx+c)Ba^4-Aa^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12\,Aa^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+8\,Ba^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+4\,Aa^4\sin(dx+c)+16\,Ba^4\sin(dx+c)+16\,Aa^4\tan(dx+c)+4\,Ba^4\tan(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(16*(d*x + c)*A*a^4 + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 + 24*(d*x
+ c)*B*a^4 - A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)
+ 1) + log(sin(d*x + c) - 1)) + 12*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) + 8*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4
*A*a^4*sin(d*x + c) + 16*B*a^4*sin(d*x + c) + 16*A*a^4*tan(d*x + c) + 4*B*a
^4*tan(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 156, normalized size = 0.96

$$\frac{2\,(8\,A+13\,B)a^4dx\cos(dx+c)^2+(13\,A+8\,B)a^4\cos(dx+c)^2\log(\sin(dx+c)+1)-(13\,A+8\,B)a^4\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\left(Ba^4\cos(dx+c)^3+2\,(A+4\,B)a^4\cos(dx+c)^2+2\,(4\,A+B)a^4\cos(dx+c)+Aa^4\right)\sin(dx+c)}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(2*(8*A + 13*B)*a^4*d*x*cos(d*x + c)^2 + (13*A + 8*B)*a^4*cos(d*x + c)^
2*log(sin(d*x + c) + 1) - (13*A + 8*B)*a^4*cos(d*x + c)^2*log(-sin(d*x + c)
 + 1) + 2*(B*a^4*cos(d*x + c)^3 + 2*(A + 4*B)*a^4*cos(d*x + c)^2 + 2*(4*A +
 B)*a^4*cos(d*x + c) + A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 230, normalized size = 1.42

$$\frac{(8\,Aa^4+13\,Ba^4)(dx+c)+(13\,Aa^4+8\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(13\,Aa^4+8\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(5\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-5\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+7\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+7\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-11\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-11\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*((8*A*a^4 + 13*B*a^4)*(d*x + c) + (13*A*a^4 + 8*B*a^4)*log(abs(tan(1/2*
d*x + 1/2*c) + 1)) - (13*A*a^4 + 8*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*(5*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 5*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 7*
A*a^4*tan(1/2*d*x + 1/2*c)^5 + 7*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^4*tan
(1/2*d*x + 1/2*c)^3 + 9*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 11*A*a^4*tan(1/2*d*x
 + 1/2*c) - 11*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/
d
```

Mupad [B]
time = 0.40, size = 243, normalized size = 1.50

$$\frac{A\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{4\,B\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{8\,A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{13\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{13\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{8\,B\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,A\,a^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{A\,a^4\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}+\frac{B\,a^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{B\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^3,x)
```

```
[Out] (A*a^4*sin(c + d*x))/d + (4*B*a^4*sin(c + d*x))/d + (8*A*a^4*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (13*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2)))/d + (13*B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d + (8*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*A*a^4*sin
(c + d*x))/(d*cos(c + d*x)) + (A*a^4*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (
B*a^4*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^4*cos(c + d*x)*sin(c + d*x))/(2
*d)
```

3.34 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=165

$$a^4(A+4B)x+\frac{a^4(12A+13B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5a^4(2A+B)\sin(c+dx)}{2d}+\frac{(11A+9B)\left(a^4+a^4\cos(c+\right.}{3d}$$

```
[Out] a^4*(A+4*B)*x+1/2*a^4*(12*A+13*B)*arctanh(sin(d*x+c))/d-5/2*a^4*(2*A+B)*sin
(d*x+c)/d+1/3*(11*A+9*B)*(a^4+a^4*cos(d*x+c))*tan(d*x+c)/d+1/2*(2*A+B)*(a^2
+a^2*cos(d*x+c))^2*sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*(a+a*cos(d*x+c))^3*sec(d
*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.34, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3054, 3047, 3102, 2814, 3855}

$$-\frac{5a^4(2A+B)\sin(c+dx)}{2d}+\frac{a^4(12A+13B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(11A+9B)\tan(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{3d}+a^4x(A+4B)+\frac{(2A+B)\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] a^4*(A + 4*B)*x + (a^4*(12*A + 13*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^4*
(2*A + B)*Sin[c + d*x])/(2*d) + ((11*A + 9*B)*(a^4 + a^4*Cos[c + d*x])*Tan[
c + d*x])/(3*d) + ((2*A + B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c
+ d*x])/(2*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3
*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx}{3d}$$

$$= \frac{(2A+B)\left(a^2+a^2\cos(c+dx)\right)^2\sec(c+dx)\tan}{2d}$$

$$= \frac{(11A+9B)\left(a^4+a^4\cos(c+dx)\right)\tan(c+dx)}{3d}$$

$$= \frac{(11A+9B)\left(a^4+a^4\cos(c+dx)\right)\tan(c+dx)}{3d}$$

$$= -\frac{5a^4(2A+B)\sin(c+dx)}{2d} + \frac{(11A+9B)\left(a^4+\right.}{}$$

$$= a^4(A+4B)x - \frac{5a^4(2A+B)\sin(c+dx)}{2d} + \frac{(11}{}$$

$$= a^4(A+4B)x + \frac{a^4(12A+13B)\tanh^{-1}(\sin(c+}{2d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 380 vs. 2(165) = 330.

time = 6.23, size = 380, normalized size = 2.30

$$a^4\left(\frac{(A+4B)(c+dx)}{d}+\frac{(-12A-13B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{(12A+13B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{13A+3B}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{A\sin\left(\frac{1}{2}(c+dx)\right)}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{A\sin\left(\frac{1}{2}(c+dx)\right)}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{-13A-3B}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4\left(5A\sin\left(\frac{1}{2}(c+dx)\right)+3B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{4\left(5A\sin\left(\frac{1}{2}(c+dx)\right)+3B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{B\sin(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] a^4*(((A + 4*B)*(c + d*x))/d + ((-12*A - 13*B)*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]])/(2*d) + ((12*A + 13*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]])/(2*d) + (13*A + 3*B)/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
A*Sin[(c + d*x)/2])/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (A*Sin[
(c + d*x)/2])/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (-13*A - 3*B)
/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(5*A*Sin[(c + d*x)/2]
+ 3*B*Sin[(c + d*x)/2]))/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*(
5*A*Sin[(c + d*x)/2] + 3*B*Sin[(c + d*x)/2]))/(3*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])) + (B*Sin[c + d*x])/d)
```

Maple [A]
time = 0.34, size = 199, normalized size = 1.21

method	result
derivativedivides	$\frac{A\,a^4(dx+c)+a^4B\sin(dx+c)+4A\,a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4B(dx+c)+6A\,a^4\tan(dx+c)+6a^4B\ln(\sec(dx+c)+t}{}$
default	$\frac{A\,a^4(dx+c)+a^4B\sin(dx+c)+4A\,a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4B(dx+c)+6A\,a^4\tan(dx+c)+6a^4B\ln(\sec(dx+c)+t}{}$
risch	$a^4xA+4a^4Bx-\frac{i e^{i(dx+c)}a^4B}{2d}+\frac{i e^{-i(dx+c)}a^4B}{2d}-\frac{i a^4\left(12A\,e^{5i(dx+c)}+3B\,e^{5i(dx+c)}-36A\,e^{4i(dx+c)}-24B\,e^{4}\right.}{}$
norman	$\frac{(-A\,a^4-4a^4B)x+(-6A\,a^4-24a^4B)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-2A\,a^4-8a^4B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-2A\,a^4-8a^4B)x\left(\tan^1\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*(d*x+c)+a^4*B*sin(d*x+c)+4*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^4
*B*(d*x+c)+6*A*a^4*tan(d*x+c)+6*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+4*A*a^4*(1/
2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+4*a^4*B*tan(d*x+c)-A
*a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^4*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/
2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 235, normalized size = 1.42

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^4+12(dx+c)Aa^4+48(dx+c)Ba^4-12Aa^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-3Ba^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+24Aa^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+36Ba^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+12Ba^4\sin(dx+c)+72Aa^4\tan(dx+c)+48Ba^4\tan(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 12*(d*x + c)*A*a^4 + 48*(
d*x + c)*B*a^4 - 12*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*a^4*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*A*a^4*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*B*a^4*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + 12*B*a^4*sin(d*x + c) + 72*A*a^4*tan(d*x + c) +
 48*B*a^4*tan(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 159, normalized size = 0.96

$$\frac{12\,(A+4\,B)a^4dx\cos\left(dx+c\right)^3+3\,(12\,A+13\,B)a^4\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(12\,A+13\,B)a^4\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(6\,Ba^4\cos\left(dx+c\right)^3+8\,(5\,A+3\,B)a^4\cos\left(dx+c\right)^2+3\,(4\,A+B)a^4\cos\left(dx+c\right)+2\,Aa^4\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(12*(A + 4*B)*a^4*d*x*cos(d*x + c)^3 + 3*(12*A + 13*B)*a^4*cos(d*x + c
)^3*log(sin(d*x + c) + 1) - 3*(12*A + 13*B)*a^4*cos(d*x + c)^3*log(-sin(d*x
 + c) + 1) + 2*(6*B*a^4*cos(d*x + c)^3 + 8*(5*A + 3*B)*a^4*cos(d*x + c)^2 +
 3*(4*A + B)*a^4*cos(d*x + c) + 2*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.51, size = 227, normalized size = 1.38

$$\frac{\frac{12\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+6\,(Aa^4+4\,Ba^4)(dx+c)+3\,(12\,Aa^4+13\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(12\,Aa^4+13\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(30\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+21\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-76\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-48\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+54\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+27\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(12*B*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(A*a^4
+ 4*B*a^4)*(d*x + c) + 3*(12*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3*(12*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30
*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 21*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 76*A*a^4*
tan(1/2*d*x + 1/2*c)^3 - 48*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 54*A*a^4*tan(1/2
*d*x + 1/2*c) + 27*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)
^3)/d
```

Mupad [B]
time = 0.41, size = 254, normalized size = 1.54

$$\frac{B a^4 \sin(c+dx)}{d}+\frac{2 A a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{12 A a^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{8 B a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{13 B a^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{20 A a^4 \sin(c+dx)}{3d\cos(c+dx)}+\frac{2 A a^4 \sin(c+dx)}{d\cos(c+dx)^2}+\frac{A a^4 \sin(c+dx)}{3d\cos(c+dx)^3}+\frac{4 B a^4 \sin(c+dx)}{d\cos(c+dx)}+\frac{B a^4 \sin(c+dx)}{2d\cos(c+dx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^4,x)
```

```
[Out] (B*a^4*sin(c + d*x))/d + (2*A*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d + (12*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*B*a^
4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (13*B*a^4*atanh(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (20*A*a^4*sin(c + d*x))/(3*d*cos(c + d*
x)) + (2*A*a^4*sin(c + d*x))/(d*cos(c + d*x)^2) + (A*a^4*sin(c + d*x))/(3*d
*cos(c + d*x)^3) + (4*B*a^4*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^4*sin(c +
 d*x))/(2*d*cos(c + d*x)^2)
```

3.35 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=173

$$a^4Bx+\frac{a^4(35A+48B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5a^4(7A+8B)\tan(c+dx)}{8d}+\frac{(35A+32B)\left(a^4+a^4\cos(c+dx)\right)}{24d}$$

```
[Out] a^4*B*x+1/8*a^4*(35*A+48*B)*arctanh(sin(d*x+c))/d+5/8*a^4*(7*A+8*B)*tan(d*x
+c)/d+1/24*(35*A+32*B)*(a^4+a^4*cos(d*x+c))*sec(d*x+c)*tan(d*x+c)/d+1/12*(7
*A+4*B)*(a^2+a^2*cos(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a*A*(a+a*cos(d
*x+c))^3*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.31, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3054, 3047, 3100, 2814, 3855}

$$\frac{5a^4(7A+8B)\tan(c+dx)}{8d}+\frac{a^4(35A+48B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(35A+32B)\tan(c+dx)\sec(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{24d}+a^4Bx+\frac{(7A+4B)\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{12d}+\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] a^4*B*x + (a^4*(35*A + 48*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^4*(7*A + 8
*B)*Tan[c + d*x])/(8*d) + ((35*A + 32*B)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d
*x]*Tan[c + d*x])/(24*d) + ((7*A + 4*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c +
d*x]^2*Tan[c + d*x])/(12*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3*Ta
n[c + d*x])/(4*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\sec^3(c+dx)\tan(c+d}{4d}$$

$$= \frac{(7A+4B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^2(c+dx)}{12d}$$

$$= \frac{(35A+32B)\left(a^4+a^4\cos(c+dx)\right)\sec(c+dx}{24d}$$

$$= \frac{(35A+32B)\left(a^4+a^4\cos(c+dx)\right)\sec(c+dx}{24d}$$

$$= \frac{5a^4(7A+8B)\tan(c+dx)}{8d} + \frac{(35A+32B)\,(a}{}$$

$$= a^4Bx + \frac{5a^4(7A+8B)\tan(c+dx)}{8d} + \frac{(35A+}{}$$

$$= a^4Bx + \frac{a^4(35A+48B)\tanh^{-1}(\sin(c+dx))}{8d}$$

Mathematica [A]
time = 1.98, size = 326, normalized size = 1.88

$$\frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^4\left(-24(35A+48B)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec(c)(72Bdx\cos(c)+48Bdx\cos(c+2dx)+48Bdx\cos(3c+2dx)+12Bdx\cos(3c+4dx)+12Bdx\cos(5c+4dx)-480A\sin(c)-480B\sin(c)+105A\sin(dx)+48B\sin(dx)+105A\sin(2c+dx)+48B\sin(2c+dx)+544A\sin(c+2dx)+496B\sin(c+2dx)-96A\sin(3c+2dx)-144B\sin(3c+2dx)+81A\sin(2c+3dx)+48B\sin(2c+3dx)+81A\sin(4c+3dx)+48B\sin(4c+3dx)+160A\sin(3c+4dx)+160B\sin(3c+4dx))\right)}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(-24*(35*A + 48*B)*Cos[c + d*x
]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]]) + Sec[c]*(72*B*d*x*Cos[c] + 48*B*d*x*Cos[c + 2*d*x] + 48*B*d
*x*Cos[3*c + 2*d*x] + 12*B*d*x*Cos[3*c + 4*d*x] + 12*B*d*x*Cos[5*c + 4*d*x]
 - 480*A*Sin[c] - 480*B*Sin[c] + 105*A*Sin[d*x] + 48*B*Sin[d*x] + 105*A*Sin
[2*c + d*x] + 48*B*Sin[2*c + d*x] + 544*A*Sin[c + 2*d*x] + 496*B*Sin[c + 2*
d*x] - 96*A*Sin[3*c + 2*d*x] - 144*B*Sin[3*c + 2*d*x] + 81*A*Sin[2*c + 3*d*
x] + 48*B*Sin[2*c + 3*d*x] + 81*A*Sin[4*c + 3*d*x] + 48*B*Sin[4*c + 3*d*x]
+ 160*A*Sin[3*c + 4*d*x] + 160*B*Sin[3*c + 4*d*x])))/(3072*d)
```

Maple [A]
time = 0.26, size = 250, normalized size = 1.45

method	result
derivativedivides	$\frac{A\,a^4\ln(\sec(dx+c)+\tan(dx+c))+a^4B(dx+c)+4A\,a^4\tan(dx+c)+4a^4B\ln(\sec(dx+c)+\tan(dx+c))+6A\,a^4\left(\frac{\sec(dx+c)\tan(d}{2}\right.}{}$
default	$\frac{A\,a^4\ln(\sec(dx+c)+\tan(dx+c))+a^4B(dx+c)+4A\,a^4\tan(dx+c)+4a^4B\ln(\sec(dx+c)+\tan(dx+c))+6A\,a^4\left(\frac{\sec(dx+c)\tan(d}{2}\right.}{}$
risch	$a^4Bx-\frac{ia^4\left(81A\,e^{7i(dx+c)}+48B\,e^{7i(dx+c)}-96A\,e^{6i(dx+c)}-144B\,e^{6i(dx+c)}+105A\,e^{5i(dx+c)}+48B\,e^{5i(dx+c)}-480A\,e^{4i}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*B*(d*x+c)+4*A*a^4*tan(d*x+c)+4*a^4
*B*ln(sec(d*x+c)+tan(d*x+c))+6*A*a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(
d*x+c)+tan(d*x+c)))+6*a^4*B*tan(d*x+c)-4*A*a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(
d*x+c)+4*a^4*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+A*
a^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d
*x+c)))-a^4*B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.26, size = 307, normalized size = 1.77

$$\frac{64\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^4+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^4+48(dx+c)Ba^4-3Aa^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-72Aa^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-48Ba^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+24Aa^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+96Ba^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+192Aa^4\tan(dx+c)+288Ba^4\tan(dx+c)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^4 + 48*(d*x + c)*B*a^4 - 3*A*a^4*(2*(3*sin(d*x + c)^3 - 5*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 72*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 48*B*a^4*(2*sin(d*x
 + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))
 + 24*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 96*B*a^4*(log
(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 192*A*a^4*tan(d*x + c) + 288*
B*a^4*tan(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 157, normalized size = 0.91

$$\frac{48\,Ba^4dx\cos\left(dx+c\right)^4+3\left(35\,A+48\,B\right)a^4\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(35\,A+48\,B\right)a^4\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(160\left(A+B\right)a^4\cos\left(dx+c\right)^3+3\left(27\,A+16\,B\right)a^4\cos\left(dx+c\right)^2+8\left(4\,A+B\right)a^4\cos\left(dx+c\right)+6\,Aa^4\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(48*B*a^4*d*x*cos(d*x + c)^4 + 3*(35*A + 48*B)*a^4*cos(d*x + c)^4*log(
sin(d*x + c) + 1) - 3*(35*A + 48*B)*a^4*cos(d*x + c)^4*log(-sin(d*x + c) +
1) + 2*(160*(A + B)*a^4*cos(d*x + c)^3 + 3*(27*A + 16*B)*a^4*cos(d*x + c)^2
 + 8*(4*A + B)*a^4*cos(d*x + c) + 6*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.54, size = 223, normalized size = 1.29

$$\frac{24\left(dx+c\right)Ba^4+3\left(35\,Aa^4+48\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(35\,Aa^4+48\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+120\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-385\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-424\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+511\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+520\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-279\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-216\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(24*(d*x + c)*B*a^4 + 3*(35*A*a^4 + 48*B*a^4)*log(abs(tan(1/2*d*x + 1/
2*c) + 1)) - 3*(35*A*a^4 + 48*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2
*(105*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 385
*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 424*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 511*A*a^
4*tan(1/2*d*x + 1/2*c)^3 + 520*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*A*a^4*tan
(1/2*d*x + 1/2*c) - 216*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 - 1)^4)/d
```

Mupad [B]
time = 0.38, size = 255, normalized size = 1.47

$$\frac{35\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}+\frac{2\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{12\,B\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{20\,A\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)}+\frac{27\,A\,a^4\,\sin\left(c+d\,x\right)}{8\,d\,{\cos\left(c+d\,x\right)}^2}+\frac{4\,A\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3}+\frac{A\,a^4\,\sin\left(c+d\,x\right)}{4\,d\,{\cos\left(c+d\,x\right)}^4}+\frac{20\,B\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)}+\frac{2\,B\,a^4\,\sin\left(c+d\,x\right)}{d\,{\cos\left(c+d\,x\right)}^2}+\frac{B\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^5,x)
```

```
[Out] (35*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*d) + (2*B*a^4*at
an(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (12*B*a^4*atanh(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2)))/d + (20*A*a^4*sin(c + d*x))/(3*d*cos(c + d*x))
+ (27*A*a^4*sin(c + d*x))/(8*d*cos(c + d*x)^2) + (4*A*a^4*sin(c + d*x))/(3*
d*cos(c + d*x)^3) + (A*a^4*sin(c + d*x))/(4*d*cos(c + d*x)^4) + (20*B*a^4*s
in(c + d*x))/(3*d*cos(c + d*x)) + (2*B*a^4*sin(c + d*x))/(d*cos(c + d*x)^2)
 + (B*a^4*sin(c + d*x))/(3*d*cos(c + d*x)^3)
```

3.36 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=198

$$\frac{7a^4(4A+5B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4(83A+100B)\tan(c+dx)}{15d}+\frac{a^4(244A+275B)\sec(c+dx)\tan(c+d$$

```
[Out] 7/8*a^4*(4*A+5*B)*arctanh(sin(d*x+c))/d+1/15*a^4*(83*A+100*B)*tan(d*x+c)/d+
1/120*a^4*(244*A+275*B)*sec(d*x+c)*tan(d*x+c)/d+1/30*(26*A+25*B)*(a^4+a^4*c
os(d*x+c))*sec(d*x+c)^2*tan(d*x+c)/d+1/20*(8*A+5*B)*(a^2+a^2*cos(d*x+c))^2*
sec(d*x+c)^3*tan(d*x+c)/d+1/5*a*A*(a+a*cos(d*x+c))^3*sec(d*x+c)^4*tan(d*x+c
)/d
```

Rubi [A]
time = 0.37, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3054, 3047, 3100, 2827, 3852, 8, 3855}

$$\frac{a^4(83A+100B)\tan(c+dx)}{15d}+\frac{7a^4(4A+5B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4(244A+275B)\tan(c+dx)\sec(c+dx)}{120d}+\frac{(26A+25B)\tan(c+dx)\sec^2(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{30d}+\frac{(8A+5B)\tan(c+dx)\sec^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{20d}+\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (7*a^4*(4*A + 5*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^4*(83*A + 100*B)*Tan[c
 + d*x])/(15*d) + (a^4*(244*A + 275*B)*Sec[c + d*x]*Tan[c + d*x])/(120*d) +
 ((26*A + 25*B)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(30*d
) + ((8*A + 5*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(2
0*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\sec^4(c+dx)\tan(c+d}{5d}$$
$$= \frac{(8A+5B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^3(c+dx)}{20d}$$
$$= \frac{(26A+25B)\left(a^4+a^4\cos(c+dx)\right)\sec^2(c+dx}{30d}$$
$$= \frac{(26A+25B)\left(a^4+a^4\cos(c+dx)\right)\sec^2(c+dx}{30d}$$
$$= \frac{a^4(244A+275B)\sec(c+dx)\tan(c+dx)}{120d} + \frac{(}{}$$
$$= \frac{a^4(244A+275B)\sec(c+dx)\tan(c+dx)}{120d} + \frac{(}{}$$
$$= \frac{7a^4(4A+5B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^4(244A}{}$$
$$= \frac{7a^4(4A+5B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^4(83A}{}$$

Mathematica [**A**]
time = 1.69, size = 306, normalized size = 1.55

$$-\frac{a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(1680(4A+5B)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(80(59A+64B)\sin(dx)-960(2A+3B)\sin(2c+dx)+1320A\sin(c+2dx)+930B\sin(c+2dx)+1320A\sin(3c+2dx)+930B\sin(3c+2dx)+3200A\sin(2c+3dx)+3520B\sin(2c+3dx)-120A\sin(4c+3dx)-480B\sin(4c+3dx)+420A\sin(3c+4dx)+405B\sin(3c+4dx)+420A\sin(5c+4dx)+405B\sin(5c+4dx)+664A\sin(4c+5dx)+800B\sin(4c+5dx))\right)}{30720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] -1/30720*(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^5*(1680*
(4*A + 5*B)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(59*A + 64*B)*Sin[d*x] -
 960*(2*A + 3*B)*Sin[2*c + d*x] + 1320*A*Sin[c + 2*d*x] + 930*B*Sin[c + 2*d
*x] + 1320*A*Sin[3*c + 2*d*x] + 930*B*Sin[3*c + 2*d*x] + 3200*A*Sin[2*c + 3
*d*x] + 3520*B*Sin[2*c + 3*d*x] - 120*A*Sin[4*c + 3*d*x] - 480*B*Sin[4*c +
3*d*x] + 420*A*Sin[3*c + 4*d*x] + 405*B*Sin[3*c + 4*d*x] + 420*A*Sin[5*c +
4*d*x] + 405*B*Sin[5*c + 4*d*x] + 664*A*Sin[4*c + 5*d*x] + 800*B*Sin[4*c +
5*d*x])))/d
```

Maple [**A**]
time = 0.27, size = 303, normalized size = 1.53

method	result

derivativedivides	$\frac{A\,a^4\tan(dx+c)+a^4B\ln(\sec(dx+c)+\tan(dx+c))+4A\,a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4a^4B\tan(dx+c}{}$
default	$\frac{A\,a^4\tan(dx+c)+a^4B\ln(\sec(dx+c)+\tan(dx+c))+4A\,a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4a^4B\tan(dx+c}{}$
risch	$-\frac{ia^4\left(420A\,e^{9i(dx+c)}+405B\,e^{9i(dx+c)}-120A\,e^{8i(dx+c)}-480B\,e^{8i(dx+c)}+1320A\,e^{7i(dx+c)}+930B\,e^{7i(dx+c)}-1920A\,e^{6i(d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*tan(d*x+c)+a^4*B*ln(sec(d*x+c)+tan(d*x+c))+4*A*a^4*(1/2*sec(d*x+
c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+4*a^4*B*tan(d*x+c)-6*A*a^4*(-2
/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+6*a^4*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(s
ec(d*x+c)+tan(d*x+c)))+4*A*a^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x
+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-4*a^4*B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+
c)-A*a^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+a^4*B*(-(-1/
4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(186) = 372$.
time = 0.27, size = 376, normalized size = 1.90

$$\frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^4+480\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^4+320\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^4-60\,Aa^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-15\,Ba^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-240\,Aa^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-360\,Ba^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+120\,Ba^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+240\,Aa^4\tan(dx+c)+960\,Ba^4\tan(dx+c)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
480*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 320*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^4 - 60*A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 15*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 240*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
 log(sin(d*x + c) - 1)) - 360*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) -
log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120*B*a^4*(log(sin(d*x + c
) + 1) - log(sin(d*x + c) - 1)) + 240*A*a^4*tan(d*x + c) + 960*B*a^4*tan(d*
x + c))/d
```

Fricas [A]
time = 0.36, size = 165, normalized size = 0.83

$$\frac{105\,(4\,A+5\,B)a^4\cos(dx+c)^5\log(\sin(dx+c)+1)-105\,(4\,A+5\,B)a^4\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(8\,(83\,A+100\,B)a^4\cos(dx+c)^4+15\,(28\,A+27\,B)a^4\cos(dx+c)^3+16\,(17\,A+10\,B)a^4\cos(dx+c)^2+30\,(4\,A+B)a^4\cos(dx+c)+24\,Aa^4\right)\sin(dx+c)}{240\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fr
icas")
```

```
[Out] 1/240*(105*(4*A + 5*B)*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*(4*A
+ 5*B)*a^4*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(83*A + 100*B)*a^4*
cos(d*x + c)^4 + 15*(28*A + 27*B)*a^4*cos(d*x + c)^3 + 16*(17*A + 10*B)*a^4
*cos(d*x + c)^2 + 30*(4*A + B)*a^4*cos(d*x + c) + 24*A*a^4)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.50, size = 246, normalized size = 1.24

$$\frac{105\,(4\,Aa^4+5\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,(4\,Aa^4+5\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(420\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+525\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-1960\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-2450\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3584\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4480\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3160\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3950\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1500\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1395\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="gi
ac")
```

```
[Out] 1/120*(105*(4*A*a^4 + 5*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(4*
A*a^4 + 5*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(420*A*a^4*tan(1/2*
d*x + 1/2*c)^9 + 525*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 1960*A*a^4*tan(1/2*d*x
+ 1/2*c)^7 - 2450*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 3584*A*a^4*tan(1/2*d*x + 1
/2*c)^5 + 4480*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 3160*A*a^4*tan(1/2*d*x + 1/2*
c)^3 - 3950*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 1500*A*a^4*tan(1/2*d*x + 1/2*c)
+ 1395*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 2.79, size = 224, normalized size = 1.13

$$\frac{7\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,A+5\,B\right)}{4\,d}-\frac{\left(7\,A\,a^4+\frac{35\,B\,a^4}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{98\,A\,a^4}{3}-\frac{245\,B\,a^4}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{896\,A\,a^4}{15}+\frac{224\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{158\,A\,a^4}{3}-\frac{395\,B\,a^4}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(25\,A\,a^4+\frac{93\,B\,a^4}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^6,x)
```

```
[Out] (7*a^4*atanh(tan(c/2 + (d*x)/2))*(4*A + 5*B))/(4*d) - (tan(c/2 + (d*x)/2)*(
25*A*a^4 + (93*B*a^4)/4) + tan(c/2 + (d*x)/2)^9*(7*A*a^4 + (35*B*a^4)/4) -
tan(c/2 + (d*x)/2)^7*((98*A*a^4)/3 + (245*B*a^4)/6) - tan(c/2 + (d*x)/2)^3*
((158*A*a^4)/3 + (395*B*a^4)/6) + tan(c/2 + (d*x)/2)^5*((896*A*a^4)/15 + (2
24*B*a^4)/3))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan
(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.37 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$

Optimal. Leaf size=229

$$\frac{7a^4(7A+8B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(72A+83B)\tan(c+dx)}{15d}+\frac{7a^4(7A+8B)\sec(c+dx)\tan(c+dx)}{16d}+$$

```
[Out] 7/16*a^4*(7*A+8*B)*arctanh(sin(d*x+c))/d+1/15*a^4*(72*A+83*B)*tan(d*x+c)/d+
7/16*a^4*(7*A+8*B)*sec(d*x+c)*tan(d*x+c)/d+1/120*a^4*(159*A+176*B)*sec(d*x+
c)^2*tan(d*x+c)/d+1/120*(73*A+72*B)*(a^4+a^4*cos(d*x+c))*sec(d*x+c)^3*tan(d
*x+c)/d+1/10*(3*A+2*B)*(a^2+a^2*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d+1/6
*a*A*(a+a*cos(d*x+c))^3*sec(d*x+c)^5*tan(d*x+c)/d
```

Rubi [A]
time = 0.40, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3054, 3047, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{a^4(72A+83B)\tan(c+dx)}{15d}+\frac{7a^4(7A+8B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(159A+176B)\tan(c+dx)\sec^2(c+dx)}{120d}+\frac{7a^4(7A+8B)\tan(c+dx)\sec(c+dx)}{16d}+\frac{(73A+72B)\tan(c+dx)\sec^3(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{120d}+\frac{(3A+2B)\tan(c+dx)\sec^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{10d}+\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

```
[Out] (7*a^4*(7*A + 8*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^4*(72*A + 83*B)*Tan[c
 + d*x])/(15*d) + (7*a^4*(7*A + 8*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a
^4*(159*A + 176*B)*Sec[c + d*x]^2*Tan[c + d*x])/(120*d) + ((73*A + 72*B)*(a
^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + ((3*A + 2*B)*
(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(10*d) + (a*A*(a +
a*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^7(c + dx)\, dx = \frac{aA(a + a\cos(c + dx))^3 \sec^5(c + dx)\tan(c + d}{6d}$$

$$= \frac{(3A + 2B)\left(a^2 + a^2\cos(c + dx)\right)^2 \sec^4(c + dx)}{10d}$$

$$= \frac{(73A + 72B)\left(a^4 + a^4\cos(c + dx)\right)\sec^3(c + dx}{120d}$$

$$= \frac{(73A + 72B)\left(a^4 + a^4\cos(c + dx)\right)\sec^3(c + dx}{120d}$$

$$= \frac{a^4(159A + 176B)\sec^2(c + dx)\tan(c + dx)}{120d} +$$

$$= \frac{a^4(159A + 176B)\sec^2(c + dx)\tan(c + dx)}{120d} +$$

$$= \frac{7a^4(7A + 8B)\sec(c + dx)\tan(c + dx)}{16d} + \frac{a^4(1}{}$$

$$= \frac{7a^4(7A + 8B)\tanh^{-1}(\sin(c + dx))}{16d} + \frac{a^4(72A}{}$$

Mathematica [A]
time = 2.21, size = 358, normalized size = 1.56

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

```
[Out] -1/122880*(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^6*(3360
*(7*A + 8*B)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-160*(72*A + 83*B)*Sin[c]
+ 30*(125*A + 88*B)*Sin[d*x] + 3750*A*Sin[2*c + d*x] + 2640*B*Sin[2*c + d*x
] + 15360*A*Sin[c + 2*d*x] + 15840*B*Sin[c + 2*d*x] - 1920*A*Sin[3*c + 2*d*
x] - 4080*B*Sin[3*c + 2*d*x] + 3845*A*Sin[2*c + 3*d*x] + 3480*B*Sin[2*c + 3
*d*x] + 3845*A*Sin[4*c + 3*d*x] + 3480*B*Sin[4*c + 3*d*x] + 6912*A*Sin[3*c
+ 4*d*x] + 7728*B*Sin[3*c + 4*d*x] - 240*B*Sin[5*c + 4*d*x] + 735*A*Sin[4*c
 + 5*d*x] + 840*B*Sin[4*c + 5*d*x] + 735*A*Sin[6*c + 5*d*x] + 840*B*Sin[6*c
 + 5*d*x] + 1152*A*Sin[5*c + 6*d*x] + 1328*B*Sin[5*c + 6*d*x]))/d
```

Maple [A]
time = 0.31, size = 365, normalized size = 1.59

method	result
derivativedivides	$\frac{Aa^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^4B\tan(dx+c)-4Aa^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4a^4B\left(\sec\right.}{}$
default	$\frac{Aa^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^4B\tan(dx+c)-4Aa^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4a^4B\left(\sec\right.}{}$
risch	$-\frac{ia^4(735A\,e^{11i(dx+c)}+840B\,e^{11i(dx+c)}-240B\,e^{10i(dx+c)}+3845A\,e^{9i(dx+c)}+3480B\,e^{9i(dx+c)}-1920A\,e^{8i(dx+c)}-4080B}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x,method=_RETURNVERBOSE)

[Out] 1/d*(A*a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^4*B*tan(d*x+c)-4*A*a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*a^4*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+6*A*a^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-6*a^4*B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-4*A*a^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+4*a^4*B*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+A*a^4*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c)))-a^4*B*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(215) = 430$.
time = 0.28, size = 464, normalized size = 2.03

$$\frac{128\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^4+640\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^4+32\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ba^4+960\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^4-5Aa^4\left(\frac{2\left(15\sin(dx+c)^5-40\sin(dx+c)^3+33\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)\right)-180Aa^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-120Ba^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-120Aa^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-480Ba^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+480Ba^4\tan(dx+c)}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="maxima")

[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 + 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 + 960*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^4 - 5*A*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 180*A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 480*B*a^4*(2*sin(d

```
*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1
)) + 480*B*a^4*tan(d*x + c))/d
```

Fricas [**A**]
time = 0.35, size = 185, normalized size = 0.81

$$\frac{105 \, {\left(7 \, A + 8 \, B\right)} a^{4} \cos \left(d x + c\right)^{6} \log \left(\sin \left(d x + c\right) + 1\right) - 105 \, {\left(7 \, A + 8 \, B\right)} a^{4} \cos \left(d x + c\right)^{6} \log \left(-\sin \left(d x + c\right) + 1\right) + 2 \, {\left(16 \, {\left(72 \, A + 83 \, B\right)} a^{4} \cos \left(d x + c\right)^{5} + 105 \, {\left(7 \, A + 8 \, B\right)} a^{4} \cos \left(d x + c\right)^{4} + 32 \, {\left(18 \, A + 17 \, B\right)} a^{4} \cos \left(d x + c\right)^{3} + 10 \, {\left(41 \, A + 24 \, B\right)} a^{4} \cos \left(d x + c\right)^{2} + 48 \, {\left(4 \, A + B\right)} a^{4} \cos \left(d x + c\right) + 40 \, A a^{4}\right)} \sin \left(d x + c\right)}{480 \, d \cos \left(d x + c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="fr
icas")
```

```
[Out] 1/480*(105*(7*A + 8*B)*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 105*(7*A
+ 8*B)*a^4*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(72*A + 83*B)*a^4*
cos(d*x + c)^5 + 105*(7*A + 8*B)*a^4*cos(d*x + c)^4 + 32*(18*A + 17*B)*a^4*
cos(d*x + c)^3 + 10*(41*A + 24*B)*a^4*cos(d*x + c)^2 + 48*(4*A + B)*a^4*cos
(d*x + c) + 40*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**7,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [**A**]
time = 0.51, size = 280, normalized size = 1.22

$$\frac{105 \, {\left(7 \, A a^{4} + 8 \, B a^{4}\right)} \log \left({\left| \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 1 \right|}\right) - 105 \, {\left(7 \, A a^{4} + 8 \, B a^{4}\right)} \log \left({\left| \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 1 \right|}\right) - \frac{2 \, {\left(735 \, A a^{4} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{11} + 840 \, B a^{4} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{11} - 4165 \, A a^{4} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{9} - 4760 \, B a^{4} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{9} + 9702 \, A a^{4} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{7} + 11088 \, B a^{4} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{7} - 11802 \, A a^{4} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{5} - 13488 \, B a^{4} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{5} + 7355 \, A a^{4} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} + 9320 \, B a^{4} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 3105 \, A a^{4} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 3000 \, B a^{4} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)\right)}}{{\left(\tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - 1\right)}^{6}}}{240 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="gi
ac")
```

```
[Out] 1/240*(105*(7*A*a^4 + 8*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(7*
A*a^4 + 8*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(735*A*a^4*tan(1/2*
d*x + 1/2*c)^11 + 840*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*A*a^4*tan(1/2*d*
x + 1/2*c)^9 - 4760*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 9702*A*a^4*tan(1/2*d*x +
 1/2*c)^7 + 11088*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*A*a^4*tan(1/2*d*x +
1/2*c)^5 - 13488*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 7355*A*a^4*tan(1/2*d*x + 1/
2*c)^3 + 9320*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*A*a^4*tan(1/2*d*x + 1/2*c
) - 3000*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B]
time = 2.84, size = 262, normalized size = 1.14

$$\frac{\left(-\frac{49\,A\,a^4}{8}-7\,B\,a^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{833\,A\,a^4}{24}+\frac{119\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{1617\,A\,a^4}{20}-\frac{462\,B\,a^4}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{1967\,A\,a^4}{20}+\frac{562\,B\,a^4}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{1471\,A\,a^4}{24}-\frac{233\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{207\,A\,a^4}{8}+25\,B\,a^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{7\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(7\,A+8\,B\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^7,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((207*A*a^4)/8 + 25*B*a^4) - tan(c/2 + (d*x)/2)^11*((49
*A*a^4)/8 + 7*B*a^4) + tan(c/2 + (d*x)/2)^9*((833*A*a^4)/24 + (119*B*a^4)/3
) - tan(c/2 + (d*x)/2)^3*((1471*A*a^4)/24 + (233*B*a^4)/3) - tan(c/2 + (d*x
)/2)^7*((1617*A*a^4)/20 + (462*B*a^4)/5) + tan(c/2 + (d*x)/2)^5*((1967*A*a^
4)/20 + (562*B*a^4)/5))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^
2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2
)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (7*a^4*atanh(tan(c/2 + (d*x)/2))*(7*A
+ 8*B))/(8*d)
```

3.38 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=153

$$-\frac{3(4A-5B)x}{8a}+\frac{4(A-B)\sin(c+dx)}{ad}-\frac{3(4A-5B)\cos(c+dx)\sin(c+dx)}{8ad}-\frac{(4A-5B)\cos^3(c+dx)\sin(}{4ad}$$

```
[Out] -3/8*(4*A-5*B)*x/a+4*(A-B)*sin(d*x+c)/a/d-3/8*(4*A-5*B)*cos(d*x+c)*sin(d*x+
c)/a/d-1/4*(4*A-5*B)*cos(d*x+c)^3*sin(d*x+c)/a/d+(A-B)*cos(d*x+c)^4*sin(d*x
+c)/d/(a+a*cos(d*x+c))-4/3*(A-B)*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3056, 2827, 2713, 2715, 8}

$$-\frac{4(A-B)\sin^3(c+dx)}{3ad}+\frac{4(A-B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(4A-5B)\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{3(4A-5B)\sin(c+dx)\cos(c+dx)}{8ad}-\frac{3x(4A-5B)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-3*(4*A - 5*B)*x)/(8*a) + (4*(A - B)*Sin[c + d*x])/(a*d) - (3*(4*A - 5*B)*
Cos[c + d*x]*Sin[c + d*x])/(8*a*d) - ((4*A - 5*B)*Cos[c + d*x]^3*Sin[c + d*
x])/(4*a*d) + ((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])
) - (4*(A - B)*Sin[c + d*x]^3)/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^3(c+dx)(4a(A-B)-a}{a^2}$$
$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(4A-5B)\int \cos^4(c+dx)\,dx}{a}$$
$$= -\frac{(4A-5B)\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{(A-B)\cos^4(c+dx)\sin(}{d(a+a\cos(c+dx)}$$
$$= \frac{4(A-B)\sin(c+dx)}{ad} - \frac{3(4A-5B)\cos(c+dx)\sin(c+dx)}{8ad} - \frac{(4A}{}$$
$$= -\frac{3(4A-5B)x}{8a} + \frac{4(A-B)\sin(c+dx)}{ad} - \frac{3(4A-5B)\cos(c+dx}{8ad}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 311 vs. 2(153) = 306.
time = 0.67, size = 311, normalized size = 2.03

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-72(4A-5B)dx\cos\left(\frac{dx}{2}\right)-72(4A-5B)dx\cos\left(c+\frac{dx}{2}\right)+552A\sin\left(\frac{dx}{2}\right)-552B\sin\left(\frac{dx}{2}\right)+168A\sin\left(c+\frac{dx}{2}\right)-168B\sin\left(c+\frac{dx}{2}\right)+144A\sin\left(c+\frac{3dx}{2}\right)-120B\sin\left(c+\frac{3dx}{2}\right)+144A\sin\left(2c+\frac{3dx}{2}\right)-120B\sin\left(2c+\frac{3dx}{2}\right)-16A\sin\left(2c+\frac{5dx}{2}\right)+40B\sin\left(2c+\frac{5dx}{2}\right)-16A\sin\left(3c+\frac{5dx}{2}\right)+40B\sin\left(3c+\frac{5dx}{2}\right)+8A\sin\left(3c+\frac{7dx}{2}\right)-5B\sin\left(3c+\frac{7dx}{2}\right)+8A\sin\left(4c+\frac{7dx}{2}\right)-5B\sin\left(4c+\frac{7dx}{2}\right)+3B\sin\left(4c+\frac{9dx}{2}\right)+3B\sin\left(5c+\frac{9dx}{2}\right)\right)}{192ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-72*(4*A - 5*B)*d*x*Cos[(d*x)/2] - 72*(4*A - 5*
B)*d*x*Cos[c + (d*x)/2] + 552*A*Sin[(d*x)/2] - 552*B*Sin[(d*x)/2] + 168*A*S
in[c + (d*x)/2] - 168*B*Sin[c + (d*x)/2] + 144*A*Sin[c + (3*d*x)/2] - 120*B
*Sin[c + (3*d*x)/2] + 144*A*Sin[2*c + (3*d*x)/2] - 120*B*Sin[2*c + (3*d*x)/
2] - 16*A*Sin[2*c + (5*d*x)/2] + 40*B*Sin[2*c + (5*d*x)/2] - 16*A*Sin[3*c +
```

```
(5*d*x)/2] + 40*B*Sin[3*c + (5*d*x)/2] + 8*A*Sin[3*c + (7*d*x)/2] - 5*B*Si
n[3*c + (7*d*x)/2] + 8*A*Sin[4*c + (7*d*x)/2] - 5*B*Sin[4*c + (7*d*x)/2] +
3*B*Sin[4*c + (9*d*x)/2] + 3*B*Sin[5*c + (9*d*x)/2]))/(192*a*d*(1 + Cos[c +
d*x]))
```

Maple [A]
time = 0.18, size = 143, normalized size = 0.93

method	result
derivativedivides	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(\left(\frac{25B}{8}-\frac{5A}{2}\right)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{115B}{24}-\frac{31A}{6}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{109B}{24}-\frac{25A}{6}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}}{da}$
default	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(\left(\frac{25B}{8}-\frac{5A}{2}\right)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{115B}{24}-\frac{31A}{6}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{109B}{24}-\frac{25A}{6}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}}{da}$
risch	$-\frac{3xA}{2a}+\frac{15Bx}{8a}-\frac{7i\mathrm{e}^{i(dx+c)}A}{8da}+\frac{7i\mathrm{e}^{i(dx+c)}B}{8da}+\frac{7i\mathrm{e}^{-i(dx+c)}A}{8da}-\frac{7i\mathrm{e}^{-i(dx+c)}B}{8da}+\frac{2iA}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}-$
norman	$\frac{(A-B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{3(4A-5B)x}{8a}+\frac{86(A-B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{15(4A-5B)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{15(4A-5B)x\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(A*tan(1/2*d*x+1/2*c)-B*tan(1/2*d*x+1/2*c)-2*((25/8*B-5/2*A)*tan(1/2*
d*x+1/2*c)^7+(115/24*B-31/6*A)*tan(1/2*d*x+1/2*c)^5+(109/24*B-25/6*A)*tan(1
/2*d*x+1/2*c)^3+(7/8*B-3/2*A)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^
4-3/4*(4*A-5*B)*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(145) = 290$.
time = 0.48, size = 394, normalized size = 2.58

$$-\frac{B\left(\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{115\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-4A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/12*(B*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 4*A*((9*sin(d*x +
 c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*
```

```
x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 3*sin(d*x + c)/(a*
(cos(d*x + c) + 1))))/d
```

Fricas [A]
time = 0.34, size = 120, normalized size = 0.78

$$-\frac{9\,(4\,A-5\,B)dx\cos\left(dx+c\right)+9\,(4\,A-5\,B)dx-\left(6\,B\cos\left(dx+c\right)^4+2\,(4\,A-B)\cos\left(dx+c\right)^3-(4\,A-13\,B)\cos\left(dx+c\right)^2+(28\,A-19\,B)\cos\left(dx+c\right)+64\,A-64\,B\right)\sin\left(dx+c\right)}{24\,(ad\cos\left(dx+c\right)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/24*(9*(4*A - 5*B)*d*x*cos(d*x + c) + 9*(4*A - 5*B)*d*x - (6*B*cos(d*x +
c)^4 + 2*(4*A - B)*cos(d*x + c)^3 - (4*A - 13*B)*cos(d*x + c)^2 + (28*A - 1
9*B)*cos(d*x + c) + 64*A - 64*B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1794 vs. $2(134) = 268$.
time = 2.20, size = 1794, normalized size = 11.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((-36*A*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a
*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x
/2)**2 + 24*a*d) - 144*A*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**
8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c
/2 + d*x/2)**2 + 24*a*d) - 216*A*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 +
d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a
*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 144*A*d*x*tan(c/2 + d*x/2)**2/(24*a*d*ta
n(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**
4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 36*A*d*x/(24*a*d*tan(c/2 + d*x/2
)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*ta
n(c/2 + d*x/2)**2 + 24*a*d) + 24*A*tan(c/2 + d*x/2)**9/(24*a*d*tan(c/2 + d*
x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d
*tan(c/2 + d*x/2)**2 + 24*a*d) + 216*A*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2
+ d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96
*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 392*A*tan(c/2 + d*x/2)**5/(24*a*d*tan(
c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4
+ 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 296*A*tan(c/2 + d*x/2)**3/(24*a*d*
tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)
**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 96*A*tan(c/2 + d*x/2)/(24*a*d*
```

```
tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)
**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 45*B*d*x*tan(c/2 + d*x/2)**8/(
24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 +
 d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 180*B*d*x*tan(c/2 + d*x
/2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*t
an(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 270*B*d*x*tan(c
/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 1
44*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 180*B*d
*x*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2
)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) +
 45*B*d*x/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*
d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 24*B*tan(c/2
 + d*x/2)**9/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144
*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 246*B*tan
(c/2 + d*x/2)**7/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 +
 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 374*B
*tan(c/2 + d*x/2)**5/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)*
*6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 3
14*B*tan(c/2 + d*x/2)**3/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x
/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d)
 - 66*B*tan(c/2 + d*x/2)/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x
/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d)
, Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**4/(a*cos(c) + a), True))
```

Giac [A]
time = 0.42, size = 181, normalized size = 1.18

$$-\frac{\frac{9\,(dx+c)(4\,A-5\,B)}{a}-\frac{24\,\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}-\frac{2\,\left(60\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-75\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+124\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-115\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+100\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-109\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+36\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/24*(9*(d*x + c)*(4*A - 5*B)/a - 24*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d
*x + 1/2*c))/a - 2*(60*A*tan(1/2*d*x + 1/2*c)^7 - 75*B*tan(1/2*d*x + 1/2*c)
^7 + 124*A*tan(1/2*d*x + 1/2*c)^5 - 115*B*tan(1/2*d*x + 1/2*c)^5 + 100*A*ta
n(1/2*d*x + 1/2*c)^3 - 109*B*tan(1/2*d*x + 1/2*c)^3 + 36*A*tan(1/2*d*x + 1/
2*c) - 21*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

Mupad [B]
time = 0.38, size = 170, normalized size = 1.11

$$\frac{15\,B\,x}{8\,a}-\frac{3\,A\,x}{2\,a}+\frac{7\,A\,\sin\left(c+d\,x\right)}{4\,a\,d}-\frac{7\,B\,\sin\left(c+d\,x\right)}{4\,a\,d}-\frac{A\,\sin\left(2\,c+2\,d\,x\right)}{4\,a\,d}+\frac{A\,\sin\left(3\,c+3\,d\,x\right)}{12\,a\,d}+\frac{A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}+\frac{B\,\sin\left(2\,c+2\,d\,x\right)}{2\,a\,d}-\frac{B\,\sin\left(3\,c+3\,d\,x\right)}{12\,a\,d}+\frac{B\,\sin\left(4\,c+4\,d\,x\right)}{32\,a\,d}-\frac{B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

```
[Out] (15*B*x)/(8*a) - (3*A*x)/(2*a) + (7*A*sin(c + d*x))/(4*a*d) - (7*B*sin(c +
d*x))/(4*a*d) - (A*sin(2*c + 2*d*x))/(4*a*d) + (A*sin(3*c + 3*d*x))/(12*a*d
) + (A*tan(c/2 + (d*x)/2))/(a*d) + (B*sin(2*c + 2*d*x))/(2*a*d) - (B*sin(3*
c + 3*d*x))/(12*a*d) + (B*sin(4*c + 4*d*x))/(32*a*d) - (B*tan(c/2 + (d*x)/2
))/(a*d)
```

3.39 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{3(A-B)x}{2a}-\frac{(3A-4B)\sin(c+dx)}{ad}+\frac{3(A-B)\cos(c+dx)\sin(c+dx)}{2ad}+\frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+$$

```
[Out] 3/2*(A-B)*x/a-(3*A-4*B)*sin(d*x+c)/a/d+3/2*(A-B)*cos(d*x+c)*sin(d*x+c)/a/d+
(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))+1/3*(3*A-4*B)*sin(d*x+c)^3
/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3056, 2827, 2715, 8, 2713}

$$\frac{(3A-4B)\sin^3(c+dx)}{3ad}-\frac{(3A-4B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3(A-B)\sin(c+dx)\cos(c+dx)}{2ad}+\frac{3x(A-B)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*(A - B)*x)/(2*a) - ((3*A - 4*B)*Sin[c + d*x])/(a*d) + (3*(A - B)*Cos[c +
 d*x]*Sin[c + d*x])/(2*a*d) + ((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(d*(a +
 a*Cos[c + d*x])) + ((3*A - 4*B)*Sin[c + d*x]^3)/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx &= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \cos^2(c+dx)(3a(A-B)-a\\ &= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(3A-4B)\int \cos^3(c+dx)\,dx}{a}\\ &= \frac{3(A-B)\cos(c+dx)\sin(c+dx)}{2ad}+\frac{(A-B)\cos^3(c+dx)\sin(c+\\ &= \frac{3(A-B)x}{2a}-\frac{(3A-4B)\sin(c+dx)}{ad}+\frac{3(A-B)\cos(c+dx)\sin(\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 249 vs. 2(122) = 244.
time = 0.60, size = 249, normalized size = 2.04

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(36(A-B)dx\cos\left(\frac{dx}{2}\right)+36(A-B)dx\cos\left(c+\frac{dx}{2}\right)-60A\sin\left(\frac{dx}{2}\right)+69B\sin\left(\frac{dx}{2}\right)-12A\sin\left(c+\frac{dx}{2}\right)+21B\sin\left(c+\frac{dx}{2}\right)-9A\sin\left(c+\frac{3dx}{2}\right)+18B\sin\left(c+\frac{3dx}{2}\right)-9A\sin\left(2c+\frac{3dx}{2}\right)+18B\sin\left(2c+\frac{3dx}{2}\right)+3A\sin\left(2c+\frac{5dx}{2}\right)-2B\sin\left(2c+\frac{5dx}{2}\right)+3A\sin\left(3c+\frac{5dx}{2}\right)-2B\sin\left(3c+\frac{5dx}{2}\right)+B\sin\left(3c+\frac{7dx}{2}\right)+B\sin\left(4c+\frac{7dx}{2}\right)\right)}{24ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(A - B)*d*x*Cos[(d*x)/2] + 36*(A - B)*d*x*Co
s[c + (d*x)/2] - 60*A*Sin[(d*x)/2] + 69*B*Sin[(d*x)/2] - 12*A*Sin[c + (d*x)
/2] + 21*B*Sin[c + (d*x)/2] - 9*A*Sin[c + (3*d*x)/2] + 18*B*Sin[c + (3*d*x)
/2] - 9*A*Sin[2*c + (3*d*x)/2] + 18*B*Sin[2*c + (3*d*x)/2] + 3*A*Sin[2*c +
(5*d*x)/2] - 2*B*Sin[2*c + (5*d*x)/2] + 3*A*Sin[3*c + (5*d*x)/2] - 2*B*Sin[
3*c + (5*d*x)/2] + B*Sin[3*c + (7*d*x)/2] + B*Sin[4*c + (7*d*x)/2]))/(24*a*
d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.15, size = 122, normalized size = 1.00

method	result
derivativedivides	$\frac{-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2\left(-\frac{3A}{2}+\frac{5B}{2}\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{8B}{3}-2A\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(-\frac{A}{2}+\frac{3B}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+3}{da}$
default	$\frac{-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2\left(-\frac{3A}{2}+\frac{5B}{2}\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{8B}{3}-2A\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(-\frac{A}{2}+\frac{3B}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+3}{da}$
risch	$\frac{3xA}{2a}-\frac{3Bx}{2a}+\frac{ie^{i(dx+c)}A}{2da}-\frac{7ie^{i(dx+c)}B}{8da}-\frac{ie^{-i(dx+c)}A}{2da}+\frac{7ie^{-i(dx+c)}B}{8da}-\frac{2iA}{da\left(e^{i(dx+c)}+1\right)}+\frac{2iB}{da\left(e^{i(dx+c)}\right.}$
norman	$\frac{\frac{3(A-B)x}{2a}-\frac{2(A-2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{(A-B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{6(A-B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{9(A-B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{6(A-B)x\left(\right.}{}}{\left(1+\tan^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)+2*((-3/2*A+5/2*B)*tan(1/2
*d*x+1/2*c)^5+(8/3*B-2*A)*tan(1/2*d*x+1/2*c)^3+(-1/2*A+3/2*B)*tan(1/2*d*x+1
/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^3+3*(A-B)*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(116) = 232$.
time = 0.49, size = 310, normalized size = 2.54

$$\frac{B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3A\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*(B*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 3*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*A*((sin(d*x + c)/(cos(d*x + c)
 + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A]
time = 0.35, size = 98, normalized size = 0.80

$$\frac{9(A-B)dx\cos(dx+c)+9(A-B)dx+\left(2B\cos(dx+c)^3+(3A-B)\cos(dx+c)^2-(3A-7B)\cos(dx+c)-12A+16B\right)\sin(dx+c)}{6(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/6*(9*(A - B)*d*x*cos(d*x + c) + 9*(A - B)*d*x + (2*B*cos(d*x + c)^3 + (3*
A - B)*cos(d*x + c)^2 - (3*A - 7*B)*cos(d*x + c) - 12*A + 16*B)*sin(d*x + c
))/(a*d*cos(d*x + c) + a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. $2(105) = 210$.
time = 1.42, size = 1161, normalized size = 9.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((9*A*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*
tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 27*A*d*x*tan(c/
2 + d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*
a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 27*A*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(
c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
 6*a*d) + 9*A*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 +
 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 6*A*tan(c/2 + d*x/2)**7/(6*a*d*tan(c
/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
6*a*d) - 36*A*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c
/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 42*A*tan(c/2 + d*x/2
)**3/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c
/2 + d*x/2)**2 + 6*a*d) - 12*A*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6
+ 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 9*B*d*
x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)*
*4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*B*d*x*tan(c/2 + d*x/2)**4/(6*
a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x
/2)**2 + 6*a*d) - 27*B*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*x/2)**6 +
 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 9*B*d*x
/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 +
 d*x/2)**2 + 6*a*d) + 6*B*tan(c/2 + d*x/2)**7/(6*a*d*tan(c/2 + d*x/2)**6 +
18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 48*B*tan
(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 +
18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 50*B*tan(c/2 + d*x/2)**3/(6*a*d*tan(c
/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
6*a*d) + 24*B*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2
+ d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*(A + B*cos
(c))*cos(c)**3/(a*cos(c) + a), True))
```

Giac [A]
time = 0.41, size = 151, normalized size = 1.24

$$\frac{\frac{9\,(dx+c)(A-B)}{a}-\frac{6\,\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}-\frac{2\,\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-16\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(9*(d*x + c)*(A - B)/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/
2*c))/a - 2*(9*A*tan(1/2*d*x + 1/2*c)^5 - 15*B*tan(1/2*d*x + 1/2*c)^5 + 12*
A*tan(1/2*d*x + 1/2*c)^3 - 16*B*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x +
1/2*c) - 9*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

Mupad [B]
time = 1.36, size = 138, normalized size = 1.13

$$\frac{3\,x\,(A-B)}{2\,a}-\frac{(3\,A-5\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(4\,A-\frac{16\,B}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(A-3\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

```
[Out] (3*x*(A - B))/(2*a) - (tan(c/2 + (d*x)/2)^5*(3*A - 5*B) + tan(c/2 + (d*x)/2
)^3*(4*A - (16*B)/3) + tan(c/2 + (d*x)/2)*(A - 3*B))/(d*(a + 3*a*tan(c/2 +
(d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 + a*tan(c/2 + (d*x)/2)^6)) - (tan(c/2
 + (d*x)/2)*(A - B))/(a*d)
```

3.40 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{(A-B)x}{a}+\frac{Bx}{2a}+\frac{(A-B)\sin(c+dx)}{ad}+\frac{B\cos(c+dx)\sin(c+dx)}{2ad}+\frac{(A-B)\sin(c+dx)}{ad(1+\cos(c+dx))}$$

```
[Out] -(A-B)*x/a+1/2*B*x/a+(A-B)*sin(d*x+c)/a/d+1/2*B*cos(d*x+c)*sin(d*x+c)/a/d+(
A-B)*sin(d*x+c)/a/d/(1+cos(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {3056, 2813}

$$\frac{2(A-B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(2A-3B)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x(2A-3B)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] -1/2*((2*A - 3*B)*x)/a + (2*(A - B)*Sin[c + d*x])/(a*d) - ((2*A - 3*B)*Cos[
c + d*x]*Sin[c + d*x])/(2*a*d) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(
a + a*Cos[c + d*x]))
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos(c+dx)(2a(A-B)-\ldots}{a^2\ldots}$$
$$= -\frac{(2A-3B)x}{2a} + \frac{2(A-B)\sin(c+dx)}{ad} - \frac{(2A-3B)\cos(c+dx)\ldots}{2ad}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 197 vs. 2(90) = 180.

time = 0.49, size = 197, normalized size = 2.19

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-4(2A-3B)dx\cos\left(\frac{dx}{2}\right)-4(2A-3B)dx\cos\left(c+\frac{dx}{2}\right)+20A\sin\left(\frac{dx}{2}\right)-20B\sin\left(\frac{dx}{2}\right)+4A\sin\left(c+\frac{dx}{2}\right)-4B\sin\left(c+\frac{dx}{2}\right)+4A\sin\left(c+\frac{3dx}{2}\right)-3B\sin\left(c+\frac{3dx}{2}\right)+4A\sin\left(2c+\frac{3dx}{2}\right)-3B\sin\left(2c+\frac{3dx}{2}\right)+B\sin\left(2c+\frac{5dx}{2}\right)+B\sin\left(3c+\frac{5dx}{2}\right)\right)}{8ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-4*(2*A - 3*B)*d*x*Cos[(d*x)/2] - 4*(2*A - 3*B)
*d*x*Cos[c + (d*x)/2] + 20*A*Sin[(d*x)/2] - 20*B*Sin[(d*x)/2] + 4*A*Sin[c +
 (d*x)/2] - 4*B*Sin[c + (d*x)/2] + 4*A*Sin[c + (3*d*x)/2] - 3*B*Sin[c + (3*
d*x)/2] + 4*A*Sin[2*c + (3*d*x)/2] - 3*B*Sin[2*c + (3*d*x)/2] + B*Sin[2*c +
 (5*d*x)/2] + B*Sin[3*c + (5*d*x)/2]))/(8*a*d*(1 + Cos[c + d*x]))
```

Maple [A]

time = 0.14, size = 105, normalized size = 1.17

method	result
derivativedivides	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(\left(\frac{3B}{2}-A\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{B}{2}-A\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-(2A-3B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
default	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(\left(\frac{3B}{2}-A\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{B}{2}-A\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-(2A-3B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
risch	$-\frac{xA}{a}+\frac{3Bx}{2a}-\frac{ie^{i(dx+c)}A}{2da}+\frac{ie^{i(dx+c)}B}{2da}+\frac{ie^{-i(dx+c)}A}{2da}-\frac{ie^{-i(dx+c)}B}{2da}+\frac{2iA}{da\left(e^{i(dx+c)}+1\right)}-\frac{2iB}{da\left(e^{i(dx+c)}+\ldots\right.}$
norman	$\frac{\frac{(A-B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{(3A-2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{(5A-6B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{(2A-3B)x}{2a}+\frac{7(A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{3(2A-3B)\ldots}{\ldots}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(A*tan(1/2*d*x+1/2*c)-B*tan(1/2*d*x+1/2*c)-2*((3/2*B-A)*tan(1/2*d*x+1
/2*c)^3+(1/2*B-A)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2-(2*A-3*B)*
arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(86) = 172$.

time = 0.48, size = 225, normalized size = 2.50

$$-\frac{B\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(B*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1))) + A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a -
 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c)
+ 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A]

time = 0.35, size = 83, normalized size = 0.92

$$-\frac{(2A-3B)dx\cos(dx+c)+(2A-3B)dx-\left(B\cos(dx+c)^2+(2A-B)\cos(dx+c)+4A-4B\right)\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2*((2*A - 3*B)*d*x*cos(d*x + c) + (2*A - 3*B)*d*x - (B*cos(d*x + c)^2 +
(2*A - B)*cos(d*x + c) + 4*A - 4*B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(68) = 136$.

time = 0.92, size = 665, normalized size = 7.39

$$\begin{cases} -\frac{2Adx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}-\frac{4Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}-\frac{2Adx}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}+\frac{2A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}+\frac{8A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}+\frac{6A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}+\frac{3Bdx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}+\frac{6Bdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}+\frac{3Bdx}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}-\frac{2B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}-\frac{10B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}-\frac{4B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} & \text{for}\ d \neq 0 \\ \frac{x(A+B\cos(c))\cos^2(c)}{a\cos(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((-2*A*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*
tan(c/2 + d*x/2)**2 + 2*a*d) - 4*A*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 +
 d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*A*d*x/(2*a*d*tan(c/2 +
d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 2*A*tan(c/2 + d*x/2)**5/(2
*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 8*A*tan(c/2
 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d
```

```
) + 6*A*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2
)**2 + 2*a*d) + 3*B*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*
a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 6*B*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c
/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 3*B*d*x/(2*a*d*tan(c/
2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*B*tan(c/2 + d*x/2)**
5/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 10*B*ta
n(c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 +
2*a*d) - 4*B*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 +
d*x/2)**2 + 2*a*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**2/(a*cos(c) + a),
True))
```

Giac [A]
time = 0.47, size = 124, normalized size = 1.38

$$-\frac{\frac{(dx+c)(2A-3B)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}-\frac{2\left(2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2*((d*x + c)*(2*A - 3*B)/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x +
 1/2*c))/a - 2*(2*A*tan(1/2*d*x + 1/2*c)^3 - 3*B*tan(1/2*d*x + 1/2*c)^3 + 2
*A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
+ 1)^2*a))/d
```

Mupad [B]
time = 0.49, size = 107, normalized size = 1.19

$$\frac{(2A-3B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(2A-B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}-\frac{x(2A-3B)}{2a}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(A-B)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(2*A - 3*B) + tan(c/2 + (d*x)/2)*(2*A - B))/(d*(a + 2
*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4)) - (x*(2*A - 3*B))/(2*a)
+ (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

3.41 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{(A-B)x}{a}+\frac{B\sin(c+dx)}{ad}-\frac{(A-B)\sin(c+dx)}{ad(1+\cos(c+dx))}$$

```
[Out] (A-B)*x/a+B*sin(d*x+c)/a/d-(A-B)*sin(d*x+c)/a/d/(1+cos(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3047, 3102, 12, 2814, 2727}

$$-\frac{(A-B)\sin(c+dx)}{ad(\cos(c+dx)+1)}+\frac{x(A-B)}{a}+\frac{B\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((A - B)*x)/a + (B*Sin[c + d*x])/(a*d) - ((A - B)*Sin[c + d*x])/(a*d*(1 + C
os[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx &= \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx\\
&= \frac{B\sin(c+dx)}{ad}+\frac{\int \frac{a(A-B)\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{a}\\
&= \frac{B\sin(c+dx)}{ad}+(A-B)\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx\\
&= \frac{(A-B)x}{a}+\frac{B\sin(c+dx)}{ad}+(-A+B)\int \frac{1}{a+a\cos(c+dx)}\,dx\\
&= \frac{(A-B)x}{a}+\frac{B\sin(c+dx)}{ad}-\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 126 vs. 2(54) = 108.
time = 0.27, size = 126, normalized size = 2.33

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(2(A-B)dx\cos\left(\frac{dx}{2}\right)+2(A-B)dx\cos\left(c+\frac{dx}{2}\right)-4A\sin\left(\frac{dx}{2}\right)+5B\sin\left(\frac{dx}{2}\right)+B\sin\left(c+\frac{dx}{2}\right)+B\sin\left(c+\frac{3dx}{2}\right)+B\sin\left(2c+\frac{3dx}{2}\right)\right)}{2ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(2*(A - B)*d*x*Cos[(d*x)/2] + 2*(A - B)*d*x*Cos[
c + (d*x)/2] - 4*A*Sin[(d*x)/2] + 5*B*Sin[(d*x)/2] + B*Sin[c + (d*x)/2] + B
*Sin[c + (3*d*x)/2] + B*Sin[2*c + (3*d*x)/2]))/(2*a*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.12, size = 76, normalized size = 1.41

method	result
derivativedivides	$\frac{-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2(A-B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$

default	$\frac{-A\tan(\frac{dx}{2}+\frac{c}{2})+B\tan(\frac{dx}{2}+\frac{c}{2})+\frac{2B\tan(\frac{dx}{2}+\frac{c}{2})}{1+\tan^2(\frac{dx}{2}+\frac{c}{2})}+2(A-B)\arctan(\tan(\frac{dx}{2}+\frac{c}{2}))}{da}$
risch	$\frac{xA}{a}-\frac{Bx}{a}-\frac{i e^{i(dx+c)}B}{2da}+\frac{i e^{-i(dx+c)}B}{2da}-\frac{2iA}{da(e^{i(dx+c)}+1)}+\frac{2iB}{da(e^{i(dx+c)}+1)}$
norman	$\frac{\frac{(A-B)x}{a}+\frac{(A-B)x(\tan^4(\frac{dx}{2}+\frac{c}{2}))}{a}-\frac{(A-3B)\tan(\frac{dx}{2}+\frac{c}{2})}{da}-\frac{2(A-2B)(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{da}-\frac{(A-B)(\tan^5(\frac{dx}{2}+\frac{c}{2}))}{da}+\frac{2(A-B)x(\tan^2(}{a}}{(1+\tan^2(\frac{dx}{2}+\frac{c}{2}))^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)+2*B*tan(1/2*d*x+1/2*c)/(1
+tan(1/2*d*x+1/2*c)^2)+2*(A-B)*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(54) = 108$.
time = 0.52, size = 143, normalized size = 2.65

$$-\frac{B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -(B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(
cos(d*x + c) + 1))) - A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(
d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A]
time = 0.34, size = 61, normalized size = 1.13

$$\frac{(A-B)dx\cos(dx+c)+(A-B)dx+(B\cos(dx+c)-A+2B)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] ((A - B)*d*x*cos(d*x + c) + (A - B)*d*x + (B*cos(d*x + c) - A + 2*B)*sin(d*
x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(39) = 78$.

time = 0.63, size = 264, normalized size = 4.89

$$\begin{cases} \frac{Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{Adx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{Bdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{Bdx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{3B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x(A+B\cos(c))\cos(c)}{a\cos(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) + A*d*
x/(a*d*tan(c/2 + d*x/2)**2 + a*d) - A*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*
x/2)**2 + a*d) - A*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d) - B*d*x
*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) - B*d*x/(a*d*tan(c/2 +
 d*x/2)**2 + a*d) + B*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**2 + a*d) +
 3*B*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*(A + B
*cos(c))*cos(c)/(a*cos(c) + a), True))
```

Giac [A]
time = 0.42, size = 78, normalized size = 1.44

$$\frac{\frac{(dx+c)(A-B)}{a} - \frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} + \frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*(A - B)/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a
+ 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

Mupad [B]
time = 0.27, size = 65, normalized size = 1.20

$$\frac{x\,(A-B)}{a} + \frac{2\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

```
[Out] (x*(A - B))/a + (2*B*tan(c/2 + (d*x)/2))/(d*(a + a*tan(c/2 + (d*x)/2)^2)) -
 (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

3.42 $\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{Bx}{a}+\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

```
[Out] B*x/a+(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2814, 2727}

$$\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}+\frac{Bx}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (B*x)/a + ((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{Bx}{a}-(-A+B)\int \frac{1}{a+a\cos(c+dx)}\,dx\\ &= \frac{Bx}{a}+\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 72 vs. 2(34) = 68.
time = 0.14, size = 72, normalized size = 2.12

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(Bdx\cos\left(\frac{dx}{2}\right)+Bdx\cos\left(c+\frac{dx}{2}\right)+2(A-B)\sin\left(\frac{dx}{2}\right)\right)}{ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(B*d*x*Cos[(d*x)/2] + B*d*x*Cos[c + (d*x)/2] + 2
*(A - B)*Sin[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.09, size = 45, normalized size = 1.32

method	result	size
derivativedivides	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2B\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	45
default	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2B\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	45
risch	$\frac{Bx}{a}+\frac{2iA}{da\left(e^{i(dx+c)}+1\right)}-\frac{2iB}{da\left(e^{i(dx+c)}+1\right)}$	54
norman	$\frac{\frac{Bx}{a}+\frac{Bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{(A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(A*tan(1/2*d*x+1/2*c)-B*tan(1/2*d*x+1/2*c)+2*B*arctan(tan(1/2*d*x+1/2
*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(34) = 68$.
time = 0.49, size = 73, normalized size = 2.15

$$\frac{B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{A\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] (B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x
+ c) + 1))) + A*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 0.35, size = 43, normalized size = 1.26

$$\frac{Bdx\cos(dx+c)+Bdx+(A-B)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] (B*d*x*cos(d*x + c) + B*d*x + (A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d
)
```

Sympy [A]
time = 0.38, size = 49, normalized size = 1.44

$$\begin{cases} \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad}+\frac{Bx}{a}-\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad} & \text{for } d\neq 0 \\ \frac{x(A+B\cos(c))}{a\cos(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)/(a*d) + B*x/a - B*tan(c/2 + d*x/2)/(a*d), Ne(
d, 0)), (x*(A + B*cos(c))/(a*cos(c) + a), True))
```

Giac [A]
time = 0.43, size = 43, normalized size = 1.26

$$\frac{\frac{(dx+c)B}{a}+\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*B/a + (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d
```

Mupad [B]
time = 0.20, size = 30, normalized size = 0.88

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a}+\frac{B\,d\,x}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x)),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(A - B))/a + (B*d*x)/a)/d
```

3.43 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

```
[Out] A*arctanh(sin(d*x+c))/a/d-(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3057, 12, 3855}

$$\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a*d) - ((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int aA\sec(c+dx)\,dx}{a^2}$$
$$= -\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{A\int \sec(c+dx)\,dx}{a}$$
$$= \frac{A\tanh^{-1}(\sin(c+dx))}{ad} - \frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(44) = 88.
time = 0.28, size = 109, normalized size = 2.48

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(A\cos\left(\frac{1}{2}(c+dx)\right)\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+(-A+B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\right)}{ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(A*Cos[(c + d*x)/2]*(-Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (-A + B)*Sec[c/2]*Sin
[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.16, size = 61, normalized size = 1.39

method	result	size
derivativedivides	$\frac{-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	61
default	$\frac{-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	61
risch	$-\frac{2iA}{da\left(e^{i(dx+c)}+1\right)}+\frac{2iB}{da\left(e^{i(dx+c)}+1\right)}+\frac{A\ln\left(e^{i(dx+c)}+i\right)}{da}-\frac{A\ln\left(e^{i(dx+c)}-i\right)}{da}$	91
norman	$\frac{-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{(A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)-A*ln(tan(1/2*d*x+1/2*c)-1
)+A*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(44) = 88.

time = 0.28, size = 99, normalized size = 2.25

$$\frac{A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{B\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] (A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
+ c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) + B*sin(d*x + c)/(a
*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 0.36, size = 74, normalized size = 1.68

$$\frac{(A\cos(dx+c)+A)\log(\sin(dx+c)+1)-(A\cos(dx+c)+A)\log(-\sin(dx+c)+1)-2(A-B)\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/2*((A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - (A*cos(d*x + c) + A)*log(
-sin(d*x + c) + 1) - 2*(A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A\sec(c+dx)}{\cos(c+dx)+1}\,dx+\int\frac{B\cos(c+dx)\sec(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*s
ec(c + d*x)/(cos(c + d*x) + 1), x))/a
```

Giac [A]
time = 0.44, size = 71, normalized size = 1.61

$$\frac{\frac{A\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] (A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - A*log(abs(tan(1/2*d*x + 1/2*c) -
1))/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d
```

Mupad [B]
time = 0.22, size = 42, normalized size = 0.95

$$\frac{2\,A\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A-B\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))),x)
```

```
[Out] (2*A*atanh(tan(c/2 + (d*x)/2)))/(a*d) - (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

3.44 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=69

$$-\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}+\frac{(2A-B)\tan(c+dx)}{ad}-\frac{(A-B)\tan(c+dx)}{d(a+a\cos(c+dx))}$$

```
[Out] -(A-B)*arctanh(sin(d*x+c))/a/d+(2*A-B)*tan(d*x+c)/a/d-(A-B)*tan(d*x+c)/d/(a
+a*cos(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3057, 2827, 3852, 8, 3855}

$$\frac{(2A-B)\tan(c+dx)}{ad}-\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(((A - B)*ArcTanh[Sin[c + d*x]])/(a*d)) + ((2*A - B)*Tan[c + d*x])/(a*d) -
 ((A - B)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{(A-B)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int(a(2A-B)-a(A-B)\cos(c+dx))\,\mathrm{s}}{a^2} \\
&= -\frac{(A-B)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{(A-B)\int\sec(c+dx)\,dx}{a}+\frac{(2A-B)\int}{} \\
&= -\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{(2A-B}{} \\
&= -\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}+\frac{(2A-B)\tan(c+dx)}{ad}-\frac{(A-B}{d(a+}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(69) = 138.
time = 1.31, size = 201, normalized size = 2.91

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\frac{A\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)\right)}{ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((A -
 B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]]) + (A*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
))))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.20, size = 100, normalized size = 1.45

method	result

derivativedivides	$\frac{-\frac{A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+(-A+B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+(A-B)\ln\left(\tan\left(\frac{dx}{2}\cdots\right.\right.}{da}$
default	$\frac{-\frac{A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+(-A+B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+(A-B)\ln\left(\tan\left(\frac{dx}{2}\cdots\right.\right.}{da}$
norman	$\frac{\frac{(A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{(3A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(A-B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}-\frac{(A-B)\ln\left(\tan\left(\frac{dx}{2}\cdots\right.\right.}{da}$
risch	$\frac{2i\left(A\,e^{2i(dx+c)}-B\,e^{2i(dx+c)}+A\,e^{i(dx+c)}+2A-B\right)}{da\left(e^{2i(dx+c)}+1\right)\left(e^{i(dx+c)}+1\right)}-\frac{A\ln\left(e^{i(dx+c)}+i\right)}{da}+\frac{\ln\left(e^{i(dx+c)}+i\right)B}{da}+\frac{A\ln\left(e^{i(dx+c)}-i\right)}{da}-\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-A/(tan(1/2*d*x+1/2*c)+1)+(-A+B)*ln(tan(1/2*d*x+1/2*c)+1)+A*tan(1/2*
d*x+1/2*c)-B*tan(1/2*d*x+1/2*c)-A/(tan(1/2*d*x+1/2*c)-1)+(A-B)*ln(tan(1/2*d
*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(69) = 138$.
time = 0.28, size = 196, normalized size = 2.84

$$-\frac{A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - B*(log(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A]
time = 0.38, size = 127, normalized size = 1.84

$$-\frac{\left((A-B)\cos(dx+c)^2+(A-B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-\left((A-B)\cos(dx+c)^2+(A-B)\cos(dx+c)\right)\log(-\sin(dx+c)+1)-2\left((2A-B)\cos(dx+c)+A\right)\sin(dx+c)}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2*(((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(sin(d*x + c) + 1)
 - ((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) -
```

```
2*((2*A - B)*cos(d*x + c) + A)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos
(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^2(c+dx)}{\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec^2(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)*sec(c + d*x)**2/(cos(c + d*x) + 1), x))/a
```

Giac [A]
time = 0.44, size = 110, normalized size = 1.59

$$-\frac{\frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} - \frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} - \frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} + \frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -((A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (A - B)*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a +
2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

Mupad [B]
time = 0.29, size = 78, normalized size = 1.13

$$\frac{2\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A-B\right)}{a\,d} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A-B\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))),x)
```

```
[Out] (2*A*tan(c/2 + (d*x)/2))/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c/
2 + (d*x)/2))*(A - B))/(a*d) + (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

3.45 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{(3A-2B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{2(A-B)\tan(c+dx)}{ad}+\frac{(3A-2B)\sec(c+dx)\tan(c+dx)}{2ad}-\frac{(A-B)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}$$

```
[Out] 1/2*(3*A-2*B)*arctanh(sin(d*x+c))/a/d-2*(A-B)*tan(d*x+c)/a/d+1/2*(3*A-2*B)*
sec(d*x+c)*tan(d*x+c)/a/d-(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3057, 2827, 3853, 3855, 3852, 8}

$$-\frac{2(A-B)\tan(c+dx)}{ad}+\frac{(3A-2B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(3A-2B)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((3*A - 2*B)*ArcTanh[Sin[c + d*x]])/(2*a*d) - (2*(A - B)*Tan[c + d*x])/(a*d
) + ((3*A - 2*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A - B)*Sec[c + d*x]
*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int (a(3A-2B)-2a(A-B)\\ &= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{(3A-2B)\int \sec^3(c+dx)\,dx}{a}\\ &= \frac{(3A-2B)\sec(c+dx)\tan(c+dx)}{2ad}-\frac{(A-B)\sec(c+dx)\tan(c+}{d(a+a\cos(c+dx))}\\ &= \frac{(3A-2B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{2(A-B)\tan(c+dx)}{ad}+\frac{(3A-\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 289 vs. 2(107) = 214.

time = 3.48, size = 289, normalized size = 2.70

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(4(-A+B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left((-6A+4B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+6A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-4B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{4(A-B)\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)\right)}{2ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(4*(-A + B)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((-6
*A + 4*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*A*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] - 4*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + A/(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - A/(Cos[(c + d*x)/2] + Sin[(c + d*x
```

```
)/2])^2 - (4*(A - B)*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)))))/(2*a*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.22, size = 142, normalized size = 1.33

method	result
derivativedivides	$\frac{-\frac{A}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{-\frac{3A}{2}+B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\left(\frac{3A}{2}-B\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{A}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{da}$
default	$\frac{-\frac{A}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{-\frac{3A}{2}+B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\left(\frac{3A}{2}-B\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{A}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{da}$
norman	$\frac{\frac{(3A-B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{(4A-3B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{(A-B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{(2A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{(3A-2B)\ln\left(\tan\left(\right.\right.}{2da}$
risch	$-\frac{i\left(3A\,e^{4i(dx+c)}-2B\,e^{4i(dx+c)}+3A\,e^{3i(dx+c)}-2B\,e^{3i(dx+c)}+5A\,e^{2i(dx+c)}-6B\,e^{2i(dx+c)}+A\,e^{i(dx+c)}-2B\,e^{i(dx+c)}+4A\right.}{da\left(e^{i(dx+c)}+1\right)\left(e^{2i(dx+c)}+1\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/2*A/(tan(1/2*d*x+1/2*c)+1)^2-(-3/2*A+B)/(tan(1/2*d*x+1/2*c)+1)+(3
/2*A-B)*ln(tan(1/2*d*x+1/2*c)+1)-A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)+
1/2*A/(tan(1/2*d*x+1/2*c)-1)^2-(-3/2*A+B)/(tan(1/2*d*x+1/2*c)-1)+(-3/2*A+B)
*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(103) = 206$.
time = 0.28, size = 282, normalized size = 2.64

$$-\frac{A\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+2B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(A*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) +
 1))) + 2*B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/
(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A]
time = 0.35, size = 156, normalized size = 1.46

$$\frac{((3A-2B)\cos(dx+c)^3+(3A-2B)\cos(dx+c)^2)\log(\sin(dx+c)+1)-((3A-2B)\cos(dx+c)^3+(3A-2B)\cos(dx+c)^2)\log(-\sin(dx+c)+1)-2(4(A-B)\cos(dx+c)^2+(A-2B)\cos(dx+c)-A)\sin(dx+c)}{4(ad\cos(dx+c)^3+ad\cos(dx+c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/4*(((3*A - 2*B)*cos(d*x + c)^3 + (3*A - 2*B)*cos(d*x + c)^2)*log(sin(d*x
+ c) + 1) - ((3*A - 2*B)*cos(d*x + c)^3 + (3*A - 2*B)*cos(d*x + c)^2)*log(-
sin(d*x + c) + 1) - 2*(4*(A - B)*cos(d*x + c)^2 + (A - 2*B)*cos(d*x + c) -
A)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^3(c+dx)}{\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^3(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)*sec(c + d*x)**3/(cos(c + d*x) + 1), x))/a
```

Giac [A]
time = 0.46, size = 157, normalized size = 1.47

$$\frac{\frac{(3A-2B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{(3A-2B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((3*A - 2*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (3*A - 2*B)*log(abs
(tan(1/2*d*x + 1/2*c) - 1))/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x +
 1/2*c))/a + 2*(3*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 - A
*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
- 1)^2*a))/d
```

Mupad [B]
time = 0.37, size = 119, normalized size = 1.11

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(3\,A-2\,B)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-2\,B)}{d\,\left(a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}+\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,A}{2}-B\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(3*A - 2*B) - tan(c/2 + (d*x)/2)*(A - 2*B))/(d*(a - 2
*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4)) + (2*atanh(tan(c/2 + (d*
x)/2))*((3*A)/2 - B))/(a*d) - (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

3.46 $\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=131

$$-\frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(4A-3B)\tan(c+dx)}{ad}-\frac{3(A-B)\sec(c+dx)\tan(c+dx)}{2ad}-\frac{(A-B)\sec^2}{d(a+}$$

```
[Out] -3/2*(A-B)*arctanh(sin(d*x+c))/a/d+(4*A-3*B)*tan(d*x+c)/a/d-3/2*(A-B)*sec(d
*x+c)*tan(d*x+c)/a/d-(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))+1/3*(
4*A-3*B)*tan(d*x+c)^3/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3057, 2827, 3852, 3853, 3855}

$$\frac{(4A-3B)\tan^3(c+dx)}{3ad}+\frac{(4A-3B)\tan(c+dx)}{ad}-\frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{3(A-B)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-3*(A - B)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((4*A - 3*B)*Tan[c + d*x])/(a*
d) - (3*(A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A - B)*Sec[c + d*x]^
2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((4*A - 3*B)*Tan[c + d*x]^3)/(3*
a*d)
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int (a(4A-3B)-3a(A-\ldots}{\ldots}$$
$$= -\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{(4A-3B)\int \sec^4(c+dx)\ldots}{a}$$
$$= -\frac{3(A-B)\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(A-B)\sec^2(c+dx)\tan(\ldots}{d(a+a\cos(c+dx)\ldots}$$
$$= -\frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad} + \frac{(4A-3B)\tan(c+dx)}{ad} - \frac{3(\ldots}{\ldots}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 490 vs. 2(131) = 262.
time = 4.48, size = 490, normalized size = 3.74

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(144(A-B)\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\frac{c}{2}\right)\sec(c)\sec^3(c+dx)\left(6(A+B)\sin\left(\frac{dx}{2}\right)+3(13A-9B)\sin\left(\frac{3dx}{2}\right)-24A\sin\left(c-\frac{dx}{2}\right)+12B\sin\left(c-\frac{dx}{2}\right)-6A\sin\left(c+\frac{dx}{2}\right)+6B\sin\left(c+\frac{dx}{2}\right)-24A\sin\left(2c+\frac{dx}{2}\right)+24B\sin\left(2c+\frac{dx}{2}\right)+21A\sin\left(c+\frac{3dx}{2}\right)-9B\sin\left(c+\frac{3dx}{2}\right)+9A\sin\left(2c+\frac{3dx}{2}\right)-9B\sin\left(2c+\frac{3dx}{2}\right)-9A\sin\left(3c+\frac{3dx}{2}\right)+9B\sin\left(3c+\frac{3dx}{2}\right)+7A\sin\left(c+\frac{5dx}{2}\right)-3B\sin\left(c+\frac{5dx}{2}\right)+A\sin\left(2c+\frac{5dx}{2}\right)+3B\sin\left(2c+\frac{5dx}{2}\right)-3A\sin\left(3c+\frac{5dx}{2}\right)+3B\sin\left(3c+\frac{5dx}{2}\right)-9A\sin\left(4c+\frac{5dx}{2}\right)+9B\sin\left(4c+\frac{5dx}{2}\right)+16A\sin\left(2c+\frac{7dx}{2}\right)-12B\sin\left(2c+\frac{7dx}{2}\right)+10A\sin\left(3c+\frac{7dx}{2}\right)-6B\sin\left(3c+\frac{7dx}{2}\right)+6A\sin\left(4c+\frac{7dx}{2}\right)-6B\sin\left(4c+\frac{7dx}{2}\right)\right)\right)}{48ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(144*(A - B)*Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sec[c
]*Sec[c + d*x]^3*(6*(A + B)*Sin[(d*x)/2] + 3*(13*A - 9*B)*Sin[(3*d*x)/2] -
24*A*Sin[c - (d*x)/2] + 12*B*Sin[c - (d*x)/2] - 6*A*Sin[c + (d*x)/2] + 6*B*
Sin[c + (d*x)/2] - 24*A*Sin[2*c + (d*x)/2] + 24*B*Sin[2*c + (d*x)/2] + 21*A
```

```
*Sin[c + (3*d*x)/2] - 9*B*Sin[c + (3*d*x)/2] + 9*A*Sin[2*c + (3*d*x)/2] - 9
*B*Sin[2*c + (3*d*x)/2] - 9*A*Sin[3*c + (3*d*x)/2] + 9*B*Sin[3*c + (3*d*x)/
2] + 7*A*Sin[c + (5*d*x)/2] - 3*B*Sin[c + (5*d*x)/2] + A*Sin[2*c + (5*d*x)/
2] + 3*B*Sin[2*c + (5*d*x)/2] - 3*A*Sin[3*c + (5*d*x)/2] + 3*B*Sin[3*c + (5
*d*x)/2] - 9*A*Sin[4*c + (5*d*x)/2] + 9*B*Sin[4*c + (5*d*x)/2] + 16*A*Sin[2
*c + (7*d*x)/2] - 12*B*Sin[2*c + (7*d*x)/2] + 10*A*Sin[3*c + (7*d*x)/2] - 6
*B*Sin[3*c + (7*d*x)/2] + 6*A*Sin[4*c + (7*d*x)/2] - 6*B*Sin[4*c + (7*d*x)/
2])))/(48*a*d*(1 + Cos[c + d*x]))
```

Maple [**A**]
time = 0.26, size = 190, normalized size = 1.45

method	result
derivativedivides	$\dfrac{-\frac{A}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-2A+B}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\left(-\frac{3A}{2}+\frac{3B}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{\frac{5A}{2}-\frac{3B}{2}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{d}{2}\right.}{da}$
default	$\dfrac{-\frac{A}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-2A+B}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\left(-\frac{3A}{2}+\frac{3B}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{\frac{5A}{2}-\frac{3B}{2}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{d}{2}\right.}{da}$
norman	$\dfrac{\frac{(A-B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{(A-3B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{2(2A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{(7A-5B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{(13A-15B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{3da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$
risch	$\dfrac{i\left(9A\,e^{6i(dx+c)}-9B\,e^{6i(dx+c)}+9A\,e^{5i(dx+c)}-9B\,e^{5i(dx+c)}+24A\,e^{4i(dx+c)}-24B\,e^{4i(dx+c)}+24A\,e^{3i(dx+c)}-12B\,e^{3i(dx+c)}\right.}{3da\left(e^{2i(dx+c)}+1\right)^3\left(e^{i(dx+c)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/3*A/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-2*A+B)/(tan(1/2*d*x+1/2*c)+1)^
2+(-3/2*A+3/2*B)*ln(tan(1/2*d*x+1/2*c)+1)-(5/2*A-3/2*B)/(tan(1/2*d*x+1/2*c)
+1)+A*tan(1/2*d*x+1/2*c)-B*tan(1/2*d*x+1/2*c)-1/3*A/(tan(1/2*d*x+1/2*c)-1)^
3-1/2*(2*A-B)/(tan(1/2*d*x+1/2*c)-1)^2+(3/2*A-3/2*B)*ln(tan(1/2*d*x+1/2*c)-
1)-(5/2*A-3/2*B)/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(125) = 250$.
time = 0.28, size = 368, normalized size = 2.81

$$\frac{A\left(\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3B\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(A*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^
```

```
2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(
d*x + c) + 1))) - 3*B*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)/a + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(
cos(d*x + c) + 1))))/d
```

Fricas [**A**]
time = 0.35, size = 168, normalized size = 1.28

$$-\frac{9\left((A-B)\cos\left(dx+c\right)^{4}+(A-B)\cos\left(dx+c\right)^{3}\right)\log\left(\sin\left(dx+c\right)+1\right)-9\left((A-B)\cos\left(dx+c\right)^{4}+(A-B)\cos\left(dx+c\right)^{3}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(4\left(4A-3B\right)\cos\left(dx+c\right)^{3}+\left(7A-3B\right)\cos\left(dx+c\right)^{2}-\left(A-3B\right)\cos\left(dx+c\right)+2A\right)\sin\left(dx+c\right)}{12\left(ad\cos\left(dx+c\right)^{4}+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/12*(9*((A - B)*cos(d*x + c)^4 + (A - B)*cos(d*x + c)^3)*log(sin(d*x + c)
 + 1) - 9*((A - B)*cos(d*x + c)^4 + (A - B)*cos(d*x + c)^3)*log(-sin(d*x +
c) + 1) - 2*(4*(4*A - 3*B)*cos(d*x + c)^3 + (7*A - 3*B)*cos(d*x + c)^2 - (A
 - 3*B)*cos(d*x + c) + 2*A)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x
 + c)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^{4}\left(c+dx\right)}{\cos\left(c+dx\right)+1}\,dx+\int \frac{B\cos\left(c+dx\right)\sec^{4}\left(c+dx\right)}{\cos\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**4/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)*sec(c + d*x)**4/(cos(c + d*x) + 1), x))/a
```

Giac [**A**]
time = 0.45, size = 182, normalized size = 1.39

$$-\frac{\frac{9\,(A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{9\,(A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{6\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}+\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-16\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+12\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(9*(A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*(A - B)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1))/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/
2*c))/a + 2*(15*A*tan(1/2*d*x + 1/2*c)^5 - 9*B*tan(1/2*d*x + 1/2*c)^5 - 16*
A*tan(1/2*d*x + 1/2*c)^3 + 12*B*tan(1/2*d*x + 1/2*c)^3 + 9*A*tan(1/2*d*x +
1/2*c) - 3*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

Mupad [B]
time = 0.64, size = 152, normalized size = 1.16

$$\frac{(5A-3B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(4B-\frac{16A}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(3A-B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}-\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(A-B)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(A-B)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^4*(a + a*cos(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(5*A - 3*B) - tan(c/2 + (d*x)/2)^3*((16*A)/3 - 4*B) +
 tan(c/2 + (d*x)/2)*(3*A - B))/(d*(a - 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c
/2 + (d*x)/2)^4 - a*tan(c/2 + (d*x)/2)^6)) - (3*atanh(tan(c/2 + (d*x)/2))*(
A - B))/(a*d) + (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

3.47 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=170

$$\frac{(7A-10B)x}{2a^2}-\frac{4(2A-3B)\sin(c+dx)}{a^2d}+\frac{(7A-10B)\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{(7A-10B)\cos^3(c+dx)\sin}{3a^2d(1+\cos(c+dx)}$$

```
[Out] 1/2*(7*A-10*B)*x/a^2-4*(2*A-3*B)*sin(d*x+c)/a^2/d+1/2*(7*A-10*B)*cos(d*x+c)
*sin(d*x+c)/a^2/d+1/3*(7*A-10*B)*cos(d*x+c)^3*sin(d*x+c)/a^2/d/(1+cos(d*x+c
))+1/3*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+4/3*(2*A-3*B)*sin
(d*x+c)^3/a^2/d
```

Rubi [A]

time = 0.22, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3056, 2827, 2715, 8, 2713}

$$\frac{4(2A-3B)\sin^3(c+dx)}{3a^2d}-\frac{4(2A-3B)\sin(c+dx)}{a^2d}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(7A-10B)\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{x(7A-10B)}{2a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((7*A - 10*B)*x)/(2*a^2) - (4*(2*A - 3*B)*Sin[c + d*x])/(a^2*d) + ((7*A - 1
0*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + ((7*A - 10*B)*Cos[c + d*x]^3*Si
n[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^4*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (4*(2*A - 3*B)*Sin[c + d*x]^3)/(3*a^2*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx &= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{\cos^3(c+dx)(4a(A-B)-3a(A-2B)\cos}{a+a\cos(c+dx)}}{3a^2} \\ &= \frac{(7A-10B)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}+\frac{(A-B)\cos^4(c+dx)\sin(c}{3d(a+a\cos(c+dx))} \\ &= \frac{(7A-10B)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}+\frac{(A-B)\cos^4(c+dx)\sin(c}{3d(a+a\cos(c+dx))} \\ &= \frac{(7A-10B)\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{(7A-10B)\cos^3(c+dx)\sin}{3a^2d(1+\cos(c+dx}\\ &= \frac{(7A-10B)x}{2a^2}-\frac{4(2A-3B)\sin(c+dx)}{a^2d}+\frac{(7A-10B)\cos(c+dx}{2a^2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 369 vs. 2(170) = 340.
time = 0.66, size = 369, normalized size = 2.17

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(36(7A-10B)dx\cos\left(\frac{dx}{2}\right)+36(7A-10B)dx\cos\left(c+\frac{dx}{2}\right)+84Adx\cos\left(c+\frac{3dx}{2}\right)-120Bdx\cos\left(c+\frac{3dx}{2}\right)+84Adx\cos\left(2c+\frac{3dx}{2}\right)-120Bdx\cos\left(2c+\frac{3dx}{2}\right)-381A\sin\left(\frac{dx}{2}\right)+516B\sin\left(\frac{dx}{2}\right)+147A\sin\left(c+\frac{dx}{2}\right)-156B\sin\left(c+\frac{dx}{2}\right)-239A\sin\left(c+\frac{3dx}{2}\right)+342B\sin\left(c+\frac{3dx}{2}\right)-63A\sin\left(2c+\frac{3dx}{2}\right)+118B\sin\left(2c+\frac{3dx}{2}\right)-15A\sin\left(2c+\frac{5dx}{2}\right)+30B\sin\left(2c+\frac{5dx}{2}\right)-15A\sin\left(3c+\frac{5dx}{2}\right)+30B\sin\left(3c+\frac{5dx}{2}\right)+3A\sin\left(3c+\frac{7dx}{2}\right)-3B\sin\left(3c+\frac{7dx}{2}\right)+3A\sin\left(4c+\frac{7dx}{2}\right)-3B\sin\left(4c+\frac{7dx}{2}\right)+B\sin\left(4c+\frac{9dx}{2}\right)+B\sin\left(5c+\frac{9dx}{2}\right)\right)}{48a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(7*A - 10*B)*d*x*Cos[(d*x)/2] + 36*(7*A - 10
*B)*d*x*Cos[c + (d*x)/2] + 84*A*d*x*Cos[c + (3*d*x)/2] - 120*B*d*x*Cos[c +
```

```
(3*d*x)/2] + 84*A*d*x*Cos[2*c + (3*d*x)/2] - 120*B*d*x*Cos[2*c + (3*d*x)/2]
 - 381*A*Sin[(d*x)/2] + 516*B*Sin[(d*x)/2] + 147*A*Sin[c + (d*x)/2] - 156*B
*Sin[c + (d*x)/2] - 239*A*Sin[c + (3*d*x)/2] + 342*B*Sin[c + (3*d*x)/2] - 6
3*A*Sin[2*c + (3*d*x)/2] + 118*B*Sin[2*c + (3*d*x)/2] - 15*A*Sin[2*c + (5*d
*x)/2] + 30*B*Sin[2*c + (5*d*x)/2] - 15*A*Sin[3*c + (5*d*x)/2] + 30*B*Sin[3
*c + (5*d*x)/2] + 3*A*Sin[3*c + (7*d*x)/2] - 3*B*Sin[3*c + (7*d*x)/2] + 3*A
*Sin[4*c + (7*d*x)/2] - 3*B*Sin[4*c + (7*d*x)/2] + B*Sin[4*c + (9*d*x)/2] +
 B*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.18, size = 154, normalized size = 0.91

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}-7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+9B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8\left(-\frac{5A}{4}+\frac{5B}{2}\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(-2A+\frac{10B}{3}\right)(\mathrm{ta}}{\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}}{2d\,a^2}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}-7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+9B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8\left(-\frac{5A}{4}+\frac{5B}{2}\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(-2A+\frac{10B}{3}\right)(\mathrm{ta}}{\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}}{2d\,a^2}$
risch	$\frac{7xA}{2a^2}-\frac{5Bx}{a^2}-\frac{i\mathrm{e}^{2i(dx+c)}A}{8a^2d}+\frac{i\mathrm{e}^{2i(dx+c)}B}{4a^2d}+\frac{i\mathrm{e}^{i(dx+c)}A}{a^2d}-\frac{15i\mathrm{e}^{i(dx+c)}B}{8a^2d}-\frac{i\mathrm{e}^{-i(dx+c)}A}{a^2d}+\frac{15i\mathrm{e}^{-i(dx+c)}B}{8a^2d}-$
norman	$\frac{\frac{(7A-10B)x}{2a}+\frac{(A-B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{5(7A-10B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{5(7A-10B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{5(7A-10B)x\left(\tan^6\left(\frac{dx}{2}+\right.\right.}{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3*A-1/3*tan(1/2*d*x+1/2*c)^3*B-7*A*tan(1/
2*d*x+1/2*c)+9*B*tan(1/2*d*x+1/2*c)+8*((-5/4*A+5/2*B)*tan(1/2*d*x+1/2*c)^5+
(-2*A+10/3*B)*tan(1/2*d*x+1/2*c)^3+(-3/4*A+3/2*B)*tan(1/2*d*x+1/2*c))/(1+ta
n(1/2*d*x+1/2*c)^2)^3+2*(7*A-10*B)*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(160) = 320.
time = 0.49, size = 372, normalized size = 2.19

$$\frac{B\left(\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-A\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(B*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
```

```
c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^2) - A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x +
c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A]
time = 0.36, size = 154, normalized size = 0.91

$$\frac{3(7A-10B)dx\cos(dx+c)^2+6(7A-10B)dx\cos(dx+c)+3(7A-10B)dx+(2B\cos(dx+c)^4+(3A-2B)\cos(dx+c)^3-6(A-2B)\cos(dx+c)^2-(43A-66B)\cos(dx+c)-32A+48B)\sin(dx+c)}{6(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(7*A - 10*B)*d*x*cos(d*x + c)^2 + 6*(7*A - 10*B)*d*x*cos(d*x + c) +
3*(7*A - 10*B)*d*x + (2*B*cos(d*x + c)^4 + (3*A - 2*B)*cos(d*x + c)^3 - 6*(
A - 2*B)*cos(d*x + c)^2 - (43*A - 66*B)*cos(d*x + c) - 32*A + 48*B)*sin(d*x
 + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. $2(155) = 310$.
time = 3.27, size = 1425, normalized size = 8.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((21*A*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*
a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 63
*A*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/
2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 63*A*d*x*tan(c/
2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4
 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 21*A*d*x/(6*a**2*d*tan(c/2 +
 d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2
+ 6*a**2*d) + A*tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2
*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 18*A*t
an(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/
2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*A*tan(c/2 + d*x/2)**
5/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d
*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 110*A*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(
c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2
```

```
)**2 + 6*a**2*d) - 39*A*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18
*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 3
0*B*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c
/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*B*d*x*tan(c
/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**
4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*B*d*x*tan(c/2 + d*x/2)**
2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d
*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 30*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**6 +
 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d)
- B*tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 +
 d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 24*B*tan(c/2 + d*x
/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a
**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 138*B*tan(c/2 + d*x/2)**5/(6*a**2*d
*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 +
d*x/2)**2 + 6*a**2*d) + 160*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2
)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a*
*2*d) + 63*B*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan
(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x
*(A + B*cos(c))*cos(c)**4/(a*cos(c) + a)**2, True))
```

Giac [A]
time = 0.45, size = 192, normalized size = 1.13

$$\frac{\frac{3\,(dx+c)(7\,A-10\,B)}{a^2}-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-30\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+24\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-18\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-21\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+27\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(d*x + c)*(7*A - 10*B)/a^2 - 2*(15*A*tan(1/2*d*x + 1/2*c)^5 - 30*B*t
an(1/2*d*x + 1/2*c)^5 + 24*A*tan(1/2*d*x + 1/2*c)^3 - 40*B*tan(1/2*d*x + 1/
2*c)^3 + 9*A*tan(1/2*d*x + 1/2*c) - 18*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*
x + 1/2*c)^2 + 1)^3*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*
x + 1/2*c)^3 - 21*A*a^4*tan(1/2*d*x + 1/2*c) + 27*B*a^4*tan(1/2*d*x + 1/2*c
))/a^6)/d
```

Mupad [B]
time = 0.33, size = 189, normalized size = 1.11

$$\frac{x\,\left(7\,A-10\,B\right)}{2\,a^2}-\frac{\left(5\,A-10\,B\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(8\,A-\frac{40\,B}{3}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(3\,A-6\,B\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)}-\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{2\,\left(A-B\right)}{a^2}+\frac{3\,A-5\,B}{2\,a^2}\right)}{d}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(A-B\right)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] (x*(7*A - 10*B))/(2*a^2) - (tan(c/2 + (d*x)/2)^5*(5*A - 10*B) + tan(c/2 + (
d*x)/2)^3*(8*A - (40*B)/3) + tan(c/2 + (d*x)/2)*(3*A - 6*B))/(d*(3*a^2*tan(
c/2 + (d*x)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 +
a^2)) - (tan(c/2 + (d*x)/2)*((2*(A - B))/a^2 + (3*A - 5*B)/(2*a^2)))/d + (t
an(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d)
```

3.48 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=147

$$-\frac{(4A-7B)x}{2a^2}+\frac{2(5A-8B)\sin(c+dx)}{3a^2d}-\frac{(4A-7B)\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{(5A-8B)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}$$

```
[Out] -1/2*(4*A-7*B)*x/a^2+2/3*(5*A-8*B)*sin(d*x+c)/a^2/d-1/2*(4*A-7*B)*cos(d*x+c
)*sin(d*x+c)/a^2/d+1/3*(5*A-8*B)*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(1+cos(d*x+c
))+1/3*(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Rubi [A]
time = 0.18, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {3056, 2813}

$$\frac{2(5A-8B)\sin(c+dx)}{3a^2d}+\frac{(5A-8B)\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{(4A-7B)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{x(4A-7B)}{2a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -1/2*((4*A - 7*B)*x)/a^2 + (2*(5*A - 8*B)*Sin[c + d*x])/(3*a^2*d) - ((4*A -
 7*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + ((5*A - 8*B)*Cos[c + d*x]^2*Si
n[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^3*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)(3a(A-B)-a(2A-5B)\cos}{a+a\cos(c+dx)}}{3a^2}$$
$$= \frac{(5A-8B)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^3(c+dx)\sin(c-}{3d(a+a\cos(c+dx))^2}$$
$$= -\frac{(4A-7B)x}{2a^2} + \frac{2(5A-8B)\sin(c+dx)}{3a^2d} - \frac{(4A-7B)\cos(c+dx}{2a^2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 315 vs. 2(147) = 294.
time = 0.83, size = 315, normalized size = 2.14

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-36(4A-7B)dx\cos\left(\frac{dx}{2}\right)-36(4A-7B)dx\cos\left(c+\frac{dx}{2}\right)-48Adx\cos\left(c+\frac{3dx}{2}\right)+84Bdx\cos\left(c+\frac{3dx}{2}\right)-48Adx\cos\left(2c+\frac{3dx}{2}\right)+84Bdx\cos\left(2c+\frac{3dx}{2}\right)+264A\sin\left(\frac{dx}{2}\right)-381B\sin\left(\frac{dx}{2}\right)-120A\sin\left(c+\frac{dx}{2}\right)+147B\sin\left(c+\frac{dx}{2}\right)+164A\sin\left(c+\frac{3dx}{2}\right)-239B\sin\left(c+\frac{3dx}{2}\right)+36A\sin\left(2c+\frac{3dx}{2}\right)-63B\sin\left(2c+\frac{3dx}{2}\right)+12A\sin\left(2c+\frac{5dx}{2}\right)-15B\sin\left(2c+\frac{5dx}{2}\right)+12A\sin\left(3c+\frac{5dx}{2}\right)-15B\sin\left(3c+\frac{5dx}{2}\right)+3B\sin\left(3c+\frac{7dx}{2}\right)+3B\sin\left(4c+\frac{7dx}{2}\right)\right)}{48a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-36*(4*A - 7*B)*d*x*Cos[(d*x)/2] - 36*(4*A - 7*
B)*d*x*Cos[c + (d*x)/2] - 48*A*d*x*Cos[c + (3*d*x)/2] + 84*B*d*x*Cos[c + (3
*d*x)/2] - 48*A*d*x*Cos[2*c + (3*d*x)/2] + 84*B*d*x*Cos[2*c + (3*d*x)/2] +
264*A*Sin[(d*x)/2] - 381*B*Sin[(d*x)/2] - 120*A*Sin[c + (d*x)/2] + 147*B*Si
n[c + (d*x)/2] + 164*A*Sin[c + (3*d*x)/2] - 239*B*Sin[c + (3*d*x)/2] + 36*A
*Sin[2*c + (3*d*x)/2] - 63*B*Sin[2*c + (3*d*x)/2] + 12*A*Sin[2*c + (5*d*x)/
2] - 15*B*Sin[2*c + (5*d*x)/2] + 12*A*Sin[3*c + (5*d*x)/2] - 15*B*Sin[3*c +
 (5*d*x)/2] + 3*B*Sin[3*c + (7*d*x)/2] + 3*B*Sin[4*c + (7*d*x)/2]))/(48*a^2
*d*(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.16, size = 135, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+5A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-7B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4\left(\left(\frac{5B}{2}-A\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3B}{2}-A\right)\tan\left(\frac{dx}{2}+\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{2d\,a^2}$
default	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+5A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-7B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4\left(\left(\frac{5B}{2}-A\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3B}{2}-A\right)\tan\left(\frac{dx}{2}+\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{2d\,a^2}$
risch	$-\frac{2xA}{a^2}+\frac{7Bx}{2a^2}-\frac{i e^{2i(dx+c)}B}{8a^2d}-\frac{i e^{i(dx+c)}A}{2a^2d}+\frac{i e^{i(dx+c)}B}{a^2d}+\frac{i e^{-i(dx+c)}A}{2a^2d}-\frac{i e^{-i(dx+c)}B}{a^2d}+\frac{i e^{-2i(dx+c)}B}{8a^2d}+$

norman	$\frac{\frac{(11A-18B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{(4A-7B)x}{2a}-\frac{(A-B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{2(4A-7B)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{3(4A-7B)x\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3*A+1/3*tan(1/2*d*x+1/2*c)^3*B+5*A*tan(1
/2*d*x+1/2*c)-7*B*tan(1/2*d*x+1/2*c)-4*((5/2*B-A)*tan(1/2*d*x+1/2*c)^3+(3/2
*B-A)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2-2*(4*A-7*B)*arctan(tan
(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(137) = 274$.
time = 0.49, size = 283, normalized size = 1.93

$$-\frac{B\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2) - A*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*si
n(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) +
 1))))/d
```

Fricas [A]
time = 0.35, size = 138, normalized size = 0.94

$$-\frac{3(4A-7B)dx\cos(dx+c)^2+6(4A-7B)dx\cos(dx+c)+3(4A-7B)dx-\left(3B\cos(dx+c)^3+6(A-B)\cos(dx+c)^2+(28A-43B)\cos(dx+c)+20A-32B\right)\sin(dx+c)}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*(4*A - 7*B)*d*x*cos(d*x + c)^2 + 6*(4*A - 7*B)*d*x*cos(d*x + c) + 3
*(4*A - 7*B)*d*x - (3*B*cos(d*x + c)^3 + 6*(A - B)*cos(d*x + c)^2 + (28*A -
 43*B)*cos(d*x + c) + 20*A - 32*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*
a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(136) = 272$.
time = 2.07, size = 843, normalized size = 5.73

$$\begin{cases} -\frac{12Adx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{24Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{12Adx}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{13A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{41A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{27A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{21Bdx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{42Bdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{21Bdx}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{19B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{71B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{39B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} & \text{for}\ d \neq 0 \\ \frac{x(A+B\cos(c))\cos^3(c)}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-12*A*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12
*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 24*A*d*x*tan(c/2 + d*x/2)**2/(6*a
**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*
A*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**
2*d) - A*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(
c/2 + d*x/2)**2 + 6*a**2*d) + 13*A*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 +
d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 41*A*tan(c/2 + d*x/
2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**
2*d) + 27*A*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(
c/2 + d*x/2)**2 + 6*a**2*d) + 21*B*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/
2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 42*B*d*x*tan(c/
2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2
 + 6*a**2*d) + 21*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 6*a**2*d) + B*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**
4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 19*B*tan(c/2 + d*x/2)**5/(6
*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 7
1*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 6*a**2*d) - 39*B*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**
4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*(A + B*cos(c))
*cos(c)**3/(a*cos(c) + a)**2, True))
```

Giac [A]
time = 0.44, size = 164, normalized size = 1.12

$$-\frac{\frac{3\,(dx+c)(4\,A-7\,B)}{a^2} - \frac{6\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2} + \frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(d*x + c)*(4*A - 7*B)/a^2 - 6*(2*A*tan(1/2*d*x + 1/2*c)^3 - 5*B*tan
(1/2*d*x + 1/2*c)^3 + 2*A*tan(1/2*d*x + 1/2*c) - 3*B*tan(1/2*d*x + 1/2*c))/
((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^
4*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^4*tan(1/2*d*x + 1/2*c) + 21*B*a^4*tan(1/2
*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 0.29, size = 152, normalized size = 1.03

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,\left(A-B\right)}{2\,a^2}+\frac{2\,A-4\,B}{2\,a^2}\right)}{d}-\frac{x\,\left(4\,A-7\,B\right)}{2\,a^2}+\frac{\left(2\,A-5\,B\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,A-3\,B\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(A-B\right)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((3*(A - B))/(2*a^2) + (2*A - 4*B)/(2*a^2)))/d - (x*(4*
A - 7*B))/(2*a^2) + (tan(c/2 + (d*x)/2)^3*(2*A - 5*B) + tan(c/2 + (d*x)/2)*
(2*A - 3*B))/(d*(2*a^2*tan(c/2 + (d*x)/2)^2 + a^2*tan(c/2 + (d*x)/2)^4 + a^
2)) - (tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d)
```

3.49 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=99

$$\frac{(A-2B)x}{a^2}-\frac{(A-4B)\sin(c+dx)}{3a^2d}-\frac{(A-2B)\sin(c+dx)}{a^2d(1+\cos(c+dx))}+\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

```
[Out] (A-2*B)*x/a^2-1/3*(A-4*B)*sin(d*x+c)/a^2/d-(A-2*B)*sin(d*x+c)/a^2/d/(1+cos(
d*x+c))+1/3*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Rubi [A]
time = 0.18, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3056, 3047, 3102, 12, 2814, 2727}

$$-\frac{(A-4B)\sin(c+dx)}{3a^2d}-\frac{(A-2B)\sin(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{x(A-2B)}{a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((A - 2*B)*x)/a^2 - ((A - 4*B)*Sin[c + d*x])/(3*a^2*d) - ((A - 2*B)*Sin[c +
 d*x])/(a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(
3*d*(a + a*Cos[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx &= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos(c+dx)(2a(A-B)-a(A-4B)\cos}{a+a\cos(c+dx)}}{3a^2} \\
&= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{2a(A-B)\cos(c+dx)-a(A-4B)\cos^2}{a+a\cos(c+dx)}}{3a^2} \\
&= -\frac{(A-4B)\sin(c+dx)}{3a^2d} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int}{} \\
&= -\frac{(A-4B)\sin(c+dx)}{3a^2d} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(A}{} \\
&= \frac{(A-2B)x}{a^2} - \frac{(A-4B)\sin(c+dx)}{3a^2d} + \frac{(A-B)\cos^2(c+dx)\sin(}{3d(a+a\cos(c+dx)} \\
&= \frac{(A-2B)x}{a^2} - \frac{(A-4B)\sin(c+dx)}{3a^2d} + \frac{(A-B)\cos^2(c+dx)\sin(}{3d(a+a\cos(c+dx)}
\end{aligned}
$$

Mathematica [A]

time = 0.76, size = 137, normalized size = 1.38

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-2(5A-8B)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left((A-2B)dx+B\sin(c+dx)\right)+(A-B)\cos\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{3a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] - 2*(5*A - 8*B)*Cos[(c +
 d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*((A - 2*B)*d*x + B*
Sin[c + d*x]) + (A - B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d
*x])^2)
```

Maple [A]
time = 0.14, size = 106, normalized size = 1.07

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}-3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+4(A-2B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{2d\,a^2}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}-3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+4(A-2B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{2d\,a^2}$
risch	$\frac{xA}{a^2}-\frac{2Bx}{a^2}-\frac{i\mathrm{e}^{i(dx+c)}B}{2a^2d}+\frac{i\mathrm{e}^{-i(dx+c)}B}{2a^2d}-\frac{2i\left(6A\,\mathrm{e}^{2i(dx+c)}-9B\,\mathrm{e}^{2i(dx+c)}+9A\,\mathrm{e}^{i(dx+c)}-15B\,\mathrm{e}^{i(dx+c)}+5A-8B\right)}{3d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^3}$
norman	$\frac{\frac{(A-2B)x}{a}+\frac{(A-2B)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{3(A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{3(A-2B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{3(A-2B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{(A-2B)}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3*A-1/3*tan(1/2*d*x+1/2*c)^3*B-3*A*tan(1/
2*d*x+1/2*c)+5*B*tan(1/2*d*x+1/2*c)+4*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1
/2*c)^2)+4*(A-2*B)*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(95) = 190$.
time = 0.48, size = 191, normalized size = 1.93

$$\frac{B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(B*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x +
 c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) -
 A*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A]
time = 0.34, size = 117, normalized size = 1.18

$$\frac{3\,(A-2\,B)dx\cos{(dx+c)}^{2}+6\,(A-2\,B)dx\cos{(dx+c)}+3\,(A-2\,B)dx+\left(3\,B\cos{(dx+c)}^{2}-(5\,A-14\,B)\cos{(dx+c)}-4\,A+10\,B\right)\sin{(dx+c)}}{3\,\left(a^{2}d\cos{(dx+c)}^{2}+2\,a^{2}d\cos{(dx+c)}+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/3*(3*(A - 2*B)*d*x*cos(d*x + c)^2 + 6*(A - 2*B)*d*x*cos(d*x + c) + 3*(A -
 2*B)*d*x + (3*B*cos(d*x + c)^2 - (5*A - 14*B)*cos(d*x + c) - 4*A + 10*B)*s
in(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(90) = 180.
time = 1.26, size = 411, normalized size = 4.15

$$\begin{cases} \frac{6Adx\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d}+\frac{6Adx}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d}+\frac{A\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d}-\frac{8A\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d}-\frac{9A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d}-\frac{12Bdx\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d}-\frac{12Bdx}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d}-\frac{B\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d}+\frac{14B\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d}+\frac{27B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d} & \text{for}\ d\neq 0 \\ \frac{x(A+B\cos{(c)})\cos^{2}{(c)}}{(a\cos{(c)}+a)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((6*A*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a*
*2*d) + 6*A*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + A*tan(c/2 + d*x
/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 8*A*tan(c/2 + d*x/2)**3/
(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 9*A*tan(c/2 + d*x/2)/(6*a**2*d*
tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*B*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*ta
n(c/2 + d*x/2)**2 + 6*a**2*d) - 12*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*
a**2*d) - B*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) +
 14*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 27*B*
tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*(
A + B*cos(c))*cos(c)**2/(a*cos(c) + a)**2, True))
```

Giac [A]
time = 0.45, size = 119, normalized size = 1.20

$$\frac{\frac{6\,(dx+c)(A-2\,B)}{a^{2}}+\frac{12\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{2}}+\frac{Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-9\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(d*x + c)*(A - 2*B)/a^2 + 12*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x +
1/2*c)^2 + 1)*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/
2*c)^3 - 9*A*a^4*tan(1/2*d*x + 1/2*c) + 15*B*a^4*tan(1/2*d*x + 1/2*c))/a^6)
/d
```

Mupad [B]
time = 0.26, size = 105, normalized size = 1.06

$$\frac{x\,\left(A-2\,B\right)}{a^2}-\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{a^2}+\frac{A-3\,B}{2\,a^2}\right)}{d}+\frac{2\,B\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a^2\right)}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(A-B\right)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] (x*(A - 2*B))/a^2 - (tan(c/2 + (d*x)/2)*((A - B)/a^2 + (A - 3*B)/(2*a^2)))/
d + (2*B*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 + a^2)) + (tan(c/
2 + (d*x)/2)^3*(A - B))/(6*a^2*d)
```

3.50 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$\frac{Bx}{a^2}+\frac{(2A-5B)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

```
[Out] B*x/a^2+1/3*(2*A-5*B)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*sin(d*x+c)/
d/(a+a*cos(d*x+c))^2
```

Rubi [A]
time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.138$, Rules used = {3047, 3098, 2814, 2727}

$$\frac{(2A-5B)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{Bx}{a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (B*x)/a^2 + ((2*A - 5*B)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A -
 B)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
```

```
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx &= \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx\\ &= -\frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{\int \frac{-2a(A-B)-3aB\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}\\ &= \frac{Bx}{a^2}-\frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{(2A-5B)\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}\\ &= \frac{Bx}{a^2}-\frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{(2A-5B)\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 153 vs. 2(70) = 140.
time = 0.38, size = 153, normalized size = 2.19

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(9Bdx\cos\left(\frac{dx}{2}\right)+9Bdx\cos\left(c+\frac{dx}{2}\right)+3Bdx\cos\left(c+\frac{3dx}{2}\right)+3Bdx\cos\left(2c+\frac{3dx}{2}\right)+6A\sin\left(\frac{dx}{2}\right)-18B\sin\left(\frac{dx}{2}\right)-6A\sin\left(c+\frac{dx}{2}\right)+12B\sin\left(c+\frac{dx}{2}\right)+4A\sin\left(c+\frac{3dx}{2}\right)-10B\sin\left(c+\frac{3dx}{2}\right)\right)}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*B*d*x*Cos[(d*x)/2] + 9*B*d*x*Cos[c + (d*x)/
2] + 3*B*d*x*Cos[c + (3*d*x)/2] + 3*B*d*x*Cos[2*c + (3*d*x)/2] + 6*A*Sin[(d
*x)/2] - 18*B*Sin[(d*x)/2] - 6*A*Sin[c + (d*x)/2] + 12*B*Sin[c + (d*x)/2] +
 4*A*Sin[c + (3*d*x)/2] - 10*B*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Maple [A]
time = 0.12, size = 74, normalized size = 1.06

method	result
derivativedivides	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4B\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$
default	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4B\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$
risch	$\frac{Bx}{a^2}+\frac{2i\left(3A\,\mathrm{e}^{2i(dx+c)}-6B\,\mathrm{e}^{2i(dx+c)}+3A\,\mathrm{e}^{i(dx+c)}-9B\,\mathrm{e}^{i(dx+c)}+2A-5B\right)}{3d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^3}$

norman	$\frac{\frac{Bx}{a}+\frac{Bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2Bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{(A-7B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{(A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}-\frac{(A-B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{(5\ldots}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3*A+1/3*tan(1/2*d*x+1/2*c)^3*B+A*tan(1/2
*d*x+1/2*c)-3*B*tan(1/2*d*x+1/2*c)+4*B*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.50, size = 120, normalized size = 1.71

$$-\frac{B\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-\frac{A\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(B*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) - A*(3*sin(d*
x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A]
time = 0.34, size = 91, normalized size = 1.30

$$\frac{3\,Bdx\cos(dx+c)^2+6\,Bdx\cos(dx+c)+3\,Bdx+((2\,A-5\,B)\cos(dx+c)+A-4\,B)\sin(dx+c)}{3\,(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*(3*B*d*x*cos(d*x + c)^2 + 6*B*d*x*cos(d*x + c) + 3*B*d*x + ((2*A - 5*B)
*cos(d*x + c) + A - 4*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [A]
time = 0.76, size = 105, normalized size = 1.50

$$\begin{cases} -\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}+\frac{Bx}{a^2}+\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}-\frac{3B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0 \\ \frac{x(A+B\cos(c))\cos(c)}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**3/(6*a**2*d) + A*tan(c/2 + d*x/2)/(2*a**2*d
) + B*x/a**2 + B*tan(c/2 + d*x/2)**3/(6*a**2*d) - 3*B*tan(c/2 + d*x/2)/(2*a
**2*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)/(a*cos(c) + a)**2, True))
```

Giac [A]
time = 0.48, size = 86, normalized size = 1.23

$$\frac{\frac{6\,(dx+c)B}{a^2}-\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(6*(d*x + c)*B/a^2 - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x
+ 1/2*c)^3 - 3*A*a^4*tan(1/2*d*x + 1/2*c) + 9*B*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

Mupad [B]
time = 0.22, size = 65, normalized size = 0.93

$$\frac{3\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\,B\,d\,x}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] (3*A*tan(c/2 + (d*x)/2) - 9*B*tan(c/2 + (d*x)/2) - A*tan(c/2 + (d*x)/2)^3 +
 B*tan(c/2 + (d*x)/2)^3 + 6*B*d*x)/(6*a^2*d)
```

3.51 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{(A+2B)\sin(c+dx)}{3d\,(a^2+a^2\cos(c+dx))}$$

```
[Out] 1/3*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+1/3*(A+2*B)*sin(d*x+c)/d/(a^2+a^2
*cos(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2829, 2727}

$$\frac{(A+2B)\sin(c+dx)}{3d\,(a^2\cos(c+dx)+a^2)}+\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((A - B)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((A + 2*B)*Sin[c + d*
x])/(3*d*(a^2 + a^2*Cos[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{(A+2B)\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}\\ &= \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{(A+2B)\sin(c+dx)}{3d\,(a^2+a^2\cos(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 76, normalized size = 1.17

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(3(A+B)\sin\left(\frac{dx}{2}\right)-3B\sin\left(c+\frac{dx}{2}\right)+(A+2B)\sin\left(c+\frac{3dx}{2}\right)\right)}{3a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(3*(A + B)*Sin[(d*x)/2] - 3*B*Sin[c + (d*x)/2] +
 (A + 2*B)*Sin[c + (3*d*x)/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.10, size = 60, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$	60
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$	60
risch	$\frac{2i\left(3B\,e^{2i(dx+c)}+3A\,e^{i(dx+c)}+3B\,e^{i(dx+c)}+A+2B\right)}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3}$	64
norman	$\frac{\frac{(A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{(2A+B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}}{a\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3*A-1/3*tan(1/2*d*x+1/2*c)^3*B+A*tan(1/2*
d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 93, normalized size = 1.43

$$\frac{\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}+\frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(A*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 + B*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/a^2)/d
```

Fricas [A]
time = 0.33, size = 58, normalized size = 0.89

$$\frac{((A+2B)\cos(dx+c)+2A+B)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((A + 2*B)*cos(d*x + c) + 2*A + B)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 +
 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A]
time = 0.56, size = 94, normalized size = 1.45

$$\begin{cases} \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}-\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0 \\ \frac{x(A+B\cos(c))}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**3/(6*a**2*d) + A*tan(c/2 + d*x/2)/(2*a**2*d)
 - B*tan(c/2 + d*x/2)**3/(6*a**2*d) + B*tan(c/2 + d*x/2)/(2*a**2*d), Ne(d,
0)), (x*(A + B*cos(c))/(a*cos(c) + a)**2, True))
```

Giac [A]
time = 0.43, size = 60, normalized size = 0.92

$$\frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{6a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(A*tan(1/2*d*x + 1/2*c)^3 - B*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x
+ 1/2*c) + 3*B*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

Mupad [B]
time = 0.19, size = 45, normalized size = 0.69

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A-B)}{6\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A+B)}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d) + (tan(c/2 + (d*x)/2)*(A + B))/(2*
a^2*d)
```

3.52 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(4A-B)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

```
[Out] A*arctanh(sin(d*x+c))/a^2/d-1/3*(4*A-B)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3
*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Rubi [A]
time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3057, 12, 3855}

$$-\frac{(4A-B)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^2*d) - ((4*A - B)*Sin[c + d*x])/(3*a^2*d*(1 +
Cos[c + d*x])) - ((A - B)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{(3aA-a(A-B)\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}\\ &= -\frac{(4A-B)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int 3a^2A\sec(c}{3a^4}\\ &= -\frac{(4A-B)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{A\int \sec(c+}{a^2}\\ &= \frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(4A-B)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\sin(c}{3d(a+a\cos(c}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(79) = 158.
time = 0.55, size = 170, normalized size = 2.15

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(6A\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+(A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+2(4A-B)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+(A-B)\cos\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{3a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(6*A*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (A - B)*Sec[c/2]*
Sin[(d*x)/2] + 2*(4*A - B)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + (A -
B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.19, size = 91, normalized size = 1.15

method	result
derivativedivides	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}-3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2d\,a^2}$
default	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}-3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2d\,a^2}$
risch	$-\frac{2i\left(3A\,e^{2i(dx+c)}+9A\,e^{i(dx+c)}-3B\,e^{i(dx+c)}+4A-B\right)}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3}+\frac{A\ln\left(e^{i(dx+c)}+i\right)}{a^2d}-\frac{A\ln\left(e^{i(dx+c)}-i\right)}{a^2d}$
norman	$\frac{-\frac{(A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{(3A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}-\frac{(5A-2B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}}{a\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2d}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3*A+1/3*tan(1/2*d*x+1/2*c)^3*B-3*A*tan(1
/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)-2*A*ln(tan(1/2*d*x+1/2*c)-1)+2*A*ln(tan(
1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.27, size = 145, normalized size = 1.84

$$-\frac{A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-\frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*(3*sin(d*x + c)/(cos(d*x + c) + 1)
+ sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A]
time = 0.35, size = 131, normalized size = 1.66

$$\frac{3\left(A\cos(dx+c)^2+2\,A\cos(dx+c)+A\right)\log(\sin(dx+c)+1)-3\left(A\cos(dx+c)^2+2\,A\cos(dx+c)+A\right)\log(-\sin(dx+c)+1)-2\left((4\,A-B)\cos(dx+c)+5\,A-2\,B\right)\sin(dx+c)}{6\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - 3*
(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*log(-sin(d*x + c) + 1) - 2*((4*A
- B)*cos(d*x + c) + 5*A - 2*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A\sec(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx+\int\frac{B\cos(c+dx)\sec(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + Integ
ral(B*cos(c + d*x)*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x))
/a**2
```

Giac [A]
time = 0.45, size = 113, normalized size = 1.43

$$\frac{\frac{6\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{6\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}-\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(6*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*A*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2
*c)^3 + 9*A*a^4*tan(1/2*d*x + 1/2*c) - 3*B*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 0.23, size = 74, normalized size = 0.94

$$\frac{2\,A\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(A-B\right)}{6\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A}{a^2}+\frac{A-B}{2\,a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] (2*A*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (tan(c/2 + (d*x)/2)^3*(A - B))/(6
*a^2*d) - (tan(c/2 + (d*x)/2)*(A/a^2 + (A - B)/(2*a^2)))/d
```

3.53 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=107

$$-\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2(5A-2B)\tan(c+dx)}{3a^2d}-\frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{(A-B)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}$$

```
[Out] -(2*A-B)*arctanh(sin(d*x+c))/a^2/d+2/3*(5*A-2*B)*tan(d*x+c)/a^2/d-(2*A-B)*t
an(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Rubi [A]
time = 0.19, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3057, 2827, 3852, 8, 3855}

$$\frac{2(5A-2B)\tan(c+dx)}{3a^2d}-\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(2A-B)\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -(((2*A - B)*ArcTanh[Sin[c + d*x]])/(a^2*d)) + (2*(5*A - 2*B)*Tan[c + d*x])
/(3*a^2*d) - ((2*A - B)*Tan[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - ((A - B)
*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{(A-B)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{(a(4A-B)-2a(A-B)\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,d}{3a^2} \\
&= -\frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{(A-B)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int (2a^2(5A-}{} \\
&= -\frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{(A-B)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{(2(5A-2B}{} \\
&= -\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{(A}{3d(a} \\
&= -\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2(5A-2B)\tan(c+dx)}{3a^2d}-\frac{(2}{a}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 264 vs. 2(107) = 214.
time = 1.80, size = 264, normalized size = 2.47

$$
\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+2(7A-4B)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left((2A-B)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\frac{A\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)+(A-B)\cos\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{3a^2d(1+\cos(c+dx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] + 2*(7*A - 4*B)*Cos[(c +
 d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*((2*A - B)*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
]) + (A*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (A - B)
*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.21, size = 134, normalized size = 1.25

method	result
derivativedivides	$\frac{(2B-4A)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+5A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$
default	$\frac{(2B-4A)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+5A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$
norman	$\frac{\frac{(A-B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{3(3A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{(5A-3B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}-\frac{(13A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a}+\frac{(2A-B)\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{a^2d}$
risch	$\frac{2i\left(6A\,e^{4i(dx+c)}-3B\,e^{4i(dx+c)}+18A\,e^{3i(dx+c)}-9B\,e^{3i(dx+c)}+22A\,e^{2i(dx+c)}-7B\,e^{2i(dx+c)}+24A\,e^{i(dx+c)}-9B\,e^{i(dx+c)}+1\right.}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d/a^2*((2*B-4*A)*ln(tan(1/2*d*x+1/2*c)+1)-2*A/(tan(1/2*d*x+1/2*c)+1)+1/
3*tan(1/2*d*x+1/2*c)^3*A-1/3*tan(1/2*d*x+1/2*c)^3*B+5*A*tan(1/2*d*x+1/2*c)-
3*B*tan(1/2*d*x+1/2*c)+(4*A-2*B)*ln(tan(1/2*d*x+1/2*c)-1)-2*A/(tan(1/2*d*x+
1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(103) = 206$.
time = 0.27, size = 244, normalized size = 2.28

$$\frac{A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(A*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) - B*((9*sin(d*x + c)/(cos
(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/a^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(103) = 206$.
time = 0.35, size = 207, normalized size = 1.93

$$-\frac{3\left((2A-B)\cos(dx+c)^3+2(2A-B)\cos(dx+c)^2+(2A-B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-3\left((2A-B)\cos(dx+c)^3+2(2A-B)\cos(dx+c)^2+(2A-B)\cos(dx+c)\right)\log(-\sin(dx+c)+1)-2\left(2(5A-2B)\cos(dx+c)^2+(14A-5B)\cos(dx+c)+3A\right)\sin(dx+c)}{6\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*((2*A - B)*cos(d*x + c)^3 + 2*(2*A - B)*cos(d*x + c)^2 + (2*A - B)*
cos(d*x + c))*log(sin(d*x + c) + 1) - 3*((2*A - B)*cos(d*x + c)^3 + 2*(2*A
- B)*cos(d*x + c)^2 + (2*A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2
*(5*A - 2*B)*cos(d*x + c)^2 + (14*A - 5*B)*cos(d*x + c) + 3*A)*sin(d*x + c)
)/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^2(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec^2(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + In
tegral(B*cos(c + d*x)*sec(c + d*x)**2/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1
), x))/a**2
```

Giac [A]
time = 0.46, size = 155, normalized size = 1.45

$$-\frac{\frac{6\,(2\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} - \frac{6\,(2\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} + \frac{12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^2} - \frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 15\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 9\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(6*(2*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*(2*A - B)*log(
abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 12*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*
x + 1/2*c)^2 - 1)*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x
+ 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x + 1/2*c) - 9*B*a^4*tan(1/2*d*x + 1/2*c))/
a^6)/d
```

Mupad [B]
time = 0.28, size = 123, normalized size = 1.15

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{a^2}+\frac{3\,A-B}{2\,a^2}\right)}{d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(A-B\right)}{6\,a^2\,d}-\frac{2\,A\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-a^2\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,A-B\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^2),x)

[Out] (tan(c/2 + (d*x)/2)*((A - B)/a^2 + (3*A - B)/(2*a^2)))/d + (tan(c/2 + (d*x)
/2)^3*(A - B))/(6*a^2*d) - (2*A*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)
/2)^2 - a^2)) - (2*atanh(tan(c/2 + (d*x)/2))*(2*A - B))/(a^2*d)
```

3.54 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$\frac{(7A-4B)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2(8A-5B)\tan(c+dx)}{3a^2d}+\frac{(7A-4B)\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{(8A-5B}{3a^2}$$

```
[Out] 1/2*(7*A-4*B)*arctanh(sin(d*x+c))/a^2/d-2/3*(8*A-5*B)*tan(d*x+c)/a^2/d+1/2*
(7*A-4*B)*sec(d*x+c)*tan(d*x+c)/a^2/d-1/3*(8*A-5*B)*sec(d*x+c)*tan(d*x+c)/a
^2/d/(1+cos(d*x+c))-1/3*(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Rubi [A]
time = 0.21, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3057, 2827, 3853, 3855, 3852, 8}

$$-\frac{2(8A-5B)\tan(c+dx)}{3a^2d}+\frac{(7A-4B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7A-4B)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{(8A-5B)\tan(c+dx)\sec(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((7*A - 4*B)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (2*(8*A - 5*B)*Tan[c + d*x]
)/(3*a^2*d) + ((7*A - 4*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - ((8*A - 5
*B)*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B)*Sec[
c + d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{(a(5A-2B)-3a(A-B)\cos(c+dx))\,\mathrm{s}}{a+a\cos(c+dx)}}{3a^2} \\
&= -\frac{(8A-5B)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\sec(c+dx)\tan(c}{3d(a+a\cos(c+dx))} \\
&= -\frac{(8A-5B)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\sec(c+dx)\tan(c}{3d(a+a\cos(c+dx))} \\
&= \frac{(7A-4B)\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{(8A-5B)\sec(c+dx)\tan(c}{3a^2d(1+\cos(c+dx))} \\
&= \frac{(7A-4B)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2(8A-5B)\tan(c+dx)}{3a^2d}+\frac{(7A}{}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 496 vs. 2(152) = 304.
time = 3.34, size = 496, normalized size = 3.26

$$-\frac{96(7A-4B)\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sec(c)\sec^2(c+dx)\left(-14(A-B)\sin\left(\frac{dx}{2}\right)+(97A-64B)\sin\left(\frac{3dx}{2}\right)-126A\sin\left(c-\frac{dx}{2}\right)+84B\sin\left(c-\frac{dx}{2}\right)+42A\sin\left(c+\frac{dx}{2}\right)-42B\sin\left(c+\frac{dx}{2}\right)-98A\sin\left(2c+\frac{dx}{2}\right)+56B\sin\left(2c+\frac{dx}{2}\right)-3A\sin\left(c+\frac{3dx}{2}\right)+6B\sin\left(c+\frac{3dx}{2}\right)+37A\sin\left(2c+\frac{3dx}{2}\right)-34B\sin\left(2c+\frac{3dx}{2}\right)-63A\sin\left(3c+\frac{3dx}{2}\right)+36B\sin\left(3c+\frac{3dx}{2}\right)+75A\sin\left(c+\frac{5dx}{2}\right)-48B\sin\left(c+\frac{5dx}{2}\right)+15A\sin\left(2c+\frac{5dx}{2}\right)-6B\sin\left(2c+\frac{5dx}{2}\right)+39A\sin\left(3c+\frac{5dx}{2}\right)-30B\sin\left(3c+\frac{5dx}{2}\right)-21A\sin\left(4c+\frac{5dx}{2}\right)+12B\sin\left(4c+\frac{5dx}{2}\right)+32A\sin\left(2c+\frac{7dx}{2}\right)-20B\sin\left(2c+\frac{7dx}{2}\right)+12A\sin\left(3c+\frac{7dx}{2}\right)-6B\sin\left(3c+\frac{7dx}{2}\right)+20A\sin\left(4c+\frac{7dx}{2}\right)-14B\sin\left(4c+\frac{7dx}{2}\right)\right)}{48a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -1/48*(96*(7*A - 4*B)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[
c/2]*Sec[c]*Sec[c + d*x]^2*(-14*(A - B)*Sin[(d*x)/2] + (97*A - 64*B)*Sin[(3
*d*x)/2] - 126*A*Sin[c - (d*x)/2] + 84*B*Sin[c - (d*x)/2] + 42*A*Sin[c + (d
*x)/2] - 42*B*Sin[c + (d*x)/2] - 98*A*Sin[2*c + (d*x)/2] + 56*B*Sin[2*c + (
d*x)/2] - 3*A*Sin[c + (3*d*x)/2] + 6*B*Sin[c + (3*d*x)/2] + 37*A*Sin[2*c +
(3*d*x)/2] - 34*B*Sin[2*c + (3*d*x)/2] - 63*A*Sin[3*c + (3*d*x)/2] + 36*B*S
in[3*c + (3*d*x)/2] + 75*A*Sin[c + (5*d*x)/2] - 48*B*Sin[c + (5*d*x)/2] + 1
5*A*Sin[2*c + (5*d*x)/2] - 6*B*Sin[2*c + (5*d*x)/2] + 39*A*Sin[3*c + (5*d*x
)/2] - 30*B*Sin[3*c + (5*d*x)/2] - 21*A*Sin[4*c + (5*d*x)/2] + 12*B*Sin[4*c
 + (5*d*x)/2] + 32*A*Sin[2*c + (7*d*x)/2] - 20*B*Sin[2*c + (7*d*x)/2] + 12*
A*Sin[3*c + (7*d*x)/2] - 6*B*Sin[3*c + (7*d*x)/2] + 20*A*Sin[4*c + (7*d*x)/
2] - 14*B*Sin[4*c + (7*d*x)/2]))/(a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.27, size = 177, normalized size = 1.16

method	result
derivativedivides	$\frac{-\frac{-5A+2B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+(7A-4B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{A}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}-7A\tan\left(\frac{dx}{2}\right.}{2d\,a^{2}}$
default	$\frac{-\frac{-5A+2B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+(7A-4B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{A}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}-7A\tan\left(\frac{dx}{2}\right.}{2d\,a^{2}}$
norman	$\frac{-\frac{(A-B)\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{(10A-7B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{(13A-9B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{2(13A-7B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{(16A-7B)\left(\tan^{3}\right.}{3da}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}a}$
risch	$-\frac{i\left(21A\,e^{6i(dx+c)}-12B\,e^{6i(dx+c)}+63A\,e^{5i(dx+c)}-36B\,e^{5i(dx+c)}+98A\,e^{4i(dx+c)}-56B\,e^{4i(dx+c)}+126A\,e^{3i(dx+c)}-84B\right.}{3d\,a^{2}\left(e^{i(dx+c)}+1\right)^{3}\left(e^{2i(dx+c)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d/a^2*(-(-5*A+2*B)/(tan(1/2*d*x+1/2*c)+1)+(7*A-4*B)*ln(tan(1/2*d*x+1/2*
c)+1)-A/(tan(1/2*d*x+1/2*c)+1)^2-1/3*tan(1/2*d*x+1/2*c)^3*A+1/3*tan(1/2*d*x
+1/2*c)^3*B-7*A*tan(1/2*d*x+1/2*c)+5*B*tan(1/2*d*x+1/2*c)+(-7*A+4*B)*ln(tan
(1/2*d*x+1/2*c)-1)-(-5*A+2*B)/(tan(1/2*d*x+1/2*c)-1)+A/(tan(1/2*d*x+1/2*c)-
1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(142) = 284.
time = 0.27, size = 336, normalized size = 2.21

$$-\frac{A\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{2}-\frac{2a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{2}}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{2}}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{2}}\right)-B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{2}}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{2}}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{2}}+\frac{12\sin(dx+c)}{\left(a^{2}-\frac{a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*((15*sin(d
*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*
log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1))))/d
```

Fricas [A]
time = 0.36, size = 228, normalized size = 1.50

$$\frac{3\left((7A-4B)\cos\left(dx+c\right)^{4}+2\left(7A-4B\right)\cos\left(dx+c\right)^{3}+\left(7A-4B\right)\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left((7A-4B)\cos\left(dx+c\right)^{4}+2\left(7A-4B\right)\cos\left(dx+c\right)^{3}+\left(7A-4B\right)\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(4\left(8A-5B\right)\cos\left(dx+c\right)^{3}+\left(43A-28B\right)\cos\left(dx+c\right)^{2}+6\left(A-B\right)\cos\left(dx+c\right)-3A\right)\sin\left(dx+c\right)}{12\left(a^{2}d\cos\left(dx+c\right)^{4}+2a^{2}d\cos\left(dx+c\right)^{3}+a^{2}d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*((7*A - 4*B)*cos(d*x + c)^4 + 2*(7*A - 4*B)*cos(d*x + c)^3 + (7*A -
 4*B)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((7*A - 4*B)*cos(d*x + c)^4
 + 2*(7*A - 4*B)*cos(d*x + c)^3 + (7*A - 4*B)*cos(d*x + c)^2)*log(-sin(d*x
+ c) + 1) - 2*(4*(8*A - 5*B)*cos(d*x + c)^3 + (43*A - 28*B)*cos(d*x + c)^2
+ 6*(A - B)*cos(d*x + c) - 3*A)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2
*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^{3}\left(c+dx\right)}{\cos^{2}\left(c+dx\right)+2\cos\left(c+dx\right)+1}\,dx+\int \frac{B\cos\left(c+dx\right)\sec^{3}\left(c+dx\right)}{\cos^{2}\left(c+dx\right)+2\cos\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + In
tegral(B*cos(c + d*x)*sec(c + d*x)**3/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1
), x))/a**2
```

Giac [A]
time = 0.51, size = 198, normalized size = 1.30

$$\frac{\frac{3\,(7\,A-4\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{2}}-\frac{3\,(7\,A-4\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{2}}+\frac{6\left(5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{2}}-\frac{Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+21\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(7*A - 4*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(7*A - 4*B)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 6*(5*A*tan(1/2*d*x + 1/2*c)^3 - 2*B
*tan(1/2*d*x + 1/2*c)^3 - 3*A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 -
B*a^4*tan(1/2*d*x + 1/2*c)^3 + 21*A*a^4*tan(1/2*d*x + 1/2*c) - 15*B*a^4*tan
(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 0.30, size = 165, normalized size = 1.09

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(5A-2B)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3A-2B)}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3(A-B)}{2a^2}+\frac{4A-2B}{2a^2}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(A-B)}{6a^2d}+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(7A-4B)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(5*A - 2*B) - tan(c/2 + (d*x)/2)*(3*A - 2*B))/(d*(a^2
*tan(c/2 + (d*x)/2)^4 - 2*a^2*tan(c/2 + (d*x)/2)^2 + a^2)) - (tan(c/2 + (d*
x)/2)*((3*(A - B))/(2*a^2) + (4*A - 2*B)/(2*a^2)))/d - (tan(c/2 + (d*x)/2)^
3*(A - B))/(6*a^2*d) + (atanh(tan(c/2 + (d*x)/2))*(7*A - 4*B))/(a^2*d)
```

3.55 $\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$-\frac{(10A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{4(3A-2B)\tan(c+dx)}{a^2d}-\frac{(10A-7B)\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{(10A-$$

```
[Out] -1/2*(10*A-7*B)*arctanh(sin(d*x+c))/a^2/d+4*(3*A-2*B)*tan(d*x+c)/a^2/d-1/2*
(10*A-7*B)*sec(d*x+c)*tan(d*x+c)/a^2/d-1/3*(10*A-7*B)*sec(d*x+c)^2*tan(d*x+
c)/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c)
)^2+4/3*(3*A-2*B)*tan(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.22, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3057, 2827, 3852, 3853, 3855}

$$\frac{4(3A-2B)\tan^3(c+dx)}{3a^2d}+\frac{4(3A-2B)\tan(c+dx)}{a^2d}-\frac{(10A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{(10A-7B)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{(10A-7B)\tan(c+dx)\sec^2(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -1/2*((10*A - 7*B)*ArcTanh[Sin[c + d*x]])/(a^2*d) + (4*(3*A - 2*B)*Tan[c +
d*x])/(a^2*d) - ((10*A - 7*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - ((10*A
 - 7*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B
)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (4*(3*A - 2*B
)*Tan[c + d*x]^3)/(3*a^2*d)
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{(3a(2A-B)-4a(A-B)\cos(c+dx}{a+a\cos(c+dx)}}{3a^2}\\ &= -\frac{(10A-7B)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\sec^2(c+dx)\,\mathrm{t}}{3d(a+a\cos(c+}\\ &= -\frac{(10A-7B)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\sec^2(c+dx)\,\mathrm{t}}{3d(a+a\cos(c+}\\ &= -\frac{(10A-7B)\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{(10A-7B)\sec^2(c+dx}{3a^2d(1+\cos(c-}\\ &= -\frac{(10A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{4(3A-2B)\tan(c+dx)}{a^2d}-\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 609 vs. 2(179) = 358.
time = 4.99, size = 609, normalized size = 3.40

$$\frac{192(10A-7B)\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sec(c)\sec^3(c+dx)\left((-6A+45B)\sin\left(\frac{dx}{2}\right)+(310A-201B)\sin\left(\frac{3dx}{2}\right)-306A\sin\left(c-\frac{dx}{2}\right)+195B\sin\left(c-\frac{dx}{2}\right)+42A\sin\left(c+\frac{dx}{2}\right)-51B\sin\left(c+\frac{dx}{2}\right)-270A\sin\left(2c+\frac{dx}{2}\right)+189B\sin\left(2c+\frac{dx}{2}\right)+50A\sin\left(c+\frac{3dx}{2}\right)-B\sin\left(c+\frac{3dx}{2}\right)+90A\sin\left(2c+\frac{3dx}{2}\right)-81B\sin\left(2c+\frac{3dx}{2}\right)-170A\sin\left(3c+\frac{3dx}{2}\right)+119B\sin\left(3c+\frac{3dx}{2}\right)+198A\sin\left(c+\frac{5dx}{2}\right)-129B\sin\left(c+\frac{5dx}{2}\right)+42A\sin\left(2c+\frac{5dx}{2}\right)-9B\sin\left(2c+\frac{5dx}{2}\right)+66A\sin\left(3c+\frac{5dx}{2}\right)-57B\sin\left(3c+\frac{5dx}{2}\right)-90A\sin\left(4c+\frac{5dx}{2}\right)+63B\sin\left(4c+\frac{5dx}{2}\right)+114A\sin\left(2c+\frac{7dx}{2}\right)-75B\sin\left(2c+\frac{7dx}{2}\right)+36A\sin\left(3c+\frac{7dx}{2}\right)-15B\sin\left(3c+\frac{7dx}{2}\right)+48A\sin\left(4c+\frac{7dx}{2}\right)-39B\sin\left(4c+\frac{7dx}{2}\right)-30A\sin\left(5c+\frac{7dx}{2}\right)+21B\sin\left(5c+\frac{7dx}{2}\right)+48A\sin\left(3c+\frac{9dx}{2}\right)-32B\sin\left(3c+\frac{9dx}{2}\right)+22A\sin\left(4c+\frac{9dx}{2}\right)-12B\sin\left(4c+\frac{9dx}{2}\right)+26A\sin\left(5c+\frac{9dx}{2}\right)-20B\sin\left(5c+\frac{9dx}{2}\right)\right)}{96a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (192*(10*A - 7*B)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]
```

```
*Sec[c]*Sec[c + d*x]^3*((-6*A + 45*B)*Sin[(d*x)/2] + (310*A - 201*B)*Sin[(3
*d*x)/2] - 306*A*Sin[c - (d*x)/2] + 195*B*Sin[c - (d*x)/2] + 42*A*Sin[c + (
d*x)/2] - 51*B*Sin[c + (d*x)/2] - 270*A*Sin[2*c + (d*x)/2] + 189*B*Sin[2*c
+ (d*x)/2] + 50*A*Sin[c + (3*d*x)/2] - B*Sin[c + (3*d*x)/2] + 90*A*Sin[2*c
+ (3*d*x)/2] - 81*B*Sin[2*c + (3*d*x)/2] - 170*A*Sin[3*c + (3*d*x)/2] + 119
*B*Sin[3*c + (3*d*x)/2] + 198*A*Sin[c + (5*d*x)/2] - 129*B*Sin[c + (5*d*x)/
2] + 42*A*Sin[2*c + (5*d*x)/2] - 9*B*Sin[2*c + (5*d*x)/2] + 66*A*Sin[3*c +
(5*d*x)/2] - 57*B*Sin[3*c + (5*d*x)/2] - 90*A*Sin[4*c + (5*d*x)/2] + 63*B*S
in[4*c + (5*d*x)/2] + 114*A*Sin[2*c + (7*d*x)/2] - 75*B*Sin[2*c + (7*d*x)/2
] + 36*A*Sin[3*c + (7*d*x)/2] - 15*B*Sin[3*c + (7*d*x)/2] + 48*A*Sin[4*c +
(7*d*x)/2] - 39*B*Sin[4*c + (7*d*x)/2] - 30*A*Sin[5*c + (7*d*x)/2] + 21*B*S
in[5*c + (7*d*x)/2] + 48*A*Sin[3*c + (9*d*x)/2] - 32*B*Sin[3*c + (9*d*x)/2]
 + 22*A*Sin[4*c + (9*d*x)/2] - 12*B*Sin[4*c + (9*d*x)/2] + 26*A*Sin[5*c + (
9*d*x)/2] - 20*B*Sin[5*c + (9*d*x)/2]))/(96*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.28, size = 222, normalized size = 1.24

method	result
derivativedivides	$(-10A+7B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{-6A+2B}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{10A-5B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{2A}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{\left(\tan^{3}\left(\frac{d}{2}\right.\right.}{3}$
default	$(-10A+7B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{-6A+2B}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{10A-5B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{2A}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{\left(\tan^{3}\left(\frac{d}{2}\right.\right.}{3}$
norman	$\frac{\frac{(A-B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{(11A-10B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{(19A-12B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{(21A-13B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{(25A-19B)\left(\tan^{9}\right.}{6da}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}a}$
risch	$\frac{i\left(30A\,\mathrm{e}^{8i(dx+c)}-21B\,\mathrm{e}^{8i(dx+c)}+90A\,\mathrm{e}^{7i(dx+c)}-63B\,\mathrm{e}^{7i(dx+c)}+170A\,\mathrm{e}^{6i(dx+c)}-119B\,\mathrm{e}^{6i(dx+c)}+270A\,\mathrm{e}^{5i(dx+c)}-189B\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d/a^2*((-10*A+7*B)*ln(tan(1/2*d*x+1/2*c)+1)-1/2*(-6*A+2*B)/(tan(1/2*d*x
+1/2*c)+1)^2-(10*A-5*B)/(tan(1/2*d*x+1/2*c)+1)-2/3*A/(tan(1/2*d*x+1/2*c)+1)
^3+1/3*tan(1/2*d*x+1/2*c)^3*A-1/3*tan(1/2*d*x+1/2*c)^3*B+9*A*tan(1/2*d*x+1/
2*c)-7*B*tan(1/2*d*x+1/2*c)-1/2*(6*A-2*B)/(tan(1/2*d*x+1/2*c)-1)^2+(10*A-7*
B)*ln(tan(1/2*d*x+1/2*c)-1)-(10*A-5*B)/(tan(1/2*d*x+1/2*c)-1)-2/3*A/(tan(1/
2*d*x+1/2*c)-1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(169) = 338$.
time = 0.28, size = 425, normalized size = 2.37

$$\frac{A\left(\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a^{2}-\frac{3a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{2}}-\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{2}}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{2}}\right)-B\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{2}-\frac{2a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{2}}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{2}}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{2}}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(A*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 30*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^2 + 30*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*
(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3
/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^
2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2))/d
```

Fricas [A]
time = 0.36, size = 247, normalized size = 1.38

$$-\frac{3\left(\left(10A-7B\right)\cos\left(dx+c\right)^{5}+2\left(10A-7B\right)\cos\left(dx+c\right)^{4}+\left(10A-7B\right)\cos\left(dx+c\right)^{3}\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\left(10A-7B\right)\cos\left(dx+c\right)^{5}+2\left(10A-7B\right)\cos\left(dx+c\right)^{4}+\left(10A-7B\right)\cos\left(dx+c\right)^{3}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(16\left(3A-2B\right)\cos\left(dx+c\right)^{4}+\left(66A-43B\right)\cos\left(dx+c\right)^{3}+6\left(2A-B\right)\cos\left(dx+c\right)^{2}-\left(2A-3B\right)\cos\left(dx+c\right)+2A\right)\sin\left(dx+c\right)}{12\left(a^{2}d\cos\left(dx+c\right)^{5}+2a^{2}d\cos\left(dx+c\right)^{4}+a^{2}d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/12*(3*((10*A - 7*B)*cos(d*x + c)^5 + 2*(10*A - 7*B)*cos(d*x + c)^4 + (10
*A - 7*B)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((10*A - 7*B)*cos(d*x +
 c)^5 + 2*(10*A - 7*B)*cos(d*x + c)^4 + (10*A - 7*B)*cos(d*x + c)^3)*log(-s
in(d*x + c) + 1) - 2*(16*(3*A - 2*B)*cos(d*x + c)^4 + (66*A - 43*B)*cos(d*x
 + c)^3 + 6*(2*A - B)*cos(d*x + c)^2 - (2*A - 3*B)*cos(d*x + c) + 2*A)*sin(
d*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x +
c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^{4}\left(c+dx\right)}{\cos^{2}\left(c+dx\right)+2\cos\left(c+dx\right)+1}\,dx+\int \frac{B\cos\left(c+dx\right)\sec^{4}\left(c+dx\right)}{\cos^{2}\left(c+dx\right)+2\cos\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**4/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + In
tegral(B*cos(c + d*x)*sec(c + d*x)**4/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1
), x))/a**2
```

Giac [A]
time = 0.48, size = 226, normalized size = 1.26

$$-\frac{\frac{3\,(10\,A-7\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{3\,(10\,A-7\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{2\left(30\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a^2}-\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-21\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(10*A - 7*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(10*A - 7*B
)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(30*A*tan(1/2*d*x + 1/2*c)^5 -
 15*B*tan(1/2*d*x + 1/2*c)^5 - 40*A*tan(1/2*d*x + 1/2*c)^3 + 24*B*tan(1/2*d
*x + 1/2*c)^3 + 18*A*tan(1/2*d*x + 1/2*c) - 9*B*tan(1/2*d*x + 1/2*c))/((tan
(1/2*d*x + 1/2*c)^2 - 1)^3*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan
(1/2*d*x + 1/2*c)^3 + 27*A*a^4*tan(1/2*d*x + 1/2*c) - 21*B*a^4*tan(1/2*d*x
+ 1/2*c))/a^6)/d
```

Mupad [B]
time = 0.34, size = 203, normalized size = 1.13

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\frac{2\,(A-B)}{a^2}+\frac{5\,A-3\,B}{2\,a^2}\right)}{d}-\frac{(10\,A-5\,B)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(8\,B-\frac{40\,A}{3}\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(6\,A-3\,B)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,(A-B)}{6\,a^2\,d}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(10\,A-7\,B)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^4*(a + a*cos(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((2*(A - B))/a^2 + (5*A - 3*B)/(2*a^2)))/d - (tan(c/2 +
 (d*x)/2)^5*(10*A - 5*B) - tan(c/2 + (d*x)/2)^3*((40*A)/3 - 8*B) + tan(c/2
+ (d*x)/2)*(6*A - 3*B))/(d*(3*a^2*tan(c/2 + (d*x)/2)^2 - 3*a^2*tan(c/2 + (d
*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 - a^2)) + (tan(c/2 + (d*x)/2)^3*(A - B)
)/(6*a^2*d) - (atanh(tan(c/2 + (d*x)/2))*(10*A - 7*B))/(a^2*d)
```

3.56 $\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=218

$$\frac{(13A-23B)x}{2a^3}-\frac{4(19A-34B)\sin(c+dx)}{5a^3d}+\frac{(13A-23B)\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{(A-B)\cos^5(c+dx)\,\mathrm{si}}{5d(a+a\cos(c+}$$

```
[Out] 1/2*(13*A-23*B)*x/a^3-4/5*(19*A-34*B)*sin(d*x+c)/a^3/d+1/2*(13*A-23*B)*cos(
d*x+c)*sin(d*x+c)/a^3/d+1/5*(A-B)*cos(d*x+c)^5*sin(d*x+c)/d/(a+a*cos(d*x+c)
)^3+1/15*(8*A-13*B)*cos(d*x+c)^4*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+1/3*(13*
A-23*B)*cos(d*x+c)^3*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))+4/15*(19*A-34*B)*sin
(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.34, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3056, 2827, 2715, 8, 2713}

$$\frac{4(19A-34B)\sin^3(c+dx)}{15a^3d}-\frac{4(19A-34B)\sin(c+dx)}{5a^3d}+\frac{(13A-23B)\sin(c+dx)\cos^3(c+dx)}{3d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(13A-23B)\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{x(13A-23B)}{2a^3}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(8A-13B)\sin(c+dx)\cos^4(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((13*A - 23*B)*x)/(2*a^3) - (4*(19*A - 34*B)*Sin[c + d*x])/(5*a^3*d) + ((13
*A - 23*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) + ((A - B)*Cos[c + d*x]^5*S
in[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((8*A - 13*B)*Cos[c + d*x]^4*Si
n[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((13*A - 23*B)*Cos[c + d*x]^3
*Sin[c + d*x])/(3*d*(a^3 + a^3*Cos[c + d*x])) + (4*(19*A - 34*B)*Sin[c + d*
x]^3)/(15*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^4(c+dx)(5a(A-B)-a(3A-8B)\cos}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(8A-13B)\cos^4(c+dx)\sin(c}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(8A-13B)\cos^4(c+dx)\sin(c}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(8A-13B)\cos^4(c+dx)\sin(c}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(13A-23B)\cos(c+dx)\sin(c+dx)}{2a^3d} + \frac{(A-B)\cos^5(c+dx)\sin(c}{5d(a+a\cos(c+dx))}$$

$$= \frac{(13A-23B)x}{2a^3} - \frac{4(19A-34B)\sin(c+dx)}{5a^3d} + \frac{(13A-23B)\cos(c}{2a^3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 491 vs. 2(218) = 436.

time = 0.96, size = 491, normalized size = 2.25

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]

[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(600*(13*A - 23*B)*d*x*Cos[(d*x)/2] + 600*(13*A - 23*B)*d*x*Cos[c + (d*x)/2] + 3900*A*d*x*Cos[c + (3*d*x)/2] - 6900*B*d*x*Cos[c + (3*d*x)/2] + 3900*A*d*x*Cos[2*c + (3*d*x)/2] - 6900*B*d*x*Cos[2*c + (3*d*x)/2] + 780*A*d*x*Cos[2*c + (5*d*x)/2] - 1380*B*d*x*Cos[2*c + (5*d*x)/2] + 780*A*d*x*Cos[3*c + (5*d*x)/2] - 1380*B*d*x*Cos[3*c + (5*d*x)/2] - 12760*A*Sin[(d*x)/2] + 20410*B*Sin[(d*x)/2] + 7560*A*Sin[c + (d*x)/2] - 11110*B*Sin[c + (d*x)/2] - 9230*A*Sin[c + (3*d*x)/2] + 15380*B*Sin[c + (3*d*x)/2] + 930*A*Sin[2*c + (3*d*x)/2] - 380*B*Sin[2*c + (3*d*x)/2] - 2782*A*Sin[2*c + (5*d*x)/2] + 4777*B*Sin[2*c + (5*d*x)/2] - 750*A*Sin[3*c + (5*d*x)/2] + 1625*B*Sin[3*c + (5*d*x)/2] - 105*A*Sin[3*c + (7*d*x)/2] + 230*B*Sin[3*c + (7*d*x)/2] - 105*A*Sin[4*c + (7*d*x)/2] + 230*B*Sin[4*c + (7*d*x)/2] + 15*A*Sin[4*c + (9*d*x)/2] - 20*B*Sin[4*c + (9*d*x)/2] + 15*A*Sin[5*c + (9*d*x)/2] - 20*B*Sin[5*c + (9*d*x)/2] + 5*B*Sin[5*c + (11*d*x)/2] + 5*B*Sin[6*c + (11*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)

Maple [A]
time = 0.22, size = 182, normalized size = 0.83

method	result
derivativedivides	$-\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}-31A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+49B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+$
default	$-\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}-31A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+49B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+$
risch	$\frac{13xA}{2a^3}-\frac{23Bx}{2a^3}-\frac{iB\,e^{3i(dx+c)}}{24a^3d}-\frac{ie^{2i(dx+c)}A}{8a^3d}+\frac{3ie^{2i(dx+c)}B}{8a^3d}+\frac{3ie^{i(dx+c)}A}{2a^3d}-\frac{27ie^{i(dx+c)}B}{8a^3d}-\frac{3ie^{-i(dx+c)}}{2a^3d}$
norman	$\frac{(13A-23B)x}{2a}-\frac{(A-B)\left(\tan^{17}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}-\frac{(9A-16B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}+\frac{(11A-16B)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30da}+\frac{3(13A-23B)x\left(\tan^2\left(\frac{dx}{2}+\right.\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/4/d/a^3*(-1/5*A*tan(1/2*d*x+1/2*c)^5+1/5*B*tan(1/2*d*x+1/2*c)^5+8/3*tan(1/2*d*x+1/2*c)^3*A-10/3*tan(1/2*d*x+1/2*c)^3*B-31*A*tan(1/2*d*x+1/2*c)+49*B*tan(1/2*d*x+1/2*c)+16*((-7/4*A+17/4*B)*tan(1/2*d*x+1/2*c)^5+(-3*A+19/3*B)*tan(1/2*d*x+1/2*c)^3+(-5/4*A+11/4*B)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^3+4*(13*A-23*B)*arctan(tan(1/2*d*x+1/2*c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(204) = 408.

time = 0.50, size = 412, normalized size = 1.89

$$\frac{B\left(\frac{20\left(\frac{33\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{76\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{51\,\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}\right)}{a^{3}+\frac{3\,a^{3}\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{3\,a^{3}\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}+\frac{a^{3}\sin\left(dx+c\right)^{6}}{\left(\cos\left(dx+c\right)+1\right)^{6}}}+\frac{\frac{735\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{50\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{3\,\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}}{a^{3}}-\frac{1380\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a^{3}}\right)-A\left(\frac{60\left(\frac{5\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{7\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}\right)}{a^{3}+\frac{2\,a^{3}\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{a^{3}\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}}+\frac{\frac{465\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{40\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{3\,\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}}{a^{3}}-\frac{780\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a^{3}}\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(B*(20*(33*sin(d*x + c)/(cos(d*x + c) + 1) + 76*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 51*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3 + 3*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (735*sin(d*x + c)/(cos(d*x + c)
 + 1) - 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5)/a^3 - 1380*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*(6
0*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 780
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A]
time = 0.35, size = 205, normalized size = 0.94

$$\frac{15\,(13\,A-23\,B)dx\cos\left(dx+c\right)^{3}+45\,(13\,A-23\,B)dx\cos\left(dx+c\right)^{2}+45\,(13\,A-23\,B)dx\cos\left(dx+c\right)+15\,(13\,A-23\,B)dx+\left(10\,B\cos\left(dx+c\right)^{5}+15\,(A-B)\cos\left(dx+c\right)^{4}-5\,(9\,A-19\,B)\cos\left(dx+c\right)^{3}-(479\,A-869\,B)\cos\left(dx+c\right)^{2}-3\,(239\,A-429\,B)\cos\left(dx+c\right)-304\,A+544\,B\right)\sin\left(dx+c\right)}{30\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*(13*A - 23*B)*d*x*cos(d*x + c)^3 + 45*(13*A - 23*B)*d*x*cos(d*x +
c)^2 + 45*(13*A - 23*B)*d*x*cos(d*x + c) + 15*(13*A - 23*B)*d*x + (10*B*cos
(d*x + c)^5 + 15*(A - B)*cos(d*x + c)^4 - 5*(9*A - 19*B)*cos(d*x + c)^3 - (
479*A - 869*B)*cos(d*x + c)^2 - 3*(239*A - 429*B)*cos(d*x + c) - 304*A + 54
4*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d
*cos(d*x + c) + a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1584 vs. $2(206) = 412$.
time = 6.80, size = 1584, normalized size = 7.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((390*A*d*x*tan(c/2 + d*x/2)**6/(60*a**3*d*tan(c/2 + d*x/2)**6 + 1
80*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
 + 1170*A*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3
*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 1170
*A*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(
c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 390*A*d*x/(
60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d
*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3*A*tan(c/2 + d*x/2)**11/(60*a**3*d*tan
(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*
x/2)**2 + 60*a**3*d) + 31*A*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)
**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*
a**3*d) - 354*A*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a*
*3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 16
98*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/
2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 2075*A*tan(c/
2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)*
*4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 765*A*tan(c/2 + d*x/2)/(
60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d
*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 690*B*d*x*tan(c/2 + d*x/2)**6/(60*a**3*
d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2
 + d*x/2)**2 + 60*a**3*d) - 2070*B*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c
/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/
2)**2 + 60*a**3*d) - 2070*B*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*
x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 +
 60*a**3*d) - 690*B*d*x/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2
 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 3*B*tan(c/2 +
d*x/2)**11/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4
+ 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 41*B*tan(c/2 + d*x/2)**9/(6
0*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*
tan(c/2 + d*x/2)**2 + 60*a**3*d) + 594*B*tan(c/2 + d*x/2)**7/(60*a**3*d*tan
(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*
x/2)**2 + 60*a**3*d) + 3078*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/
2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 6
0*a**3*d) + 3675*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180
*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) +
 1395*B*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/
2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d), Ne(d, 0)), (x*
(A + B*cos(c))*cos(c)**5/(a*cos(c) + a)**3, True))
```

Giac [A]
time = 0.47, size = 228, normalized size = 1.05

$$\frac{\frac{30\,(dx+c)(13\,A-23\,B)}{a^3}-\frac{20\,\left(21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-51\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+36\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-76\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-33\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+50\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+465\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-735\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(30*(d*x + c)*(13*A - 23*B)/a^3 - 20*(21*A*tan(1/2*d*x + 1/2*c)^5 - 51
*B*tan(1/2*d*x + 1/2*c)^5 + 36*A*tan(1/2*d*x + 1/2*c)^3 - 76*B*tan(1/2*d*x
+ 1/2*c)^3 + 15*A*tan(1/2*d*x + 1/2*c) - 33*B*tan(1/2*d*x + 1/2*c))/((tan(1
/2*d*x + 1/2*c)^2 + 1)^3*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12
*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 50*B*a^12*tan(
1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2*d*x + 1/2*c) - 735*B*a^12*tan(1/2*d
*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 0.33, size = 238, normalized size = 1.09

$$\frac{x\,\left(13\,A-23\,B\right)}{2\,a^3}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,\left(A-B\right)}{2\,a^3}+\frac{4\,A-6\,B}{a^3}+\frac{5\,A-15\,B}{4\,a^3}\right)}{d}-\frac{\left(7\,A-17\,B\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(12\,A-\frac{76\,B}{3}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(5\,A-11\,B\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+3\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a^3\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{A-B}{3\,a^3}+\frac{4\,A-6\,B}{12\,a^3}\right)}{d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(A-B\right)}{20\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (x*(13*A - 23*B))/(2*a^3) - (tan(c/2 + (d*x)/2)*((5*(A - B))/(2*a^3) + (4*A
 - 6*B)/a^3 + (5*A - 15*B)/(4*a^3)))/d - (tan(c/2 + (d*x)/2)^5*(7*A - 17*B)
 + tan(c/2 + (d*x)/2)^3*(12*A - (76*B)/3) + tan(c/2 + (d*x)/2)*(5*A - 11*B)
)/(d*(3*a^3*tan(c/2 + (d*x)/2)^2 + 3*a^3*tan(c/2 + (d*x)/2)^4 + a^3*tan(c/2
 + (d*x)/2)^6 + a^3)) + (tan(c/2 + (d*x)/2)^3*((A - B)/(3*a^3) + (4*A - 6*B
)/(12*a^3)))/d - (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d)
```

3.57 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=193

$$-\frac{(6A-13B)x}{2a^3}+\frac{8(9A-19B)\sin(c+dx)}{15a^3d}-\frac{(6A-13B)\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{(A-B)\cos^4(c+dx)\sin}{5d(a+a\cos(c+dx}$$

```
[Out] -1/2*(6*A-13*B)*x/a^3+8/15*(9*A-19*B)*sin(d*x+c)/a^3/d-1/2*(6*A-13*B)*cos(d
*x+c)*sin(d*x+c)/a^3/d+1/5*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))
^3+1/15*(6*A-11*B)*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+4/15*(9*A
-19*B)*cos(d*x+c)^2*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A]
time = 0.30, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {3056, 2813}

$$\frac{8(9A-19B)\sin(c+dx)}{15a^3d}+\frac{4(9A-19B)\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A-13B)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{x(6A-13B)}{2a^3}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(6A-11B)\sin(c+dx)\cos^3(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -1/2*((6*A - 13*B)*x)/a^3 + (8*(9*A - 19*B)*Sin[c + d*x])/(15*a^3*d) - ((6*
A - 13*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) + ((A - B)*Cos[c + d*x]^4*Si
n[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((6*A - 11*B)*Cos[c + d*x]^3*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + (4*(9*A - 19*B)*Cos[c + d*x]^2
*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^3(c+dx)(4a(A-B)-a(2A-7B)\cos}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(6A-11B)\cos^3(c+dx)\sin(c}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(6A-11B)\cos^3(c+dx)\sin(c}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(6A-13B)x}{2a^3} + \frac{8(9A-19B)\sin(c+dx)}{15a^3d} - \frac{(6A-13B)\cos(c+}{2a^3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 435 vs. 2(193) = 386.
time = 0.85, size = 435, normalized size = 2.25

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-600(6A-13B)dx\cos\left(\frac{dx}{2}\right)-600(6A-13B)dx\cos\left(c+\frac{dx}{2}\right)-1800Adx\cos\left(c+\frac{3dx}{2}\right)+3900Bdx\cos\left(c+\frac{3dx}{2}\right)-1800Adx\cos\left(2c+\frac{3dx}{2}\right)+3900Bdx\cos\left(2c+\frac{3dx}{2}\right)-360Adx\cos\left(2c+\frac{5dx}{2}\right)+780Bdx\cos\left(2c+\frac{5dx}{2}\right)-360Adx\cos\left(3c+\frac{5dx}{2}\right)+780Bdx\cos\left(3c+\frac{5dx}{2}\right)+7020A\sin\left(\frac{dx}{2}\right)-12760B\sin\left(\frac{dx}{2}\right)-4500A\sin\left(c+\frac{dx}{2}\right)+7560B\sin\left(c+\frac{dx}{2}\right)+4860A\sin\left(c+\frac{3dx}{2}\right)-9230B\sin\left(c+\frac{3dx}{2}\right)-900A\sin\left(2c+\frac{3dx}{2}\right)+930B\sin\left(2c+\frac{3dx}{2}\right)+1452A\sin\left(2c+\frac{5dx}{2}\right)-2782B\sin\left(2c+\frac{5dx}{2}\right)+300A\sin\left(3c+\frac{5dx}{2}\right)-750B\sin\left(3c+\frac{5dx}{2}\right)+60A\sin\left(3c+\frac{7dx}{2}\right)-105B\sin\left(3c+\frac{7dx}{2}\right)+60A\sin\left(4c+\frac{7dx}{2}\right)-105B\sin\left(4c+\frac{7dx}{2}\right)+15B\sin\left(4c+\frac{9dx}{2}\right)+15B\sin\left(5c+\frac{9dx}{2}\right)\right)}{480a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-600*(6*A - 13*B)*d*x*Cos[(d*x)/2] - 600*(6*A -
 13*B)*d*x*Cos[c + (d*x)/2] - 1800*A*d*x*Cos[c + (3*d*x)/2] + 3900*B*d*x*Co
s[c + (3*d*x)/2] - 1800*A*d*x*Cos[2*c + (3*d*x)/2] + 3900*B*d*x*Cos[2*c + (
3*d*x)/2] - 360*A*d*x*Cos[2*c + (5*d*x)/2] + 780*B*d*x*Cos[2*c + (5*d*x)/2]
 - 360*A*d*x*Cos[3*c + (5*d*x)/2] + 780*B*d*x*Cos[3*c + (5*d*x)/2] + 7020*A
*Sin[(d*x)/2] - 12760*B*Sin[(d*x)/2] - 4500*A*Sin[c + (d*x)/2] + 7560*B*Sin
[c + (d*x)/2] + 4860*A*Sin[c + (3*d*x)/2] - 9230*B*Sin[c + (3*d*x)/2] - 900
*A*Sin[2*c + (3*d*x)/2] + 930*B*Sin[2*c + (3*d*x)/2] + 1452*A*Sin[2*c + (5*
d*x)/2] - 2782*B*Sin[2*c + (5*d*x)/2] + 300*A*Sin[3*c + (5*d*x)/2] - 750*B*
Sin[3*c + (5*d*x)/2] + 60*A*Sin[3*c + (7*d*x)/2] - 105*B*Sin[3*c + (7*d*x)/
2] + 60*A*Sin[4*c + (7*d*x)/2] - 105*B*Sin[4*c + (7*d*x)/2] + 15*B*Sin[4*c
+ (9*d*x)/2] + 15*B*Sin[5*c + (9*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]
time = 0.20, size = 163, normalized size = 0.84

method	result

derivativedivides	$\frac{\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+17A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-31B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{}}{4d\,a^3}$
default	$\frac{\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+17A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-31B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{}}{4d\,a^3}$
risch	$-\frac{3xA}{a^3}+\frac{13Bx}{2a^3}-\frac{i\mathrm{e}^{2i(dx+c)}B}{8a^3d}-\frac{i\mathrm{e}^{i(dx+c)}A}{2a^3d}+\frac{3i\mathrm{e}^{i(dx+c)}B}{2a^3d}+\frac{i\mathrm{e}^{-i(dx+c)}A}{2a^3d}-\frac{3i\mathrm{e}^{-i(dx+c)}B}{2a^3d}+\frac{i\mathrm{e}^{-2i(dx+c)}B}{8a^3d}$
norman	$-\frac{(6A-13B)x}{2a}+\frac{(A-B)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}-\frac{(3A-5B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}-\frac{5(6A-13B)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{5(6A-13B)x\left(\tan^{4}\left(\frac{dx}{2}+\right.\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/d/a^3*(1/5*A*tan(1/2*d*x+1/2*c)^5-1/5*B*tan(1/2*d*x+1/2*c)^5-2*tan(1/2*
d*x+1/2*c)^3*A+8/3*tan(1/2*d*x+1/2*c)^3*B+17*A*tan(1/2*d*x+1/2*c)-31*B*tan(
1/2*d*x+1/2*c)-16*((-1/2*A+7/4*B)*tan(1/2*d*x+1/2*c)^3+(-1/2*A+5/4*B)*tan(1
/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2-4*(6*A-13*B)*arctan(tan(1/2*d*x+1
/2*c)))
```

Maxima [A]
time = 0.51, size = 322, normalized size = 1.67

$$-\frac{B\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-3A\left(\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(B*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - 3*A*(40*sin(d*x
+ c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) +
 (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^3))/d
```

Fricas [A]
time = 0.34, size = 190, normalized size = 0.98

$$-\frac{15(6A-13B)dx\cos(dx+c)^3+45(6A-13B)dx\cos(dx+c)^2+45(6A-13B)dx\cos(dx+c)+15(6A-13B)dx-\left(15B\cos(dx+c)^4+15(2A-3B)\cos(dx+c)^3+(234A-479B)\cos(dx+c)^2+3(114A-239B)\cos(dx+c)+144A-304B\right)\sin(dx+c)}{30\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*(6*A - 13*B)*d*x*cos(d*x + c)^3 + 45*(6*A - 13*B)*d*x*cos(d*x + c
)^2 + 45*(6*A - 13*B)*d*x*cos(d*x + c) + 15*(6*A - 13*B)*d*x - (15*B*cos(d*
x + c)^4 + 15*(2*A - 3*B)*cos(d*x + c)^3 + (234*A - 479*B)*cos(d*x + c)^2 +
 3*(114*A - 239*B)*cos(d*x + c) + 144*A - 304*B)*sin(d*x + c))/(a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 966 vs. $2(178) = 356$.

time = 4.42, size = 966, normalized size = 5.01

$$\begin{cases} -\frac{180Adx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} - \frac{360Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} - \frac{180Adx}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{3A\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} - \frac{24A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{198A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{600A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{375A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{390Bdx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{780Bdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{390Bdx}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} - \frac{3B\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{34B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} - \frac{388B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} - \frac{1310B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} - \frac{765B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} & \text{for}\: d \neq 0 \\ \frac{x\left(A+B\cos(c)\right)\cos^4(c)}{\left(a\cos(c)+a\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-180*A*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 +
120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 360*A*d*x*tan(c/2 + d*x/2)**2
/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*
d) - 180*A*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)
**2 + 60*a**3*d) + 3*A*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4 +
 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 24*A*tan(c/2 + d*x/2)**7/(60
*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) +
 198*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(
c/2 + d*x/2)**2 + 60*a**3*d) + 600*A*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2
 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 375*A*tan(c/2
+ d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 +
60*a**3*d) + 390*B*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 +
 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 780*B*d*x*tan(c/2 + d*x/2)**
2/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3
*d) + 390*B*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2
)**2 + 60*a**3*d) - 3*B*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4
+ 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 34*B*tan(c/2 + d*x/2)**7/(6
0*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
- 388*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan
(c/2 + d*x/2)**2 + 60*a**3*d) - 1310*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c
/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 765*B*tan(c/
2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2
+ 60*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**4/(a*cos(c) + a)**3, Tru
e))
```

Giac [**A**]

time = 0.45, size = 200, normalized size = 1.04

$$-\frac{\frac{30\,(dx+c)(6\,A-13\,B)}{a^3}-\frac{60\,\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-30\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+255\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-465\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(30*(d*x + c)*(6*A - 13*B)/a^3 - 60*(2*A*tan(1/2*d*x + 1/2*c)^3 - 7*B
*tan(1/2*d*x + 1/2*c)^3 + 2*A*tan(1/2*d*x + 1/2*c) - 5*B*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5
 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 40*
B*a^12*tan(1/2*d*x + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 465*B*a^1
2*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]

time = 0.27, size = 203, normalized size = 1.05

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,(A-B)}{2\,a^3}+\frac{3\,(3\,A-5\,B)}{4\,a^3}+\frac{2\,A-10\,B}{4\,a^3}\right)}{d}-\frac{x\,(6\,A-13\,B)}{2\,a^3}+\frac{(2\,A-7\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(2\,A-5\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{4\,a^3}+\frac{3\,A-5\,B}{12\,a^3}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A-B)}{20\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((3*(A - B))/(2*a^3) + (3*(3*A - 5*B))/(4*a^3) + (2*A -
 10*B)/(4*a^3)))/d - (x*(6*A - 13*B))/(2*a^3) + (tan(c/2 + (d*x)/2)^3*(2*A
- 7*B) + tan(c/2 + (d*x)/2)*(2*A - 5*B))/(d*(2*a^3*tan(c/2 + (d*x)/2)^2 + a
^3*tan(c/2 + (d*x)/2)^4 + a^3)) - (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^3) +
(3*A - 5*B)/(12*a^3)))/d + (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d)
```

3.58 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=147

$$\frac{(A-3B)x}{a^3}-\frac{(7A-27B)\sin(c+dx)}{15a^3d}+\frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(4A-9B)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

```
[Out] (A-3*B)*x/a^3-1/15*(7*A-27*B)*sin(d*x+c)/a^3/d+1/5*(A-B)*cos(d*x+c)^3*sin(d
*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(4*A-9*B)*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*
cos(d*x+c))^2-(A-3*B)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A]
time = 0.29, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3056, 3047, 3102, 12, 2814, 2727}

$$-\frac{(7A-27B)\sin(c+dx)}{15a^3d}-\frac{(A-3B)\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{x(A-3B)}{a^3}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(4A-9B)\sin(c+dx)\cos^2(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((A - 3*B)*x)/a^3 - ((7*A - 27*B)*Sin[c + d*x])/(15*a^3*d) + ((A - B)*Cos[c
 + d*x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((4*A - 9*B)*Cos[c +
 d*x]^2*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((A - 3*B)*Sin[c +
d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^2(c+dx)(3a(A-B)-a(A-6B)\text{co}}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A-9B)\cos^2(c+dx)\sin(c}{15ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A-9B)\cos^2(c+dx)\sin(c}{15ad(a+a\cos(c+dx))}$$

$$= -\frac{(7A-27B)\sin(c+dx)}{15a^3d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} +$$

$$= -\frac{(7A-27B)\sin(c+dx)}{15a^3d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} +$$

$$= \frac{(A-3B)x}{a^3} - \frac{(7A-27B)\sin(c+dx)}{15a^3d} + \frac{(A-B)\cos^3(c+dx)\text{si}}{5d(a+a\cos(c+d}$$

$$= \frac{(A-3B)x}{a^3} - \frac{(7A-27B)\sin(c+dx)}{15a^3d} + \frac{(A-B)\cos^3(c+dx)\text{si}}{5d(a+a\cos(c+d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 361 vs. $2(147) = 294$.
time = 0.98, size = 361, normalized size = 2.46

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(300(A-3B)dx\cos\left(\frac{dx}{2}\right)+300(A-3B)dx\cos\left(c+\frac{dx}{2}\right)+150Adx\cos\left(c+\frac{3dx}{2}\right)-450Bdx\cos\left(c+\frac{3dx}{2}\right)+150Adx\cos\left(2c+\frac{3dx}{2}\right)-450Bdx\cos\left(2c+\frac{3dx}{2}\right)+30Adx\cos\left(2c+\frac{5dx}{2}\right)-90Bdx\cos\left(2c+\frac{5dx}{2}\right)+30Adx\cos\left(3c+\frac{5dx}{2}\right)-90Bdx\cos\left(3c+\frac{5dx}{2}\right)-740A\sin\left(\frac{dx}{2}\right)+1755B\sin\left(\frac{dx}{2}\right)+540A\sin\left(c+\frac{dx}{2}\right)-1125B\sin\left(c+\frac{dx}{2}\right)-460A\sin\left(c+\frac{3dx}{2}\right)+1215B\sin\left(c+\frac{3dx}{2}\right)+180A\sin\left(2c+\frac{3dx}{2}\right)-225B\sin\left(2c+\frac{3dx}{2}\right)-128A\sin\left(2c+\frac{5dx}{2}\right)+363B\sin\left(2c+\frac{5dx}{2}\right)+75B\sin\left(3c+\frac{5dx}{2}\right)+15B\sin\left(3c+\frac{7dx}{2}\right)+15B\sin\left(4c+\frac{7dx}{2}\right)\right)}{120a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(300*(A - 3*B)*d*x*Cos[(d*x)/2] + 300*(A - 3*B)*
d*x*Cos[c + (d*x)/2] + 150*A*d*x*Cos[c + (3*d*x)/2] - 450*B*d*x*Cos[c + (3*
d*x)/2] + 150*A*d*x*Cos[2*c + (3*d*x)/2] - 450*B*d*x*Cos[2*c + (3*d*x)/2] +
 30*A*d*x*Cos[2*c + (5*d*x)/2] - 90*B*d*x*Cos[2*c + (5*d*x)/2] + 30*A*d*x*C
os[3*c + (5*d*x)/2] - 90*B*d*x*Cos[3*c + (5*d*x)/2] - 740*A*Sin[(d*x)/2] +
1755*B*Sin[(d*x)/2] + 540*A*Sin[c + (d*x)/2] - 1125*B*Sin[c + (d*x)/2] - 46
0*A*Sin[c + (3*d*x)/2] + 1215*B*Sin[c + (3*d*x)/2] + 180*A*Sin[2*c + (3*d*x
)/2] - 225*B*Sin[2*c + (3*d*x)/2] - 128*A*Sin[2*c + (5*d*x)/2] + 363*B*Sin[
2*c + (5*d*x)/2] + 75*B*Sin[3*c + (5*d*x)/2] + 15*B*Sin[3*c + (7*d*x)/2] +
15*B*Sin[4*c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]
time = 0.18, size = 134, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B-7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+17B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8B}{1+}}{4d\,a^3}$
default	$\frac{-\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B-7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+17B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8B}{1+}}{4d\,a^3}$
risch	$\frac{xA}{a^3}-\frac{3Bx}{a^3}-\frac{i e^{i(dx+c)}B}{2a^3d}+\frac{i e^{-i(dx+c)}B}{2a^3d}-\frac{2i\left(45A\,e^{4i(dx+c)}-90B\,e^{4i(dx+c)}+135A\,e^{3i(dx+c)}-300B\,e^{3i(dx+c)}+185\right.}{15d\,a^3\left(e^{i}\right.}$
norman	$\frac{\frac{(A-3B)x}{a}+\frac{(A-3B)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{4(A-3B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{6(A-3B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{4(A-3B)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{(A-}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/d/a^3*(-1/5*A*tan(1/2*d*x+1/2*c)^5+1/5*B*tan(1/2*d*x+1/2*c)^5+4/3*tan(1
/2*d*x+1/2*c)^3*A-2*tan(1/2*d*x+1/2*c)^3*B-7*A*tan(1/2*d*x+1/2*c)+17*B*tan(
1/2*d*x+1/2*c)+8*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+8*(A-3*B)*ar
ctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.48, size = 231, normalized size = 1.57

$$\frac{3\,B\left(\frac{40\,\sin\left(dx+c\right)}{\left(a^{3}+\frac{a^{3}\,\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}\right)\left(\cos\left(dx+c\right)+1\right)}+\frac{\frac{85\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{10\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}}{a^{3}}-\frac{120\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a^{3}}\right)-A\left(\frac{\frac{105\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{20\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{3\,\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}}{a^{3}}-\frac{120\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a^{3}}\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*B*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*((105*sin(d*x + c)/(cos(d*x
 + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A]
time = 0.34, size = 165, normalized size = 1.12

$$\frac{15\,(A-3\,B)dx\cos\left(dx+c\right)^{3}+45\,(A-3\,B)dx\cos\left(dx+c\right)^{2}+45\,(A-3\,B)dx\cos\left(dx+c\right)+15\,(A-3\,B)dx+\left(15\,B\cos\left(dx+c\right)^{3}-(32\,A-117\,B)\cos\left(dx+c\right)^{2}-3\,(17\,A-57\,B)\cos\left(dx+c\right)-22\,A+72\,B\right)\sin\left(dx+c\right)}{15\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*(A - 3*B)*d*x*cos(d*x + c)^3 + 45*(A - 3*B)*d*x*cos(d*x + c)^2 + 4
5*(A - 3*B)*d*x*cos(d*x + c) + 15*(A - 3*B)*d*x + (15*B*cos(d*x + c)^3 - (3
2*A - 117*B)*cos(d*x + c)^2 - 3*(17*A - 57*B)*cos(d*x + c) - 22*A + 72*B)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(134) = 268$.
time = 2.72, size = 496, normalized size = 3.37

$$\begin{cases} \frac{60Adx\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}+\frac{60Adx}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}-\frac{3A\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}+\frac{17A\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}-\frac{85A\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}-\frac{105A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}-\frac{180Bdx\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}-\frac{180Bdx}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}+\frac{3B\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}-\frac{27B\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}+\frac{225B\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}+\frac{375B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d} & \text{for}\: d \neq 0 \\ \frac{x\left(A+B\cos\left(c\right)\right)\cos^{3}\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((60*A*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60
*a**3*d) + 60*A*d*x/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3*A*tan(c
/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 17*A*tan(c/2 +
 d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 85*A*tan(c/2 + d*x
```

```
/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 105*A*tan(c/2 + d*x/2)
/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*B*d*x*tan(c/2 + d*x/2)**
2/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*B*d*x/(60*a**3*d*tan(c/
2 + d*x/2)**2 + 60*a**3*d) + 3*B*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d
*x/2)**2 + 60*a**3*d) - 27*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2
)**2 + 60*a**3*d) + 225*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**
2 + 60*a**3*d) + 375*B*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60
*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**3/(a*cos(c) + a)**3, True))
```

Giac [A]
time = 0.43, size = 155, normalized size = 1.05

$$\frac{\frac{60\,(dx+c)(A-3\,B)}{a^3}+\frac{120\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+30\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-255\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*(d*x + c)*(A - 3*B)/a^3 + 120*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x
 + 1/2*c)^2 + 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2
*d*x + 1/2*c)^5 - 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 30*B*a^12*tan(1/2*d*x
+ 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 255*B*a^12*tan(1/2*d*x + 1/2
*c))/a^15)/d
```

Mupad [B]
time = 0.26, size = 152, normalized size = 1.03

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{6\,a^3}+\frac{2\,A-4\,B}{12\,a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,(A-B)}{4\,a^3}-\frac{3\,B}{2\,a^3}+\frac{2\,A-4\,B}{2\,a^3}\right)}{d}+\frac{x\,(A-3\,B)}{a^3}+\frac{2\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A-B)}{20\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((A - B)/(6*a^3) + (2*A - 4*B)/(12*a^3)))/d - (tan(c/
2 + (d*x)/2)*((3*(A - B))/(4*a^3) - (3*B)/(2*a^3) + (2*A - 4*B)/(2*a^3)))/d
 + (x*(A - 3*B))/a^3 + (2*B*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d*x)/2)^
2 + a^3)) - (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d)
```

3.59 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3} dx$

Optimal. Leaf size=116

$$\frac{Bx}{a^3}+\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(2A-7B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{(4A-29B)\sin(c+dx)}{15d\,(a^3+a^3\cos(c+dx))}$$

```
[Out] B*x/a^3+1/5*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-1/15*(2*A-7*
B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+1/15*(4*A-29*B)*sin(d*x+c)/d/(a^3+a^3*
cos(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3056, 3047, 3098, 2814, 2727}

$$\frac{(4A-29B)\sin(c+dx)}{15d\,(a^3\cos(c+dx)+a^3)}+\frac{Bx}{a^3}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(2A-7B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (B*x)/a^3 + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])
^3) - ((2*A - 7*B)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((4*A -
29*B)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx &= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{\cos(c+dx)(2a(A-B)+5aB\cos(c+dx))}{(a+a\cos(c+dx))^2}}{5a^2} \\
&= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{2a(A-B)\cos(c+dx)+5aB\cos^2(c+dx)}{(a+a\cos(c+dx))^2}}{5a^2} \\
&= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(2A-7B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}-\int \\
&= \frac{Bx}{a^3}+\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(2A-7B)\sin(c+dx)}{15ad(a+a\cos(c+dx))} \\
&= \frac{Bx}{a^3}+\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(2A-7B)\sin(c+dx)}{15ad(a+a\cos(c+dx))}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(116) = 232$.
time = 0.60, size = 241, normalized size = 2.08

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(150Bdx\cos\left(\frac{dx}{2}\right)+150Bdx\cos\left(c+\frac{dx}{2}\right)+75Bdx\cos\left(c+\frac{3dx}{2}\right)+75Bdx\cos\left(2c+\frac{3dx}{2}\right)+15Bdx\cos\left(2c+\frac{5dx}{2}\right)+15Bdx\cos\left(3c+\frac{5dx}{2}\right)+80A\sin\left(\frac{dx}{2}\right)-370B\sin\left(\frac{dx}{2}\right)-60A\sin\left(c+\frac{dx}{2}\right)+270B\sin\left(c+\frac{dx}{2}\right)+40A\sin\left(c+\frac{3dx}{2}\right)-230B\sin\left(c+\frac{3dx}{2}\right)-30A\sin\left(2c+\frac{3dx}{2}\right)+90B\sin\left(2c+\frac{3dx}{2}\right)+14A\sin\left(2c+\frac{5dx}{2}\right)-64B\sin\left(2c+\frac{5dx}{2}\right)\right)}{480a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(150*B*d*x*Cos[(d*x)/2] + 150*B*d*x*Cos[c + (d*x)/2] + 75*B*d*x*Cos[c + (3*d*x)/2] + 75*B*d*x*Cos[2*c + (3*d*x)/2] + 15*B*d*x*Cos[2*c + (5*d*x)/2] + 15*B*d*x*Cos[3*c + (5*d*x)/2] + 80*A*Sin[(d*x)/2] - 370*B*Sin[(d*x)/2] - 60*A*Sin[c + (d*x)/2] + 270*B*Sin[c + (d*x)/2] + 40*A*Sin[c + (3*d*x)/2] - 230*B*Sin[c + (3*d*x)/2] - 30*A*Sin[2*c + (3*d*x)/2] + 90*B*Sin[2*c + (3*d*x)/2] + 14*A*Sin[2*c + (5*d*x)/2] - 64*B*Sin[2*c + (5*d*x)/2]))/(480*a^3*d)

Maple [A]
time = 0.15, size = 102, normalized size = 0.88

method	result
derivativedivides	$\frac{\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-7B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8B\,\mathrm{arc}}{4d\,a^3}$
default	$\frac{\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-7B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8B\,\mathrm{arc}}{4d\,a^3}$
risch	$\frac{Bx}{a^3}+\frac{2i\left(15A\,e^{4i(dx+c)}-45B\,e^{4i(dx+c)}+30A\,e^{3i(dx+c)}-135B\,e^{3i(dx+c)}+40A\,e^{2i(dx+c)}-185B\,e^{2i(dx+c)}+20A\,e^{i(dx+c}}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5}$
norman	$\frac{\frac{Bx}{a}+\frac{Bx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{3Bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{3Bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{(A-11B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60da}+\frac{(A-7B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{(A-}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/4/d/a^3*(1/5*A*tan(1/2*d*x+1/2*c)^5-1/5*B*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3*A+4/3*tan(1/2*d*x+1/2*c)^3*B+A*tan(1/2*d*x+1/2*c)-7*B*tan(1/2*d*x+1/2*c)+8*B*arctan(tan(1/2*d*x+1/2*c)))

Maxima [A]
time = 0.48, size = 160, normalized size = 1.38

$$-\frac{B\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-\frac{A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] -1/60*(B*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d

Fricas [A]
time = 0.34, size = 137, normalized size = 1.18

$$\frac{15\,Bdx\cos\left(dx+c\right)^3+45\,Bdx\cos\left(dx+c\right)^2+45\,Bdx\cos\left(dx+c\right)+15\,Bdx+\left(\left(7\,A-32\,B\right)\cos\left(dx+c\right)^2+3\left(2\,A-17\,B\right)\cos\left(dx+c\right)+2\,A-22\,B\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*B*d*x*cos(d*x + c)^3 + 45*B*d*x*cos(d*x + c)^2 + 45*B*d*x*cos(d*x
+ c) + 15*B*d*x + ((7*A - 32*B)*cos(d*x + c)^2 + 3*(2*A - 17*B)*cos(d*x + c
) + 2*A - 22*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A]
time = 1.58, size = 148, normalized size = 1.28

$$\begin{cases} \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}-\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d}+\frac{Bx}{a^3}-\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d}-\frac{7B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for}\: d\neq 0 \\ \frac{x(A+B\cos\left(c\right))\cos^2\left(c\right)}{\left(a\cos\left(c\right)+a\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**5/(20*a**3*d) - A*tan(c/2 + d*x/2)**3/(6*a**
3*d) + A*tan(c/2 + d*x/2)/(4*a**3*d) + B*x/a**3 - B*tan(c/2 + d*x/2)**5/(20
*a**3*d) + B*tan(c/2 + d*x/2)**3/(3*a**3*d) - 7*B*tan(c/2 + d*x/2)/(4*a**3*
d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**2/(a*cos(c) + a)**3, True))
```

Giac [A]
time = 0.48, size = 120, normalized size = 1.03

$$\frac{\frac{60\,(dx+c)B}{a^3}+\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-105\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*(d*x + c)*B/a^3 + (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(
1/2*d*x + 1/2*c)^5 - 10*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 20*B*a^12*tan(1/2*d
*x + 1/2*c)^3 + 15*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x + 1
/2*c))/a^15)/d
```

Mupad [B]
time = 0.38, size = 134, normalized size = 1.16

$$\frac{B\,x}{a^3}-\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}-\frac{B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}\right)-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{7\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)-\frac{A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20}+\frac{B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20}}{a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (B*x)/a^3 - (cos(c/2 + (d*x)/2)^2*((A*sin(c/2 + (d*x)/2)^3)/6 - (B*sin(c/2
+ (d*x)/2)^3)/3) - cos(c/2 + (d*x)/2)^4*((A*sin(c/2 + (d*x)/2))/4 - (7*B*si
n(c/2 + (d*x)/2))/4) - (A*sin(c/2 + (d*x)/2)^5)/20 + (B*sin(c/2 + (d*x)/2)^
5)/20)/(a^3*d*cos(c/2 + (d*x)/2)^5)
```

3.60 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$-\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(3A-8B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{(3A+7B)\sin(c+dx)}{15d\,(a^3+a^3\cos(c+dx))}$$

```
[Out] -1/5*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(3*A-8*B)*sin(d*x+c)/a/d/(a
+a*cos(d*x+c))^2+1/15*(3*A+7*B)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3047, 3098, 2829, 2727}

$$\frac{(3A+7B)\sin(c+dx)}{15d\,(a^3\cos(c+dx)+a^3)}+\frac{(3A-8B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -1/5*((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) + ((3*A - 8*B)*Sin[c
 + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((3*A + 7*B)*Sin[c + d*x])/(15*d
*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx &= \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx\\ &= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{\int \frac{-3a(A-B)-5aB\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}\\ &= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(3A-8B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{(3A+7B}{}\\ &= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(3A-8B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{(3A+7I}{15d\,(a^3+}\end{aligned}$$

Mathematica [A]

time = 0.37, size = 135, normalized size = 1.32

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(5(3A+8B)\sin\left(\frac{dx}{2}\right)-15(A+2B)\sin\left(c+\frac{dx}{2}\right)+15A\sin\left(c+\frac{3dx}{2}\right)+20B\sin\left(c+\frac{3dx}{2}\right)-15B\sin\left(2c+\frac{3dx}{2}\right)+3A\sin\left(2c+\frac{5dx}{2}\right)+7B\sin\left(2c+\frac{5dx}{2}\right)\right)}{30a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(3*A + 8*B)*Sin[(d*x)/2] - 15*(A + 2*B)*Sin[c
 + (d*x)/2] + 15*A*Sin[c + (3*d*x)/2] + 20*B*Sin[c + (3*d*x)/2] - 15*B*Sin[
2*c + (3*d*x)/2] + 3*A*Sin[2*c + (5*d*x)/2] + 7*B*Sin[2*c + (5*d*x)/2]))/(3
0*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]

time = 0.17, size = 64, normalized size = 0.63

method	result
derivativedivides	$\frac{\frac{(-A+B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$
default	$\frac{\frac{(-A+B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$

risch	$\frac{2i(15B\,e^{4i(dx+c)}+15A\,e^{3i(dx+c)}+30B\,e^{3i(dx+c)}+15A\,e^{2i(dx+c)}+40B\,e^{2i(dx+c)}+15A\,e^{i(dx+c)}+20B\,e^{i(dx+c)}+3A+7B)}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5}$
norman	$\frac{-\frac{(A-B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}+\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{(3A+2B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{(3A+2B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30da}+\frac{(6A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(1/5*(-A+B)*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3*B+A*tan
(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 115, normalized size = 1.13

$$\frac{\frac{B\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3\,A\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(B*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*A*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A]
time = 0.34, size = 93, normalized size = 0.91

$$\frac{\left((3\,A+7\,B)\cos\left(dx+c\right)^2+3\,(3\,A+2\,B)\cos\left(dx+c\right)+3\,A+2\,B\right)\sin\left(dx+c\right)}{15\,\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*((3*A + 7*B)*cos(d*x + c)^2 + 3*(3*A + 2*B)*cos(d*x + c) + 3*A + 2*B)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [A]
time = 1.17, size = 117, normalized size = 1.15

$$\begin{cases} -\frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d}+\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}-\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d\neq 0 \\ \frac{x(A+B\cos(c))\cos(c)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**5/(20*a**3*d) + A*tan(c/2 + d*x/2)/(4*a**3*
d) + B*tan(c/2 + d*x/2)**5/(20*a**3*d) - B*tan(c/2 + d*x/2)**3/(6*a**3*d) +
 B*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)/(a*cos(
c) + a)**3, True))
```

Giac [A]
time = 0.45, size = 75, normalized size = 0.74

$$-\frac{3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+10\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] -1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 10*B*tan(1
/2*d*x + 1/2*c)^3 - 15*A*tan(1/2*d*x + 1/2*c) - 15*B*tan(1/2*d*x + 1/2*c))/
(a^3*d)
```

Mupad [B]
time = 0.21, size = 66, normalized size = 0.65

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(15\,A+15\,B-3\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-10\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}{60\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(15*A + 15*B - 3*A*tan(c/2 + (d*x)/2)^4 - 10*B*tan(c/2
+ (d*x)/2)^2 + 3*B*tan(c/2 + (d*x)/2)^4))/(60*a^3*d)
```

3.61 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(2A+3B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{(2A+3B)\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}$$

```
[Out] 1/5*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(2*A+3*B)*sin(d*x+c)/a/d/(a+
a*cos(d*x+c))^2+1/15*(2*A+3*B)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2829, 2729, 2727}

$$\frac{(2A+3B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(2A+3B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((A - B)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((2*A + 3*B)*Sin[c +
d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((2*A + 3*B)*Sin[c + d*x])/(15*d*(a
^3 + a^3*Cos[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(2A+3B)\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{5a}$$
$$= \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(2A+3B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(2A+3B)\int \frac{1}{a+a\cos(c+dx}}{15a^2}$$
$$= \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(2A+3B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(2A+3B)\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx\right.}$$

Mathematica [A]
time = 0.29, size = 96, normalized size = 0.94

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(5(4A+3B)\sin\left(\frac{dx}{2}\right)-15B\sin\left(c+\frac{dx}{2}\right)+(2A+3B)\left(5\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)\right)\right)}{30a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(4*A + 3*B)*Sin[(d*x)/2] - 15*B*Sin[c + (d*x)
/2] + (2*A + 3*B)*(5*Sin[c + (3*d*x)/2] + Sin[2*c + (5*d*x)/2])))/(30*a^3*d
*(1 + Cos[c + d*x])^3)
```

Maple [A]
time = 0.14, size = 64, normalized size = 0.63

method	result	size
derivativedivides	$\frac{\frac{(A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$	64
default	$\frac{\frac{(A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$	64
risch	$\frac{2i\left(15B\,e^{3i(dx+c)}+20A\,e^{2i(dx+c)}+15B\,e^{2i(dx+c)}+10A\,e^{i(dx+c)}+15B\,e^{i(dx+c)}+2A+3B\right)}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5}$	90
norman	$\frac{\frac{(A-B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}+\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{(5A+3B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}+\frac{(13A-3B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(1/5*(A-B)*tan(1/2*d*x+1/2*c)^5+2/3*tan(1/2*d*x+1/2*c)^3*A+A*tan(
1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 115, normalized size = 1.13

$$\frac{\frac{A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3B\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(A*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*B*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A]
time = 0.34, size = 93, normalized size = 0.91

$$\frac{\left((2A+3B)\cos(dx+c)^2+3(2A+3B)\cos(dx+c)+7A+3B\right)\sin(dx+c)}{15\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*((2*A + 3*B)*cos(d*x + c)^2 + 3*(2*A + 3*B)*cos(d*x + c) + 7*A + 3*B)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [A]
time = 0.89, size = 114, normalized size = 1.12

$$\begin{cases} \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d}-\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d\neq 0 \\ \frac{x(A+B\cos(c))}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**5/(20*a**3*d) + A*tan(c/2 + d*x/2)**3/(6*a**
3*d) + A*tan(c/2 + d*x/2)/(4*a**3*d) - B*tan(c/2 + d*x/2)**5/(20*a**3*d) +
B*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))/(a*cos(c) + a)*
*3, True))
```

Giac [A]
time = 0.43, size = 75, normalized size = 0.74

$$\frac{3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+10A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+15B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 10*A*tan(1/
2*d*x + 1/2*c)^3 + 15*A*tan(1/2*d*x + 1/2*c) + 15*B*tan(1/2*d*x + 1/2*c))/(
a^3*d)
```

Mupad [B]
time = 0.20, size = 66, normalized size = 0.65

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(15\,A+15\,B+10\,A\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\,A\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-3\,B\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}{60\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(15*A + 15*B + 10*A*tan(c/2 + (d*x)/2)^2 + 3*A*tan(c/2
+ (d*x)/2)^4 - 3*B*tan(c/2 + (d*x)/2)^4))/(60*a^3*d)
```

3.62 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3} dx$

Optimal. Leaf size=117

$$\frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(7A-2B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}-\frac{2(11A-B)\sin(c+dx)}{15d\,(a^3+a^3\cos(c+dx))}$$

```
[Out] A*arctanh(sin(d*x+c))/a^3/d-1/5*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-1/15*
(7*A-2*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2-2/15*(11*A-B)*sin(d*x+c)/d/(a^3
+a^3*cos(d*x+c))
```

Rubi [A]
time = 0.20, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3057, 12, 3855}

$$-\frac{2(11A-B)\sin(c+dx)}{15d\,(a^3\cos(c+dx)+a^3)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(7A-2B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((A - B)*Sin[c + d*x])/(5*d*(a + a*Cos[
c + d*x])^3) - ((7*A - 2*B)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) -
 (2*(11*A - B)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(5aA-2a(A-B)\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-2B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{(15a^2A-a}{}}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-2B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{2(11A-}{15d\,(a^3+}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-2B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{2(11A-}{15d\,(a^3+}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-2B)\sin}{15ad(a+a\cos}$$

Mathematica [A]

time = 0.99, size = 197, normalized size = 1.68

$$\frac{-240A\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-5(29A-4B)\sin\left(\frac{dx}{2}\right)+75A\sin\left(c+\frac{dx}{2}\right)-95A\sin\left(c+\frac{3dx}{2}\right)+10B\sin\left(c+\frac{3dx}{2}\right)+15A\sin\left(2c+\frac{3dx}{2}\right)-22A\sin\left(2c+\frac{5dx}{2}\right)+2B\sin\left(2c+\frac{5dx}{2}\right)\right)}{30a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-240*A*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-5*(29*A
 - 4*B)*Sin[(d*x)/2] + 75*A*Sin[c + (d*x)/2] - 95*A*Sin[c + (3*d*x)/2] + 10
*B*Sin[c + (3*d*x)/2] + 15*A*Sin[2*c + (3*d*x)/2] - 22*A*Sin[2*c + (5*d*x)/
2] + 2*B*Sin[2*c + (5*d*x)/2]))/(30*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]

time = 0.21, size = 119, normalized size = 1.02

method	result
derivativedivides	$\frac{-\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+4A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-4A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+\frac{2\left(\tan^3\right.}{}}{4d\,a^3}$
default	$\frac{-\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+4A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-4A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+\frac{2\left(\tan^3\right.}{}}{4d\,a^3}$
risch	$-\frac{2i\left(15A\,e^{4i(dx+c)}+75A\,e^{3i(dx+c)}+145A\,e^{2i(dx+c)}-20B\,e^{2i(dx+c)}+95A\,e^{i(dx+c)}-10B\,e^{i(dx+c)}+22A-2B\right)}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5}+\frac{A\ln(e}{}$
norman	$\frac{-\frac{(A-B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}-\frac{5(5A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}-\frac{(7A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}-\frac{(23A-13B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(-1/5*A*tan(1/2*d*x+1/2*c)^5+1/5*B*tan(1/2*d*x+1/2*c)^5+4*A*ln(ta
n(1/2*d*x+1/2*c)+1)-4*A*ln(tan(1/2*d*x+1/2*c)-1)-4/3*tan(1/2*d*x+1/2*c)^3*A
+2/3*tan(1/2*d*x+1/2*c)^3*B-7*A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 187, normalized size = 1.60

$$-\frac{A\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-\frac{B\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] -1/60*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^3) - B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A]
time = 0.36, size = 185, normalized size = 1.58

$$\frac{15\,(A\cos(dx+c)^3+3\,A\cos(dx+c)^2+3\,A\cos(dx+c)+A)\log(\sin(dx+c)+1)-15\,(A\cos(dx+c)^3+3\,A\cos(dx+c)^2+3\,A\cos(dx+c)+A)\log(-\sin(dx+c)+1)-2\,(2\,(11\,A-B)\cos(dx+c)^2+3\,(17\,A-2\,B)\cos(dx+c)+32\,A-7\,B)\sin(dx+c)}{30\,(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/30*(15*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*log
(sin(d*x + c) + 1) - 15*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*
x + c) + A)*log(-sin(d*x + c) + 1) - 2*(2*(11*A - B)*cos(d*x + c)^2 + 3*(17
*A - 2*B)*cos(d*x + c) + 32*A - 7*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d
*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)/(cos(c + d*x)**3 + 3*co
s(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.46, size = 148, normalized size = 1.26

$$\frac{\frac{60\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-10\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] 1/60*(60*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*A*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2*
d*x + 1/2*c)^5 + 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 10*B*a^12*tan(1/2*d*x +
 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 15*B*a^12*tan(1/2*d*x + 1/2*c
))/a^15)/d
```

Mupad [B]
time = 0.25, size = 130, normalized size = 1.11

$$\frac{2\,A\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{4\,a^3}+\frac{3\,A+B}{4\,a^3}+\frac{3\,A-B}{4\,a^3}\right)}{d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(A-B\right)}{20\,a^3\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{A-B}{12\,a^3}+\frac{3\,A-B}{12\,a^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] (2*A*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - (tan(c/2 + (d*x)/2)*((A - B)/(4*a
^3) + (3*A + B)/(4*a^3) + (3*A - B)/(4*a^3)))/d - (tan(c/2 + (d*x)/2)^5*(A
- B))/(20*a^3*d) - (tan(c/2 + (d*x)/2)^3*((A - B)/(12*a^3) + (3*A - B)/(12*
a^3)))/d
```

3.63 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$-\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{2(36A-11B)\tan(c+dx)}{15a^3d}-\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(9A-4B)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

```
[Out] -(3*A-B)*arctanh(sin(d*x+c))/a^3/d+2/15*(36*A-11*B)*tan(d*x+c)/a^3/d-1/5*(A
-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^3-1/15*(9*A-4*B)*tan(d*x+c)/a/d/(a+a*cos(
d*x+c))^2-(3*A-B)*tan(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A]

time = 0.31, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3057, 2827, 3852, 8, 3855}

$$\frac{2(36A-11B)\tan(c+dx)}{15a^3d}-\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(3A-B)\tan(c+dx)}{d\,(a^3\cos(c+dx)+a^3)}-\frac{(9A-4B)\tan(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -(((3*A - B)*ArcTanh[Sin[c + d*x]])/(a^3*d)) + (2*(36*A - 11*B)*Tan[c + d*x
])/(15*a^3*d) - ((A - B)*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((9*A
 - 4*B)*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((3*A - B)*Tan[c +
d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(a(6A-B)-3a(A-B)\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,d}{5a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-4B)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{(a^2(27A}{}}{}$$

$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-4B)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(3A-B}{d\,(a^3+a}$$

$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-4B)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(3A-B}{d\,(a^3+a}$$

$$= -\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9}{15a}$$

$$= -\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{2(36A-11B)\tan(c+dx)}{15a^3d} -$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 482 vs. 2(145) = 290.

time = 3.18, size = 482, normalized size = 3.32

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (960*(3*A - B)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Se
c[c]*Sec[c + d*x]*(-5*(51*A - 32*B)*Sin[(d*x)/2] + (567*A - 167*B)*Sin[(3*d
```

```
*x)/2] - 600*A*Sin[c - (d*x)/2] + 170*B*Sin[c - (d*x)/2] + 375*A*Sin[c + (d
*x)/2] - 170*B*Sin[c + (d*x)/2] - 480*A*Sin[2*c + (d*x)/2] + 160*B*Sin[2*c
+ (d*x)/2] - 60*A*Sin[c + (3*d*x)/2] + 75*B*Sin[c + (3*d*x)/2] + 402*A*Sin[
2*c + (3*d*x)/2] - 167*B*Sin[2*c + (3*d*x)/2] - 225*A*Sin[3*c + (3*d*x)/2]
+ 75*B*Sin[3*c + (3*d*x)/2] + 315*A*Sin[c + (5*d*x)/2] - 95*B*Sin[c + (5*d*
x)/2] + 30*A*Sin[2*c + (5*d*x)/2] + 15*B*Sin[2*c + (5*d*x)/2] + 240*A*Sin[3
*c + (5*d*x)/2] - 95*B*Sin[3*c + (5*d*x)/2] - 45*A*Sin[4*c + (5*d*x)/2] + 1
5*B*Sin[4*c + (5*d*x)/2] + 72*A*Sin[2*c + (7*d*x)/2] - 22*B*Sin[2*c + (7*d*
x)/2] + 15*A*Sin[3*c + (7*d*x)/2] + 57*A*Sin[4*c + (7*d*x)/2] - 22*B*Sin[4*
c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]
time = 0.24, size = 162, normalized size = 1.12

method	result
derivativedivides	$\frac{(-12A+4B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{4A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A-\frac{4\left(\tan^3\left(\frac{dx}{2} \cdots\right.\right.}{3} \cdots}{4d\,a^3}$
default	$\frac{(-12A+4B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{4A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A-\frac{4\left(\tan^3\left(\frac{dx}{2} \cdots\right.\right.}{3} \cdots}{4d\,a^3}$
norman	$\frac{\frac{(A-B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}+\frac{(3A-2B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{(15A-2B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{(25A-7B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{(42A-17B)\left(\tan^5\left(\frac{dx}{2}+ \cdots\right.\right.}{10da} \cdots}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^2}$
risch	$\frac{2i\left(45A\,e^{6i(dx+c)}-15B\,e^{6i(dx+c)}+225A\,e^{5i(dx+c)}-75B\,e^{5i(dx+c)}+480A\,e^{4i(dx+c)}-160B\,e^{4i(dx+c)}+600A\,e^{3i(dx+c)}-170 \cdots\right.}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5\left(e^{2i(dx+c)}+1 \cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/d/a^3*((-12*A+4*B)*ln(tan(1/2*d*x+1/2*c)+1)-4*A/(tan(1/2*d*x+1/2*c)+1)+
1/5*A*tan(1/2*d*x+1/2*c)^5-1/5*B*tan(1/2*d*x+1/2*c)^5+2*tan(1/2*d*x+1/2*c)^
3*A-4/3*tan(1/2*d*x+1/2*c)^3*B+17*A*tan(1/2*d*x+1/2*c)-7*B*tan(1/2*d*x+1/2*
c)+(12*A-4*B)*ln(tan(1/2*d*x+1/2*c)-1)-4*A/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(139) = 278.
time = 0.27, size = 286, normalized size = 1.97

$$\frac{3A\left(\frac{40\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-B\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*A*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^3) - B*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 6
0*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^3))/d
```

Fricas [A]
time = 0.35, size = 272, normalized size = 1.88

$$-\frac{15\left((3A-B)\cos(dx+c)^4+3(3A-B)\cos(dx+c)^3+3(3A-B)\cos(dx+c)^2+(3A-B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-15\left((3A-B)\cos(dx+c)^4+3(3A-B)\cos(dx+c)^3+3(3A-B)\cos(dx+c)^2+(3A-B)\cos(dx+c)\right)\log(-\sin(dx+c)+1)-2\left(2(36A-11B)\cos(dx+c)^3+3(57A-17B)\cos(dx+c)^2+(117A-32B)\cos(dx+c)+15A\right)\sin(dx+c)}{30\left(a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+a^3d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*((3*A - B)*cos(d*x + c)^4 + 3*(3*A - B)*cos(d*x + c)^3 + 3*(3*A -
 B)*cos(d*x + c)^2 + (3*A - B)*cos(d*x + c))*log(sin(d*x + c) + 1) - 15*((3
*A - B)*cos(d*x + c)^4 + 3*(3*A - B)*cos(d*x + c)^3 + 3*(3*A - B)*cos(d*x +
 c)^2 + (3*A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(36*A - 11*B)
*cos(d*x + c)^3 + 3*(57*A - 17*B)*cos(d*x + c)^2 + (117*A - 32*B)*cos(d*x +
 c) + 15*A)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^2(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^2(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c
+ d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)**2/(cos(c + d*x)**3
+ 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.47, size = 190, normalized size = 1.31

$$-\frac{\frac{60(3A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{60(3A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{120A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^3}-\frac{3Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+30Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-20Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+255Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-105Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(60*(3*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*(3*A - B)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 120*A*tan(1/2*d*x + 1/2*c)/((tan(1/
2*d*x + 1/2*c)^2 - 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*ta
n(1/2*d*x + 1/2*c)^5 + 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 20*B*a^12*tan(1/2
*d*x + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x
+ 1/2*c))/a^15)/d
```

Mupad [B]
time = 0.28, size = 168, normalized size = 1.16

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{6\,a^3}+\frac{4\,A-2\,B}{12\,a^3}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,A}{2\,a^3}+\frac{3\,(A-B)}{4\,a^3}+\frac{4\,A-2\,B}{2\,a^3}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A-B)}{20\,a^3\,d}-\frac{2\,A\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^3\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(3\,A-B)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((A - B)/(6*a^3) + (4*A - 2*B)/(12*a^3)))/d + (tan(c/
2 + (d*x)/2)*((3*A)/(2*a^3) + (3*(A - B))/(4*a^3) + (4*A - 2*B)/(2*a^3)))/d
 + (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d) - (2*A*tan(c/2 + (d*x)/2))/(d*
(a^3*tan(c/2 + (d*x)/2)^2 - a^3)) - (2*atanh(tan(c/2 + (d*x)/2))*(3*A - B))
/(a^3*d)
```

3.64 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=196

$$\frac{(13A-6B)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{8(19A-9B)\tan(c+dx)}{15a^3d}+\frac{(13A-6B)\sec(c+dx)\tan(c+dx)}{2a^3d}-\frac{(A-}{5}$$

```
[Out] 1/2*(13*A-6*B)*arctanh(sin(d*x+c))/a^3/d-8/15*(19*A-9*B)*tan(d*x+c)/a^3/d+1
/2*(13*A-6*B)*sec(d*x+c)*tan(d*x+c)/a^3/d-1/5*(A-B)*sec(d*x+c)*tan(d*x+c)/d
/(a+a*cos(d*x+c))^3-1/15*(11*A-6*B)*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+
c))^2-4/15*(19*A-9*B)*sec(d*x+c)*tan(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A]

time = 0.33, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3057, 2827, 3853, 3855, 3852, 8}

$$-\frac{8(19A-9B)\tan(c+dx)}{15a^3d}+\frac{(13A-6B)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{(13A-6B)\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{4(19A-9B)\tan(c+dx)\sec(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(11A-6B)\tan(c+dx)\sec(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((13*A - 6*B)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (8*(19*A - 9*B)*Tan[c + d*
x])/(15*a^3*d) + ((13*A - 6*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((A -
 B)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((11*A - 6*B)
*Sec[c + d*x]*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (4*(19*A - 9*
B)*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(a(7A-2B)-4a(A-B)\cos(c+dx))\,\mathrm{s}}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(11A-6B)\sec(c+dx)\tan(}{15ad(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(11A-6B)\sec(c+dx)\tan(}{15ad(a+a\cos(c+dx)}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(11A-6B)\sec(c+dx)\tan(}{15ad(a+a\cos(c+dx)}$$

$$= \frac{(13A-6B)\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{(A-B)\sec(c+dx)\tan(c}{5d(a+a\cos(c+dx))^3}$$

$$= \frac{(13A-6B)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{8(19A-9B)\tan(c+dx)}{15a^3d} + \frac{(1}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 610 vs. 2(196) = 392.
time = 4.99, size = 610, normalized size = 3.11

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -1/480*(1920*(13*A - 6*B)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*
Sec[c/2]*Sec[c]*Sec[c + d*x]^2*((-1235*A + 870*B)*Sin[(d*x)/2] + 5*(761*A -
 366*B)*Sin[(3*d*x)/2] - 4329*A*Sin[c - (d*x)/2] + 2094*B*Sin[c - (d*x)/2]
+ 1989*A*Sin[c + (d*x)/2] - 1314*B*Sin[c + (d*x)/2] - 3575*A*Sin[2*c + (d*x
)/2] + 1650*B*Sin[2*c + (d*x)/2] - 475*A*Sin[c + (3*d*x)/2] + 450*B*Sin[c +
 (3*d*x)/2] + 2005*A*Sin[2*c + (3*d*x)/2] - 1230*B*Sin[2*c + (3*d*x)/2] - 2
275*A*Sin[3*c + (3*d*x)/2] + 1050*B*Sin[3*c + (3*d*x)/2] + 2673*A*Sin[c + (
5*d*x)/2] - 1278*B*Sin[c + (5*d*x)/2] + 105*A*Sin[2*c + (5*d*x)/2] + 90*B*S
in[2*c + (5*d*x)/2] + 1593*A*Sin[3*c + (5*d*x)/2] - 918*B*Sin[3*c + (5*d*x)
/2] - 975*A*Sin[4*c + (5*d*x)/2] + 450*B*Sin[4*c + (5*d*x)/2] + 1325*A*Sin[
2*c + (7*d*x)/2] - 630*B*Sin[2*c + (7*d*x)/2] + 255*A*Sin[3*c + (7*d*x)/2]
- 60*B*Sin[3*c + (7*d*x)/2] + 875*A*Sin[4*c + (7*d*x)/2] - 480*B*Sin[4*c +
(7*d*x)/2] - 195*A*Sin[5*c + (7*d*x)/2] + 90*B*Sin[5*c + (7*d*x)/2] + 304*A
*Sin[3*c + (9*d*x)/2] - 144*B*Sin[3*c + (9*d*x)/2] + 90*A*Sin[4*c + (9*d*x)
/2] - 30*B*Sin[4*c + (9*d*x)/2] + 214*A*Sin[5*c + (9*d*x)/2] - 114*B*Sin[5*
c + (9*d*x)/2]))/(a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]

time = 0.28, size = 206, normalized size = 1.05

method	result
derivativedivides	$\frac{-\frac{-14A+4B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+(26A-12B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2A}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{8\left(\tan^3\left(\right.\right.}{}}{}$
default	$\frac{-\frac{-14A+4B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+(26A-12B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2A}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{8\left(\tan^3\left(\right.\right.}{}}{}$
norman	$\frac{-\frac{(A-B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}-\frac{(37A-27B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60da}-\frac{(51A-25B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{(109A-45B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}-\frac{(211A-111B}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2a^2}$
risch	$-\frac{i\left(195A\,e^{8i(dx+c)}-90B\,e^{8i(dx+c)}+975A\,e^{7i(dx+c)}-450B\,e^{7i(dx+c)}+2275A\,e^{6i(dx+c)}-1050B\,e^{6i(dx+c)}+3575A\,e^{5i(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/d/a^3*(-(-14*A+4*B)/(tan(1/2*d*x+1/2*c)+1)+(26*A-12*B)*ln(tan(1/2*d*x+1
/2*c)+1)-2*A/(tan(1/2*d*x+1/2*c)+1)^2-1/5*A*tan(1/2*d*x+1/2*c)^5+1/5*B*tan(
1/2*d*x+1/2*c)^5-8/3*tan(1/2*d*x+1/2*c)^3*A+2*tan(1/2*d*x+1/2*c)^3*B-31*A*t
an(1/2*d*x+1/2*c)+17*B*tan(1/2*d*x+1/2*c)+(-26*A+12*B)*ln(tan(1/2*d*x+1/2*c
)-1)-(-14*A+4*B)/(tan(1/2*d*x+1/2*c)-1)+2*A/(tan(1/2*d*x+1/2*c)-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(184) = 368$.
time = 0.27, size = 377, normalized size = 1.92

$$-\frac{A\left(\frac{60\left(\frac{5\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{7\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}\right)}{a^3-\frac{2a^3\sin\left(dx+c\right)^2}{\left(\cos\left(dx+c\right)+1\right)^2}+\frac{a^3\sin\left(dx+c\right)^4}{\left(\cos\left(dx+c\right)+1\right)^4}}+\frac{\frac{465\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{40\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}+\frac{3\sin\left(dx+c\right)^5}{\left(\cos\left(dx+c\right)+1\right)^5}}{a^3}-\frac{390\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-1\right)}{a^3}\right)-3B\left(\frac{40\sin\left(dx+c\right)}{\left(a^3-\frac{a^3\sin\left(dx+c\right)^2}{\left(\cos\left(dx+c\right)+1\right)^2}\right)\left(\cos\left(dx+c\right)+1\right)}+\frac{\frac{85\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{10\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}+\frac{\sin\left(dx+c\right)^5}{\left(\cos\left(dx+c\right)+1\right)^5}}{a^3}-\frac{60\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-1\right)}{a^3}\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(A*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3) - 3*B*(40*sin(d*x + c)/((a^3 - a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(
d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(co
s(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 +
60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3))/d
```

Fricas [A]
time = 0.38, size = 295, normalized size = 1.51

$$\frac{15\left(\left(13A-6B\right)\cos\left(dx+c\right)^5+3\left(13A-6B\right)\cos\left(dx+c\right)^4+3\left(13A-6B\right)\cos\left(dx+c\right)^3+\left(13A-6B\right)\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\left(13A-6B\right)\cos\left(dx+c\right)^5+3\left(13A-6B\right)\cos\left(dx+c\right)^4+3\left(13A-6B\right)\cos\left(dx+c\right)^3+\left(13A-6B\right)\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(16\left(19A-9B\right)\cos\left(dx+c\right)^4+3\left(239A-114B\right)\cos\left(dx+c\right)^3+\left(479A-234B\right)\cos\left(dx+c\right)^2+15\left(3A-2B\right)\cos\left(dx+c\right)-15A\right)\sin\left(dx+c\right)}{60\left(a^3d\cos\left(dx+c\right)^5+3a^3d\cos\left(dx+c\right)^4+3a^3d\cos\left(dx+c\right)^3+a^3d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*((13*A - 6*B)*cos(d*x + c)^5 + 3*(13*A - 6*B)*cos(d*x + c)^4 + 3*(
13*A - 6*B)*cos(d*x + c)^3 + (13*A - 6*B)*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1) - 15*((13*A - 6*B)*cos(d*x + c)^5 + 3*(13*A - 6*B)*cos(d*x + c)^4 + 3*
(13*A - 6*B)*cos(d*x + c)^3 + (13*A - 6*B)*cos(d*x + c)^2)*log(-sin(d*x + c
) + 1) - 2*(16*(19*A - 9*B)*cos(d*x + c)^4 + 3*(239*A - 114*B)*cos(d*x + c)
^3 + (479*A - 234*B)*cos(d*x + c)^2 + 15*(3*A - 2*B)*cos(d*x + c) - 15*A)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d
*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^3\left(c+dx\right)}{\cos^3\left(c+dx\right)+3\cos^2\left(c+dx\right)+3\cos\left(c+dx\right)+1}\,dx+\int \frac{B\cos\left(c+dx\right)\sec^3\left(c+dx\right)}{\cos^3\left(c+dx\right)+3\cos^2\left(c+dx\right)+3\cos\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c
+ d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)**3/(cos(c + d*x)**3
+ 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.49, size = 233, normalized size = 1.19

$$\frac{\frac{30\,(13\,A-6\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{30\,(13\,A-6\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{60\left(7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+40\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-30\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+465\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-255\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(30*(13*A - 6*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(13*A - 6
*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 60*(7*A*tan(1/2*d*x + 1/2*c)^3
 - 2*B*tan(1/2*d*x + 1/2*c)^3 - 5*A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x
+ 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1/
2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^12*tan(1/2*d*x + 1/2*c)^3
 - 30*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2*d*x + 1/2*c) - 255
*B*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 0.28, size = 216, normalized size = 1.10

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(7\,A-2\,B)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(5\,A-2\,B)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,(A-B)}{2\,a^3}+\frac{3\,(5\,A-3\,B)}{4\,a^3}+\frac{10\,A-2\,B}{4\,a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{4\,a^3}+\frac{5\,A-3\,B}{12\,a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A-B)}{20\,a^3\,d}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(13\,A-6\,B)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(7*A - 2*B) - tan(c/2 + (d*x)/2)*(5*A - 2*B))/(d*(a^3
*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2)^2 + a^3)) - (tan(c/2 + (d*
x)/2)*((3*(A - B))/(2*a^3) + (3*(5*A - 3*B))/(4*a^3) + (10*A - 2*B)/(4*a^3)
))/d - (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^3) + (5*A - 3*B)/(12*a^3)))/d -
(tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d) + (atanh(tan(c/2 + (d*x)/2))*(13*
A - 6*B))/(a^3*d)
```

3.65 $\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=229

$$-\frac{(8A-21B)x}{2a^4}+\frac{8(83A-216B)\sin(c+dx)}{105a^4d}-\frac{(8A-21B)\cos(c+dx)\sin(c+dx)}{2a^4d}+\frac{(52A-129B)\cos^3(c+}{105a^4d(1+\cos(c}$$

```
[Out] -1/2*(8*A-21*B)*x/a^4+8/105*(83*A-216*B)*sin(d*x+c)/a^4/d-1/2*(8*A-21*B)*co
s(d*x+c)*sin(d*x+c)/a^4/d+1/105*(52*A-129*B)*cos(d*x+c)^3*sin(d*x+c)/a^4/d/
(1+cos(d*x+c))^2+4/105*(83*A-216*B)*cos(d*x+c)^2*sin(d*x+c)/a^4/d/(1+cos(d*
x+c))+1/7*(A-B)*cos(d*x+c)^5*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+1/5*(A-2*B)*co
s(d*x+c)^4*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A]

time = 0.41, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {3056, 2813}

$$\frac{8(83A-216B)\sin(c+dx)}{105a^4d}+\frac{(52A-129B)\sin(c+dx)\cos^3(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{4(83A-216B)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(8A-21B)\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{x(8A-21B)}{2a^4}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{(A-2B)\sin(c+dx)\cos^4(c+dx)}{5ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] -1/2*((8*A - 21*B)*x)/a^4 + (8*(83*A - 216*B)*Sin[c + d*x])/(105*a^4*d) - (
(8*A - 21*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) + ((52*A - 129*B)*Cos[c +
 d*x]^3*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + (4*(83*A - 216*B)*
Cos[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[
c + d*x]^5*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((A - 2*B)*Cos[c +
d*x]^4*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^4(c+dx)(5a(A-B)-a(2A-9B)\,c}{(a+a\cos(c+dx))^3}}{7a^2}$$

$$= \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(A-2B)\cos^4(c+dx)\sin(c+}{5ad(a+a\cos(c+dx))^3}$$

$$= \frac{(52A-129B)\cos^3(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^5(c+dx)\,\mathrm{s}}{7d(a+a\cos(c+}$$

$$= \frac{(52A-129B)\cos^3(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^5(c+dx)\,\mathrm{s}}{7d(a+a\cos(c+}$$

$$= -\frac{(8A-21B)x}{2a^4} + \frac{8(83A-216B)\sin(c+dx)}{105a^4d} - \frac{(8A-21B)\cos}{2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 555 vs. 2(229) = 458.
time = 1.31, size = 555, normalized size = 2.42

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-14700*(8*A - 21*B)*d*x*Cos[(d*x)/2] - 14700*(8
*A - 21*B)*d*x*Cos[c + (d*x)/2] - 70560*A*d*x*Cos[c + (3*d*x)/2] + 185220*B
*d*x*Cos[c + (3*d*x)/2] - 70560*A*d*x*Cos[2*c + (3*d*x)/2] + 185220*B*d*x*C
os[2*c + (3*d*x)/2] - 23520*A*d*x*Cos[2*c + (5*d*x)/2] + 61740*B*d*x*Cos[2*
c + (5*d*x)/2] - 23520*A*d*x*Cos[3*c + (5*d*x)/2] + 61740*B*d*x*Cos[3*c + (
5*d*x)/2] - 3360*A*d*x*Cos[3*c + (7*d*x)/2] + 8820*B*d*x*Cos[3*c + (7*d*x)/
2] - 3360*A*d*x*Cos[4*c + (7*d*x)/2] + 8820*B*d*x*Cos[4*c + (7*d*x)/2] + 24
3320*A*Sin[(d*x)/2] - 539490*B*Sin[(d*x)/2] - 184520*A*Sin[c + (d*x)/2] + 3
86190*B*Sin[c + (d*x)/2] + 184464*A*Sin[c + (3*d*x)/2] - 422478*B*Sin[c + (
3*d*x)/2] - 72240*A*Sin[2*c + (3*d*x)/2] + 132930*B*Sin[2*c + (3*d*x)/2] +
77168*A*Sin[2*c + (5*d*x)/2] - 181461*B*Sin[2*c + (5*d*x)/2] - 8400*A*Sin[3
*c + (5*d*x)/2] + 3675*B*Sin[3*c + (5*d*x)/2] + 15164*A*Sin[3*c + (7*d*x)/2
] - 36003*B*Sin[3*c + (7*d*x)/2] + 2940*A*Sin[4*c + (7*d*x)/2] - 9555*B*Sin
[4*c + (7*d*x)/2] + 420*A*Sin[4*c + (9*d*x)/2] - 945*B*Sin[4*c + (9*d*x)/2]
 + 420*A*Sin[5*c + (9*d*x)/2] - 945*B*Sin[5*c + (9*d*x)/2] + 105*B*Sin[5*c
```

```
+ (11*d*x)/2] + 105*B*Sin[6*c + (11*d*x)/2]))/(6720*a^4*d*(1 + Cos[c + d*x]
)^4)
```

Maple [A]
time = 0.16, size = 191, normalized size = 0.83

method	result
derivativedivides	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}+\frac{7A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{9B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{23\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{}$
default	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}+\frac{7A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{9B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{23\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{}$
risch	$-\frac{4xA}{a^4}+\frac{21Bx}{2a^4}-\frac{iB\,e^{2i(dx+c)}}{8a^4d}-\frac{ie^{i(dx+c)}A}{2a^4d}+\frac{2ie^{i(dx+c)}B}{a^4d}+\frac{ie^{-i(dx+c)}A}{2a^4d}-\frac{2ie^{-i(dx+c)}B}{a^4d}+\frac{iB\,e^{-2i(dx+c)}}{8a^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7*A+1/7*tan(1/2*d*x+1/2*c)^7*B+7/5*A*tan
(1/2*d*x+1/2*c)^5-9/5*B*tan(1/2*d*x+1/2*c)^5-23/3*tan(1/2*d*x+1/2*c)^3*A+13
*tan(1/2*d*x+1/2*c)^3*B+49*A*tan(1/2*d*x+1/2*c)-111*B*tan(1/2*d*x+1/2*c)-16
*((-A+9/2*B)*tan(1/2*d*x+1/2*c)^3+(-A+7/2*B)*tan(1/2*d*x+1/2*c))/(1+tan(1/2
*d*x+1/2*c)^2)^2-8*(8*A-21*B)*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.48, size = 364, normalized size = 1.59

$$-\frac{3\,B\left(\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-A\left(\frac{1680\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(3*B*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
- 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4) - A*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*
x + c) + 1) - 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4))/d
```

Fricas [A]
time = 0.34, size = 238, normalized size = 1.04

$$-\frac{105\,(8\,A-21\,B)dx\cos\left(dx+c\right)^{4}+420\,(8\,A-21\,B)dx\cos\left(dx+c\right)^{3}+630\,(8\,A-21\,B)dx\cos\left(dx+c\right)^{2}+420\,(8\,A-21\,B)dx\cos\left(dx+c\right)+105\,(8\,A-21\,B)dx-\left(105\,B\cos\left(dx+c\right)^{5}+210\,(A-2\,B)\cos\left(dx+c\right)^{4}+4\,(592\,A-1509\,B)\cos\left(dx+c\right)^{3}+4\,(1318\,A-3411\,B)\cos\left(dx+c\right)^{2}+(4472\,A-11619\,B)\cos\left(dx+c\right)+1328\,A-3456\,B\right)\sin\left(dx+c\right)}{210\,\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] -1/210*(105*(8*A - 21*B)*d*x*cos(d*x + c)^4 + 420*(8*A - 21*B)*d*x*cos(d*x
+ c)^3 + 630*(8*A - 21*B)*d*x*cos(d*x + c)^2 + 420*(8*A - 21*B)*d*x*cos(d*x
 + c) + 105*(8*A - 21*B)*d*x - (105*B*cos(d*x + c)^5 + 210*(A - 2*B)*cos(d*
x + c)^4 + 4*(592*A - 1509*B)*cos(d*x + c)^3 + 4*(1318*A - 3411*B)*cos(d*x
+ c)^2 + (4472*A - 11619*B)*cos(d*x + c) + 1328*A - 3456*B)*sin(d*x + c))/(
a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. 2(216) = 432.
time = 9.28, size = 1085, normalized size = 4.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-3360*A*d*x*tan(c/2 + d*x/2)**4/(840*a**4*d*tan(c/2 + d*x/2)**4
+ 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 6720*A*d*x*tan(c/2 + d*x/
2)**2/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 8
40*a**4*d) - 3360*A*d*x/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c
/2 + d*x/2)**2 + 840*a**4*d) - 15*A*tan(c/2 + d*x/2)**11/(840*a**4*d*tan(c/
2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 117*A*tan(c
/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/
2)**2 + 840*a**4*d) - 526*A*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2
)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 3682*A*tan(c/2 + d*x
/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 +
840*a**4*d) + 11165*A*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**4 +
 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*A*tan(c/2 + d*x/2)/(8
40*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*
d) + 8820*B*d*x*tan(c/2 + d*x/2)**4/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*
a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 17640*B*d*x*tan(c/2 + d*x/2)**2/
(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**
4*d) + 8820*B*d*x/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d
*x/2)**2 + 840*a**4*d) + 15*B*tan(c/2 + d*x/2)**11/(840*a**4*d*tan(c/2 + d*
x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 159*B*tan(c/2 + d
```

```
*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2
+ 840*a**4*d) + 1002*B*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**4
+ 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 9114*B*tan(c/2 + d*x/2)**
5/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a
**4*d) - 29505*B*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680
*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 17535*B*tan(c/2 + d*x/2)/(840*a
**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d),
Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**5/(a*cos(c) + a)**4, True))
```

Giac [A]
time = 0.44, size = 233, normalized size = 1.02

$$-\frac{\frac{420\,(dx+c)(8\,A-21\,B)}{a^4}-\frac{840\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-147\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+189\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+805\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1365\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5145\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11655\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(420*(d*x + c)*(8*A - 21*B)/a^4 - 840*(2*A*tan(1/2*d*x + 1/2*c)^3 -
9*B*tan(1/2*d*x + 1/2*c)^3 + 2*A*tan(1/2*d*x + 1/2*c) - 7*B*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*
c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*A*a^24*tan(1/2*d*x + 1/2*c)^5
 + 189*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)^3 -
1365*B*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*A*a^24*tan(1/2*d*x + 1/2*c) + 116
55*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B]
time = 0.31, size = 259, normalized size = 1.13

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{5\,(A-B)}{4\,a^4}-\frac{5\,B}{2\,a^4}+\frac{3\,(4\,A-6\,B)}{4\,a^4}+\frac{3\,(5\,A-15\,B)}{8\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{A-B}{4\,a^4}+\frac{4\,A-6\,B}{8\,a^4}+\frac{5\,A-15\,B}{24\,a^4}\right)}{d}-\frac{x\,(8\,A-21\,B)}{2\,a^4}+\frac{(2\,A-9\,B)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(2\,A-7\,B)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^4\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{3\,(A-B)}{40\,a^4}+\frac{4\,A-6\,B}{40\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7(A-B)}{56\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((5*(A - B))/(4*a^4) - (5*B)/(2*a^4) + (3*(4*A - 6*B))/
(4*a^4) + (3*(5*A - 15*B))/(8*a^4)))/d - (tan(c/2 + (d*x)/2)^3*((A - B)/(4*
a^4) + (4*A - 6*B)/(8*a^4) + (5*A - 15*B)/(24*a^4)))/d - (x*(8*A - 21*B))/(
2*a^4) + (tan(c/2 + (d*x)/2)^3*(2*A - 9*B) + tan(c/2 + (d*x)/2)*(2*A - 7*B)
)/(d*(2*a^4*tan(c/2 + (d*x)/2)^2 + a^4*tan(c/2 + (d*x)/2)^4 + a^4)) + (tan(
c/2 + (d*x)/2)^5*((3*(A - B))/(40*a^4) + (4*A - 6*B)/(40*a^4)))/d - (tan(c/
2 + (d*x)/2)^7*(A - B))/(56*a^4*d)
```

3.66 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=185

$$\frac{(A-4B)x}{a^4}-\frac{(55A-244B)\sin(c+dx)}{105a^4d}+\frac{(25A-88B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2}-\frac{(A-4B)\sin(c+dx)}{a^4d(1+\cos(c+dx))}+(A$$

```
[Out] (A-4*B)*x/a^4-1/105*(55*A-244*B)*sin(d*x+c)/a^4/d+1/105*(25*A-88*B)*cos(d*x
+c)^2*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2-(A-4*B)*sin(d*x+c)/a^4/d/(1+cos(d*x
+c))+1/7*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+1/35*(5*A-12*B)
*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A]

time = 0.40, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3056, 3047, 3102, 12, 2814, 2727}

$$-\frac{(55A-244B)\sin(c+dx)}{105a^4d}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(A-4B)\sin(c+dx)}{a^4d(\cos(c+dx)+1)}+\frac{x(A-4B)}{a^4}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{(5A-12B)\sin(c+dx)\cos^3(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] ((A - 4*B)*x)/a^4 - ((55*A - 244*B)*Sin[c + d*x])/(105*a^4*d) + ((25*A - 88
*B)*Cos[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) - ((A - 4
*B)*Sin[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^4*Sin[
c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((5*A - 12*B)*Cos[c + d*x]^3*Sin[c
 + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^3(c+dx)(4a(A-B)-a(A-8B)\mathrm{co}}{(a+a\cos(c+dx))^3}}{7a^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(5A-12B)\cos^3(c+dx)\sin(}{35ad(a+a\cos(c+dx)}$$

$$= \frac{(25A-88B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^4(c+dx)\sin}{7d(a+a\cos(c+d}$$

$$= \frac{(25A-88B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^4(c+dx)\sin}{7d(a+a\cos(c+d}$$

$$= -\frac{(55A-244B)\sin(c+dx)}{105a^4d} + \frac{(25A-88B)\cos^2(c+dx)\sin(c+}{105a^4d(1+\cos(c+dx))^2}$$

$$= -\frac{(55A-244B)\sin(c+dx)}{105a^4d} + \frac{(25A-88B)\cos^2(c+dx)\sin(c+}{105a^4d(1+\cos(c+dx))^2}$$

$$= \frac{(A-4B)x}{a^4} - \frac{(55A-244B)\sin(c+dx)}{105a^4d} + \frac{(25A-88B)\cos^2(c+}{105a^4d(1+\cos}$$

$$= \frac{(A-4B)x}{a^4} - \frac{(55A-244B)\sin(c+dx)}{105a^4d} + \frac{(25A-88B)\cos^2(c+}{105a^4d(1+\cos}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 481 vs. 2(185) = 370.
time = 0.92, size = 481, normalized size = 2.60

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(7350*(A - 4*B)*d*x*Cos[(d*x)/2] + 7350*(A - 4*B
)*d*x*Cos[c + (d*x)/2] + 4410*A*d*x*Cos[c + (3*d*x)/2] - 17640*B*d*x*Cos[c
+ (3*d*x)/2] + 4410*A*d*x*Cos[2*c + (3*d*x)/2] - 17640*B*d*x*Cos[2*c + (3*d
*x)/2] + 1470*A*d*x*Cos[2*c + (5*d*x)/2] - 5880*B*d*x*Cos[2*c + (5*d*x)/2]
+ 1470*A*d*x*Cos[3*c + (5*d*x)/2] - 5880*B*d*x*Cos[3*c + (5*d*x)/2] + 210*A
*d*x*Cos[3*c + (7*d*x)/2] - 840*B*d*x*Cos[3*c + (7*d*x)/2] + 210*A*d*x*Cos[
4*c + (7*d*x)/2] - 840*B*d*x*Cos[4*c + (7*d*x)/2] - 19880*A*Sin[(d*x)/2] +
60830*B*Sin[(d*x)/2] + 16520*A*Sin[c + (d*x)/2] - 46130*B*Sin[c + (d*x)/2]
- 14280*A*Sin[c + (3*d*x)/2] + 46116*B*Sin[c + (3*d*x)/2] + 7560*A*Sin[2*c
+ (3*d*x)/2] - 18060*B*Sin[2*c + (3*d*x)/2] - 5600*A*Sin[2*c + (5*d*x)/2] +
 19292*B*Sin[2*c + (5*d*x)/2] + 1680*A*Sin[3*c + (5*d*x)/2] - 2100*B*Sin[3*
```

```
c + (5*d*x)/2] - 1040*A*Sin[3*c + (7*d*x)/2] + 3791*B*Sin[3*c + (7*d*x)/2]
+ 735*B*Sin[4*c + (7*d*x)/2] + 105*B*Sin[4*c + (9*d*x)/2] + 105*B*Sin[5*c +
 (9*d*x)/2]))/(1680*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A]

time = 0.21, size = 162, normalized size = 0.88

method	result
derivativedivides	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}-A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{7B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{23\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}-1}{8d\,a^4}$
default	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}-A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{7B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{23\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}-1}{8d\,a^4}$
risch	$\frac{xA}{a^4}-\frac{4Bx}{a^4}-\frac{i\mathrm{e}^{i(dx+c)}B}{2a^4d}+\frac{i\mathrm{e}^{-i(dx+c)}B}{2a^4d}-\frac{4i\left(210A\,\mathrm{e}^{6i(dx+c)}-525B\,\mathrm{e}^{6i(dx+c)}+945A\,\mathrm{e}^{5i(dx+c)}-2625B\,\mathrm{e}^{5i(dx+c)}+\right.}{}$
norman	$\frac{(A-4B)x}{a}+\frac{(A-4B)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{(A-22B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{84da}+\frac{5(A-4B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{10(A-4B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7*A-1/7*tan(1/2*d*x+1/2*c)^7*B-A*tan(1/2*
d*x+1/2*c)^5+7/5*B*tan(1/2*d*x+1/2*c)^5+11/3*tan(1/2*d*x+1/2*c)^3*A-23/3*ta
n(1/2*d*x+1/2*c)^3*B-15*A*tan(1/2*d*x+1/2*c)+49*B*tan(1/2*d*x+1/2*c)+16*B*t
an(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+16*(A-4*B)*arctan(tan(1/2*d*x+1/
2*c)))
```

Maxima [A]

time = 0.49, size = 271, normalized size = 1.46

$$\frac{B\left(\frac{1680\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-5A\left(\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(B*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a^4) - 5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*s
in(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^4))/d
```

Fricas [A]
time = 0.34, size = 213, normalized size = 1.15

$$\frac{105 (A-4 B)dx \cos(dx+c)^4+420 (A-4 B)dx \cos(dx+c)^3+630 (A-4 B)dx \cos(dx+c)^2+420 (A-4 B)dx \cos(dx+c)+105 (A-4 B)dx+\left(105 B \cos(dx+c)^4-4 (65 A-296 B) \cos(dx+c)^3-4 (155 A-659 B) \cos(dx+c)^2-(535 A-2236 B) \cos(dx+c)-160 A+664 B\right) \sin(dx+c)}{105 \left(a^4 d \cos(dx+c)^4+4 a^4 d \cos(dx+c)^3+6 a^4 d \cos(dx+c)^2+4 a^4 d \cos(dx+c)+a^4 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*(105*(A - 4*B)*d*x*cos(d*x + c)^4 + 420*(A - 4*B)*d*x*cos(d*x + c)^3
+ 630*(A - 4*B)*d*x*cos(d*x + c)^2 + 420*(A - 4*B)*d*x*cos(d*x + c) + 105*(
A - 4*B)*d*x + (105*B*cos(d*x + c)^4 - 4*(65*A - 296*B)*cos(d*x + c)^3 - 4*
(155*A - 659*B)*cos(d*x + c)^2 - (535*A - 2236*B)*cos(d*x + c) - 160*A + 66
4*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d
*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(172) = 344$.
time = 5.91, size = 578, normalized size = 3.12

$$\begin{cases} \frac{840 A dx \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d}+\frac{840 A dx}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d}+\frac{15 A \tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d}-\frac{90 A \tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d}+\frac{280 A \tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d}-\frac{1190 A \tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d}-\frac{1575 A \tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d}-\frac{3360 B dx \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d}-\frac{3360 B dx}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d}-\frac{15 B \tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d}+\frac{132 B \tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d}-\frac{658 B \tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d}+\frac{4340 B \tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d}+\frac{6825 B \tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840 a^4 d} & \text{for}\ d \neq 0 \\ \frac{x(A+B\cos(c))\cos^4(c)}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((840*A*d*x*tan(c/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 +
840*a**4*d) + 840*A*d*x/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 15*
A*tan(c/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 90*A*
tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 280*A*t
an(c/2 + d*x/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 1190*A*t
an(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 1575*A*t
an(c/2 + d*x/2)/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 3360*B*d*x*
tan(c/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 3360*B*
d*x/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 15*B*tan(c/2 + d*x/2)**
9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 132*B*tan(c/2 + d*x/2)**7
/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 658*B*tan(c/2 + d*x/2)**5/
(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 4340*B*tan(c/2 + d*x/2)**3/
(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*B*tan(c/2 + d*x/2)/(84
0*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d), Ne(d, 0)), (x*(A + B*cos(c))*co
s(c)**4/(a*cos(c) + a)**4, True))
```

Giac [A]
time = 0.48, size = 188, normalized size = 1.02

$$\frac{\frac{840 (dx+c)(A-4 B)}{a^4}+\frac{1680 B \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+1\right) a^4}+\frac{15 A a^{24} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7-15 B a^{24} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7-105 A a^{24} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5+147 B a^{24} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5+385 A a^{24} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-805 B a^{24} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-1575 A a^{24} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+5145 B a^{24} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{28}}}{840 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(840*(d*x + c)*(A - 4*B)/a^4 + 1680*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*
d*x + 1/2*c)^2 + 1)*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*ta
n(1/2*d*x + 1/2*c)^7 - 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 147*B*a^24*tan(1
/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 805*B*a^24*tan(1/2*
d*x + 1/2*c)^3 - 1575*A*a^24*tan(1/2*d*x + 1/2*c) + 5145*B*a^24*tan(1/2*d*x
 + 1/2*c))/a^28)/d
```

Mupad [B]
time = 0.39, size = 201, normalized size = 1.09

$$\frac{A\,d\,x-4\,B\,d\,x}{a^4\,d}-\frac{\left(\frac{52\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}-\frac{764\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{105}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{143\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{105}-\frac{16\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{5\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{28}-\frac{8\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{56}+\frac{B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{56}}{a^4\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}+\frac{2\,B\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

```
[Out] (A*d*x - 4*B*d*x)/(a^4*d) - ((B*sin(c/2 + (d*x)/2))/56 - (A*sin(c/2 + (d*x)
/2))/56 + cos(c/2 + (d*x)/2)^2*((5*A*sin(c/2 + (d*x)/2))/28 - (8*B*sin(c/2
+ (d*x)/2))/35) - cos(c/2 + (d*x)/2)^4*((16*A*sin(c/2 + (d*x)/2))/21 - (143
*B*sin(c/2 + (d*x)/2))/105) + cos(c/2 + (d*x)/2)^6*((52*A*sin(c/2 + (d*x)/2
))/21 - (764*B*sin(c/2 + (d*x)/2))/105))/(a^4*d*cos(c/2 + (d*x)/2)^7) + (2*
B*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))/(a^4*d)
```

3.67 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=154

$$\frac{Bx}{a^4}-\frac{(6A-55B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2}+\frac{(12A-215B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))}+\frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{(3A-10B}{35ad}$$

```
[Out] B*x/a^4-1/105*(6*A-55*B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2+1/105*(12*A-215*
B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c))+1/7*(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a
*cos(d*x+c))^4+1/35*(3*A-10*B)*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))
^3
```

Rubi [A]

time = 0.28, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3056, 3047, 3098, 2814, 2727}

$$\frac{(12A-215B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(6A-55B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{Bx}{a^4}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{(3A-10B)\sin(c+dx)\cos^2(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (B*x)/a^4 - ((6*A - 55*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) +
((12*A - 215*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos
[c + d*x]^3*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((3*A - 10*B)*Cos[
c + d*x]^2*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^2(c+dx)(3a(A-B)+7aB\cos(c+dx}{(a+a\cos(c+dx))^3}}{7a^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A-10B)\cos^2(c+dx)\sin(c}{35ad(a+a\cos(c+dx))^3}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A-10B)\cos^2(c+dx)\sin(c}{35ad(a+a\cos(c+dx))^3}$$

$$= -\frac{(6A-55B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} +$$

$$= \frac{Bx}{a^4} - \frac{(6A-55B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^3(c+dx)\sin(c+d}{7d(a+a\cos(c+dx))^4}$$

$$= \frac{Bx}{a^4} - \frac{(6A-55B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^3(c+dx)\sin(c+d}{7d(a+a\cos(c+dx))^4}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(154) = 308.
time = 0.80, size = 329, normalized size = 2.14

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(3675Bdx\cos\left(\frac{dx}{2}\right)+3675Bdx\cos\left(c+\frac{dx}{2}\right)+2205Bdx\cos\left(c+\frac{3dx}{2}\right)+2205Bdx\cos\left(2c+\frac{3dx}{2}\right)+735Bdx\cos\left(2c+\frac{5dx}{2}\right)+735Bdx\cos\left(3c+\frac{5dx}{2}\right)+105Bdx\cos\left(3c+\frac{7dx}{2}\right)+105Bdx\cos\left(4c+\frac{7dx}{2}\right)+1260A\sin\left(\frac{dx}{2}\right)-9940B\sin\left(\frac{dx}{2}\right)-1260A\sin\left(c+\frac{dx}{2}\right)+8260B\sin\left(c+\frac{dx}{2}\right)+882A\sin\left(c+\frac{3dx}{2}\right)-7140B\sin\left(c+\frac{3dx}{2}\right)-630A\sin\left(2c+\frac{3dx}{2}\right)+3780B\sin\left(2c+\frac{3dx}{2}\right)+294A\sin\left(2c+\frac{5dx}{2}\right)-2800B\sin\left(2c+\frac{5dx}{2}\right)-210A\sin\left(3c+\frac{5dx}{2}\right)+840B\sin\left(3c+\frac{5dx}{2}\right)+72A\sin\left(3c+\frac{7dx}{2}\right)-520B\sin\left(3c+\frac{7dx}{2}\right)\right)}{13440a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(3675*B*d*x*Cos[(d*x)/2] + 3675*B*d*x*Cos[c +
(d*x)/2] + 2205*B*d*x*Cos[c + (3*d*x)/2] + 2205*B*d*x*Cos[2*c + (3*d*x)/2]
+ 735*B*d*x*Cos[2*c + (5*d*x)/2] + 735*B*d*x*Cos[3*c + (5*d*x)/2] + 105*B*d
*x*Cos[3*c + (7*d*x)/2] + 105*B*d*x*Cos[4*c + (7*d*x)/2] + 1260*A*Sin[(d*x)
/2] - 9940*B*Sin[(d*x)/2] - 1260*A*Sin[c + (d*x)/2] + 8260*B*Sin[c + (d*x)/
2] + 882*A*Sin[c + (3*d*x)/2] - 7140*B*Sin[c + (3*d*x)/2] - 630*A*Sin[2*c +
 (3*d*x)/2] + 3780*B*Sin[2*c + (3*d*x)/2] + 294*A*Sin[2*c + (5*d*x)/2] - 28
00*B*Sin[2*c + (5*d*x)/2] - 210*A*Sin[3*c + (5*d*x)/2] + 840*B*Sin[3*c + (5
*d*x)/2] + 72*A*Sin[3*c + (7*d*x)/2] - 520*B*Sin[3*c + (7*d*x)/2]))/(13440*
a^4*d)
```

Maple [A]

time = 0.19, size = 130, normalized size = 0.84

method	result
derivativedivides	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}+\frac{3A\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-B\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A+\frac{11\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}}{8d\,a^{4}}$
default	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}+\frac{3A\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-B\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A+\frac{11\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3}}{8d\,a^{4}}$
risch	$\frac{Bx}{a^{4}}+\frac{2i\left(105A\,e^{6i(dx+c)}-420B\,e^{6i(dx+c)}+315A\,e^{5i(dx+c)}-1890B\,e^{5i(dx+c)}+630A\,e^{4i(dx+c)}-4130B\,e^{4i(dx+c)}+630A\right.}{105d\,a^{4}\left(e^{i(\right.}}$
norman	$\frac{\frac{Bx}{a}+\frac{Bx\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{4Bx\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{6Bx\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{4Bx\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{(A-15B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{(A-15B}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7*A+1/7*tan(1/2*d*x+1/2*c)^7*B+3/5*A*tan
(1/2*d*x+1/2*c)^5-B*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^3*A+11/3*tan(1/
2*d*x+1/2*c)^3*B+A*tan(1/2*d*x+1/2*c)-15*B*tan(1/2*d*x+1/2*c)+16*B*arctan(t
an(1/2*d*x+1/2*c)))
```

Maxima [A]

time = 0.48, size = 201, normalized size = 1.31

$$-\frac{5\,B\left(\frac{\frac{315\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{77\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{21\,\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}-\frac{3\,\sin\left(dx+c\right)^{7}}{\left(\cos\left(dx+c\right)+1\right)^{7}}}{a^{4}}-\frac{336\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a^{4}}\right)-\frac{3\,A\left(\frac{35\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{35\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{21\,\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}-\frac{5\,\sin\left(dx+c\right)^{7}}{\left(\cos\left(dx+c\right)+1\right)^{7}}\right)}{a^{4}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(5*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^
4) - 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A]
time = 0.35, size = 180, normalized size = 1.17

$$\frac{105\,Bdx\cos\left(dx+c\right)^{4}+420\,Bdx\cos\left(dx+c\right)^{3}+630\,Bdx\cos\left(dx+c\right)^{2}+420\,Bdx\cos\left(dx+c\right)+105\,Bdx+\left(4\left(9\,A-65\,B\right)\cos\left(dx+c\right)^{3}+\left(39\,A-620\,B\right)\cos\left(dx+c\right)^{2}+\left(24\,A-535\,B\right)\cos\left(dx+c\right)+6\,A-160\,B\right)\sin\left(dx+c\right)}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*(105*B*d*x*cos(d*x + c)^4 + 420*B*d*x*cos(d*x + c)^3 + 630*B*d*x*cos(
d*x + c)^2 + 420*B*d*x*cos(d*x + c) + 105*B*d*x + (4*(9*A - 65*B)*cos(d*x +
 c)^3 + (39*A - 620*B)*cos(d*x + c)^2 + (24*A - 535*B)*cos(d*x + c) + 6*A -
 160*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^
4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [A]
time = 3.62, size = 192, normalized size = 1.25

$$\begin{cases} -\frac{A\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d}+\frac{3A\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{4}d}-\frac{A\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d}+\frac{Bx}{a^{4}}+\frac{B\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d}-\frac{B\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d}+\frac{11B\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^{4}d}-\frac{15B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d} & \text{for}\: d\neq 0 \\ \frac{x(A+B\cos\left(c\right))\cos^{3}\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*A*tan(c/2 + d*x/2)**5/(40
*a**4*d) - A*tan(c/2 + d*x/2)**3/(8*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
 + B*x/a**4 + B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/(8*
a**4*d) + 11*B*tan(c/2 + d*x/2)**3/(24*a**4*d) - 15*B*tan(c/2 + d*x/2)/(8*a
**4*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**3/(a*cos(c) + a)**4, True))
```

Giac [A]
time = 0.43, size = 155, normalized size = 1.01

$$\frac{\frac{840\,(dx+c)B}{a^{4}}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-63\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+105\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-385\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1575\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(840*(d*x + c)*B/a^4 - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*
tan(1/2*d*x + 1/2*c)^7 - 63*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 105*B*a^24*tan(
1/2*d*x + 1/2*c)^5 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*tan(1/2
*d*x + 1/2*c)^3 - 105*A*a^24*tan(1/2*d*x + 1/2*c) + 1575*B*a^24*tan(1/2*d*x
 + 1/2*c))/a^28)/d
```

Mupad [B]
time = 0.34, size = 162, normalized size = 1.05

$$\frac{B\,x}{a^4}+\frac{\left(\frac{12\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}-\frac{52\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{16\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}-\frac{23\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{70}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{9\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{70}-\frac{5\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{28}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{56}+\frac{B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{56}}{a^4\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

```
[Out] (B*x)/a^4 + ((B*sin(c/2 + (d*x)/2))/56 - (A*sin(c/2 + (d*x)/2))/56 + cos(c/
2 + (d*x)/2)^2*((9*A*sin(c/2 + (d*x)/2))/70 - (5*B*sin(c/2 + (d*x)/2))/28)
+ cos(c/2 + (d*x)/2)^6*((12*A*sin(c/2 + (d*x)/2))/35 - (52*B*sin(c/2 + (d*x
)/2))/21) - cos(c/2 + (d*x)/2)^4*((23*A*sin(c/2 + (d*x)/2))/70 - (16*B*sin(
c/2 + (d*x)/2))/21))/(a^4*d*cos(c/2 + (d*x)/2)^7)
```

3.68 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4} dx$

Optimal. Leaf size=136

$$-\frac{2(A+27B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2}+\frac{(13A+36B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))}+\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{(A-8B)\sin(c}{35ad(a+a\cos(c}$$

```
[Out] -2/105*(A+27*B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2+1/105*(13*A+36*B)*sin(d*x
+c)/a^4/d/(1+cos(d*x+c))+1/7*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c
))^4-1/35*(A-8*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A]
time = 0.22, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3056, 3047, 3098, 2829, 2727}

$$\frac{(13A+36B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{2(A+27B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{(A-8B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-2*(A + 27*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + ((13*A + 36
*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^2*
Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((A - 8*B)*Sin[c + d*x])/(35*a
*d*(a + a*Cos[c + d*x])^3)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos(c+dx)(2a(A-B)+a(A+6B)\cos}{(a+a\cos(c+dx))^3}}{7a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{2a(A-B)\cos(c+dx)+a(A+6B)\cos^2}{(a+a\cos(c+dx))^3}}{7a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(A-8B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} -$$

$$= -\frac{2(A+27B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} -$$

$$= -\frac{2(A+27B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} -$$

Mathematica [A]
time = 0.50, size = 193, normalized size = 1.42

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(70(4A+9B)\sin\left(\frac{dx}{2}\right)-35(5A+18B)\sin\left(c+\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)+441B\sin\left(c+\frac{3dx}{2}\right)-105A\sin\left(2c+\frac{3dx}{2}\right)-315B\sin\left(2c+\frac{3dx}{2}\right)+91A\sin\left(2c+\frac{5dx}{2}\right)+147B\sin\left(2c+\frac{5dx}{2}\right)-105B\sin\left(3c+\frac{5dx}{2}\right)+13A\sin\left(3c+\frac{7dx}{2}\right)+36B\sin\left(3c+\frac{7dx}{2}\right)\right)}{420a^4d(1+\cos(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(70*(4*A + 9*B)*Sin[(d*x)/2] - 35*(5*A + 18*B)*S
in[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 441*B*Sin[c + (3*d*x)/2] - 105
*A*Sin[2*c + (3*d*x)/2] - 315*B*Sin[2*c + (3*d*x)/2] + 91*A*Sin[2*c + (5*d*
x)/2] + 147*B*Sin[2*c + (5*d*x)/2] - 105*B*Sin[3*c + (5*d*x)/2] + 13*A*Sin[
3*c + (7*d*x)/2] + 36*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c + d*x]
)^4)
```

Maple [A]
time = 0.19, size = 90, normalized size = 0.66

method	result
derivativedivides	$\frac{\frac{(A-B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(-A+3B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(-A-3B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$
default	$\frac{\frac{(A-B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(-A+3B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(-A-3B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$
risch	$\frac{2i\left(105B\,e^{6i(dx+c)}+105A\,e^{5i(dx+c)}+315B\,e^{5i(dx+c)}+175A\,e^{4i(dx+c)}+630B\,e^{4i(dx+c)}+280A\,e^{3i(dx+c)}+630B\,e^{3i(dx+c)}+\right.}{105d\,a^{4}\left(e^{i(dx+c)}+1\right)^{7}}$
norman	$\frac{\frac{(A-B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56da}+\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{3(3A+B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40da}+\frac{(4A+3B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}-\frac{(4A+3B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{70da}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d/a^4*(1/7*(A-B)*tan(1/2*d*x+1/2*c)^7+1/5*(-A+3*B)*tan(1/2*d*x+1/2*c)^5
+1/3*(-A-3*B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c
))
```

Maxima [A]
time = 0.27, size = 175, normalized size = 1.29

$$\frac{\frac{A\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{21\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{a^{4}}+\frac{3B\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{21\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{5\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{a^{4}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(
```

d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d

Fricas [A]
time = 0.36, size = 124, normalized size = 0.91

$$\frac{\left((13\,A+36\,B)\cos\left(dx+c\right)^{3}+13\,(4\,A+3\,B)\cos\left(dx+c\right)^{2}+8\,(4\,A+3\,B)\cos\left(dx+c\right)+8\,A+6\,B\right)\sin\left(dx+c\right)}{105\,\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")

[Out] 1/105*((13*A + 36*B)*cos(d*x + c)^3 + 13*(4*A + 3*B)*cos(d*x + c)^2 + 8*(4*
A + 3*B)*cos(d*x + c) + 8*A + 6*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a
^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d
)

Sympy [A]
time = 2.54, size = 182, normalized size = 1.34

$$\begin{cases} \frac{A\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d}-\frac{A\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{4}d}-\frac{A\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^{4}d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d}-\frac{B\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d}+\frac{3B\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{4}d}-\frac{B\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d} & \text{for}\ d\neq 0 \\ \frac{x(A+B\cos\left(c\right))\cos^{2}\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)

[Out] Piecewise((A*tan(c/2 + d*x/2)**7/(56*a**4*d) - A*tan(c/2 + d*x/2)**5/(40*a*
*4*d) - A*tan(c/2 + d*x/2)**3/(24*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d) -
 B*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*B*tan(c/2 + d*x/2)**5/(40*a**4*d) -
B*tan(c/2 + d*x/2)**3/(8*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0))
, (x*(A + B*cos(c))*cos(c)**2/(a*cos(c) + a)**4, True))

Giac [A]
time = 0.44, size = 117, normalized size = 0.86

$$\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+63\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-105\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+105\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+105\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")

[Out] 1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 - 21*A*tan
(1/2*d*x + 1/2*c)^5 + 63*B*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2*
c)^3 - 105*B*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/2*c) + 105*B*ta
n(1/2*d*x + 1/2*c))/(a^4*d)

Mupad [B]
time = 0.25, size = 86, normalized size = 0.63

$$-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A+3\,B)}{24\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A-3\,B)}{40\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(A-B)}{56\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A+B)}{8\,a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

```
[Out] -((tan(c/2 + (d*x)/2)^3*(A + 3*B))/(24*a^4) + (tan(c/2 + (d*x)/2)^5*(A - 3*
B))/(40*a^4) - (tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4) - (tan(c/2 + (d*x)/2
)*(A + B))/(8*a^4))/d
```

3.69 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$-\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{(4A-11B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}+\frac{(8A+13B)\sin(c+dx)}{105d\,(a^2+a^2\cos(c+dx))^2}+\frac{(8A+13B)\sin(c+dx}{105d\,(a^4+a^4\cos(c+dx}$$

```
[Out] -1/7*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+1/35*(4*A-11*B)*sin(d*x+c)/a/d/(
a+a*cos(d*x+c))^3+1/105*(8*A+13*B)*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+1/10
5*(8*A+13*B)*sin(d*x+c)/d/(a^4+a^4*cos(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$,
Rules used = {3047, 3098, 2829, 2729, 2727}

$$\frac{(8A+13B)\sin(c+dx)}{105d\,(a^4\cos(c+dx)+a^4)}+\frac{(8A+13B)\sin(c+dx)}{105d\,(a^2\cos(c+dx)+a^2)^2}+\frac{(4A-11B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] -1/7*((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^4) + ((4*A - 11*B)*Sin[
c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + ((8*A + 13*B)*Sin[c + d*x])/(10
5*d*(a^2 + a^2*Cos[c + d*x])^2) + ((8*A + 13*B)*Sin[c + d*x])/(105*d*(a^4 +
 a^4*Cos[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{-4a(A-B)-7aB\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$
$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(4A-11B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(8A+13B)}{}$$
$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(4A-11B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(8A+13}{105d\,(a^2+}$$
$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(4A-11B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(8A+13}{105d\,(a^2+}$$

Mathematica [A]
time = 0.40, size = 163, normalized size = 1.18

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(140(A+2B)\sin\left(\frac{dx}{2}\right)-35(4A+5B)\sin\left(c+\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)+168B\sin\left(c+\frac{3dx}{2}\right)-105B\sin\left(2c+\frac{3dx}{2}\right)+56A\sin\left(2c+\frac{5dx}{2}\right)+91B\sin\left(2c+\frac{5dx}{2}\right)+8A\sin\left(3c+\frac{7dx}{2}\right)+13B\sin\left(3c+\frac{7dx}{2}\right)\right)}{420a^4d(1+\cos(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(140*(A + 2*B)*Sin[(d*x)/2] - 35*(4*A + 5*B)*Sin
[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3*d*x)/2] - 105*B
```

```
*Sin[2*c + (3*d*x)/2] + 56*A*Sin[2*c + (5*d*x)/2] + 91*B*Sin[2*c + (5*d*x)/
2] + 8*A*Sin[3*c + (7*d*x)/2] + 13*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 +
 Cos[c + d*x])^4)
```

Maple [A]
time = 0.20, size = 88, normalized size = 0.64

method	result
derivativedivides	$\frac{\frac{(-A+B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(-A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$
default	$\frac{\frac{(-A+B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(-A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$
risch	$\frac{2i\left(105B\,e^{5i(dx+c)}+140A\,e^{4i(dx+c)}+175B\,e^{4i(dx+c)}+140A\,e^{3i(dx+c)}+280B\,e^{3i(dx+c)}+168A\,e^{2i(dx+c)}+168B\,e^{2i(dx+c)}\right.}{105d\,a^4\left(e^{i(dx+c)}+1\right)^7}$
norman	$\frac{-\frac{(A-B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56da}+\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{(7A+5B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24da}+\frac{(11A+B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60da}-\frac{(11A+31B)\left(\tan^7\left(\frac{dx}{2}\right.\right.}{420da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(1/7*(-A+B)*tan(1/2*d*x+1/2*c)^7+1/5*(-A-B)*tan(1/2*d*x+1/2*c)^5+
1/3*(A-B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 174, normalized size = 1.26

$$\frac{\frac{A\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{B\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + B*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A]
time = 0.38, size = 124, normalized size = 0.90

$$\frac{\left((8\,A+13\,B)\cos\left(dx+c\right)^3+4\,(8\,A+13\,B)\cos\left(dx+c\right)^2+4\,(13\,A+8\,B)\cos\left(dx+c\right)+13\,A+8\,B\right)\sin\left(dx+c\right)}{105\,\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] 1/105*((8*A + 13*B)*cos(d*x + c)^3 + 4*(8*A + 13*B)*cos(d*x + c)^2 + 4*(13*
A + 8*B)*cos(d*x + c) + 13*A + 8*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

Sympy [A]
time = 1.97, size = 178, normalized size = 1.29

$$\begin{cases} -\frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} + \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{for}\ d \neq 0 \\ \frac{x(A+B\cos(c))\cos(c)}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**7/(56*a**4*d) - A*tan(c/2 + d*x/2)**5/(40*a
**4*d) + A*tan(c/2 + d*x/2)**3/(24*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
+ B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/(40*a**4*d) - B
*tan(c/2 + d*x/2)**3/(24*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0))
, (x*(A + B*cos(c))*cos(c)/(a*cos(c) + a)**4, True))
```

Giac [A]
time = 0.43, size = 117, normalized size = 0.85

$$-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 35\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 105\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 105\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 21*A*ta
n(1/2*d*x + 1/2*c)^5 + 21*B*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2
*c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 - 105*A*tan(1/2*d*x + 1/2*c) - 105*B*ta
n(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B]
time = 0.25, size = 84, normalized size = 0.61

$$-\frac{\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(A+B\right)}{40\,a^4}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(A-B\right)}{24\,a^4}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(A-B\right)}{56\,a^4}-\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A+B\right)}{8\,a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

```
[Out] -((tan(c/2 + (d*x)/2)^5*(A + B))/(40*a^4) - (tan(c/2 + (d*x)/2)^3*(A - B))/
(24*a^4) + (tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4) - (tan(c/2 + (d*x)/2)*(A
 + B))/(8*a^4))/d
```

3.70 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{(3A+4B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}+\frac{2(3A+4B)\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2}+\frac{2(3A+4B)\sin(c+dx)}{105d\left(a^4+a^4\cos(c+dx)\right)}$$

```
[Out] 1/7*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+1/35*(3*A+4*B)*sin(d*x+c)/a/d/(a+
a*cos(d*x+c))^3+2/105*(3*A+4*B)*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+2/105*(
3*A+4*B)*sin(d*x+c)/d/(a^4+a^4*cos(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2829, 2729, 2727}

$$\frac{2(3A+4B)\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{2(3A+4B)\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{(3A+4B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] ((A - B)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((3*A + 4*B)*Sin[c +
d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + (2*(3*A + 4*B)*Sin[c + d*x])/(105*d
*(a^2 + a^2*Cos[c + d*x])^2) + (2*(3*A + 4*B)*Sin[c + d*x])/(105*d*(a^4 + a
^4*Cos[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B)\int \frac{1}{(a+a\cos(c+dx))^3}\,dx}{7a}$$
$$= \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(2(3A+4B))\int \frac{1}{(a+a\cos(}}{35a^2}$$
$$= \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{2(3A+4B)\sin(c+d}{105d\,(a^2+a^2\cos(c+d}$$
$$= \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{2(3A+4B)\sin(c+d}{105d\,(a^2+a^2\cos(c+d}$$

Mathematica [A]
time = 0.37, size = 109, normalized size = 0.79

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(35(3A+2B)\sin\left(\frac{dx}{2}\right)-70B\sin\left(c+\frac{dx}{2}\right)+(3A+4B)\left(21\sin\left(c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{7dx}{2}\right)\right)\right)}{210a^4d(1+\cos(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^4,x]
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(35*(3*A + 2*B)*Sin[(d*x)/2] - 70*B*Sin[c + (d*x
)/2] + (3*A + 4*B)*(21*Sin[c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2] + Sin[3*
c + (7*d*x)/2])))/(210*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A]
time = 0.15, size = 88, normalized size = 0.64

method	result
derivativedivides	$\frac{\frac{(A-B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(3A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(3A+B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$
default	$\frac{\frac{(A-B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(3A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(3A+B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$
risch	$\frac{4i\left(70B\,e^{4i(dx+c)}+105A\,e^{3i(dx+c)}+70B\,e^{3i(dx+c)}+63A\,e^{2i(dx+c)}+84B\,e^{2i(dx+c)}+21A\,e^{i(dx+c)}+28B\,e^{i(dx+c)}+3A+4B\right.}{105d\,a^4\left(e^{i(dx+c)}+1\right)^7}$
norman	$\frac{\frac{(A-B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56da}+\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{(3A+2B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}+\frac{(12A+B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60da}+\frac{(13A-6B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{140da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(1/7*(A-B)*tan(1/2*d*x+1/2*c)^7+1/5*(3*A-B)*tan(1/2*d*x+1/2*c)^5+
1/3*(3*A+B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 175, normalized size = 1.27

$$\frac{\frac{B\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3\,A\left(\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(B*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A]
time = 0.34, size = 125, normalized size = 0.91

$$\frac{\left(2\,(3\,A+4\,B)\cos\left(dx+c\right)^3+8\,(3\,A+4\,B)\cos\left(dx+c\right)^2+13\,(3\,A+4\,B)\cos\left(dx+c\right)+36\,A+13\,B\right)\sin\left(dx+c\right)}{105\,\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*A + 4*B)*cos(d*x + c)^3 + 8*(3*A + 4*B)*cos(d*x + c)^2 + 13*(3*
A + 4*B)*cos(d*x + c) + 36*A + 13*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4
*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4
*d)
```

Sympy [A]
time = 1.66, size = 177, normalized size = 1.28

$$\begin{cases} \frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d}+\frac{3A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d}+\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}-\frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d}-\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d}+\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{for}\ d\neq 0 \\ \frac{x(A+B\cos(c))}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*A*tan(c/2 + d*x/2)**5/(40*
a**4*d) + A*tan(c/2 + d*x/2)**3/(8*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
- B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/(40*a**4*d) + B
```

```
*tan(c/2 + d*x/2)**3/(24*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0))
, (x*(A + B*cos(c))/(a*cos(c) + a)**4, True))
```

Giac [A]
time = 0.45, size = 117, normalized size = 0.85

$$\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+63\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+105\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+35\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+105\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+105\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 63*A*tan
(1/2*d*x + 1/2*c)^5 - 21*B*tan(1/2*d*x + 1/2*c)^5 + 105*A*tan(1/2*d*x + 1/2
*c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/2*c) + 105*B*ta
n(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B]
time = 0.24, size = 87, normalized size = 0.63

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\,A+B\right)}{24\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(A-B\right)}{56\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A+B\right)}{8\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(3\,A-B\right)}{40\,a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^4,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(3*A + B))/(24*a^4) + (tan(c/2 + (d*x)/2)^7*(A - B))
/(56*a^4) + (tan(c/2 + (d*x)/2)*(A + B))/(8*a^4) + (tan(c/2 + (d*x)/2)^5*(3
*A - B))/(40*a^4))/d
```

3.71 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=147

$$\frac{A\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2}-\frac{2(80A-3B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))}-\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{(10A}{35ad}$$

```
[Out] A*arctanh(sin(d*x+c))/a^4/d-1/105*(55*A-6*B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c)
)^2-2/105*(80*A-3*B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c))-1/7*(A-B)*sin(d*x+c)/d
/(a+a*cos(d*x+c))^4-1/35*(10*A-3*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A]
time = 0.29, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3057, 12, 3855}

$$-\frac{2(80A-3B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(55A-6B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(10A-3B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((55*A - 6*B)*Sin[c + d*x])/(105*a^4*d*
(1 + Cos[c + d*x])^2) - (2*(80*A - 3*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c
 + d*x])) - ((A - B)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((10*A -
3*B)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(7aA-3a(A-B)\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{(35a^2A-2}{}}{}$$

$$= -\frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3}{35ad(a+}$$

$$= -\frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3}{35ad(a+}$$

$$= -\frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3}{35ad(a+}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\,\mathrm{si}}{7d(a+a\,\mathrm{co}}$$

Mathematica [A]
time = 1.55, size = 239, normalized size = 1.63

$$\frac{-6720A\cos^8\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-70(49A-3B)\sin\left(\frac{dx}{2}\right)+2170A\sin\left(c+\frac{dx}{2}\right)-2625A\sin\left(c+\frac{3dx}{2}\right)+126B\sin\left(c+\frac{3dx}{2}\right)+735A\sin\left(2c+\frac{3dx}{2}\right)-1015A\sin\left(2c+\frac{5dx}{2}\right)+42B\sin\left(2c+\frac{5dx}{2}\right)+105A\sin\left(3c+\frac{5dx}{2}\right)-160A\sin\left(3c+\frac{7dx}{2}\right)+6B\sin\left(3c+\frac{7dx}{2}\right)\right)}{420a^4d(1+\cos(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-6720*A*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-70*(49
*A - 3*B)*Sin[(d*x)/2] + 2170*A*Sin[c + (d*x)/2] - 2625*A*Sin[c + (3*d*x)/2
] + 126*B*Sin[c + (3*d*x)/2] + 735*A*Sin[2*c + (3*d*x)/2] - 1015*A*Sin[2*c
+ (5*d*x)/2] + 42*B*Sin[2*c + (5*d*x)/2] + 105*A*Sin[3*c + (5*d*x)/2] - 160
*A*Sin[3*c + (7*d*x)/2] + 6*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c
+ d*x])^4)
```

Maple [A]
time = 0.23, size = 146, normalized size = 0.99

method	result

derivativedivides	$\frac{-A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{3B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+8A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-8A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^7\left(\frac{dx}{2}\right.\right.}{7}}{8d\,a^4}$
default	$\frac{-A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{3B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+8A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-8A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^7\left(\frac{dx}{2}\right.\right.}{7}}{8d\,a^4}$
risch	$-\frac{2i\left(105A\,e^{6i(dx+c)}+735A\,e^{5i(dx+c)}+2170A\,e^{4i(dx+c)}+3430A\,e^{3i(dx+c)}-210B\,e^{3i(dx+c)}+2625A\,e^{2i(dx+c)}-126B\,e^{2i(dx\right.}}{105d\,a^4\left(e^{i(dx+c)}+1\right)^7}$
norman	$\frac{-\frac{(A-B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56da}-\frac{(15A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{(20A-13B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{140da}-\frac{(28A-3B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}-\frac{(35A-12B)\left(\tan^5\left(\frac{d}{}\right.\right.}{60da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(-A*tan(1/2*d*x+1/2*c)^5+3/5*B*tan(1/2*d*x+1/2*c)^5+8*A*ln(tan(1/
2*d*x+1/2*c)+1)-8*A*ln(tan(1/2*d*x+1/2*c)-1)-1/7*tan(1/2*d*x+1/2*c)^7*A+1/7
*tan(1/2*d*x+1/2*c)^7*B-11/3*tan(1/2*d*x+1/2*c)^3*A+tan(1/2*d*x+1/2*c)^3*B-
15*A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A]

time = 0.28, size = 228, normalized size = 1.55

$$-\frac{5\,A\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)-\frac{3\,B\left(\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] -1/840*(5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
^4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - 3*B*(35*sin(d*x +
c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/
d
```

Fricas [A]

time = 0.38, size = 236, normalized size = 1.61

$$\frac{105\,\left(A\cos\left(dx+c\right)^4+4\,A\cos\left(dx+c\right)^3+6\,A\cos\left(dx+c\right)^2+4\,A\cos\left(dx+c\right)+A\right)\log\left(\sin\left(dx+c\right)+1\right)-105\,\left(A\cos\left(dx+c\right)^4+4\,A\cos\left(dx+c\right)^3+6\,A\cos\left(dx+c\right)^2+4\,A\cos\left(dx+c\right)+A\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(2\,\left(80\,A-3\,B\right)\cos\left(dx+c\right)^3+\left(535\,A-24\,B\right)\cos\left(dx+c\right)^2+\left(620\,A-39\,B\right)\cos\left(dx+c\right)+260\,A-36\,B\right)\sin\left(dx+c\right)}{210\,\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] 1/210*(105*(A*cos(d*x + c)^4 + 4*A*cos(d*x + c)^3 + 6*A*cos(d*x + c)^2 + 4*
A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - 105*(A*cos(d*x + c)^4 + 4*A*cos
(d*x + c)^3 + 6*A*cos(d*x + c)^2 + 4*A*cos(d*x + c) + A)*log(-sin(d*x + c)
+ 1) - 2*(2*(80*A - 3*B)*cos(d*x + c)^3 + (535*A - 24*B)*cos(d*x + c)^2 + (
620*A - 39*B)*cos(d*x + c) + 260*A - 36*B)*sin(d*x + c))/(a^4*d*cos(d*x + c
)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c
) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d
*x)**2 + 4*cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)/(co
s(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x) + 1)
, x))/a**4
```

Giac [A]
time = 0.47, size = 182, normalized size = 1.24

$$\frac{\frac{840\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{840\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-63\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+385\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-105\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1575\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-105\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] 1/840*(840*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*A*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^4 - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan
(1/2*d*x + 1/2*c)^7 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 63*B*a^24*tan(1/2
*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 105*B*a^24*tan(1/2*d*
x + 1/2*c)^3 + 1575*A*a^24*tan(1/2*d*x + 1/2*c) - 105*B*a^24*tan(1/2*d*x +
1/2*c))/a^28)/d
```

Mupad [B]
time = 0.36, size = 199, normalized size = 1.35

$$\frac{2\,A\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^4\,d}-\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{11\,A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}-\frac{B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8}-\frac{3\,B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{40}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{15\,A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}-\frac{B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}\right)+\frac{A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56}-\frac{B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56}}{a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^4),x)
```

```
[Out] (2*A*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^4*d) - (cos(c/2 + (d*
x)/2)^4*((11*A*sin(c/2 + (d*x)/2)^3)/24 - (B*sin(c/2 + (d*x)/2)^3)/8) + cos
(c/2 + (d*x)/2)^2*((A*sin(c/2 + (d*x)/2)^5)/8 - (3*B*sin(c/2 + (d*x)/2)^5)/
40) + cos(c/2 + (d*x)/2)^6*((15*A*sin(c/2 + (d*x)/2))/8 - (B*sin(c/2 + (d*x
)/2))/8) + (A*sin(c/2 + (d*x)/2)^7)/56 - (B*sin(c/2 + (d*x)/2)^7)/56)/(a^4*
d*cos(c/2 + (d*x)/2)^7)
```

3.72 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=175

$$-\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{8(83A-20B)\tan(c+dx)}{105a^4d}-\frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2}-\frac{(4A-B)\tan(c+dx)}{a^4d(1+\cos(c+dx))}$$

```
[Out] -(4*A-B)*arctanh(sin(d*x+c))/a^4/d+8/105*(83*A-20*B)*tan(d*x+c)/a^4/d-1/105
*(88*A-25*B)*tan(d*x+c)/a^4/d/(1+cos(d*x+c))^2-(4*A-B)*tan(d*x+c)/a^4/d/(1+
cos(d*x+c))-1/7*(A-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^4-1/35*(12*A-5*B)*tan(d
*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A]

time = 0.42, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3057, 2827, 3852, 8, 3855}

$$\frac{8(83A-20B)\tan(c+dx)}{105a^4d}-\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(4A-B)\tan(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{(88A-25B)\tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(12A-5B)\tan(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] -(((4*A - B)*ArcTanh[Sin[c + d*x]])/(a^4*d)) + (8*(83*A - 20*B)*Tan[c + d*x
])/(105*a^4*d) - ((88*A - 25*B)*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])
^2) - ((4*A - B)*Tan[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) - ((A - B)*Tan[c
+ d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((12*A - 5*B)*Tan[c + d*x])/(35*a*d*
(a + a*Cos[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(a(8A-B)-4a(A-B)\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5B)\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{(2a^2(26A-}{}}{}$$

$$= -\frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5}{35ad(a+}$$

$$= -\frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5}{35ad(a+}$$

$$= -\frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5}{35ad(a+}$$

$$= -\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(}{7a}$$

$$= -\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{8(83A-20B)\tan(c+dx)}{105a^4d} - \frac{(}{1}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 595 vs. 2(175) = 350.
time = 5.42, size = 595, normalized size = 3.40

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (26880*(4*A - B)*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*
Sec[c]*Sec[c + d*x]*(-245*(44*A - 17*B)*Sin[(d*x)/2] + 7*(2684*A - 635*B)*S
in[(3*d*x)/2] - 20524*A*Sin[c - (d*x)/2] + 4795*B*Sin[c - (d*x)/2] + 14644*
A*Sin[c + (d*x)/2] - 4795*B*Sin[c + (d*x)/2] - 16660*A*Sin[2*c + (d*x)/2] +
 4165*B*Sin[2*c + (d*x)/2] - 4690*A*Sin[c + (3*d*x)/2] + 2275*B*Sin[c + (3*
d*x)/2] + 14378*A*Sin[2*c + (3*d*x)/2] - 4445*B*Sin[2*c + (3*d*x)/2] - 9100
*A*Sin[3*c + (3*d*x)/2] + 2275*B*Sin[3*c + (3*d*x)/2] + 11668*A*Sin[c + (5*
d*x)/2] - 2785*B*Sin[c + (5*d*x)/2] - 630*A*Sin[2*c + (5*d*x)/2] + 735*B*Si
n[2*c + (5*d*x)/2] + 9358*A*Sin[3*c + (5*d*x)/2] - 2785*B*Sin[3*c + (5*d*x)
/2] - 2940*A*Sin[4*c + (5*d*x)/2] + 735*B*Sin[4*c + (5*d*x)/2] + 4228*A*Sin
[2*c + (7*d*x)/2] - 1015*B*Sin[2*c + (7*d*x)/2] + 315*A*Sin[3*c + (7*d*x)/2
] + 105*B*Sin[3*c + (7*d*x)/2] + 3493*A*Sin[4*c + (7*d*x)/2] - 1015*B*Sin[4
*c + (7*d*x)/2] - 420*A*Sin[5*c + (7*d*x)/2] + 105*B*Sin[5*c + (7*d*x)/2] +
 664*A*Sin[3*c + (9*d*x)/2] - 160*B*Sin[3*c + (9*d*x)/2] + 105*A*Sin[4*c +
(9*d*x)/2] + 559*A*Sin[5*c + (9*d*x)/2] - 160*B*Sin[5*c + (9*d*x)/2]))/(168
0*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A]

time = 0.26, size = 190, normalized size = 1.09

method	result
derivativedivides	$\frac{(-32A+8B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{8A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}+\frac{7A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-B\left(\tan^5\left(\right.\right.}{}$
default	$\frac{(-32A+8B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{8A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}+\frac{7A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-B\left(\tan^5\left(\right.\right.}{}$
norman	$\frac{\frac{(A-B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56da}+\frac{(7A-5B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40da}-\frac{5(13A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{7(17A-5B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}-\frac{(71A-11B)\left(\tan^3\right.}{24da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^3}$
risch	$\frac{2i\left(420A\,e^{8i(dx+c)}-105B\,e^{8i(dx+c)}+2940A\,e^{7i(dx+c)}-735B\,e^{7i(dx+c)}+9100A\,e^{6i(dx+c)}-2275B\,e^{6i(dx+c)}+16660A\,e^{5i(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d/a^4*((-32*A+8*B)*ln(tan(1/2*d*x+1/2*c)+1)-8*A/(tan(1/2*d*x+1/2*c)+1)+
1/7*tan(1/2*d*x+1/2*c)^7*A-1/7*tan(1/2*d*x+1/2*c)^7*B+7/5*A*tan(1/2*d*x+1/2
*c)^5-B*tan(1/2*d*x+1/2*c)^5+23/3*tan(1/2*d*x+1/2*c)^3*A-11/3*tan(1/2*d*x+1
/2*c)^3*B+49*A*tan(1/2*d*x+1/2*c)-15*B*tan(1/2*d*x+1/2*c)+(32*A-8*B)*ln(tan
(1/2*d*x+1/2*c)-1)-8*A/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [A]

time = 0.27, size = 326, normalized size = 1.86

$$\frac{A\left(\frac{1680 \sin(dx+c)}{\left(a^4-\frac{a^4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145 \sin(dx+c)}{\cos(dx+c)+1}+\frac{805 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{3360 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)-5 B\left(\frac{\frac{315 \sin(dx+c)}{\cos(dx+c)+1}+\frac{77 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)}{840\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(A*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - 5
*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 168*
log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(167) = 334$.
time = 0.37, size = 337, normalized size = 1.93

$$-\frac{105\left((4A-B)\cos(dx+c)^5+4(4A-B)\cos(dx+c)^4+6(4A-B)\cos(dx+c)^3+4(4A-B)\cos(dx+c)^2+(4A-B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-105\left((4A-B)\cos(dx+c)^5+4(4A-B)\cos(dx+c)^4+6(4A-B)\cos(dx+c)^3+4(4A-B)\cos(dx+c)^2+(4A-B)\cos(dx+c)\right)\log(-\sin(dx+c)+1)-2\left(8(83A-20B)\cos(dx+c)^4+(2236A-535B)\cos(dx+c)^3+4(659A-155B)\cos(dx+c)^2+4(296A-65B)\cos(dx+c)+105A\right)\sin(dx+c)}{210\left(a^4d\cos(dx+c)^5+4a^4d\cos(dx+c)^4+6a^4d\cos(dx+c)^3+4a^4d\cos(dx+c)^2+a^4d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] -1/210*(105*((4*A - B)*cos(d*x + c)^5 + 4*(4*A - B)*cos(d*x + c)^4 + 6*(4*A
 - B)*cos(d*x + c)^3 + 4*(4*A - B)*cos(d*x + c)^2 + (4*A - B)*cos(d*x + c))
*log(sin(d*x + c) + 1) - 105*((4*A - B)*cos(d*x + c)^5 + 4*(4*A - B)*cos(d*
x + c)^4 + 6*(4*A - B)*cos(d*x + c)^3 + 4*(4*A - B)*cos(d*x + c)^2 + (4*A -
 B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(8*(83*A - 20*B)*cos(d*x + c)^
4 + (2236*A - 535*B)*cos(d*x + c)^3 + 4*(659*A - 155*B)*cos(d*x + c)^2 + 4*
(296*A - 65*B)*cos(d*x + c) + 105*A)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 +
4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 +
a^4*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^2(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\, dx+\int \frac{B \cos(c+dx) \sec^2(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\, dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c
+ d*x)**2 + 4*cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)*
*2/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x
) + 1), x))/a**4
```

Giac [A]
time = 0.47, size = 224, normalized size = 1.28

$$-\frac{\frac{840\,(4\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{840\,(4\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{1680\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+147\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-105\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+805\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-385\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5145\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1575\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(840*(4*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*(4*A - B
)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 1680*A*tan(1/2*d*x + 1/2*c)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)*a^4) - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a
^24*tan(1/2*d*x + 1/2*c)^7 + 147*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 105*B*a^24
*tan(1/2*d*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*ta
n(1/2*d*x + 1/2*c)^3 + 5145*A*a^24*tan(1/2*d*x + 1/2*c) - 1575*B*a^24*tan(1
/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B]
time = 0.28, size = 236, normalized size = 1.35

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{8\,a^4}+\frac{5\,A-3\,B}{12\,a^4}+\frac{10\,A-2\,B}{24\,a^4}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{A-B}{20\,a^4}+\frac{5\,A-3\,B}{40\,a^4}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{2\,a^4}+\frac{3\,(5\,A-3\,B)}{8\,a^4}+\frac{10\,A-2\,B}{4\,a^4}+\frac{10\,A+2\,B}{8\,a^4}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(A-B)}{56\,a^4\,d}-\frac{2\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^4\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(4\,A-B)}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^4),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((A - B)/(8*a^4) + (5*A - 3*B)/(12*a^4) + (10*A - 2*B
)/(24*a^4)))/d + (tan(c/2 + (d*x)/2)^5*((A - B)/(20*a^4) + (5*A - 3*B)/(40*
a^4)))/d + (tan(c/2 + (d*x)/2)*((A - B)/(2*a^4) + (3*(5*A - 3*B))/(8*a^4) +
 (10*A - 2*B)/(4*a^4) + (10*A + 2*B)/(8*a^4)))/d + (tan(c/2 + (d*x)/2)^7*(A
 - B))/(56*a^4*d) - (2*A*tan(c/2 + (d*x)/2))/(d*(a^4*tan(c/2 + (d*x)/2)^2 -
 a^4)) - (2*atanh(tan(c/2 + (d*x)/2))*(4*A - B))/(a^4*d)
```

3.73 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=232

$$\frac{(21A-8B)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{8(216A-83B)\tan(c+dx)}{105a^4d}+\frac{(21A-8B)\sec(c+dx)\tan(c+dx)}{2a^4d}-\frac{(129A$$

```
[Out] 1/2*(21*A-8*B)*arctanh(sin(d*x+c))/a^4/d-8/105*(216*A-83*B)*tan(d*x+c)/a^4/
d+1/2*(21*A-8*B)*sec(d*x+c)*tan(d*x+c)/a^4/d-1/105*(129*A-52*B)*sec(d*x+c)*
tan(d*x+c)/a^4/d/(1+cos(d*x+c))^2-4/105*(216*A-83*B)*sec(d*x+c)*tan(d*x+c)/
a^4/d/(1+cos(d*x+c))-1/7*(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^4-1
/5*(2*A-B)*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A]
time = 0.43, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3057, 2827, 3853, 3855, 3852, 8}

$$-\frac{8(216A-83B)\tan(c+dx)}{105a^4d}+\frac{(21A-8B)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(21A-8B)\tan(c+dx)\sec(c+dx)}{2a^4d}-\frac{4(216A-83B)\tan(c+dx)\sec(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(129A-52B)\tan(c+dx)\sec(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(2A-B)\tan(c+dx)\sec(c+dx)}{5ad(a\cos(c+dx)+a)^3}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] ((21*A - 8*B)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (8*(216*A - 83*B)*Tan[c +
d*x])/(105*a^4*d) + ((21*A - 8*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - ((
129*A - 52*B)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) -
 (4*(216*A - 83*B)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])
) - ((A - B)*Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((2*
A - B)*Sec[c + d*x]*Tan[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
```

```
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(a(9A-2B)-5a(A-B)\cos(c+dx)}{(a+a\cos(c+dx))^3}}{7a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(2A-B)\sec(c+dx)\tan(c}{5ad(a+a\cos(c+dx))}$$

$$= -\frac{(129A-52B)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sec(c+dx)\,\mathrm{t}}{7d(a+a\cos(c+}$$

$$= -\frac{(129A-52B)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sec(c+dx)\,\mathrm{t}}{7d(a+a\cos(c+}$$

$$= -\frac{(129A-52B)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sec(c+dx)\,\mathrm{t}}{7d(a+a\cos(c+}$$

$$= \frac{(21A-8B)\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{(129A-52B)\sec(c+dx}{105a^4d(1+\cos(c+}$$

$$= \frac{(21A-8B)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{8(216A-83B)\tan(c+dx)}{105a^4d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 798 vs. 2(232) = 464.
time = 6.50, size = 798, normalized size = 3.44

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-8*(21*A - 8*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2]])/(d*(a + a*Cos[c + d*x])^4) + (8*(21*A - 8*B)*Cos[c/2 + (d*x)/2]^8*
Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^4) +
(Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(73206*A*Sin[(d*x)/2] -
38668*B*Sin[(d*x)/2] - 166668*A*Sin[(3*d*x)/2] + 64384*B*Sin[(3*d*x)/2] + 1
83162*A*Sin[c - (d*x)/2] - 70896*B*Sin[c - (d*x)/2] - 100842*A*Sin[c + (d*x
)/2] + 50316*B*Sin[c + (d*x)/2] + 155526*A*Sin[2*c + (d*x)/2] - 59248*B*Sin
[2*c + (d*x)/2] + 37380*A*Sin[c + (3*d*x)/2] - 22820*B*Sin[c + (3*d*x)/2] -
 101148*A*Sin[2*c + (3*d*x)/2] + 48004*B*Sin[2*c + (3*d*x)/2] + 102900*A*Si
n[3*c + (3*d*x)/2] - 39200*B*Sin[3*c + (3*d*x)/2] - 119364*A*Sin[c + (5*d*x
)/2] + 46032*B*Sin[c + (5*d*x)/2] + 8820*A*Sin[2*c + (5*d*x)/2] - 8750*B*Si
n[2*c + (5*d*x)/2] - 78204*A*Sin[3*c + (5*d*x)/2] + 35742*B*Sin[3*c + (5*d*
x)/2] + 49980*A*Sin[4*c + (5*d*x)/2] - 19040*B*Sin[4*c + (5*d*x)/2] - 64053
*A*Sin[2*c + (7*d*x)/2] + 24664*B*Sin[2*c + (7*d*x)/2] - 3885*A*Sin[3*c + (
7*d*x)/2] - 1050*B*Sin[3*c + (7*d*x)/2] - 44733*A*Sin[4*c + (7*d*x)/2] + 19
834*B*Sin[4*c + (7*d*x)/2] + 15435*A*Sin[5*c + (7*d*x)/2] - 5880*B*Sin[5*c
+ (7*d*x)/2] - 21987*A*Sin[3*c + (9*d*x)/2] + 8456*B*Sin[3*c + (9*d*x)/2] -
 3675*A*Sin[4*c + (9*d*x)/2] + 630*B*Sin[4*c + (9*d*x)/2] - 16107*A*Sin[5*c
 + (9*d*x)/2] + 6986*B*Sin[5*c + (9*d*x)/2] + 2205*A*Sin[6*c + (9*d*x)/2] -
 840*B*Sin[6*c + (9*d*x)/2] - 3456*A*Sin[4*c + (11*d*x)/2] + 1328*B*Sin[4*c
 + (11*d*x)/2] - 840*A*Sin[5*c + (11*d*x)/2] + 210*B*Sin[5*c + (11*d*x)/2]
- 2616*A*Sin[6*c + (11*d*x)/2] + 1118*B*Sin[6*c + (11*d*x)/2]))/(6720*d*(a
+ a*Cos[c + d*x])^4)
```

Maple [A]
time = 0.32, size = 234, normalized size = 1.01

method	result
derivativedivides	$-\frac{-36A+8B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+(84A-32B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{4A}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}-\frac{9A\left(\tan^5\left(\right.\right.}{5}$
default	$-\frac{-36A+8B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+(84A-32B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{4A}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}-\frac{9A\left(\tan^5\left(\right.\right.}{5}$
norman	$\frac{-\frac{(A-B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56da}-\frac{(29A-22B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{140da}-\frac{(167A-65B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{(171A-62B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}-\frac{(1161A-643}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2 a}$

risch	$-\frac{i\left(2205\,\mathrm{e}^{10i(dx+c)}A-840B\,\mathrm{e}^{10i(dx+c)}+15435A\,\mathrm{e}^{9i(dx+c)}-5880B\,\mathrm{e}^{9i(dx+c)}+49980A\,\mathrm{e}^{8i(dx+c)}-19040B\,\mathrm{e}^{8i(dx+c)}+10\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d/a^4*(-(-36*A+8*B)/(tan(1/2*d*x+1/2*c)+1)+(84*A-32*B)*ln(tan(1/2*d*x+1
/2*c)+1)-4*A/(tan(1/2*d*x+1/2*c)+1)^2-1/7*tan(1/2*d*x+1/2*c)^7*A+1/7*tan(1/
2*d*x+1/2*c)^7*B-9/5*A*tan(1/2*d*x+1/2*c)^5+7/5*B*tan(1/2*d*x+1/2*c)^5-13*t
an(1/2*d*x+1/2*c)^3*A+23/3*tan(1/2*d*x+1/2*c)^3*B-111*A*tan(1/2*d*x+1/2*c)+
49*B*tan(1/2*d*x+1/2*c)+(-84*A+32*B)*ln(tan(1/2*d*x+1/2*c)-1)-(-36*A+8*B)/(
tan(1/2*d*x+1/2*c)-1)+4*A/(tan(1/2*d*x+1/2*c)-1)^2)
```

Maxima [A]
time = 0.28, size = 419, normalized size = 1.81

$$-\frac{3\,A\left(\frac{280\left(\frac{7\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{2940\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)-B\left(\frac{1680\,\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{3360\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(3*A*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
+ 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) -
1)/a^4) - B*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d
*x + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))
/d
```

Fricas [A]
time = 0.36, size = 360, normalized size = 1.55

$$\frac{105\left((21\,A-8\,B)\cos(dx+c)^6+4\,(21\,A-8\,B)\cos(dx+c)^5+6\,(21\,A-8\,B)\cos(dx+c)^4+4\,(21\,A-8\,B)\cos(dx+c)^3+(21\,A-8\,B)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-105\left((21\,A-8\,B)\cos(dx+c)^6+4\,(21\,A-8\,B)\cos(dx+c)^5+6\,(21\,A-8\,B)\cos(dx+c)^4+4\,(21\,A-8\,B)\cos(dx+c)^3+(21\,A-8\,B)\cos(dx+c)^2\right)\log(-\sin(dx+c)+1)-2\left(16\,(216\,A-83\,B)\cos(dx+c)^5+(11619\,A-4472\,B)\cos(dx+c)^4+4\,(3411\,A-1318\,B)\cos(dx+c)^3+4\,(1509\,A-592\,B)\cos(dx+c)^2+210\,(2\,A-B)\cos(dx+c)-105\,A\right)\sin(dx+c)}{420\left(a^4d\cos(dx+c)^6+4\,a^4d\cos(dx+c)^5+6\,a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+a^4d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/420*(105*((21*A - 8*B)*cos(d*x + c)^6 + 4*(21*A - 8*B)*cos(d*x + c)^5 + 6
*(21*A - 8*B)*cos(d*x + c)^4 + 4*(21*A - 8*B)*cos(d*x + c)^3 + (21*A - 8*B)
*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 105*((21*A - 8*B)*cos(d*x + c)^6 +
 4*(21*A - 8*B)*cos(d*x + c)^5 + 6*(21*A - 8*B)*cos(d*x + c)^4 + 4*(21*A -
8*B)*cos(d*x + c)^3 + (21*A - 8*B)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) -
 2*(16*(216*A - 83*B)*cos(d*x + c)^5 + (11619*A - 4472*B)*cos(d*x + c)^4 +
4*(3411*A - 1318*B)*cos(d*x + c)^3 + 4*(1509*A - 592*B)*cos(d*x + c)^2 + 21
0*(2*A - B)*cos(d*x + c) - 105*A)*sin(d*x + c))/(a^4*d*cos(d*x + c)^6 + 4*a
^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + a^4
*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^3(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^3(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c
+ d*x)**2 + 4*cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)*
*3/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x
) + 1), x))/a**4
```

Giac [A]
time = 0.50, size = 267, normalized size = 1.15

$$\frac{\frac{420\,(21\,A-8\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{420\,(21\,A-8\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{840\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+189\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-147\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1365\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-805\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+11655\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5145\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(420*(21*A - 8*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 420*(21*A
- 8*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 840*(9*A*tan(1/2*d*x + 1/2*
c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 - 7*A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*
d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (15*A*a^24*tan(1/2*d*x
 + 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 + 189*A*a^24*tan(1/2*d*x + 1
/2*c)^5 - 147*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 1365*A*a^24*tan(1/2*d*x + 1/2
*c)^3 - 805*B*a^24*tan(1/2*d*x + 1/2*c)^3 + 11655*A*a^24*tan(1/2*d*x + 1/2*
c) - 5145*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B]
time = 0.29, size = 273, normalized size = 1.18

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(9\,A-2\,B)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(7\,A-2\,B)}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^4\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,A}{2\,a^4}+\frac{5\,(A-B)}{4\,a^4}+\frac{3\,(6\,A-4\,B)}{4\,a^4}+\frac{3\,(15\,A-5\,B)}{8\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{4\,a^4}+\frac{6\,A-4\,B}{8\,a^4}+\frac{15\,A-5\,B}{24\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{3\,(A-B)}{40\,a^4}+\frac{6\,A-4\,B}{40\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(A-B)}{56\,a^4\,d}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(21\,A-8\,B)}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^4),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(9*A - 2*B) - tan(c/2 + (d*x)/2)*(7*A - 2*B))/(d*(a^4
*tan(c/2 + (d*x)/2)^4 - 2*a^4*tan(c/2 + (d*x)/2)^2 + a^4)) - (tan(c/2 + (d*
x)/2)*((5*A)/(2*a^4) + (5*(A - B))/(4*a^4) + (3*(6*A - 4*B))/(4*a^4) + (3*(
15*A - 5*B))/(8*a^4)))/d - (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^4) + (6*A -
4*B)/(8*a^4) + (15*A - 5*B)/(24*a^4)))/d - (tan(c/2 + (d*x)/2)^5*((3*(A - B
))/(40*a^4) + (6*A - 4*B)/(40*a^4)))/d - (tan(c/2 + (d*x)/2)^7*(A - B))/(56
*a^4*d) + (atanh(tan(c/2 + (d*x)/2))*(21*A - 8*B))/(a^4*d)
```

3.74 $\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=187

$$\frac{4a(9A+8B)\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}}+\frac{2a(9A+8B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}}+\frac{2aB\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}}-\frac{8(9A+8B)}{}$$

```
[Out] 4/105*(9*A+8*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+4/45*a*(9*A+8*B)*sin(
d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/63*a*(9*A+8*B)*cos(d*x+c)^3*sin(d*x+c)/d/
(a+a*cos(d*x+c))^(1/2)+2/9*a*B*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(
1/2)-8/315*(9*A+8*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3060, 2849, 2838, 2830, 2725}

$$\frac{2a(9A+8B)\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{4(9A+8B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105ad}-\frac{8(9A+8B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{4a(9A+8B)\sin(c+dx)}{45d\sqrt{a\cos(c+dx)+a}}+\frac{2aB\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*a*(9*A + 8*B)*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(9*A
+ 8*B)*Cos[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
B*Cos[c + d*x]^4*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (8*(9*A + 8
*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (4*(9*A + 8*B)*(a + a*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*a*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{2aB\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{1}{9}(9A+8B)$$

$$= \frac{2a(9A+8B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aB}{9}$$

$$= \frac{2a(9A+8B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aB}{9}$$

$$= \frac{2a(9A+8B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aB}{9}$$

$$= \frac{4a(9A+8B)\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} + \frac{2a(9A+8B)\cos}{63d\sqrt{a+}}$$

Mathematica [A]
time = 0.70, size = 103, normalized size = 0.55

$$\frac{\sqrt{a(1+\cos(c+dx))}\,(1368A+1321B+94(9A+8B)\cos(c+dx)+4(54A+83B)\cos(2(c+dx))+90A\cos(3(c+dx))+80B\cos(3(c+dx))+35B\cos(4(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(1368*A + 1321*B + 94*(9*A + 8*B)*Cos[c + d*x]
+ 4*(54*A + 83*B)*Cos[2*(c + d*x)] + 90*A*Cos[3*(c + d*x)] + 80*B*Cos[3*(c
+ d*x)] + 35*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A]
time = 0.22, size = 121, normalized size = 0.65

method	result
default	$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(560B\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-360A-1440B\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(756A+1512B\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-630A-840\right.}{315\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/315*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(560*B*sin(1/2*d*x+1/2*c)^8+(
-360*A-1440*B)*sin(1/2*d*x+1/2*c)^6+(756*A+1512*B)*sin(1/2*d*x+1/2*c)^4+(-6
30*A-840*B)*sin(1/2*d*x+1/2*c)^2+315*A+315*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)
^2)^(1/2)/d
```

Maxima [A]
time = 0.57, size = 145, normalized size = 0.78

$$\frac{18\left(5\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+35\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+105\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+\left(35\sqrt{2}\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+45\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+252\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+420\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+1890\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)B\sqrt{a}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/2520*(18*(5*sqrt(2)*sin(7/2*d*x + 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c)
 + 35*sqrt(2)*sin(3/2*d*x + 3/2*c) + 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*sq
rt(a) + (35*sqrt(2)*sin(9/2*d*x + 9/2*c) + 45*sqrt(2)*sin(7/2*d*x + 7/2*c)
+ 252*sqrt(2)*sin(5/2*d*x + 5/2*c) + 420*sqrt(2)*sin(3/2*d*x + 3/2*c) + 189
0*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A]
time = 0.34, size = 99, normalized size = 0.53

$$\frac{2\left(35\,B\cos\left(dx+c\right)^{4}+5\left(9\,A+8\,B\right)\cos\left(dx+c\right)^{3}+6\left(9\,A+8\,B\right)\cos\left(dx+c\right)^{2}+8\left(9\,A+8\,B\right)\cos\left(dx+c\right)+144\,A+128\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/315*(35*B*cos(d*x + c)^4 + 5*(9*A + 8*B)*cos(d*x + c)^3 + 6*(9*A + 8*B)*c
os(d*x + c)^2 + 8*(9*A + 8*B)*cos(d*x + c) + 144*A + 128*B)*sqrt(a*cos(d*x
+ c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.81, size = 182, normalized size = 0.97

$$\frac{\sqrt{2}\left(35\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+45\left(2\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+126\left(A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+2\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+210\left(3\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+2\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+1890\left(A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*B*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) + 45*(2
*A*sgn(cos(1/2*d*x + 1/2*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(7/2*d*x + 7
/2*c) + 126*(A*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*sgn(cos(1/2*d*x + 1/2*c)))*s
in(5/2*d*x + 5/2*c) + 210*(3*A*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*sgn(cos(1/2*
d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c) + 1890*(A*sgn(cos(1/2*d*x + 1/2*c)) + B
*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

3.75 $\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=144

$$\frac{2a(7A+6B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2aB\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}}-\frac{4(7A+6B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{105d}+\frac{2(7A$$

```
[Out] 2/35*(7*A+6*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+2/15*a*(7*A+6*B)*sin(d
*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a*B*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d
*x+c))^(1/2)-4/105*(7*A+6*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3060, 2838, 2830, 2725}

$$\frac{2(7A+6B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35ad}-\frac{4(7A+6B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(7A+6B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2aB\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a*(7*A + 6*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*B*Cos
[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) - (4*(7*A + 6*B)*S
qrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A + 6*B)*(a + a*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(35*a*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
```

```
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{2aB\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}(7A+6B)$$

$$= \frac{2aB\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2(7A+6B)(}{}$$

$$= \frac{2aB\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{4(7A+6B)}{}$$

$$= \frac{2a(7A+6B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\cos^3(c+dx)}{7d\sqrt{a+a\cos}}$$

Mathematica [A]
time = 0.39, size = 80, normalized size = 0.56

$$\frac{\sqrt{a(1+\cos(c+dx))}\,(266A+228B+(112A+141B)\cos(c+dx)+6(7A+6B)\cos(2(c+dx))+15B\cos(3(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(266*A + 228*B + (112*A + 141*B)*Cos[c + d*x] +
 6*(7*A + 6*B)*Cos[2*(c + d*x)] + 15*B*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/
(210*d)
```

Maple [A]
time = 0.22, size = 102, normalized size = 0.71

method	result
default	$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-120B\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(84A+252B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-140A-210B)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+105A+105B\right)}{105\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/105*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(-120*B*sin(1/2*d*x+1/2*c)^6+
(84*A+252*B)*sin(1/2*d*x+1/2*c)^4+(-140*A-210*B)*sin(1/2*d*x+1/2*c)^2+105*A
+105*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A]
time = 0.56, size = 118, normalized size = 0.82

$$\frac{14\left(3\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+30\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+3\left(5\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+35\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+105\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)B\sqrt{a}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/420*(14*(3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c)
+ 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 3*(5*sqrt(2)*sin(7/2*d*x + 7
/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c) + 35*sqrt(2)*sin(3/2*d*x + 3/2*c) +
105*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A]
time = 0.33, size = 82, normalized size = 0.57

$$\frac{2\left(15\,B\cos\left(dx+c\right)^3+3\left(7\,A+6\,B\right)\cos\left(dx+c\right)^2+4\left(7\,A+6\,B\right)\cos\left(dx+c\right)+56\,A+48\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/105*(15*B*cos(d*x + c)^3 + 3*(7*A + 6*B)*cos(d*x + c)^2 + 4*(7*A + 6*B)*c
os(d*x + c) + 56*A + 48*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x
 + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)\right)\cos^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*cos(c + d*x)**2, x
)
```

Giac [A]
time = 0.58, size = 147, normalized size = 1.02

$$\frac{\sqrt{2}\,\left(15\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\,\left(2\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+35\,\left(2\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+3\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+105\,\left(4\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+3\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] 1/420*sqrt(2)*(15*B*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 21*(2*
A*sgn(cos(1/2*d*x + 1/2*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(5/2*d*x + 5/
2*c) + 35*(2*A*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*sgn(cos(1/2*d*x + 1/2*c)))*s
in(3/2*d*x + 3/2*c) + 105*(4*A*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*sgn(cos(1/2*
d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

3.76 $\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{2a(5A+7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2(5A-2B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15d}+\frac{2B(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5ad}$$

```
[Out] 2/5*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+2/15*a*(5*A+7*B)*sin(d*x+c)/d/(
a+a*cos(d*x+c))^(1/2)+2/15*(5*A-2*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3047, 3102, 2830, 2725}

$$\frac{2(5A-2B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a(5A+7B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a*(5*A + 7*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(5*A -
2*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*B*(a + a*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(5*a*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \int \sqrt{a+a\cos(c+dx)}\,(A\cos(c+dx)+B\cos$$
$$= \frac{2B(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5ad} + \frac{2\int\sqrt{}}{}$$
$$= \frac{2(5A-2B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15d}$$
$$= \frac{2a(5A+7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2(5A-2B)\sqrt{a+}}{}$$

Mathematica [A]
time = 0.23, size = 64, normalized size = 0.63

$$\frac{\sqrt{a(1+\cos(c+dx))}\,(20A+19B+2(5A+4B)\cos(c+dx)+3B\cos(2(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(20*A + 19*B + 2*(5*A + 4*B)*Cos[c + d*x] + 3*B
*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d)
```

Maple [A]
time = 0.18, size = 83, normalized size = 0.82

method	result	size
default	$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(12B\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-10A-20B)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15A+15B\right)\sqrt{2}}{15\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/15*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(12*B*sin(1/2*d*x+1/2*c)^4+(-1
0*A-20*B)*sin(1/2*d*x+1/2*c)^2+15*A+15*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^
(1/2)/d
```

Maxima [A]
time = 0.55, size = 88, normalized size = 0.87

$$\frac{10\left(\sqrt{2}\,\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sqrt{2}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+\left(3\,\sqrt{2}\,\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}\,\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+30\,\sqrt{2}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/30*(10*(sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*
sqrt(a) + (3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c)
+ 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A]
time = 0.35, size = 64, normalized size = 0.63

$$\frac{2\left(3\,B\cos\left(dx+c\right)^{2}+\left(5\,A+4\,B\right)\cos\left(dx+c\right)+10\,A+8\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/15*(3*B*cos(d*x + c)^2 + (5*A + 4*B)*cos(d*x + c) + 10*A + 8*B)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\left(A+B\cos\left(c+dx\right)\right)\cos\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*cos(c + d*x), x)
```

Giac [A]
time = 0.47, size = 107, normalized size = 1.06

$$\frac{\sqrt{2}\left(3\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\left(2\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+30\left(A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/30*sqrt(2)*(3*B*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 5*(2*A*s
gn(cos(1/2*d*x + 1/2*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c
) + 30*(A*sgn(cos(1/2*d*x + 1/2*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*
d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

3.77 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=62

$$\frac{2a(3A+B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2B\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2/3*a*(3*A+B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)*(a+a*cos
(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2830, 2725}

$$\frac{2a(3A+B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a*(3*A + B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*B*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx &= \frac{2B\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{1}{3}(3A+B)\int \sqrt{a+\\ &= \frac{2a(3A+B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2B\sqrt{a+a\cos(c+dx)}\,\sin(c+}{3d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 46, normalized size = 0.74

$$\frac{2\sqrt{a(1+\cos(c+dx))}\,(3A+2B+B\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(3*A + 2*B + B*Cos[c + d*x])*Tan[(c + d*x)/2]
)/(3*d)
```

Maple [A]
time = 0.18, size = 62, normalized size = 1.00

method	result	size
default	$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2B\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A+B\right)\sqrt{2}}{3\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(2*B*cos(1/2*d*x+1/2*c)^2+3*A+B
)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A]
time = 0.54, size = 57, normalized size = 0.92

$$\frac{6\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\left(\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)B\sqrt{a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(6*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*sin(3/2*d*x + 3/2*
c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A]
time = 0.34, size = 47, normalized size = 0.76

$$\frac{2\left(B\cos\left(dx+c\right)+3A+2B\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/3*(B*cos(d*x + c) + 3*A + 2*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*c
os(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\cos(c+dx)+1)}\,(A+B\cos(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x)), x)
```

Giac [A]
time = 0.45, size = 70, normalized size = 1.13

$$\frac{\sqrt{2}\left(B\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\left(2\,A\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+B\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(B*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 3*(2*A*sgn(
cos(1/2*d*x + 1/2*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*
sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

3.78 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 2*A*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+2*a*B*sin(
d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3060, 2852, 212}

$$\frac{2\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[a]*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
```

```
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + A\int \sqrt{a+a\cos(c+\ldots$$

$$= \frac{2aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(2aA)\text{Subst}\left(\int \frac{1}{a-x^2}\,d\ldots\right.}{\ldots}$$

$$= \frac{2\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2\ldots}{d\sqrt{\ldots}}$$

Mathematica [A]
time = 0.10, size = 66, normalized size = 1.00

$$\frac{\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\,A\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*A*ArcTanh[Sqrt[2]*Sin
[(c + d*x)/2]] + 2*B*Sin[(c + d*x)/2]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(58) = 116.
time = 0.33, size = 212, normalized size = 3.21

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(A\ln\left(\frac{4a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{a}\,\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)a+A\ln\left(-\frac{4\left(a\ldots\right.}{\ldots}\right.\right.}{\sqrt{a}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\ldots\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/a^(1/2)*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(A*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(
1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a
*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-
2*a))*a+2*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2
*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [**A**]
time = 0.52, size = 21, normalized size = 0.32

$$\frac{2\sqrt{2}\,B\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] 2*sqrt(2)*B*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(58) = 116.
time = 0.36, size = 127, normalized size = 1.92

$$\frac{(A\cos(dx+c)+A)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,\sqrt{a\cos(dx+c)+a}\,B\sin(dx+c)}{2\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/2*((A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*B*sin(d*
x + c))/(d*cos(d*x + c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\,(\cos(c+dx)+1)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sec(c + d*x), x)
```

Giac [A]
time = 0.46, size = 89, normalized size = 1.35

$$-\frac{\sqrt{2}\left(\sqrt{2}\,A\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-4\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*A*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*
sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) - 4*B*sgn(cos(
1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x), x)
```

3.79 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a}\,(A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{aA\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

```
[Out] (A+2*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+a*A*ta
n(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3059, 2852, 212}

$$\frac{\sqrt{a}\,(A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aA\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (Sqrt[a]*(A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]
)/d + (a*A*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
```

```
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{1}{2}(A+2B)\int \sqrt{a+$$

$$= \frac{aA\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(a(A+2B))\text{Subst}\Big(}{}$$

$$= \frac{\sqrt{a}\,(A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$

Mathematica [A]
time = 0.23, size = 85, normalized size = 1.25

$$\frac{\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{2}\,(A+2B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos(c+dx)+2A\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*(A + 2*B
)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*A*Sin[(c + d*x)/2]))/(
2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(60) = 120$.
time = 0.37, size = 650, normalized size = 9.56

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-2a\left(A\ln\left(\frac{4a\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{a}\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+A\ln\left(-\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(-2*a*(A*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*
d*x+1/2*c)^2*a)^(1/2)+2*a))+A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))+
2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/
2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))+2*B*ln(-4/(2*cos(1/2*d*x+1/
2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/
2*c)^2*a)^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^2+2*A*a^(1/2)*2^(1/2)*(sin(1/2*d*
x+1/2*c)^2*a)^(1/2)+A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+A*ln(-4
/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/
2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*
a)^(1/2)+2*a))*a+2*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a)/a^(1/2
)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x
+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(60) = 120$.
time = 0.55, size = 710, normalized size = 10.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*cos(3/2*d
*x + 3/2*c)*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2
*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*lo
g(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*
x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(
1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
 - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arct
```

```
an2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*
x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqr
t(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(
d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*sin(5/2*d*x + 5/2*c) + 4*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)
*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c))*A*sqrt(a)/(
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(60) = 120$.
time = 0.38, size = 153, normalized size = 2.25

$$\frac{\left((A+2\,B)\cos\left(dx+c\right)^{2}+(A+2\,B)\cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\,\sqrt{a\cos\left(dx+c\right)+a}\,A\sin\left(dx+c\right)}{4\,\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] 1/4*(((A + 2*B)*cos(d*x + c)^2 + (A + 2*B)*cos(d*x + c))*sqrt(a)*log((a*cos
(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(
d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sq
rt(a*cos(d*x + c) + a)*A*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\left(A+B\cos\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sec(c + d*x)**2, x
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(60) = 120$.
time = 0.45, size = 121, normalized size = 1.78

$$-\frac{\sqrt{2}\left(\sqrt{2}\,\left(A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+2\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}\right)\sqrt{a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*(A*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*sgn(cos(1/2*d*x +
1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin
(1/2*d*x + 1/2*c))) + 4*A*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/(2
*sin(1/2*d*x + 1/2*c)^2 - 1))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^2, x)
```

3.80 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{a}\,(3A+4B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}+\frac{a(3A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{aA\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 1/4*(3*A+4*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+
1/4*a*(3*A+4*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a*A*sec(d*x+c)*tan(
d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3059, 2851, 2852, 212}

$$\frac{a(3A+4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\,(3A+4B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (Sqrt[a]*(3*A + 4*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) + (a*(3*A + 4*B)*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (
a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{4}(3A+4B)\int$$

$$= \frac{a(3A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sec(c+dx)\,\text{ta}}{2d\sqrt{a+a\cos(}}$$

$$= \frac{a(3A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sec(c+dx)\,\text{ta}}{2d\sqrt{a+a\cos(}}$$

$$= \frac{\sqrt{a}\,(3A+4B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right.}{4d}$$

Mathematica [A]
time = 0.88, size = 101, normalized size = 0.86

$$\frac{\sqrt{a(1+\cos(c+dx))}\left(3\sqrt{2}\,(3A+4B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)+6(2A+(3A+4B)\cos(c+dx))\sec^2(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(3*Sqrt[2]*(3*A + 4*B)*ArcTanh[Sqrt[2]*Sin[(c +
 d*x)/2]]*Sec[(c + d*x)/2] + 6*(2*A + (3*A + 4*B)*Cos[c + d*x])*Sec[c + d*x
]^2*Tan[(c + d*x)/2]))/(24*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(101) = 202$.
time = 0.39, size = 1003, normalized size = 8.57

method	result	size
default	Expression too large to display	1003

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/2*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(4*a*(3*A*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin
(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))+3*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(
a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)
-2*a))+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c
)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))+4*B*ln(-4/(2*cos(1/2
*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2
*d*x+1/2*c)^2*a)^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^4-4*(3*A*a^(1/2)*2^(1/2)*(
sin(1/2*d*x+1/2*c)^2*a)^(1/2)+4*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^
(1/2)+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)
+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+3*A*ln(-4/(2*cos(1/
2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/
2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*
2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2
*a))*a+4*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*
c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*
c)^2+10*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+3*A*ln(-4/(2*cos(1
/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1
/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+
2*a))*a+8*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+4*B*ln(-4/(2*cos
(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin
(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2
)+2*a))*a)/a^(1/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3352 vs. $2(101) = 202$.
time = 3.46, size = 3352, normalized size = 28.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm ="maxima")

[Out] 1/16*((3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 24*sqrt(2)*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 8*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 2*(6*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 6*sqrt(2)*sin(7/2*d*x + 7/2*c) + 2*sqrt(2)*sin(5/2*d*x + 5/2*c) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) - 6*sqrt(2)*sin(1/2*d*x + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x

```
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2))*cos(4*d*x + 4*c) - 4*(2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)
*sin(1/2*d*x + 1/2*c) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*(3*(log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c) - 3*sqrt(2)*cos(7/2*d
*x + 7/2*c) - sqrt(2)*cos(5/2*d*x + 5/2*c) + sqrt(2)*cos(3/2*d*x + 3/2*c) +
 3*sqrt(2)*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) + 12*(2*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(7/2*d*x + 7/2*c) + 4*(2*sqrt(2)*cos(2*d*x + 2*c) + s
qrt(2))*sin(5/2*d*x + 5/2*c) + 8*(sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*
cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) - 4*sqrt...
```

Fricas [A]
time = 0.38, size = 178, normalized size = 1.52

$$\frac{\left((3A+4B)\cos(dx+c)^3+(3A+4B)\cos(dx+c)^2\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left((3A+4B)\cos(dx+c)+2A\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{16\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] 1/16*(((3*A + 4*B)*cos(d*x + c)^3 + (3*A + 4*B)*cos(d*x + c)^2)*sqrt(a)*log
((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
 + 4*((3*A + 4*B)*cos(d*x + c) + 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\left(A+B\cos\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sec(c + d*x)**3, x
)
```

Giac [A]
time = 0.47, size = 195, normalized size = 1.67

$$-\frac{\sqrt{2}\left(\sqrt{2}\left(3\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+4\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\left(6\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+8\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}\right)\sqrt{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] -1/16*sqrt(2)*(sqrt(2)*(3*A*sgn(cos(1/2*d*x + 1/2*c)) + 4*B*sgn(cos(1/2*d*x
 + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*
sin(1/2*d*x + 1/2*c))) + 4*(6*A*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2
*c)^3 + 8*B*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 5*A*sgn(cos(
1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) - 4*B*sgn(cos(1/2*d*x + 1/2*c))*sin(
1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^3, x)
```

3.81 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a}\,(5A+6B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}+\frac{a(5A+6B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}+\frac{a(5A+6B)\sec(c+dx)\tan(c+}{12d\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 1/8*(5*A+6*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+
1/8*a*(5*A+6*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/12*a*(5*A+6*B)*sec(d*
x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d/
(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3059, 2851, 2852, 212}

$$\frac{a(5A+6B)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\,(5A+6B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(5A+6B)\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (Sqrt[a]*(5*A + 6*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(8*d) + (a*(5*A + 6*B)*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (
a*(5*A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) +
(a*A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{1}{6}(5A+6B)$$

$$= \frac{a(5A+6B)\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}}+\frac{aA\,\mathrm{se}}{3d}$$

$$= \frac{a(5A+6B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}+\frac{a(5A+6B)\sec(c}{12d\sqrt{a+a}}$$

$$= \frac{a(5A+6B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}+\frac{a(5A+6B)\sec(c}{12d\sqrt{a+a}}$$

$$= \frac{\sqrt{a}\,(5A+6B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right.}{8d}$$

Mathematica [A]

time = 1.94, size = 129, normalized size = 0.81

$$\frac{\sqrt{a(1+\cos(c+dx))}\,\sec^3(c+dx)\left(3\sqrt{2}\,(5A+6B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\cos^3(c+dx)\sec\left(\tfrac{1}{2}(c+dx)\right)+(31A+18B+4(5A+6B)\cos(c+dx)+3(5A+6B)\cos(2(c+dx)))\tan\left(\tfrac{1}{2}(c+dx)\right)\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[c + d*x]^3*(3*Sqrt[2]*(5*A + 6*B)*ArcTanh[S
qrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3*Sec[(c + d*x)/2] + (31*A + 18*B + 4
*(5*A + 6*B)*Cos[c + d*x] + 3*(5*A + 6*B)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2
]))/(48*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1326 vs. 2(140) = 280.
time = 0.46, size = 1327, normalized size = 8.29

method	result	size
default	Expression too large to display	1327

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/6*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(-24*a*(5*A*ln(-4/(2*
cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(
sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))+5*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/
2)+2*a))+6*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/
2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))+6*B*ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1
/2*d*x+1/2*c)^2*a)^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^6+12*(10*A*a^(1/2)*2^(1/
2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+12*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/
2)*a^(1/2)+15*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x
+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+15*A*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*
(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+18*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(
1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a
)^(1/2)-2*a))*a+18*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2
*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a)*sin(1/2
*d*x+1/2*c)^4-2*(80*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+96*B*2
^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+45*A*ln(-4/(2*cos(1/2*d*x+1/2
*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2
*c)^2*a)^(1/2)-2*a))*a+45*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*
cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+5
4*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1
/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+54*B*ln(4/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x
+1/2*c)^2*a)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+66*A*a^(1/2)*2^(1/2)*(sin(
1/2*d*x+1/2*c)^2*a)^(1/2)+15*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*
a+15*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a
^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+60*B*2^(1/2)*(sin(1/2
```

```
*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+18*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*
2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2
*a))*a+18*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2
*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a)/a^(1/2)/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/sin(1/2*d*x+1/2*
c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 5021 vs. $2(140) = 280$.
time = 3.55, size = 5021, normalized size = 31.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] -1/96*((120*(sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 3*sin(2*d*x + 2*c))*co
s(13/2*d*x + 13/2*c) - 8*(15*sin(11/2*d*x + 11/2*c) + 50*sin(9/2*d*x + 9/2*
c) + 42*sin(7/2*d*x + 7/2*c) + 3*sin(5/2*d*x + 5/2*c) - 5*sin(3/2*d*x + 3/2
*c))*cos(6*d*x + 6*c) + 360*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*cos(11/2*
d*x + 11/2*c) + 1200*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*cos(9/2*d*x + 9/
2*c) - 24*(42*sin(7/2*d*x + 7/2*c) + 3*sin(5/2*d*x + 5/2*c) - 5*sin(3/2*d*x
 + 3/2*c))*cos(4*d*x + 4*c) - 15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*co
s(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^
2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*
sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*c
os(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)
^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2
*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x +
6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 15*(sqrt(2)*cos(6*d*x + 6*c)^2
 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*si
n(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*
```

```
c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x
 + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x +
4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 2) + 15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^
2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*s
in(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2
)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 120*(cos(6*d*x
+ 6*c) + 3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(13/2*d*x + 13/2*c
) + 8*(15*cos(11/2*d*x + 11/2*c) + 50*cos(9/2*d*x + 9/2*c) + 42*cos(7/2*d*x
 + 7/2*c) + 3*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c))*sin(6*d*x + 6*
c) - 120*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(11/2*d*x + 11/2*
c) - 400*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(9/2*d*x + 9/2*c)
 + 24*(42*cos(7/2*d*x + 7/2*c) + 3*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3
/2*c))*sin(4*d*x + 4*c) - 336*(3*cos(2*d*x + 2*c) + 1)*sin(7/2*d*x + 7/2*c)
 - 24*(3*cos(2*d*x + 2*c) + 1)*sin(5/2*d*x + 5/2*c) + 1008*cos(7/2*d*x + 7/
2*c)*sin(2*d*x + 2*c) + 72*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 120*cos(
3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 120*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*
c) + 120*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
 1)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 40*sin(3/2*d*x + 3/2*c))
*A*sqrt(a)/(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*s
qrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*
x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2
*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(
4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sq...
```

Fricas [A]

time = 0.39, size = 197, normalized size = 1.23

$$\frac{3\left(\left(5\,A+6\,B\right)\cos\left(dx+c\right)^4+\left(5\,A+6\,B\right)\cos\left(dx+c\right)^3\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(3\left(5\,A+6\,B\right)\cos\left(dx+c\right)^2+2\left(5\,A+6\,B\right)\cos\left(dx+c\right)+8\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{96\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] 1/96*(3*((5*A + 6*B)*cos(d*x + c)^4 + (5*A + 6*B)*cos(d*x + c)^3)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2
)) + 4*(3*(5*A + 6*B)*cos(d*x + c)^2 + 2*(5*A + 6*B)*cos(d*x + c) + 8*A)*sq
rt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.51, size = 244, normalized size = 1.52

$$-\frac{\sqrt{2}\left(3\,\sqrt{2}\,\left(5\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+6\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\left(60\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+72\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-80\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-96\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+33\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+30\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}\right)\sqrt{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] -1/96*sqrt(2)*(3*sqrt(2)*(5*A*sgn(cos(1/2*d*x + 1/2*c)) + 6*B*sgn(cos(1/2*d
*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) +
4*sin(1/2*d*x + 1/2*c))) + 4*(60*A*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x +
1/2*c)^5 + 72*B*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 80*A*sgn
(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 96*B*sgn(cos(1/2*d*x + 1/2*
c))*sin(1/2*d*x + 1/2*c)^3 + 33*A*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1
/2*c) + 30*B*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x
 + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^4,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^4, x)
```

3.82 $\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=234

$$\frac{4a^2(187A+168B)\sin(c+dx)}{495d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(187A+168B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(11A+12B)\cos^4(c+dx)\,\mathrm{s}}{99d\sqrt{a+a\cos(c+dx}}$$

```
[Out] 4/1155*(187*A+168*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+4/495*a^2*(187*A+1
68*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/693*a^2*(187*A+168*B)*cos(d*x+c
)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/99*a^2*(11*A+12*B)*cos(d*x+c)^4*s
in(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-8/3465*a*(187*A+168*B)*sin(d*x+c)*(a+a*c
os(d*x+c))^(1/2)/d+2/11*a*B*cos(d*x+c)^4*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/
d
```

Rubi [A]
time = 0.33, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3055, 3060, 2849, 2838, 2830, 2725}

$$\frac{2a^2(11A+12B)\sin(c+dx)\cos^4(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(187A+168B)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(187A+168B)\sin(c+dx)}{495d\sqrt{a\cos(c+dx)+a}}+\frac{4(187A+168B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{1155d}-\frac{8a(187A+168B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3465d}+\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*a^2*(187*A + 168*B)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
a^2*(187*A + 168*B)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*a^2*(11*A + 12*B)*Cos[c + d*x]^4*Sin[c + d*x])/(99*d*Sqrt[a + a
*Cos[c + d*x]]) - (8*a*(187*A + 168*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x
])/(3465*d) + (2*a*B*Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/
(11*d) + (4*(187*A + 168*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(1155*
d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx &= \frac{2aB\cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,\sin(c+\ldots}{11d} \\ &= \frac{2a^2(11A+12B)\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}}+\ldots \\ &= \frac{2a^2(187A+168B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}+\ldots \\ &= \frac{2a^2(187A+168B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}+\ldots \\ &= \frac{2a^2(187A+168B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}+\ldots \\ &= \frac{4a^2(187A+168B)\sin(c+dx)}{495d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(187A+\ldots}{69\ldots}\end{aligned}$$

Mathematica [A]
time = 1.03, size = 125, normalized size = 0.53

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,(59158A+55482B+(35156A+34734B)\cos(c+dx)+8(1507A+1743B)\cos(2(c+dx))+3740A\cos(3(c+dx))+4935B\cos(3(c+dx))+770A\cos(4(c+dx))+1470B\cos(4(c+dx))+315B\cos(5(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{27720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(59158*A + 55482*B + (35156*A + 34734*B)*Cos[
c + d*x] + 8*(1507*A + 1743*B)*Cos[2*(c + d*x)] + 3740*A*Cos[3*(c + d*x)] +
 4935*B*Cos[3*(c + d*x)] + 770*A*Cos[4*(c + d*x)] + 1470*B*Cos[4*(c + d*x)]
 + 315*B*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])/(27720*d)
```

Maple [A]
time = 0.24, size = 142, normalized size = 0.61

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-5040B\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(3080A+18480B)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-9900A-27720B)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(12\ldots\right.}{3465\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 4/3465*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(-5040*B*sin(1/2*d*x+1/2*c
)^10+(3080*A+18480*B)*sin(1/2*d*x+1/2*c)^8+(-9900*A-27720*B)*sin(1/2*d*x+1/
2*c)^6+(12474*A+22176*B)*sin(1/2*d*x+1/2*c)^4+(-8085*A-10395*B)*sin(1/2*d*x
+1/2*c)^2+3465*A+3465*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A]
time = 0.59, size = 185, normalized size = 0.79

$$\frac{22 \left(35 \sqrt{2} a \sin \left(\frac{9}{2} dx + \frac{9}{2} c\right) + 135 \sqrt{2} a \sin \left(\frac{7}{2} dx + \frac{7}{2} c\right) + 378 \sqrt{2} a \sin \left(\frac{5}{2} dx + \frac{5}{2} c\right) + 1050 \sqrt{2} a \sin \left(\frac{3}{2} dx + \frac{3}{2} c\right) + 3780 \sqrt{2} a \sin \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) A \sqrt{a} + 21 \left(15 \sqrt{2} a \sin \left(\frac{11}{2} dx + \frac{11}{2} c\right) + 55 \sqrt{2} a \sin \left(\frac{9}{2} dx + \frac{9}{2} c\right) + 165 \sqrt{2} a \sin \left(\frac{7}{2} dx + \frac{7}{2} c\right) + 429 \sqrt{2} a \sin \left(\frac{5}{2} dx + \frac{5}{2} c\right) + 990 \sqrt{2} a \sin \left(\frac{3}{2} dx + \frac{3}{2} c\right) + 3630 \sqrt{2} a \sin \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) B \sqrt{a}}{55440 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/55440*(22*(35*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*sin(7/2*d*x
+ 7/2*c) + 378*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) + 3780*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 21*(15*sqrt(2)*
a*sin(11/2*d*x + 11/2*c) + 55*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 165*sqrt(2)*
a*sin(7/2*d*x + 7/2*c) + 429*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 990*sqrt(2)*a
*sin(3/2*d*x + 3/2*c) + 3630*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A]
time = 0.35, size = 125, normalized size = 0.53

$$\frac{2 \left(315 B a \cos \left(dx + c\right)^{5} + 35 \left(11 A + 21 B\right) a \cos \left(dx + c\right)^{4} + 5 \left(187 A + 168 B\right) a \cos \left(dx + c\right)^{3} + 6 \left(187 A + 168 B\right) a \cos \left(dx + c\right)^{2} + 8 \left(187 A + 168 B\right) a \cos \left(dx + c\right) + 16 \left(187 A + 168 B\right) a\right) \sqrt{a \cos \left(dx + c\right) + a} \sin \left(dx + c\right)}{3465 \left(d \cos \left(dx + c\right) + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/3465*(315*B*a*cos(d*x + c)^5 + 35*(11*A + 21*B)*a*cos(d*x + c)^4 + 5*(187
*A + 168*B)*a*cos(d*x + c)^3 + 6*(187*A + 168*B)*a*cos(d*x + c)^2 + 8*(187*
A + 168*B)*a*cos(d*x + c) + 16*(187*A + 168*B)*a)*sqrt(a*cos(d*x + c) + a)*
sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [A]
time = 2.23, size = 235, normalized size = 1.00

$$\frac{\sqrt{2} \left(315 B a \mathrm{sgn}\left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin \left(\frac{11}{2} dx + \frac{11}{2} c\right) + 385 \left(2 A a \mathrm{sgn}\left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + 3 B a \mathrm{sgn}\left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)\right) \sin \left(\frac{9}{2} dx + \frac{9}{2} c\right) + 495 \left(6 A a \mathrm{sgn}\left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + 7 B a \mathrm{sgn}\left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)\right) \sin \left(\frac{7}{2} dx + \frac{7}{2} c\right) + 693 \left(12 A a \mathrm{sgn}\left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + 13 B a \mathrm{sgn}\left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)\right) \sin \left(\frac{5}{2} dx + \frac{5}{2} c\right) + 2310 \left(10 A a \mathrm{sgn}\left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + 9 B a \mathrm{sgn}\left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)\right) \sin \left(\frac{3}{2} dx + \frac{3}{2} c\right) + 6930 \left(12 A a \mathrm{sgn}\left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + 11 B a \mathrm{sgn}\left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)\right) \sin \left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sqrt{a}}{55440 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] 1/55440*sqrt(2)*(315*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(11/2*d*x + 11/2*c) +
 385*(2*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*a*sgn(cos(1/2*d*x + 1/2*c)))*si
n(9/2*d*x + 9/2*c) + 495*(6*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 7*B*a*sgn(cos(1
/2*d*x + 1/2*c)))*sin(7/2*d*x + 7/2*c) + 693*(12*A*a*sgn(cos(1/2*d*x + 1/2*
c)) + 13*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(5/2*d*x + 5/2*c) + 2310*(10*A*a
*sgn(cos(1/2*d*x + 1/2*c)) + 9*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x +
 3/2*c) + 6930*(12*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 11*B*a*sgn(cos(1/2*d*x +
 1/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^3\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

3.83 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=189

$$\frac{2a^2(39A+34B)\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(9A+10B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}}-\frac{4a(39A+34B)\sqrt{a+a\cos(c+dx}}{315d}$$

```
[Out] 2/105*(39*A+34*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/45*a^2*(39*A+34*B)*
sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/63*a^2*(9*A+10*B)*cos(d*x+c)^3*sin(d*
x+c)/d/(a+a*cos(d*x+c))^(1/2)-4/315*a*(39*A+34*B)*sin(d*x+c)*(a+a*cos(d*x+c
))^(1/2)/d+2/9*a*B*cos(d*x+c)^3*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.28, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3055, 3060, 2838, 2830, 2725}

$$\frac{2a^2(9A+10B)\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(39A+34B)\sin(c+dx)}{45d\sqrt{a\cos(c+dx)+a}}+\frac{2(39A+34B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105d}-\frac{4a(39A+34B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{2aB\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a^2*(39*A + 34*B)*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(9*A + 10*B)*Cos[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]])
- (4*a*(39*A + 34*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*
B*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d) + (2*(39*A +
34*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2aB\cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,\sin(c+}{9d}$$

$$= \frac{2a^2(9A+10B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a}{}$$

$$= \frac{2a^2(9A+10B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a}{}$$

$$= \frac{2a^2(9A+10B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} - \frac{4a}{}$$

$$= \frac{2a^2(39A+34B)\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(9A+10B}{63d\sqrt{}}$$

Mathematica [A]

time = 0.61, size = 103, normalized size = 0.54

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,(2964A+2689B+2(759A+799B)\cos(c+dx)+(468A+548B)\cos(2(c+dx))+90A\cos(3(c+dx))+170B\cos(3(c+dx))+35B\cos(4(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(2964*A + 2689*B + 2*(759*A + 799*B)*Cos[c +
d*x] + (468*A + 548*B)*Cos[2*(c + d*x)] + 90*A*Cos[3*(c + d*x)] + 170*B*Cos
[3*(c + d*x)] + 35*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A]

time = 0.20, size = 123, normalized size = 0.65

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(280B\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-180A-900B)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(504A+1134B)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-525A-7\right.}{315\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 4/315*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(280*B*sin(1/2*d*x+1/2*c)^8
+(-180*A-900*B)*sin(1/2*d*x+1/2*c)^6+(504*A+1134*B)*sin(1/2*d*x+1/2*c)^4+(-
525*A-735*B)*sin(1/2*d*x+1/2*c)^2+315*A+315*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c
)^2)^(1/2)/d
```

Maxima [A]

time = 0.58, size = 154, normalized size = 0.81

$$\frac{6\left(15\sqrt{2}a\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+63\sqrt{2}a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+175\sqrt{2}a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+735\sqrt{2}a\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+\left(35\sqrt{2}a\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+135\sqrt{2}a\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+378\sqrt{2}a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+1050\sqrt{2}a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3780\sqrt{2}a\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)B\sqrt{a}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/2520*(6*(15*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5
/2*c) + 175*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/
2*c))*A*sqrt(a) + (35*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*sin(7/
2*d*x + 7/2*c) + 378*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*sin(3/
2*d*x + 3/2*c) + 3780*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A]
time = 0.34, size = 107, normalized size = 0.57

$$\frac{2 \, {\left(35 \, B a \cos\left(d x + c\right)^{4} + 5 \, {\left(9 \, A + 17 \, B\right)} a \cos\left(d x + c\right)^{3} + 3 \, {\left(39 \, A + 34 \, B\right)} a \cos\left(d x + c\right)^{2} + 4 \, {\left(39 \, A + 34 \, B\right)} a \cos\left(d x + c\right) + 8 \, {\left(39 \, A + 34 \, B\right)} a\right)} \sqrt{a \cos\left(d x + c\right) + a} \sin\left(d x + c\right)}{315 \, {\left(d \cos\left(d x + c\right) + d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/315*(35*B*a*cos(d*x + c)^4 + 5*(9*A + 17*B)*a*cos(d*x + c)^3 + 3*(39*A +
34*B)*a*cos(d*x + c)^2 + 4*(39*A + 34*B)*a*cos(d*x + c) + 8*(39*A + 34*B)*a
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [A]
time = 1.05, size = 191, normalized size = 1.01

$$\frac{\sqrt{2} \, {\left(35 \, B a \mathrm{sgn}\left(\cos\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)\right) \sin\left(\frac{9}{2} \, d x + \frac{9}{2} \, c\right) + 45 \, {\left(2 \, A a \mathrm{sgn}\left(\cos\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)\right) + 3 \, B a \mathrm{sgn}\left(\cos\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)\right)\right)} \sin\left(\frac{7}{2} \, d x + \frac{7}{2} \, c\right) + 378 \, {\left(A a \mathrm{sgn}\left(\cos\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)\right) + B a \mathrm{sgn}\left(\cos\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)\right)\right)} \sin\left(\frac{5}{2} \, d x + \frac{5}{2} \, c\right) + 1050 \, {\left(A a \mathrm{sgn}\left(\cos\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)\right) + B a \mathrm{sgn}\left(\cos\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)\right)\right)} \sin\left(\frac{3}{2} \, d x + \frac{3}{2} \, c\right) + 630 \, {\left(7 \, A a \mathrm{sgn}\left(\cos\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)\right) + 6 \, B a \mathrm{sgn}\left(\cos\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)\right)\right)} \sin\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)\right)} \sqrt{a}}{2520 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) + 45*
(2*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(7/2
*d*x + 7/2*c) + 378*(A*a*sgn(cos(1/2*d*x + 1/2*c)) + B*a*sgn(cos(1/2*d*x +
1/2*c)))*sin(5/2*d*x + 5/2*c) + 1050*(A*a*sgn(cos(1/2*d*x + 1/2*c)) + B*a*s
gn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c) + 630*(7*A*a*sgn(cos(1/2*d*x
 + 1/2*c)) + 6*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

3.84 $\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{8a^2(21A+19B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2a(21A+19B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{105d}+\frac{2(7A-2B)(a+a\cos(c+}{35d}$$

```
[Out] 2/35*(7*A-2*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(a+a*cos(d*x+c))^(
5/2)*sin(d*x+c)/a/d+8/105*a^2*(21*A+19*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/
2)+2/105*a*(21*A+19*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3047, 3102, 2830, 2726, 2725}

$$\frac{8a^2(21A+19B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2(7A-2B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{2a(21A+19B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*a^2*(21*A + 19*B)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
(21*A + 19*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A - 2*
B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*B*(a + a*Cos[c + d*
x])^(5/2)*Sin[c + d*x])/(7*a*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^{3/2}\left(A\cos(c+dx)+B\cos\right.$$
$$= \frac{2B(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7ad} + \frac{2\int(a}{}$$
$$= \frac{2(7A-2B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d}$$
$$= \frac{2a(21A+19B)\sqrt{a+a\cos(c+dx)}\,\sin(c+d}{105d}$$
$$= \frac{8a^2(21A+19B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a(21A+19B}{}$$

Mathematica [A]
time = 0.41, size = 81, normalized size = 0.59

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,(546A+494B+(252A+253B)\cos(c+dx)+6(7A+13B)\cos(2(c+dx))+15B\cos(3(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(546*A + 494*B + (252*A + 253*B)*Cos[c + d*x]
 + 6*(7*A + 13*B)*Cos[2*(c + d*x)] + 15*B*Cos[3*(c + d*x)])*Tan[(c + d*x)/2
])/(210*d)
```

Maple [A]
time = 0.20, size = 104, normalized size = 0.75

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-60B\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(42A+168B)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-105A-175B)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+105A+105B}{105\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] 4/105*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(-60*B*sin(1/2*d*x+1/2*c)^6
+(42*A+168*B)*sin(1/2*d*x+1/2*c)^4+(-105*A-175*B)*sin(1/2*d*x+1/2*c)^2+105*
A+105*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A]
time = 0.55, size = 123, normalized size = 0.89

$$\frac{42\left(\sqrt{2}\,a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+\left(15\,\sqrt{2}\,a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+63\,\sqrt{2}\,a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+175\,\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+735\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/420*(42*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*c
) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + (15*sqrt(2)*a*sin(7/2*d*
x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 175*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A]
time = 0.35, size = 88, normalized size = 0.64

$$\frac{2\left(15\,Ba\cos\left(dx+c\right)^{3}+3\,(7\,A+13\,B)a\cos\left(dx+c\right)^{2}+(63\,A+52\,B)a\cos\left(dx+c\right)+2\,(63\,A+52\,B)a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{105\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/105*(15*B*a*cos(d*x + c)^3 + 3*(7*A + 13*B)*a*cos(d*x + c)^2 + (63*A + 52
*B)*a*cos(d*x + c) + 2*(63*A + 52*B)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.63, size = 155, normalized size = 1.12

$$\frac{\sqrt{2}\left(15\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\left(2\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+3\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+35\left(6\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+5\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+105\left(8\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+7\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/420*sqrt(2)*(15*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 21*(
2*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(5/2*
d*x + 5/2*c) + 35*(6*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 5*B*a*sgn(cos(1/2*d*x
+ 1/2*c)))*sin(3/2*d*x + 3/2*c) + 105*(8*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 7*
B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

3.85 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{8a^2(5A+3B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a(5A+3B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15d}+\frac{2B(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

```
[Out] 2/5*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+8/15*a^2*(5*A+3*B)*sin(d*x+c)/d/(
a+a*cos(d*x+c))^(1/2)+2/15*a*(5*A+3*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2830, 2726, 2725}

$$\frac{8a^2(5A+3B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a(5A+3B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*a^2*(5*A + 3*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(5*
A + 3*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*B*(a + a*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2B(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}(5A+3B)\int (a \cdots$$

$$= \frac{2a(5A+3B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2B(a+a \cdots}{}$$

$$= \frac{8a^2(5A+3B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a(5A+3B)\sqrt{a+a\cos(c+ \cdots}}{15d}$$

Mathematica [A]
time = 0.21, size = 65, normalized size = 0.64

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,(50A+39B+2(5A+9B)\cos(c+dx)+3B\cos(2(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(50*A + 39*B + 2*(5*A + 9*B)*Cos[c + d*x] + 3
*B*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d)
```

Maple [A]
time = 0.16, size = 85, normalized size = 0.84

method	result	size
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(6B\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-5A-15B)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15A+15B\right)\sqrt{2}}{15\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/15*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(6*B*sin(1/2*d*x+1/2*c)^4+(-
5*A-15*B)*sin(1/2*d*x+1/2*c)^2+15*A+15*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^
(1/2)/d
```

Maxima [A]
time = 0.54, size = 93, normalized size = 0.92

$$\frac{10\left(\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+3\left(\sqrt{2}\,a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(10*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c)
)*A*sqrt(a) + 3*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x +
 3/2*c) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [**A**]
time = 0.36, size = 69, normalized size = 0.68

$$\frac{2\left(3\,Ba\cos\left(dx+c\right)^{2}+(5\,A+9\,B)a\cos\left(dx+c\right)+(25\,A+18\,B)a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/15*(3*B*a*cos(d*x + c)^2 + (5*A + 9*B)*a*cos(d*x + c) + (25*A + 18*B)*a)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a(\cos\left(c+dx\right)+1)\right)^{\frac{3}{2}}\left(A+B\cos\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)*(A + B*cos(c + d*x)), x)
```

Giac [**A**]
time = 0.50, size = 115, normalized size = 1.14

$$\frac{\sqrt{2}\left(3\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\left(2\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+3\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+30\left(3\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+2\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(2)*(3*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 5*(2*A
*a*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x
 + 3/2*c) + 30*(3*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*a*sgn(cos(1/2*d*x + 1
/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

3.86 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=105

$$\frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^2(3A+4B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2aB\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2*a^(3/2)*A*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2/3*a^2*(3
*A+4*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*a*B*sin(d*x+c)*(a+a*cos(d*x
+c))^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3055, 3060, 2852, 212}

$$\frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2(3A+4B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*a^(3/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^2*(3*A + 4*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*B*Sq
rt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx &= \frac{2aB\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2}{3}\int \\ &= \frac{2a^2(3A+4B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2aB\sqrt{a+a\cos}}{} \\ &= \frac{2a^2(3A+4B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2aB\sqrt{a+a\cos}}{} \\ &= \frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a}{3}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 85, normalized size = 0.81

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}\,A\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2(3A+5B+B\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*A*ArcTanh[Sqrt[2]
*Sin[(c + d*x)/2]] + 2*(3*A + 5*B + B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(3*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(91) = 182$.
time = 0.33, size = 274, normalized size = 2.61

method	result
default	$\frac{\sqrt{a}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-4B\sqrt{a}\,\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6A\sqrt{a}\,\sqrt{2}\,\sqrt{a\left(\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/3*a^(1/2)*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(-4*B*a^(1/2)
*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*A*a^(1/2)*2^
(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1
/2)+2*a))*a+3*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x
+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+12*B*2^(1/2)
*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+
1/2*c)^2)^(1/2)/d
```

Maxima [A]
time = 0.51, size = 39, normalized size = 0.37

$$\frac{\left(\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*s
qrt(a)/d
```

Fricas [A]
time = 0.39, size = 149, normalized size = 1.42

$$\frac{3\left(Aa\cos\left(dx+c\right)+Aa\right)\sqrt{a}\,\log\left(\frac{a\cos\left(dx+c\right)^{3}-7\,a\cos\left(dx+c\right)^{2}-4\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,\left(\cos\left(dx+c\right)-2\right)\sin\left(dx+c\right)+8\,a}{\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}}\right)+4\left(Ba\cos\left(dx+c\right)+\left(3\,A+5\,B\right)a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{6\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/6*(3*(A*a*cos(d*x + c) + A*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x
 + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c
) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(B*a*cos(d*x + c) + (3*A +
5*B)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**A**]
time = 0.53, size = 140, normalized size = 1.33

$$-\frac{\sqrt{2}\left(8\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,\sqrt{2}\,Aa\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-12\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] -1/6*sqrt(2)*(8*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 3*sq
rt(2)*A*a*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*si
n(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) - 12*A*a*sgn(cos(1/2*d*x + 1
/2*c))*sin(1/2*d*x + 1/2*c) - 24*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x
+ 1/2*c))*sqrt(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}{\cos\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x), x)
```

3.87 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{a^{3/2}(3A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}-\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{aA\sqrt{a+a\cos(c+dx)}\tan(c+d}{}$$

```
[Out] a^(3/2)*(3*A+2*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d-a^2*
(A-2*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+a*A*(a+a*cos(d*x+c))^(1/2)*tan(
d*x+c)/d
```

Rubi [A]
time = 0.18, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$,
Rules used = {3054, 3060, 2852, 212}

$$\frac{a^{3/2}(3A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^(3/2)*(3*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d - (a^2*(A - 2*B)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (a*A*Sq
rt[a + a*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\frac{aA\sqrt{a+a\cos(c+dx)}\,\tan(c+dx)}{d}+\int \sqrt{}$$

$$=-\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{aA\sqrt{a+a\cos(}}{}$$

$$=-\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{aA\sqrt{a+a\cos(}}{}$$

$$=\frac{a^{3/2}(3A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right.}{d}$$

Mathematica [A]
time = 0.33, size = 98, normalized size = 0.95

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{2}\,(3A+2B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos(c+dx)+2(A+2B\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*(3*A +
 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(A + 2*B*Cos[c + d
*x])*Sin[(c + d*x)/2]))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(93) = 186$.
time = 0.36, size = 704, normalized size = 6.83

method	result
default	$\frac{\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\left(\left(-6A\ln\left(\frac{4a\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{a}\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)a-6A\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] a^(1/2)*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*((-6*A*ln(4/(2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(si
n(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a-6*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)
)*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1
/2)-2*a))*a-8*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)-4*B*ln(4/(2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(
sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a-4*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/
2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^
(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^2+2*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)
^2*a)^(1/2)+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+
1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+3*A*ln(-4/(2*
cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(
sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+4*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^
(1/2)*a^(1/2)+2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*
x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+2*B*ln(-4/(
2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)
*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*
cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/
2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(93) = 186$.
time = 0.56, size = 1315, normalized size = 12.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 6*sqrt(2)*a*cos(5
/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 6*sq
rt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (2*sqrt(2)*a*sin(
3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c
)^2 - 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) -
 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
))*cos(2*d*x + 2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 2*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin
(7/2*d*x + 7/2*c) - 6*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/2*d*x
+ 5/2*c) + 2*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + sqrt(2)*a*cos(1/2*d*x + 1/
2*c))*sin(2*d*x + 2*c))*A*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A]

time = 0.38, size = 172, normalized size = 1.67

$$\frac{\left((3\,A+2\,B)a\cos\left(dx+c\right)^{2}+(3\,A+2\,B)a\cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\left(2\,Ba\cos\left(dx+c\right)+Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{4\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] 1/4*(((3*A + 2*B)*a*cos(d*x + c)^2 + (3*A + 2*B)*a*cos(d*x + c))*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(
a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)
) + 4*(2*B*a*cos(d*x + c) + A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*
cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [A]
time = 0.53, size = 149, normalized size = 1.45

$$\frac{\sqrt{2}\left(8\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{4\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}-\sqrt{2}\left(3\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+2\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\right)\sqrt{a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] 1/4*sqrt(2)*(8*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) - 4*A*a*s
gn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/(2*sin(1/2*d*x + 1/2*c)^2 - 1
) - sqrt(2)*(3*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*a*sgn(cos(1/2*d*x + 1/2*
c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2
*d*x + 1/2*c))))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^2, x)
```

3.88 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{a^{3/2}(7A+12B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}+\frac{a^2(5A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{aA\sqrt{a+a\cos(c+dx)}\,\sec(c$$

```
[Out] 1/4*a^(3/2)*(7*A+12*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d
+1/4*a^2*(5*A+4*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a*A*sec(d*x+c)*(
a+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]

time = 0.22, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3054, 3059, 2852, 212}

$$\frac{a^{3/2}(7A+12B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2(5A+4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^(3/2)*(7*A + 12*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(4*d) + (a^2*(5*A + 4*B)*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]])
+ (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA\sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\tan(c+d}{2d}$$

$$= \frac{a^2(5A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+a\cos(c}{}$$

$$= \frac{a^2(5A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+a\cos(c}{}$$

$$= \frac{a^{3/2}(7A+12B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right.}{4d}$$

Mathematica [A]

time = 0.56, size = 109, normalized size = 0.92

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(\sqrt{2}\,(7A+12B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^2(c+dx)+2(2A+(7A+4B)\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(Sqrt[2]*(7*A
 + 12*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*(2*A + (7*A +
 4*B)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(103) = 206$.
time = 0.40, size = 1003, normalized size = 8.43

method	result	size
default	Expression too large to display	1003

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/2*a^(1/2)*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(4*a*(7*A*ln(
-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(
1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))+7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*
a)^(1/2)+2*a))+12*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2
*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))+12*B*ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2
)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^4-4*(7*A*a^(1/2)
*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+4*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a
)^(1/2)*a^(1/2)+7*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2
*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+7*A*ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/
2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+12*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-
2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^
2*a)^(1/2)-2*a))*a+12*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a)*sin(
1/2*d*x+1/2*c)^2+18*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+7*A*ln
(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^
(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+7*A*ln(4/(2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)
^2*a)^(1/2)+2*a))*a+8*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+12*B
*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)
*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+12*B*ln(4/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/
2*c)^2*a)^(1/2)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3339 vs. $2(103) = 206$.
time = 0.71, size = 3339, normalized size = 28.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] -1/16*((12*a*cos(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 48*a*cos(2*d*x + 2*c
)^2*sin(3/2*d*x + 3/2*c) + 12*a*sin(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 4
8*a*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 160*a*cos(7/2*d*x + 7/2*c)*si
n(2*d*x + 2*c) + 168*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 72*a*cos(3/2
*d*x + 3/2*c)*sin(2*d*x + 2*c) - 24*a*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c)
 - 4*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 1
2*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 48*(
a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) + 4*(12*a*c
os(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 20*a*sin(7/2*d*x + 7/2*c) - 21*a*sin
(5/2*d*x + 5/2*c) - 3*a*sin(3/2*d*x + 3/2*c))*cos(4*d*x + 4*c) - 7*(sqrt(2)
*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*
x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*si
n(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x
+ 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*
sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(
2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*
d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 2) - 7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)
^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*
(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*lo
g(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(sqrt(2)*a*
cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x +
 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2
*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2
*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/
```

```
2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x +
 2*c) + a)*sin(13/2*d*x + 13/2*c) - 12*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x
+ 2*c) + a)*sin(11/2*d*x + 11/2*c) - 48*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x
 + 2*c) + a)*sin(9/2*d*x + 9/2*c) + 4*(12*a*sin(2*d*x + 2*c)*sin(3/2*d*x +
3/2*c) + 20*a*cos(7/2*d*x + 7/2*c) + 21*a*cos(5/2*d*x + 5/2*c) + 9*a*cos(3/
2*d*x + 3/2*c))*sin(4*d*x + 4*c) - 80*(2*a*cos(2*d*x + 2*c) + a)*sin(7/2*d*
x + 7/2*c) - 84*(2*a*cos(2*d*x + 2*c) + a)*sin(5/2*d*x + 5/2*c) - 24*a*sin(
3/2*d*x + 3/2*c) - 4*(a*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin
(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2
*c)^2 + 2*(2*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c
) + a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 56*(a
*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin(4*d*x + 4*c)^2 + 4*a*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2*c)^2 + 2*(2*a*cos(2*d*
x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(sqrt(2)*cos(4*d*x
+ 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sq
rt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*
(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d
*x + 2*c) + sqrt(2)) + 4*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c)
 + 6*sqrt(2)*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2
*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*...
```

Fricas [**A**]
time = 0.38, size = 182, normalized size = 1.53

$$\frac{\left((7\,A+12\,B)a\cos\left(dx+c\right)^{3}+(7\,A+12\,B)a\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\left((7\,A+4\,B)a\cos\left(dx+c\right)+2\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{16\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] 1/16*(((7*A + 12*B)*a*cos(d*x + c)^3 + (7*A + 12*B)*a*cos(d*x + c)^2)*sqrt(
a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*
sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x +
c)^2)) + 4*((7*A + 4*B)*a*cos(d*x + c) + 2*A*a)*sqrt(a*cos(d*x + c) + a)*si
n(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [A]
time = 0.53, size = 201, normalized size = 1.69

$$-\frac{\sqrt{2}\left(\sqrt{2}\left(7\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+12\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\left(14\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+8\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-9\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}\right)\sqrt{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] -1/16*sqrt(2)*(sqrt(2)*(7*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 12*B*a*sgn(cos(1/
2*d*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2)
 + 4*sin(1/2*d*x + 1/2*c))) + 4*(14*A*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d
*x + 1/2*c)^3 + 8*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 9*
A*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) - 4*B*a*sgn(cos(1/2*d*x
+ 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^3, x)
```

3.89 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=164

$$\frac{a^{3/2}(11A+14B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}+\frac{a^2(11A+14B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(7A+6B)\sec(c+dx)\,t}{12d\sqrt{a+a\cos(c+}}$$

```
[Out] 1/8*a^(3/2)*(11*A+14*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/
d+1/8*a^2*(11*A+14*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/12*a^2*(7*A+6*B
)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a*A*sec(d*x+c)^2*(a+a*
cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.27, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3054, 3059, 2851, 2852, 212}

$$\frac{a^{3/2}(11A+14B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(11A+14B)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(7A+6B)\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a^(3/2)*(11*A + 14*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*d) + (a^2*(11*A + 14*B)*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]
]) + (a^2*(7*A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d
*x]]) + (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA\sqrt{a+a\cos(c+dx)}\,\sec^2(c+dx)\tan(c+}{3d}$$
$$= \frac{a^2(7A+6B)\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{aA}{}$$
$$= \frac{a^2(11A+14B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(7A+6B)}{12d\sqrt{a}}$$
$$= \frac{a^2(11A+14B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(7A+6B)}{12d\sqrt{a}}$$
$$= \frac{a^{3/2}(11A+14B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+}}\right.}{8d}$$

Mathematica [A]
time = 0.99, size = 132, normalized size = 0.80

$$\frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(3\sqrt{2}(11A+14B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3(c+dx)+(7(7A+6B)+4(11A+6B)\cos(c+dx)+(33A+42B)\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(3*Sqrt[2]*(1
1*A + 14*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (7*(7*A + 6*
B) + 4*(11*A + 6*B)*Cos[c + d*x] + (33*A + 42*B)*Cos[2*(c + d*x)])*Sin[(c +
 d*x)/2]))/(48*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1325 vs. 2(144) = 288.
time = 0.41, size = 1326, normalized size = 8.09

method	result	size
default	Expression too large to display	1326

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/6*a^(1/2)*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(-24*a*(11*A*
ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*
```

```
2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))+11*A*ln(4/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c
)^2*a)^(1/2)+2*a))+14*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))+14*B*l
n(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^
(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^6+12*(22*A*a
^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+28*B*2^(1/2)*(sin(1/2*d*x+1/2
*c)^2*a)^(1/2)*a^(1/2)+33*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)
*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+
33*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1
/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+42*B*ln(-4/(2*cos(1/2*d*
x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*
x+1/2*c)^2*a)^(1/2)-2*a))*a+42*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)
)*a)*sin(1/2*d*x+1/2*c)^4+(-352*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^
(1/2)-198*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*
c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a-198*A*ln(-4/(2*co
s(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(si
n(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a-384*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^
(1/2)*a^(1/2)-252*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*
d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a-252*B*ln(
-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(
1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^2+126*A*a^(
1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+33*A*ln(4/(2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)
^2*a)^(1/2)+2*a))*a+33*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*co
s(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+108
*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+42*B*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+
1/2*c)^2*a)^(1/2)+2*a))*a+42*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1
/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))
*a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/sin(1
/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 7567 vs. $2(144) = 288$.
time = 153.57, size = 7567, normalized size = 46.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] -1/96*((774*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 162*sqrt(2)*a
*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (14*sqrt(2)*a*sin(3/2*d*x + 3/2*c)
```

+ 90*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(6*d*x + 6*c)^2 + 9*(14*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 90*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 + 9*(14*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 90*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (14*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 90*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(6*d*x + 6*c)^2 + 9*(14*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 90*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 9*(14*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 90*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x

```
 + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 44
*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 132*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 14*(
sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin
(2*d*x + 2*c))*cos(15/2*d*x + 15/2*c) + 90*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*
sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(13/2*d*x + 1
3/2*c) - 2*(87*sqrt(2)*a*sin(11/2*d*x + 11/2*c) + 157*sqrt(2)*a*sin(9/2*d*x
 + 9/2*c) + 129*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 27*sqrt(2)*a*sin(5/2*d*x +
 5/2*c) - 29*sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 111*sqrt(2)*a*sin(1/2*d*x + 1
/2*c) - 3*(14*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 90*sqrt(2)*a*sin(1/2*d*x + 1
/2*c) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 33*a*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + ...
```

Fricas [**A**]
time = 0.40, size = 202, normalized size = 1.23

$$\frac{3\left((11A+14B)a\cos(dx+c)^4+(11A+14B)a\cos(dx+c)^3\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(3(11A+14B)a\cos(dx+c)^2+2(11A+6B)a\cos(dx+c)+8Aa\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{96\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] 1/96*(3*((11*A + 14*B)*a*cos(d*x + c)^4 + (11*A + 14*B)*a*cos(d*x + c)^3)*s
qrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) +
 a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)) + 4*(3*(11*A + 14*B)*a*cos(d*x + c)^2 + 2*(11*A + 6*B)*a*cos(d*x
 + c) + 8*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^4 + d
*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 252, normalized size = 1.54

$$-\frac{\sqrt{2}\left(3\sqrt{2}\left(11\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+14\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\left(132\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+168\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-176\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-192\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+63\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+54\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}\right)\sqrt{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] -1/96*sqrt(2)*(3*sqrt(2)*(11*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 14*B*a*sgn(cos
(1/2*d*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt
(2) + 4*sin(1/2*d*x + 1/2*c))) + 4*(132*A*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1
/2*d*x + 1/2*c)^5 + 168*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^
5 - 176*A*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 192*B*a*sgn(
cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 63*A*a*sgn(cos(1/2*d*x + 1/2
*c))*sin(1/2*d*x + 1/2*c) + 54*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x +
1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^4,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^4, x)
```

3.90 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=209

$$\frac{a^{3/2}(75A+88B)\tanh^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d}+\frac{a^2(75A+88B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(75A+88B)\sec(c+dx)}{96d\sqrt{a+a\cos(c+}}$$

```
[Out] 1/64*a^(3/2)*(75*A+88*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))
/d+1/64*a^2*(75*A+88*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/96*a^2*(75*A+
88*B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/24*a^2*(9*A+8*B)*sec
(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a*A*sec(d*x+c)^3*(a+a*cos
(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.32, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3054, 3059, 2851, 2852, 212}

$$\frac{a^{3/2}(75A+88B)\tanh^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(75A+88B)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(9A+8B)\tan(c+dx)\sec^2(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(75A+88B)\tan(c+dx)\sec(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{aA\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a^(3/2)*(75*A + 88*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(64*d) + (a^2*(75*A + 88*B)*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*
x]]) + (a^2*(75*A + 88*B)*Sec[c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*Cos[c
 + d*x]]) + (a^2*(9*A + 8*B)*Sec[c + d*x]^2*Tan[c + d*x])/(24*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d*x])
/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA\sqrt{a+a\cos(c+dx)}\,\sec^3(c+dx)\tan(c+ }{4d}$$

$$= \frac{a^2(9A+8B)\sec^2(c+dx)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aA}{}$$

$$= \frac{a^2(75A+88B)\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(}{}$$

$$= \frac{a^2(75A+88B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(75A+88B}{96d\sqrt{a}}$$

$$= \frac{a^2(75A+88B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(75A+88B}{96d\sqrt{a}}$$

$$= \frac{a^{3/2}(75A+88B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx}}\right.}{64d}$$

Mathematica [A]
time = 1.59, size = 151, normalized size = 0.72

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(6\sqrt{2}\,(75A+88B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^4(c+dx)+(492A+352B+(1155A+1048B)\cos(c+dx)+4(75A+88B)\cos(2(c+dx))+225A\cos(3(c+dx))+264B\cos(3(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^4*(6*Sqrt[2]*(7
5*A + 88*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (492*A + 352
*B + (1155*A + 1048*B)*Cos[c + d*x] + 4*(75*A + 88*B)*Cos[2*(c + d*x)] + 22
5*A*Cos[3*(c + d*x)] + 264*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1650 vs. 2(185) = 370.
time = 0.48, size = 1651, normalized size = 7.90

method	result	size
default	Expression too large to display	1651

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/24*a^(1/2)*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(48*a*(75*A*
ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*
2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))+75*A*ln(4/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c
)^2*a)^(1/2)+2*a))+88*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))+88*B*l
n(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^
(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^8-48*(75*A*a
^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+88*B*2^(1/2)*(sin(1/2*d*x+1/2
*c)^2*a)^(1/2)*a^(1/2)+150*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2
)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a
+150*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^
(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+176*B*ln(-4/(2*cos(1/2
*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2
*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+176*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+
2*a))*a)*sin(1/2*d*x+1/2*c)^6+8*(825*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^
2*a)^(1/2)+968*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+675*A*ln(-4
/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/
2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+675*A*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^
2*a)^(1/2)+2*a))*a+792*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*co
s(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+792
*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2
)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-4*(1
095*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+1208*B*2^(1/2)*(sin(1/
2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+450*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*
(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2
)-2*a))*a+450*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+
1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+528*B*ln(-4/(
2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)
*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+528*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*
a)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+1086*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+
1/2*c)^2*a)^(1/2)+225*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+225*A
*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)
*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+1008*B*2^(1/2)*(sin(1/2*d*x
+1/2*c)^2*a)^(1/2)*a^(1/2)+264*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)
)*a+264*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c
)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a)/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))^4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4/sin(1/2*d*x+1/2*c)/(a*cos(
1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 10504 vs. $2(185) = 370$.
time = 154.60, size = 10504, normalized size = 50.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="maxima")
```

```
[Out] -1/768*(3*(140*a*cos(8*d*x + 8*c)^2*sin(3/2*d*x + 3/2*c) + 2240*a*cos(6*d*x
 + 6*c)^2*sin(3/2*d*x + 3/2*c) + 5040*a*cos(4*d*x + 4*c)^2*sin(3/2*d*x + 3/
2*c) + 2240*a*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 140*a*sin(8*d*x + 8
*c)^2*sin(3/2*d*x + 3/2*c) + 2240*a*sin(6*d*x + 6*c)^2*sin(3/2*d*x + 3/2*c)
 + 5040*a*sin(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 2240*a*sin(2*d*x + 2*c)
^2*sin(3/2*d*x + 3/2*c) + 4064*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 33
6*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 240*a*cos(3/2*d*x + 3/2*c)*sin(
2*d*x + 2*c) + 1360*a*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 36*(a*sin(8*d
*x + 8*c) + 4*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + 4*a*sin(2*d*x + 2
*c))*cos(21/2*d*x + 21/2*c) + 140*(a*sin(8*d*x + 8*c) + 4*a*sin(6*d*x + 6*c
) + 6*a*sin(4*d*x + 4*c) + 4*a*sin(2*d*x + 2*c))*cos(19/2*d*x + 19/2*c) + 4
56*(a*sin(8*d*x + 8*c) + 4*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + 4*a*
sin(2*d*x + 2*c))*cos(17/2*d*x + 17/2*c) + 4*(280*a*cos(6*d*x + 6*c)*sin(3/
2*d*x + 3/2*c) + 420*a*cos(4*d*x + 4*c)*sin(3/2*d*x + 3/2*c) + 280*a*cos(2*
d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 290*a*sin(15/2*d*x + 15/2*c) - 596*a*sin(
13/2*d*x + 13/2*c) - 780*a*sin(11/2*d*x + 11/2*c) - 750*a*sin(9/2*d*x + 9/2
*c) - 254*a*sin(7/2*d*x + 7/2*c) - 21*a*sin(5/2*d*x + 5/2*c) + 85*a*sin(3/2
*d*x + 3/2*c))*cos(8*d*x + 8*c) + 2320*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*
x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(15/2*d*x + 15/2*c) + 4768*(2*a*sin(6*d
*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/
2*c) + 16*(420*a*cos(4*d*x + 4*c)*sin(3/2*d*x + 3/2*c) + 280*a*cos(2*d*x +
2*c)*sin(3/2*d*x + 3/2*c) - 780*a*sin(11/2*d*x + 11/2*c) - 750*a*sin(9/2*d*
x + 9/2*c) - 254*a*sin(7/2*d*x + 7/2*c) - 21*a*sin(5/2*d*x + 5/2*c) + 85*a*
sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) + 6240*(3*a*sin(4*d*x + 4*c) + 2*a*s
in(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 6000*(3*a*sin(4*d*x + 4*c) + 2*a*
sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) + 24*(280*a*cos(2*d*x + 2*c)*sin(3/2
*d*x + 3/2*c) - 254*a*sin(7/2*d*x + 7/2*c) - 21*a*sin(5/2*d*x + 5/2*c) + 85
*a*sin(3/2*d*x + 3/2*c))*cos(4*d*x + 4*c) - 75*(sqrt(2)*a*cos(8*d*x + 8*c)^
2 + 16*sqrt(2)*a*cos(6*d*x + 6*c)^2 + 36*sqrt(2)*a*cos(4*d*x + 4*c)^2 + 16*
sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(8*d*x + 8*c)^2 + 16*sqrt(2)*a*
sin(6*d*x + 6*c)^2 + 36*sqrt(2)*a*sin(4*d*x + 4*c)^2 + 48*sqrt(2)*a*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 16*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 8*sqrt(2)*a*
cos(2*d*x + 2*c) + 2*(4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x
+ 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(8*d*x + 8*c) + 8*(6*
sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(
```

```
6*d*x + 6*c) + 12*(4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d*x + 4*
c) + 4*(2*sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqr
t(2)*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*sqrt(2)*a*sin(4*d*x + 4*c
) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sqrt(2)*a)*log(2*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 75*(sqrt(2)*a*cos(8*d*
x + 8*c)^2 + 16*sqrt(2)*a*cos(6*d*x + 6*c)^2 + 36*sqrt(2)*a*cos(4*d*x + 4*c
)^2 + 16*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(8*d*x + 8*c)^2 + 16*s
qrt(2)*a*sin(6*d*x + 6*c)^2 + 36*sqrt(2)*a*sin(4*d*x + 4*c)^2 + 48*sqrt(2)*
a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 8*s
qrt(2)*a*cos(2*d*x + 2*c) + 2*(4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*c
os(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(8*d*x + 8*c
) + 8*(6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2
)*a)*cos(6*d*x + 6*c) + 12*(4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4
*d*x + 4*c) + 4*(2*sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c
) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*sqrt(2)*a*sin(4*
d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sqrt(2)*a)*lo
g(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(sqrt(2)*a
*cos(8*d*x + 8*c)^2 + 16*sqrt(2)*a*cos(6*d*x + 6*c)^2 + 36*sqrt(2)*a*cos(4*
d*x + 4*c)^2 + 16*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(8*d*x + 8*c)
^2 + 16*sqrt(2)*a*sin(6*d*x + 6*c)^2 + 36*sqrt(2)*a*sin(4*d*x + 4*c)^2 + 48
*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sqrt(2)*a*sin(2*d*x + 2*c
)^2 + 8*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sq
rt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(8*
d*x + 8*c) + 8*(6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c)
 + sqrt(2)*a)*cos(6*d*x + 6*c) + 12*(4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)
*a)*cos(4*d*x + 4*c) + 4*(2*sqrt(2)*a*sin(6*d*x...
```

Fricas [A]

time = 0.39, size = 220, normalized size = 1.05

$$\frac{3\left((75A+88B)a\cos(dx+c)^5+(75A+88B)a\cos(dx+c)^4\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(3(75A+88B)a\cos(dx+c)^3+2(75A+88B)a\cos(dx+c)^2+8(15A+8B)a\cos(dx+c)+48Aa\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{768\left(d\cos(dx+c)^5+d\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="fricas")
```

```
[Out] 1/768*(3*((75*A + 88*B)*a*cos(d*x + c)^5 + (75*A + 88*B)*a*cos(d*x + c)^4)*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c)
+ a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d
```

```
*x + c)^2)) + 4*(3*(75*A + 88*B)*a*cos(d*x + c)^3 + 2*(75*A + 88*B)*a*cos(d
*x + c)^2 + 8*(15*A + 8*B)*a*cos(d*x + c) + 48*A*a)*sqrt(a*cos(d*x + c) + a
)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.64, size = 302, normalized size = 1.44

$$-\frac{\sqrt{2}\left(3\sqrt{2}\left(75\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+88\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\left(1800\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+2112\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3300\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3872\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+2190\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2416\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-543\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-504\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{4}}\right)\sqrt{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="giac")
```

```
[Out] -1/768*sqrt(2)*(3*sqrt(2)*(75*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 88*B*a*sgn(co
s(1/2*d*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqr
t(2) + 4*sin(1/2*d*x + 1/2*c))) + 4*(1800*A*a*sgn(cos(1/2*d*x + 1/2*c))*sin
(1/2*d*x + 1/2*c)^7 + 2112*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*
c)^7 - 3300*A*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 3872*B*a
*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 + 2190*A*a*sgn(cos(1/2*d*
x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 2416*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin
(1/2*d*x + 1/2*c)^3 - 543*A*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c
) - 504*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x
+ 1/2*c)^2 - 1)^4)*sqrt(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+a\,\cos(c+d\,x)\right)^{3/2}}{\cos(c+d\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^5,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^5, x)
```

3.91 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=237

$$\frac{2a^3(803A+710B)\sin(c+dx)}{495d\sqrt{a+a\cos(c+dx)}}+\frac{2a^3(209A+194B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}-\frac{4a^2(803A+710B)\sqrt{a+a\,\mathrm{co}}}{3465d}$$

```
[Out] 2/1155*a*(803*A+710*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/11*a*B*cos(d*x
+c)^3*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/495*a^3*(803*A+710*B)*sin(d*x+c
)/d/(a+a*cos(d*x+c))^(1/2)+2/693*a^3*(209*A+194*B)*cos(d*x+c)^3*sin(d*x+c)/
d/(a+a*cos(d*x+c))^(1/2)-4/3465*a^2*(803*A+710*B)*sin(d*x+c)*(a+a*cos(d*x+c
))^(1/2)/d+2/99*a^2*(11*A+14*B)*cos(d*x+c)^3*sin(d*x+c)*(a+a*cos(d*x+c))^(1
/2)/d
```

Rubi [A]
time = 0.41, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3055, 3060, 2838, 2830, 2725}

$$\frac{2a^3(209A+194B)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(803A+710B)\sin(c+dx)}{495d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(11A+14B)\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{99d}-\frac{4a^2(803A+710B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3465d}+\frac{2a(803A+710B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{1155d}+\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a^3*(803*A + 710*B)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
a^3*(209*A + 194*B)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c +
d*x]]) - (4*a^2*(803*A + 710*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(346
5*d) + (2*a^2*(11*A + 14*B)*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
 d*x])/(99*d) + (2*a*(803*A + 710*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x
])/(1155*d) + (2*a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x]
)/(11*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{2aB\cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c}{11d}$$
$$= \frac{2a^2(11A+14B)\cos^3(c+dx)\sqrt{a+a\cos(c}}{99d}$$
$$= \frac{2a^3(209A+194B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}$$
$$= \frac{2a^3(209A+194B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}$$
$$= \frac{2a^3(209A+194B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}$$
$$= \frac{2a^3(803A+710B)\sin(c+dx)}{495d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(209A}{6}$$

Mathematica [A]
time = 1.10, size = 127, normalized size = 0.54

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,(124366A+114640B+(68552A+69890B)\cos(c+dx)+16(1397A+1625B)\cos(2(c+dx))+5720A\cos(3(c+dx))+8675B\cos(3(c+dx))+770A\cos(4(c+dx))+2240B\cos(4(c+dx))+315B\cos(5(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{27720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(124366*A + 114640*B + (68552*A + 69890*B)*
Cos[c + d*x] + 16*(1397*A + 1625*B)*Cos[2*(c + d*x)] + 5720*A*Cos[3*(c + d*
x)] + 8675*B*Cos[3*(c + d*x)] + 770*A*Cos[4*(c + d*x)] + 2240*B*Cos[4*(c +
d*x)] + 315*B*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])/(27720*d)
```

Maple [A]
time = 0.19, size = 142, normalized size = 0.60

method	result
default	$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-2520B\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(1540A+10780B)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-5940A-18810B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\right.}{3465\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 8/3465*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(-2520*B*sin(1/2*d*x+1/2*c
)^10+(1540*A+10780*B)*sin(1/2*d*x+1/2*c)^8+(-5940*A-18810*B)*sin(1/2*d*x+1/
```

```
2*c)^6+(9009*A+17325*B)*sin(1/2*d*x+1/2*c)^4+(-6930*A-9240*B)*sin(1/2*d*x+1
/2*c)^2+3465*A+3465*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A]
time = 0.60, size = 207, normalized size = 0.87

$$\frac{22 \left(35 \sqrt{2} a^2 \sin\left(\frac{9}{2} dx + \frac{9}{2} c\right) + 225 \sqrt{2} a^2 \sin\left(\frac{7}{2} dx + \frac{7}{2} c\right) + 756 \sqrt{2} a^2 \sin\left(\frac{5}{2} dx + \frac{5}{2} c\right) + 2100 \sqrt{2} a^2 \sin\left(\frac{3}{2} dx + \frac{3}{2} c\right) + 8190 \sqrt{2} a^2 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) A\sqrt{a} + 5 \left(63 \sqrt{2} a^2 \sin\left(\frac{11}{2} dx + \frac{11}{2} c\right) + 385 \sqrt{2} a^2 \sin\left(\frac{9}{2} dx + \frac{9}{2} c\right) + 1287 \sqrt{2} a^2 \sin\left(\frac{7}{2} dx + \frac{7}{2} c\right) + 3465 \sqrt{2} a^2 \sin\left(\frac{5}{2} dx + \frac{5}{2} c\right) + 8778 \sqrt{2} a^2 \sin\left(\frac{3}{2} dx + \frac{3}{2} c\right) + 31878 \sqrt{2} a^2 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) B\sqrt{a}}{55440\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/55440*(22*(35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)*a^2*sin(7/2*
d*x + 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqrt(2)*a^2*sin(
3/2*d*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 5*(63
*sqrt(2)*a^2*sin(11/2*d*x + 11/2*c) + 385*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c)
+ 1287*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 3465*sqrt(2)*a^2*sin(5/2*d*x + 5/
2*c) + 8778*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 31878*sqrt(2)*a^2*sin(1/2*d*
x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A]
time = 0.34, size = 137, normalized size = 0.58

$$\frac{2 \left(315 \, B a^2 \cos\left(dx + c\right)^5 + 35 \left(11 \, A + 32 \, B\right) a^2 \cos\left(dx + c\right)^4 + 5 \left(286 \, A + 355 \, B\right) a^2 \cos\left(dx + c\right)^3 + 3 \left(803 \, A + 710 \, B\right) a^2 \cos\left(dx + c\right)^2 + 4 \left(803 \, A + 710 \, B\right) a^2 \cos\left(dx + c\right) + 8 \left(803 \, A + 710 \, B\right) a^2\right) \sqrt{a \cos\left(dx + c\right) + a} \sin\left(dx + c\right)}{3465 \left(d \cos\left(dx + c\right) + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/3465*(315*B*a^2*cos(d*x + c)^5 + 35*(11*A + 32*B)*a^2*cos(d*x + c)^4 + 5*
(286*A + 355*B)*a^2*cos(d*x + c)^3 + 3*(803*A + 710*B)*a^2*cos(d*x + c)^2 +
 4*(803*A + 710*B)*a^2*cos(d*x + c) + 8*(803*A + 710*B)*a^2)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.04, size = 257, normalized size = 1.08

$$\frac{\sqrt{2} \left(315 \, B a^2 \mathrm{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{11}{2} dx + \frac{11}{2} c\right) + 385 \left(2 \, A a^2 \mathrm{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + 5 \, B a^2 \mathrm{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)\right) \sin\left(\frac{9}{2} dx + \frac{9}{2} c\right) + 495 \left(10 \, A a^2 \mathrm{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + 13 \, B a^2 \mathrm{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)\right) \sin\left(\frac{7}{2} dx + \frac{7}{2} c\right) + 693 \left(24 \, A a^2 \mathrm{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + 25 \, B a^2 \mathrm{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)\right) \sin\left(\frac{5}{2} dx + \frac{5}{2} c\right) + 2310 \left(20 \, A a^2 \mathrm{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + 19 \, B a^2 \mathrm{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)\right) \sin\left(\frac{3}{2} dx + \frac{3}{2} c\right) + 6930 \left(26 \, A a^2 \mathrm{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + 23 \, B a^2 \mathrm{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sqrt{a}}{55440 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] 1/55440*sqrt(2)*(315*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(11/2*d*x + 11/2*c)
 + 385*(2*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 5*B*a^2*sgn(cos(1/2*d*x + 1/2*c
)))*sin(9/2*d*x + 9/2*c) + 495*(10*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 13*B*a
^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(7/2*d*x + 7/2*c) + 693*(24*A*a^2*sgn(cos(
1/2*d*x + 1/2*c)) + 25*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(5/2*d*x + 5/2*c
) + 2310*(20*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 19*B*a^2*sgn(cos(1/2*d*x + 1
/2*c)))*sin(3/2*d*x + 3/2*c) + 6930*(26*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 2
3*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

3.92 $\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=175

$$\frac{64a^3(15A+13B)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}+\frac{16a^2(15A+13B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{315d}+\frac{2a(15A+13B)(a+a\cos}{10}$$

```
[Out] 2/105*a*(15*A+13*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/63*(9*A-2*B)*(a+a
*cos(d*x+c))^(5/2)*sin(d*x+c)/d+2/9*B*(a+a*cos(d*x+c))^(7/2)*sin(d*x+c)/a/d
+64/315*a^3*(15*A+13*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+16/315*a^2*(15*
A+13*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3047, 3102, 2830, 2726, 2725}

$$\frac{64a^3(15A+13B)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2(15A+13B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{2(9A-2B)\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{63d}+\frac{2a(15A+13B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105d}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (64*a^3*(15*A + 13*B)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (16*
a^2*(15*A + 13*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*(15
*A + 13*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*d) + (2*(9*A - 2*B
)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (2*B*(a + a*Cos[c + d*x
])^(7/2)*Sin[c + d*x])/(9*a*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^{5/2}\left(A\cos(c+dx)+B\right.$$
$$= \frac{2B(a+a\cos(c+dx))^{7/2}\sin(c+dx)}{9ad} + \frac{2\int(}{}$$
$$= \frac{2(9A-2B)(a+a\cos(c+dx))^{5/2}\sin(c+dx}{63d}$$
$$= \frac{2a(15A+13B)(a+a\cos(c+dx))^{3/2}\sin(c+}{105d}$$
$$= \frac{16a^2(15A+13B)\sqrt{a+a\cos(c+dx)}\,\sin(c}{315d}$$
$$= \frac{64a^3(15A+13B)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2(15A+}{}$$

Mathematica [A]
time = 0.75, size = 105, normalized size = 0.60

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,(6240A+5653B+(3030A+3116B)\cos(c+dx)+8(90A+127B)\cos(2(c+dx))+90A\cos(3(c+dx))+260B\cos(3(c+dx))+35B\cos(4(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(6240*A + 5653*B + (3030*A + 3116*B)*Cos[c
+ d*x] + 8*(90*A + 127*B)*Cos[2*(c + d*x)] + 90*A*Cos[3*(c + d*x)] + 260*B*
Cos[3*(c + d*x)] + 35*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A]
time = 0.18, size = 123, normalized size = 0.70

method	result
default	$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{3}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(140B\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-90A-540B\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(315A+819B\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-420A-630B\right.}{315\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] 8/315*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(140*B*sin(1/2*d*x+1/2*c)^8
+(-90*A-540*B)*sin(1/2*d*x+1/2*c)^6+(315*A+819*B)*sin(1/2*d*x+1/2*c)^4+(-42
0*A-630*B)*sin(1/2*d*x+1/2*c)^2+315*A+315*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^
2)^(1/2)/d
```

Maxima [A]
time = 0.58, size = 172, normalized size = 0.98

$$\frac{30\left(3\sqrt{2}a^{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+21\sqrt{2}a^{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+77\sqrt{2}a^{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+315\sqrt{2}a^{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+\left(35\sqrt{2}a^{2}\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+225\sqrt{2}a^{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+756\sqrt{2}a^{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+2100\sqrt{2}a^{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+8190\sqrt{2}a^{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)B\sqrt{a}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/2520*(30*(3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x
 + 5/2*c) + 77*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c))*A*sqrt(a) + (35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)
*a^2*sin(7/2*d*x + 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqr
t(2)*a^2*sin(3/2*d*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sq
rt(a))/d
```

Fricas [A]
time = 0.35, size = 116, normalized size = 0.66

$$\frac{2\left(35\,Ba^{2}\cos\left(dx+c\right)^{4}+5\left(9\,A+26\,B\right)a^{2}\cos\left(dx+c\right)^{3}+3\left(60\,A+73\,B\right)a^{2}\cos\left(dx+c\right)^{2}+\left(345\,A+292\,B\right)a^{2}\cos\left(dx+c\right)+2\left(345\,A+292\,B\right)a^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/315*(35*B*a^2*cos(d*x + c)^4 + 5*(9*A + 26*B)*a^2*cos(d*x + c)^3 + 3*(60*
A + 73*B)*a^2*cos(d*x + c)^2 + (345*A + 292*B)*a^2*cos(d*x + c) + 2*(345*A
+ 292*B)*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [**A**]
time = 1.17, size = 213, normalized size = 1.22

$$\frac{\sqrt{2}\,\left(35\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+45\,\left(2\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+5\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+126\,\left(5\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+6\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+210\,\left(11\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+10\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+630\,\left(15\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+13\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/2520*sqrt(2)*(35*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) + 4
5*(2*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 5*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*s
in(7/2*d*x + 7/2*c) + 126*(5*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 6*B*a^2*sgn(
cos(1/2*d*x + 1/2*c)))*sin(5/2*d*x + 5/2*c) + 210*(11*A*a^2*sgn(cos(1/2*d*x
 + 1/2*c)) + 10*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c) + 630
*(15*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 13*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*
sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

3.93 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{64a^3(7A+5B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{16a^2(7A+5B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{105d}+\frac{2a(7A+5B)(a+a\cos(c+d}{35d}$$

```
[Out] 2/35*a*(7*A+5*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(a+a*cos(d*x+c))
^(5/2)*sin(d*x+c)/d+64/105*a^3*(7*A+5*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2
)+16/105*a^2*(7*A+5*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2830, 2726, 2725}

$$\frac{64a^3(7A+5B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2(7A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(7A+5B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (64*a^3*(7*A + 5*B)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a^
2*(7*A + 5*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*a*(7*A +
5*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*B*(a + a*Cos[c +
d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx &= \frac{2B(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d}+\frac{1}{7}(7A+5B)\int (a \\ &= \frac{2a(7A+5B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d}+\frac{2B(a-}{} \\ &= \frac{16a^2(7A+5B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d}+\frac{2a(7A}{} \\ &= \frac{64a^3(7A+5B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{16a^2(7A+5B)\sqrt{a+a\cos}}{105d}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 83, normalized size = 0.60

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,(1246A+1040B+(392A+505B)\cos(c+dx)+6(7A+20B)\cos(2(c+dx))+15B\cos(3(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(1246*A + 1040*B + (392*A + 505*B)*Cos[c +
d*x] + 6*(7*A + 20*B)*Cos[2*(c + d*x)] + 15*B*Cos[3*(c + d*x)])*Tan[(c + d*
x)/2])/(210*d)
```

Maple [A]
time = 0.16, size = 104, normalized size = 0.75

method	result
default	$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-30B\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(21A+105B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-70A-140B)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+105A+105B\right)}{105\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/105*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(-30*B*sin(1/2*d*x+1/2*c)^6
+(21*A+105*B)*sin(1/2*d*x+1/2*c)^4+(-70*A-140*B)*sin(1/2*d*x+1/2*c)^2+105*A
+105*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A]
time = 0.56, size = 139, normalized size = 1.01

$$\frac{14\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+5\left(3\sqrt{2}a^2\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+21\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+77\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+315\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)B\sqrt{a}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*(14*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x
+ 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 5*(3*sqrt(2)*a
^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 77*sqrt(2)*
a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt(a))
/d
```

Fricas [**A**]
time = 0.34, size = 95, normalized size = 0.69

$$\frac{2\left(15\,Ba^{2}\cos\left(dx+c\right)^{3}+3\left(7\,A+20\,B\right)a^{2}\cos\left(dx+c\right)^{2}+\left(98\,A+115\,B\right)a^{2}\cos\left(dx+c\right)+\left(301\,A+230\,B\right)a^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/105*(15*B*a^2*cos(d*x + c)^3 + 3*(7*A + 20*B)*a^2*cos(d*x + c)^2 + (98*A
+ 115*B)*a^2*cos(d*x + c) + (301*A + 230*B)*a^2)*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**A**]
time = 0.73, size = 169, normalized size = 1.22

$$\frac{\sqrt{2}\left(15\,Ba^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\left(2\,Aa^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+5\,Ba^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+35\left(10\,Aa^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+11\,Ba^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+525\left(4\,Aa^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+3\,Ba^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*sqrt(2)*(15*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 21
*(2*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 5*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*si
n(5/2*d*x + 5/2*c) + 35*(10*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 11*B*a^2*sgn(
cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c) + 525*(4*A*a^2*sgn(cos(1/2*d*x
+ 1/2*c)) + 3*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a
)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos (c+dx))\,(a+a\cos (c+dx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
[Out] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

3.94 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=142

$$\frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^3(35A+32B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(5A+8B)\sqrt{a+a\cos(c+dx)}\,\mathrm{si}}{15d}$$

```
[Out] 2*a^(5/2)*A*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2/5*a*B*(a
+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/15*a^3*(35*A+32*B)*sin(d*x+c)/d/(a+a*co
s(d*x+c))^(1/2)+2/15*a^2*(5*A+8*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.27, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3055, 3060, 2852, 212}

$$\frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^3(35A+32B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(5A+8B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*a^(5/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^3*(35*A + 32*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(5*A + 8*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*B*(a + a*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2}{5}\int$$

$$=\frac{2a^2(5A+8B)\sqrt{a+a\cos(c+dx)}\,\sin(c+d}{15d}$$

$$=\frac{2a^3(35A+32B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(5A+8B}{}$$

$$=\frac{2a^3(35A+32B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(5A+8B}{}$$

$$=\frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a}{}$$

Mathematica [A]
time = 0.43, size = 104, normalized size = 0.73

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(15\sqrt{2}\,A\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+(80A+89B+2(5A+14B)\cos(c+dx)+3B\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*A*ArcTanh[Sqrt
[2]*Sin[(c + d*x)/2]] + (80*A + 89*B + 2*(5*A + 14*B)*Cos[c + d*x] + 3*B*Co
s[2*(c + d*x)])*Sin[(c + d*x)/2]))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(124) = 248$.
time = 0.33, size = 313, normalized size = 2.20

method	result
default	$\frac{a^{\frac{3}{2}} \cos\left(\frac{dx}{2}+\frac{c}{2}\right) \sqrt{a \left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} \left(24 B \sqrt{2}\, \sqrt{a \left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, \sqrt{a}\, \left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20 \sqrt{a}\, \sqrt{2}\, \sqrt{a \left(\sin\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/15*a^(3/2)*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(24*B*2^(1/2
)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4-20*a^(1/2)*2^
(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(A+4*B)*sin(1/2*d*x+1/2*c)^2+90*A*a^(1
/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+15*A*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^
2*a)^(1/2)+2*a))*a+15*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+120*
B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos
(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A]
time = 0.53, size = 61, normalized size = 0.43

$$\frac{\left(3\,\sqrt{2}\,a^{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\,\sqrt{2}\,a^{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\,\sqrt{2}\,a^{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt(a)/d
```

Fricas [A]
time = 0.36, size = 177, normalized size = 1.25

$$\frac{15\left(Aa^{2}\cos\left(dx+c\right)+Aa^{2}\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\left(3\,Ba^{2}\cos\left(dx+c\right)^{2}+(5\,A+14\,B)a^{2}\cos\left(dx+c\right)+(40\,A+43\,B)a^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{30\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/30*(15*(A*a^2*cos(d*x + c) + A*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*c
os(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d
*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*B*a^2*cos(d*x + c)
^2 + (5*A + 14*B)*a^2*cos(d*x + c) + (40*A + 43*B)*a^2)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.09, size = 202, normalized size = 1.42

$$\frac{\sqrt{2}\left(48\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-160\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,\sqrt{2}\,Aa^2\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+180\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+240\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] 1/30*sqrt(2)*(48*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 4
0*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 160*B*a^2*sgn(co
s(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 15*sqrt(2)*A*a^2*log(abs(-2*sq
rt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sg
n(cos(1/2*d*x + 1/2*c)) + 180*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x +
 1/2*c) + 240*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)
/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x), x)
```

3.95 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{a^{5/2}(5A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}-\frac{a^2(3A-2B)\sqrt{a+a\cos(c+dx)}}{3d}$$

```
[Out] a^(5/2)*(5*A+2*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+1/3*
a^3*(3*A+14*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-1/3*a^2*(3*A-2*B)*sin(d*
x+c)*(a+a*cos(d*x+c))^(1/2)/d+a*A*(a+a*cos(d*x+c))^(3/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.29, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3054, 3055, 3060, 2852, 212}

$$\frac{a^{5/2}(5A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(3A-2B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^(5/2)*(5*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d + (a^3*(3*A + 14*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) - (a
^2*(3*A - 2*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (a*A*(a + a*C
os[c + d*x])^(3/2)*Tan[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^{3/2}\tan(c+dx)}{d} + \int (a$$

$$= -\frac{a^2(3A-2B)\sqrt{a+a\cos(c+dx)}\,\sin(c+d$$

$$= \frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(3A-2B)\sqrt{a}$$

$$= \frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(3A-2B)\sqrt{a}$$

$$= \frac{a^{5/2}(5A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right.}{d}$$

Mathematica [A]
time = 0.56, size = 120, normalized size = 0.83

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(3\sqrt{2}(5A+2B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos(c+dx)+2(3A+B+2(3A+8B)\cos(c+dx)+B\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(3*Sqrt[2]*(5
*A + 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(3*A + B + 2*(
3*A + 8*B)*Cos[c + d*x] + B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(128) = 256$.
time = 0.38, size = 764, normalized size = 5.31

method	result
default	$\frac{a^{\frac{3}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(16B\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-30A\ln\left(\frac{4a\sqrt{2}\cos(}{}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/3*a^(3/2)*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(16*B*2^(1/2)
*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4+(-30*A*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*
(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a-30*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(
1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a
)^(1/2)-2*a))*a-24*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-12*B*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(
1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a-12*B*ln(-4/(2*cos(1/2*d*x+1/2*c
)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c
)^2*a)^(1/2)-2*a))*a-80*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))*s
in(1/2*d*x+1/2*c)^2+15*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+15*A
*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)
*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+18*A*a^(1/2)*2^(1/2)*(sin(1
/2*d*x+1/2*c)^2*a)^(1/2)+6*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)
*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+
6*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1
/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+36*B*2^(1/2)*(sin(1/2*d*
x+1/2*c)^2*a)^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 8114 vs. $2(128) = 256$.
time = 0.84, size = 8114, normalized size = 56.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] -1/252*(1449*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^3*sin(2*d*x + 2*c) - 1260*sqr
t(2)*a^2*sin(1/2*d*x + 1/2*c)^3 - 1449*(sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt
(2)*a^2)*sin(5/2*d*x + 5/2*c)^3 + 21*(25*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin
(3/2*d*x + 3/2*c) + 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c)
- 60*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 5*(5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*
c) - 12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + (25*sqrt(2)*a^
2*cos(3/2*d*x + 3/2*c) + 198*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x +
2*c))*cos(5/2*d*x + 5/2*c)^2 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 25
*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*
sin(3/2*d*x + 3/2*c))*cos(2*d*x + 2*c)^2 + 21*(25*sqrt(2)*a^2*cos(2*d*x + 2
*c)^2*sin(3/2*d*x + 3/2*c) + 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x
+ 3/2*c) + 69*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 198*sqrt(
2)*a^2*sin(1/2*d*x + 1/2*c) + (25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 198*sq
rt(2)*a^2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 5*(5*sqrt(2)*a^2*cos(3/2
```

```
*d*x + 3/2*c) + 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sin(
5/2*d*x + 5/2*c)^2 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 25*(sqrt(2)*
a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*
x + 3/2*c))*sin(2*d*x + 2*c)^2 - 35*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin
(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin
(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqr
t(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt
(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*
d*x + 2*c))*cos(13/2*d*x + 13/2*c) - 135*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^
2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c
)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) +
2*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) +
(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*s
in(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) - 98*(sqrt(2)*a^2*cos(5/2*d*x + 5/2
*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1
/2*c)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c
) + 2*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c
) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^
2)*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) + 390*(sqrt(2)*a^2*cos(5/2*d*x +
5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x
+ 1/2*c)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x +
2*c) + 2*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/
2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*
c)^2)*sin(2*d*x + 2*c))*cos(7/2*d*x + 7/2*c) + 21*(50*sqrt(2)*a^2*cos(2*d*x
 + 2*c)^2*cos(1/2*d*x + 1/2*c)*sin(3/2*d*x + 3/2*c) + 50*sqrt(2)*a^2*cos(1/
2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) - 120*sqrt(2)*a^2*co
s(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 10*(5*sqrt(2)*a^2*cos(1/2*d*x + 1
/2*c)*sin(3/2*d*x + 3/2*c) - 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*
x + 1/2*c))*cos(2*d*x + 2*c) + (50*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2
*d*x + 1/2*c) + 189*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 69*sqrt(2)*a^2*sin
(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c))*cos(5/2*d*x + 5/2*c) - 21*(60*sqrt(2
)*a^2*sin(1/2*d*x + 1/2*c)^3 - 25*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqr
t(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c) + 12*(5*sqrt(2)*a^2*c
os(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*a^2)*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*
c) - 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*co
s(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*c
os(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2
*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^
2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x
 + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2
*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2
*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x
 + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2
*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*
```

```
sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(
2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 315*(a^2*cos(1/
2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a
^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^
2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d...
```

Fricas [A]
time = 0.38, size = 202, normalized size = 1.40

$$\frac{3\left((5A+2B)a^2\cos(dx+c)^2+(5A+2B)a^2\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(2Ba^2\cos(dx+c)^2+2(3A+8B)a^2\cos(dx+c)+3Aa^2\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{12\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] 1/12*(3*((5*A + 2*B)*a^2*cos(d*x + c)^2 + (5*A + 2*B)*a^2*cos(d*x + c))*sqr
t(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a
)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)) + 4*(2*B*a^2*cos(d*x + c)^2 + 2*(3*A + 8*B)*a^2*cos(d*x + c) + 3*A
*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x
+ c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.78, size = 209, normalized size = 1.45

$$-\frac{\sqrt{2}\left(16Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-24Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)-72Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{12Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}+3\sqrt{2}\left(5Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)+2Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\right)\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\right)\sqrt{a}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] -1/12*sqrt(2)*(16*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 -
24*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) - 72*B*a^2*sgn(cos(
```

```
1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) + 12*A*a^2*sgn(cos(1/2*d*x + 1/2*c))
*sin(1/2*d*x + 1/2*c)/(2*sin(1/2*d*x + 1/2*c)^2 - 1) + 3*sqrt(2)*(5*A*a^2*s
gn(cos(1/2*d*x + 1/2*c)) + 2*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*log(abs(-2*sq
rt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))))*s
qrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^2,x)
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^2, x)
```

3.96 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=156

$$\frac{a^{5/2}(19A+20B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}-\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(7A+4B)\sqrt{a+a\cos(c+}{4d}$$

```
[Out] 1/4*a^(5/2)*(19*A+20*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/
d-1/4*a^3*(9*A-4*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a*A*(a+a*cos(d*
x+c))^(3/2)*sec(d*x+c)*tan(d*x+c)/d+1/4*a^2*(7*A+4*B)*(a+a*cos(d*x+c))^(1/2
)*tan(d*x+c)/d
```

Rubi [A]

time = 0.31, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3054, 3060, 2852, 212}

$$\frac{a^{5/2}(19A+20B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(7A+4B)\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^(5/2)*(19*A + 20*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(4*d) - (a^3*(9*A - 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]])
 + (a^2*(7*A + 4*B)*Sqrt[a + a*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a*A*(a
+ a*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx &= \frac{aA(a+a\cos(c+dx))^{3/2}\sec(c+dx)\tan(c+\cdots}{2d} \\
&= \frac{a^2(7A+4B)\sqrt{a+a\cos(c+dx)}\,\tan(c+dx\cdots}{4d} \\
&= -\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(7A+4B)\sqrt{\cdots}}{\cdots} \\
&= -\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(7A+4B)\sqrt{\cdots}}{\cdots} \\
&= \frac{a^{5/2}(19A+20B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx\cdots}}\right.}{4d}
\end{aligned}$$

Mathematica [A]
time = 0.67, size = 126, normalized size = 0.81

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(\sqrt{2}\,(19A+20B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^2(c+dx)+2((11A+4B)\cos(c+dx)+2(A+2B+2B\cos(2(c+dx))))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(Sqrt[2]*(1
9*A + 20*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*((11*A + 4
*B)*Cos[c + d*x] + 2*(A + 2*B + 2*B*Cos[2*(c + d*x)]))*Sin[(c + d*x)/2]))/(
8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(136) = 272$.
time = 0.41, size = 1028, normalized size = 6.59

method	result	size
default	Expression too large to display	1028

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/2*a^(3/2)*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*((76*A*ln(-4/
(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2
)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*
a)^(1/2)+2*a))*a+64*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+80*B*l
n(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2
^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+80*B*ln(4/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*
c)^2*a)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-44*A*a^(1/2)*2^(1/2)*(sin(1/2
*d*x+1/2*c)^2*a)^(1/2)-76*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)
*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a-
76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1
/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a-80*B*2^(1/2)*(sin(1/2*d*
x+1/2*c)^2*a)^(1/2)*a^(1/2)-80*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^
(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a
))*a-80*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)
+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)
^2+26*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+19*A*ln(-4/(2*cos(1/
2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/
2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+19*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+
2*a))*a+24*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+20*B*ln(-4/(2*c
os(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(s
in(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+20*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(
1/2)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 11782 vs. $2(136) = 272$.

time = 3.73, size = 11782, normalized size = 75.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] -1/1008*(63*(150*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 154*sq
rt(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*
x + 3/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - (3*sqrt(2)*a^2*sin(7/2*d
*x + 7/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) - 17*sqrt(2)*a^2*sin(3/2*d
*x + 3/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4
*d*x + 4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sqrt(2)*a^2*sin
(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - (3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)
 - 17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c
) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
```

```
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x +
3/2*c) + 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x
+ 2*c)^2 - 3*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c)
)*cos(15/2*d*x + 15/2*c) - 5*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*
sin(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 11*(sqrt(2)*a^2*sin(4*d*x + 4*c)
 + 2*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 45*(sqrt(2)*a^2
*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) -
(11*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 99*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)
+ 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*a^2*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sqrt(2)*a^
2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*
a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -...
```

Fricas [A]

time = 0.38, size = 204, normalized size = 1.31

$$\frac{\left((19\,A+20\,B)a^2\cos\left(dx+c\right)^3+(19\,A+20\,B)a^2\cos\left(dx+c\right)^2\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(8\,Ba^2\cos\left(dx+c\right)^2+(11\,A+4\,B)a^2\cos\left(dx+c\right)+2\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{16\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] 1/16*(((19*A + 20*B)*a^2*cos(d*x + c)^3 + (19*A + 20*B)*a^2*cos(d*x + c)^2)
*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c)
 + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(
```

```
d*x + c)^2)) + 4*(8*B*a^2*cos(d*x + c)^2 + (11*A + 4*B)*a^2*cos(d*x + c) +
2*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d
*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.80, size = 239, normalized size = 1.53

$$\frac{\sqrt{2}\left(32\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{2}\left(19\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+20\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)-\frac{4\left(22\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-13\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}\right)\sqrt{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] 1/16*sqrt(2)*(32*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) - sqr
t(2)*(19*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 20*B*a^2*sgn(cos(1/2*d*x + 1/2*c
)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*
d*x + 1/2*c))) - 4*(22*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)
^3 + 8*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 13*A*a^2*sg
n(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) - 4*B*a^2*sgn(cos(1/2*d*x + 1/
2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^3, x)
```

3.97 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=164

$$\frac{a^{5/2}(25A+38B)\tanh^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}+\frac{a^3(49A+54B)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(3A+2B)\sqrt{a+a\cos(}{}$$

```
[Out] 1/8*a^(5/2)*(25*A+38*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/
d+1/3*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2*tan(d*x+c)/d+1/24*a^3*(49*A+5
4*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a^2*(3*A+2*B)*sec(d*x+c)*(a+a*
cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]

time = 0.33, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3054, 3059, 2852, 212}

$$\frac{a^{5/2}(25A+38B)\tanh^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^3(49A+54B)\tan(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(3A+2B)\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a^(5/2)*(25*A + 38*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*d) + (a^3*(49*A + 54*B)*Tan[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x
]]) + (a^2*(3*A + 2*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/
(4*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\tan(c+}{3d}$$

$$= \frac{a^2(3A+2B)\sqrt{a+a\cos(c+dx)}\ \sec(c+dx)}{4d}$$

$$= \frac{a^3(49A+54B)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(3A+2B)\sqrt{}}{}$$

$$= \frac{a^3(49A+54B)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(3A+2B)\sqrt{}}{}$$

$$= \frac{a^{5/2}(25A+38B)\tanh^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a+a\cos(c+dx}}\right.}{8d}$$

Mathematica [A]
time = 1.13, size = 131, normalized size = 0.80

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\ \sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(3\sqrt{2}\,(25A+38B)\tanh^{-1}\left(\sqrt{2}\ \sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3(c+dx)+(91A+66B+4(17A+6B)\cos(c+dx)+(75A+66B)\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(3*Sqrt[2]*
(25*A + 38*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (91*A + 66
*B + 4*(17*A + 6*B)*Cos[c + d*x] + (75*A + 66*B)*Cos[2*(c + d*x)])*Sin[(c +
 d*x)/2]))/(48*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1325 vs. $2(144) = 288$.
time = 0.42, size = 1326, normalized size = 8.09

method	result	size
default	Expression too large to display	1326

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/6*a^(3/2)*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(-24*a*(25*A*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2
^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))+25*A*ln(-4/(2*cos(1/2*d*x+1/2*c
)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c
)^2*a)^(1/2)-2*a))+38*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))+38*B*ln
(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^
(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^6+12*(50*A*a
^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+44*B*2^(1/2)*(sin(1/2*d*x+1/2
*c)^2*a)^(1/2)*a^(1/2)+75*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*
cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+7
5*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1
/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+114*B*ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*
x+1/2*c)^2*a)^(1/2)+2*a))*a+114*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2
^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*
a))*a)*sin(1/2*d*x+1/2*c)^4+(-736*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a
)^(1/2)-450*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/
2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a-450*A*ln(-4/(2*
cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(
sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a-576*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a
)^(1/2)*a^(1/2)-684*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a-684*B*l
n(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2
^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^2+234*A*a
^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+75*A*ln(4/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*
c)^2*a)^(1/2)+2*a))*a+75*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*
```

```
cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+1
56*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+114*B*ln(4/(2*cos(1/2*d
*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d
*x+1/2*c)^2*a)^(1/2)+2*a))*a+114*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*
2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2
*a))*a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/s
in(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 7994 vs. $2(144) = 288$.
time = 3.87, size = 7994, normalized size = 48.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] -1/96*((1530*a^2*cos(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 1530*a^2*cos(2*d
*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 1530*a^2*sin(4*d*x + 4*c)^2*sin(3/2*d*x
+ 3/2*c) + 1530*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 4176*a^2*cos(
7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 2430*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x
 + 2*c) + 678*a^2*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 342*a^2*cos(2*d*x
 + 2*c)*sin(3/2*d*x + 3/2*c) + 10*(a^2*sin(9/2*d*x + 9/2*c) + 17*a^2*sin(3/
2*d*x + 3/2*c))*cos(6*d*x + 6*c)^2 + 10*(a^2*sin(9/2*d*x + 9/2*c) + 17*a^2*
sin(3/2*d*x + 3/2*c))*sin(6*d*x + 6*c)^2 - 56*a^2*sin(3/2*d*x + 3/2*c) + 10
*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2*c))*c
os(21/2*d*x + 21/2*c) - 30*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) +
 3*a^2*sin(2*d*x + 2*c))*cos(19/2*d*x + 19/2*c) - 48*(a^2*sin(6*d*x + 6*c)
+ 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2*c))*cos(17/2*d*x + 17/2*c) +
 80*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2*c)
)*cos(15/2*d*x + 15/2*c) + 396*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*
c) + 3*a^2*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 6*(170*a^2*cos(4*d*x
+ 4*c)*sin(3/2*d*x + 3/2*c) + 170*a^2*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c)
 - 170*a^2*sin(11/2*d*x + 11/2*c) - 232*a^2*sin(7/2*d*x + 7/2*c) - 135*a^2*
sin(5/2*d*x + 5/2*c) + 19*a^2*sin(3/2*d*x + 3/2*c) + 10*(a^2*cos(4*d*x + 4*
c) + a^2*cos(2*d*x + 2*c) - 25*a^2)*sin(9/2*d*x + 9/2*c))*cos(6*d*x + 6*c)
+ 3060*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c)
 + 4560*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c)
+ 18*(170*a^2*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 232*a^2*sin(7/2*d*x +
 7/2*c) - 135*a^2*sin(5/2*d*x + 5/2*c) + 19*a^2*sin(3/2*d*x + 3/2*c))*cos(4
*d*x + 4*c) - 75*(sqrt(2)*a^2*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*cos(4*d*x
+ 4*c)^2 + 9*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(6*d*x + 6*c)^
2 + 9*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) + 9*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 6*sqrt(2)*a^2*cos(2*d*x +
```

```
 2*c) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*cos
(2*d*x + 2*c) + sqrt(2)*a^2)*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*a^2*cos(2*d*x
+ 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 6*(sqrt(2)*a^2*sin(4*d*x + 4*c) +
sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 75*(sqrt(2)*a^2*cos(6*d*x + 6*c)^2 + 9
*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2
)*a^2*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*a^
2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 6*
sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(4*d*x + 4
*c) + 3*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(6*d*x + 6*c) + 6*(3
*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 6*(sqrt(2)*
a^2*sin(4*d*x + 4*c) + sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(
2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(sqrt(2)*a^2
*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*a^2*cos(
2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*sin(4*d*x +
 4*c)^2 + 18*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*a^2*
sin(2*d*x + 2*c)^2 + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(3*sq
rt(2)*a^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*
cos(6*d*x + 6*c) + 6*(3*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d
*x + 4*c) + 6*(sqrt(2)*a^2*sin(4*d*x + 4*c) + sqrt(2)*a^2*sin(2*d*x + 2*c))
*sin(6*d*x + 6*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 2) + 75*(sqrt(2)*a^2*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*cos(4*d*x + 4*c)
^2 + 9*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(6*d*x + 6*c)^2 + 9*
sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 9*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 6*sqrt(2)*a^2*cos(2*d*x + 2*c)
+ sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*cos(2*d*x
 + 2*c) + sqrt(2)*a^2)*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*a^2*cos(2*d*x + 2*c)
 + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 6*(sqrt(2)*a^2*sin(4*d*x + 4*c) + sqrt(2
)*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3...
```

Fricas [A]
time = 0.38, size = 212, normalized size = 1.29

$$\frac{3\left((25A+38B)a^2\cos(dx+c)^4+(25A+38B)a^2\cos(dx+c)^3\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(3(25A+22B)a^2\cos(dx+c)^2+2(17A+6B)a^2\cos(dx+c)+8Aa^2\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{96\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] 1/96*(3*((25*A + 38*B)*a^2*cos(d*x + c)^4 + (25*A + 38*B)*a^2*cos(d*x + c)^
3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x +
c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + co
s(d*x + c)^2)) + 4*(3*(25*A + 22*B)*a^2*cos(d*x + c)^2 + 2*(17*A + 6*B)*a^2
*cos(d*x + c) + 8*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x
+ c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.78, size = 268, normalized size = 1.63

$$-\frac{\sqrt{2}\left(3\sqrt{2}\left(25\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+38\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\left(300\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+264\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-368\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-288\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+117\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+78\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}\right)\sqrt{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] -1/96*sqrt(2)*(3*sqrt(2)*(25*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 38*B*a^2*sgn
(cos(1/2*d*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*
sqrt(2) + 4*sin(1/2*d*x + 1/2*c))) + 4*(300*A*a^2*sgn(cos(1/2*d*x + 1/2*c))
*sin(1/2*d*x + 1/2*c)^5 + 264*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x +
 1/2*c)^5 - 368*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 28
8*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 117*A*a^2*sgn(co
s(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) + 78*B*a^2*sgn(cos(1/2*d*x + 1/2*c
))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^4,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^4, x)
```

3.98 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=209

$$\frac{a^{5/2}(163A+200B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d}+\frac{a^3(163A+200B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}}+\frac{a^3(95A+104B)\sec(}{96d\sqrt{a+a\,c}}$$

```
[Out] 1/64*a^(5/2)*(163*A+200*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2
))/d+1/4*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3*tan(d*x+c)/d+1/64*a^3*(163
*A+200*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/96*a^3*(95*A+104*B)*sec(d*x
+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/24*a^2*(11*A+8*B)*sec(d*x+c)^2*(a
+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.39, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3054, 3059, 2851, 2852, 212}

$$\frac{a^{5/2}(163A+200B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(163A+200B)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(95A+104B)\tan(c+dx)\sec(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(11A+8B)\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{24d}+\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a^(5/2)*(163*A + 200*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(64*d) + (a^3*(163*A + 200*B)*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c
+ d*x]]) + (a^3*(95*A + 104*B)*Sec[c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*
Cos[c + d*x]]) + (a^2*(11*A + 8*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*
Tan[c + d*x])/(24*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\tan(c}{4d}$$

$$= \frac{a^2(11A+8B)\sqrt{a+a\cos(c+dx)}\,\sec^2(c+}{24d}$$

$$= \frac{a^3(95A+104B)\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + $$

$$= \frac{a^3(163A+200B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(95A+}{96}$$

$$= \frac{a^3(163A+200B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(95A+}{96}$$

$$= \frac{a^{5/2}(163A+200B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+d}{\sqrt{a+a\cos(c-}}\right.}{64d}$$

Mathematica [**A**]
time = 1.77, size = 152, normalized size = 0.73

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(6\sqrt{2}\,(163A+200B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^4(c+dx)+(844A+544B+(2203A+2056B)\cos(c+dx)+(652A+544B)\cos(2(c+dx))+489A\cos(3(c+dx))+600B\cos(3(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^4*(6*Sqrt[2]*
(163*A + 200*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (844*A +
 544*B + (2203*A + 2056*B)*Cos[c + d*x] + (652*A + 544*B)*Cos[2*(c + d*x)]
+ 489*A*Cos[3*(c + d*x)] + 600*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*
d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1649 vs. $2(185) = 370$.
time = 0.48, size = 1650, normalized size = 7.89

method	result	size
default	Expression too large to display	1650

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/24*a^(3/2)*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(48*a*(163*A
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*
2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))+163*A*ln(-4/(2*cos(1/2*d*x+1/2
*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2
*c)^2*a)^(1/2)-2*a))+200*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*c
os(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))+200*
B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2
)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^8-48*(16
3*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+200*B*2^(1/2)*(sin(1/2*d
*x+1/2*c)^2*a)^(1/2)*a^(1/2)+326*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2
^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*
a))*a+326*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2
*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+400*B*ln(4/(2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(si
n(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+400*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/
2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^
(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^6+8*(1793*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+
1/2*c)^2*a)^(1/2)+2072*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+146
7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/
2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+1467*A*ln(-4/(2*cos(1/2*d
*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d
*x+1/2*c)^2*a)^(1/2)-2*a))*a+1800*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*
2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2
*a))*a+1800*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1
/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a)*sin(1/2*d*x+1
/2*c)^4+(-3912*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*
x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a-3912*A*ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/
2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a-9212*A*a^(1/2)*2^(1/2)*(sin(1/2*d
*x+1/2*c)^2*a)^(1/2)-4800*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)
*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a-
4800*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^
(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a-9632*B*2^(1/2)*(sin(1/
2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+489*A*ln(-4/(2*cos(1/
2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/
2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+489*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)
+2*a))*a+2094*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+600*B*ln(-4/
(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2
)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+600*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2
*a)^(1/2)+2*a))*a+1872*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))^4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4/sin(1/2*d*x
+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 0.41, size = 232, normalized size = 1.11

$$\frac{3\left((163\,A+200\,B)a^2\cos\left(dx+c\right)^5+(163\,A+200\,B)a^2\cos\left(dx+c\right)^4\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(3\,(163\,A+200\,B)a^2\cos\left(dx+c\right)^3+2\,(163\,A+136\,B)a^2\cos\left(dx+c\right)^2+8\,(23\,A+8\,B)a^2\cos\left(dx+c\right)+48\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{768\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="fricas")
```

```
[Out] 1/768*(3*((163*A + 200*B)*a^2*cos(d*x + c)^5 + (163*A + 200*B)*a^2*cos(d*x
+ c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d
*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)) + 4*(3*(163*A + 200*B)*a^2*cos(d*x + c)^3 + 2*(163*A +
136*B)*a^2*cos(d*x + c)^2 + 8*(23*A + 8*B)*a^2*cos(d*x + c) + 48*A*a^2)*sqr
t(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.91, size = 322, normalized size = 1.54

$$-\frac{\sqrt{2}\left(3\,\sqrt{2}\,\left(163\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+200\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\left(3912\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+4800\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-7172\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-8288\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4606\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4816\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1047\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-936\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}\right)\sqrt{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="giac")
```

```
[Out] -1/768*sqrt(2)*(3*sqrt(2)*(163*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 200*B*a^2*
sgn(cos(1/2*d*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs
(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))) + 4*(3912*A*a^2*sgn(cos(1/2*d*x + 1/2
*c))*sin(1/2*d*x + 1/2*c)^7 + 4800*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*
d*x + 1/2*c)^7 - 7172*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^
5 - 8288*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 + 4606*A*a^
2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 4816*B*a^2*sgn(cos(1/2
*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 1047*A*a^2*sgn(cos(1/2*d*x + 1/2*c)
)*sin(1/2*d*x + 1/2*c) - 936*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x +
1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^4)*sqrt(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+a\,\cos(c+d\,x)\right)^{5/2}}{\cos(c+d\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^5,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^5, x)
```

3.99 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=254

$$\frac{a^{5/2}(283A+326B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{128d}+\frac{a^3(283A+326B)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}}+\frac{a^3(283A+326B)\sec}{192d\sqrt{a+a}}$$

```
[Out] 1/128*a^(5/2)*(283*A+326*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/
2))/d+1/5*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4*tan(d*x+c)/d+1/128*a^3*(2
83*A+326*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/192*a^3*(283*A+326*B)*sec
(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/240*a^3*(157*A+170*B)*sec(d*x
+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/40*a^2*(13*A+10*B)*sec(d*x+c)^3
*(a+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.44, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3054, 3059, 2851, 2852, 212}

$$\frac{a^{5/2}(283A+326B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^3(283A+326B)\tan(c+dx)}{128d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(157A+170B)\tan(c+dx)\sec^2(c+dx)}{240d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(283A+326B)\tan(c+dx)\sec(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(13A+10B)\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{40d}+\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (a^(5/2)*(283*A + 326*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(128*d) + (a^3*(283*A + 326*B)*Tan[c + d*x])/(128*d*Sqrt[a + a*Cos[
c + d*x]]) + (a^3*(283*A + 326*B)*Sec[c + d*x]*Tan[c + d*x])/(192*d*Sqrt[a
+ a*Cos[c + d*x]]) + (a^3*(157*A + 170*B)*Sec[c + d*x]^2*Tan[c + d*x])/(240
*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(13*A + 10*B)*Sqrt[a + a*Cos[c + d*x]]*
Sec[c + d*x]^3*Tan[c + d*x])/(40*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c
 + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
```

```
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\tan(c}{5d}$$
$$= \frac{a^2(13A+10B)\sqrt{a+a\cos(c+dx)}\,\sec^3(c+}{40d}$$
$$= \frac{a^3(157A+170B)\sec^2(c+dx)\tan(c+dx)}{240d\sqrt{a+a\cos(c+dx)}}$$
$$= \frac{a^3(283A+326B)\sec(c+dx)\tan(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} +$$
$$= \frac{a^3(283A+326B)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(283A+}{19}$$
$$= \frac{a^3(283A+326B)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(283A+}{19}$$
$$= \frac{a^{5/2}(283A+326B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+d}{\sqrt{a+a\cos(c-}}\right.}{128d}$$

Mathematica [A]
time = 2.15, size = 176, normalized size = 0.69

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(60\sqrt{2}\,(283A+326B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5(c+dx)+(24863A+22030B+36(781A+650B)\cos(c+dx)+4(6509A+6730B)\cos(2(c+dx))+5660A\cos(3(c+dx))+6520B\cos(3(c+dx))+4245A\cos(4(c+dx))+4890B\cos(4(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{15360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^5*(60*Sqrt[2]
*(283*A + 326*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (24863*
A + 22030*B + 36*(781*A + 650*B)*Cos[c + d*x] + 4*(6509*A + 6730*B)*Cos[2*(
c + d*x)] + 5660*A*Cos[3*(c + d*x)] + 6520*B*Cos[3*(c + d*x)] + 4245*A*Cos[
4*(c + d*x)] + 4890*B*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(15360*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1974 vs. $2(226) = 452$.
time = 0.54, size = 1975, normalized size = 7.78

method	result	size
default	Expression too large to display	1975

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/120*a^(3/2)*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(-480*a*(28
3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/
2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))+283*A*ln(-4/(2*cos(1/2*d*x+
1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+
1/2*c)^2*a)^(1/2)-2*a))+326*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2
)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))+3
26*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(
1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^10+24
0*(566*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+652*B*2^(1/2)*(sin(
1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+1415*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1
/2)+2*a))*a+1415*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*
d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+1630*B*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(
1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+1630*B*ln(-4/(2*cos(1/2*d*x+1/2
*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2
*c)^2*a)^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^8-80*(3962*A*a^(1/2)*2^(1/2)*(si
n(1/2*d*x+1/2*c)^2*a)^(1/2)+4564*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a
^(1/2)+4245*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/
2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+4245*A*ln(-4/(2
*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*
(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+4890*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*
a)^(1/2)+2*a))*a+4890*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a)*sin
(1/2*d*x+1/2*c)^6+8*(36224*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)
+40960*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+21225*A*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin
(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+21225*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)
^(1/2)-2*a))*a+24450*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1
/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+24450*
B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2
)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^4-10*(
12556*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+13400*B*2^(1/2)*(sin
(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+4245*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(
1/2)+2*a))*a+4245*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2
*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+4890*B*l
```

```
n(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^
(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+4890*B*ln(-4/(2*cos(1/2*d*x+1/
2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/
2*c)^2*a)^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^2+22230*A*a^(1/2)*2^(1/2)*(sin(
1/2*d*x+1/2*c)^2*a)^(1/2)+4245*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)
)*a+4245*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*
c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+20940*B*2^(1/2)*(
sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)+4890*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(
1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a
)^(1/2)+2*a))*a+4890*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a)/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))^5/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^5/sin(1/2*d*x+1
/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.42, size = 252, normalized size = 0.99

$$\frac{15\left((283A+326B)a^2\cos(dx+c)^6+(283A+326B)a^2\cos(dx+c)^5\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(15(283A+326B)a^2\cos(dx+c)^4+10(283A+326B)a^2\cos(dx+c)^3+8(283A+230B)a^2\cos(dx+c)^2+48(29A+10B)a^2\cos(dx+c)+384Aa^2\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{7680\left(d\cos(dx+c)^6+d\cos(dx+c)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm
="fricas")
```

```
[Out] 1/7680*(15*((283*A + 326*B)*a^2*cos(d*x + c)^6 + (283*A + 326*B)*a^2*cos(d*
x + c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos
(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)
^3 + cos(d*x + c)^2)) + 4*(15*(283*A + 326*B)*a^2*cos(d*x + c)^4 + 10*(283*
A + 326*B)*a^2*cos(d*x + c)^3 + 8*(283*A + 230*B)*a^2*cos(d*x + c)^2 + 48*(
29*A + 10*B)*a^2*cos(d*x + c) + 384*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x
 + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.98, size = 376, normalized size = 1.48

$$-\frac{\sqrt{2}\left(15\sqrt{2}\left(283\,Aa^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+326\,Ba^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\left(67920\,Aa^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+78240\,Ba^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-158480\,Aa^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-182560\,Ba^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+144896\,Aa^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+163840\,Ba^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-62780\,Aa^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-67000\,Ba^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+11115\,Aa^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+10470\,Ba^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{5}}\right)\sqrt{a}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm
="giac")
```

```
[Out] -1/7680*sqrt(2)*(15*sqrt(2)*(283*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 326*B*a^
2*sgn(cos(1/2*d*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/a
bs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))) + 4*(67920*A*a^2*sgn(cos(1/2*d*x +
1/2*c))*sin(1/2*d*x + 1/2*c)^9 + 78240*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(
1/2*d*x + 1/2*c)^9 - 158480*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1
/2*c)^7 - 182560*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^7 + 1
44896*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 + 163840*B*a^2
*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 62780*A*a^2*sgn(cos(1/2
*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 67000*B*a^2*sgn(cos(1/2*d*x + 1/2*c
))*sin(1/2*d*x + 1/2*c)^3 + 11115*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d
*x + 1/2*c) + 10470*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))/(
2*sin(1/2*d*x + 1/2*c)^2 - 1)^5)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^6,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^6, x)
```

3.100 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2(7A-B)\cos^2(c+dx)\,\mathrm{si}}{35d\sqrt{a+a\cos(c+}}$$

```
[Out] -(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/
2)/d/a^(1/2)+4/105*(49*A-37*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/35*(7*
A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*B*cos(d*x+c)^3*si
n(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-2/105*(7*A-31*B)*sin(d*x+c)*(a+a*cos(d*x+
c))^(1/2)/a/d
```

Rubi [A]
time = 0.37, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3062, 3047, 3102, 2830, 2728, 212}

$$\frac{2(7A-B)\sin(c+dx)\cos^2(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}-\frac{2(7A-31B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105ad}+\frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (4*(49*A - 37*B)*Sin[c + d*x])/(105*d*Sqrt[a + a
*Cos[c + d*x]]) + (2*(7*A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(35*d*Sqrt[a +
a*Cos[c + d*x]]) + (2*B*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c
+ d*x]]) - (2*(7*A - 31*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2B\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\cos^2(c+dx)\left(3aB+\frac{1}{2}a(7A-B)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}}{7a}$$

$$= \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^3(c+dx)\sin(c+dx}{7d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^3(c+dx)\sin(c+dx}{7d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^3(c+dx)\sin(c+dx}{7d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{4(49A-3}{105d\sqrt{a+}}$$

Mathematica [A]
time = 0.71, size = 111, normalized size = 0.55

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(-420(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2(406A-178B+(-28A+169B)\cos(c+dx)+6(7A-B)\cos(2(c+dx))+15B\cos(3(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{210d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(-420*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + 2*(406*A - 178*
B + (-28*A + 169*B)*Cos[c + d*x] + 6*(7*A - B)*Cos[2*(c + d*x)] + 15*B*Cos[
3*(c + d*x)])*Sin[(c + d*x)/2]))/(210*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A]
time = 0.31, size = 281, normalized size = 1.39

method	result

default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-240B\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+168\sqrt{2}\sqrt{a}\sqrt{a\left(\sin\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/105*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(-240*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+168*2^(1/2)*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(A+2*B)*sin(1/2*d*x+1/2*c)^4-140*2^(1/2)*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(A+2*B)*sin(1/2*d*x+1/2*c)^2-105*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a*A+105*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a*B+210*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2))/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1604723 vs. $2(177) = 354$.
time = 37.38, size = 1604723, normalized size = 7944.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/5040*(3*(84*(sqrt(2)*cos(3/2*d*x + 3/2*c)^2*sin(d*x + c) + 2*sqrt(2)*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + sqrt(2)*sin(3/2*d*x + 3/2*c)^2*sin(d*x + c) + 2*sqrt(2)*sin(3/2*d*x + 3/2*c)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c))*cos(7/2*d*x + 7/2*c)^3 - 84*((sqrt(2)*cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(7/2*d*x + 7/2*c)^3 - 24*((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3

/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c)^3 + 3*(420*sqrt(2)*cos(3/2*d*x + 3/2*c)^3*sin(d*x + c) - 420*(sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^3 - 280*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 + 35*((3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + (3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(d*x + c)^2 + 24*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 2*(3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 - 35*(8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 + 4*(2*sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2))*sin(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + 105*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 35*((3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + (3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(d*x + c)^2 + 12*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 2*(3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 20*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 3*sqrt(2)*log(cos(1/2*d

```
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sq
rt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
 1/2*c) + 1) - 32*sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 - 35
*(8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*co
s(1/2*d*x + 1/2*c)^2 - 3*(sqrt(2)*log(cos(1/2*d...
```

Fricas [**A**]
time = 0.36, size = 184, normalized size = 0.91

$$\frac{4\,(15\,B\cos(dx+c)^3+3\,(7\,A-B)\cos(dx+c)^2-(7\,A-31\,B)\cos(dx+c)+91\,A-43\,B)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)-\frac{105\,\sqrt{2}\,((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\,\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{210\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/210*(4*(15*B*cos(d*x + c)^3 + 3*(7*A - B)*cos(d*x + c)^2 - (7*A - 31*B)*c
os(d*x + c) + 91*A - 43*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) - 105*sqrt
(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*s
qrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.81, size = 217, normalized size = 1.07

$$-\frac{\frac{105\,\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log(\sin(\frac{1}{2}\,dx+\frac{1}{2}\,c)+1)}{a\mathrm{sgn}(\cos(\frac{1}{2}\,dx+\frac{1}{2}\,c))}-\frac{105\,\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log(-\sin(\frac{1}{2}\,dx+\frac{1}{2}\,c)+1)}{a\mathrm{sgn}(\cos(\frac{1}{2}\,dx+\frac{1}{2}\,c))}+\frac{4\,\sqrt{2}\left(120\,Ba^{\frac{13}{2}}\sin(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7-84\,Aa^{\frac{13}{2}}\sin(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-168\,Ba^{\frac{13}{2}}\sin(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5+70\,Aa^{\frac{13}{2}}\sin(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3+140\,Ba^{\frac{13}{2}}\sin(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-105\,Aa^{\frac{13}{2}}\sin(\frac{1}{2}\,dx+\frac{1}{2}\,c)\right)}{a^7\mathrm{sgn}(\cos(\frac{1}{2}\,dx+\frac{1}{2}\,c))}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/210*(105*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(
a*sgn(cos(1/2*d*x + 1/2*c))) - 105*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(-sin
```

```
(1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) + 4*sqrt(2)*(120*B*a^(
13/2)*sin(1/2*d*x + 1/2*c)^7 - 84*A*a^(13/2)*sin(1/2*d*x + 1/2*c)^5 - 168*B
*a^(13/2)*sin(1/2*d*x + 1/2*c)^5 + 70*A*a^(13/2)*sin(1/2*d*x + 1/2*c)^3 + 1
40*B*a^(13/2)*sin(1/2*d*x + 1/2*c)^3 - 105*A*a^(13/2)*sin(1/2*d*x + 1/2*c))
/(a^7*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\,(A+B\,\cos(c+d\,x))}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2), x)
```

3.101 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=159

$$\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}+\frac{2}{}$$

```
[Out] (A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2
)/d/a^(1/2)-4/15*(5*A-7*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*B*cos(d*
x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/15*(5*A-B)*sin(d*x+c)*(a+a*cos
(d*x+c))^(1/2)/a/d
```

Rubi [A]

time = 0.25, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3062, 3047, 3102, 2830, 2728, 212}

$$\frac{2(5A-B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15ad}-\frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(Sqrt[a]*d) - (4*(5*A - 7*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c
 + d*x]]) + (2*B*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]
) + (2*(5*A - B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\cos(c+dx)\left(2aB+\frac{1}{2}a(5A-B)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,d}{5a}$$

$$= \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{2aB\cos(c+dx)+\frac{1}{2}a(5A-B)\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{5a}$$

$$= \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2(5A-B)\sqrt{a+a\cos(c+dx)}\,\sin(}{15ad}$$

$$= -\frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2(5A}{}$$

$$= -\frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2(5A}{}$$

$$= \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{4(5A-7B)\mathrm{s}}{15d\sqrt{a+a\,\mathrm{co}}}$$

Mathematica [A]
time = 0.36, size = 94, normalized size = 0.59

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(15(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+(-10A+29B+2(5A-B)\cos(c+dx)+3B\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{15d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(15*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + (-10*A + 29*B +
 2*(5*A - B)*Cos[c + d*x] + 3*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(15*d*
Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A]
time = 0.28, size = 240, normalized size = 1.51

method	result

default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24B\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\sqrt{2}\sqrt{a}\sqrt{a\left(\sin^2\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/15*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(24*B*2^(1/2)*(sin(1
/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4-20*2^(1/2)*a^(1/2)*(s
in(1/2*d*x+1/2*c)^2*a)^(1/2)*(A+B)*sin(1/2*d*x+1/2*c)^2+15*2^(1/2)*ln(4/cos
(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a*A-15*2^(1/2)*
ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a*B+30*
B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/a^(3/2)/sin(1/2*d*x+1/2*c
)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 927957 vs. $2(138) = 276$.
time = 18.75, size = 927957, normalized size = 5836.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] 1/1680*(28*(20*(cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^3 + 8*(cos(d*x + c)^
2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^3 - 20*cos(5/
2*d*x + 5/2*c)^3*sin(d*x + c) + 2*(15*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + 15
*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*
d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 30*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 4*(
cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)
- 20*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 15*log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 15*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(5/2*d*x
 + 5/2*c)^2 + 30*((log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
```

c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + (log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3/2*d*x + 3/2*c)^2 + 2*(10*(cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^3 + 4*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^3 - 10*cos(5/2*d*x + 5/2*c)^3*sin(d*x + c) + (15*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + 15*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 30*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 4*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - 20*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 15*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 15*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(5/2*d*x + 5/2*c)^2 + 15*((log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + (log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3/2*d*x + 3/2*c)^2 + (15*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + 15*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 30*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 4*(cos(d*x + c)^2 + sin(d*x + c)^2 + 7*cos(d*x + c) + 6)*sin(3/2*d*x + 3/2*c) - 10*cos(5/2*d*x + 5/2*c)*sin(d*x + c) + 15*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 15*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(5/2*d*x + 5/2*c)^2 + 15*((log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1

```
/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + (log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*
(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*...
```

Fricas [A]
time = 0.37, size = 166, normalized size = 1.04

$$\frac{4\left(3\,B\cos\left(dx+c\right)^{2}+\left(5\,A-B\right)\cos\left(dx+c\right)-5\,A+13\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)-\frac{15\,\sqrt{2}\,\left(\left(A-B\right)a\cos\left(dx+c\right)+\left(A-B\right)a\right)\log\left(-\frac{\cos\left(dx+c\right)^{2}+\frac{2\,\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{\sqrt{a}}-2\,\cos\left(dx+c\right)-3}{\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1}\right)}{\sqrt{a}}}{30\,\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/30*(4*(3*B*cos(d*x + c)^2 + (5*A - B)*cos(d*x + c) - 5*A + 13*B)*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c) - 15*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B
)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(
a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\cos\left(c+dx\right)\right)\cos^{2}\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)**2/sqrt(a*(cos(c + d*x) + 1)), x
)
```

Giac [A]
time = 0.57, size = 183, normalized size = 1.15

$$\frac{\frac{15\,\sqrt{2}\,\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{15\,\sqrt{2}\,\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{4\,\sqrt{2}\,\left(12\,Ba^{\frac{9}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-10\,Aa^{\frac{9}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-10\,Ba^{\frac{9}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,Ba^{\frac{9}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{5}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/30*(15*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(a*s
gn(cos(1/2*d*x + 1/2*c))) - 15*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(-sin(1/2
*d*x + 1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) + 4*sqrt(2)*(12*B*a^(9/2)*
sin(1/2*d*x + 1/2*c)^5 - 10*A*a^(9/2)*sin(1/2*d*x + 1/2*c)^3 - 10*B*a^(9/2)
*sin(1/2*d*x + 1/2*c)^3 + 15*B*a^(9/2)*sin(1/2*d*x + 1/2*c))/(a^5*sgn(cos(1
/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2), x)
```

3.102 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}+\frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2B\sqrt{a+a\cos(c+dx)}\sin(}{3ad}$$

```
[Out] -(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/
2)/d/a^(1/2)+2/3*(3*A-2*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*B*sin(d*
x+c)*(a+a*cos(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3047, 3102, 2830, 2728, 212}

$$\frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (2*(3*A - 2*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos
[c + d*x]]) + (2*B*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2B\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{2\int \frac{\frac{aB}{2}+\frac{1}{2}a(3A-2B)\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2B\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3ad} + (-$$

$$= \frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2B\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{(2}{}$$

$$= -\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2(3A-2B)\,\mathrm{s}}{3d\sqrt{a+a\,\mathrm{co}}}$$

Mathematica [A]
time = 0.18, size = 78, normalized size = 0.66

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-3(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6A\sin\left(\frac{1}{2}(c+dx)\right)-4B\sin^3\left(\frac{1}{2}(c+dx)\right)\right)}{3d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(-3*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + 6*A*Sin[(c + d*
x)/2] - 4*B*Sin[(c + d*x)/2]^3))/(3*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A]
time = 0.28, size = 194, normalized size = 1.64

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-4B\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6A\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{3a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/3*cos(1/2*d*x+1/2*c)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(-4*B*a^(1/2)
*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*A*a^(1/2)*(sin(1/2*d
*x+1/2*c)^2*a)^(1/2)-3*A*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*
c)^2*a)^(1/2)+a))*a+3*B*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c
)^2*a)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/
2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38386 vs. $2(101) = 202$.
time = 1.06, size = 38386, normalized size = 325.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/60*((20*(cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^3 + 8*(cos(d*x + c)^2 + s
in(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^3 - 20*cos(5/2*d*x
 + 5/2*c)^3*sin(d*x + c) + 2*(15*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + 15*(log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
 1/2*c) + 1))*sin(d*x + c)^2 + 30*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
```

```
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 4*(cos(d
*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - 20*
cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 15*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 15*log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(5/2*d*x + 5/
2*c)^2 + 30*((log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + (log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x +
 c)^2 + 2*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c
)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3/2*d*x + 3
/2*c)^2 + 2*(10*(cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^3 + 4*(cos(d*x + c)
^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^3 - 10*cos(5
/2*d*x + 5/2*c)^3*sin(d*x + c) + (15*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + 15*
(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d
*x + 1/2*c) + 1))*sin(d*x + c)^2 + 30*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 4*(c
os(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) -
 20*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 15*log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 15*log(cos(1/2*d*x + 1/
2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(5/2*d*x
+ 5/2*c)^2 + 15*((log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + (log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d
*x + c)^2 + 2*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3/2*d*x
 + 3/2*c)^2 + (15*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + 15*(log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*s
```

```
in(d*x + c)^2 + 30*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 4*(cos(d*x + c)^2 + sin
(d*x + c)^2 + 7*cos(d*x + c) + 6)*sin(3/2*d*x + 3/2*c) - 10*cos(5/2*d*x + 5
/2*c)*sin(d*x + c) + 15*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - 15*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(5/2*d*x + 5/2*c)^2 + 15*((l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*cos(d*x + c)^2 + (log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +...
```

Fricas [A]
time = 0.37, size = 149, normalized size = 1.26

$$\frac{4\left(B\cos\left(dx+c\right)+3\,A-B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)-\frac{3\,\sqrt{2}\,\left((A-B)a\cos(dx+c)+(A-B)a\right)\log\left(-\frac{\cos(dx+c)^2-\frac{2\,\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sin(dx+c)}{\sqrt{a}}-2\,\cos(dx+c)-3}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)}{\sqrt{a}}}{6\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/6*(4*(B*cos(d*x + c) + 3*A - B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) - 3
*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 - 2*sqrt
(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\cos\left(c+dx\right)\right)\cos\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [A]
time = 0.48, size = 149, normalized size = 1.26

$$-\frac{\frac{3\,\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{3\,\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{4\,\sqrt{2}\left(2\,Ba^{\frac{5}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Aa^{\frac{5}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] -1/6*(3*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(a*sg
n(cos(1/2*d*x + 1/2*c))) - 3*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(-sin(1/2*d
*x + 1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) + 4*sqrt(2)*(2*B*a^(5/2)*sin
(1/2*d*x + 1/2*c)^3 - 3*A*a^(5/2)*sin(1/2*d*x + 1/2*c))/(a^3*sgn(cos(1/2*d*
x + 1/2*c))))/d
```

Mupad [B]
time = 0.38, size = 160, normalized size = 1.36

$$\frac{2\,A\,\left(2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)-\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\right)\,\sqrt{\frac{a+a\,\cos\left(c+d\,x\right)}{2\,a}}}{d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}+\frac{2\,B\,\sin\left(c+d\,x\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{3\,a\,d}-\frac{2\,B\,\left(4\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)-3\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\right)\,\sqrt{\frac{a+a\,\cos\left(c+d\,x\right)}{2\,a}}}{3\,a^2\,d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*A*(2*ellipticE(c/2 + (d*x)/2, 1) - ellipticF(c/2 + (d*x)/2, 1))*((a + a*
cos(c + d*x))/(2*a))^(1/2))/(d*(a + a*cos(c + d*x))^(1/2)) + (2*B*sin(c + d
*x)*(a + a*cos(c + d*x))^(1/2))/(3*a*d) - (2*B*(4*a^2*ellipticE(c/2 + (d*x)
/2, 1) - 3*a^2*ellipticF(c/2 + (d*x)/2, 1))*((a + a*cos(c + d*x))/(2*a))^(1
/2))/(3*a^2*d*(a + a*cos(c + d*x))^(1/2))
```

3.103 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

```
[Out] (A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2
)/d/a^(1/2)+2*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2830, 2728, 212}

$$\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(Sqrt[a]*d) + (2*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+(A-B)\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}-\frac{(2(A-B))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [**A**]
time = 0.08, size = 60, normalized size = 0.77

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
[Out] (2*Cos[(c + d*x)/2]*((A - B)*ArcTanh[Sin[(c + d*x)/2]] + 2*B*Sin[(c + d*x)/
2]))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(67) = 134.
time = 0.23, size = 160, normalized size = 2.05

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(A\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a-B\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right.\right.}{a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] cos(1/2*d*x+1/2*c)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(A*ln(4/cos(1/2*d
*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a-B*ln(4/cos(1/2*d*x+
```

```
1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a+2*B*a^(1/2)*(sin(1/2*d
*x+1/2*c)^2*a)^(1/2))/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(
1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 19040 vs. $2(67) = 134$.

time = 0.78, size = 19040, normalized size = 244.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(6*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*A/sqrt(a) - (12*sqrt(2)*cos(3/2*d
*x + 3/2*c)^3*sin(d*x + c) - 12*(sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*
x + 3/2*c)^3 - 8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 + ((3*sqrt(2)*log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sq
rt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + (3*sqrt(2)*
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(d*x + c)^2
+ 24*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 2*(3*sqrt(2)*log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sq
rt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 3*sqrt(2)*log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin
(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*
c)^2 - (8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
1))*cos(1/2*d*x + 1/2*c)^2 - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*
x + 1/2*c)^2 + 4*(2*sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2))*sin(1/2*d*x +
 1/2*c))*cos(d*x + c)^2 + 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x +
 1/2*c)^2 + (12*sqrt(2)*cos(3/2*d*x + 3/2*c)^3*sin(d*x + c) - 12*(sqrt(2)*c
os(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^3 - 8*sqrt(2)*sin(1/2*d*x + 1/2
*c)^3 + ((3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
```

```
*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2
*c))*cos(d*x + c)^2 + (3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c
)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(
1/2*d*x + 1/2*c))*sin(d*x + c)^2 + 24*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x
+ c) + 2*(3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2
*c))*cos(d*x + c) + 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2
*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 - (8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 -
 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 - 3*(sqrt(2)*lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
+ 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(
1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 + 4*(2*sqrt(2)*cos(1/2*d*x +
1/2*c)^2 + sqrt(2))*sin(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + 3*(sqrt(2)*log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2
*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + ((3*sqrt(2)*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*
c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + (3*sqrt(2)*log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1
/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(d*x + c)^2 + 12*
sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 2*(3*sqrt(2)*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*
c) + 1) - 20*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 3*sqrt(2)*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2
*d*x + 1/2*c) + 1) - 32*sqrt(2)*sin(1/2*d*x + 1...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(67) = 134$.

time = 0.36, size = 135, normalized size = 1.73

$$\frac{4\sqrt{a\cos\left(dx+c\right)+a}\,B\sin\left(dx+c\right)-\frac{\sqrt{2}\,\left((A-B)a\cos(dx+c)+(A-B)a\right)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(a*cos(d*x + c) + a)*B*sin(d*x + c) - sqrt(2)*((A - B)*a*cos(d*x
 + c) + (A - B)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a
)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [A]
time = 0.45, size = 125, normalized size = 1.60

$$\frac{\frac{4\sqrt{2}\,B\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*sqrt(2)*B*sin(1/2*d*x + 1/2*c)/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))) +
 sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(1
/2*d*x + 1/2*c))) - sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(-sin(1/2*d*x + 1/2*
c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [B]
time = 0.35, size = 112, normalized size = 1.44

$$\frac{A\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\cos\left(c+d\,x\right)\right)}{a}}+2\,B\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\cos\left(c+d\,x\right)\right)}{a}}-B\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\cos\left(c+d\,x\right)\right)}{a}}}{d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (A*ellipticF(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2) + 2*B*ell
ipticE(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2) - B*ellipticF(c
/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2))/(d*(a + a*cos(c + d*x)
)^(1/2))
```

3.104 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*A*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-(A-B)*arct
anh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2
)
```

Rubi [A]
time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3064, 2728, 212, 2852}

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\int \sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\,dx}{a} - (A-B)\int \frac{1}{\sqrt{a+a\cos}}$$

$$= -\frac{(2A)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{(2(A-B)}{}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{}{\sqrt{2}}\right.}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.09, size = 72, normalized size = 0.79

$$-\frac{2\left((A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}\,A\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-2*((A - B)*ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*A*ArcTanh[Sqrt[2]*Sin[(c +
 d*x)/2]])*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(76) = 152$.
time = 0.34, size = 270, normalized size = 2.97

method	result

default	$-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)A-\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(2^(1/2)*ln(4/cos(1/2*d*
x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*A-2^(1/2)*ln(4/cos(1/2
*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*B-A*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2
*d*x+1/2*c)^2*a)^(1/2)+2*a))-A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1
/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))
)/a^(1/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A]
time = 0.53, size = 91, normalized size = 1.00

$$\frac{\left(\sqrt{2}\,\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\,\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\right)B}{2\,\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B/(sqrt(a)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(76) = 152.
time = 0.36, size = 171, normalized size = 1.88

$$-\frac{\sqrt{2}\,(A-B)\sqrt{a}\,\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sin(dx+c)}{\sqrt{a}}-2\,\cos(dx+c)-3}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)-A\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] -1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d
*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) - A*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
- 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(76) = 152.
time = 0.51, size = 165, normalized size = 1.81

$$-\frac{\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{2\,A\log\left(\left|\frac{1}{2}\,\sqrt{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{2\,A\log\left(\left|-\frac{1}{2}\,\sqrt{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] -1/2*(sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(a*sgn(
cos(1/2*d*x + 1/2*c))) - sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(-sin(1/2*d*x +
 1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) - 2*A*log(abs(1/2*sqrt(2) + sin(
1/2*d*x + 1/2*c)))/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))) + 2*A*log(abs(-1/2*s
qrt(2) + sin(1/2*d*x + 1/2*c)))/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos(c + d\,x)}{\cos(c + d\,x)\,\sqrt{a + a\,\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2)), x)
```

3.105 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=119

$$-\frac{(A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

```
[Out] -(A-2*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)+(A-B)
*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)+A*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3063, 3064, 2728, 212, 2852}

$$-\frac{(A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -(((A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqr
t[a]*d)) + (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Cos[c + d*x]])])/(Sqrt[a]*d) + (A*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d
*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{1}{2}a(A-2B)+\frac{1}{2}aA\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$
$$= \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(A-2B)\int \sqrt{a+a\cos(c+dx)}\,\sec(c-$$
$$= \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{(A-2B)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a}{\sqrt{a+c}}\right.}{d}$$
$$= -\frac{(A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\,(A-B)\tanh$$

Mathematica [A]

time = 0.38, size = 95, normalized size = 0.80

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}\,(A-2B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\sec(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(2*(A - B)*ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*(A - 2*B)*
ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*A*Sec[c + d*x]*Sin[(c + d*x)/2]))/(d*
Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(102) = 204$.
time = 0.39, size = 820, normalized size = 6.89

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-2a\left(2\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)A-2\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\,(\mathrm{si}}{\cos}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(-2*a*(2*2^(1/2)*ln(4/cos
(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*A-2*2^(1/2)*ln(
4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*B-A*ln(-4/
(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2
)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))-A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1
/2)+2*a))+2*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1
/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))+2*B*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(
1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*ln(4/cos(1/
2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a*A-2*2^(1/2)*ln(4
/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a*B+2*A*a^(
1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2
^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2
*a)^(1/2)-2*a))*a-A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*
d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a+2*B*ln(-4
/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/
```

```
2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*
a)^(1/2)+2*a))*a)/a^(3/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2
*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 18436 vs. $2(102) = 204$.
time = 0.71, size = 18436, normalized size = 154.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] 1/4*((2*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 2*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(d*x + c)^4 + (2*sqrt(2)*log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 2*sq
rt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
 1/2*c) + 1) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2))*sin(d*x + c)^4 + 4*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^3 + 4*(2*
sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c) + 1) - 2*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*sin(1/2*d*x + 1/2*c) - log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(d*x + c)^3 + (
```

```
(2*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - 2*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2))*cos(d*x + c)^2 + (2*sqrt(2)*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 2*sqrt(2)
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*
c) + 1) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*
sin(d*x + c)^2 - 4*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 2*(2*sqrt(2)
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c) + 1) - 2*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) - log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(d*x + c) + 2*sqrt(2
)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - 2*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1) + 4*sqrt(2)*sin(1/2*d*x + 1/2*c) - log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(102) = 204$.

time = 0.41, size = 259, normalized size = 2.18

$$-\frac{\left((A-2\,B)\cos\left(dx+c\right)^2+(A-2\,B)\cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)-4\,\sqrt{a\cos\left(dx+c\right)+a}\,A\sin\left(dx+c\right)+\frac{2\,\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\log\left(-\frac{\cos(dx+c)^2+\frac{2\,\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sin(dx+c)}{\sqrt{a}}-2\,\cos(dx+c)-3}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)}{\sqrt{a}}}{4\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/4*(((A - 2*B)*cos(d*x + c)^2 + (A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*co
s(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos
(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*s
qrt(a*cos(d*x + c) + a)*A*sin(d*x + c) + 2*sqrt(2)*((A - B)*a*cos(d*x + c)^
2 + (A - B)*a*cos(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/sqrt(a*(cos(c + d*x) + 1)), x
)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(102) = 204.
time = 0.53, size = 233, normalized size = 1.96

$$\frac{\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\left(A\sqrt{a}-2\,B\sqrt{a}\right)\log\left(\left|\frac{1}{2}\,\sqrt{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{\left(A\sqrt{a}-2\,B\sqrt{a}\right)\log\left(\left|-\frac{1}{2}\,\sqrt{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{2\,\sqrt{2}\,A\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/2*(sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(a*sgn(c
os(1/2*d*x + 1/2*c))) - sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(-sin(1/2*d*x +
1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) - (A*sqrt(a) - 2*B*sqrt(a))*log(a
bs(1/2*sqrt(2) + sin(1/2*d*x + 1/2*c)))/(a*sgn(cos(1/2*d*x + 1/2*c))) + (A*
sqrt(a) - 2*B*sqrt(a))*log(abs(-1/2*sqrt(2) + sin(1/2*d*x + 1/2*c)))/(a*sgn
(cos(1/2*d*x + 1/2*c))) - 2*sqrt(2)*A*sin(1/2*d*x + 1/2*c)/((2*sin(1/2*d*x
+ 1/2*c)^2 - 1)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos\left(c + d\,x\right)}{{\cos\left(c + d\,x\right)}^2\,\sqrt{a + a\,\cos\left(c + d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2)),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2)), x)
```

3.106 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=165

$$\frac{(7A-4B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{(A-4B)}{4d\sqrt{a+a}}$$

```
[Out] 1/4*(7*A-4*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-
(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2
)/d/a^(1/2)-1/4*(A-4*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*A*sec(d*x+c
)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3063, 3064, 2728, 212, 2852}

$$-\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{(7A-4B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((7*A - 4*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*S
qrt[a]*d) - (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(Sqrt[a]*d) - ((A - 4*B)*Tan[c + d*x])/(4*d*Sqrt[a +
a*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*
x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{1}{2}a(A-4B)+\frac{3}{2}aA\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a}$$

$$= -\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{4}a^2(7A \cdots}{\cdots}$$

$$= -\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{(7A-4\cdots}{\cdots}$$

$$= -\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{(7A-4\cdots}{\cdots}$$

$$= \frac{(7A-4B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}\,d} - \frac{\sqrt{2}\,(A-B)\tanh^{-1}\cdots}{\cdots}$$

Mathematica [A]
time = 0.86, size = 114, normalized size = 0.69

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(-8(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}\,(7A-4B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sec(c+dx)(-A+4B+2A\sec(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(-8*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + Sqrt[2]*(7*A - 4*
B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(-A + 4*B + 2*A*Sec[c
 + d*x])*Sin[(c + d*x)/2]))/(4*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1251 vs. $2(140) = 280$.
time = 0.47, size = 1252, normalized size = 7.59

method	result	size
default	Expression too large to display	1252

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(4*a*(8*2^(1/2)*ln(4
/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*A-8*2^(1/2)
*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*B-7*A*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2
^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))-7*A*ln(-4/(2*cos(1/2*d*x+1/2*c)
-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)
^2*a)^(1/2)-2*a))+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))+4*B*ln(-4
/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/
2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^4-4*(A*a^(1/2)*
2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a
^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a*A-4*B*2^(1/2)*(sin(1/2*d*x+1/2*
c)^2*a)^(1/2)*a^(1/2)-8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d
*x+1/2*c)^2*a)^(1/2)+a))*a*B-7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)
)*a-7*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-
a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+4*B*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*
d*x+1/2*c)^2*a)^(1/2)+2*a))*a+4*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2
^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*
a))*a)*sin(1/2*d*x+1/2*c)^2+8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin
```

```
(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a*A-8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/
2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a*B-2*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+
1/2*c)^2*a)^(1/2)-7*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1
/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a-7*A*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(
1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a-8*B*2^(1/2)*(sin(1/2*d*x+1/2*c)
^2*a)^(1/2)*a^(1/2)+4*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*a+4*B*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2
^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*a)/a^(3/2)/(2*cos(1/2*d*x+1/2*c
)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2
*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 76209 vs. $2(140) = 280$.
time = 3.88, size = 76209, normalized size = 461.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/16*((4*sqrt(2)*cos(6*d*x + 6*c)^2*sin(3/2*d*x + 3/2*c) + 16*sqrt(2)*cos(
5*d*x + 5*c)^2*sin(3/2*d*x + 3/2*c) + 36*sqrt(2)*cos(4*d*x + 4*c)^2*sin(3/2
*d*x + 3/2*c) + 64*sqrt(2)*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + 36*sqr
t(2)*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*sin(6*d*x + 6*c)^2
*sin(3/2*d*x + 3/2*c) + 16*sqrt(2)*sin(5*d*x + 5*c)^2*sin(3/2*d*x + 3/2*c)
+ 36*sqrt(2)*sin(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 64*sqrt(2)*sin(3*d*x
 + 3*c)^2*sin(3/2*d*x + 3/2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2*sin(3/2*d*x
+ 3/2*c) - 8*(3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 3*sqrt(2)*c
os(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 2*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(d
*x + c) + (sqrt(2)*sin(9/2*d*x + 9/2*c) + 3*sqrt(2)*sin(7/2*d*x + 7/2*c) -
3*sqrt(2)*sin(5/2*d*x + 5/2*c) - sqrt(2)*sin(3/2*d*x + 3/2*c))*cos(6*d*x +
6*c) + 2*(sqrt(2)*sin(9/2*d*x + 9/2*c) + 3*sqrt(2)*sin(7/2*d*x + 7/2*c) - 3
*sqrt(2)*sin(5/2*d*x + 5/2*c) - sqrt(2)*sin(3/2*d*x + 3/2*c))*cos(5*d*x + 5
*c) - (3*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(3*d*x + 3*c) + 3*sqrt(2)*
sin(2*d*x + 2*c) + 2*sqrt(2)*sin(d*x + c))*cos(9/2*d*x + 9/2*c) + 3*(3*sqrt
(2)*sin(7/2*d*x + 7/2*c) - 3*sqrt(2)*sin(5/2*d*x + 5/2*c) - sqrt(2)*sin(3/2
*d*x + 3/2*c))*cos(4*d*x + 4*c) - 3*(4*sqrt(2)*sin(3*d*x + 3*c) + 3*sqrt(2)
*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(d*x + c))*cos(7/2*d*x + 7/2*c) - 4*(3*sqr
t(2)*sin(5/2*d*x + 5/2*c) + sqrt(2)*sin(3/2*d*x + 3/2*c))*cos(3*d*x + 3*c)
+ 3*(3*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(d*x + c))*cos(5/2*d*x + 5/2
*c) - (sqrt(2)*cos(9/2*d*x + 9/2*c) + 3*sqrt(2)*cos(7/2*d*x + 7/2*c) - 3*sq
rt(2)*cos(5/2*d*x + 5/2*c) - sqrt(2)*cos(3/2*d*x + 3/2*c))*sin(6*d*x + 6*c)
```

- 2*(sqrt(2)*cos(9/2*d*x + 9/2*c) + 3*sqrt(2)*cos(7/2*d*x + 7/2*c) - 3*sqrt(2)*cos(5/2*d*x + 5/2*c) - sqrt(2)*cos(3/2*d*x + 3/2*c))*sin(5*d*x + 5*c) + (3*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(3*d*x + 3*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(9/2*d*x + 9/2*c) - 3*(3*sqrt(2)*cos(7/2*d*x + 7/2*c) - 3*sqrt(2)*cos(5/2*d*x + 5/2*c) - sqrt(2)*cos(3/2*d*x + 3/2*c))*sin(4*d*x + 4*c) + 3*(4*sqrt(2)*cos(3*d*x + 3*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(7/2*d*x + 7/2*c) + 4*(3*sqrt(2)*cos(5/2*d*x + 5/2*c) + sqrt(2)*cos(3/2*d*x + 3/2*c))*sin(3*d*x + 3*c) - 3*(3*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(5/2*d*x + 5/2*c) - (2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 32*(3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 3*sqrt(2)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 2*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + (sqrt(2)*sin(9/2*d*x + 9/2*c) + 3*sqrt(2)*sin(7/2*d*x + 7/2*c) - 3*sqrt(2)*sin(5/2*d*x + 5/2*c) - sqrt(2)*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) + 2*(sqrt(2)*sin(9/2*d*x + 9/2*c) + 3*sqrt(2)*sin(7/2*d*x + 7/2*c) - 3*sqrt(2)*sin(5/2*d*x + 5/2*c) - sqrt(2)*sin(3/2*d*x + 3/2*c))*cos(5*d*x + 5*c) - (3*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(3*d*x + 3*c) + 3*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(d*x + c))*cos(9/2*d*x + 9/2*c) + 3*(3*sqrt(2)*sin(7/2*d*x + 7/2*c) - 3*sqrt(2)*sin(5/2*d*x + 5/2*c) - sqrt(2)*sin(3/2*d*x + 3/2*c))*cos(4*d*x + 4*c) - 3*(4*sqrt(2)*sin(3*d*x + 3*c) + 3*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(d*x + c))*cos(7/2*d*x + 7/2*c) - 4*(3*sqrt(2)*sin(5/2*d*x + 5/2*c) + sqrt(2)*sin(3/2*d*x + 3/2*c))*cos(3*d*x + 3*c) + 3*(3*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(d*x + c))*cos(5/2*d*x + 5/2*c) - (sqrt(2)*cos(9/2*d*x + 9/2*c) + 3*sqrt(2)*cos(7/2*d*x + 7/2*c) - 3*sqrt(2)*cos(5/2*d*x + 5/2*c) - sqrt(2)*cos(3/2*d*x + 3/2*c))*sin(6*d*x + 6*c) - 2*(sqrt(2)*cos(9/2*d*x + 9/2*c) + 3*sqrt(2)*cos(7/2*d*x + 7/2*c) - 3*sqrt(2)*cos(5/2*d*x + 5/2*c) - sqrt(2)*cos(3/2*d*x + 3/2*c))*sin(5*d*x + 5*c) + (3*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(3*d*x + 3*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(9/2*d*x + 9/2*c) - 3*(3*sqrt(2)*cos(7/2*d*x + 7/2*c) - 3*sqrt(2)*cos(5/2*d*x + 5/2*c) - sqrt(2)*cos(3/2*d*x + 3/2*c))*sin(4*d*x + 4*c) + 3*(4*sqrt(2)*cos(3*d*x + 3*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(7/2*d*x + 7/2*c) + 4*(3*sqrt(2)*cos(5/2*d*x + 5/2*c) + sqrt(2)*cos(3/2*d*x + 3/2*c))*sin(3*d*x + 3*c) - 3*(3*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(5/2*d*x + 5/2*c) - (2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sqrt(2)*cos(d*x + c) + sqrt(2))*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 8*(3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 3*sqrt(2)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 2*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + (sqrt(2)*sin(9/2*d*x + 9/2*c) + 3*sqrt(2)*sin(7/2*d*x + 7/2*c) - 3*sqrt(2)*sin(5/2*d*x + 5/2*c) - sqrt(2)*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) + 2*(sqrt(2)*sin(9/2*d*x + 9/2*c) + 3*sqrt(2)*sin(7/2*d*x + 7/2*c) - 3*sqrt(2)*sin(5/2*d*x + 5/2*c) - sqrt(2)*sin(3/2*d*x + 3/2*c))*cos(5*d*x + 5*c) - (3*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(3*d*x + 3...

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(140) = 280$.
time = 0.41, size = 284, normalized size = 1.72

$$-\frac{((7A-4B)\cos(dx+c)^3+(7A-4B)\cos(dx+c)^2)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4((A-4B)\cos(dx+c)-2A)\sqrt{a\cos(dx+c)+a}\sin(dx+c)+\frac{8\sqrt{2}((A-B)a\cos(dx+c)^3+(A-B)a\cos(dx+c)^2)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{16(ad\cos(dx+c)^3+ad\cos(dx+c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/16*(((7*A - 4*B)*cos(d*x + c)^3 + (7*A - 4*B)*cos(d*x + c)^2)*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(
a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)
) + 4*((A - 4*B)*cos(d*x + c) - 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
+ 8*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*log(-(cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*c
os(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d
*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/sqrt(a*(cos(c + d*x) + 1)), x
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(140) = 280$.
time = 0.49, size = 299, normalized size = 1.81

$$-\frac{\frac{4\sqrt{2}(A\sqrt{a}-B\sqrt{a})\log(\sin(\frac{1}{2}dx+\frac{1}{2}c)+1)}{a\mathrm{sgn}(\cos(\frac{1}{2}dx+\frac{1}{2}c))}-\frac{4\sqrt{2}(A\sqrt{a}-B\sqrt{a})\log(-\sin(\frac{1}{2}dx+\frac{1}{2}c)+1)}{a\mathrm{sgn}(\cos(\frac{1}{2}dx+\frac{1}{2}c))}-\frac{(7A\sqrt{a}-4B\sqrt{a})\log(|\frac{1}{2}\sqrt{2}+\sin(\frac{1}{2}dx+\frac{1}{2}c)|)}{a\mathrm{sgn}(\cos(\frac{1}{2}dx+\frac{1}{2}c))}+\frac{(7A\sqrt{a}-4B\sqrt{a})\log(|-\frac{1}{2}\sqrt{2}+\sin(\frac{1}{2}dx+\frac{1}{2}c)|)}{a\mathrm{sgn}(\cos(\frac{1}{2}dx+\frac{1}{2}c))}-\frac{2(2\sqrt{2}A\sqrt{a}\sin(\frac{1}{2}dx+\frac{1}{2}c)^3-8\sqrt{2}B\sqrt{a}\sin(\frac{1}{2}dx+\frac{1}{2}c)^3+\sqrt{2}A\sqrt{a}\sin(\frac{1}{2}dx+\frac{1}{2}c)+4\sqrt{2}B\sqrt{a}\sin(\frac{1}{2}dx+\frac{1}{2}c))}{(2\sin(\frac{1}{2}dx+\frac{1}{2}c)^2-1)^2a\mathrm{sgn}(\cos(\frac{1}{2}dx+\frac{1}{2}c))}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/8*(4*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(a*sg
n(cos(1/2*d*x + 1/2*c))) - 4*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(-sin(1/2*d
*x + 1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) - (7*A*sqrt(a) - 4*B*sqrt(a)
```

```
)*log(abs(1/2*sqrt(2) + sin(1/2*d*x + 1/2*c)))/(a*sgn(cos(1/2*d*x + 1/2*c))
) + (7*A*sqrt(a) - 4*B*sqrt(a))*log(abs(-1/2*sqrt(2) + sin(1/2*d*x + 1/2*c)
))/(a*sgn(cos(1/2*d*x + 1/2*c))) - 2*(2*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2
*c)^3 - 8*sqrt(2)*B*sqrt(a)*sin(1/2*d*x + 1/2*c)^3 + sqrt(2)*A*sqrt(a)*sin(
1/2*d*x + 1/2*c) + 4*sqrt(2)*B*sqrt(a)*sin(1/2*d*x + 1/2*c))/((2*sin(1/2*d*
x + 1/2*c)^2 - 1)^2*a*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cos(c+dx)}{\cos(c+dx)^3\sqrt{a+a\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2)),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2)), x)
```

3.107 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=261

$$-\frac{(15A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{(651A-799B)\sin(}{105ad\sqrt{a+a\cos(c}}$$

```
[Out] 1/2*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-1/4*(15*A-19*B)*
arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^
(1/2)+1/105*(651*A-799*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/70*(63*A-
67*B)*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/14*(7*A-11*B)*co
s(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/210*(273*A-397*B)*sin(d*
x+c)*(a+a*cos(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.52, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3056, 3062, 3047, 3102, 2830, 2728, 212}

$$-\frac{(15A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(273A-397B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{210a^2d}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{(7A-11B)\sin(c+dx)\cos^3(c+dx)}{14ad\sqrt{a\cos(c+dx)+a}}+\frac{(63A-67B)\sin(c+dx)\cos^2(c+dx)}{70ad\sqrt{a\cos(c+dx)+a}}+\frac{(651A-799B)\sin(c+dx)}{105ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -1/2*((15*A - 19*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[
c + d*x]])])/(Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(2
*d*(a + a*Cos[c + d*x])^(3/2)) + ((651*A - 799*B)*Sin[c + d*x])/(105*a*d*Sq
rt[a + a*Cos[c + d*x]]) + ((63*A - 67*B)*Cos[c + d*x]^2*Sin[c + d*x])/(70*a
*d*Sqrt[a + a*Cos[c + d*x]]) - ((7*A - 11*B)*Cos[c + d*x]^3*Sin[c + d*x])/(
14*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((273*A - 397*B)*Sqrt[a + a*Cos[c + d*x]
]*Sin[c + d*x])/(210*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos^3(c+dx)\left(4a(A-B)-\frac{1}{2}a(7A-11B)\right.}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(7A-11B)\cos^3(c+dx)\sin(c}{14ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(63A-67B)\cos^2(c+dx)\sin(}{70ad\sqrt{a+a\cos(c+dx}}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(63A-67B)\cos^2(c+dx)\sin(}{70ad\sqrt{a+a\cos(c+dx}}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(63A-67B)\cos^2(c+dx)\sin(}{70ad\sqrt{a+a\cos(c+dx}}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(651A-799B)\sin(c+dx)}{105ad\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(651A-799B)\sin(c+dx)}{105ad\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{(15A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\cos}{2d(a+}$$

Mathematica [A]
time = 1.13, size = 167, normalized size = 0.64

$$\frac{105(15A-19B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right) - \frac{1}{2}\cos^3\left(\frac{1}{2}(c+dx)\right)(1974A-2161B+6(273A-277B)\cos(c+dx)+(-84A+256B)\cos(2(c+dx))+42A\cos(3(c+dx))-18B\cos(3(c+dx))+15B\cos(4(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)}{105d(a(1+\cos(c+dx)))^{3/2}\left(-1+\sin^2\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (105*(15*A - 19*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - (Cos[(c +
 d*x)/2]^3*(1974*A - 2161*B + 6*(273*A - 277*B)*Cos[c + d*x] + (-84*A + 256
*B)*Cos[2*(c + d*x)] + 42*A*Cos[3*(c + d*x)] - 18*B*Cos[3*(c + d*x)] + 15*B
```

```
*Cos[4*(c + d*x)])*Sin[(c + d*x)/2])/2)/(105*d*(a*(1 + Cos[c + d*x]))^(3/2)
*(-1 + Sin[(c + d*x)/2]^2))
```

Maple [A]
time = 0.40, size = 448, normalized size = 1.72

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(960B\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/420/cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(960*B*2^(1/2)*(sin
(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^8-96*2^(1/2)*(sin(1/2
*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*(7*A+17*B)*sin(1/2*d*x+1/2*c)^6+224*2^(1/2)*
(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*(3*A+8*B)*sin(1/2*d*x+1/2*c)^4+35*2^
(1/2)*(45*A*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)
+a))*a-48*A*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-57*B*ln(4/cos(1/2*d*x+1/
2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a+16*B*a^(1/2)*(sin(1/2*d*
x+1/2*c)^2*a)^(1/2))*sin(1/2*d*x+1/2*c)^2-1575*2^(1/2)*ln(4/cos(1/2*d*x+1/2
*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a*A+1995*2^(1/2)*ln(4/cos(1
/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a*B+1785*A*a^(1/2
)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-1785*B*2^(1/2)*(sin(1/2*d*x+1/2*c)
^2*a)^(1/2)*a^(1/2))/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1
/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.37, size = 241, normalized size = 0.92

$$-\frac{105\sqrt{2}\left((15A-19B)\cos(dx+c)^2+2(15A-19B)\cos(dx+c)+15A-19B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(60B\cos(dx+c)^4+12(7A-3B)\cos(dx+c)^3-28(3A-7B)\cos(dx+c)^2+12(63A-67B)\cos(dx+c)+1029A-1201B\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{840\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/840*(105*sqrt(2)*((15*A - 19*B)*cos(d*x + c)^2 + 2*(15*A - 19*B)*cos(d*x
 + c) + 15*A - 19*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(
d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)) - 4*(60*B*cos(d*x + c)^4 + 12*(7*A - 3*B)*cos(d*x
+ c)^3 - 28*(3*A - 7*B)*cos(d*x + c)^2 + 12*(63*A - 67*B)*cos(d*x + c) + 10
29*A - 1201*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2
 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x)
```

3.108 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=216

$$\frac{(11A-15B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}-\frac{(65A-93B)\sin(c}{15ad\sqrt{a+a\cos(c+}}$$

```
[Out] 1/2*(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/4*(11*A-15*B)*
arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^
(1/2)-1/15*(65*A-93*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/10*(5*A-9*B)
*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/30*(35*A-39*B)*sin(d*
x+c)*(a+a*cos(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.39, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3056, 3062, 3047, 3102, 2830, 2728, 212}

$$\frac{(11A-15B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(35A-39B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{30a^2d}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{(5A-9B)\sin(c+dx)\cos^2(c+dx)}{10ad\sqrt{a\cos(c+dx)+a}}-\frac{(65A-93B)\sin(c+dx)}{15ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((11*A - 15*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(2*d*
(a + a*Cos[c + d*x])^(3/2)) - ((65*A - 93*B)*Sin[c + d*x])/(15*a*d*Sqrt[a +
 a*Cos[c + d*x]]) - ((5*A - 9*B)*Cos[c + d*x]^2*Sin[c + d*x])/(10*a*d*Sqrt[
a + a*Cos[c + d*x]]) + ((35*A - 39*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x]
)/(30*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)\left(3a(A-B)-\frac{1}{2}a(5A-9B)\right.}{\sqrt{a+a\cos(c+dx}}}{2a^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(5A-9B)\cos^2(c+dx)\sin(c}{10ad\sqrt{a+a\cos(c+dx}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(5A-9B)\cos^2(c+dx)\sin(c}{10ad\sqrt{a+a\cos(c+dx}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(5A-9B)\cos^2(c+dx)\sin(c}{10ad\sqrt{a+a\cos(c+dx}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(65A-93B)\sin(c+dx)}{15ad\sqrt{a+a\cos(c+dx)}} -$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(65A-93B)\sin(c+dx)}{15ad\sqrt{a+a\cos(c+dx)}} -$$

$$= \frac{(11A-15B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\cos}{2d(a+}$$

Mathematica [A]
time = 0.94, size = 142, normalized size = 0.66

$$\frac{-15(11A-15B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)+\cos^3\left(\frac{1}{2}(c+dx)\right)(85A-141B+3(20A-39B)\cos(c+dx)+(-10A+6B)\cos(2(c+dx))-3B\cos(3(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)}{15d(a(1+\cos(c+dx)))^{3/2}\left(-1+\sin^2\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (-15*(11*A - 15*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + Cos[(c +
d*x)/2]^3*(85*A - 141*B + 3*(20*A - 39*B)*Cos[c + d*x] + (-10*A + 6*B)*Cos[
2*(c + d*x)] - 3*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2])/(15*d*(a*(1 + Cos[c
+ d*x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(189) = 378.
time = 0.33, size = 407, normalized size = 1.88

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-96B\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+16\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(-96*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)
^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+16*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/
2)*a^(1/2)*(5*A+6*B)*sin(1/2*d*x+1/2*c)^4-5*2^(1/2)*(33*A*ln(4/cos(1/2*d*x+
1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+a))*a-8*A*a^(1/2)*(sin(1/2*d
*x+1/2*c)^2*a)^(1/2)-45*B*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2
*c)^2*a)^(1/2)+a))*a+48*B*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2))*sin(1/2*d
*x+1/2*c)^2+165*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*d*x+1/2*c
)^2*a)^(1/2)+a))*a*A-225*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(sin(1/2*
d*x+1/2*c)^2*a)^(1/2)+a))*a*B-135*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a
)^(1/2)+255*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/cos(1/2*d*x+1
/2*c)/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.36, size = 224, normalized size = 1.04

$$-\frac{15\sqrt{2}\left((11A-15B)\cos(dx+c)^2+2(11A-15B)\cos(dx+c)+11A-15B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(12B\cos(dx+c)^3+4(5A-3B)\cos(dx+c)^2-12(5A-9B)\cos(dx+c)-95A+147B\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{120\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/120*(15*sqrt(2)*((11*A - 15*B)*cos(d*x + c)^2 + 2*(11*A - 15*B)*cos(d*x
+ c) + 11*A - 15*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d
```

```
*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2
 + 2*cos(d*x + c) + 1)) - 4*(12*B*cos(d*x + c)^3 + 4*(5*A - 3*B)*cos(d*x +
c)^2 - 12*(5*A - 9*B)*cos(d*x + c) - 95*A + 147*B)*sqrt(a*cos(d*x + c) + a)
*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x)
```

3.109 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{(7A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{(9A-13B)\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 1/2*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-1/4*(7*A-11*B)*a
rctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(
1/2)+1/3*(9*A-13*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/6*(3*A-7*B)*sin
(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a^2/d
```

Rubi [A]

time = 0.27, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3056, 3047, 3102, 2830, 2728, 212}

$$-\frac{(7A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(3A-7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{6a^2d}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{(9A-13B)\sin(c+dx)}{3ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -1/2*((7*A - 11*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(2*
d*(a + a*Cos[c + d*x])^(3/2)) + ((9*A - 13*B)*Sin[c + d*x])/(3*a*d*Sqrt[a +
 a*Cos[c + d*x]]) - ((3*A - 7*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(6*
a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(2a(A-B)-\frac{1}{2}a(3A-7B)\cos\right.}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{2a(A-B)\cos(c+dx)-\frac{1}{2}a(3A-7B)\cos}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(3A-7B)\sqrt{a+a\cos(c+dx}}{6a^2d}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(9A-13B)\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} - \frac{(3.}{}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(9A-13B)\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} - \frac{(3.}{}$$

$$= -\frac{(7A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\cos^2}{2d(a+a}$$

Mathematica [A]
time = 0.77, size = 97, normalized size = 0.57

$$\frac{-3(7A-11B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)+(15A-17B+12(A-B)\cos(c+dx)+2B\cos(2(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{6ad\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (-3*(7*A - 11*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (15*A - 17*B
+ 12*(A - B)*Cos[c + d*x] + 2*B*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(6*a*d*
Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(148) = 296.
time = 0.32, size = 327, normalized size = 1.91

method	result

default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(16B\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-21A\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/12/cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(16*B*2^(1/2)*(sin(1
/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-21*A*ln(2*(2*a^(1/2)*
(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x
+1/2*c)^2*a+33*B*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/
2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+24*A*a^(1/2)*2^(1/2)*(sin(1/2*
d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2-40*B*a^(1/2)*2^(1/2)*(sin(1/2*d*
x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2+3*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1
/2*c)^2*a)^(1/2)-3*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/a^(5/2
)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

<u>Maxima</u> [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

<u>Fricas</u> [**A**]
time = 0.35, size = 205, normalized size = 1.20

$$-\frac{3\sqrt{2}\left((7A-11B)\cos\left(dx+c\right)^2+2(7A-11B)\cos\left(dx+c\right)+7A-11B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos\left(dx+c\right)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(4B\cos\left(dx+c\right)^2+12(A-B)\cos\left(dx+c\right)+15A-19B\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{24\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/24*(3*sqrt(2)*((7*A - 11*B)*cos(d*x + c)^2 + 2*(7*A - 11*B)*cos(d*x + c)
 + 7*A - 11*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x +
c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) - 4*(4*B*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) + 15*A
```

```
 - 19*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x)
```

3.110 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{(3A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}}$$

```
[Out] -1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/4*(3*A-7*B)*arctanh(1/2*si
n(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+2*B*sin(
d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3047, 3098, 2830, 2728, 212}

$$\frac{(3A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{2B\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((3*A - 7*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*
x])^(3/2)) + (2*B*Sin[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{-\frac{3}{2}a(A-B)-2aB\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{(3A-7B)}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} - \frac{(3A-7B)S}{}$$

$$= \frac{(3A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{(A-B)\sin(c}{2d(a+a\cos(c+}$$

Mathematica [A]
time = 0.44, size = 104, normalized size = 0.88

$$\frac{-\left((3A-7B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\right)+\cos^3\left(\frac{1}{2}(c+dx)\right)(A-5B-4B\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)}{d(a(1+\cos(c+dx)))^{3/2}\left(-1+\sin^2\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-((3*A - 7*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5) + Cos[(c + d*x
)/2]^3*(A - 5*B - 4*B*Cos[c + d*x])*Sin[(c + d*x)/2])/(d*(a*(1 + Cos[c + d*
x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(101) = 202$.
time = 0.28, size = 256, normalized size = 2.17

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(3A\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\,\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-7B\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\right.}}{\cos\left(\frac{dx}{2}\right.}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/4*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(3*A*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)
^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-7*B*ln(
2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2
)*cos(1/2*d*x+1/2*c)^2*a+8*B*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)
*cos(1/2*d*x+1/2*c)^2-A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+B*2^
(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)/a^(5/2)/si
n(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.36, size = 189, normalized size = 1.60

$$-\frac{\sqrt{2}\left((3A-7B)\cos\left(dx+c\right)^2+2(3A-7B)\cos\left(dx+c\right)+3A-7B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(4B\cos\left(dx+c\right)-A+5B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{8\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/8*(sqrt(2)*((3*A - 7*B)*cos(d*x + c)^2 + 2*(3*A - 7*B)*cos(d*x + c) + 3*
A - 7*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a
)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) - 4*(4*B*cos(d*x + c) - A + 5*B)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)})\cos{(c + dx)}}{(a(\cos{(c + dx)} + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2), x
)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x)
```

3.111 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

```
[Out] 1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/4*(A+3*B)*arctanh(1/2*sin(d
*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2829, 2728, 212}

$$\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((A + 3*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x]
)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(A+3B)\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A+3B)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+}}\right.}{2ad}$$

$$= \frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.21, size = 63, normalized size = 0.72

$$\frac{(A+3B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+\frac{1}{2}(A-B)\sin(c+dx)}{d(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((A + 3*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + ((A - B)*Sin[c +
d*x])/2)/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(72) = 144.
time = 0.26, size = 220, normalized size = 2.53

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(A\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\,\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+3B\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{}{}\right.\right.}{\cos\left(\frac{dx}{2}+\frac{c}{}\right.}\right.\right.}{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(A*ln(2*(2*a^(1/2)*(s
in(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1
```

```
/2*c)^2*a+3*B*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d
*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/
2*c)^2*a)^(1/2)-B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/a^(5/2)/s
in(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 62254 vs. $2(72) = 144$.
time = 3.35, size = 62254, normalized size = 715.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*((3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 4*sqrt(2)*sin(5/2*d*x + 5/2*c))
*cos(3*d*x + 3*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)
^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + (3*(sqrt(2)*log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1))*cos(3*d*x + 3*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
1))*cos(2*d*x + 2*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^
2 + 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 27*(sqrt(2)*lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
+ 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(
1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*sin(d*x + c)^2 - 96*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(d*x + c) - 60*sq
rt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 2*(9*(sqrt(2)*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(2*d*x + 2*c) + 9*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*
```

```
c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) +
 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sqrt(2)*sin(5/2*d*x + 5/2*c) + 16
*sqrt(2)*sin(3/2*d*x + 3/2*c) + 10*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(3*d*x
+ 3*c) + 60*(sqrt(2)*sin(2*d*x + 2*c) + sqrt(2)*sin(d*x + c))*cos(5/2*d*x +
 5/2*c) + 6*(9*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*sqrt(2)*l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1) + 16*sqrt(2)*sin(3/2*d*x + 3/2*c) + 10*sqrt(2)*sin
(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 6*(3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
) + 10*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 2*(9*(sqrt(2)*log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*sin(2*d*x + 2*c) + 9*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(
d*x + c) + 10*sqrt(2)*cos(5/2*d*x + 5/2*c) - 16*sqrt(2)*cos(3/2*d*x + 3/2*c
) - 10*sqrt(2)*cos(1/2*d*x + 1/2*c))*sin(3*d*x + 3*c) - 20*(3*sqrt(2)*cos(2
*d*x + 2*c) + 3*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(5/2*d*x + 5/2*c) + 6*(9
*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) - 16*sqrt(2)*cos(3/2*d*x +
3/2*c) - 10*sqrt(2)*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) + 32*(3*sqrt(2)*
cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*
c) + 1) + 20*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c)))^2 + 9*(3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos
(3*d*x + 3*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x +...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(72) = 144$.
time = 0.34, size = 172, normalized size = 1.98

$$\frac{\sqrt{2}\left((A+3B)\cos(dx+c)^2+2(A+3B)\cos(dx+c)+A+3B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\sqrt{a\cos(dx+c)+a}(A-B)\sin(dx+c)}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*((A + 3*B)*cos(d*x + c)^2 + 2*(A + 3*B)*cos(d*x + c) + A + 3*B
)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)) + 4*sqrt(a*cos(d*x + c) + a)*(A - B)*sin(d*x + c))/(a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos\left(c + dx\right)}{\left(a\left(\cos\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(a*(cos(c + d*x) + 1))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(72) = 144.
time = 0.49, size = 160, normalized size = 1.84

$$\frac{\frac{\sqrt{2}\left(A\sqrt{a} + 3\,B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right)}{a^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)} - \frac{\sqrt{2}\left(A\sqrt{a} + 3\,B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right)}{a^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)} - \frac{2\,\sqrt{2}\left(A\sqrt{a}\,\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - B\sqrt{a}\,\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{\left(\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{2} - 1\right)a^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*(A*sqrt(a) + 3*B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(a^2*s
gn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*(A*sqrt(a) + 3*B*sqrt(a))*log(-sin(1/2*
d*x + 1/2*c) + 1)/(a^2*sgn(cos(1/2*d*x + 1/2*c))) - 2*sqrt(2)*(A*sqrt(a)*si
n(1/2*d*x + 1/2*c) - B*sqrt(a)*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/2*c)
^2 - 1)*a^2*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos\left(c + d\,x\right)}{{\left(a + a\,\cos\left(c + d\,x\right)\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(3/2), x)
```

3.112 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d}-\frac{(5A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3}}$$

```
[Out] 2*A*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d-1/2*(A-B)*
sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-1/4*(5*A-B)*arctanh(1/2*sin(d*x+c)*a^(1
/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3057, 3064, 2728, 212, 2852}

$$-\frac{(5A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d)
- ((5*A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*
x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(2aA-\frac{1}{2}a(A-B)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{A\int \sqrt{a+a\cos(c+dx)}\,\sec(c+dx)}{a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(2A)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin}{\sqrt{a+a}}\right)}{ad}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-B)\tanh^{-1}\left(\frac{}{\sqrt{2}\,\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}}$$

Mathematica [A]

time = 0.68, size = 131, normalized size = 1.03

$$\frac{(5A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right) - 4\sqrt{2}\,A\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right) + (A-B)\cos^3\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{1}{2}(c+dx)\right)}{d(a(1+\cos(c+dx)))^{3/2}\left(-1+\sin^2\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((5*A - B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - 4*Sqrt[2]*A*ArcTa
nh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (A - B)*Cos[(c + d*x)/2]^
3*Sin[(c + d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^
2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(106) = 212$.
time = 0.39, size = 374, normalized size = 2.94

method	result
default	$-\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(5A\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\,\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-B\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\right.}}{\cos\left(\frac{dx}{2}\right.}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/4/a^(5/2)/cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(5*A*ln(2*(2
*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*co
s(1/2*d*x+1/2*c)^2*a-B*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/
cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-4*A*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+
1/2*c)^2*a)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a-4*A*ln(-4/(2*cos(1/2*d*x+1/2
*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2
*c)^2*a)^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^2*a+A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+
1/2*c)^2*a)^(1/2)-B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/sin(1/2
*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 15722 vs. $2(106) = 212$.
time = 1.16, size = 15722, normalized size = 123.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] 1/4*(32*(cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d
*x + 3/2*c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(
d*x + c))*cos(3*d*x + 3*c)^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) +
3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x
+ 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3
/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3
*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1
)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*
x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 32*(cos(3/2*d*x
+ 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + cos(d*x
 + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(3*d*x +
 3*c)^2 + 32*(6*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 9
6*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 96*sin(2*d*x + 2*c)^2*sin(3/2*d
*x + 3/2*c) + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3
/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*
d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) +
 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*c
os(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3
/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*
d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(2/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*(2*(3*cos(d*x + c) + 1)*cos(2*
d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c)
 + 3*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 2*(3*sin(3/2*d*x + 3/2*c)*si
n(d*x + c) + cos(3/2*d*x + 3/2*c))*sin(2*d*x + 2*c) + (3*cos(d*x + c)^2 + 3
*sin(d*x + c)^2 + 2*cos(d*x + c))*sin(3/2*d*x + 3/2*c) + 2*cos(3/2*d*x + 3/
2*c)*sin(d*x + c))*cos(3*d*x + 3*c) - 4*(6*(sin(2*d*x + 2*c) + sin(d*x + c)
)*sin(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^3 + (2*(3*cos(2*d*x + 2*c) + 3*cos(
d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1
)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 9*sin(2*d*x
+ 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x
+ c) + 1)*sin(3*d*x + 3*c) + 3*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)
*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x +
 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c) + sin
(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^2 + 1
8*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*(2*(3*cos(2*
d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*
(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x +
c)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3
*c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x
 + c)^2 + 6*cos(d*x + c) + 1)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
```

```
)) - 4*(8*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*cos(2*d*x + 2*c)^2*s
in(3/2*d*x + 3/2*c) - 144*cos(2*d*x + 2*c)*cos(d*x + c)*sin(3/2*d*x + 3/2*c
) - 8*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*sin(2*d*x + 2*c)^2*sin(3
/2*d*x + 3/2*c) - 16*(3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 3*cos(3/2*d
*x + 3/2*c)*sin(d*x + c) - sin(3/2*d*x + 3/2*c))*cos(3*d*x + 3*c) - 48*(cos
(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c
) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*
x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*
c)*sin(d*x + c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 16*(cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) + 3*sin(2*d*x + 2*c)*sin(3/2
*d*x + 3/2*c) + 3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x + 3/2*c))
*sin(3*d*x + 3*c) - 48*(3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x +
 3/2*c))*sin(2*d*x + 2*c) - 8*(9*cos(d*x + c)^2 + 9*sin(d*x + c)^2 - 1)*sin
(3/2*d*x + 3/2*c) - 48*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 3*(2*(3*cos(2*d*
x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3
*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)
^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c
)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x +
 c)^2 + 6*cos(d*x + c) + 1)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 - 4*(8*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*cos(2*d*x + 2*c)^2*sin
(3/2*d*x + 3/2*c) - 144*cos(2*d*x + 2*c)*cos(d*...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(106) = 212$.
time = 0.38, size = 281, normalized size = 2.21

$$-\frac{\sqrt{2}\left((5A-B)\cos\left(dx+c\right)^{2}+2\left(5A-B\right)\cos\left(dx+c\right)+5A-B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\cos\left(dx+c\right)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)-4\left(A\cos\left(dx+c\right)^{2}+2A\cos\left(dx+c\right)+A\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}-4\sqrt{a\cos\left(dx+c\right)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\sqrt{a\cos\left(dx+c\right)+a}\left(A-B\right)\sin\left(dx+c\right)}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/8*(sqrt(2)*((5*A - B)*cos(d*x + c)^2 + 2*(5*A - B)*cos(d*x + c) + 5*A -
B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1)) - 4*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x
 + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x
+ c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a
*cos(d*x + c) + a)*(A - B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*co
s(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(A+B\cos\left(c+dx\right))\sec\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2), x
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(106) = 212.
time = 0.84, size = 220, normalized size = 1.73

$$\frac{\frac{8\,A\log\left(\frac{\left|-2\,\sqrt{2}-4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}-4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}\left(5\,A\sqrt{a}-B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{\sqrt{2}\left(5\,A\sqrt{a}-B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{2\,\sqrt{2}\left(A\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] 1/8*(8*A*log(abs(-2*sqrt(2) - 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) - 4*sin
(1/2*d*x + 1/2*c)))/(a^(3/2)*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*(5*A*sqrt
(a) - B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(1/2*d*x + 1/2*c
))) + sqrt(2)*(5*A*sqrt(a) - B*sqrt(a))*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^2
*sgn(cos(1/2*d*x + 1/2*c))) + 2*sqrt(2)*(A*sqrt(a)*sin(1/2*d*x + 1/2*c) - B
*sqrt(a)*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/2*c)^2 - 1)*a^2*sgn(cos(1/
2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2)), x)
```

3.113 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{(3A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d}+\frac{(9A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\,\mathrm{t}}{2d(a+a\cos}$$

```
[Out] -(3*A-2*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d+1/4
*(9*A-5*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a
^(3/2)/d*2^(1/2)-1/2*(A-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/2*(3*A-B)*
tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3057, 3063, 3064, 2728, 212, 2852}

$$-\frac{(3A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(9A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -(((3*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a
^(3/2)*d)) + ((9*A - 5*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Tan[c + d*x])/(2*d*(a +
 a*Cos[c + d*x])^(3/2)) + ((3*A - B)*Tan[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c
+ d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(a(3A-B)-\frac{3}{2}a(A-B)\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{(-a^2(3A}{}}{}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{(9A-5B}{}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{(9A-5B}{}$$

$$= -\frac{(3A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{(9A-5B)\tanh^{-1}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.39, size = 608, normalized size = 3.58

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(2(-9A+5B)\log\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)+2(9A-5B)\log\left(\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)\right)-2\sqrt{2}(3A-2B)\log\left(\sqrt{2}+2\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{2i(3A-2B)\text{ArcTan}\left(\frac{\cos\left(\frac{1}{4}(c+dx)\right)-\left(-1+\sqrt{2}\right)\sin\left(\frac{1}{4}(c+dx)\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}\right)\left(\sqrt{2}-2\sin\left(\frac{c}{2}\right)\right)}{-1+\sqrt{2}\sin\left(\frac{c}{2}\right)}-\frac{2i(3A-2B)\text{ArcTan}\left(\frac{\cos\left(\frac{1}{4}(c+dx)\right)-\left(1+\sqrt{2}\right)\sin\left(\frac{1}{4}(c+dx)\right)}{\left(-1+\sqrt{2}\right)\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}\right)\left(\sqrt{2}-2\sin\left(\frac{c}{2}\right)\right)}{-1+\sqrt{2}\sin\left(\frac{c}{2}\right)}-\frac{(3A-2B)\log\left(2-\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sqrt{2}-2\sin\left(\frac{c}{2}\right)\right)}{-1+\sqrt{2}\sin\left(\frac{c}{2}\right)}-\frac{(3A-2B)\log\left(2+\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sqrt{2}-2\sin\left(\frac{c}{2}\right)\right)}{-1+\sqrt{2}\sin\left(\frac{c}{2}\right)}+\frac{A-B}{\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}+\frac{-A+B}{\left(\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}+\frac{4A}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-\frac{4A}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{2d(a(1+\cos(c+dx)))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(2*(-9*A + 5*B)*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]
] + 2*(9*A - 5*B)*Log[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] - 2*Sqrt[2]*(3*A
 - 2*B)*Log[Sqrt[2] + 2*Sin[(c + d*x)/2]] - ((2*I)*(3*A - 2*B)*ArcTan[(Cos[
(c + d*x)/4] - (-1 + Sqrt[2])*Sin[(c + d*x)/4])/((1 + Sqrt[2])*Cos[(c + d*x
)/4] - Sin[(c + d*x)/4])]*(Sqrt[2] - 2*Sin[c/2]))/(-1 + Sqrt[2]*Sin[c/2]) -
 ((2*I)*(3*A - 2*B)*ArcTan[(Cos[(c + d*x)/4] - (1 + Sqrt[2])*Sin[(c + d*x)/
4])/((-1 + Sqrt[2])*Cos[(c + d*x)/4] - Sin[(c + d*x)/4])]*(Sqrt[2] - 2*Sin[
c/2]))/(-1 + Sqrt[2]*Sin[c/2]) - ((3*A - 2*B)*Log[2 - Sqrt[2]*Cos[(c + d*x)
/2] - Sqrt[2]*Sin[(c + d*x)/2]]*(Sqrt[2] - 2*Sin[c/2]))/(-1 + Sqrt[2]*Sin[c
/2]) - ((3*A - 2*B)*Log[2 + Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x
)/2]]*(Sqrt[2] - 2*Sin[c/2]))/(-1 + Sqrt[2]*Sin[c/2]) + (A - B)/(Cos[(c + d
*x)/4] - Sin[(c + d*x)/4])^2 + (-A + B)/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4
])^2 + (4*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (4*A)/(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(145) = 290$.

time = 0.44, size = 1051, normalized size = 6.18

method	result	size
default	Expression too large to display	1051

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/2*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(18*A*2^(1/2)*ln(2*(2*a^(1/2)*(sin(1/2*d
*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-10*B*2
^(1/2)*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*
c))*cos(1/2*d*x+1/2*c)^4*a-12*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a)
)*cos(1/2*d*x+1/2*c)^4*a-12*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2
)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*c
os(1/2*d*x+1/2*c)^4*a+8*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*c
os(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*cos(
1/2*d*x+1/2*c)^4*a+8*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1
/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*cos(1/2*
d*x+1/2*c)^4*a-9*A*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(
1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+5*B*ln(2*(2*a^(1/2)*(sin(1/2
*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^
2*a+6*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2
+6*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(
1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^2*a+6*
A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)
*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a-2*B*a^
(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2-4*B*ln(-4
/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/
2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^2*a-4*B*ln(4/(2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(
sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a-A*a^(1/2)*2^(1/2
)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a
^(1/2))/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/
2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47933 vs. $2(145) = 290$.

time = 1.99, size = 47933, normalized size = 281.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")

[Out] 1/4*(128*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^3 + 1152*cos(3/2*d*x + 3/2*c
)*sin(2*d*x + 2*c)^3 - 128*cos(3*d*x + 3*c)^3*sin(3/2*d*x + 3/2*c) - 1152*c
os(2*d*x + 2*c)^3*sin(3/2*d*x + 3/2*c) + 32*(4*cos(3/2*d*x + 3/2*c)*sin(2*d
*x + 2*c) - 9*(3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - 28*cos(2*d*x + 2*
c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(3*d*x +
3*c)^2 - 96*(11*(3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - 9*cos(3/2*d*x +
 3/2*c)*sin(d*x + c))*cos(2*d*x + 2*c)^2 + 32*(28*cos(3/2*d*x + 3/2*c)*sin(
2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - 4*cos(3*d*x + 3*
c)*sin(3/2*d*x + 3/2*c) - 4*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 27*cos(
3/2*d*x + 3/2*c)*sin(d*x + c))*sin(3*d*x + 3*c)^2 - 288*((3*cos(d*x + c) +
1)*sin(3/2*d*x + 3/2*c) + 4*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 11*cos(
3/2*d*x + 3/2*c)*sin(d*x + c))*sin(2*d*x + 2*c)^2 - 32*(cos(3*d*x + 3*c)^2*
sin(3/2*d*x + 3/2*c) + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*x
+ 3/2*c) + 9*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*s
in(3/2*d*x + 3/2*c) + 9*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 18*sin(2*
d*x + 2*c)*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + 2*((3*cos(d*x + c) + 1)*sin(
3/2*d*x + 3/2*c) + 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c))*cos(3*d*x + 3*c
) + 6*(sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + sin(3/2*d*x + 3/2*c)*sin(d*x
 + c))*sin(3*d*x + 3*c) + (9*cos(d*x + c)^2 + 9*sin(d*x + c)^2 + 6*cos(d*x
+ c) + 1)*sin(3/2*d*x + 3/2*c))*cos(5*d*x + 5*c) - 96*(cos(3*d*x + 3*c)^2*s
in(3/2*d*x + 3/2*c) + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*x +
 3/2*c) + 9*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*si
n(3/2*d*x + 3/2*c) + 9*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 18*sin(2*d
*x + 2*c)*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + 2*((3*cos(d*x + c) + 1)*sin(3
/2*d*x + 3/2*c) + 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c))*cos(3*d*x + 3*c)
 + 6*(sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + sin(3/2*d*x + 3/2*c)*sin(d*x
+ c))*sin(3*d*x + 3*c) + (9*cos(d*x + c)^2 + 9*sin(d*x + c)^2 + 6*cos(d*x +
 c) + 1)*sin(3/2*d*x + 3/2*c))*cos(4*d*x + 4*c) - 64*(30*cos(2*d*x + 2*c)^2
*sin(3/2*d*x + 3/2*c) + 18*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) - 3*(3*c
os(d*x + c) + 1)*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + (19*(3*cos(d*x + c) +
1)*sin(3/2*d*x + 3/2*c) - 9*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(2*d*x +
2*c) - 4*((3*cos(d*x + c) + 1)*cos(3/2*d*x + 3/2*c) + 3*cos(2*d*x + 2*c)*co
s(3/2*d*x + 3/2*c) - 9*sin(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(2*d*x + 2*c)
+ 3*(9*cos(d*x + c)^2 + 6*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*sin(3/2*d*x
+ 3/2*c))*cos(3*d*x + 3*c) + 64*(9*(3*cos(d*x + c) + 1)*cos(3/2*d*x + 3/2*c
)*sin(d*x + c) - (45*cos(d*x + c)^2 + 18*sin(d*x + c)^2 + 30*cos(d*x + c) +
 5)*sin(3/2*d*x + 3/2*c))*cos(2*d*x + 2*c) + 96*(9*sin(d*x + c)^3 + (9*cos(
d*x + c)^2 + 6*cos(d*x + c) + 1)*sin(d*x + c))*cos(3/2*d*x + 3/2*c) - 12*((
sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 3*sin(d*x + c))*cos(5*d*x + 5*c)^2
+ 9*(sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 3*sin(d*x + c))*cos(4*d*x + 4*
c)^2 + 48*(sin(2*d*x + 2*c) + sin(d*x + c))*cos(3*d*x + 3*c)^2 + (sin(3*d*x
 + 3*c) + 3*sin(2*d*x + 2*c) + 3*sin(d*x + c))*sin(5*d*x + 5*c)^2 + 9*(sin(
```

```
3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 3*sin(d*x + c))*sin(4*d*x + 4*c)^2 + 8*
(10*sin(2*d*x + 2*c) + 9*sin(d*x + c))*sin(3*d*x + 3*c)^2 + 16*sin(3*d*x +
3*c)^3 + 48*sin(2*d*x + 2*c)^3 + 24*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*s
in(d*x + c) + 48*cos(2*d*x + 2*c)^2*sin(d*x + c) + 120*sin(2*d*x + 2*c)^2*s
in(d*x + c) + 27*sin(d*x + c)^3 + 2*(3*(sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*
c) + 3*sin(d*x + c))*cos(4*d*x + 4*c) + 12*(sin(2*d*x + 2*c) + sin(d*x + c)
)*cos(3*d*x + 3*c) + (4*cos(3*d*x + 3*c) + 4*cos(2*d*x + 2*c) + 3*cos(d*x +
 c) + 1)*sin(3*d*x + 3*c) + 3*(4*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*sin
(2*d*x + 2*c) + 3*(3*cos(d*x + c) + 1)*sin(d*x + c) + 12*cos(2*d*x + 2*c)*s
in(d*x + c))*cos(5*d*x + 5*c) + 6*(12*(sin(2*d*x + 2*c) + sin(d*x + c))*cos
(3*d*x + 3*c) + (4*cos(3*d*x + 3*c) + 4*cos(2*d*x + 2*c) + 3*cos(d*x + c) +
 1)*sin(3*d*x + 3*c) + 3*(4*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*sin(2*d*
x + 2*c) + 3*(3*cos(d*x + c) + 1)*sin(d*x + c) + 12*cos(2*d*x + 2*c)*sin(d*
x + c))*cos(4*d*x + 4*c) + 24*((4*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*si
n(2*d*x + 2*c) + (3*cos(d*x + c) + 1)*sin(d*x + c) + 4*cos(2*d*x + 2*c)*sin
(d*x + c))*cos(3*d*x + 3*c) + 2*(3*(sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) +
 3*sin(d*x + c))*sin(4*d*x + 4*c) + (16*sin(2*d*x + 2*c) + 15*sin(d*x + c))
*sin(3*d*x + 3*c) + 4*sin(3*d*x + 3*c)^2 + 12*sin(2*d*x + 2*c)^2 + 21*sin(2
*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2)*sin(5*d*x + 5*c) + 6*((16*sin(
2*d*x + 2*c) + 15*sin(d*x + c))*sin(3*d*x + 3*c) + 4*sin(3*d*x + 3*c)^2 + 1
2*sin(2*d*x + 2*c)^2 + 21*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2)
*sin(4*d*x + 4*c) + (8*(4*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x
+ 3*c) + 16*cos(3*d*x + 3*c)^2 + 8*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) +
16*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 112*sin(2*d*x + 2*c)^2 + 192*sin
(2*d*x + 2*c)*sin(d*x + c) + 81*sin(d*x + c)^2 ...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(145) = 290$.
time = 0.40, size = 339, normalized size = 1.99

$$-\frac{\sqrt{2}\left((9A-5B)\cos(dx+c)^3+2(9A-5B)\cos(dx+c)^2+(9A-5B)\cos(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+2\left((3A-2B)\cos(dx+c)^3+2(3A-2B)\cos(dx+c)^2+(3A-2B)\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)-4\left((3A-B)\cos(dx+c)+2A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{8\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/8*(sqrt(2)*((9*A - 5*B)*cos(d*x + c)^3 + 2*(9*A - 5*B)*cos(d*x + c)^2 +
(9*A - 5*B)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a
*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x
+ c)^2 + 2*cos(d*x + c) + 1)) + 2*((3*A - 2*B)*cos(d*x + c)^3 + 2*(3*A - 2*
B)*cos(d*x + c)^2 + (3*A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3
 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((3*A - B)*co
s(d*x + c) + 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c
)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a*(cos(c + d*x) + 1))**(3/2)
, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos(c + d\,x)}{\cos(c + d\,x)^2\,(a + a\,\cos(c + d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2)), x)
```

3.114 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=221

$$\frac{(19A-12B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{3/2}d}-\frac{(13A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(7A-6\ldots}{4ad\sqrt{a\ldots}}$$

```
[Out] 1/4*(19*A-12*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/
d-1/4*(13*A-9*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1
/2))/a^(3/2)/d*2^(1/2)-1/2*(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(
3/2)-1/4*(7*A-6*B)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/2*(2*A-B)*sec(d*
x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3057, 3063, 3064, 2728, 212, 2852}

$$\frac{(19A-12B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a\cos(c+dx)+a}}+\frac{(2A-B)\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((19*A - 12*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4
*a^(3/2)*d) - ((13*A - 9*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((7*A - 6*B)*Tan[c + d*x])/(4*
a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*d*(a
 + a*Cos[c + d*x])^(3/2)) + ((2*A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sq
rt[a + a*Cos[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{(2a(2A-B)-\frac{5}{2}a(A-B)\cos(c+dx)}{\sqrt{a+a\cos(c+d}}}{2a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A-B)\sec(c+dx)\tan(c}{2ad\sqrt{a+a\cos(c+dx}}$$

$$= -\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} +$$

$$= \frac{(19A-12B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{3/2}d} - \frac{(13A-9B)\tan}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.59, size = 1402, normalized size = 6.34

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] ((13*A - 9*B)*Cos[c/2 + (d*x)/2]^3*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)
/4]])/(d*(a*(1 + Cos[c + d*x]))^(3/2)) + ((-13*A + 9*B)*Cos[c/2 + (d*x)/2]^
3*Log[Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4]])/(d*(a*(1 + Cos[c + d*x]))^(
3/2)) + ((19*A - 12*B)*Cos[c/2 + (d*x)/2]^3*Log[Sqrt[2] + 2*Sin[c/2 + (d*x)
/2]])/(2*Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(3/2)) + ((I/4)*ArcTan[(Cos[c/4 +
 (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])/(-Cos[c/4 + (d
*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*Cos[c/2 + (d*x)/
2]^3*(19*Sqrt[2]*A - 12*Sqrt[2]*B - 38*A*Sin[c/2] + 24*B*Sin[c/2]))/(d*(a*(
1 + Cos[c + d*x]))^(3/2)*(-1 + Sqrt[2]*Sin[c/2])) + ((I/4)*ArcTan[(Cos[c/4
+ (d*x)/4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])/(Cos[c/4 + (d
*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*Cos[c/2 + (d*x)/
2]^3*(19*Sqrt[2]*A - 12*Sqrt[2]*B - 38*A*Sin[c/2] + 24*B*Sin[c/2]))/(d*(a*(
```

```
1 + Cos[c + d*x]))^(3/2)*(-1 + Sqrt[2]*Sin[c/2])) + (Cos[c/2 + (d*x)/2]^3*L
og[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]]*(19*Sqrt[2]
*A - 12*Sqrt[2]*B - 38*A*Sin[c/2] + 24*B*Sin[c/2]))/(8*d*(a*(1 + Cos[c + d*
x]))^(3/2)*(-1 + Sqrt[2]*Sin[c/2])) + (Cos[c/2 + (d*x)/2]^3*Log[2 + Sqrt[2]
*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]]*(19*Sqrt[2]*A - 12*Sqrt[2
]*B - 38*A*Sin[c/2] + 24*B*Sin[c/2]))/(8*d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1
 + Sqrt[2]*Sin[c/2])) + ((-A + B)*Cos[c/2 + (d*x)/2]^3)/(2*d*(a*(1 + Cos[c
+ d*x]))^(3/2)*(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])^2) + ((A - B)*Cos[
c/2 + (d*x)/2]^3)/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/4 + (d*x)/4] + S
in[c/4 + (d*x)/4])^2) + (A*Cos[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(d*(a*(1 + Co
s[c + d*x]))^(3/2)*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2])^2) + (Cos[c/2 + (d*x)/2]^3*(-5*A*Cos[c/2] + 4*B*Cos[c/2] + 7*A*Sin[
c/2] - 4*B*Sin[c/2]))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/2] - Sin[c/2
])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (A*Cos[c/2 + (d*x)/2]^3*Sin
[(d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/2] + Sin[c/2])*(Cos[c/2 +
 (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c/2 + (d*x)/2]^3*(5*A*Cos[c/2] -
4*B*Cos[c/2] + 7*A*Sin[c/2] - 4*B*Sin[c/2]))/(2*d*(a*(1 + Cos[c + d*x]))^(3
/2)*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1539 vs. $2(190) = 380$.
time = 0.53, size = 1540, normalized size = 6.97

method	result	size
default	Expression too large to display	1540

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(104*A*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2
*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-72*B
*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^
(1/2)*cos(1/2*d*x+1/2*c)^6*a-76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^
(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a
))*cos(1/2*d*x+1/2*c)^6*a-76*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1
/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))
*cos(1/2*d*x+1/2*c)^6*a+48*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)
*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*co
s(1/2*d*x+1/2*c)^6*a+48*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*c
os(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*cos(
1/2*d*x+1/2*c)^6*a-104*A*2^(1/2)*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(
1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+72*B*2^(1/2)*ln(2*(2*a
^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+
1/2*c)^4*a+28*A*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*cos(1/2*d*x+
```

```
1/2*c)^4+76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/
2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*cos(1/2*d*x+1/2*c
)^4*a+76*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*
c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^
4*a-24*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^
4-48*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^
(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a-4
8*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1
/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^4*a+26*
A*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2
^(1/2)*cos(1/2*d*x+1/2*c)^2*a-18*B*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)
^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-22*A*a^(1/2)
*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2-19*A*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(s
in(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a-19*A*ln(-4/(2*cos
(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin
(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^2*a+16*B*a^(1/2)*2^(1/2
)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2+12*B*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*
d*x+1/2*c)^2*a)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a+12*B*ln(-4/(2*cos(1/2*d*
x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*
x+1/2*c)^2*a)^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^2*a+2*A*a^(1/2)*2^(1/2)*(sin(1
/2*d*x+1/2*c)^2*a)^(1/2)-2*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)
)/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 0.42, size = 361, normalized size = 1.63

$$-\frac{2\,\sqrt{2}\left(\left(13\,A-9\,B\right)\cos\left(dx+c\right)^{4}+2\left(13\,A-9\,B\right)\cos\left(dx+c\right)^{3}+\left(13\,A-9\,B\right)\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)+\left(\left(19\,A-12\,B\right)\cos\left(dx+c\right)^{4}+2\left(19\,A-12\,B\right)\cos\left(dx+c\right)^{3}+\left(19\,A-12\,B\right)\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}+4\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\left(\left(7\,A-6\,B\right)\cos\left(dx+c\right)^{2}+\left(3\,A-4\,B\right)\cos\left(dx+c\right)-2\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{16\left(a^{2}d\cos\left(dx+c\right)^{4}+2\,a^{2}d\cos\left(dx+c\right)^{3}+a^{2}d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/16*(2*sqrt(2)*((13*A - 9*B)*cos(d*x + c)^4 + 2*(13*A - 9*B)*cos(d*x + c)
^3 + (13*A - 9*B)*cos(d*x + c)^2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((19*A - 12*B)*cos(d*x + c)^4 + 2*(
19*A - 12*B)*cos(d*x + c)^3 + (19*A - 12*B)*cos(d*x + c)^2)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(c
os(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4
*((7*A - 6*B)*cos(d*x + c)^2 + (3*A - 4*B)*cos(d*x + c) - 2*A)*sqrt(a*cos(d
*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 +
 a^2*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a*(cos(c + d*x) + 1))**(3/2)
, x)
```

Giac [A]
time = 0.61, size = 366, normalized size = 1.66

$$-\frac{\frac{\sqrt{2}\left(13A\sqrt{a}-9B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\left(13A\sqrt{a}-9B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\left(19A\sqrt{a}-12B\sqrt{a}\right)\log\left(\left|\frac{1}{2}\sqrt{2}+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{\left(19A\sqrt{a}-12B\sqrt{a}\right)\log\left(\left|-\frac{1}{2}\sqrt{2}+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\left(A\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\left(10\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-8\sqrt{2}B\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4\sqrt{2}B\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/8*(sqrt(2)*(13*A*sqrt(a) - 9*B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(a
^2*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*(13*A*sqrt(a) - 9*B*sqrt(a))*log(-s
in(1/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(1/2*d*x + 1/2*c))) - (19*A*sqrt(a) -
12*B*sqrt(a))*log(abs(1/2*sqrt(2) + sin(1/2*d*x + 1/2*c)))/(a^2*sgn(cos(1/2
*d*x + 1/2*c))) + (19*A*sqrt(a) - 12*B*sqrt(a))*log(abs(-1/2*sqrt(2) + sin(
1/2*d*x + 1/2*c)))/(a^2*sgn(cos(1/2*d*x + 1/2*c))) - 2*sqrt(2)*(A*sqrt(a)*s
in(1/2*d*x + 1/2*c) - B*sqrt(a)*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/2*c
)^2 - 1)*a^2*sgn(cos(1/2*d*x + 1/2*c))) - 2*(10*sqrt(2)*A*sqrt(a)*sin(1/2*d
*x + 1/2*c)^3 - 8*sqrt(2)*B*sqrt(a)*sin(1/2*d*x + 1/2*c)^3 - 3*sqrt(2)*A*sq
rt(a)*sin(1/2*d*x + 1/2*c) + 4*sqrt(2)*B*sqrt(a)*sin(1/2*d*x + 1/2*c))/((2*
sin(1/2*d*x + 1/2*c)^2 - 1)^2*a^2*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,\cos(c + d\,x)}{\cos(c + d\,x)^3\,\left(a + a\,\cos(c + d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2)),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2)), x)
```

3.115 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=261

$$\frac{(163A-283B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{(13A-21B)\cos^3(c}{16ad(a+a\cos}$$

```
[Out] 1/4*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/16*(13*A-21*B)
*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+1/32*(163*A-283*B)*arct
anh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2
)-1/120*(985*A-1729*B)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)-1/80*(85*A-1
57*B)*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)+1/240*(475*A-787
*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a^3/d
```

Rubi [A]
time = 0.54, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3056, 3062, 3047, 3102, 2830, 2728, 212}

$$\frac{(163A-283B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(475A-787B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{240a^3d}-\frac{(85A-157B)\sin(c+dx)\cos^2(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(985A-1729B)\sin(c+dx)}{120a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{(13A-21B)\sin(c+dx)\cos^3(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((163*A - 283*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) + ((13*A - 21*B)*Cos[c + d*x]^3*Sin[c + d*x]
)/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) - ((985*A - 1729*B)*Sin[c + d*x])/(12
0*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((85*A - 157*B)*Cos[c + d*x]^2*Sin[c +
d*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]]) + ((475*A - 787*B)*Sqrt[a + a*Cos
[c + d*x]]*Sin[c + d*x])/(240*a^3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^3(c+dx)\left(4a(A-B)-\frac{1}{2}a(5A-13B)\right.}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(}{16ad(a+a\cos(c+dx))^3}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(}{16ad(a+a\cos(c+dx))^3}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(}{16ad(a+a\cos(c+dx))^3}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(}{16ad(a+a\cos(c+dx))^3}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(}{16ad(a+a\cos(c+dx))^3}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(}{16ad(a+a\cos(c+dx))^3}$$

$$= \frac{(163A-283B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{(A-B)\cos}{4d(a+}$$

Mathematica [A]
time = 1.53, size = 139, normalized size = 0.53

$$\frac{30(163A-283B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+(-1895A+3491B-5(479A-887B)\cos(c+dx)+(-400A+832B)\cos(2(c+dx))+40A\cos(3(c+dx))-40B\cos(3(c+dx))+12B\cos(4(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{240ad(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (30*(163*A - 283*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-1895*A
 + 3491*B - 5*(479*A - 887*B)*Cos[c + d*x] + (-400*A + 832*B)*Cos[2*(c + d*
x)] + 40*A*Cos[3*(c + d*x)] - 40*B*Cos[3*(c + d*x)] + 12*B*Cos[4*(c + d*x)]
)*Tan[(c + d*x)/2])/(240*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(230) = 460$.
time = 0.35, size = 467, normalized size = 1.79

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(768B\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+640A\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/480*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(768*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a
)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^8+640*A*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)
^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-2176*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)
^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+2445*A*2^(1/2)*ln(2*(2*a^(1/2)*(sin(1/2
*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-4245
*B*2^(1/2)*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+
1/2*c))*cos(1/2*d*x+1/2*c)^4*a-2560*A*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2
)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+5248*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2
)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-435*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*
a)^(1/2)*cos(1/2*d*x+1/2*c)^2+555*B*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a
)^(1/2)*cos(1/2*d*x+1/2*c)^2+30*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^
(1/2)-30*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*
c)^3/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.36, size = 270, normalized size = 1.03

$$-\frac{15\sqrt{2}\left((163A-283B)\cos(dx+c)^3+3(163A-283B)\cos(dx+c)^2+3(163A-283B)\cos(dx+c)+163A-283B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(96B\cos(dx+c)^4+160(A-B)\cos(dx+c)^3-32(25A-49B)\cos(dx+c)^2-5(503A-911B)\cos(dx+c)-1495A+2671B\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{960\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/960*(15*sqrt(2)*((163*A - 283*B)*cos(d*x + c)^3 + 3*(163*A - 283*B)*cos(
d*x + c)^2 + 3*(163*A - 283*B)*cos(d*x + c) + 163*A - 283*B)*sqrt(a)*log(-(
a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c)
- 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(96*B*
cos(d*x + c)^4 + 160*(A - B)*cos(d*x + c)^3 - 32*(25*A - 49*B)*cos(d*x + c)
^2 - 5*(503*A - 911*B)*cos(d*x + c) - 1495*A + 2671*B)*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d
*cos(d*x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 8.39, size = 309, normalized size = 1.18

$$\frac{\frac{15\sqrt{2}\left(163\,A\sqrt{a}-283\,B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{15\sqrt{2}\left(163\,A\sqrt{a}-283\,B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{30\left(29\sqrt{2}\,A\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-37\sqrt{2}\,B\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-27\sqrt{2}\,A\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+35\sqrt{2}\,B\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{256\sqrt{2}\left(6\,Ba^{\frac{25}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-5\,Aa^{\frac{25}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+5\,Ba^{\frac{25}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,Aa^{\frac{25}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+30\,Ba^{\frac{25}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{15}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/960*(15*sqrt(2)*(163*A*sqrt(a) - 283*B*sqrt(a))*log(sin(1/2*d*x + 1/2*c)
+ 1)/(a^3*sgn(cos(1/2*d*x + 1/2*c))) - 15*sqrt(2)*(163*A*sqrt(a) - 283*B*sq
rt(a))*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^3*sgn(cos(1/2*d*x + 1/2*c))) + 30*
(29*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c)^3 - 37*sqrt(2)*B*sqrt(a)*sin(1/2
*d*x + 1/2*c)^3 - 27*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + 35*sqrt(2)*B*
sqrt(a)*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/2*c)^2 - 1)^2*a^3*sgn(cos(1
/2*d*x + 1/2*c))) + 256*sqrt(2)*(6*B*a^(25/2)*sin(1/2*d*x + 1/2*c)^5 - 5*A*
a^(25/2)*sin(1/2*d*x + 1/2*c)^3 + 5*B*a^(25/2)*sin(1/2*d*x + 1/2*c)^3 - 15*
A*a^(25/2)*sin(1/2*d*x + 1/2*c) + 30*B*a^(25/2)*sin(1/2*d*x + 1/2*c))/(a^15
*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^4\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x)
```

3.116 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{(75A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{2}\ \sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\ a^{5/2}d}+\frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{(9A-17B)\cos^2(c}{16ad(a+a\,\mathrm{co}}$$

```
[Out] 1/4*(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/16*(9*A-17*B)*
cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)-1/32*(75*A-163*B)*arctan
h(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+
1/24*(93*A-197*B)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)-1/48*(39*A-95*B)*
sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a^3/d
```

Rubi [A]
time = 0.41, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3056, 3047, 3102, 2830, 2728, 212}

$$-\frac{(75A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{2}\ \sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\ a^{5/2}d}-\frac{(39A-95B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{48a^3d}+\frac{(93A-197B)\sin(c+dx)}{24a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{(9A-17B)\sin(c+dx)\cos^2(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] -1/16*((75*A - 163*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Co
s[c + d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/
(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((9*A - 17*B)*Cos[c + d*x]^2*Sin[c + d*x
])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((93*A - 197*B)*Sin[c + d*x])/(24*
a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((39*A - 95*B)*Sqrt[a + a*Cos[c + d*x]]*S
in[c + d*x])/(48*a^3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^2(c+dx)\left(3a(A-B)-\frac{1}{2}a(3A-11B)\right.}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c}{16ad(a+a\cos(c+dx))^{3/}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c}{16ad(a+a\cos(c+dx))^{3/}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c}{16ad(a+a\cos(c+dx))^{3/}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c}{16ad(a+a\cos(c+dx))^{3/}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c}{16ad(a+a\cos(c+dx))^{3/}}$$

$$= -\frac{(75A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\,\mathrm{co}}{4d(a-}$$

Mathematica [A]
time = 1.05, size = 117, normalized size = 0.54

$$\frac{-6(75A-163B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+(195A-379B+(255A-479B)\cos(c+dx)+16(3A-5B)\cos(2(c+dx))+8B\cos(3(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{48ad(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (-6*(75*A - 163*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (195*A -
379*B + (255*A - 479*B)*Cos[c + d*x] + 16*(3*A - 5*B)*Cos[2*(c + d*x)] + 8*
B*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(48*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(189) = 378.
time = 0.41, size = 397, normalized size = 1.84

method	result

default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(128B\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-225A\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\right.\right.}}{\cos\left(\frac{dx}{2}+\right.}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/96/cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(128*B*2^(1/2)*(si
n(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-225*A*2^(1/2)*ln(2
*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2
*d*x+1/2*c)^4*a+489*B*2^(1/2)*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2
)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+192*A*2^(1/2)*(sin(1/2*d*
x+1/2*c)^2*a)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-512*B*2^(1/2)*(sin(1/2*d*x
+1/2*c)^2*a)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+63*A*a^(1/2)*2^(1/2)*(sin(1
/2*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2-87*B*a^(1/2)*2^(1/2)*(sin(1/2
*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2-6*A*a^(1/2)*2^(1/2)*(sin(1/2*d*
x+1/2*c)^2*a)^(1/2)+6*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/a^(
7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 0.43, size = 254, normalized size = 1.18

$$-\frac{3\sqrt{2}\left((75A-163B)\cos(dx+c)^3+3(75A-163B)\cos(dx+c)^2+3(75A-163B)\cos(dx+c)+75A-163B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(32B\cos(dx+c)^3+32(3A-5B)\cos(dx+c)^2+(255A-503B)\cos(dx+c)+147A-299B\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{192\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/192*(3*sqrt(2)*((75*A - 163*B)*cos(d*x + c)^3 + 3*(75*A - 163*B)*cos(d*x
 + c)^2 + 3*(75*A - 163*B)*cos(d*x + c) + 75*A - 163*B)*sqrt(a)*log(-(a*cos
```

```
(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a
*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*B*cos(d
*x + c)^3 + 32*(3*A - 5*B)*cos(d*x + c)^2 + (255*A - 503*B)*cos(d*x + c) +
147*A - 299*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [A]
time = 4.82, size = 263, normalized size = 1.22

$$-\frac{\frac{3\sqrt{2}\left(75A\sqrt{a}-163B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{3\sqrt{2}\left(75A\sqrt{a}-163B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{6\left(21\sqrt{2}A\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-29\sqrt{2}B\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-19\sqrt{2}A\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+27\sqrt{2}B\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{128\sqrt{2}\left(2Ba^{\frac{13}{2}}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3Aa^{\frac{13}{2}}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6Ba^{\frac{13}{2}}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^9\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/192*(3*sqrt(2)*(75*A*sqrt(a) - 163*B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) +
 1)/(a^3*sgn(cos(1/2*d*x + 1/2*c))) - 3*sqrt(2)*(75*A*sqrt(a) - 163*B*sqrt(
a))*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^3*sgn(cos(1/2*d*x + 1/2*c))) + 6*(21*
sqrt(2)*A*sin(1/2*d*x + 1/2*c)^3 - 29*sqrt(2)*B*sin(1/2*d*x + 1/2*c)^3 - 19
*sqrt(2)*A*sin(1/2*d*x + 1/2*c) + 27*sqrt(2)*B*sin(1/2*d*x + 1/2*c))/((sin(
1/2*d*x + 1/2*c)^2 - 1)^2*a^(5/2)*sgn(cos(1/2*d*x + 1/2*c))) + 128*sqrt(2)*
(2*B*a^(13/2)*sin(1/2*d*x + 1/2*c)^3 - 3*A*a^(13/2)*sin(1/2*d*x + 1/2*c) +
6*B*a^(13/2)*sin(1/2*d*x + 1/2*c))/(a^9*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^3\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x)
```

3.117 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=169

$$\frac{(19A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}-\frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

```
[Out] 1/4*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(5*A-13*B)*
sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+1/32*(19*A-75*B)*arctanh(1/2*sin(d*x+
c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*(A-9*B)*si
n(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3056, 3047, 3098, 2830, 2728, 212}

$$\frac{(19A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-9B)\sin(c+dx)}{4a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{(5A-13B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((19*A - 75*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(4*d
*(a + a*Cos[c + d*x])^(5/2)) - ((5*A - 13*B)*Sin[c + d*x])/(16*a*d*(a + a*C
os[c + d*x])^(3/2)) - ((A - 9*B)*Sin[c + d*x])/(4*a^2*d*Sqrt[a + a*Cos[c +
d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos(c+dx)(2a(A-B)-\frac{1}{2}a(A-9B)\text{co}}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{2a(A-B)\cos(c+dx)-\frac{1}{2}a(A-9B)\text{co}}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} -$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} -$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} -$$

$$= \frac{(19A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{(A-B)\cos}{4d(a+}$$

Mathematica [A]
time = 0.72, size = 100, normalized size = 0.59

$$\frac{2(19A-75B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+(-9A+65B+(-13A+85B)\cos(c+dx)+16B\cos(2(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{16ad(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (2*(19*A - 75*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-9*A + 65*
B + (-13*A + 85*B)*Cos[c + d*x] + 16*B*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/
(16*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(146) = 292$.
time = 0.31, size = 327, normalized size = 1.93

method	result

default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(19A\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-75B\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\,(}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/32*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(19*A*2^(1/2)*ln(2*(2*a^(1/2)*(sin(1/2*
d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-75*B*
2^(1/2)*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2
*c))*cos(1/2*d*x+1/2*c)^4*a+64*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(
1/2)*cos(1/2*d*x+1/2*c)^4-13*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/
2)*cos(1/2*d*x+1/2*c)^2+21*B*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)
*cos(1/2*d*x+1/2*c)^2+2*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*
B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)^3/a^(7
/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.37, size = 237, normalized size = 1.40

$$-\frac{\sqrt{2}\left((19A-75B)\cos(dx+c)^3+3(19A-75B)\cos(dx+c)^2+3(19A-75B)\cos(dx+c)+19A-75B\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(32B\cos(dx+c)^2-(13A-85B)\cos(dx+c)-9A+49B\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/64*(sqrt(2)*((19*A - 75*B)*cos(d*x + c)^3 + 3*(19*A - 75*B)*cos(d*x + c)
^2 + 3*(19*A - 75*B)*cos(d*x + c) + 19*A - 75*B)*sqrt(a)*log(-(a*cos(d*x +
c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*
x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*B*cos(d*x + c)
```

```
^2 - (13*A - 85*B)*cos(d*x + c) - 9*A + 49*B)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
+ c) + a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [A]
time = 2.69, size = 224, normalized size = 1.33

$$\frac{\frac{128\sqrt{2}B\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{\sqrt{2}\left(19A\sqrt{a}-75B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\left(19A\sqrt{a}-75B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{2\left(13\sqrt{2}A\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-21\sqrt{2}B\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-11\sqrt{2}A\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+19\sqrt{2}B\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/64*(128*sqrt(2)*B*sin(1/2*d*x + 1/2*c)/(a^(5/2)*sgn(cos(1/2*d*x + 1/2*c))
) + sqrt(2)*(19*A*sqrt(a) - 75*B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(a^
3*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*(19*A*sqrt(a) - 75*B*sqrt(a))*log(-s
in(1/2*d*x + 1/2*c) + 1)/(a^3*sgn(cos(1/2*d*x + 1/2*c))) + 2*(13*sqrt(2)*A*
sin(1/2*d*x + 1/2*c)^3 - 21*sqrt(2)*B*sin(1/2*d*x + 1/2*c)^3 - 11*sqrt(2)*A
*sin(1/2*d*x + 1/2*c) + 19*sqrt(2)*B*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x +
1/2*c)^2 - 1)^2*a^(5/2)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x)
```

3.118 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(5A+19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

```
[Out] -1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/16*(5*A-13*B)*sin(d*x+c)/a
/d/(a+a*cos(d*x+c))^(3/2)+1/32*(5*A+19*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^
(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3047, 3098, 2829, 2728, 212}

$$\frac{(5A+19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(5A-13B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((5*A + 19*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c +
d*x])^(5/2)) + ((5*A - 13*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{-\frac{5}{2}a(A-B)-4aB\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(5A+}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(5A+}{}$$

$$= \frac{(5A+19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sin}{4d(a+a\cos(c}$$

Mathematica [A]
time = 0.59, size = 87, normalized size = 0.69

$$\frac{2(5A+19B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+(A-9B+(5A-13B)\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)}{16ad(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(5*A + 19*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (A - 9*B + (
5*A - 13*B)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a*d*(a*(1 + Cos[c + d*x]))^
(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(107) = 214$.
time = 0.32, size = 292, normalized size = 2.32

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(5A\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+19B\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\right.}}{\mathrm{c}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(5*A*2^(1/2)*ln(2*
(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*
d*x+1/2*c)^4*a+19*B*2^(1/2)*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+
2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+5*A*a^(1/2)*2^(1/2)*(sin(1/
2*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2-13*B*a^(1/2)*2^(1/2)*(sin(1/2*
d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2-2*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x
+1/2*c)^2*a)^(1/2)+2*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/a^(7
/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(107) = 214$.
time = 0.35, size = 223, normalized size = 1.77

$$\frac{\sqrt{2}\left((5A+19B)\cos(dx+c)^3+3(5A+19B)\cos(dx+c)^2+3(5A+19B)\cos(dx+c)+5A+19B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left((5A-13B)\cos(dx+c)+A-9B\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/64*(sqrt(2)*((5*A + 19*B)*cos(d*x + c)^3 + 3*(5*A + 19*B)*cos(d*x + c)^2
+ 3*(5*A + 19*B)*cos(d*x + c) + 5*A + 19*B)*sqrt(a)*log(-(a*cos(d*x + c)^2
- 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c
) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((5*A - 13*B)*cos(d*x +
 c) + A - 9*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.31, size = 194, normalized size = 1.54

$$\frac{\frac{\sqrt{2}\left(5\,A\sqrt{a}+19\,B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}\left(5\,A\sqrt{a}+19\,B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{2\left(5\,\sqrt{2}\,A\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-13\,\sqrt{2}\,B\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,\sqrt{2}\,A\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\,\sqrt{2}\,B\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] 1/64*(sqrt(2)*(5*A*sqrt(a) + 19*B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(a
^3*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*(5*A*sqrt(a) + 19*B*sqrt(a))*log(-s
in(1/2*d*x + 1/2*c) + 1)/(a^3*sgn(cos(1/2*d*x + 1/2*c))) - 2*(5*sqrt(2)*A*s
in(1/2*d*x + 1/2*c)^3 - 13*sqrt(2)*B*sin(1/2*d*x + 1/2*c)^3 - 3*sqrt(2)*A*s
in(1/2*d*x + 1/2*c) + 11*sqrt(2)*B*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/
2*c)^2 - 1)^2*a^(5/2)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x)
```

3.119 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{(3A+5B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

```
[Out] 1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/16*(3*A+5*B)*sin(d*x+c)/a/d
/(a+a*cos(d*x+c))^(3/2)+1/32*(3*A+5*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/
2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2829, 2729, 2728, 212}

$$\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(3A+5B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((3*A + 5*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d
*x])^(5/2)) + ((3*A + 5*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B)\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(3A+5B)\int \frac{}{\sqrt{a-}}}{32}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(3A+5B)\text{Subst}\Big(}{}$$

$$= \frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}$$

Mathematica [A]
time = 0.49, size = 80, normalized size = 0.63

$$\frac{4(3A+5B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)+(7A+B+(3A+5B)\cos(c+dx))\sin(c+dx)}{16d(a(1+\cos(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
[Out] (4*(3*A + 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (7*A + B + (3
*A + 5*B)*Cos[c + d*x])*Sin[c + d*x])/(16*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(107) = 214$.
time = 0.30, size = 292, normalized size = 2.32

method	result

default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(3A\sqrt{2}\ \ln\left(\frac{4\sqrt{a}\ \sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+5B\sqrt{2}\ \ln\left(\frac{4\sqrt{a}\ \sqrt{a\left(\mathrm{s}\right.}}{\mathrm{co}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(3*A*2^(1/2)*ln(2*
(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*
d*x+1/2*c)^4*a+5*B*2^(1/2)*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2
*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+3*A*a^(1/2)*2^(1/2)*(sin(1/2
*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2+5*B*a^(1/2)*2^(1/2)*(sin(1/2*d*
x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2+2*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1
/2*c)^2*a)^(1/2)-2*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/a^(7/2
)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(107) = 214$.
time = 0.36, size = 223, normalized size = 1.77

$$\frac{\sqrt{2}\left((3A+5B)\cos(dx+c)^3+3(3A+5B)\cos(dx+c)^2+3(3A+5B)\cos(dx+c)+3A+5B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left((3A+5B)\cos(dx+c)+7A+B\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*A + 5*B)*cos(d*x + c)^3 + 3*(3*A + 5*B)*cos(d*x + c)^2 +
3*(3*A + 5*B)*cos(d*x + c) + 3*A + 5*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*
sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) -
3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((3*A + 5*B)*cos(d*x + c) +
 7*A + B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*
a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(a*(cos(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 0.79, size = 194, normalized size = 1.54

$$\frac{\frac{\sqrt{2}\left(3\,A\sqrt{a}+5\,B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}\left(3\,A\sqrt{a}+5\,B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{2\left(3\,\sqrt{2}\,A\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+5\,\sqrt{2}\,B\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5\,\sqrt{2}\,A\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,\sqrt{2}\,B\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(2)*(3*A*sqrt(a) + 5*B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(a^
3*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*(3*A*sqrt(a) + 5*B*sqrt(a))*log(-sin
(1/2*d*x + 1/2*c) + 1)/(a^3*sgn(cos(1/2*d*x + 1/2*c))) - 2*(3*sqrt(2)*A*sin
(1/2*d*x + 1/2*c)^3 + 5*sqrt(2)*B*sin(1/2*d*x + 1/2*c)^3 - 5*sqrt(2)*A*sin(
1/2*d*x + 1/2*c) - 3*sqrt(2)*B*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/2*c)
^2 - 1)^2*a^(5/2)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos(c + d\,x)}{(a + a\,\cos(c + d\,x))^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(5/2), x)
```

3.120 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=164

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d}-\frac{(43A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sin(c+d}{4d(a+a\cos(c+dx)}$$

```
[Out] 2*A*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d-1/4*(A-B)*
sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(11*A-3*B)*sin(d*x+c)/a/d/(a+a*cos
(d*x+c))^(3/2)-1/32*(43*A-3*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*
cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3057, 3064, 2728, 212, 2852}

$$-\frac{(43A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-3B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d)
- ((43*A - 3*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c
+ d*x])^(5/2)) - ((11*A - 3*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(
3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(4aA-\frac{3}{2}a(A-B)\cos(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{(8a^2 \cdots}{\cdots}}{\cdots}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{A\int \sqrt{\cdots}}{\cdots}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(2A)\mathrm{S}\cdots}{\cdots}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-3B)\tanh^{-1}\left(\frac{\cdots}{\sqrt{2}\,\sqrt{\cdots}}\right)}{16\sqrt{2}\,a^{5\cdots}}$$

Mathematica [A]
time = 1.45, size = 126, normalized size = 0.77

$$\frac{-2(43A-3B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+64\sqrt{2}A\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+(-15A+7B+(-11A+3B)\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)}{16ad(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(43*A - 3*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + 64*Sqrt[2]*
A*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-15*A + 7*B + (-1
1*A + 3*B)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a*d*(a*(1 + Cos[c + d*x]))^(
3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(139) = 278.
time = 0.42, size = 445, normalized size = 2.71

method	result
default	$-\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(43A\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-3B\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(43*A*2^(
1/2)*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c)
)*cos(1/2*d*x+1/2*c)^4*a-3*B*2^(1/2)*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*
a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-32*A*ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1
/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a-32*A*ln(-4/(2*cos(1/2
*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2
*d*x+1/2*c)^2*a)^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^4*a+11*A*a^(1/2)*2^(1/2)*(s
in(1/2*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2-3*B*a^(1/2)*2^(1/2)*(sin(
1/2*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2+2*A*a^(1/2)*2^(1/2)*(sin(1/2
*d*x+1/2*c)^2*a)^(1/2)-2*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2))/
sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84333 vs. 2(139) = 278.
time = 23.66, size = 84333, normalized size = 514.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/32*(512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) + (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(5*d*x + 5*c)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 - 512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) + (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(5*d*x + 5*c)^2 + 2560*cos(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) + 1024*(20*cos(2*d*x + 2*c) + 10*cos(d*x + c) + 1)*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + 10240*cos(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) + 2560*sin(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) + 10240*sin(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x +

```
4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(
8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*si
n(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*
sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x
 + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin
(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4
*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(6/
5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(
2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*si
n(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x +
 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5
/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c
)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(4/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2560*(5*(2*sin(2*d
*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5
*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/
2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*
d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*s
in(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(2/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 512*(5*cos(4*d*x + 4*
c)^2*sin(5/2*d*x + 5/2*c) + 4*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*co
s(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + 20*cos(3*d*x + 3*c)^2*sin(5/2*d*x + 5
/2*c) + 5*sin(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) + 20*sin(3*d*x + 3*c)^2*s
in(5/2*d*x + 5/2*c) + 2*((10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2
*d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c)
+ 2*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 2*(5*(2*sin(
2*d*x + 2*c) + sin(d*x + c))*sin(5/2*d*x + 5/2*...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(139) = 278$.
time = 0.41, size = 339, normalized size = 2.07

$$-\frac{\sqrt{2}\,\left((43\,A-3\,B)\cos\left(dx+c\right)^{3}+3\,(43\,A-3\,B)\cos\left(dx+c\right)^{2}+3\,(43\,A-3\,B)\cos\left(dx+c\right)+43\,A-3\,B\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)-32\,\left(A\cos\left(dx+c\right)^{3}+3\,A\cos\left(dx+c\right)^{2}+3\,A\cos\left(dx+c\right)+A\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\,\left((11\,A-3\,B)\cos\left(dx+c\right)+15\,A-7\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{64\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/64*(sqrt(2)*((43*A - 3*B)*cos(d*x + c)^3 + 3*(43*A - 3*B)*cos(d*x + c)^2
 + 3*(43*A - 3*B)*cos(d*x + c) + 43*A - 3*B)*sqrt(a)*log(-(a*cos(d*x + c)^2
 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x +
c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 32*(A*cos(d*x + c)^3 + 3
*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*s
in(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((11*A - 3*B)*cos
```

```
(d*x + c) + 15*A - 7*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.27, size = 264, normalized size = 1.61

$$-\frac{\frac{64\,A\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{\sqrt{2}\left(43\,A\sqrt{a}-3\,B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}\left(43\,A\sqrt{a}-3\,B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{2\left(11\,\sqrt{2}\,A\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,\sqrt{2}\,B\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-13\,\sqrt{2}\,A\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,\sqrt{2}\,B\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{3}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] -1/64*(64*A*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*
sin(1/2*d*x + 1/2*c)))/(a^(5/2)*sgn(cos(1/2*d*x + 1/2*c))) + sqrt(2)*(43*A*
sqrt(a) - 3*B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/(a^3*sgn(cos(1/2*d*x +
 1/2*c))) - sqrt(2)*(43*A*sqrt(a) - 3*B*sqrt(a))*log(-sin(1/2*d*x + 1/2*c)
+ 1)/(a^3*sgn(cos(1/2*d*x + 1/2*c))) - 2*(11*sqrt(2)*A*sqrt(a)*sin(1/2*d*x
+ 1/2*c)^3 - 3*sqrt(2)*B*sqrt(a)*sin(1/2*d*x + 1/2*c)^3 - 13*sqrt(2)*A*sqrt
(a)*sin(1/2*d*x + 1/2*c) + 5*sqrt(2)*B*sqrt(a)*sin(1/2*d*x + 1/2*c))/((sin(
1/2*d*x + 1/2*c)^2 - 1)^2*a^3*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2)), x)
```

3.121 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=207

$$-\frac{(5A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d}+\frac{(115A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-}{4d(a+}$$

```
[Out] -(5*A-2*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d+1/3
2*(115*A-43*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2
))/a^(5/2)/d*2^(1/2)-1/4*(A-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(15
*A-7*B)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+1/16*(35*A-11*B)*tan(d*x+c)/a
^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3057, 3063, 3064, 2728, 212, 2852}

$$-\frac{(5A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(115A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(35A-11B)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(15A-7B)\tan(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] -(((5*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a
^(5/2)*d)) + ((115*A - 43*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Tan[c + d*x])/(4*d*
(a + a*Cos[c + d*x])^(5/2)) - ((15*A - 7*B)*Tan[c + d*x])/(16*a*d*(a + a*Co
s[c + d*x])^(3/2)) + ((35*A - 11*B)*Tan[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[
c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(a(5A-B)-\frac{5}{2}a(A-B)\cos(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \left(\tfrac{1}{2}a^2(\right.}{}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(35A}{16a^2d}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(35A}{16a^2d}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(35A}{16a^2d}$$

$$= -\frac{(5A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{(115A-43B)\tan}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.98, size = 632, normalized size = 3.05

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (Cos[(c + d*x)/2]*(8*(-115*A + 43*B)*Cos[(c + d*x)/2]^4*Log[Cos[(c + d*x)/4
] - Sin[(c + d*x)/4]] + 8*(115*A - 43*B)*Cos[(c + d*x)/2]^4*Log[Cos[(c + d*
x)/4] + Sin[(c + d*x)/4]] - 64*Sqrt[2]*(5*A - 2*B)*Cos[(c + d*x)/2]^4*Log[S
qrt[2] + 2*Sin[(c + d*x)/2]] - ((64*I)*(5*A - 2*B)*ArcTan[(Cos[(c + d*x)/4]
 - (-1 + Sqrt[2])*Sin[(c + d*x)/4])/((1 + Sqrt[2])*Cos[(c + d*x)/4] - Sin[(
c + d*x)/4])]*Cos[(c + d*x)/2]^4*(Sqrt[2] - 2*Sin[c/2]))/(-1 + Sqrt[2]*Sin[
c/2]) - ((64*I)*(5*A - 2*B)*ArcTan[(Cos[(c + d*x)/4] - (1 + Sqrt[2])*Sin[(c
 + d*x)/4])/((-1 + Sqrt[2])*Cos[(c + d*x)/4] - Sin[(c + d*x)/4])]*Cos[(c +
d*x)/2]^4*(Sqrt[2] - 2*Sin[c/2]))/(-1 + Sqrt[2]*Sin[c/2]) - (32*(5*A - 2*B)
*Cos[(c + d*x)/2]^4*Log[2 - Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x
)/2]]*(Sqrt[2] - 2*Sin[c/2]))/(-1 + Sqrt[2]*Sin[c/2]) - (32*(5*A - 2*B)*Cos
[(c + d*x)/2]^4*Log[2 + Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]
```

```
]*(Sqrt[2] - 2*Sin[c/2]))/(-1 + Sqrt[2]*Sin[c/2]) + 2*(67*A - 11*B + 10*(11
*A - 3*B)*Cos[c + d*x] + (35*A - 11*B)*Cos[2*(c + d*x)])*Sec[c + d*x]*Sin[(
c + d*x)/2]))/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1121 vs. $2(178) = 356$.
time = 0.53, size = 1122, normalized size = 5.42

method	result	size
default	Expression too large to display	1122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/16*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(230*A*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2
*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-86*B
*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^
(1/2)*cos(1/2*d*x+1/2*c)^6*a-160*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2
^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*
a))*cos(1/2*d*x+1/2*c)^6*a-160*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^
(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a
))*cos(1/2*d*x+1/2*c)^6*a+64*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*
cos(1/2*d*x+1/2*c)^6*a+64*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)
*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*co
s(1/2*d*x+1/2*c)^6*a-115*A*2^(1/2)*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)
^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+43*B*2^(1/2)*ln(2*(2
*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*
x+1/2*c)^4*a+70*A*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*cos(1/2*d*
x+1/2*c)^4+80*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+
1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*cos(1/2*d*x+1/2
*c)^4*a+80*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/
2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*cos(1/2*d*x+1/2*c
)^4*a-22*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c
)^4-32*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+
a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a
-32*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^
(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^4*a-1
5*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2+7*B
*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2-2*A*a^
(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*B*2^(1/2)*(sin(1/2*d*x+1/2*c
)^2*a)^(1/2)*a^(1/2))/a^(7/2)/cos(1/2*d*x+1/2*c)^3/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2
*c)^2)^(1/2)/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(178) = 356$.
time = 0.46, size = 404, normalized size = 1.95

$$-\frac{\sqrt{2}\left((115A-43B)\cos(dx+c)^4+3(115A-43B)\cos(dx+c)^3+3(115A-43B)\cos(dx+c)^2+(115A-43B)\cos(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+16\left((5A-2B)\cos(dx+c)^4+3(5A-2B)\cos(dx+c)^3+3(5A-2B)\cos(dx+c)^2+(5A-2B)\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)-4\left((35A-11B)\cos(dx+c)^2+5(11A-3B)\cos(dx+c)+16A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{64\left(a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+a^3d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/64*(sqrt(2)*((115*A - 43*B)*cos(d*x + c)^4 + 3*(115*A - 43*B)*cos(d*x +
c)^3 + 3*(115*A - 43*B)*cos(d*x + c)^2 + (115*A - 43*B)*cos(d*x + c))*sqrt(
a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(
d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) +
 16*((5*A - 2*B)*cos(d*x + c)^4 + 3*(5*A - 2*B)*cos(d*x + c)^3 + 3*(5*A - 2
*B)*cos(d*x + c)^2 + (5*A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^
3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((35*A - 11*
B)*cos(d*x + c)^2 + 5*(11*A - 3*B)*cos(d*x + c) + 16*A)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*
d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.88, size = 347, normalized size = 1.68

$$\frac{\frac{\sqrt{2}\left(115A\sqrt{a}-43B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\left(115A\sqrt{a}-43B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{32\left(5A\sqrt{a}-2B\sqrt{a}\right)\log\left(\left|\frac{1}{2}\sqrt{2}+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{32\left(5A\sqrt{a}-2B\sqrt{a}\right)\log\left(\left|-\frac{1}{2}\sqrt{2}+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{64\sqrt{2}A\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\left(19\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-11\sqrt{2}B\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-21\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13\sqrt{2}B\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/64*(sqrt(2)*(115*A*sqrt(a) - 43*B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1)/
(a^3*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*(115*A*sqrt(a) - 43*B*sqrt(a))*lo
g(-sin(1/2*d*x + 1/2*c) + 1)/(a^3*sgn(cos(1/2*d*x + 1/2*c))) - 32*(5*A*sqrt
(a) - 2*B*sqrt(a))*log(abs(1/2*sqrt(2) + sin(1/2*d*x + 1/2*c)))/(a^3*sgn(co
s(1/2*d*x + 1/2*c))) + 32*(5*A*sqrt(a) - 2*B*sqrt(a))*log(abs(-1/2*sqrt(2)
+ sin(1/2*d*x + 1/2*c)))/(a^3*sgn(cos(1/2*d*x + 1/2*c))) - 64*sqrt(2)*A*sin
(1/2*d*x + 1/2*c)/((2*sin(1/2*d*x + 1/2*c)^2 - 1)*a^(5/2)*sgn(cos(1/2*d*x +
 1/2*c))) - 2*(19*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c)^3 - 11*sqrt(2)*B*s
qrt(a)*sin(1/2*d*x + 1/2*c)^3 - 21*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) +
 13*sqrt(2)*B*sqrt(a)*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/2*c)^2 - 1)^2
*a^3*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^2\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2)), x)
```

3.122 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=264

$$\frac{(39A-20B)\tanh^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{5/2}d}-\frac{(219A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{2}\ \sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\ a^{5/2}d}-\frac{7(9A}{16a^2d}$$

```
[Out] 1/4*(39*A-20*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/
d-1/32*(219*A-115*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c)
)^(1/2))/a^(5/2)/d*2^(1/2)-1/4*(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c
))^(5/2)-1/16*(19*A-11*B)*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)-
7/16*(9*A-5*B)*tan(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)+1/16*(31*A-15*B)*sec
(d*x+c)*tan(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.61, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3057, 3063, 3064, 2728, 212, 2852}

$$\frac{(39A-20B)\tanh^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{2}\ \sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\ a^{5/2}d}-\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(31A-15B)\tan(c+dx)\sec(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(19A-11B)\tan(c+dx)\sec(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((39*A - 20*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4
*a^(5/2)*d) - ((219*A - 115*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (7*(9*A - 5*B)*Tan[c + d*x
])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]*Tan[c + d*x]
)/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((19*A - 11*B)*Sec[c + d*x]*Tan[c + d*
x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((31*A - 15*B)*Sec[c + d*x]*Tan[c
 + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{(2a(3A-B)-\frac{7}{2}a(A-B)\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-11B)\sec(c+dx)\tan}{16ad(a+a\cos(c+dx))}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-11B)\sec(c+dx)\tan}{16ad(a+a\cos(c+dx))}$$

$$= -\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} -$$

$$= -\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} -$$

$$= -\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} -$$

$$= \frac{(39A-20B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{5/2}d} - \frac{(219A-115B)\,\mathrm{ta}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.26, size = 656, normalized size = 2.48

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (Cos[(c + d*x)/2]*(8*(219*A - 115*B)*Cos[(c + d*x)/2]^4*Log[Cos[(c + d*x)/4
] - Sin[(c + d*x)/4]] + 8*(-219*A + 115*B)*Cos[(c + d*x)/2]^4*Log[Cos[(c +
d*x)/4] + Sin[(c + d*x)/4]] + 16*Sqrt[2]*(39*A - 20*B)*Cos[(c + d*x)/2]^4*L
og[Sqrt[2] + 2*Sin[(c + d*x)/2]] + ((16*I)*(39*A - 20*B)*ArcTan[(Cos[(c + d
*x)/4] - (-1 + Sqrt[2])*Sin[(c + d*x)/4])/((1 + Sqrt[2])*Cos[(c + d*x)/4] -
 Sin[(c + d*x)/4])]*Cos[(c + d*x)/2]^4*(Sqrt[2] - 2*Sin[c/2]))/(-1 + Sqrt[2
]*Sin[c/2]) + ((16*I)*(39*A - 20*B)*ArcTan[(Cos[(c + d*x)/4] - (1 + Sqrt[2]
)*Sin[(c + d*x)/4])/((-1 + Sqrt[2])*Cos[(c + d*x)/4] - Sin[(c + d*x)/4])]*C
os[(c + d*x)/2]^4*(Sqrt[2] - 2*Sin[c/2]))/(-1 + Sqrt[2]*Sin[c/2]) + (8*(39*
```

```
A - 20*B)*Cos[(c + d*x)/2]^4*Log[2 - Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin
[(c + d*x)/2]]*(Sqrt[2] - 2*Sin[c/2]))/(-1 + Sqrt[2]*Sin[c/2]) + (8*(39*A -
 20*B)*Cos[(c + d*x)/2]^4*Log[2 + Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c
 + d*x)/2]]*(Sqrt[2] - 2*Sin[c/2]))/(-1 + Sqrt[2]*Sin[c/2]) - (158*A - 110*
B + (269*A - 169*B)*Cos[c + d*x] + 10*(19*A - 11*B)*Cos[2*(c + d*x)] + 63*A
*Cos[3*(c + d*x)] - 35*B*Cos[3*(c + d*x)])*Sec[c + d*x]^2*Sin[(c + d*x)/2])
)/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1609 vs. $2(229) = 458$.

time = 0.57, size = 1610, normalized size = 6.10

method	result	size
default	Expression too large to display	1610

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/8*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*(-624*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^
(1/2)+2*a))*cos(1/2*d*x+1/2*c)^6*a-624*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(
1/2)-2*a))*cos(1/2*d*x+1/2*c)^6*a+320*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/
2)+2*a))*cos(1/2*d*x+1/2*c)^6*a+320*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*
(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2
)-2*a))*cos(1/2*d*x+1/2*c)^6*a-876*A*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*
a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^8*a+460*B*ln(2
*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)
*cos(1/2*d*x+1/2*c)^8*a-252*A*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2
)*cos(1/2*d*x+1/2*c)^6+140*B*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)
*cos(1/2*d*x+1/2*c)^6-320*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)
*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*co
s(1/2*d*x+1/2*c)^8*a+624*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*c
os(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*cos(
1/2*d*x+1/2*c)^8*a-320*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a))*cos(1/
2*d*x+1/2*c)^8*a+624*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*cos(1/2
*d*x+1/2*c)^8*a-219*A*2^(1/2)*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2
)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+115*B*2^(1/2)*ln(2*(2*a^(
1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/
2*c)^4*a-2*A*a^(1/2)*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)-19*A*a^(1/2)*2^
(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2+11*B*a^(1/2)*2^(1
```

```
/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*cos(1/2*d*x+1/2*c)^2-100*B*2^(1/2)*(sin(
1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+156*A*ln(-4/(2*cos(1
/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1
/2*d*x+1/2*c)^2*a)^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^4*a+156*A*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2
*d*x+1/2*c)^2*a)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a-80*B*ln(-4/(2*cos(1/2*d
*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(sin(1/2*d
*x+1/2*c)^2*a)^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^4*a-80*B*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(sin(1/2*d*x+
1/2*c)^2*a)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a+2*B*2^(1/2)*(sin(1/2*d*x+1/2
*c)^2*a)^(1/2)*a^(1/2)+876*A*ln(2*(2*a^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)
+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-460*B*ln(2*(2*a^(1
/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2
*d*x+1/2*c)^6*a+188*A*2^(1/2)*(sin(1/2*d*x+1/2*c)^2*a)^(1/2)*a^(1/2)*cos(1/
2*d*x+1/2*c)^4)/a^(7/2)/cos(1/2*d*x+1/2*c)^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))
^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c
)^2)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 0.52, size = 428, normalized size = 1.62

$$-\frac{\sqrt{2}\left((219A-115B)\cos(dx+c)^5+3(219A-115B)\cos(dx+c)^4+3(219A-115B)\cos(dx+c)^3+(219A-115B)\cos(dx+c)^2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left((39A-20B)\cos(dx+c)^5+3(39A-20B)\cos(dx+c)^4+3(39A-20B)\cos(dx+c)^3+(39A-20B)\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(7(9A-5B)\cos(dx+c)^3+5(19A-11B)\cos(dx+c)^2+4(5A-4B)\cos(dx+c)-8A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{64\left(a^3d\cos(dx+c)^5+3a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+a^3d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/64*(sqrt(2)*((219*A - 115*B)*cos(d*x + c)^5 + 3*(219*A - 115*B)*cos(d*x
+ c)^4 + 3*(219*A - 115*B)*cos(d*x + c)^3 + (219*A - 115*B)*cos(d*x + c)^2)
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1)) + 4*((39*A - 20*B)*cos(d*x + c)^5 + 3*(39*A - 20*B)*cos(d*x + c)^4 + 3
*(39*A - 20*B)*cos(d*x + c)^3 + (39*A - 20*B)*cos(d*x + c)^2)*sqrt(a)*log((
a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) +
```

```
4*(7*(9*A - 5*B)*cos(d*x + c)^3 + 5*(19*A - 11*B)*cos(d*x + c)^2 + 4*(5*A
- 4*B)*cos(d*x + c) - 8*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*co
s(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(
d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.86, size = 374, normalized size = 1.42

$$-\frac{\frac{\sqrt{2}\left(219\,A\sqrt{a}-115\,B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}\left(219\,A\sqrt{a}-115\,B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{8\left(39\,A\sqrt{a}-20\,B\sqrt{a}\right)\log\left(\left|\frac{1}{2}\sqrt{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{8\left(39\,A\sqrt{a}-20\,B\sqrt{a}\right)\log\left(\left|-\frac{1}{2}\sqrt{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{2\sqrt{2}\left(252\,A\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-140\,B\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-568\,A\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+320\,B\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+399\,A\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-231\,B\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-85\,A\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+53\,B\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/64*(sqrt(2)*(219*A*sqrt(a) - 115*B*sqrt(a))*log(sin(1/2*d*x + 1/2*c) + 1
)/(a^3*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*(219*A*sqrt(a) - 115*B*sqrt(a))
*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^3*sgn(cos(1/2*d*x + 1/2*c))) - 8*(39*A*s
qrt(a) - 20*B*sqrt(a))*log(abs(1/2*sqrt(2) + sin(1/2*d*x + 1/2*c)))/(a^3*sg
n(cos(1/2*d*x + 1/2*c))) + 8*(39*A*sqrt(a) - 20*B*sqrt(a))*log(abs(-1/2*sqr
t(2) + sin(1/2*d*x + 1/2*c)))/(a^3*sgn(cos(1/2*d*x + 1/2*c))) - 2*sqrt(2)*(
252*A*sqrt(a)*sin(1/2*d*x + 1/2*c)^7 - 140*B*sqrt(a)*sin(1/2*d*x + 1/2*c)^7
 - 568*A*sqrt(a)*sin(1/2*d*x + 1/2*c)^5 + 320*B*sqrt(a)*sin(1/2*d*x + 1/2*c
)^5 + 399*A*sqrt(a)*sin(1/2*d*x + 1/2*c)^3 - 231*B*sqrt(a)*sin(1/2*d*x + 1/
2*c)^3 - 85*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + 53*B*sqrt(a)*sin(1/2*d*x + 1/2
*c))/((2*sin(1/2*d*x + 1/2*c)^4 - 3*sin(1/2*d*x + 1/2*c)^2 + 1)^2*a^3*sgn(c
os(1/2*d*x + 1/2*c))))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(5/2)), x)
```

3.123 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=159

$$\frac{2a(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{10a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{10a(A+B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2a(9A$$

```
[Out] 2/15*a*(9*A+7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/d+10/21*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*a*(9*A+7*B)*c
os(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a*(A+B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*
a*B*cos(d*x+c)^(7/2)*sin(d*x+c)/d+10/21*a*(A+B)*sin(d*x+c)*cos(d*x+c)^(1/2)
/d
```

Rubi [A]
time = 0.14, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3047, 3102, 2827, 2715, 2719, 2720}

$$\frac{10a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(A+B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a(9A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{10a(A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a*(9*A + 7*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*a*(A + B)*Elliptic
F[(c + d*x)/2, 2])/(21*d) + (10*a*(A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(21*d) + (2*a*(9*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*a*(A
 + B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a*B*Cos[c + d*x]^(7/2)*Si
n[c + d*x])/(9*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^{\frac{5}{2}}(c+dx)\left(aA+(aA+aB)\cos(c+dx)\right.$$

$$= \frac{2aB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2}{9}\int \cos^{\frac{5}{2}}(c+$$

$$= \frac{2aB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + (a(A+B))\int$$

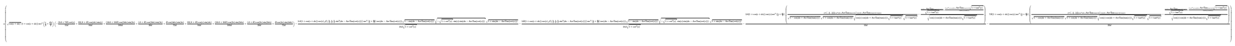
$$= \frac{2a(9A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{2a(A}{}$$

$$= \frac{2a(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10a(A+B)\sqrt{}}{}$$

$$= \frac{2a(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10a(A+B)F}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 6.33, size = 914, normalized size = 5.75



Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-1/15*((9*A
+ 7*B)*Cot[c])/d + (23*(A + B)*Cos[d*x]*Sin[c])/(84*d) + ((18*A + 19*B)*Cos
[2*d*x]*Sin[2*c])/(180*d) + ((A + B)*Cos[3*d*x]*Sin[3*c])/(28*d) + (B*Cos[4
*d*x]*Sin[4*c])/(72*d) + (23*(A + B)*Cos[c]*Sin[d*x])/(84*d) + ((18*A + 19*
B)*Cos[2*c]*Sin[2*d*x])/(180*d) + ((A + B)*Cos[3*c]*Sin[3*d*x])/(28*d) + (B
*Cos[4*c]*Sin[4*d*x])/(72*d)) - (5*A*(1 + Cos[c + d*x])*Csc[c]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*
Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[C
ot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (5*B*(1 + Cos[c + d*x])*Csc[c]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)
/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(
Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Ar
cTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*
Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Arc
Tan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTa
n[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan
[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (7*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 +
 (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(30*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(191) = 382$.
time = 0.31, size = 411, normalized size = 2.58

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(-1120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720A+2960B)\left(\sin^8\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A+2960*B)*sin(1/2*d*x+1/2*c)^8
*cos(1/2*d*x+1/2*c)+(-1584*A-3152*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c
)+(1344*A+1792*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-366*A-408*B)*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 193, normalized size = 1.21

$$\frac{-75i\sqrt{2}(A+B)a\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}(A+B)a\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21i\sqrt{2}(9A+7B)a\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}(9A+7B)a\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(35Ba\cos(dx+c)^3+45(A+B)a\cos(dx+c)^2+7(9A+7B)a\cos(dx+c)+75(A+B)a\right)\sqrt{\cos(dx+c)}\sin(dx+c)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/315*(-75*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 75*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x +
```

```
 c) - I*sin(d*x + c)) + 21*I*sqrt(2)*(9*A + 7*B)*a*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(9
*A + 7*B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) + 2*(35*B*a*cos(d*x + c)^3 + 45*(A + B)*a*cos(d*x + c)^2
 + 7*(9*A + 7*B)*a*cos(d*x + c) + 75*(A + B)*a)*sqrt(cos(d*x + c))*sin(d*x
+ c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [B]
time = 1.09, size = 177, normalized size = 1.11

$$-\frac{2\,A\,a\,\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,A\,a\,\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,a\,\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,a\,\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] - (2*A*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
+ d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a*cos(c + d*x)^(9/2)*sin(c +
 d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1
/2)) - (2*B*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, c
os(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a*cos(c + d*x)^(11/2)*s
in(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*
x)^2)^(1/2))
```

3.124 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=132

$$\frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(7A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(7A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2a(\ldots}{\ldots}$$

```
[Out] 6/5*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(7*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*(A+B)*cos(d*x+
c)^(3/2)*sin(d*x+c)/d+2/7*a*B*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/21*a*(7*A+5*B
)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3047, 3102, 2827, 2715, 2720, 2719}

$$\frac{2a(7A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a(7A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*a*(A + B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(7*A + 5*B)*EllipticF[
(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(21*d) + (2*a*(A + B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*B*Cos[
c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^{\frac{3}{2}}(c+dx)\left(aA+(aA+aB)\cos(c+dx)+\right.$$

$$= \frac{2aB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \cos^{\frac{3}{2}}(c+d$$

$$= \frac{2aB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + (a(A+B))\int$$

$$= \frac{2a(7A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2a(A}{}$$

$$= \frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{2a(7A+5B)F\left(\right.}{21d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.27, size = 872, normalized size = 6.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((-3*(A + B)*
Cot[c])/(5*d) + ((28*A + 23*B)*Cos[d*x]*Sin[c])/(84*d) + ((A + B)*Cos[2*d*x
]*Sin[2*c])/(10*d) + (B*Cos[3*d*x]*Sin[3*c])/(28*d) + ((28*A + 23*B)*Cos[c]
*Sin[d*x])/(84*d) + ((A + B)*Cos[2*c]*Sin[2*d*x])/(10*d) + (B*Cos[3*c]*Sin[
3*d*x])/(28*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*S
qrt[1 + Cot[c]^2]) - (5*B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
21*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2
]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqr
t[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])
/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]))/(10*d) - (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hyp
ergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan
[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^
2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/
(10*d))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(168) = 336$.
time = 0.33, size = 383, normalized size = 2.90

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-528B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-528*B)*sin(1/2*d*x+1/2*c)^6*co
```

```
s(1/2*d*x+1/2*c)+(308*A+448*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-11
2*A-122*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 179, normalized size = 1.36

$$\frac{-5i\sqrt{2}(7A+5B)a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(7A+5B)a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+63i\sqrt{2}(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-63i\sqrt{2}(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(15Ba\cos(dx+c)^2+21(A+B)a\cos(dx+c)+5(7A+5B)a\right)\sqrt{\cos(dx+c)}\sin(dx+c)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(7*A + 5*B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 5*I*sqrt(2)*(7*A + 5*B)*a*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*(A + B)*a*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*
(A + B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(15*B*a*cos(d*x + c)^2 + 21*(A + B)*a*cos(d*x + c) + 5
*(7*A + 5*B)*a)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 0.61, size = 166, normalized size = 1.26

$$\frac{2\,A\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}-\frac{2\,A\,a\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,B\,a\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,B\,a\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] (2*A*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) - (2*A*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos
(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a*cos(c + d*x)^(7/2)*sin(
c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)
^(1/2)) - (2*B*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4
, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.125 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2aB\cos}{}$$

```
[Out] 2/5*a*(5*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*B*cos(d*x+c)^(3
/2)*sin(d*x+c)/d+2/3*a*(A+B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3047, 3102, 2827, 2719, 2715, 2720}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a*(5*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF[
(c + d*x)/2, 2])/(3*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d
) + (2*a*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \sqrt{\cos(c+dx)}\,(aA+(aA+aB)\cos(c+$$

$$= \frac{2aB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\cos(}$$

$$= \frac{2aB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + (a(A+B))$$

$$= \frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{2a(A+B)}{}$$

$$= \frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{2a(A+B)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.24, size = 830, normalized size = 8.22

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-1/5*((5*A +
 3*B)*Cot[c])/d + ((A + B)*Cos[d*x]*Sin[c])/(3*d) + (B*Cos[2*d*x]*Sin[2*c])
/(10*d) + ((A + B)*Cos[c]*Sin[d*x])/(3*d) + (B*Cos[2*c]*Sin[2*d*x])/(10*d))
 - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^
2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[
c]^2]) - (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2
*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^
2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (3*
B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2,
 -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]
*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^
2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(141) = 282$.
time = 0.26, size = 355, normalized size = 3.51

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+44B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+44*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-10*A-16*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
```

```
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 161, normalized size = 1.59

$$\frac{-5i\sqrt{2}(A+B)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(A+B)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}(5A+3B)a\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}(5A+3B)a\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2(3Ba\cos(dx+c)+5(A+B)a)\sqrt{\cos(dx+c)}\sin(dx+c)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 3*I*sqrt(2)*(5*A + 3*B)*a*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(5*A +
3*B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) + 2*(3*B*a*cos(d*x + c) + 5*(A + B)*a)*sqrt(cos(d*x + c))*sin
(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 0.52, size = 128, normalized size = 1.27

$$\frac{2\,A\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}-\frac{2\,B\,a\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] (2*A*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) + (2*B*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2))
)/(3*d) + (2*A*a*ellipticE(c/2 + (d*x)/2, 2))/d - (2*B*a*cos(c + d*x)^(7/2)
*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*
x)^2)^(1/2))
```

3.126 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=70

$$\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+2/3*a*(3*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*B*sin(d*x+c)*cos(d*
x+c)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3047, 3102, 2827, 2720, 2719}

$$\frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + B)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2aB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}a(3A+B)+\frac{3}{2}a(A}{\sqrt{\cos(c+}}$$

$$= \frac{2aB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + (a(A+B))\int \sqrt{\cos(c+}$$

$$= \frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.19, size = 309, normalized size = 4.41

$$\frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-6(A+B)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\text{ArcTan}(\tan(c)))\right)\sec(c)\sin(dx+\text{ArcTan}(\tan(c)))+\left(9(A+B)\cos(c-dx-\text{ArcTan}(\tan(c)))\csc(c)\sec(c)+3A\cos(c+dx+\text{ArcTan}(\tan(c)))\csc(c)\sec(c)+3B\cos(c+dx+\text{ArcTan}(\tan(c)))\csc(c)\sec(c)-12A\cos(c+dx)\cot(c)\sqrt{\sec^2(c)}-12B\cos(c+dx)\cot(c)\sqrt{\sec^2(c)}-4(3A+B)\cos(c+dx)\sqrt{\cos^2(dx-\text{ArcTan}(\cot(c)))}\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sqrt{\sec^2(c)}\sec(dx-\text{ArcTan}(\cot(c)))\sin(c)+4B\cos(c+dx)\sqrt{\sec^2(c)}\sin(c+dx)\right)\sqrt{\sin^2(dx+\text{ArcTan}(\tan(c)))}\right)}{12d\sqrt{\cos(c+dx)}\sqrt{\sec^2(c)}\sqrt{\sin^2(dx+\text{ArcTan}(\tan(c)))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-6*(A + B)*HypergeometricPFQ[{-1/
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c
]]] + (9*(A + B)*Cos[c - d*x - ArcTan[Tan[c]]]*Csc[c]*Sec[c] + 3*A*Cos[c +
d*x + ArcTan[Tan[c]]]*Csc[c]*Sec[c] + 3*B*Cos[c + d*x + ArcTan[Tan[c]]]*Csc
[c]*Sec[c] - 12*A*Cos[c + d*x]*Cot[c]*Sqrt[Sec[c]^2] - 12*B*Cos[c + d*x]*Co
t[c]*Sqrt[Sec[c]^2] - 4*(3*A + B)*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]
]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sqrt[Sec[c]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + 4*B*Cos[c + d
```

```
*x]*Sqrt[Sec[c]^2]*Sin[c + d*x])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(116) = 232$.
time = 0.27, size = 321, normalized size = 4.59

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\,(}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*B*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 142, normalized size = 2.03

$$\frac{2\,Ba\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-i\,\sqrt{2}\,(3\,A+B)a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}\,(3\,A+B)a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+3i\,\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-3i\,\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*B*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*(3*A + B)*a*weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(3*A + B)*a*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(A +
 B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) - 3*I*sqrt(2)*(A + B)*a*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{A}{\sqrt{\cos(c+dx)}}\,dx+\int A\sqrt{\cos(c+dx)}\,dx+\int B\sqrt{\cos(c+dx)}\,dx+\int B\cos^{\frac{3}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A/sqrt(cos(c + d*x)), x) + Integral(A*sqrt(cos(c + d*x)), x) +
Integral(B*sqrt(cos(c + d*x)), x) + Integral(B*cos(c + d*x)**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 0.53, size = 79, normalized size = 1.13

$$\frac{2\,B\,a\left(\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*B*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) + (2*B*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*a*(ellipticE(c/2 + (d*x)/
2, 2) + ellipticF(c/2 + (d*x)/2, 2)))/d
```

3.127 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=66

$$-\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*a*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*A*sin(d*x+c)/d/cos(d*x+c
)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3047, 3100, 2827, 2720, 2719}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + B)*EllipticF[(c + d*
x)/2, 2])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+2\int \frac{\frac{1}{2}a(A+B)-\frac{1}{2}a(A-B)\cos(c+dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-(a(A-B))\int \sqrt{\cos(c+dx)}\,dx+(a(A\\ &= -\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.09, size = 256, normalized size = 3.88

$$\frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc(c)\left(-3(A-B)\cos(c-dx-\text{ArcTan}(\tan(c)))\sec(c)-(A-B)\cos(c+dx+\text{ArcTan}(\tan(c)))\sec(c)+2((2A-B)\cos(dx)-B\cos(2c+dx))\sqrt{\sec^2(c)}\right)}{\sqrt{\sec^2(c)}}-4(A+B)\cos(c+dx)\sqrt{\cos^2(dx-\text{ArcTan}(\cot(c)))}\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sec(dx-\text{ArcTan}(\cot(c)))\sin(c)+\frac{2(A-B)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\text{ArcTan}(\tan(c)))\right)\sec(c)\sin(dx+\text{ArcTan}(\tan(c)))}{\sqrt{\sec^2(c)}\sqrt{\sin^2(dx+\text{ArcTan}(\tan(c)))}}\right)}{4d\sqrt{\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((Csc[c]*(-3*(A - B)*Cos[c - d*x -
 ArcTan[Tan[c]]]*Sec[c] - (A - B)*Cos[c + d*x + ArcTan[Tan[c]]]*Sec[c] + 2*
((2*A - B)*Cos[d*x] - B*Cos[2*c + d*x])*Sqrt[Sec[c]^2]))/Sqrt[Sec[c]^2] - 4
*(A + B)*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sin[c] + (2*(A - B)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c]]])/(Sqrt[Sec[c]^2]
*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(4*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(116) = 232$.
time = 0.31, size = 242, normalized size = 3.67

method	result
default	$\frac{2a\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-A$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2*a*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 173, normalized size = 2.62

$$\frac{-i\sqrt{2}(A+B)a\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(A+B)a\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-i\sqrt{2}(A-B)a\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}(A-B)a\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2Aa\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] (-I*sqrt(2)*(A + B)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*(A + B)*a*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*(A - B)*a*cos(d*x + c)*weie
```

```
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + I*sqrt(2)*(A - B)*a*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*A*a*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Mupad [**B**]
time = 0.96, size = 90, normalized size = 1.36

$$\frac{2\,A\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}+\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*A*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a*(ellipticE(c/2 + (d*x)/2, 2)
 + ellipticF(c/2 + (d*x)/2, 2)))/d + (2*A*a*sin(c + d*x)*hypergeom([-1/4, 1
/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.128 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+2/3*a*(A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*A*sin(d*x+c)/d/cos
(d*x+c)^(3/2)+2*a*(A+B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3047, 3100, 2827, 2716, 2719, 2720}

$$\frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(A
 + B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{3}{2}a(A+B)+\frac{1}{2}a(A+3B)\cos(c+d}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{3}(a(A$$

$$= \frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d} + \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(A+}{d\sqrt{\mathrm{co}}}$$

$$= -\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|\,2\right.}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.37, size = 813, normalized size = 8.56

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((A + B)*Csc
[c]*Sec[c])/d + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (Sec[c]*Sec[c +
d*x]*(A*Sin[c] + 3*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d)) - (A*(1 + Cos[c + d*x
])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (B*(1 + Cos[c +
d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]
^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTa
n[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sq
rt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) + (A*(1 + Cos[c +
 d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) + (B*(1 + Cos[c + d*x])*Csc[c
]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(139) = 278$.

time = 0.47, size = 399, normalized size = 4.20

method	result
default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(\frac{B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\cdots\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1
/2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2*A+1/2*B)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 196, normalized size = 2.06

$$\frac{-i\sqrt{2}(A+3B)a\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(A+3B)a\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}(A+B)a\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}(A+B)a\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2(3(A+B)a\cos(dx+c)+Aa)\sqrt{\cos(dx+c)}\sin(dx+c)}{3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(A + 3*B)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*B)*a*cos(d*x + c)^2*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(A + B)*a*cos
(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + 3*I*sqrt(2)*(A + B)*a*cos(d*x + c)^2*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*(A
+ B)*a*cos(d*x + c) + A*a)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3881 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Mupad [B]
time = 1.30, size = 150, normalized size = 1.58

$$\frac{2\,B\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,B\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*B*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a*sin(c + d*x)*hypergeom([-1/4
, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
 + (2*A*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*co
s(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a*sin(c + d*x)*hypergeom([-
1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/
2))
```

3.129 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=132

$$-\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a}{}$$

```
[Out] -2/5*a*(3*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*A*sin(d*x+c)/d
/cos(d*x+c)^(5/2)+2/3*a*(A+B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*a*(3*A+5*B)
*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3047, 3100, 2827, 2716, 2720, 2719}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(3A+5B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*a*(3*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF
[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A + 5*B)*Sin[c
 + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{5}{2}a(A+B)+\frac{1}{2}a(3A+5B)\cos(c+ \cdots}{\cos^{\frac{5}{2}}(c+dx)}$$
$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + \frac{1}{5}(a(3 \cdots$$
$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(3A+5B) \cdots}{5d\sqrt{\cos(\cdots}}$$
$$= -\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)F\left(\frac{1}{2}(c+dx) \cdots\right.}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.43, size = 865, normalized size = 6.55

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((3*A + 5*B)
*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*
Sec[c + d*x]^2*(3*A*Sin[c] + 5*A*Sin[d*x] + 5*B*Sin[d*x]))/(15*d) + (Sec[c]
*Sec[c + d*x]*(5*A*Sin[c] + 5*B*Sin[c] + 9*A*Sin[d*x] + 15*B*Sin[d*x]))/(15
*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot
[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 +
Cot[c]^2]) + (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + S
in[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d
) + (B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{
-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*
Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[
c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Ta
n[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[
c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(168) = 336$.
time = 0.66, size = 634, normalized size = 4.80

method	result
default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(\left(\frac{A}{2}+\frac{B}{2}\right)\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2}}{6\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*((1/2*A+1/2*
B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/10*A/(8*sin(1/2*d*x+1/2*c)^6
-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/
2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)+1/2*B/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 219, normalized size = 1.66

$$\frac{-5i\sqrt{2}\,(A+B)a\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 5i\sqrt{2}\,(A+B)a\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)) - 3i\sqrt{2}\,(3A+5B)a\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) + 3i\sqrt{2}\,(3A+5B)a\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))) + 2\left(3(3A+5B)a\cos(dx+c)^2+5(A+B)a\cos(dx+c)+3Aa\right)\sqrt{\cos(dx+c)}\,\sin(dx+c)}{15\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*(A + B)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(A + B)*a*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(3*A + 5*B)*
a*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(3*A + 5*B)*a*cos(d*x + c)^3*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) +
```

```
2*(3*(3*A + 5*B)*a*cos(d*x + c)^2 + 5*(A + B)*a*cos(d*x + c) + 3*A*a)*sqrt
(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Mupad [**B**]
time = 1.61, size = 177, normalized size = 1.34

$$\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{5\,d\,{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,B\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,B\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*A*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c
 + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*a*sin(c + d*x)*hypergeom([-5/4
, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/
2)) + (2*B*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*c
os(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a*sin(c + d*x)*hypergeom([
-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^
(1/2))
```

3.130 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=194

$$\frac{4a^2(9A+8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(6A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(6A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}$$

```
[Out] 4/15*a^2*(9*A+8*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^2*(6*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/45*a^2*(9
*A+8*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/63*a^2*(9*A+11*B)*cos(d*x+c)^(5/2)*
sin(d*x+c)/d+2/9*B*cos(d*x+c)^(5/2)*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d+4/21*
a^2*(6*A+5*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3055, 3047, 3102, 2827, 2715, 2720, 2719}

$$\frac{4a^2(6A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(9A+8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(9A+11B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}+\frac{4a^2(9A+8B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{4a^2(6A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*a^2*(9*A + 8*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(6*A + 5*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(6*A + 5*B)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(21*d) + (4*a^2*(9*A + 8*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4
5*d) + (2*a^2*(9*A + 11*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*B*C
os[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2B\cos^{\frac{5}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+}{9d}$$

$$= \frac{2B\cos^{\frac{5}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+}{9d}$$

$$= \frac{2a^2(9A+11B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2}{}$$

$$= \frac{2a^2(9A+11B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2}{}$$

$$= \frac{4a^2(6A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{4}{}$$

$$= \frac{4a^2(9A+8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^2(6A+5}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.30, size = 944, normalized size = 4.87

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/15*((9*A
 + 8*B)*Cot[c])/d + ((51*A + 46*B)*Cos[d*x]*Sin[c])/(168*d) + ((36*A + 37*B
)*Cos[2*d*x]*Sin[2*c])/(360*d) + ((A + 2*B)*Cos[3*d*x]*Sin[3*c])/(56*d) + (
B*Cos[4*d*x]*Sin[4*c])/(144*d) + ((51*A + 46*B)*Cos[c]*Sin[d*x])/(168*d) +
((36*A + 37*B)*Cos[2*c]*Sin[2*d*x])/(360*d) + ((A + 2*B)*Cos[3*c]*Sin[3*d*x
])/(56*d) + (B*Cos[4*c]*Sin[4*d*x])/(144*d)) - (2*A*(a + a*Cos[c + d*x])^2*
Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Se
c[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[
c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
 Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c +
 d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c
]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*A*(a +
 a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -
1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]
```

```
) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (4*B*(a + a*Co
s[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4},
 {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqr
t[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - (
(Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d)
```

Maple [A]
time = 0.28, size = 413, normalized size = 2.13

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2\left(-560B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(360A+1840B)\left(\sin^8\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(360*A+1840*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-1044*A-2368*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1134*A+1568*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-351*A-387*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+90*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 223, normalized size = 1.15

$$-\frac{2 \left(15 i \sqrt{2} (6 A+5 B) a^2 \mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 15 i \sqrt{2} (6 A+5 B) a^2 \mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)) - 21 i \sqrt{2} (9 A+8 B) a^2 \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) + 21 i \sqrt{2} (9 A+8 B) a^2 \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))) - \left(35 B a^2 \cos(dx+c)^3 + 45 (A+2 B) a^2 \cos(dx+c)^2 + 14 (9 A+8 B) a^2 \cos(dx+c) + 30 (6 A+5 B) a^2\right) \sqrt{\cos(dx+c)} \sin(dx+c)\right)}{315 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -2/315*(15*I*sqrt(2)*(6*A + 5*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - 15*I*sqrt(2)*(6*A + 5*B)*a^2*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(9*A + 8*B)*a^2*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 2
1*I*sqrt(2)*(9*A + 8*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - (35*B*a^2*cos(d*x + c)^3 + 45*(A + 2*B
)*a^2*cos(d*x + c)^2 + 14*(9*A + 8*B)*a^2*cos(d*x + c) + 30*(6*A + 5*B)*a^2
)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

Mupad [B]
time = 1.07, size = 266, normalized size = 1.37

$$\frac{2 A a^2 \left(\sqrt{\cos(c+d x)}\,\sin(c+d x)+\mathrm{F}\left(\frac{c}{2}+\frac{d x}{2}\middle|2\right)\right)}{3 d}-\frac{4 A a^2 \cos(c+d x)^{7/2}\sin(c+d x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d x)^2\right)}{7 d\sqrt{\sin(c+d x)^2}}-\frac{2 A a^2 \cos(c+d x)^{9/2}\sin(c+d x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+d x)^2\right)}{9 d\sqrt{\sin(c+d x)^2}}-\frac{2 B a^2 \cos(c+d x)^{7/2}\sin(c+d x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d x)^2\right)}{7 d\sqrt{\sin(c+d x)^2}}-\frac{4 B a^2 \cos(c+d x)^{9/2}\sin(c+d x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+d x)^2\right)}{9 d\sqrt{\sin(c+d x)^2}}-\frac{2 B a^2 \cos(c+d x)^{11/2}\sin(c+d x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos(c+d x)^2\right)}{11 d\sqrt{\sin(c+d x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(
3*d) - (4*A*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4,
 cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a^2*cos(c + d*x)^(9/2
)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d
*x)^2)^(1/2)) - (2*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/
4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*B*a^2*cos(c +
d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(
sin(c + d*x)^2)^(1/2)) - (2*B*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeo
m([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

3.131 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=161

$$\frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(7A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(7A+6B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}$$

```
[Out] 4/5*a^2*(4*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^2*(7*A+6*B)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/35*a^2*(7*
A+9*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*B*cos(d*x+c)^(3/2)*(a^2+a^2*cos(d*
x+c))*sin(d*x+c)/d+4/21*a^2*(7*A+6*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.20, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3055, 3047, 3102, 2827, 2719, 2715, 2720}

$$\frac{4a^2(7A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(7A+9B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d}+\frac{4a^2(7A+6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*a^2*(4*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(7*A + 6*B)*El
lipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(7*A + 6*B)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(21*d) + (2*a^2*(7*A + 9*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35
*d) + (2*B*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2B\cos^{\frac{3}{2}}(c+dx)\,(a^2+a^2\cos(c+dx))\sin(c}{7d}$$

$$= \frac{2B\cos^{\frac{3}{2}}(c+dx)\,(a^2+a^2\cos(c+dx))\sin(c}{7d}$$

$$= \frac{2a^2(7A+9B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + $$

$$= \frac{2a^2(7A+9B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + $$

$$= \frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(7A+}{}$$

$$= \frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(7A+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.26, size = 898, normalized size = 5.58

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/5*((4*A
+ 3*B)*Cot[c])/d + ((56*A + 51*B)*Cos[d*x]*Sin[c])/(168*d) + ((A + 2*B)*Cos
[2*d*x]*Sin[2*c])/(20*d) + (B*Cos[3*d*x]*Sin[3*c])/(56*d) + ((56*A + 51*B)*
Cos[c]*Sin[d*x])/(168*d) + ((A + 2*B)*Cos[2*c]*Sin[2*d*x])/(20*d) + (B*Cos[
3*c]*Sin[3*d*x])/(56*d)) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec
[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[
c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)
/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(
Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Ar
cTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (2*A*(a + a*Cos[c + d*x])^2*Csc[
c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x +
ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]
```

```
*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d) - (3*B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec
[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan
[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]
]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]))/(10*d)
```

Maple [A]
time = 0.30, size = 385, normalized size = 2.39

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^{2}\left(120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-84A-348B\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-348*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(224*A+378*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-9
1*A-117*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-84*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+30*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 203, normalized size = 1.26

$$-\frac{2\left(5i\sqrt{2}\,(7A+6B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,(7A+6B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21i\sqrt{2}\,(4A+3B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}\,(4A+3B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-(15Ba^2\cos(dx+c)^2+21(A+2B)a^2\cos(dx+c)+10(7A+6B)a^2)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/105*(5*I*sqrt(2)*(7*A + 6*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 5*I*sqrt(2)*(7*A + 6*B)*a^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(4*A + 3*B)*a^2*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*
I*sqrt(2)*(4*A + 3*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - (15*B*a^2*cos(d*x + c)^2 + 21*(A + 2*B)*
a^2*cos(d*x + c) + 10*(7*A + 6*B)*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

Mupad [B]
time = 1.01, size = 231, normalized size = 1.43

$$\frac{2Aa^2\left(\frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3}+\frac{2\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{3}\right)}{d}+\frac{2Ba^2\left(\sqrt{\cos(c+dx)}\,\sin(c+dx)+\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)\right)}{3d}+\frac{2Aa^2\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}-\frac{2Aa^2\cos(c+dx)^{7/2}\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+dx)^2\right)}{7d\sqrt{\sin(c+dx)^2}}-\frac{4Ba^2\cos(c+dx)^{7/2}\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+dx)^2\right)}{7d\sqrt{\sin(c+dx)^2}}-\frac{2Ba^2\cos(c+dx)^{9/2}\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+dx)^2\right)}{9d\sqrt{\sin(c+dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/
2, 2))/3))/d + (2*B*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 +
(d*x)/2, 2)))/(3*d) + (2*A*a^2*ellipticE(c/2 + (d*x)/2, 2))/d - (2*A*a^2*co
s(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/
(7*d*(sin(c + d*x)^2)^(1/2)) - (4*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hyp
ergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2
*B*a^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c +
d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.132 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=126

$$\frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(5A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d}+$$

```
[Out] 4/5*a^2*(5*A+4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/15*a^2*(5*A+7
*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+2/5*B*(a^2+a^2*cos(d*x+c))*sin(d*x+c)*cos
(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3055, 3047, 3102, 2827, 2720, 2719}

$$\frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\,(a^2\cos(c+dx)+a^2)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (4*a^2*(5*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(2*A + B)*Elli
pticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(15*d) + (2*B*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d
*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2B\sqrt{\cos(c+dx)}\,(a^2+a^2\cos(c+dx))\sin(c+dx)}{5d}+\frac{2}{5}\int$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,(a^2+a^2\cos(c+dx))\sin(c+dx)}{5d}+\frac{2}{5}\int$$

$$= \frac{2a^2(5A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d}+\frac{2B\sqrt{\cos(c+d}}{}$$

$$= \frac{2a^2(5A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d}+\frac{2B\sqrt{\cos(c+d}}{}$$

$$= \frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx\right.}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 6.31, size = 852, normalized size = 6.76

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/5*((5*A
+ 4*B)*Cot[c])/d + ((A + 2*B)*Cos[d*x]*Sin[c])/(6*d) + (B*Cos[2*d*x]*Sin[2*
c])/(20*d) + ((A + 2*B)*Cos[c]*Sin[d*x])/(6*d) + (B*Cos[2*c]*Sin[2*d*x])/(2
0*d)) - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[
1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*
d*Sqrt[1 + Cot[c]^2]) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]
^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt
[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/
(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]))/(2*d) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((H
ypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + T
an[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c
]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])
)/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(166) = 332$.

time = 0.33, size = 357, normalized size = 2.83

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2\left(-12B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(10A+32B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(10*A+32*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-5*A-13*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-12*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 179, normalized size = 1.42

$$-\frac{2\left(5i\sqrt{2}(2A+B)a^{2}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(2A+B)a^{2}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}(5A+4B)a^{2}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}(5A+4B)a^{2}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-(3Ba^{2}\cos(dx+c)+5(A+2B)a^{2})\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*(2*A + B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) - 5*I*sqrt(2)*(2*A + B)*a^2*weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(5*A + 4*B)*a^2*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(
2)*(5*A + 4*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) - (3*B*a^2*cos(d*x + c) + 5*(A + 2*B)*a^2)*sqrt(c
os(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

Mupad [B]
time = 1.00, size = 153, normalized size = 1.21

$$\frac{2\,B\,a^2\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,A\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+6\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}-\frac{2\,B\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*B*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/
2, 2))/3))/d + (2*A*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + 6*ellipticE(c/2
+ (d*x)/2, 2) + 4*ellipticF(c/2 + (d*x)/2, 2)))/(3*d) + (2*B*a^2*ellipticE(
c/2 + (d*x)/2, 2))/d - (2*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([
1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.133 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{4a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4a^2(3A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a^2(3A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2A(a^2+a}{}$$

```
[Out] 4*a^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+4/3*a^2*(3*A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*A*(a^2+a^2*cos(d*x+c)
)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-2/3*a^2*(3*A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)
/d
```

Rubi [A]
time = 0.18, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3054, 3047, 3102, 2827, 2720, 2719}

$$\frac{4a^2(3A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}+\frac{4a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*B*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(3*A + 2*B)*EllipticF[(c + d
*x)/2, 2])/(3*d) - (2*a^2*(3*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
+ (2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a^2+a^2\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+a\cos(c+$$

$$= \frac{2A(a^2+a^2\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}a^2(3A+B)}{}$$

$$= -\frac{2a^2(3A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2A(a^2+a^2\text{c}}{d\sqrt{}}$$

$$= -\frac{2a^2(3A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2A(a^2+a^2\text{c}}{d\sqrt{}}$$

$$= \frac{4a^2BE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{4a^2(3A+2B)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.39, size = 623, normalized size = 5.28

$$\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{(-A+2B+A\cos(2c)+2B\cos(2c))\csc(c)\sec(c)}{4d}+\frac{B\cos(dx)\sin(c)}{6d}+\frac{B\cos(c)\sin(dx)}{6d}+\frac{A\sec(c)\sec(c+dx)\sin(dx)}{2d}\right)-\frac{A(a+a\cos(c+dx))^2\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(dx-\text{ArcTan}(\cot(c)))\sqrt{1-\sin(dx-\text{ArcTan}(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\,\sin(c)\sin(dx-\text{ArcTan}(\cot(c)))}\sqrt{1+\sin(dx-\text{ArcTan}(\cot(c)))}}{d\sqrt{1+\cot^2(c)}}-\frac{2B(a+a\cos(c+dx))^2\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(dx-\text{ArcTan}(\cot(c)))\sqrt{1-\sin(dx-\text{ArcTan}(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\,\sin(c)\sin(dx-\text{ArcTan}(\cot(c)))}\sqrt{1+\sin(dx-\text{ArcTan}(\cot(c)))}}{3d\sqrt{1+\cot^2(c)}}-\frac{B(a+a\cos(c+dx))^2\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\text{ArcTan}(\tan(c)))\right)\sin(dx+\text{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\text{ArcTan}(\tan(c)))}\sqrt{1+\cos(dx+\text{ArcTan}(\tan(c)))}\sqrt{\cos(c)\cos(dx+\text{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^2(c)}}-\frac{\frac{\sin(dx+\text{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1+\tan^2(c)}}+\frac{2\cos^2(c)\cos(dx+\text{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}{\cos^2(c)+\sin^2(c)}}{\sqrt{\cos(c)\cos(dx+\text{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}}\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/4*((-A +
 2*B + A*Cos[2*c] + 2*B*Cos[2*c])*Csc[c]*Sec[c])/d + (B*Cos[d*x]*Sin[c])/(6
*d) + (B*Cos[c]*Sin[d*x])/(6*d) + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/(2*d)) -
 (A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^
2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1
+ Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
2*d)
```

Maple [A]
time = 0.37, size = 244, normalized size = 2.07

method	result
default	$\frac{4a^2\left(-2B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 4/3*a^2*(-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
```

```
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 198, normalized size = 1.68

$$-\frac{2\left(i\sqrt{2}\,(3A+2B)a^2\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}\,(3A+2B)a^2\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}\,Ba^2\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\,Ba^2\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-(Ba^2\cos(dx+c)+3Aa^2)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] -2/3*(I*sqrt(2)*(3*A + 2*B)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) - I*sqrt(2)*(3*A + 2*B)*a^2*cos(d*x + c)*weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*a^2*cos
(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c))) + 3*I*sqrt(2)*B*a^2*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (B*a^2*cos(d*x +
 c) + 3*A*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

Mupad [B]
time = 1.14, size = 134, normalized size = 1.14

$$\frac{2\,B\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+6\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,A\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*B*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + 6*ellipticE(c/2 + (d*x)/2, 2) +
 4*ellipticF(c/2 + (d*x)/2, 2)))/(3*d) + (2*A*a^2*ellipticE(c/2 + (d*x)/2,
2))/d + (4*A*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^2*sin(c + d*x)*hyp
ergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*
x)^2)^(1/2))
```

3.134 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=120

$$-\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(5A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2A(a^2+a^2\cos(c+d}{3d\cos^{\frac{3}{2}}(c-}$$

```
[Out] -4*a^2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+4/3*a^2*(2*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*A*(a^2+a^2*cos(d*x
+c))*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/3*a^2*(5*A+3*B)*sin(d*x+c)/d/cos(d*x+c
)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3054, 3047, 3100, 2827, 2720, 2719}

$$\frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(5A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}-\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^2*A*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(2*A + 3*B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a^2*(5*A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]
]) + (2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2A(a^2+a^2\cos(c+dx))\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{(a+a\cos(c+\\
&= \frac{2A(a^2+a^2\cos(c+dx))\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\frac{1}{2}a^2(5A+3B)\\
&= \frac{2a^2(5A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2A(a^2+a^2\cos(c+dx))\sin(c}{3d\cos^{\frac{3}{2}}(c+dx)}\\
&= \frac{2a^2(5A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2A(a^2+a^2\cos(c+dx))\sin(c}{3d\cos^{\frac{3}{2}}(c+dx)}\\
&= -\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.43, size = 624, normalized size = 5.20

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/4*((-4*A
 - B + B*Cos[2*c])*Csc[c]*Sec[c])/d + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(6
*d) + (Sec[c]*Sec[c + d*x]*(A*Sin[c] + 6*A*Sin[d*x] + 3*B*Sin[d*x]))/(6*d))
 - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + C
ot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[
1 + Cot[c]^2]) + (A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hy
pergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
 ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[
d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Ta
n[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]
^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))
/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(162) = 324$.
time = 0.37, size = 513, normalized size = 4.28

method	result
default	$-\frac{4\left(6\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(2A+B)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/3*(6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A+B)*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
```

```
c)^2)^(1/2)*(7*A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2)))*sin(1/2*d*x+1/2*c)^2+2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))*a^2/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*
x+1/2*c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 210, normalized size = 1.75

$$-\frac{2\left(i\sqrt{2}\,(2A+3B)a^2\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}\,(2A+3B)a^2\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}\,Aa^2\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,Aa^2\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-(3(2A+B)a^2\cos(dx+c)+Aa^2)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] -2/3*(I*sqrt(2)*(2*A + 3*B)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) - I*sqrt(2)*(2*A + 3*B)*a^2*cos(d*x + c)^2*we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*A*a^2
*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*a^2*cos(d*x + c)^2*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*(2*A
+ B)*a^2*cos(d*x + c) + A*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
+ c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5990 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x
)
```

Mupad [B]
time = 1.69, size = 196, normalized size = 1.63

$$\frac{2\,A\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,B\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,B\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*A*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a^2*ellipticE(c/2 + (d*x)/2,
 2))/d + (4*B*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (4*A*a^2*sin(c + d*x)*hy
pergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d
*x)^2)^(1/2)) + (2*A*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d
*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a^2*sin(c +
d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(si
n(c + d*x)^2)^(1/2))
```

3.135 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=159

$$-\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(4A+5B)}{5d\sqrt{\cos(c}}$$

```
[Out] -4/5*a^2*(4*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/15*a^2*(7*A+
5*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*A*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d/
cos(d*x+c)^(5/2)+4/5*a^2*(4*A+5*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3054, 3047, 3100, 2827, 2716, 2719, 2720}

$$\frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(4A+5B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-4*a^2*(4*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 2*B)*Ell
ipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(7*A + 5*B)*Sin[c + d*x])/(15*d*Cos[
c + d*x]^(3/2)) + (4*a^2*(4*A + 5*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a^2+a^2\cos(c+dx))\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\cos(c+}{}$$

$$= \frac{2A(a^2+a^2\cos(c+dx))\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}a^2(7A+5B)}{}$$

$$= \frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a^2+a^2\cos(c+dx))\sin(c}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a^2+a^2\cos(c+dx))\sin(c}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(A+2B)F\left(\frac{1}{2}(c+\right.}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.54, size = 883, normalized size = 5.55

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(((4*A + 5*B
)*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(10*d) + (Sec[c
]*Sec[c + d*x]^2*(3*A*Sin[c] + 10*A*Sin[d*x] + 5*B*Sin[d*x]))/(30*d) + (Sec
[c]*Sec[c + d*x]*(10*A*Sin[c] + 5*B*Sin[c] + 24*A*Sin[d*x] + 30*B*Sin[d*x])
)/(30*d)) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqr
t[1 + Cot[c]^2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]
^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(3*d*Sqrt[1 + Cot[c]^2]) + (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d
*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]
^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*
Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*S
```

```
qrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c]
)/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2]]))/(5*d) + (B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(195) = 390$.
time = 0.73, size = 714, normalized size = 4.49

method	result
default	$-\frac{8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2 \left(\frac{B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{4\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+1/20*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c
)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1
/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/2*A+1/4*B)*(-1/6*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*
c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))+(1/4*A+1/2*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2
-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
```

```
*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 239, normalized size = 1.50

$$-\frac{2\left(5i\sqrt{2}(A+2B)a^2\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(A+2B)a^2\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}(4A+5B)a^2\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}(4A+5B)a^2\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-(6(4A+5B)a^2\cos(dx+c)^2+5(2A+B)a^2\cos(dx+c)+3Aa^2)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{15d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*(A + 2*B)*a^2*cos(d*x + c)^3*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(A + 2*B)*a^2*cos(d*x + c)^3*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(4*A
 + 5*B)*a^2*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(4*A + 5*B)*a^2*cos(d*x + c
)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) - (6*(4*A + 5*B)*a^2*cos(d*x + c)^2 + 5*(2*A + B)*a^2*cos(d*x +
c) + 3*A*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x
)
```

Mupad [B]
time = 1.97, size = 229, normalized size = 1.44

$$\frac{6\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)+20\,A\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)+30\,A\,a^2\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{15\,d\,{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{1-{\cos\left(c+d\,x\right)}^2}}+\frac{2\,B\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,B\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,B\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(7/2),x)
```

```
[Out] (6*A*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*A*a
^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) +
30*A*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*
x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (2*B*a^2*elli
pticF(c/2 + (d*x)/2, 2))/d + (4*B*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3
/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a
^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d
*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

3.136 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=194

$$-\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2(6A+7B}{21d\cos^{\frac{3}{2}}}$$

```
[Out] -4/5*a^2*(3*A+4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^2*(6*A+7*B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/35*a^2*(9
*A+7*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+4/21*a^2*(6*A+7*B)*sin(d*x+c)/d/cos(d
*x+c)^(3/2)+2/7*A*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(7/2)+4/5*a^
2*(3*A+4*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3054, 3047, 3100, 2827, 2716, 2720, 2719}

$$\frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(6A+7B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2(3A+4B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-4*a^2*(3*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(6*A + 7*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(9*A + 7*B)*Sin[c + d*x])/(35*d*C
os[c + d*x]^(5/2)) + (4*a^2*(6*A + 7*B)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3
/2)) + (4*a^2*(3*A + 4*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*A*(a^
2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2A(a^2+a^2\cos(c+dx))\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2}{7}\int \frac{(a+a\cos(c+ \\
&= \frac{2A(a^2+a^2\cos(c+dx))\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2}{7}\int \frac{\frac{1}{2}a^2(9A+7B)}{} \\
&= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2A(a^2+a^2\cos(c+dx))\sin(c}{7d\cos^{\frac{7}{2}}(c+dx)} \\
&= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2A(a^2+a^2\cos(c+dx))\sin(c}{7d\cos^{\frac{7}{2}}(c+dx)} \\
&= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2(6A+7B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+ \\
&= -\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+\right.}{21d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.63, size = 925, normalized size = 4.77

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(((3*A + 4*B
)*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(14*d) + (Sec[c
]*Sec[c + d*x]^3*(5*A*Sin[c] + 14*A*Sin[d*x] + 7*B*Sin[d*x]))/(70*d) + (Sec
[c]*Sec[c + d*x]^2*(42*A*Sin[c] + 21*B*Sin[c] + 60*A*Sin[d*x] + 70*B*Sin[d*
x]))/(210*d) + (Sec[c]*Sec[c + d*x]*(30*A*Sin[c] + 35*B*Sin[c] + 63*A*Sin[d
*x] + 84*B*Sin[d*x]))/(105*d)) - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/
2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(S
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
 (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (3*A*(a + a*Cos[c + d*x])^
2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*
```

```
x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) + (2*B*(a + a*Cos[c + d*x])^2*Csc
[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x +
 ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan
[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(226) = 452$.
time = 0.80, size = 824, normalized size = 4.25

method	result	size
default	Expression too large to display	824

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/5*(1/2*
A+1/4*B)/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)
^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/4*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2
*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2)))+1/4*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d
*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2)))+1/4*B/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2
-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
```

```
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(9/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 263, normalized size = 1.36

$$-\frac{2\left(5i\sqrt{2}(6A+7B)a^2\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(6A+7B)a^2\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21i\sqrt{2}(3A+4B)a^2\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}(3A+4B)a^2\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\left(42(3A+4B)a^2\cos(dx+c)^3+10(6A+7B)a^2\cos(dx+c)^2+21(2A+B)a^2\cos(dx+c)+15Aa^2\right)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{105d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] -2/105*(5*I*sqrt(2)*(6*A + 7*B)*a^2*cos(d*x + c)^4*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(6*A + 7*B)*a^2*cos(d*x + c
)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sqrt(2
)*(3*A + 4*B)*a^2*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(3*A + 4*B)*a^2*cos(
d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) - (42*(3*A + 4*B)*a^2*cos(d*x + c)^3 + 10*(6*A + 7*B)*a^2
*cos(d*x + c)^2 + 21*(2*A + B)*a^2*cos(d*x + c) + 15*A*a^2)*sqrt(cos(d*x +
c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(9/2), x
)
```

Mupad [B]
time = 2.30, size = 235, normalized size = 1.21

$$\frac{30\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)+84\,A\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+70\,A\,a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{105\,d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{1-\cos\left(c+d\,x\right)^2}}+\frac{6\,B\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+20\,B\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+30\,B\,a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{15\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{1-\cos\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(9/2),x)
```

```
[Out] (30*A*a^2*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 84*A*
a^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2)
+ 70*A*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c +
d*x)^2))/(105*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2)) + (6*B*a^2*s
in(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*B*a^2*cos(c +
 d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 30*B*a^2*c
os(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(15
*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.137 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=237

$$\frac{4a^3(17A+15B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(121A+105B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(121A+105B)\sqrt{\cos(c+dx)}\,\text{si}}{231d}$$

```
[Out] 4/15*a^3*(17*A+15*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/231*a^3*(121*A+105*B)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/45
*a^3*(17*A+15*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+20/693*a^3*(22*A+21*B)*cos(d
*x+c)^(5/2)*sin(d*x+c)/d+2/11*a*B*cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2*sin(d
*x+c)/d+2/99*(11*A+15*B)*cos(d*x+c)^(5/2)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d
+4/231*a^3*(121*A+105*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.32, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3055, 3047, 3102, 2827, 2715, 2720, 2719}

$$\frac{4a^3(121A+105B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(17A+15B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{20a^3(22A+21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d}+\frac{4a^3(17A+15B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2(11A+15B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{99d}+\frac{4a^3(121A+105B)\sin(c+dx)\sqrt{\cos(c+dx)}}{231d}+\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*a^3*(17*A + 15*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(121*A + 10
5*B)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(121*A + 105*B)*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(17*A + 15*B)*Cos[c + d*x]^(3/2)*Si
n[c + d*x])/(45*d) + (20*a^3*(22*A + 21*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(693*d) + (2*a*B*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(
11*d) + (2*(11*A + 15*B)*Cos[c + d*x]^(5/2)*(a^3 + a^3*Cos[c + d*x])*Sin[c
+ d*x])/(99*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{2aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+d}{11d}$$

$$= \frac{2aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+d}{11d}$$

$$= \frac{2aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+d}{11d}$$

$$= \frac{20a^3(22A+21B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d} + \frac{2}{}$$

$$= \frac{20a^3(22A+21B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d} + \frac{2}{}$$

$$= \frac{4a^3(121A+105B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{231d}$$

$$= \frac{4a^3(17A+15B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d} + \frac{4a^3(121A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.35, size = 990, normalized size = 4.18

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/30*((17*
A + 15*B)*Cot[c])/d + ((2134*A + 1953*B)*Cos[d*x]*Sin[c])/(7392*d) + ((73*A
 + 75*B)*Cos[2*d*x]*Sin[2*c])/(720*d) + (3*(44*A + 63*B)*Cos[3*d*x]*Sin[3*c
])/(4928*d) + ((A + 3*B)*Cos[4*d*x]*Sin[4*c])/(288*d) + (B*Cos[5*d*x]*Sin[5
*c])/(704*d) + ((2134*A + 1953*B)*Cos[c]*Sin[d*x])/(7392*d) + ((73*A + 75*B
)*Cos[2*c]*Sin[2*d*x])/(720*d) + (3*(44*A + 63*B)*Cos[3*c]*Sin[3*d*x])/(492
8*d) + ((A + 3*B)*Cos[4*c]*Sin[4*d*x])/(288*d) + (B*Cos[5*c]*Sin[5*d*x])/(7
04*d)) - (11*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sq
rt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x
- ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c
]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]
```

```
)/(22*d*Sqrt[1 + Cot[c]^2]) - (17*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 +
 (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(60*d) - (B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/
2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*
Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt
[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sq
rt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
)/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2]]))/(4*d)
```

Maple [A]
time = 0.30, size = 441, normalized size = 1.86

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(10080B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-6160A-43680B)\left(\sin^{1}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(10080*
B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-6160*A-43680*B)*sin(1/2*d*x+1/
2*c)^10*cos(1/2*d*x+1/2*c)+(24200*A+77280*B)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d
*x+1/2*c)+(-37532*A-72240*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(29722
*A+39270*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-8118*A-8820*B)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1815*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3927*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+1575*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3465*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 243, normalized size = 1.03

$$-\frac{2\left(15i\sqrt{2}\,(121A+105B)a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-15i\sqrt{2}\,(121A+105B)a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-231i\sqrt{2}\,(17A+15B)a^3\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+231i\sqrt{2}\,(17A+15B)a^3\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-(315Ba^3\cos(dx+c)^4+385(A+3B)a^3\cos(dx+c)^3+135(11A+14B)a^3\cos(dx+c)^2+154(17A+15B)a^3\cos(dx+c)+30(121A+105B)a^3)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{3465d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -2/3465*(15*I*sqrt(2)*(121*A + 105*B)*a^3*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) - 15*I*sqrt(2)*(121*A + 105*B)*a^3*weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 231*I*sqrt(2)*(17*A + 15*B)*a^
3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) + 231*I*sqrt(2)*(17*A + 15*B)*a^3*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (315*B*a^3*cos(d*x + c)
^4 + 385*(A + 3*B)*a^3*cos(d*x + c)^3 + 135*(11*A + 14*B)*a^3*cos(d*x + c)^
2 + 154*(17*A + 15*B)*a^3*cos(d*x + c) + 30*(121*A + 105*B)*a^3)*sqrt(cos(d
*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x
)
```

Mupad [B]
time = 1.31, size = 360, normalized size = 1.52

$$\frac{A\,a^3\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}-\frac{6\,A\,a^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,A\,a^3\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,A\,a^3\,{\cos\left(c+d\,x\right)}^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,a^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,a^3\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{6\,B\,a^3\,{\cos\left(c+d\,x\right)}^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,a^3\,{\cos\left(c+d\,x\right)}^{13/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{13}{4};\ \frac{17}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{13\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (A*a^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d - (6*A*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4],
 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a^3*cos(c + d*x
)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin
(c + d*x)^2)^(1/2)) - (2*A*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([
1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3
*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2
))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*
hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2)) -
 (6*B*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos
(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(13/2)*
sin(c + d*x)*hypergeom([1/2, 13/4], 17/4, cos(c + d*x)^2))/(13*d*(sin(c + d
*x)^2)^(1/2))
```

3.138 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=204

$$\frac{4a^3(21A+17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(13A+11B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(13A+11B)\sqrt{\cos(c+dx)}\,\sin(c+$$

```
[Out] 4/15*a^3*(21*A+17*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^3*(13*A+11*B)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/105*a
^3*(24*A+23*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/9*a*B*cos(d*x+c)^(3/2)*(a+a*
cos(d*x+c))^2*sin(d*x+c)/d+2/63*(9*A+13*B)*cos(d*x+c)^(3/2)*(a^3+a^3*cos(d*
x+c))*sin(d*x+c)/d+4/21*a^3*(13*A+11*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.30, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3055, 3047, 3102, 2827, 2719, 2715, 2720}

$$\frac{4a^3(13A+11B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(21A+17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(24A+23B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(9A+13B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{63d}+\frac{4a^3(13A+11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*a^3*(21*A + 17*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(13*A + 11*
B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(13*A + 11*B)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(21*d) + (4*a^3*(24*A + 23*B)*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(105*d) + (2*a*B*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*
x])/(9*d) + (2*(9*A + 13*B)*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x])*Sin
[c + d*x])/(63*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{2aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+}{9d}$$

$$= \frac{2aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+}{9d}$$

$$= \frac{2aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+}{9d}$$

$$= \frac{4a^3(24A+23B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} +$$

$$= \frac{4a^3(24A+23B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} +$$

$$= \frac{4a^3(21A+17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(13A}{}$$

$$= \frac{4a^3(21A+17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(13A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.30, size = 944, normalized size = 4.63

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/30*((21*
A + 17*B)*Cot[c])/d + ((107*A + 97*B)*Cos[d*x]*Sin[c])/(336*d) + ((54*A + 7
3*B)*Cos[2*d*x]*Sin[2*c])/(720*d) + ((A + 3*B)*Cos[3*d*x]*Sin[3*c])/(112*d)
 + (B*Cos[4*d*x]*Sin[4*c])/(288*d) + ((107*A + 97*B)*Cos[c]*Sin[d*x])/(336*
d) + ((54*A + 73*B)*Cos[2*c]*Sin[2*d*x])/(720*d) + ((A + 3*B)*Cos[3*c]*Sin[
3*d*x])/(112*d) + (B*Cos[4*c]*Sin[4*d*x])/(288*d)) - (13*A*(a + a*Cos[c + d
*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcT
an[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*S
qrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (11*B*(a +
a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - Arc
Tan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) -
(7*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ
```

```
[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]
]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Ta
n[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 +
Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Co
s[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))
/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) - (17*B
*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]
]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[
c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]
^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d)
```

Maple [A]
time = 0.29, size = 413, normalized size = 2.02

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(-560B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(360A+2200B)\left(\sin^8\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(360*A+2200*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-1296*A-3412*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1806*A+2702*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-624*A-738*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+195*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-441*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+165*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x
)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 223, normalized size = 1.09

$$-\frac{2\left(15i\sqrt{2}\,(13\,A+11\,B)a^{3}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-15i\sqrt{2}\,(13\,A+11\,B)a^{3}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-21i\sqrt{2}\,(21\,A+17\,B)a^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+21i\sqrt{2}\,(21\,A+17\,B)a^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-(35\,Ba^{3}\cos\left(dx+c\right)^{3}+45\,(A+3\,B)a^{3}\cos\left(dx+c\right)^{2}+7\,(27\,A+34\,B)a^{3}\cos\left(dx+c\right)+30\,(13\,A+11\,B)a^{3})\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/315*(15*I*sqrt(2)*(13*A + 11*B)*a^3*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 15*I*sqrt(2)*(13*A + 11*B)*a^3*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(21*A + 17*B)*a^3*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + 21*I*sqrt(2)*(21*A + 17*B)*a^3*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*B*a^3*cos(d*x + c)^3 + 45*
(A + 3*B)*a^3*cos(d*x + c)^2 + 7*(27*A + 34*B)*a^3*cos(d*x + c) + 30*(13*A
+ 11*B)*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x
)
```

Mupad [B]
time = 1.07, size = 323, normalized size = 1.58

$$\frac{2\left(A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+A\,a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}+\frac{B\,a^3\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}-\frac{6\,A\,a^3\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,A\,a^3\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{6\,B\,a^3\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,B\,a^3\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,B\,a^3\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (2*(A*a^3*ellipticE(c/2 + (d*x)/2, 2) + A*a^3*ellipticF(c/2 + (d*x)/2, 2) +
 A*a^3*cos(c + d*x)^(1/2)*sin(c + d*x)))/d + (B*a^3*((2*cos(c + d*x)^(1/2)*
sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d - (6*A*a^3*cos(c +
d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(
sin(c + d*x)^2)^(1/2)) - (2*A*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom
([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (6*B*a^3
*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2
))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*
hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2)) -
 (2*B*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos
(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

3.139 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=171

$$\frac{4a^3(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(21A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(42A+41B)\sqrt{\cos(c+dx)}\,\sin(c+d}{105d}$$

```
[Out] 4/5*a^3*(9*A+7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^3*(21*A+13*B)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/105*a^3*
(42*A+41*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+2/7*a*B*(a+a*cos(d*x+c))^2*sin(d*
x+c)*cos(d*x+c)^(1/2)/d+2/35*(7*A+11*B)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)*cos
(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.29, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3055, 3047, 3102, 2827, 2720, 2719}

$$\frac{4a^3(21A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(42A+41B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{2(7A+11B)\sin(c+dx)\sqrt{\cos(c+dx)}\,(a^3\cos(c+dx)+a^3)}{35d}+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (4*a^3*(9*A + 7*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(21*A + 13*B)*
EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(42*A + 41*B)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(105*d) + (2*a*B*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*S
in[c + d*x])/(7*d) + (2*(7*A + 11*B)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c +
d*x])*Sin[c + d*x])/(35*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2aB\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2}{7}\int$$

$$= \frac{2aB\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2(7A}{}$$

$$= \frac{2aB\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2(7A}{}$$

$$= \frac{4a^3(42A+41B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{105d} + \frac{2aB\sqrt{\cos(c}}{}$$

$$= \frac{4a^3(42A+41B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{105d} + \frac{2aB\sqrt{\cos(c}}{}$$

$$= \frac{4a^3(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{4a^3(21A+13B)F\left(\frac{1}{2}(c+\right.}{21d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.36, size = 898, normalized size = 5.25

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/10*((9*A
 + 7*B)*Cot[c])/d + ((84*A + 107*B)*Cos[d*x]*Sin[c])/(336*d) + ((A + 3*B)*C
os[2*d*x]*Sin[2*c])/(40*d) + (B*Cos[3*d*x]*Sin[3*c])/(112*d) + ((84*A + 107
*B)*Cos[c]*Sin[d*x])/(336*d) + ((A + 3*B)*Cos[2*c]*Sin[2*d*x])/(40*d) + (B*
Cos[3*c]*Sin[3*d*x])/(112*d)) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^
6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (13*B*(a + a*Cos[c + d*x])^3*Csc[c]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
 (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (9*A*(a + a*Cos[c + d*x])
^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d
```

```
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) - (7*B*(a + a*Cos[c + d*x])^3*Cs
c[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
+ ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]
]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[
Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [A]
time = 0.28, size = 385, normalized size = 2.25

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-84A-432B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-432*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(294*A+602*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
26*A-208*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+65*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 203, normalized size = 1.19

$$-\frac{2\left(5i\sqrt{2}(21A+13B)a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(21A+13B)a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21i\sqrt{2}(9A+7B)a^3\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}(9A+7B)a^3\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-(15Ba^3\cos(dx+c)^2+21(A+3B)a^3\cos(dx+c)+5(21A+26B)a^3)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/105*(5*I*sqrt(2)*(21*A + 13*B)*a^3*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(21*A + 13*B)*a^3*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(9*A + 7*B)*a^3*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 21*I*sqrt(2)*(9*A + 7*B)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*B*a^3*cos(d*x + c)^2 + 21*(A + 3
*B)*a^3*cos(d*x + c) + 5*(21*A + 26*B)*a^3)*sqrt(cos(d*x + c))*sin(d*x + c)
)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x
)
```

Mupad [B]
time = 1.00, size = 255, normalized size = 1.49

$$\frac{2\left(Ba^3\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)+Ba^3\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)+Ba^3\sqrt{\cos(c+dx)}\,\sin(c+dx)\right)}{d}+\frac{6Aa^3\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}+\frac{4Aa^3\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}+\frac{2Aa^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d}-\frac{2Aa^3\cos(c+dx)^{7/2}\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+dx)^2\right)}{7d\sqrt{\sin(c+dx)^2}}-\frac{6Ba^3\cos(c+dx)^{7/2}\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+dx)^2\right)}{7d\sqrt{\sin(c+dx)^2}}-\frac{2Ba^3\cos(c+dx)^{9/2}\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+dx)^2\right)}{9d\sqrt{\sin(c+dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*(B*a^3*ellipticE(c/2 + (d*x)/2, 2) + B*a^3*ellipticF(c/2 + (d*x)/2, 2) +
 B*a^3*cos(c + d*x)^(1/2)*sin(c + d*x)))/d + (6*A*a^3*ellipticE(c/2 + (d*x)
/2, 2))/d + (4*A*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^3*cos(c + d*x)
^(1/2)*sin(c + d*x))/d - (2*A*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom
([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (6*B*a^3
*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2
))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*
hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.140 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=169

$$\frac{4a^3(5A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(5A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(5A-6B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d}+$$

```
[Out] 4/5*a^3*(5*A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^3*(5*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*A*(a+a*co
s(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)-4/15*a^3*(5*A-6*B)*sin(d*x+c)*cos
(d*x+c)^(1/2)/d-2/5*(5*A-B)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^(1/2
)/d
```

Rubi [A]

time = 0.28, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3054, 3055, 3047, 3102, 2827, 2720, 2719}

$$\frac{4a^3(5A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(5A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(5A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}-\frac{2(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\,(a^3\cos(c+dx)+a^3)}{5d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (4*a^3*(5*A + 9*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(5*A + 3*B)*El
lipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(5*A - 6*B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(15*d) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]]) - (2*(5*A - B)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c
 + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+a\cos(c+$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(5A-B)\sqrt{\cos(c-}}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(5A-B)\sqrt{\cos(c-}}{}$$

$$= -\frac{4a^3(5A-6B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2aA(a+a\,\mathrm{co}}{d\sqrt{}}$$

$$= -\frac{4a^3(5A-6B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2aA(a+a\,\mathrm{co}}{d\sqrt{}}$$

$$= \frac{4a^3(5A+9B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{4a^3(5A+3B)F\left(\frac{1}{2}(c+d\right.}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 6.47, size = 888, normalized size = 5.25

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/40*((5*A
 + 18*B + 15*A*Cos[2*c] + 18*B*Cos[2*c])*Csc[c]*Sec[c])/d + ((A + 3*B)*Cos[
d*x]*Sin[c])/(12*d) + (B*Cos[2*d*x]*Sin[2*c])/(40*d) + ((A + 3*B)*Cos[c]*Si
n[d*x])/(12*d) + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/(4*d) + (B*Cos[2*c]*Sin[2
*d*x])/(40*d)) - (5*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
6*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPF
Q[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[
d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c
]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 +
 (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
```

```
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(4*d) - (9*B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)
/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]
*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]))/(20*d)
```

Maple [A]
time = 0.34, size = 337, normalized size = 1.99

method	result
default	$-\frac{4a^3\left(-12B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+42B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/15*a^3*(-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+10*A*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^4+42*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*
A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+25*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-18*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+15*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-27*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x
)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 229, normalized size = 1.36

$$-\frac{2\left(5i\sqrt{2}(5A+3B)a^3\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 5i\sqrt{2}(5A+3B)a^3\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)) - 3i\sqrt{2}(5A+9B)a^3\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) + 3i\sqrt{2}(5A+9B)a^3\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))) - \left(3Ba^3\cos(dx+c)^2+5(A+3B)a^3\cos(dx+c)+15Aa^3\right)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{15d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")

[Out] -2/15*(5*I*sqrt(2)*(5*A + 3*B)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(5*A + 3*B)*a^3*cos(d*x + c)*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(5*A
 + 9*B)*a^3*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(5*A + 9*B)*a^3*cos(d*x + c)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c))) - (3*B*a^3*cos(d*x + c)^2 + 5*(A + 3*B)*a^3*cos(d*x + c) + 15*A*a^3)
*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4850 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")

[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x
)

Mupad [B]
time = 1.04, size = 229, normalized size = 1.36

$$\frac{A\,a^3\left(\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{6\,A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a^3\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{d}+\frac{2\,A\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}-\frac{2\,B\,a^3\,\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(3/2),x)
```

```
[Out] (A*a^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d + (6*A*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (6*A*a^3*ellipticF(c
/2 + (d*x)/2, 2))/d + (6*B*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (4*B*a^3*el
lipticF(c/2 + (d*x)/2, 2))/d + (2*B*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/d
+ (2*A*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos
(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(7/2)*sin(c
 + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^
(1/2))
```

3.141 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$-\frac{4a^3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{20a^3(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(4A+B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2a\ldots}{}$$

```
[Out] -4*a^3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/d+20/3*a^3*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*A*(a+a*cos(d*x+
c))^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/3*(7*A+3*B)*(a^3+a^3*cos(d*x+c))*sin(
d*x+c)/d/cos(d*x+c)^(1/2)-4/3*a^3*(4*A+B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.28, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3054, 3047, 3102, 2827, 2720, 2719}

$$\frac{20a^3(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{4a^3(4A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2(7A+3B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{3d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^3*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*(A + B)*EllipticF[(c
 + d*x)/2, 2])/(3*d) - (4*a^3*(4*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
+ (2*(7*A + 3*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d
*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+a\cos(c+\cdots}{\cdots}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(7A+3B)\left(a^3+a\cdots\right.}{3d\sqrt{c\cdots}}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(7A+3B)\left(a^3+a\cdots\right.}{3d\sqrt{c\cdots}}$$

$$= -\frac{4a^3(4A+B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aA(a+a\cos\cdots}{3d\,c\cdots}$$

$$= -\frac{4a^3(4A+B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aA(a+a\cos\cdots}{3d\,c\cdots}$$

$$= -\frac{4a^3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{20a^3(A+B)F\left(\frac{1}{2}(c+dx\cdots\right.}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.56, size = 879, normalized size = 5.46

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/8*((-5*A
 + B + A*Cos[2*c] + 3*B*Cos[2*c])*Csc[c]*Sec[c])/d + (B*Cos[d*x]*Sin[c])/(1
2*d) + (B*Cos[c]*Sin[d*x])/(12*d) + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(12*
d) + (Sec[c]*Sec[c + d*x]*(A*Sin[c] + 9*A*Sin[d*x] + 3*B*Sin[d*x]))/(12*d))
 - (5*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + C
ot[c]^2]) - (5*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*S
qrt[1 + Cot[c]^2]) + (A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*
((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + T
```

```
an[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
+ Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Co
s[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]))/(4*d) - (B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hyperg
eometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Arc
Tan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
+ ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]
^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 +
 Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*
d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(199) = 398$.
time = 0.37, size = 654, normalized size = 4.06

method	result
default	$-\frac{4\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/3*(-4*B*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(9*A+5*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+2*B)*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
*sin(1/2*d*x+1/2*c)^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-3*B*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*a^3/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d
*x+1/2*c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 223, normalized size = 1.39

$$-\frac{2\left(5i\sqrt{2}(A+B)a^3\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(A+B)a^3\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}(A-B)a^3\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}(A-B)a^3\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\left(Ba^3\cos(dx+c)^2+3(3A+B)a^3\cos(dx+c)+Aa^3\right)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] -2/3*(5*I*sqrt(2)*(A + B)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(A + B)*a^3*cos(d*x + c)^2*weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(A - B)*a
^3*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(A - B)*a^3*cos(d*x + c)^2*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) -
(B*a^3*cos(d*x + c)^2 + 3*(3*A + B)*a^3*cos(d*x + c) + A*a^3)*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x
)
```

Mupad [B]

time = 1.63, size = 251, normalized size = 1.56

$$\frac{2\left(A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+3\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}+\frac{B\,a^3\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{6\,B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,B\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*(A*a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*A*a^3*ellipticF(c/2 + (d*x)/2, 2)
))/d + (B*a^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (
d*x)/2, 2))/3))/d + (6*B*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (6*B*a^3*elli
pticF(c/2 + (d*x)/2, 2))/d + (6*A*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3
/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*a
^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d
*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a^3*sin(c + d*x)*hypergeom([-1/4,
1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.142 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=171

$$-\frac{4a^3(9A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(3A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(21A+20B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}+\frac{2aA(a+a$$

```
[Out] -4/5*a^3*(9*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^3*(3*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*A*(a+a
*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/15*(9*A+5*B)*(a^3+a^3*cos(d*
x+c))*sin(d*x+c)/d/cos(d*x+c)^(3/2)+4/15*a^3*(21*A+20*B)*sin(d*x+c)/d/cos(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3054, 3047, 3100, 2827, 2720, 2719}

$$\frac{4a^3(3A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(9A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(9A+5B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^3(21A+20B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-4*a^3*(9*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(3*A + 5*B)*E
llipticF[(c + d*x)/2, 2])/(3*d) + (4*a^3*(21*A + 20*B)*Sin[c + d*x])/(15*d*
Sqrt[Cos[c + d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[
c + d*x]^(5/2)) + (2*(9*A + 5*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15
*d*Cos[c + d*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\cos(c+$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(9A+5B)\left(a^3+a\right.}{15d\,\mathrm{co}}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(9A+5B)\left(a^3+a\right.}{15d\,\mathrm{co}}$$

$$= \frac{4a^3(21A+20B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{2aA(a+a\cos(c+dx))^2\sin}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{4a^3(21A+20B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{2aA(a+a\cos(c+dx))^2\sin}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{4a^3(9A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(3A+5B)F\left(\frac{1}{2}(c+\right.}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.62, size = 890, normalized size = 5.20

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/40*((-36
*A - 25*B + 5*B*Cos[2*c])*Csc[c]*Sec[c])/d + (A*Sec[c]*Sec[c + d*x]^3*Sin[d
*x])/(20*d) + (Sec[c]*Sec[c + d*x]^2*(3*A*Sin[c] + 15*A*Sin[d*x] + 5*B*Sin[
d*x]))/(60*d) + (Sec[c]*Sec[c + d*x]*(15*A*Sin[c] + 5*B*Sin[c] + 54*A*Sin[d
*x] + 45*B*Sin[d*x]))/(60*d)) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^
6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c + d*x])^3*Csc[c]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
(d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sq
rt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
 - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) + (9*A*(a + a*Cos[c + d*x])^3
*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
```

```
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) + (B*(a + a*Cos[c + d*x])^3*Csc[c]
*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(207) = 414$.
time = 0.69, size = 916, normalized size = 5.36

method	result	size
default	Expression too large to display	916

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin(
1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*
d*x+1/2*c)^3*(216*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*A*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-108*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4+180*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-100*B*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-60*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d
*x+1/2*c)^4-246*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*A*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+108*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^2-190*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+100*B*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+60*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2+72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+50*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
```

```
+1/2*c)^2-25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 243, normalized size = 1.42

$$-\frac{2\left(5i\sqrt{2}\,(3A+5B)a^3\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 5i\sqrt{2}\,(3A+5B)a^3\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)) + 3i\sqrt{2}\,(9A+5B)a^3\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - 3i\sqrt{2}\,(9A+5B)a^3\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))) - (9(6A+5B)a^3\cos(dx+c)^2+5(3A+B)a^3\cos(dx+c)+3Aa^3)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{15d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*(3*A + 5*B)*a^3*cos(d*x + c)^3*weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(3*A + 5*B)*a^3*cos(d*x + c)
^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*
(9*A + 5*B)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(9*A + 5*B)*a^3*cos(d*x
 + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c))) - (9*(6*A + 5*B)*a^3*cos(d*x + c)^2 + 5*(3*A + B)*a^3*cos(d*
x + c) + 3*A*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x
)
```

Mupad [B]
time = 2.50, size = 287, normalized size = 1.68

$$\frac{2\left(B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+3\,B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}+\frac{2\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\,{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{6\,B\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*(B*a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*B*a^3*ellipticF(c/2 + (d*x)/2, 2)
))/d + (2*A*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*A*a^3*sin(c + d*x)*hype
rgeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x
)^2)^(1/2)) + (2*A*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x
)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*a^3*sin(c + d*x)
*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin
(c + d*x)^2)^(1/2)) + (6*B*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos
(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a^3*sin(
c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/
2)*(sin(c + d*x)^2)^(1/2))
```

3.143 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=204

$$-\frac{4a^3(7A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(13A+21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^3(7A\cdots}{5d\sqrt{\cdots}}$$

```
[Out] -4/5*a^3*(7*A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^3*(13*A+21*B)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/105*a^3
*(41*A+42*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/7*a*A*(a+a*cos(d*x+c))^2*sin(d
*x+c)/d/cos(d*x+c)^(7/2)+2/35*(11*A+7*B)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d/
cos(d*x+c)^(5/2)+4/5*a^3*(7*A+9*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3054, 3047, 3100, 2827, 2716, 2719, 2720}

$$\frac{4a^3(13A+21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(7A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(11A+7B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^3(7A+9B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-4*a^3*(7*A + 9*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(13*A + 21*B)
*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(41*A + 42*B)*Sin[c + d*x])/(10
5*d*Cos[c + d*x]^(3/2)) + (4*a^3*(7*A + 9*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c
+ d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7
/2)) + (2*(11*A + 7*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(35*d*Cos[c +
 d*x]^(5/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\cos(c+}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(11A+7B)\left(a^3+\right.}{35d\,\mathrm{c}}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(11A+7B)\left(a^3+\right.}{35d\,\mathrm{c}}$$

$$= \frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))^2\sin}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))^2\sin}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{4a^3(13A+21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{4a^3(41A+42B)\sin(c}{105d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^3(7A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(13A+21B)F\left(\frac{1}{2}(c\right.}{21d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.69, size = 925, normalized size = 4.53

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(((7*A + 9*B
)*Csc[c]*Sec[c])/(10*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(28*d) + (Sec[
c]*Sec[c + d*x]^3*(5*A*Sin[c] + 21*A*Sin[d*x] + 7*B*Sin[d*x]))/(140*d) + (S
ec[c]*Sec[c + d*x]^2*(63*A*Sin[c] + 21*B*Sin[c] + 130*A*Sin[d*x] + 105*B*Si
n[d*x]))/(420*d) + (Sec[c]*Sec[c + d*x]*(130*A*Sin[c] + 105*B*Sin[c] + 294*
A*Sin[d*x] + 378*B*Sin[d*x]))/(420*d)) - (13*A*(a + a*Cos[c + d*x])^3*Csc[c
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]
*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[
d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])
^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
```

```
1 + Sin[d*x - ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) + (7*A*(a + a*Cos[
c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[
1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) + (9*B*(a + a*Cos[c + d
*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(236) = 472$.
time = 0.84, size = 902, normalized size = 4.42

method	result	size
default	Expression too large to display	902

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1/5*(3/8*A+1/8*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+
8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(3/8*A+3/8*B)*(-1/6*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(
1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/
```

```
2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/8*A+3/8*B)/sin(1/2*d*x+1/2*c)^2/(2*s
in(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 263, normalized size = 1.29

$$-\frac{2\left(5i\sqrt{2}(13A+21B)a^3\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(13A+21B)a^3\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21i\sqrt{2}(7A+9B)a^3\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}(7A+9B)a^3\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\left(42(7A+9B)a^3\cos(dx+c)^3+5(26A+21B)a^3\cos(dx+c)^2+21(3A+B)a^3\cos(dx+c)+15Aa^3\right)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{105\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] -2/105*(5*I*sqrt(2)*(13*A + 21*B)*a^3*cos(d*x + c)^4*weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(13*A + 21*B)*a^3*cos(d*x
 + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sq
rt(2)*(7*A + 9*B)*a^3*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(7*A + 9*B)*a^3*
cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c))) - (42*(7*A + 9*B)*a^3*cos(d*x + c)^3 + 5*(26*A + 21*B
)*a^3*cos(d*x + c)^2 + 21*(3*A + B)*a^3*cos(d*x + c) + 15*A*a^3)*sqrt(cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x
)
```

Mupad [B]
time = 2.67, size = 307, normalized size = 1.50

$$\frac{2\,B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{\frac{2\,A\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos(c+d\,x)^2\right)}{7}+\frac{6\,A\,a^3\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+d\,x)^2\right)}{5}+2\,A\,a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+2\,A\,a^3\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{1-\cos\left(c+d\,x\right)^2}}+\frac{6\,B\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(9/2),x)
```

```
[Out] (2*B*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + ((2*A*a^3*sin(c + d*x)*hypergeom(
[-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + (6*A*a^3*cos(c + d*x)*sin(c + d*x)*
hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/5 + 2*A*a^3*cos(c + d*x)^2*si
n(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 2*A*a^3*cos(c + d*
x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d
*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2)) + (6*B*a^3*sin(c + d*x)*hypergeom([-1
/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2
)) + (2*B*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*
cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a^3*sin(c + d*x)*hypergeo
m([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)
^2)^(1/2))
```

3.144 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$-\frac{4a^3(17A+21B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(11A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(23A+24B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^3(1\ldots}{\ldots}$$

```
[Out] -4/15*a^3*(17*A+21*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^3*(11*A+13*B)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/105*
a^3*(23*A+24*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+4/21*a^3*(11*A+13*B)*sin(d*x+
c)/d/cos(d*x+c)^(3/2)+2/9*a*A*(a+a*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(9
/2)+2/63*(13*A+9*B)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(7/2)+4/15
*a^3*(17*A+21*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3054, 3047, 3100, 2827, 2716, 2720, 2719}

$$\frac{4a^3(11A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(17A+21B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(11A+13B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^3(23A+24B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(13A+9B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{63d\cos^{\frac{7}{2}}(c+dx)}+\frac{4a^3(17A+21B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x]
```

```
[Out] (-4*a^3*(17*A + 21*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(11*A + 13
*B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(23*A + 24*B)*Sin[c + d*x])/
(105*d*Cos[c + d*x]^(5/2)) + (4*a^3*(11*A + 13*B)*Sin[c + d*x])/(21*d*Cos[c
 + d*x]^(3/2)) + (4*a^3*(17*A + 21*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]
]) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) +
 (2*(13*A + 9*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(63*d*Cos[c + d*x]^
(7/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+a\cos(c+\ldots}{\ldots}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(13A+9B)\left(a^3+\ldots\right.}{63d\,c\ldots}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(13A+9B)\left(a^3+\ldots\right.}{63d\,c\ldots}$$

$$= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))^2\sin\ldots}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))^2\sin\ldots}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^3(11A+13B)\sin(c+dx\ldots}{21d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^3(17A+21B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d} + \frac{4a^3(11A+13B)F\left(\frac{1}{2}\ldots\right.}{21d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.78, size = 967, normalized size = 4.08

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2)
,x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(((17*A + 21
*B)*Csc[c]*Sec[c])/(30*d) + (A*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(36*d) + (Se
c[c]*Sec[c + d*x]^4*(7*A*Sin[c] + 27*A*Sin[d*x] + 9*B*Sin[d*x]))/(252*d) +
(Sec[c]*Sec[c + d*x]*(55*A*Sin[c] + 65*B*Sin[c] + 119*A*Sin[d*x] + 147*B*Si
n[d*x]))/(210*d) + (Sec[c]*Sec[c + d*x]^3*(135*A*Sin[c] + 45*B*Sin[c] + 238
*A*Sin[d*x] + 189*B*Sin[d*x]))/(1260*d) + (Sec[c]*Sec[c + d*x]^2*(238*A*Sin
[c] + 189*B*Sin[c] + 330*A*Sin[d*x] + 390*B*Sin[d*x]))/(1260*d)) - (11*A*(a
 + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2])
 - (13*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
```

```
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 +
 Cot[c]^2]) + (17*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hy
pergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
 ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[
d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Ta
n[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]
^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))
/(60*d) + (7*B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hyperge
ometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
 ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]
*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^
2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 +
Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*
d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1150 vs. $2(265) = 530$.
time = 1.11, size = 1151, normalized size = 4.86

method	result	size
default	Expression too large to display	1151

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/5*(3/8
*A+3/8*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2
*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1
2*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/8*A+3/8*B)*(-1/6*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/
2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2)))+(3/8*A+1/8*B)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+
cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
```

```
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*A*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-
7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+1/8*B/sin(1/2*d*x+1/2*c)^2/(2
*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorith
m="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(11/2),
x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 283, normalized size = 1.19

$$-\frac{2\left(15i\sqrt{2}(11A+13B)a^3\cos(dx+c)^5\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-15i\sqrt{2}(11A+13B)a^3\cos(dx+c)^5\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21i\sqrt{2}(17A+21B)a^3\cos(dx+c)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}(17A+21B)a^3\cos(dx+c)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\left(42(17A+21B)a^3\cos(dx+c)^4+30(11A+13B)a^3\cos(dx+c)^3+7(34A+27B)a^3\cos(dx+c)^2+45(3A+B)a^3\cos(dx+c)+35Aa^3\right)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{315d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorith
m="fricas")
```

```
[Out] -2/315*(15*I*sqrt(2)*(11*A + 13*B)*a^3*cos(d*x + c)^5*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) - 15*I*sqrt(2)*(11*A + 13*B)*a^3*cos(d
*x + c)^5*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*
sqrt(2)*(17*A + 21*B)*a^3*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(17*A + 21*B
)*a^3*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c))) - (42*(17*A + 21*B)*a^3*cos(d*x + c)^4 + 30*(11
*A + 13*B)*a^3*cos(d*x + c)^3 + 7*(34*A + 27*B)*a^3*cos(d*x + c)^2 + 45*(3*
```

```
A + B)*a^3*cos(d*x + c) + 35*A*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos
(d*x + c)^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorith
m="giac")
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(11/2),
x)
```

Mupad [**B**]
time = 3.03, size = 552, normalized size = 2.33

$$\frac{2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)\left(\frac{19\,A\,a^3\sin(c+d\,x)}{\cos(c+d\,x)^{3/2}\sqrt{1-\cos(c+d\,x)^2}}+\frac{9\,A\,a^3\sin(c+d\,x)}{\cos(c+d\,x)^{7/2}\sqrt{1-\cos(c+d\,x)^2}}+\frac{25\,B\,a^3\sin(c+d\,x)}{\cos(c+d\,x)^{3/2}\sqrt{1-\cos(c+d\,x)^2}}+\frac{3\,B\,a^3\sin(c+d\,x)}{\cos(c+d\,x)^{7/2}\sqrt{1-\cos(c+d\,x)^2}}\right)}{21\,d}-\frac{8\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{7}{4};\ \cos(c+d\,x)^2\right)\left(\frac{34\,A\,a^3\sin(c+d\,x)}{\sqrt{\cos(c+d\,x)}\sqrt{1-\cos(c+d\,x)^2}}+\frac{5\,A\,a^3\sin(c+d\,x)}{\cos(c+d\,x)^{5/2}\sqrt{1-\cos(c+d\,x)^2}}+\frac{27\,B\,a^3\sin(c+d\,x)}{\sqrt{\cos(c+d\,x)}\sqrt{1-\cos(c+d\,x)^2}}\right)}{135\,d}+\frac{8\left(\frac{3\,A\,a^3\sin(c+d\,x)}{\cos(c+d\,x)^{3/2}\sqrt{1-\cos(c+d\,x)^2}}+\frac{B\,a^3\sin(c+d\,x)}{\cos(c+d\,x)^{3/2}\sqrt{1-\cos(c+d\,x)^2}}\right)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{5}{4};\ \cos(c+d\,x)^2\right)}{21\,d}+\frac{2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)\left(\frac{136\,A\,a^3\sin(c+d\,x)}{\sqrt{\cos(c+d\,x)}\sqrt{1-\cos(c+d\,x)^2}}+\frac{39\,A\,a^3\sin(c+d\,x)}{\cos(c+d\,x)^{5/2}\sqrt{1-\cos(c+d\,x)^2}}+\frac{5\,A\,a^3\sin(c+d\,x)}{\cos(c+d\,x)^{9/2}\sqrt{1-\cos(c+d\,x)^2}}+\frac{153\,B\,a^3\sin(c+d\,x)}{\sqrt{\cos(c+d\,x)}\sqrt{1-\cos(c+d\,x)^2}}+\frac{27\,B\,a^3\sin(c+d\,x)}{\cos(c+d\,x)^{5/2}\sqrt{1-\cos(c+d\,x)^2}}\right)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(11/2),x)
[Out] (2*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2)*((19*A*a^3*sin(c + d*x))/(co
s(c + d*x)^(3/2)*(1 - cos(c + d*x)^2)^(1/2)) + (9*A*a^3*sin(c + d*x))/(cos(
c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2)) + (25*B*a^3*sin(c + d*x))/(cos(c
 + d*x)^(3/2)*(1 - cos(c + d*x)^2)^(1/2)) + (3*B*a^3*sin(c + d*x))/(cos(c +
 d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))))/(21*d) - (8*hypergeom([-1/4, 1/2]
, 7/4, cos(c + d*x)^2)*((34*A*a^3*sin(c + d*x))/(cos(c + d*x)^(1/2)*(1 - co
s(c + d*x)^2)^(1/2)) + (5*A*a^3*sin(c + d*x))/(cos(c + d*x)^(5/2)*(1 - cos(
c + d*x)^2)^(1/2)) + (27*B*a^3*sin(c + d*x))/(cos(c + d*x)^(1/2)*(1 - cos(c
 + d*x)^2)^(1/2))))/(135*d) + (8*((3*A*a^3*sin(c + d*x))/(cos(c + d*x)^(3/2
)*(1 - cos(c + d*x)^2)^(1/2)) + (B*a^3*sin(c + d*x))/(cos(c + d*x)^(3/2)*(1
 - cos(c + d*x)^2)^(1/2)))*hypergeom([-3/4, 1/2], 5/4, cos(c + d*x)^2))/(21
*d) + (2*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2)*((136*A*a^3*sin(c + d*
x))/(cos(c + d*x)^(1/2)*(1 - cos(c + d*x)^2)^(1/2)) + (39*A*a^3*sin(c + d*x
))/(cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (5*A*a^3*sin(c + d*x))
/(cos(c + d*x)^(9/2)*(1 - cos(c + d*x)^2)^(1/2)) + (153*B*a^3*sin(c + d*x))
/(cos(c + d*x)^(1/2)*(1 - cos(c + d*x)^2)^(1/2)) + (27*B*a^3*sin(c + d*x))/
(cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))))/(45*d)
```

3.145 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=156

$$-\frac{3(5A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}+\frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{5(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad}-\frac{(5A-7B}{}$$

```
[Out] -3/5*(5*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))/a/d+5/3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-1/5*(5*A-7*B)*cos(d*
x+c)^(3/2)*sin(d*x+c)/a/d+(A-B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+
c))+5/3*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d
```

Rubi [A]

time = 0.13, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3056, 2827, 2715, 2720, 2719}

$$\frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(5A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(5A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad}+\frac{5(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-3*(5*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(5*a*d) + (5*(A - B)*EllipticF[(
c + d*x)/2, 2])/(3*a*d) + (5*(A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*
d) - ((5*A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) + ((A - B)*Cos[c
 + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)\right.\cdots}{\cdots}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(5A-7B)\int \cos^{\frac{5}{2}}(c+dx)\,d\cdots}{2a}$$

$$= \frac{5(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{(5A-7B)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}\cdots}{5ad}$$

$$= -\frac{3(5A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} + \frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{5(\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.67, size = 1182, normalized size = 7.58

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
```

```
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
(((21*I)/20)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
 (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((2*(5*A - 5*B + 10*A*Cos[c] - 16
*B*Cos[c])*Csc[c])/(5*d) + (4*(A - B)*Cos[d*x]*Sin[c])/(3*d) + (2*B*Cos[2*d
*x]*Sin[2*c])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Si
n[(d*x)/2]))/d + (4*(A - B)*Cos[c]*Sin[d*x])/(3*d) + (2*B*Cos[2*c]*Sin[2*d*
x])/(5*d)))/(a + a*Cos[c + d*x]) - (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d
*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Co
t[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]
]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (5*B*Cos[c/2 + (d*x)/2
]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]
^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*
Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d
*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A]
time = 0.31, size = 281, normalized size = 1.80

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(2\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+
1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25*A*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*A*EllipticE(cos(1/2*d*x+1/2*c),2^(
```

```
1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2)))+48*B*sin(1/2*d*x+1/2*c)^8+(-40*A-56*B)*sin(1/2*d*x+1/2*c)
^6+(90*A-30*B)*sin(1/2*d*x+1/2*c)^4+(-35*A+23*B)*sin(1/2*d*x+1/2*c)^2)/a/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 269, normalized size = 1.72

$$\frac{2\left(6B\cos\left(dx+c\right)^{2}+2\left(5A-2B\right)\cos\left(dx+c\right)+25A-25B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-25\left(\sqrt{2}\left(iA-iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(iA-iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-25\left(\sqrt{2}\left(-iA+iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-iA+iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-9\left(\sqrt{2}\left(5iA-7iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(5iA-7iB\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-9\left(\sqrt{2}\left(-5iA+7iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-5iA+7iB\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{30\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/30*(2*(6*B*cos(d*x + c)^2 + 2*(5*A - 2*B)*cos(d*x + c) + 25*A - 25*B)*sqr
t(cos(d*x + c))*sin(d*x + c) - 25*(sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(
2)*(I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) -
 25*(sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*(sqrt(2)*(5*I*A - 7*I*B)*
cos(d*x + c) + sqrt(2)*(5*I*A - 7*I*B))*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*(sqrt(2)*(-5*I*A + 7*I*B
)*cos(d*x + c) + sqrt(2)*(-5*I*A + 7*I*B))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)), x)
```

3.146 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=123

$$\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{(3A-5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad}+\frac{(A-B)\,\mathrm{c}}{d(a}$$

```
[Out] 3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a/d-1/3*(3*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+(A-B)*cos(d*x+c)^(3/2)*
sin(d*x+c)/d/(a+a*cos(d*x+c))-1/3*(3*A-5*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d
```

Rubi [A]

time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3056, 2827, 2719, 2715, 2720}

$$-\frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(3A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((3*A - 5*B)*EllipticF[(c + d
*x)/2, 2])/(3*a*d) - ((3*A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
+ ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int\sqrt{\cos(c+dx)}\,\left(\frac{3}{2}a(A-B)\right.}{}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(3A-5B)\int\cos^{\frac{3}{2}}(c+dx)\,dx}{2a}$$

$$= \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(3A-5B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{3ad}$$

$$= \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{(3A-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.58, size = 1129, normalized size = 9.18

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (((3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
```

```
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - (
((3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (C
os[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(A - B)*(1 + 2*Cos[c])*Csc[c])/
d + (4*B*Cos[d*x]*Sin[c])/(3*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*
x)/2] - B*Sin[(d*x)/2]))/d + (4*B*Cos[c]*Sin[d*x])/(3*d)))/(a + a*Cos[c + d
*x]) + (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*S
qrt[1 + Cot[c]^2]) - (5*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Co
t[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]])/(3*d*(a +
 a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A]

time = 0.30, size = 262, normalized size = 2.13

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(3\right.\right.}{3a\cos\left(\frac{dx}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/
2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*A*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))+8*B*sin(1/2*d*x+1/2*c)^6+(6*A-18*B)*sin(1/2*d*x+1/2*c)^4+(-3*A+
```

7*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 250, normalized size = 2.03

$$\frac{2\left(2B\cos\left(dx+c\right)-3A+5B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\sqrt{2}\left(3iA-5iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(3iA-5iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\left(\sqrt{2}\left(-3iA+5iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-3iA+5iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-9\left(\sqrt{2}\left(-iA+iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-iA+iB\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-9\left(\sqrt{2}\left(iA-iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(iA-iB\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{6\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
fricas")

[Out] 1/6*(2*(2*B*cos(d*x + c) - 3*A + 5*B)*sqrt(cos(d*x + c))*sin(d*x + c) + (sq
rt(2)*(3*I*A - 5*I*B)*cos(d*x + c) + sqrt(2)*(3*I*A - 5*I*B))*weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(-3*I*A + 5*I*B)*co
s(d*x + c) + sqrt(2)*(-3*I*A + 5*I*B))*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - 9*(sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt(2)*(-I*
A + I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c))) - 9*(sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B
))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c))))/(a*d*cos(d*x + c) + a*d)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)), x)
```

3.147 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=85

$$-\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}$$

```
[Out] -(A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a/d+(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+(A-B)*sin(d*x+c)*cos(d*x+c)^(1
/2)/d/(a+a*cos(d*x+c))
```

Rubi [A]

time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3056, 2827, 2720, 2719}

$$\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(((A - 3*B)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((A - B)*EllipticF[(c + d*
x)/2, 2])/(a*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c
 + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(A-B)-\frac{1}{2}a(A-3B)\cos(c+}{\sqrt{\cos(c+dx)}}}{a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(A-3B)\int \sqrt{\cos(c+d}}{2a}$$

$$= -\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.49, size = 1098, normalized size = 12.92

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((-1/4*I)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + ((
(3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperge
ometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(
1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*S
```

```
qrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E
^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric
2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^
((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
+ E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)
*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(a + a*Cos[c + d*x]) + (Co
s[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(-A + B + 2*B*Cos[c])*Csc[c])/d
+ (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d))/(a
+ a*Cos[c + d*x]) - (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Co
s[c + d*x])*Sqrt[1 + Cot[c]^2]) + (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x
- ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c
]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]
)/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A]
time = 0.29, size = 244, normalized size = 2.87

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\cdots\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\cdots\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 237, normalized size = 2.79

$$\frac{2\,(A-B)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\left(\sqrt{2}\,(-i\,A+i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(-i\,A+i\,B)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\left(\sqrt{2}\,(i\,A-i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(i\,A-i\,B)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+\left(\sqrt{2}\,(-i\,A+3i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(-i\,A+3i\,B)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+\left(\sqrt{2}\,(i\,A-3i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(i\,A-3i\,B)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/2*(2*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(-I*A + I*B)*cos(
d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + (sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (sqrt(2)*(-I*A
+ 3*I*B)*cos(d*x + c) + sqrt(2)*(-I*A + 3*I*B))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + (sqrt(2)*(I*A - 3*
I*B)*cos(d*x + c) + sqrt(2)*(I*A - 3*I*B))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)), x)
```

3.148 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=83

$$\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}$$

```
[Out] (A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a/d+(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)
/d/(a+a*cos(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3057, 2827, 2720, 2719}

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

```
[Out] ((A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((A + B)*EllipticF[(c + d*x)/2,
 2])/(a*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*
x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(A+B)+\frac{1}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}}{a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{(A-B)\int \sqrt{\cos(c+dx)}}{2a}$$

$$= \frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-B)}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.52, size = 1094, normalized size = 13.18

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

```
[Out] ((I/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - ((I/4
)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeometric
2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
```

```
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)
*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*
Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[c/2 +
 (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(A - B)*Csc[c])/d - (2*Sec[c/2]*Sec[c/2
 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d))/(a + a*Cos[c + d*x]) - (
A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 +
Cot[c]^2]) - (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d
*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A]
time = 0.28, size = 243, normalized size = 2.93

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\,(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 241, normalized size = 2.90

$$-\frac{2(A-B)\sqrt{\cos(dx+c)}\sin(dx+c)-\left(\sqrt{2}(-iA-iB)\cos(dx+c)+\sqrt{2}(-iA-iB)\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-\left(\sqrt{2}(iA+iB)\cos(dx+c)+\sqrt{2}(iA+iB)\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-\left(\sqrt{2}(iA-iB)\cos(dx+c)+\sqrt{2}(iA-iB)\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-\left(\sqrt{2}(-iA+iB)\cos(dx+c)+\sqrt{2}(-iA+iB)\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] -1/2*(2*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(-I*A - I*B)*cos
(d*x + c) + sqrt(2)*(-I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) - (sqrt(2)*(I*A + I*B)*cos(d*x + c) + sqrt(2)*(I*A + I*B))
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (sqrt(2)*(I*A
- I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - (sqrt(2)*(-I*A + I*B)*c
os(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos^{\frac{3}{2}}(c+dx)+\sqrt{\cos(c+dx)}}\,dx+\int \frac{B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(cos(c + d*x)**(3/2) + sqrt(cos(c + d*x))), x) + Integral(B*cos
(c + d*x)/(cos(c + d*x)**(3/2) + sqrt(cos(c + d*x))), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))), x)
```

3.149 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=119

$$-\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}$$

```
[Out] -(3*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a/d-(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+(3*A-B)*sin(d*x+c)/a/d/cos(d*x
+c)^(1/2)-(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3057, 2827, 2716, 2719, 2720}

$$-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] -(((3*A - B)*EllipticE[(c + d*x)/2, 2])/(a*d)) - ((A - B)*EllipticF[(c + d*
x)/2, 2])/(a*d) + ((3*A - B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A -
 B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx &= -\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}+\frac{\int \frac{\frac{1}{2}a(3A-B)-\frac{1}{2}a(A-B)\cos(c+d}{\cos^{\frac{3}{2}}(c+dx)}}{a^2} \\ &= -\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}-\frac{(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}}}{2a} \\ &= -\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A}{d\sqrt{\cos(c+}} \\ &= -\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-}{ad\sqrt{}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.74, size = 1130, normalized size = 9.50

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
```

```
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
((I/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[
c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(((2*A + A*Cos[c] - B*Cos[c])*Csc[c/2]*
Sec[c/2]*Sec[c])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin
[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/(a + a*Cos[c + d*x])
 + (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[
1 + Cot[c]^2]) - (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c
 + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A]
time = 0.39, size = 319, normalized size = 2.68

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d
```

```
*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A-B)*sin(1/2*d*x+1/2*c)^4+(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A-B)*sin(1/2*d*x+1/2*c)^2
)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 290, normalized size = 2.44

$$\frac{2\left((3A-B)\cos(dx+c)+2A\right)\sqrt{\cos(dx+c)}\,\sin(dx+c)+\left(\sqrt{2}\,(iA-iB)\cos(dx+c)^2+\sqrt{2}\,(iA-iB)\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\left(\sqrt{2}\,(-iA+iB)\cos(dx+c)^2+\sqrt{2}\,(-iA+iB)\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+\left(\sqrt{2}\,(-3iA+iB)\cos(dx+c)^2+\sqrt{2}\,(-3iA+iB)\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+\left(\sqrt{2}\,(3iA-iB)\cos(dx+c)^2+\sqrt{2}\,(3iA-iB)\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/2*(2*((3*A - B)*cos(d*x + c) + 2*A)*sqrt(cos(d*x + c))*sin(d*x + c) + (sq
rt(2)*(I*A - I*B)*cos(d*x + c)^2 + sqrt(2)*(I*A - I*B)*cos(d*x + c))*weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(-I*A + I*B)
*cos(d*x + c)^2 + sqrt(2)*(-I*A + I*B)*cos(d*x + c))*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + (sqrt(2)*(-3*I*A + I*B)*cos(d*x + c)^
2 + sqrt(2)*(-3*I*A + I*B)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + (sqrt(2)*(3*I*A - I*B)*co
s(d*x + c)^2 + sqrt(2)*(3*I*A - I*B)*cos(d*x + c))*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c
)^2 + a*d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))), x)
```

3.150 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=153

$$\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{(5A-3B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3(A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-$$

```
[Out] 3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a/d+1/3*(5*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+1/3*(5*A-3*B)*sin(d*x+c
)/a/d/cos(d*x+c)^(3/2)-(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
-3*(A-B)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3057, 2827, 2716, 2720, 2719}

$$\frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(5A-3B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3(A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((5*A - 3*B)*EllipticF[(c + d
*x)/2, 2])/(3*a*d) + ((5*A - 3*B)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
- (3*(A - B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A - B)*Sin[c + d*x]
)/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(5A-3B)-\frac{3}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}}{a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{(5A-3B)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{2a} -$$

$$= \frac{(5A-3B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3(A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A-B)\,\mathrm{s}}{d\cos^{\frac{3}{2}}(c+dx)(a}$$

$$= \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(5A-}{3ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.16, size = 1167, normalized size = 7.63

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (((3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
```

```
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - (
((3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (C
os[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(-(((A - B)*(2 + Cos[c])*Csc[c/2]*Se
c[c/2]*Sec[c])/d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[
(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c
 + d*x]*(A*Sin[c] - 3*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d)))/(a + a*Cos[c + d*
x]) - (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x]
)*Sqrt[1 + Cot[c]^2]) + (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a +
a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(195) = 390$.

time = 0.60, size = 466, normalized size = 3.05

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{(A-B)\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\right.}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((A-B)*(cos(1/
2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*(-1/6*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2)))+(-2*A+2*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 320, normalized size = 2.09

$$-\frac{2\left(9\left(A-B\right)\cos\left(dx+c\right)^{2}+2\left(2A-3B\right)\cos\left(dx+c\right)-2A\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\sqrt{2}\left(-5iA+3iB\right)\cos\left(dx+c\right)^{3}+\sqrt{2}\left(-5iA+3iB\right)\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-\left(\sqrt{2}\left(5iA-3iB\right)\cos\left(dx+c\right)^{3}+\sqrt{2}\left(5iA-3iB\right)\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+9\left(\sqrt{2}\left(-iA+iB\right)\cos\left(dx+c\right)^{3}+\sqrt{2}\left(-iA+iB\right)\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+9\left(\sqrt{2}\left(iA-iB\right)\cos\left(dx+c\right)^{3}+\sqrt{2}\left(iA-iB\right)\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{6\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/6*(2*(9*(A - B)*cos(d*x + c)^2 + 2*(2*A - 3*B)*cos(d*x + c) - 2*A)*sqrt(
cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(-5*I*A + 3*I*B)*cos(d*x + c)^3 + sqr
t(2)*(-5*I*A + 3*I*B)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - (sqrt(2)*(5*I*A - 3*I*B)*cos(d*x + c)^3 + sqrt(2)*(5
*I*A - 3*I*B)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c)) + 9*(sqrt(2)*(-I*A + I*B)*cos(d*x + c)^3 + sqrt(2)*(-I*A + I*B
)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) + 9*(sqrt(2)*(I*A - I*B)*cos(d*x + c)^3 + sqrt(2)*
(I*A - I*B)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^
2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))), x)
```

3.151 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=203

$$-\frac{7(5A-8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}+\frac{5(2A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{5(2A-3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d}-\frac{7(5A}{}$$

```
[Out] -7/5*(5*A-8*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))/a^2/d+5/3*(2*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-7/15*(5*A-8*
B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d+(2*A-3*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/
a^2/d/(1+cos(d*x+c))+1/3*(A-B)*cos(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c
))^2+5/3*(2*A-3*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.22, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3056, 2827, 2715, 2720, 2719}

$$\frac{5(2A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7(5A-8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}+\frac{(2A-3B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{7(5A-8B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{15a^2d}+\frac{5(2A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-7*(5*A - 8*B)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) + (5*(2*A - 3*B)*Ellip
ticF[(c + d*x)/2, 2])/(3*a^2*d) + (5*(2*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(3*a^2*d) - (7*(5*A - 8*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*
d) + ((2*A - 3*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]
)) + ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx &= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7}{2}a(A-B)-\frac{1}{2}a(5A-1\cdots}{a+a\cos(c+dx)}}{3a^2} \\
&= \frac{(2A-3B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(\cdots}{3d(a+a\cos(c+dx\cdots} \\
&= \frac{(2A-3B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(\cdots}{3d(a+a\cos(c+dx\cdots} \\
&= \frac{5(2A-3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d} - \frac{7(5A-8B)\cos^{\frac{3}{2}}(c+\cdots}{15a^2d} \\
&= -\frac{7(5A-8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} + \frac{5(2A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \cdots
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.93, size = 1262, normalized size = 6.22

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] (((-7*I)/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2
+ (((28*I)/5)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
2 - (20*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])
^2*Sqrt[1 + Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcT
an[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(
a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[
c + d*x]]*((4*(15*A - 20*B + 20*A*Cos[c] - 36*B*Cos[c])*Csc[c])/(5*d) + (8*
(A - 2*B)*Cos[d*x]*Sin[c])/(3*d) + (4*B*Cos[2*d*x]*Sin[2*c])/(5*d) + (4*Sec
[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] - 4*B*Sin[(d*x)/2]))/d - (2*Sec[
c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (8*(A
- 2*B)*Cos[c]*Sin[d*x])/(3*d) + (4*B*Cos[2*c]*Sin[2*d*x])/(5*d) - (2*(A - B
)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [A]
time = 0.36, size = 465, normalized size = 2.29

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(96B\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+80A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-352B\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNVE
```

```
RBOSE)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*B*cos(1/2
*d*x+1/2*c)^10+80*A*cos(1/2*d*x+1/2*c)^8-352*B*cos(1/2*d*x+1/2*c)^8+60*A*co
s(1/2*d*x+1/2*c)^6+100*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+210
*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+120*B*cos(1/2*d*x+1/2*c)^
6-150*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*B*cos(1/2*d*x+1/
2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-240*A*cos(1/2*d*x+1/2*c)^4+266*B*cos(1/2*d
*x+1/2*c)^4+105*A*cos(1/2*d*x+1/2*c)^2-135*B*cos(1/2*d*x+1/2*c)^2-5*A+5*B)/
a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 383, normalized size = 1.89

$$\frac{2\left(6 B\cos\left(dx+c\right)^3+2\left(5A-4B\right)\cos\left(dx+c\right)^2+\left(65A-94B\right)\cos\left(dx+c\right)+50A-75B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-25\left(\sqrt{2}\left(2iA-3iB\right)\cos\left(dx+c\right)^2+2\sqrt{2}\left(2iA-3iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(2iA-3iB\right)\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-25\left(\sqrt{2}\left(-2iA+3iB\right)\cos\left(dx+c\right)^2+2\sqrt{2}\left(-2iA+3iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-2iA+3iB\right)\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-21\left(\sqrt{2}\left(5iA-8iB\right)\cos\left(dx+c\right)^2+2\sqrt{2}\left(5iA-8iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(5iA-8iB\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-21\left(\sqrt{2}\left(-5iA+8iB\right)\cos\left(dx+c\right)^2+2\sqrt{2}\left(-5iA+8iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-5iA+8iB\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{30\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] 1/30*(2*(6*B*cos(d*x + c)^3 + 2*(5*A - 4*B)*cos(d*x + c)^2 + (65*A - 94*B)*
cos(d*x + c) + 50*A - 75*B)*sqrt(cos(d*x + c))*sin(d*x + c) - 25*(sqrt(2)*(
2*I*A - 3*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(2*I*A - 3*I*B)*cos(d*x + c) + sq
rt(2)*(2*I*A - 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 25*(sqrt(2)*(-2*I*A + 3*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-2*I*A + 3
*I*B)*cos(d*x + c) + sqrt(2)*(-2*I*A + 3*I*B))*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 21*(sqrt(2)*(5*I*A - 8*I*B)*cos(d*x + c)^2
+ 2*sqrt(2)*(5*I*A - 8*I*B)*cos(d*x + c) + sqrt(2)*(5*I*A - 8*I*B))*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
```

```
- 21*(sqrt(2)*(-5*I*A + 8*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-5*I*A + 8*I*B)*
cos(d*x + c) + sqrt(2)*(-5*I*A + 8*I*B))*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*
a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{7/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2, x)
```

3.152 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=166

$$\frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{5(A-2B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d}+\frac{(4A-7\cdots}{3a\cdots}$$

```
[Out] (4*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/a^2/d-5/3*(A-2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+1/3*(4*A-7*B)*cos(d
*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))+1/3*(A-B)*cos(d*x+c)^(5/2)*sin(
d*x+c)/d/(a+a*cos(d*x+c))^2-5/3*(A-2*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

Rubi [A]

time = 0.21, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3056, 2827, 2719, 2715, 2720}

$$-\frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{5(A-2B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((4*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*(A - 2*B)*EllipticF[(c
 + d*x)/2, 2])/(3*a^2*d) - (5*(A - 2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*a^2*d) + ((4*A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c
 + d*x])) + ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d
*x])^2)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx &= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{3}{2}a(A-3B)\right.}{a+a\cos(c+dx)}}{3a^2} \\
&= \frac{(4A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}+\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c}{3d(a+a\cos(c+dx))} \\
&= \frac{(4A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}+\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c}{3d(a+a\cos(c+dx))} \\
&= \frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5(A-2B)\sqrt{\cos(c+dx)}\sin(c+d}{3a^2d} \\
&= \frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{5(A}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.82, size = 1218, normalized size = 7.34

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] ((2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (((
7*I)/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + (1
0*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqr
t[1 + Cot[c]^2]) - (20*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a +
a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c +
d*x]]*((-4*(2*A - 3*B + 2*A*Cos[c] - 4*B*Cos[c])*Csc[c])/d + (8*B*Cos[d*x]*
Sin[c])/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(2*A*Sin[(d*x)/2] - 3*B*Sin[
(d*x)/2]))/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*
x)/2]))/(3*d) + (8*B*Cos[c]*Sin[d*x])/(3*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(204) = 408$.

time = 0.32, size = 435, normalized size = 2.62

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-16B\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNVE
```

```
RBOSE)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*B*cos(1/2*
d*x+1/2*c)^8+24*A*cos(1/2*d*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1
/2*d*x+1/2*c)^3+24*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*co
s(1/2*d*x+1/2*c)^6-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-42*B
*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1/2*c)^4+4
8*B*cos(1/2*d*x+1/2*c)^4+15*A*cos(1/2*d*x+1/2*c)^2-21*B*cos(1/2*d*x+1/2*c)^
2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 367, normalized size = 2.21

$$\frac{2\left(2B\cos(dx+c)^2-(6A-13B)\cos(dx+c)-5A+10B\right)\sqrt{\cos(dx+c)}\sin(dx+c)-5\left(\sqrt{2}(-iA+2iB)\cos(dx+c)^2+2\sqrt{2}(-iA+2iB)\cos(dx+c)+\sqrt{2}(-iA+2iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5\left(\sqrt{2}(iA-2iB)\cos(dx+c)^2+2\sqrt{2}(iA-2iB)\cos(dx+c)+\sqrt{2}(iA-2iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\left(\sqrt{2}(-4iA+7iB)\cos(dx+c)^2+2\sqrt{2}(-4iA+7iB)\cos(dx+c)+\sqrt{2}(-4iA+7iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\left(\sqrt{2}(4iA-7iB)\cos(dx+c)^2+2\sqrt{2}(4iA-7iB)\cos(dx+c)+\sqrt{2}(4iA-7iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] 1/6*(2*(2*B*cos(d*x + c)^2 - (6*A - 13*B)*cos(d*x + c) - 5*A + 10*B)*sqrt(c
os(d*x + c))*sin(d*x + c) - 5*(sqrt(2)*(-I*A + 2*I*B)*cos(d*x + c)^2 + 2*sq
rt(2)*(-I*A + 2*I*B)*cos(d*x + c) + sqrt(2)*(-I*A + 2*I*B))*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*(sqrt(2)*(I*A - 2*I*B)*cos(d
*x + c)^2 + 2*sqrt(2)*(I*A - 2*I*B)*cos(d*x + c) + sqrt(2)*(I*A - 2*I*B))*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(-4*I
*A + 7*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-4*I*A + 7*I*B)*cos(d*x + c) + sqrt
(2)*(-4*I*A + 7*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c))) - 3*(sqrt(2)*(4*I*A - 7*I*B)*cos(d*x + c)^2 +
2*sqrt(2)*(4*I*A - 7*I*B)*cos(d*x + c) + sqrt(2)*(4*I*A - 7*I*B))*weierstra
```

```
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(
a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2, x)
```

3.153 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=136

$$-\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(2A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))}+\frac{(A-B)\cos\ldots}{3d(\ldots}$$

```
[Out] -(A-4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a^2/d+1/3*(2*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+1/3*(A-B)*cos(d*x+
c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+1/3*(2*A-5*B)*sin(d*x+c)*cos(d*x+c
)^(1/2)/a^2/d/(1+cos(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3056, 2827, 2720, 2719}

$$\frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(2A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -(((A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((2*A - 5*B)*EllipticF[(
c + d*x)/2, 2])/(3*a^2*d) + ((2*A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3*a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*
d*(a + a*Cos[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-B)-\frac{1}{2}\right.}{a+a\cos(c+dx)}}{3a^2}$$

$$= \frac{(2A-5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin}{3d(a+a\cos(c+d}$$

$$= \frac{(2A-5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin}{3d(a+a\cos(c+d}$$

$$= -\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(2A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.70, size = 1184, normalized size = 8.71

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] ((-1/2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
```

)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + ((2*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (4*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*((-4*(-A + 2*B + 2*B*Cos[c])*Csc[c])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - 2*B*Sin[(d*x)/2]))/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(178) = 356$.
time = 0.32, size = 421, normalized size = 3.10

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(12A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*B*cos(1/2*d*x+1/2*c)^6-10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-24*B*cos(1/2*d*x+1/2*c)^3*(sin(1

```
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4+38*B*cos(1/2*d*x+1/2*c)^4+9*A
*cos(1/2*d*x+1/2*c)^2-15*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)
^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 352, normalized size = 2.59

$$\frac{2\left(3\left(A-2B\right)\cos\left(dx+c\right)+2A-5B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\sqrt{2}\left(-2iA+5iB\right)\cos\left(dx+c\right)^{2}-2\sqrt{2}\left(2iA-5iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-2iA+5iB\right)\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\left(\sqrt{2}\left(2iA-5iB\right)\cos\left(dx+c\right)^{2}-2\sqrt{2}\left(-2iA+5iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(2iA-5iB\right)\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-3\left(\sqrt{2}\left(iA-4iB\right)\cos\left(dx+c\right)^{2}+2\sqrt{2}\left(iA-4iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(iA-4iB\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-3\left(\sqrt{2}\left(-iA+4iB\right)\cos\left(dx+c\right)^{2}+2\sqrt{2}\left(-iA+4iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-iA+4iB\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] 1/6*(2*(3*(A - 2*B)*cos(d*x + c) + 2*A - 5*B)*sqrt(cos(d*x + c))*sin(d*x +
c) + (sqrt(2)*(-2*I*A + 5*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(2*I*A - 5*I*B)*c
os(d*x + c) + sqrt(2)*(-2*I*A + 5*I*B))*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + (sqrt(2)*(2*I*A - 5*I*B)*cos(d*x + c)^2 - 2*sqrt(2
)*(-2*I*A + 5*I*B)*cos(d*x + c) + sqrt(2)*(2*I*A - 5*I*B))*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(I*A - 4*I*B)*cos(d*
x + c)^2 + 2*sqrt(2)*(I*A - 4*I*B)*cos(d*x + c) + sqrt(2)*(I*A - 4*I*B))*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 3*(sqrt(2)*(-I*A + 4*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-I*A + 4*I*B)*
cos(d*x + c) + sqrt(2)*(-I*A + 4*I*B))*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^
2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2, x)
```

3.154 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=121

$$-\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))}+\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

```
[Out] -B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a^2/d+1/3*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+B*sin(d*x+c)*cos(d*x+c)^(1
/2)/a^2/d/(1+cos(d*x+c))+1/3*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d
*x+c))^2
```

Rubi [A]
time = 0.18, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3056, 3057, 2827, 2720, 2719}

$$\frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -((B*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((A + 2*B)*EllipticF[(c + d*x)/2
, 2])/(3*a^2*d) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d
*x])) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])
^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{1}{2}a(A+5B)\cos(c+\cdots}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+\cdots}}{3a^2}$$

$$= \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+\cdots}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+\cdots}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{B\sqrt{\cos(c+\cdots}}{a^2d(1+\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.56, size = 815, normalized size = 6.74

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] ((-1/2*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 -
(2*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sq
rt[1 + Cot[c]^2]) - (4*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a +
a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c +
d*x]]*((4*B*Csc[c])/d + (4*B*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d +
(2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) +
 (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(165) = 330$.
time = 0.33, size = 350, normalized size = 2.89

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{Ellipti}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+12*B*cos(1/2*d*x+1/2*c)^6+4*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
```

```
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4-20*B*cos(1/2*d*x+1/2*c)^4-3*A*cos(
1/2*d*x+1/2*c)^2+9*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 314, normalized size = 2.60

$$\frac{2\,(3\,B\cos(dx+c)+A+2\,B)\sqrt{\cos(dx+c)}\,\sin(dx+c)+\left(\sqrt{2}\,(-i\,A-2i\,B)\cos(dx+c)^{2}-2\,\sqrt{2}\,(i\,A+2i\,B)\cos(dx+c)+\sqrt{2}\,(-i\,A-2i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\left(\sqrt{2}\,(i\,A+2i\,B)\cos(dx+c)^{2}-2\,\sqrt{2}\,(-i\,A-2i\,B)\cos(dx+c)+\sqrt{2}\,(i\,A+2i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3\left(i\,\sqrt{2}\,B\cos(dx+c)^{2}+2i\,\sqrt{2}\,B\cos(dx+c)+i\,\sqrt{2}\,B\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3\left(-i\,\sqrt{2}\,B\cos(dx+c)^{2}-2i\,\sqrt{2}\,B\cos(dx+c)-i\,\sqrt{2}\,B\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{6\,\left(a^{2}d\cos(dx+c)^{2}+2\,a^{2}d\cos(dx+c)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] 1/6*(2*(3*B*cos(d*x + c) + A + 2*B)*sqrt(cos(d*x + c))*sin(d*x + c) + (sqrt
(2)*(-I*A - 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(I*A + 2*I*B)*cos(d*x + c) +
sqrt(2)*(-I*A - 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + (sqrt(2)*(I*A + 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-I*A - 2*I*B)*c
os(d*x + c) + sqrt(2)*(I*A + 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c)) - 3*(I*sqrt(2)*B*cos(d*x + c)^2 + 2*I*sqrt(2)*B*cos(d*x
 + c) + I*sqrt(2)*B)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*B*cos(d*x + c)^2 - 2*I*sqrt(2)*
B*cos(d*x + c) - I*sqrt(2)*B)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2, x)
```

3.155 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=121

$$\frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{A\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c}{3d(a+a\cos(c+dx))^2}$$

```
[Out] A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a^2/d+1/3*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-A*sin(d*x+c)*cos(d*x+c)^(1/
2)/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*
x+c))^2
```

Rubi [A]
time = 0.18, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3057, 2827, 2720, 2719}

$$\frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] (A*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((2*A + B)*EllipticF[(c + d*x)/2, 2
])/(3*a^2*d) - (A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]
)) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(5A+B)-\frac{1}{2}a(A-B)\cos}{\sqrt{\cos(c+dx)}\,(a+a\cos}}{3a^2}$$

$$= -\frac{A\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{A\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{A\sqrt{\cos(c+}}{a^2d(1+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.59, size = 815, normalized size = 6.74

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),
x]
```

```
[Out] ((I/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
```

```
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(a + a*Cos[c + d*x])^2 - (4*
A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[
1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*C
os[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x
]]*((-4*A*Csc[c])/d - (4*A*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (2
*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) - (
2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(165) = 330$.
time = 0.35, size = 350, normalized size = 2.89

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*d
*x+1/2*c)^6-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2*
d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^3-16*A*cos(1/2*d*x+1/2*c)^4-2*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1
/2*d*x+1/2*c)^2+3*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 318, normalized size = 2.63

$$-\frac{2\,(3\,A\cos(dx+c)+4\,A-B)\sqrt{\cos(dx+c)}\,\sin(dx+c)-\left(\sqrt{2}\,(-2i\,A-i\,B)\cos(dx+c)^2-2\,\sqrt{2}\,(2i\,A+i\,B)\cos(dx+c)+\sqrt{2}\,(-2i\,A-i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-\left(\sqrt{2}\,(2i\,A+i\,B)\cos(dx+c)^2-2\,\sqrt{2}\,(-2i\,A-i\,B)\cos(dx+c)+\sqrt{2}\,(2i\,A+i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+3\left(-i\,\sqrt{2}\,A\cos(dx+c)^2-2i\,\sqrt{2}\,A\cos(dx+c)-i\,\sqrt{2}\,A\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+3\left(i\,\sqrt{2}\,A\cos(dx+c)^2+2i\,\sqrt{2}\,A\cos(dx+c)+i\,\sqrt{2}\,A\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{6\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/6*(2*(3*A*cos(d*x + c) + 4*A - B)*sqrt(cos(d*x + c))*sin(d*x + c) - (sqr
t(2)*(-2*I*A - I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(2*I*A + I*B)*cos(d*x + c) +
 sqrt(2)*(-2*I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - (sqrt(2)*(2*I*A + I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-2*I*A - I*B)*
cos(d*x + c) + sqrt(2)*(2*I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*A*cos(d*x + c)^2 - 2*I*sqrt(2)*A*cos(d
*x + c) - I*sqrt(2)*A)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*A*cos(d*x + c)^2 + 2*I*sqrt(2)
*A*cos(d*x + c) + I*sqrt(2)*A)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2), x)
```

3.156 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=168

$$-\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{(5A-2B)\sin(c}{3a^2d\sqrt{\cos(c+dx)}\,(1+}$$

```
[Out] -(4*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a^2/d-1/3*(5*A-2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+(4*A-B)*sin(d*x+c)
/a^2/d/cos(d*x+c)^(1/2)-1/3*(5*A-2*B)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))/cos(d
*x+c)^(1/2)-1/3*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3057, 2827, 2716, 2719, 2720}

$$-\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{(5A-2B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(\cos(c+dx)+1)}-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] -(((4*A - B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) - ((5*A - 2*B)*EllipticF[(
c + d*x)/2, 2])/(3*a^2*d) + ((4*A - B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*
x]]) - ((5*A - 2*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c +
d*x])) - ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]
)^2)
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(7A-B)-\frac{3}{2}a(A-B)\cos(c\cdots}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx\cdots}}{3a^2}$$

$$= -\frac{(5A-2B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\cos(c+dx))} - \frac{(A-B)\sin(c+\cdots}{3d\sqrt{\cos(c+dx)}\,(a+a\cdots}$$

$$= -\frac{(5A-2B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\cos(c+dx))} - \frac{(A-B)\sin(c+\cdots}{3d\sqrt{\cos(c+dx)}\,(a+a\cdots}$$

$$= -\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(4A-B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(5\cdots}{3a^2d\sqrt{\cos\cdots}}$$

$$= -\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(4A\cdots}{a^2d\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.85, size = 1217, normalized size = 7.24

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),
x]
[Out] ((-2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + ((
I/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2
*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[
-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*
I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^
((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x
))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + (10*A
*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1
 + Cot[c]^2]) - (4*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Co
s[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]
]*((2*(2*A + 2*A*Cos[c] - B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/d + (2*Sec[c/
2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (4*Sec[c
/2]*Sec[c/2 + (d*x)/2]*(2*A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d + (8*A*Sec[c]
*Sec[c + d*x]*Sin[d*x])/d + (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)
))/(a + a*Cos[c + d*x])^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(208) = 416$.

time = 0.43, size = 494, normalized size = 2.94

method	result
default	$-\frac{2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(5A\,\mathrm{EllipticF}\left(\cos(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(4*A-B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A-10*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(37*A-7*B)*sin(1/2*d*x+1/2*c)^2)/
a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)),
 x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 407, normalized size = 2.42

$$\frac{2\left(3\left(4A-B\right)\cos\left(dx+c\right)^{2}+\left(19A-4B\right)\cos\left(dx+c\right)+6A\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\sqrt{2}\left(5iA-2iB\right)\cos\left(dx+c\right)^{3}-2\sqrt{2}\left(-5iA+2iB\right)\cos\left(dx+c\right)^{2}+\sqrt{2}\left(5iA-2iB\right)\cos\left(dx+c\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\left(\sqrt{2}\left(-5iA+2iB\right)\cos\left(dx+c\right)^{3}-2\sqrt{2}\left(5iA-2iB\right)\cos\left(dx+c\right)^{2}+\sqrt{2}\left(-5iA+2iB\right)\cos\left(dx+c\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-3\left(\sqrt{2}\left(4iA-iB\right)\cos\left(dx+c\right)^{3}+2\sqrt{2}\left(4iA-iB\right)\cos\left(dx+c\right)^{2}+\sqrt{2}\left(4iA-iB\right)\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-3\left(\sqrt{2}\left(-4iA+iB\right)\cos\left(dx+c\right)^{3}+2\sqrt{2}\left(-4iA+iB\right)\cos\left(dx+c\right)^{2}+\sqrt{2}\left(-4iA+iB\right)\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{6\left(a^{2}d\cos\left(dx+c\right)^{3}+2a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] 1/6*(2*(3*(4*A - B)*cos(d*x + c)^2 + (19*A - 4*B)*cos(d*x + c) + 6*A)*sqrt(
cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(5*I*A - 2*I*B)*cos(d*x + c)^3 - 2*sq
rt(2)*(-5*I*A + 2*I*B)*cos(d*x + c)^2 + sqrt(2)*(5*I*A - 2*I*B)*cos(d*x + c
))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(-5
*I*A + 2*I*B)*cos(d*x + c)^3 - 2*sqrt(2)*(5*I*A - 2*I*B)*cos(d*x + c)^2 + s
qrt(2)*(-5*I*A + 2*I*B)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) - 3*(sqrt(2)*(4*I*A - I*B)*cos(d*x + c)^3 + 2*sqrt(2)*
```

```
(4*I*A - I*B)*cos(d*x + c)^2 + sqrt(2)*(4*I*A - I*B)*cos(d*x + c))*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
 3*(sqrt(2)*(-4*I*A + I*B)*cos(d*x + c)^3 + 2*sqrt(2)*(-4*I*A + I*B)*cos(d*
x + c)^2 + sqrt(2)*(-4*I*A + I*B)*cos(d*x + c))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)
^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2), x)
```

3.157 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=201

$$\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{5(2A-B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{(7A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-$$

```
[Out] (7*A-4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/a^2/d+5/3*(2*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+5/3*(2*A-B)*sin(d*x
+c)/a^2/d/cos(d*x+c)^(3/2)-1/3*(7*A-4*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)/
(1+cos(d*x+c))-1/3*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2-(
7*A-4*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3057, 2827, 2716, 2720, 2719}

$$\frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A-4B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{5(2A-B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{(7A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] ((7*A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (5*(2*A - B)*EllipticF[(c
 + d*x)/2, 2])/(3*a^2*d) + (5*(2*A - B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]
^(3/2)) - ((7*A - 4*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - ((7*A - 4
*B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])) - ((A - B
)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}+\frac{\int \frac{\frac{3}{2}a(3A-B)-\frac{5}{2}a(A-B)\cos(c+}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx}}{3a^2}$$

$$= -\frac{(7A-4B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))}-\frac{(A-B)\sin(c+}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos}$$

$$= -\frac{(7A-4B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))}-\frac{(A-B)\sin(c+}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos}$$

$$= \frac{5(2A-B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{(7A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{(7A-}{3a^2d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2d}+\frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3a^2d}+\frac{5(2}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.49, size = 1258, normalized size = 6.26

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),
x]
```

```
[Out] (((7*I)/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 -
 ((2*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (2
0*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqr
t[1 + Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a +
a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c +
d*x]]*((-2*(4*A - 2*B + 3*A*Cos[c] - 2*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/
d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] - 2*B*Sin[(d*x)/2]))/d
 - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d
) + (8*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (8*Sec[c]*Sec[c + d*x]*(A*
Sin[c] - 6*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d) - (2*(A - B)*Sec[c/2 + (d*x)/2
]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(237) = 474$.
time = 0.72, size = 723, normalized size = 3.60

method	result	size
default	Expression too large to display	723

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*((4*A-2*
B)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*A*(-1/6*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2
*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2)))+(-8*A+4*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/
2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/3*(A-B)*(2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^6+20*sin(
1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1+sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 436, normalized size = 2.17

$$-\frac{2\left(3\left(7A-4B\right)\cos\left(dx+c\right)^{3}+\left(32A-19B\right)\cos\left(dx+c\right)^{2}+2\left(4A-3B\right)\cos\left(dx+c\right)-2A\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(\sqrt{2}\left(2iA-iB\right)\cos\left(dx+c\right)^{4}+2\sqrt{2}\left(2iA-iB\right)\cos\left(dx+c\right)^{3}+\sqrt{2}\left(2iA-iB\right)\cos\left(dx+c\right)^{2}\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\left(\sqrt{2}\left(-2iA+iB\right)\cos\left(dx+c\right)^{4}+2\sqrt{2}\left(-2iA+iB\right)\cos\left(dx+c\right)^{3}+\sqrt{2}\left(-2iA+iB\right)\cos\left(dx+c\right)^{2}\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+3\left(\sqrt{2}\left(-7iA+4iB\right)\cos\left(dx+c\right)^{4}+2\sqrt{2}\left(-7iA+4iB\right)\cos\left(dx+c\right)^{3}+\sqrt{2}\left(-7iA+4iB\right)\cos\left(dx+c\right)^{2}\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\left(\sqrt{2}\left(7iA-4iB\right)\cos\left(dx+c\right)^{4}+2\sqrt{2}\left(7iA-4iB\right)\cos\left(dx+c\right)^{3}+\sqrt{2}\left(7iA-4iB\right)\cos\left(dx+c\right)^{2}\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{6\left(a^{2}d\cos\left(dx+c\right)^{4}+2a^{2}d\cos\left(dx+c\right)^{3}+a^{2}d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] -1/6*(2*(3*(7*A - 4*B)*cos(d*x + c)^3 + (32*A - 19*B)*cos(d*x + c)^2 + 2*(4
*A - 3*B)*cos(d*x + c) - 2*A)*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(sqrt(2)*
```

```
(2*I*A - I*B)*cos(d*x + c)^4 + 2*sqrt(2)*(2*I*A - I*B)*cos(d*x + c)^3 + sqr
t(2)*(2*I*A - I*B)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 5*(sqrt(2)*(-2*I*A + I*B)*cos(d*x + c)^4 + 2*sqrt(2)*(-
2*I*A + I*B)*cos(d*x + c)^3 + sqrt(2)*(-2*I*A + I*B)*cos(d*x + c)^2)*weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(-7*I*A +
4*I*B)*cos(d*x + c)^4 + 2*sqrt(2)*(-7*I*A + 4*I*B)*cos(d*x + c)^3 + sqrt(2)
*(-7*I*A + 4*I*B)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(7*I*A - 4*I*B)*cos(d
*x + c)^4 + 2*sqrt(2)*(7*I*A - 4*I*B)*cos(d*x + c)^3 + sqrt(2)*(7*I*A - 4*I
*B)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3
+ a^2*d*cos(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2), x)
```

3.158 $\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=254

$$-\frac{7(17A-33B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(11A-21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{(11A-21B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a^3d}$$

```
[Out] -7/10*(17*A-33*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/2*(11*A-21*B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-7/30*(1
7*A-33*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a^3/d+1/5*(A-B)*cos(d*x+c)^(9/2)*sin(
d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(7*A-12*B)*cos(d*x+c)^(7/2)*sin(d*x+c)/a/d
/(a+a*cos(d*x+c))^2+3/10*(11*A-21*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a^3+a^3
*cos(d*x+c))+1/2*(11*A-21*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.36, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3056, 2827, 2715, 2720, 2719}

$$\frac{(11A-21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{7(17A-33B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{3(11A-21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{7(17A-33B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30a^3d}+\frac{(11A-21B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(7A-12B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-7*(17*A - 33*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((11*A - 21*B)*El
lipticF[(c + d*x)/2, 2])/(2*a^3*d) + ((11*A - 21*B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(2*a^3*d) - (7*(17*A - 33*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30
*a^3*d) + ((A - B)*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x
])^3) + ((7*A - 12*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c
 + d*x])^2) + (3*(11*A - 21*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3
+ a^3*Cos[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(\frac{9}{2}a(A-B)-\frac{5}{2}a(A-3B)\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(7A-12B)\cos^{\frac{7}{2}}(c+dx)\sin(}{15ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(7A-12B)\cos^{\frac{7}{2}}(c+dx)\sin(}{15ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(7A-12B)\cos^{\frac{7}{2}}(c+dx)\sin(}{15ad(a+a\cos(c+dx))}$$

$$= \frac{(11A-21B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} - \frac{7(17A-33B)\cos^{\frac{3}{2}}(c+}{30a^3d}$$

$$= -\frac{7(17A-33B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(11A-21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 7.23, size = 1346, normalized size = 5.30

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] (((-119*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*H
ypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqr
t[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d
*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*
(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeo
metric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(
1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*S
qrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^
((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])
^3 + (((231*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*
x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I
)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hype
rgeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
+ E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d
*x])^3 - (22*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*
x])^3*Sqrt[1 + Cot[c]^2]) + (42*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[C
os[c + d*x]]*((4*(59*A - 99*B + 60*A*Cos[c] - 132*B*Cos[c])*Csc[c])/(5*d) +
 (16*(A - 3*B)*Cos[d*x]*Sin[c])/(3*d) + (8*B*Cos[2*d*x]*Sin[2*c])/(5*d) + (
4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(59*A*Sin[(d*x)/2] - 99*B*Sin[(d*x)/2]))/(5*d
) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(19*A*Sin[(d*x)/2] - 24*B*Sin[(d*x)/2]
))/(15*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/
2]))/(5*d) + (16*(A - 3*B)*Cos[c]*Sin[d*x])/(3*d) + (8*B*Cos[2*c]*Sin[2*d*x
])/(5*d) - (4*(19*A - 24*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A -
 B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [A]
time = 0.37, size = 493, normalized size = 1.94

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(192B\left(\cos^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+160A\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-864B\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(192*B*cos(1/
2*d*x+1/2*c)^12+160*A*cos(1/2*d*x+1/2*c)^10-864*B*cos(1/2*d*x+1/2*c)^10+468
*A*cos(1/2*d*x+1/2*c)^8+330*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^
5+714*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-228*B*cos(1/2*d*x+1/
2*c)^8-630*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-1386*B*cos(1/2*
d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*A*cos(1/2*d*x+1/2*c)^6+1590*B*c
os(1/2*d*x+1/2*c)^6+474*A*cos(1/2*d*x+1/2*c)^4-744*B*cos(1/2*d*x+1/2*c)^4-4
7*A*cos(1/2*d*x+1/2*c)^2+57*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x
+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 495, normalized size = 1.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] 1/60*(2*(12*B*cos(d*x + c)^4 + 4*(5*A - 6*B)*cos(d*x + c)^3 + 3*(79*A - 147
*B)*cos(d*x + c)^2 + 2*(188*A - 357*B)*cos(d*x + c) + 165*A - 315*B)*sqrt(c
os(d*x + c))*sin(d*x + c) - 15*(sqrt(2)*(11*I*A - 21*I*B)*cos(d*x + c)^3 +
3*sqrt(2)*(11*I*A - 21*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(11*I*A - 21*I*B)*co
s(d*x + c) + sqrt(2)*(11*I*A - 21*I*B))*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - 15*(sqrt(2)*(-11*I*A + 21*I*B)*cos(d*x + c)^3 + 3*
sqrt(2)*(-11*I*A + 21*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-11*I*A + 21*I*B)*co
s(d*x + c) + sqrt(2)*(-11*I*A + 21*I*B))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) - 21*(sqrt(2)*(17*I*A - 33*I*B)*cos(d*x + c)^3 + 3*
sqrt(2)*(17*I*A - 33*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(17*I*A - 33*I*B)*cos(
d*x + c) + sqrt(2)*(17*I*A - 33*I*B))*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*(sqrt(2)*(-17*I*A + 33*I*
B)*cos(d*x + c)^3 + 3*sqrt(2)*(-17*I*A + 33*I*B)*cos(d*x + c)^2 + 3*sqrt(2)
*(-17*I*A + 33*I*B)*cos(d*x + c) + sqrt(2)*(-17*I*A + 33*I*B))*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3
*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{9/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(9/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(9/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

3.159 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=219

$$\frac{7(7A-17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-33B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(13A-33B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6a^3d}+\frac{(A$$

```
[Out] 7/10*(7*A-17*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/6*(13*A-33*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*(A-B)
*cos(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/3*(A-2*B)*cos(d*x+c)^(5
/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+7/30*(7*A-17*B)*cos(d*x+c)^(3/2)*sin(
d*x+c)/d/(a^3+a^3*cos(d*x+c))-1/6*(13*A-33*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a
^3/d
```

Rubi [A]
time = 0.33, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3056, 2827, 2719, 2715, 2720}

$$-\frac{(13A-33B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(7A-17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{7(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(13A-33B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6a^3d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(A-2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (7*(7*A - 17*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 33*B)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) - ((13*A - 33*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(6*a^3*d) + ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*C
os[c + d*x])^3) + ((A - 2*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*a*d*(a + a
*Cos[c + d*x])^2) + (7*(7*A - 17*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*d*
(a^3 + a^3*Cos[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx &= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7}{2}a(A-B)-\frac{1}{2}a(3A-1}{(a+a\cos(c+dx))^2}}{5a^2} \\
&= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c}{3ad(a+a\cos(c+dx))} \\
&= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c}{3ad(a+a\cos(c+dx))} \\
&= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c}{3ad(a+a\cos(c+dx))} \\
&= \frac{7(7A-17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-33B)\sqrt{\cos(c+dx)}\,\sin}{6a^3d} \\
&= \frac{7(7A-17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-33B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} -
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 7.05, size = 1306, normalized size = 5.96

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] (((49*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
 - (((119*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)
*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*
d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 +
E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x
])^3 + (26*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*
x])^3*Sqrt[1 + Cot[c]^2]) - (22*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[C
os[c + d*x]]*((-4*(29*A - 59*B + 20*A*Cos[c] - 60*B*Cos[c])*Csc[c])/(5*d) +
 (16*B*Cos[d*x]*Sin[c])/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(29*A*Sin[(d
*x)/2] - 59*B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(14*A
*Sin[(d*x)/2] - 19*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]
^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) + (16*B*Cos[c]*Sin[d*x])/(3*d)
+ (4*(14*A - 19*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(A - B)*Sec[c
/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [A]
time = 0.33, size = 465, normalized size = 2.12

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-160B\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+348A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130A\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-160*B*cos(1/
2*d*x+1/2*c)^10+348*A*cos(1/2*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
cos(1/2*d*x+1/2*c)^5+294*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-4
68*B*cos(1/2*d*x+1/2*c)^8-330*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c
)^5-714*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+
1/2*c)^6+1058*B*cos(1/2*d*x+1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-474*B*cos(1
/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2+47*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B
)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 478, normalized size = 2.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] 1/60*(2*(20*B*cos(d*x + c)^3 - 3*(29*A - 79*B)*cos(d*x + c)^2 - 2*(73*A - 1
88*B)*cos(d*x + c) - 65*A + 165*B)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(sqr
t(2)*(-13*I*A + 33*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-13*I*A + 33*I*B)*cos(d
*x + c)^2 + 3*sqrt(2)*(-13*I*A + 33*I*B)*cos(d*x + c) + sqrt(2)*(-13*I*A +
33*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*(sqr
t(2)*(13*I*A - 33*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(13*I*A - 33*I*B)*cos(d*x
 + c)^2 + 3*sqrt(2)*(13*I*A - 33*I*B)*cos(d*x + c) + sqrt(2)*(13*I*A - 33*I
*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*(sqrt(2
)*(-7*I*A + 17*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-7*I*A + 17*I*B)*cos(d*x +
c)^2 + 3*sqrt(2)*(-7*I*A + 17*I*B)*cos(d*x + c) + sqrt(2)*(-7*I*A + 17*I*B)
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) - 21*(sqrt(2)*(7*I*A - 17*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(7*I*A -
 17*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(7*I*A - 17*I*B)*cos(d*x + c) + sqrt(2)
*(7*I*A - 17*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{7/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

3.160 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=188

$$-\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(3A-8\ldots}{15\ldots}$$

```
[Out] -1/10*(9*A-49*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(3*A-13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*(A-B)
*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(3*A-8*B)*cos(d*x+c)
^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+1/6*(3*A-13*B)*sin(d*x+c)*cos(d*x+
c)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A]

time = 0.31, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3056, 2827, 2720, 2719}

$$\frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(3A-8B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -1/10*((9*A - 49*B)*EllipticE[(c + d*x)/2, 2])/(a^3*d) + ((3*A - 13*B)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) + ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x]
)/(5*d*(a + a*Cos[c + d*x])^3) + ((3*A - 8*B)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((3*A - 13*B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{1}{2}a(A-11B)\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\cos^{\frac{3}{2}}(c+dx)\sin(c}{15ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\cos^{\frac{3}{2}}(c+dx)\sin(c}{15ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\cos^{\frac{3}{2}}(c+dx)\sin(c}{15ad(a+a\cos(c+dx))}$$

$$= -\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.95, size = 1273, normalized size = 6.77

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] (((-9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
```

```
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
 + (((49*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*
Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sq
rt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*
d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d
*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperge
ometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E
^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]
)^3 - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^
3*Sqrt[1 + Cot[c]^2]) + (26*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Si
n[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*
(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos
[c + d*x]]*((-4*(-9*A + 29*B + 20*B*Cos[c])*Csc[c])/(5*d) + (4*Sec[c/2]*Sec
[c/2 + (d*x)/2]*(9*A*Sin[(d*x)/2] - 29*B*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]
*Sec[c/2 + (d*x)/2]^3*(9*A*Sin[(d*x)/2] - 14*B*Sin[(d*x)/2]))/(15*d) + (2*S
ec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) - (4*
(9*A - 14*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (
d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(224) = 448$.

time = 0.35, size = 451, normalized size = 2.40

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(108A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+30A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{d}{2}\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348*B*cos(1/2*d*x+1/2*c)^8-130*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-294*B*cos(1/2*d*x+1/2*c)^5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6+578*B*cos(1/2*d*x+1/2
*c)^6+114*A*cos(1/2*d*x+1/2*c)^4-264*B*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*
x+1/2*c)^2+37*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 467, normalized size = 2.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] 1/60*(2*(3*(9*A - 29*B)*cos(d*x + c)^2 + 2*(18*A - 73*B)*cos(d*x + c) + 15*
A - 65*B)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(sqrt(2)*(3*I*A - 13*I*B)*cos
(d*x + c)^3 + 3*sqrt(2)*(3*I*A - 13*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(3*I*A
- 13*I*B)*cos(d*x + c) + sqrt(2)*(3*I*A - 13*I*B))*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) - 5*(sqrt(2)*(-3*I*A + 13*I*B)*cos(d*x +
c)^3 + 3*sqrt(2)*(-3*I*A + 13*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-3*I*A + 13*
I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A + 13*I*B))*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(9*I*A - 49*I*B)*cos(d*x + c)^3
+ 3*sqrt(2)*(9*I*A - 49*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(9*I*A - 49*I*B)*co
s(d*x + c) + sqrt(2)*(9*I*A - 49*I*B))*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(sqrt(2)*(-9*I*A + 49*I*B
```

```
)*cos(d*x + c)^3 + 3*sqrt(2)*(-9*I*A + 49*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(
-9*I*A + 49*I*B)*cos(d*x + c) + sqrt(2)*(-9*I*A + 49*I*B))*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

3.161 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$-\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(A-6B)\sqrt{\cos}}{15ad(a+}$$

```
[Out] -1/10*(A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*(A-B)*cos(d
*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(A-6*B)*sin(d*x+c)*cos(d*x
+c)^(1/2)/a/d/(a+a*cos(d*x+c))^2+1/10*(A+9*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
/(a^3+a^3*cos(d*x+c))
```

Rubi [A]
time = 0.30, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3056, 3057, 2827, 2720, 2719}

$$\frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -1/10*((A + 9*B)*EllipticE[(c + d*x)/2, 2])/(a^3*d) + ((A + 3*B)*EllipticF[
(c + d*x)/2, 2])/(6*a^3*d) + ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d
*(a + a*Cos[c + d*x])^3) + ((A - 6*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*
a*d*(a + a*Cos[c + d*x])^2) + ((A + 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-B)+\frac{1}{2}\cdots\right)}{(a+a\cos(c+dx)\cdots}}{5a^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\,\sin\cdots}{15ad(a+a\cos(c+dx\cdots}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\,\sin\cdots}{15ad(a+a\cos(c+dx\cdots}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\,\sin\cdots}{15ad(a+a\cos(c+dx\cdots}$$

$$= -\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(A-\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 6.89, size = 1265, normalized size = 7.03

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] ((-1/10*I)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 -
 (((9*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
 - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[
Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^3
*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a +
 a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c +
 d*x]]*((4*(A + 9*B)*Csc[c])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(4*A*
Sin[(d*x)/2] - 9*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5
*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*
(A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/(5*d) + (4*(4*A - 9*B)*Sec[c/2 + (d*x)
/2]^2*Tan[c/2])/(15*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/
(a + a*Cos[c + d*x])^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(216) = 432$.
time = 0.33, size = 451, normalized size = 2.51

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}\cdots\right.\right.}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8+30*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*B*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6-198*B*cos(1/2*d*x+1/2*c)^6-
24*A*cos(1/2*d*x+1/2*c)^4+114*B*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c
)^2-27*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 465, normalized size = 2.58

$$\frac{2\left(3\left(A+9B\right)\cos\left(dx+c\right)^{2}+2\left(7A+18B\right)\cos\left(dx+c\right)+5A+15B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\left(\sqrt{2}\left(iA+3iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(iA+3iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(iA+3iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(iA+3iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-5\left(\sqrt{2}\left(-iA-3iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(-iA-3iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(-iA-3iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-iA-3iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-3\left(\sqrt{2}\left(iA+9iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(iA+9iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(iA+9iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(iA+9iB\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-3\left(\sqrt{2}\left(-iA-9iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(-iA-9iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(-iA-9iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-iA-9iB\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{60\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] 1/60*(2*(3*(A + 9*B)*cos(d*x + c)^2 + 2*(7*A + 18*B)*cos(d*x + c) + 5*A + 1
5*B)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(sqrt(2)*(I*A + 3*I*B)*cos(d*x + c
)^3 + 3*sqrt(2)*(I*A + 3*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*A + 3*I*B)*cos(
d*x + c) + sqrt(2)*(I*A + 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) - 5*(sqrt(2)*(-I*A - 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I
*A - 3*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-I*A - 3*I*B)*cos(d*x + c) + sqrt(2
)*(-I*A - 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 - 3*(sqrt(2)*(I*A + 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A + 9*I*B)*cos(d*
x + c)^2 + 3*sqrt(2)*(I*A + 9*I*B)*cos(d*x + c) + sqrt(2)*(I*A + 9*I*B))*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 3*(sqrt(2)*(-I*A - 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A - 9*I*B)*
cos(d*x + c)^2 + 3*sqrt(2)*(-I*A - 9*I*B)*cos(d*x + c) + sqrt(2)*(-I*A - 9*
I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 7319 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

3.162 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(A+4B)\sqrt{\cos(c+dx)}}{15ad(a+}$$

```
[Out] 1/10*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*(A-B)*sin(d*x+c)
*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^3+1/15*(A+4*B)*sin(d*x+c)*cos(d*x+c)^(
1/2)/a/d/(a+a*cos(d*x+c))^2-1/10*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a
^3*cos(d*x+c))
```

Rubi [A]
time = 0.29, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3056, 3057, 2827, 2720, 2719}

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((A - B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + B)*EllipticF[(c + d*
x)/2, 2])/(6*a^3*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a
*Cos[c + d*x])^3) + ((A + 4*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*(a
+ a*Cos[c + d*x])^2) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*d*(a^3
 + a^3*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{1}{2}a(3A+7B)\cos(c\ldots}{\sqrt{\cos(c+dx)}\,(a+a\cos(c\ldots}}{5a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A+4B)\sqrt{\cos(c+dx)}\,\mathrm{s}\ldots}{15ad(a+a\cos(c+\ldots}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A+4B)\sqrt{\cos(c+dx)}\,\mathrm{s}\ldots}{15ad(a+a\cos(c+\ldots}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A+4B)\sqrt{\cos(c+dx)}\,\mathrm{s}\ldots}{15ad(a+a\cos(c+\ldots}$$

$$= \frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{10a^3d} + \frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{6a^3d} + \frac{(A-B\ldots}{5\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 6.77, size = 1264, normalized size = 7.10

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] ((I/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 - ((
I/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeome
tric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((
2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1
[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2
*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E
^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*
x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 - (2*A
*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^3*Sqrt[1
 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Co
s[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]
]*((-4*(A - B)*Csc[c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/
2] - B*Sin[(d*x)/2]))/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)
/2] - B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x
)/2] + 4*B*Sin[(d*x)/2]))/(15*d) + (4*(A + 4*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/
2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c
 + d*x])^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(214) = 428$.

time = 0.32, size = 451, normalized size = 2.53

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8-10*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*B*cos(1/2*d*x+1/2*c)^5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6+2*B*cos(1/2*d*x+1/2*c)^6+6*A*
cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/2*c)^2-17*
B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 465, normalized size = 2.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(2*(3*(A - B)*cos(d*x + c)^2 + 2*(2*A - 7*B)*cos(d*x + c) - 5*A - 5*B
)*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(sqrt(2)*(I*A + I*B)*cos(d*x + c)^3 +
 3*sqrt(2)*(I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*A + I*B)*cos(d*x + c)
+ sqrt(2)*(I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) + 5*(sqrt(2)*(-I*A - I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A - I*B)*cos
(d*x + c)^2 + 3*sqrt(2)*(-I*A - I*B)*cos(d*x + c) + sqrt(2)*(-I*A - I*B))*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(-I*A
 + I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*
(-I*A + I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(I*A
- I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I
*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

3.163 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=182

$$\frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(6A-B)\sqrt{\cos}}{15ad(a+}$$

```
[Out] 1/10*(9*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(3*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*(A-B)*sin(d*
x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^3-1/15*(6*A-B)*sin(d*x+c)*cos(d*x+
c)^(1/2)/a/d/(a+a*cos(d*x+c))^2-1/10*(9*A+B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/
(a^3+a^3*cos(d*x+c))
```

Rubi [A]
time = 0.30, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3057, 2827, 2720, 2719}

$$\frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] ((9*A + B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + B)*EllipticF[(c
+ d*x)/2, 2])/(6*a^3*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a
 + a*Cos[c + d*x])^3) - ((6*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d
*(a + a*Cos[c + d*x])^2) - ((9*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*
d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(9A+B)-\frac{3}{2}a(A-B)\,\text{co}}{\sqrt{\cos(c+dx)}\,(a+a\,\text{co}}}{5a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B)\sqrt{\cos(c+dx}}{15ad(a+a\cos(c}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B)\sqrt{\cos(c+dx}}{15ad(a+a\cos(c}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B)\sqrt{\cos(c+dx}}{15ad(a+a\cos(c}$$

$$= \frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.85, size = 1265, normalized size = 6.95

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),
x]
```

```
[Out] (((9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
```

```
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
+ ((I/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 -
(2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^3*Sqrt
[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*
Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*
x]]*((-4*(9*A + B)*Csc[c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[
(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(6*A*S
in[(d*x)/2] - B*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(9*A
*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(5*d) - (4*(6*A - B)*Sec[c/2 + (d*x)/2]^2*
Tan[c/2])/(15*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a
*Cos[c + d*x])^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(218) = 436$.
time = 0.37, size = 451, normalized size = 2.48

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(108A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-30A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/2
*d*x+1/2*c)^8-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos(
1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8-10*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*B*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-138*A*cos(1/2*d*x+1/2*c)^6-22*B*cos(1/2*d*x+1/2*c)^6+
24*A*cos(1/2*d*x+1/2*c)^4+6*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*x+1/2*c)^2
+7*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 465, normalized size = 2.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/60*(2*(3*(9*A + B)*cos(d*x + c)^2 + 2*(33*A + 2*B)*cos(d*x + c) + 45*A -
 5*B)*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(sqrt(2)*(3*I*A + I*B)*cos(d*x +
c)^3 + 3*sqrt(2)*(3*I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(3*I*A + I*B)*cos
(d*x + c) + sqrt(2)*(3*I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 5*(sqrt(2)*(-3*I*A - I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-
3*I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-3*I*A - I*B)*cos(d*x + c) + sqrt(
2)*(-3*I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) + 3*(sqrt(2)*(-9*I*A - I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-9*I*A - I*B)*cos
(d*x + c)^2 + 3*sqrt(2)*(-9*I*A - I*B)*cos(d*x + c) + sqrt(2)*(-9*I*A - I*B
))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) + 3*(sqrt(2)*(9*I*A + I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(9*I*A + I*
B)*cos(d*x + c)^2 + 3*sqrt(2)*(9*I*A + I*B)*cos(d*x + c) + sqrt(2)*(9*I*A +
```

```
 I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos
(d*x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3), x)
```

3.164 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(49A-9B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{(A-B)\,\mathrm{s}}{5d\sqrt{\cos(c+dx)}\,(}$$

```
[Out] -1/10*(49*A-9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/6*(13*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/10*(49*
A-9*B)*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)-1/5*(A-B)*sin(d*x+c)/d/(a+a*cos(d*
x+c))^3/cos(d*x+c)^(1/2)-1/15*(8*A-3*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2/c
os(d*x+c)^(1/2)-1/6*(13*A-3*B)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))/cos(d*x+c)
^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3057, 2827, 2716, 2719, 2720}

$$-\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(49A-9B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{(13A-3B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3\cos(c+dx)+a^3)}-\frac{(8A-3B)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] -1/10*((49*A - 9*B)*EllipticE[(c + d*x)/2, 2])/(a^3*d) - ((13*A - 3*B)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) + ((49*A - 9*B)*Sin[c + d*x])/(10*a^3*d*Sq
rt[Cos[c + d*x]]) - ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*(a + a*C
os[c + d*x])^3) - ((8*A - 3*B)*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a
+ a*Cos[c + d*x])^2) - ((13*A - 3*B)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*
(a^3 + a^3*Cos[c + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3}+\frac{\int \frac{\frac{1}{2}a(11A-B)-\frac{5}{2}a(A-B)\cos(c}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx}}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3}-\frac{(8A-3B)\sin(}{15ad\sqrt{\cos(c+dx)}\,(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3}-\frac{(8A-3B)\sin(}{15ad\sqrt{\cos(c+dx)}\,(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3}-\frac{(8A-3B)\sin(}{15ad\sqrt{\cos(c+dx)}\,(a+}$$

$$= -\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(49A-9B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{}{5d\sqrt{c}}$$

$$= -\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 7.18, size = 1305, normalized size = 5.90

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),
x]
```

```
[Out] (((-49*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
3 + (((9*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*
Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sq
rt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*
d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d
*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperge
ometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E
^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]
)^3 + (26*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/
4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x
])^3*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*
(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos
[c + d*x]]*((2*(20*A + 29*A*Cos[c] - 9*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/
(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(29*A*Sin[(d*x)/2] - 9*B*Sin[(d*x)/2
]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(11*A*Sin[(d*x)/2] - 6*B*Sin[(
d*x)/2]))/(15*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin
[(d*x)/2]))/(5*d) + (16*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (4*(11*A - 6*B)
*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Ta
n[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(253) = 506$.

time = 0.45, size = 685, normalized size = 3.10

method	result
default	$-\frac{-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(65A\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x
+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(49*A-9*B)*
sin(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(817*A-147*B)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(248*A-43*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(439*A-69*B)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*
d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 521, normalized size = 2.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] 1/60*(2*(3*(49*A - 9*B)*cos(d*x + c)^3 + 2*(188*A - 33*B)*cos(d*x + c)^2 +
5*(59*A - 9*B)*cos(d*x + c) + 60*A)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(sq
rt(2)*(-13*I*A + 3*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(-13*I*A + 3*I*B)*cos(d*
x + c)^3 + 3*sqrt(2)*(-13*I*A + 3*I*B)*cos(d*x + c)^2 + sqrt(2)*(-13*I*A +
3*I*B)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) - 5*(sqrt(2)*(13*I*A - 3*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(13*I*A - 3*I*
B)*cos(d*x + c)^3 + 3*sqrt(2)*(13*I*A - 3*I*B)*cos(d*x + c)^2 + sqrt(2)*(13
*I*A - 3*I*B)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) - 3*(sqrt(2)*(49*I*A - 9*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(49*I*A
 - 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(49*I*A - 9*I*B)*cos(d*x + c)^2 + sqrt
(2)*(49*I*A - 9*I*B)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(sqrt(2)*(-49*I*A + 9*I*B)*co
s(d*x + c)^4 + 3*sqrt(2)*(-49*I*A + 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-49*
I*A + 9*I*B)*cos(d*x + c)^2 + sqrt(2)*(-49*I*A + 9*I*B)*cos(d*x + c))*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
+ a^3*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3), x)
```

3.165 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=254

$$\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(33A-13B)\sin(c+dx)}{6a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{7(17A-7B)\sin}{10a^3d\sqrt{\cos(c}}$$

```
[Out] 7/10*(17*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(33*A-13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(33*A
-13*B)*sin(d*x+c)/a^3/d/cos(d*x+c)^(3/2)-1/5*(A-B)*sin(d*x+c)/d/cos(d*x+c)^
(3/2)/(a+a*cos(d*x+c))^3-1/3*(2*A-B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*c
os(d*x+c))^2-7/30*(17*A-7*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a^3+a^3*cos(d*x
+c))-7/10*(17*A-7*B)*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3057, 2827, 2716, 2720, 2719}

$$\frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{7(17A-7B)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(33A-13B)\sin(c+dx)}{6a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{7(17A-7B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{(2A-B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] (7*(17*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((33*A - 13*B)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) + ((33*A - 13*B)*Sin[c + d*x])/(6*a^3*d*Co
s[c + d*x]^(3/2)) - (7*(17*A - 7*B)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*
x]]) - ((A - B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^
3) - ((2*A - B)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]
)^2) - (7*(17*A - 7*B)*Sin[c + d*x])/(30*d*Cos[c + d*x]^(3/2)*(a^3 + a^3*Co
s[c + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(13A-3B)-\frac{7}{2}a(A-B)\cos(c\cdots}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\cdots}}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{(2A-B)\sin(c+\cdots}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}\cdots}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{(2A-B)\sin(c+\cdots}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}\cdots}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{(2A-B)\sin(c+\cdots}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}\cdots}$$

$$= \frac{(33A-13B)\sin(c+dx)}{6a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{7(17A-7B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{(A\cdots}{5d\cos^{\frac{3}{2}}(c\cdots}$$

$$= \frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.95, size = 1346, normalized size = 5.30

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),
x]
```

```
[Out] (((119*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
3 - (((49*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)
*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*
d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 +
E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x
])^3 - (22*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x]
)^3*Sqrt[1 + Cot[c]^2]) + (26*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*
d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[C
os[c + d*x]]*((-2*(60*A - 20*B + 59*A*Cos[c] - 29*B*Cos[c])*Csc[c/2]*Sec[c/
2]*Sec[c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(59*A*Sin[(d*x)/2] - 29*B
*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(16*A*Sin[(d*x)/2]
 - 11*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*
x)/2] - B*Sin[(d*x)/2]))/(5*d) + (16*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d
) + (16*Sec[c]*Sec[c + d*x]*(A*Sin[c] - 9*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d)
 - (4*(16*A - 11*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(A - B)*Sec[
c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 875 vs. 2(282) = 564.
time = 0.63, size = 876, normalized size = 3.45

method	result	size
default	Expression too large to display	876

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60*(4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(165*A*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-357*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-10*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(165*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*A*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(165*A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-357*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(165*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*A*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-168*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(17*A-7*B)*sin(1/2*d*x+1/2*c)^10+8*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(1167*A-482*B)*sin(1/2*d*x+1/2*c)
^8-10*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(1111*A-461*B)*s
in(1/2*d*x+1/2*c)^6+14*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(404*A-169*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(1029*A-439*B)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)^5
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 548, normalized size = 2.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(2*(21*(17*A - 7*B)*cos(d*x + c)^4 + 2*(453*A - 188*B)*cos(d*x + c)^3
 + 5*(139*A - 59*B)*cos(d*x + c)^2 + 60*(2*A - B)*cos(d*x + c) - 20*A)*sqrt
(cos(d*x + c))*sin(d*x + c) + 5*(sqrt(2)*(33*I*A - 13*I*B)*cos(d*x + c)^5 +
 3*sqrt(2)*(33*I*A - 13*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(33*I*A - 13*I*B)*c
os(d*x + c)^3 + sqrt(2)*(33*I*A - 13*I*B)*cos(d*x + c)^2)*weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)*(-33*I*A + 13*I*B)*co
s(d*x + c)^5 + 3*sqrt(2)*(-33*I*A + 13*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(-33
*I*A + 13*I*B)*cos(d*x + c)^3 + sqrt(2)*(-33*I*A + 13*I*B)*cos(d*x + c)^2)*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*(sqrt(2)*(-1
7*I*A + 7*I*B)*cos(d*x + c)^5 + 3*sqrt(2)*(-17*I*A + 7*I*B)*cos(d*x + c)^4
+ 3*sqrt(2)*(-17*I*A + 7*I*B)*cos(d*x + c)^3 + sqrt(2)*(-17*I*A + 7*I*B)*co
s(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c))) + 21*(sqrt(2)*(17*I*A - 7*I*B)*cos(d*x + c)^5 + 3*sqrt
(2)*(17*I*A - 7*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(17*I*A - 7*I*B)*cos(d*x +
c)^3 + sqrt(2)*(17*I*A - 7*I*B)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)
^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^3), x)
```

3.166 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=221

$$\frac{5\sqrt{a}\,(8A+7B)\mathrm{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d}+\frac{5a(8A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}}+\frac{5a(8A+7B)\,\mathrm{c}}{96d\sqrt{a}}$$

```
[Out] 5/64*(8*A+7*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+
5/96*a*(8*A+7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/24*
a*(8*A+7*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a*B*co
s(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+5/64*a*(8*A+7*B)*sin(d*x
+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3060, 2849, 2853, 222}

$$\frac{5\sqrt{a}\,(8A+7B)\mathrm{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a(8A+7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{5a(8A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{5a(8A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{64d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (5*Sqrt[a]*(8*A + 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(64*d) + (5*a*(8*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqrt[
a + a*Cos[c + d*x]]) + (5*a*(8*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(9
6*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(8*A + 7*B)*Cos[c + d*x]^(5/2)*Sin[c + d
*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(7/2)*Sin[c + d*x]
)/(4*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{1}{8}(8A+7B)\int$$

$$= \frac{a(8A+7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos}{4d}$$

$$= \frac{5a(8A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a(8A}{}$$

$$= \frac{5a(8A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{5a}{}$$

$$= \frac{5a(8A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{5a}{}$$

$$= \frac{5\sqrt{a}\,(8A+7B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d}$$

Mathematica [**A**]

time = 1.07, size = 135, normalized size = 0.61

$$\frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(15\sqrt{2}\,(8A+7B)\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\,(152A+133B+2(40A+53B)\cos(c+dx)+4(8A+7B)\cos(2(c+dx))+12B\cos(3(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(8*A + 7*B)*ArcSin
[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(152*A + 133*B + 2*(40*A
+ 53*B)*Cos[c + d*x] + 4*(8*A + 7*B)*Cos[2*(c + d*x)] + 12*B*Cos[3*(c + d*x
)])*Sin[(c + d*x)/2]))/(384*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(189) = 378.
time = 3.00, size = 428, normalized size = 1.94

method	result
default	$\frac{(-1+\cos(dx+c))^4\left(64A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^3(dx+c))\sin(dx+c)+144A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^2(dx+c))\sin(dx+c)+48B\sin(dx+c)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/192/d*(-1+cos(d*x+c))^4*(64*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c
)^3*sin(d*x+c)+144*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*sin(d*x
+c)+48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+200*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+c)+56*B*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+120*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(3/2)*sin(d*x+c)+70*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x
+c)^2+105*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+120*A*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+1
05*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*
x+c))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(5/2)/sin(d*x+c)^8/(cos(d*x+c)/(1
+cos(d*x+c)))^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 8220 vs. 2(189) = 378.
time = 1.08, size = 8220, normalized size = 37.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*si
n(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(
arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) -
cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2
*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
```

```
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*A + (2*(cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)^(3/4)*((156*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - 2*cos(4*d*
x + 4*c) + 1)*sin(4*d*x + 4*c))*cos(1/2*arctan2...
```

Fricas [A]
time = 0.45, size = 151, normalized size = 0.68

$$\frac{\left(48\,B\cos\left(dx+c\right)^{3}+8\left(8\,A+7\,B\right)\cos\left(dx+c\right)^{2}+10\left(8\,A+7\,B\right)\cos\left(dx+c\right)+120\,A+105\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-15\left(\left(8\,A+7\,B\right)\cos\left(dx+c\right)+8\,A+7\,B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)}{192\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/192*((48*B*cos(d*x + c)^3 + 8*(8*A + 7*B)*cos(d*x + c)^2 + 10*(8*A + 7*B)
*cos(d*x + c) + 120*A + 105*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c) - 15*((8*A + 7*B)*cos(d*x + c) + 8*A + 7*B)*sqrt(a)*arctan(sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*
x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

3.167 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=176

$$\frac{\sqrt{a}\,(6A+5B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}+\frac{a(6A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}+\frac{a(6A+5B)\cos^{\frac{3}{2}}(}{12d\sqrt{a+a}}$$

```
[Out] 1/8*(6*A+5*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+1
/12*a*(6*A+5*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a*
B*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/8*a*(6*A+5*B)*sin(
d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3060, 2849, 2853, 222}

$$\frac{\sqrt{a}\,(6A+5B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(6A+5B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{a(6A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(8*d) + (a*(6*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*(6*A + 5*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt
[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a +
 a*Cos[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{6}(6A+5B)\int$$

$$= \frac{a(6A+5B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{aB\,\mathrm{co}}{3d}$$

$$= \frac{a(6A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a(6}{}$$

$$= \frac{a(6A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a(6}{}$$

$$= \frac{\sqrt{a}\,(6A+5B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}$$

Mathematica [A]
time = 0.58, size = 118, normalized size = 0.67

$$\frac{\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}\,(6A+5B)\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\,(18A+19B+2(6A+5B)\cos(c+dx)+4B\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(6*A + 5*B)*ArcSin[
Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(18*A + 19*B + 2*(6*A + 5*
B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(150) = 300.
time = 0.37, size = 356, normalized size = 2.02

method	result
default	$-\frac{(-1+\cos(dx+c))^3\left(12A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^2(dx+c))\sin(dx+c)+30A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\cos(dx+c)\sin(dx+c)+8B\sin(dx+c)\sqrt{\ldots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^3*(12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c
)^2*sin(d*x+c)+30*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+c)
+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+18*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+10*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)^2+15*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)+18*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c))*cos(d*x+c)+15*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c))*cos(d*x+c))*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
^6/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2981 vs. 2(150) = 300.
time = 0.83, size = 2981, normalized size = 16.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
```

```
2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - 1)))*A + (4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin
(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 5*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
- 4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sq
rt(a)*(arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(s
```

```
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2
(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(co
s(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*...
```

Fricas [A]
time = 0.41, size = 134, normalized size = 0.76

$$\frac{\left(8\,B\cos\left(dx+c\right)^{2}+2\left(6\,A+5\,B\right)\cos\left(dx+c\right)+18\,A+15\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-3\left(\left(6\,A+5\,B\right)\cos\left(dx+c\right)+6\,A+5\,B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/24*((8*B*cos(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 18*A + 15*B)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((6*A + 5*B)*cos(d*
x + c) + 6*A + 5*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

3.168 $\int \sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{a}\,(4A+3B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}+\frac{a(4A+3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{aB\cos^{\frac{3}{2}}(c+dx)\,s}{2d\sqrt{a+a\cos(}}$$

```
[Out] 1/4*(4*A+3*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+1
/2*a*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a*(4*A+3*B)
*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3060, 2849, 2853, 222}

$$\frac{\sqrt{a}\,(4A+3B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(4A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(4*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(4*d) + (a*(4*A + 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[
c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}+\frac{1}{4}(4A+3B)$$

$$= \frac{a(4A+3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{aI}{}$$

$$= \frac{a(4A+3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{aI}{}$$

$$= \frac{\sqrt{a}\,(4A+3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right.}{4d}$$

Mathematica [A]
time = 0.32, size = 100, normalized size = 0.76

$$\frac{\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\,(4A+3B)\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\,(4A+3B+2B\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(4*A + 3*B)*ArcSin[Sq
rt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(4*A + 3*B + 2*B*Cos[c + d*x
])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(111) = 222$.
time = 0.32, size = 284, normalized size = 2.17

method	result
default	$\frac{(-1+\cos(dx+c))^2 \left(4A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}} \cos(dx+c)\sin(dx+c)+4A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}} \sin(dx+c)+2B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ (\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))^2*(4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*s
in(d*x+c)+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+2*B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+3*B*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*cos(d*x+c)+4*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+3*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))
^(1/2)/sin(d*x+c)^4/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1851 vs. $2(111) = 222$.
time = 0.71, size = 1851, normalized size = 14.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
```

```
s(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A + (2*(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2
*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x +
 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d
*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2
*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - cos(2*d*
x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqr
t(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

Fricas [A]
time = 0.41, size = 117, normalized size = 0.89

$$\frac{(2\,B\cos{(dx+c)}+4\,A+3\,B)\sqrt{a\cos{(dx+c)}+a}\,\sqrt{\cos{(dx+c)}}\,\sin{(dx+c)}-((4\,A+3\,B)\cos{(dx+c)}+4\,A+3\,B)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos{(dx+c)}+a}\,\sqrt{\cos{(dx+c)}}}{\sqrt{a}\,\sin{(dx+c)}}\right)}{4\,(d\cos{(dx+c)}+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/4*((2*B*cos(d*x + c) + 4*A + 3*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))*sin(d*x + c) - ((4*A + 3*B)*cos(d*x + c) + 4*A + 3*B)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(
d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cos{(c+dx)}+1\right)}\,(A+B\cos{(c+dx)})\,\sqrt{\cos{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sqrt(cos(c + d*x))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos{(c+d\,x)}}\,\left(A+B\,\cos{(c+d\,x)}\right)\,\sqrt{a+a\,\cos{(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

3.169 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{a}\,(2A+B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{aB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

```
[Out] (2*A+B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+a*B*sin
(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3060, 2853, 222}

$$\frac{\sqrt{a}\,(2A+B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(2*A + B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])
/d + (a*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
```

```
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{aB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{1}{2}(2A+B)\int \frac{\sqrt{a+a\,\mathrm{c}}}{\sqrt{\cos(}}$$

$$= \frac{aB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(2A+B)\mathrm{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{}$$

$$= \frac{\sqrt{a}\,(2A+B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{aB\sqrt{\cos}}{d\sqrt{a}}$$

Mathematica [A]
time = 0.17, size = 83, normalized size = 1.06

$$\frac{\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\,(2A+B)\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sqrt{\cos(c+dx)}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(2*A + B)*ArcSin[Sqrt
[2]*Sin[(c + d*x)/2]] + 2*B*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(68) = 136$.
time = 0.39, size = 164, normalized size = 2.10

method	result
default	$-\frac{(-1+\cos(dx+c))\left(B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+2A\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+B\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+}{1+\cos(dx}}}{\cos(dx+c)}\right.\right.}{d\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/d*(-1+cos(d*x+c))*(B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(68) = 136$.
time = 0.66, size = 939, normalized size = 12.04

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/4*(4*A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2

```
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

Fricas [A]
time = 0.42, size = 97, normalized size = 1.24

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}\,B\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\left((2\,A+B)\cos\left(dx+c\right)+2\,A+B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A + B)*co
s(d*x + c) + 2*A + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\left(A+B\cos\left(c+dx\right)\right)}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/sqrt(cos(c + d*x))
, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2), x
)
```

3.170 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{2\sqrt{a}\,B\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+2*a*A*sin(d
*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3059, 2853, 222}

$$\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}\,B\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a]*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
```

```
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + B\int \frac{\sqrt{a+a\cos(c}}{\sqrt{\cos(c+}}$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} - \frac{(2B)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{}$$

$$= \frac{2\sqrt{a}\,B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2aA\,\text{si}}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.19, size = 86, normalized size = 1.13

$$\frac{\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\,B\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2A\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*A*Sin[(c + d*x)/2]))/(d*Sqrt[Cos[c + d
*x]])
```

Maple [A]
time = 0.38, size = 109, normalized size = 1.43

method	result

default	$-\frac{2 \sqrt{a \left(1+\cos \left(d x +c \right)\right)}\, \left(-B \sqrt{\frac{\cos \left(d x +c \right)}{1+\cos \left(d x +c \right)}}\, \arctan \left(\frac{\sin \left(d x +c \right) \sqrt{\frac{\cos \left(d x +c \right)}{1+\cos \left(d x +c \right)}}}{\cos \left(d x +c \right)}\right) \sin \left(d x +c \right)+A \cos \left(d x +c \right)-A \right)}{d \sin \left(d x +c \right) \sqrt{\cos \left(d x +c \right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/d*(a*(1+cos(d*x+c)))^(1/2)*(-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+A*cos(d
*x+c)-A)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(66) = 132$.
time = 0.61, size = 245, normalized size = 3.22

$$\frac{B\sqrt{a}\arctan\left(\left(\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)\right)+\sin\left(dx+c\right),\left(\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)\right)+\cos\left(dx+c\right)\right)+\frac{2\,A\left(\frac{\sqrt{2}\sqrt{a}\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{\sqrt{2}\sqrt{a}\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}\right)}{\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)^{\frac{3}{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] (B*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos
(d*x + c)) + 2*A*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)))/d
```

Fricas [A]
time = 0.38, size = 109, normalized size = 1.43

$$\frac{2\left(\sqrt{a\cos\left(dx+c\right)+a}\,A\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\left(B\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin\left(dx+c\right)}\right)\right)}{d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 2*(sqrt(a*cos(d*x + c) + a)*A*sqrt(cos(d*x + c))*sin(d*x + c) - (B*cos(d*x
+ c)^2 + B*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\left(A+B\cos\left(c+dx\right)\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/cos(c + d*x)**(3/2
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2), x
)
```

3.171 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a(2A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 2/3*a*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+2/3*a*(2*A+3*B
)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3059, 2850}

$$\frac{2a(2A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
*a*(2*A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x
]])
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{3}(2A+3B)\int \frac{\sqrt{\cdot}}{}$$
$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(2A+3B}{3d\sqrt{\cos(c+dx)}\,\sqrt{}}$$

Mathematica [A]
time = 0.17, size = 57, normalized size = 0.67

$$\frac{2\sqrt{a(1+\cos(c+dx))}\,(A+(2A+3B)\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(A + (2*A + 3*B)*Cos[c + d*x])*Tan[(c + d*x)/
2])/(3*d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.50, size = 62, normalized size = 0.73

method	result	size
default	$-\frac{2(-1+\cos(dx+c))(2A\cos(dx+c)+3B\cos(dx+c)+A)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}}{3d\sin(dx+c)\cos(dx+c)^{\frac{3}{2}}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(2*A*cos(d*x+c)+3*B*cos(d*x+c)+A)*(a*(1+cos(d*x+c)))
^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(73) = 146.
time = 0.54, size = 289, normalized size = 3.40

$$\frac{2\left(\frac{3B\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}+\frac{A\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*B*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c) + 1) + 1
)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)) + A*(3*sqrt(2)*sqrt(a
)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)*(sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) +
1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [**A**]
time = 0.38, size = 67, normalized size = 0.79

$$\frac{2\left(\left(2\,A+3\,B\right)\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 2/3*((2*A + 3*B)*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\left(A+B\cos\left(c+dx\right)\right)}{\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/cos(c + d*x)**(5/2
), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

[Out] Timed out

Mupad [B]
time = 1.56, size = 112, normalized size = 1.32

$$\frac{2\sqrt{a\left(\cos\left(c+d\,x\right)+1\right)}\,\left(2\,A\,\sin\left(c+d\,x\right)+3\,B\,\sin\left(c+d\,x\right)+2\,A\,\sin\left(2\,c+2\,d\,x\right)+2\,A\,\sin\left(3\,c+3\,d\,x\right)+3\,B\,\sin\left(3\,c+3\,d\,x\right)\right)}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(3\,\cos\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*(a*(cos(c + d*x) + 1))^(1/2)*(2*A*sin(c + d*x) + 3*B*sin(c + d*x) + 2*A*
sin(2*c + 2*d*x) + 2*A*sin(3*c + 3*d*x) + 3*B*sin(3*c + 3*d*x)))/(3*d*cos(c
 + d*x)^(1/2)*(3*cos(c + d*x) + 2*cos(2*c + 2*d*x) + cos(3*c + 3*d*x) + 2))
```

3.172 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a(4A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{4a(4A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 2/5*a*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+2/15*a*(4*A+5*
B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+4/15*a*(4*A+5*B)*si
n(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3059, 2851, 2850}

$$\frac{2a(4A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a(4A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*
x]]) + (4*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{5}(4A+5B)\int \frac{\sqrt{a}}{}$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(4A+5B}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(4A+5B}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{}}$$

Mathematica [A]
time = 0.27, size = 78, normalized size = 0.60

$$\frac{2\sqrt{a(1+\cos(c+dx))}\,(7A+5B+(4A+5B)\cos(c+dx)+(4A+5B)\cos(2(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(7*A + 5*B + (4*A + 5*B)*Cos[c + d*x] + (4*A
+ 5*B)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.37, size = 86, normalized size = 0.66

method	result	siz
default	$-\frac{2(-1+\cos(dx+c))\left(8A\left(\cos^2(dx+c)\right)+10B\left(\cos^2(dx+c)\right)+4A\cos(dx+c)+5B\cos(dx+c)+3A\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}}{15d\sin(dx+c)\cos(dx+c)^{\frac{5}{2}}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(8*A*cos(d*x+c)^2+10*B*cos(d*x+c)^2+4*A*cos(d*x+c)+
5*B*cos(d*x+c)+3*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(112) = 224$.
time = 0.54, size = 428, normalized size = 3.29

$$\frac{2\left(\frac{5\,B\left(\frac{3\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\,\sqrt{a}\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}+\frac{A\left(\frac{15\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{17\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{7\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(5*B*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sq
rt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x
 + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 + 1)) + A*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*
sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*
(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [**A**]
time = 0.37, size = 86, normalized size = 0.66

$$\frac{2\left(2\left(4\,A+5\,B\right)\cos\left(dx+c\right)^2+\left(4\,A+5\,B\right)\cos\left(dx+c\right)+3\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(2*(4*A + 5*B)*cos(d*x + c)^2 + (4*A + 5*B)*cos(d*x + c) + 3*A)*sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*co
s(d*x + c)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.25, size = 194, normalized size = 1.49

$$\frac{4\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(14\,A\,\sin\left(c+d\,x\right)+10\,B\,\sin\left(c+d\,x\right)+8\,A\,\sin\left(2\,c+2\,d\,x\right)+18\,A\,\sin\left(3\,c+3\,d\,x\right)+4\,A\,\sin\left(4\,c+4\,d\,x\right)+4\,A\,\sin\left(5\,c+5\,d\,x\right)+10\,B\,\sin\left(2\,c+2\,d\,x\right)+15\,B\,\sin\left(3\,c+3\,d\,x\right)+5\,B\,\sin\left(4\,c+4\,d\,x\right)+5\,B\,\sin\left(5\,c+5\,d\,x\right)\right)}{15\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(5\,c+5\,d\,x\right)+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2),x)
```

```
[Out] (4*(a*(cos(c + d*x) + 1))^(1/2)*(14*A*sin(c + d*x) + 10*B*sin(c + d*x) + 8*
A*sin(2*c + 2*d*x) + 18*A*sin(3*c + 3*d*x) + 4*A*sin(4*c + 4*d*x) + 4*A*sin
(5*c + 5*d*x) + 10*B*sin(2*c + 2*d*x) + 15*B*sin(3*c + 3*d*x) + 5*B*sin(4*c
 + 4*d*x) + 5*B*sin(5*c + 5*d*x)))/(15*d*cos(c + d*x)^(1/2)*(10*cos(c + d*x
) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos(4*c + 4*d*x) + cos(5*c
+ 5*d*x) + 6))
```

3.173 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a(6A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{8a(6A+7B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 2/7*a*A*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+2/35*a*(6*A+7*
B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+8/105*a*(6*A+7*B)*s
in(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+16/105*a*(6*A+7*B)*sin(
d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3059, 2851, 2850}

$$\frac{8a(6A+7B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a(6A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*a*A*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
*a*(6*A + 7*B)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*
x]]) + (8*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*
Cos[c + d*x]]) + (16*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}(6A+7B)\int \frac{\sqrt{}}{}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(6A+7B}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{}}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(6A+7B}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{}}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(6A+7B}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{}}$$

Mathematica [A]
time = 0.42, size = 102, normalized size = 0.58

$$\frac{2\sqrt{a(1+\cos(c+dx))}\,(27A+14B+9(6A+7B)\cos(c+dx)+2(6A+7B)\cos(2(c+dx))+12A\cos(3(c+dx))+14B\cos(3(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{105d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2
),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(27*A + 14*B + 9*(6*A + 7*B)*Cos[c + d*x] + 2
*(6*A + 7*B)*Cos[2*(c + d*x)] + 12*A*Cos[3*(c + d*x)] + 14*B*Cos[3*(c + d*x
)])*Tan[(c + d*x)/2])/(105*d*Cos[c + d*x]^(7/2))
```

Maple [A]
time = 0.38, size = 108, normalized size = 0.62

method	result

default	$-\frac{2(-1+\cos(dx+c))\left(48A\left(\cos^3(dx+c)\right)+56B\left(\cos^3(dx+c)\right)+24A\left(\cos^2(dx+c)\right)+28B\left(\cos^2(dx+c)\right)+18A\cos(dx+c)+21B\cos(dx+c)}{105d\sin(dx+c)\cos(dx+c)^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(48*A*cos(d*x+c)^3+56*B*cos(d*x+c)^3+24*A*cos(d*x+
c)^2+28*B*cos(d*x+c)^2+18*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*(a*(1+cos(d*x+
c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(151) = 302$.
time = 0.54, size = 522, normalized size = 2.98

$$\frac{2\left(\frac{7B\left(\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{7\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}+\frac{3A\left(\frac{35\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{70\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{58\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{9\sqrt{2}\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 2/105*(7*B*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 3*A*(35*sqrt(2)*sqrt(a)*sin(d*x
+ c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 58*sqrt(2)
*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x +
c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(co
s(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A]
time = 0.39, size = 104, normalized size = 0.59

$$\frac{2\left(8\left(6A+7B\right)\cos\left(dx+c\right)^3+4\left(6A+7B\right)\cos\left(dx+c\right)^2+3\left(6A+7B\right)\cos\left(dx+c\right)+15A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(8*(6*A + 7*B)*cos(d*x + c)^3 + 4*(6*A + 7*B)*cos(d*x + c)^2 + 3*(6*A
 + 7*B)*cos(d*x + c) + 15*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*si
n(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**B**]
time = 6.23, size = 479, normalized size = 2.74

$$\frac{\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\left(\frac{(96\,A+112\,B)\,1i}{105\,d}-\frac{B\,e^{c\,3i+d\,x\,3i}\,8i}{3\,d}+\frac{B\,e^{c\,4i+d\,x\,4i}\,8i}{3\,d}-\frac{e^{c\,7i+d\,x\,7i}\,(96\,A+112\,B)\,1i}{105\,d}+\frac{e^{c\,2i+d\,x\,2i}\,(336\,A+392\,B)\,1i}{105\,d}-\frac{e^{c\,5i+d\,x\,5i}\,(336\,A+392\,B)\,1i}{105\,d}\right)}{\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+e^{c\,1i+d\,x\,1i}\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+3\,e^{c\,2i+d\,x\,2i}\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+3\,e^{c\,3i+d\,x\,3i}\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+3\,e^{c\,4i+d\,x\,4i}\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+3\,e^{c\,5i+d\,x\,5i}\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+e^{c\,6i+d\,x\,6i}\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+e^{c\,7i+d\,x\,7i}\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] ((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(((96*A + 11
2*B)*1i)/(105*d) - (B*exp(c*3i + d*x*3i)*8i)/(3*d) + (B*exp(c*4i + d*x*4i)*
8i)/(3*d) - (exp(c*7i + d*x*7i)*(96*A + 112*B)*1i)/(105*d) + (exp(c*2i + d*
x*2i)*(336*A + 392*B)*1i)/(105*d) - (exp(c*5i + d*x*5i)*(336*A + 392*B)*1i)
/(105*d)))/((exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*1
i + d*x*1i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c
*2i + d*x*2i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp
```

```
(c*3i + d*x*3i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*e
xp(c*4i + d*x*4i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3
*exp(c*5i + d*x*5i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) +
 exp(c*6i + d*x*6i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) +
 exp(c*7i + d*x*7i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2))
```

3.174 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=227

$$\frac{a^{3/2}(88A+75B)\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d}+\frac{a^2(88A+75B)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(88A+75B)}{96d\sqrt{}}$$

```
[Out] 1/64*a^(3/2)*(88*A+75*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/
d+1/96*a^2*(88*A+75*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
+1/24*a^2*(8*A+9*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/
64*a^2*(88*A+75*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/4
*a*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.33, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3055, 3060, 2849, 2853, 222}

$$\frac{a^{3/2}(88A+75B)\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)\sqrt{\cos(c+dx)}}{64d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(64*d) + (a^2*(88*A + 75*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqr
t[a + a*Cos[c + d*x]]) + (a^2*(88*A + 75*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x]
)/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(8*A + 9*B)*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(5/2)*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,\sin(c+}{4d}$$

$$= \frac{a^2(8A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aB}{}$$

$$= \frac{a^2(88A+75B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a}{}$$

$$= \frac{a^2(88A+75B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{a^2(88A+75B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{a^{3/2}(88A+75B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right.}{64d}$$

Mathematica [A]
time = 1.20, size = 136, normalized size = 0.60

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}\,(88A+75B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\,(296A+285B+2(88A+93B)\cos(c+dx)+4(8A+15B)\cos(2(c+dx))+12B\cos(3(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(88*A + 75*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(296*A + 285*B + 2*(88
*A + 93*B)*Cos[c + d*x] + 4*(8*A + 15*B)*Cos[2*(c + d*x)] + 12*B*Cos[3*(c +
 d*x)])*Sin[(c + d*x)/2]))/(384*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(195) = 390$.
time = 0.28, size = 429, normalized size = 1.89

method	result

default	$-\frac{a(-1+\cos(dx+c))^3\left(64A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^3(dx+c))\sin(dx+c)+240A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^2(dx+c))\sin(dx+c)+48B\sin(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/192/d*a*(-1+cos(d*x+c))^3*(64*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*
x+c)^3*sin(d*x+c)+240*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*sin(
d*x+c)+48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+440*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+c)+120*B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+264*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*sin(d*x+c)+150*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)^2+225*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+26
4*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x
+c)+225*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*c
os(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x
+c)))^(5/2)/sin(d*x+c)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 8904 vs. $2(195) = 390$.
time = 1.09, size = 8904, normalized size = 39.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*
d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
```

```
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqr
t(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/
4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(si
n(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1)
 - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*arc
tan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*s
```

```
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqrt(
a))*A + 3*(2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((9*a*cos(4*d*x + 4*c)^2*sin(4*
d*x + 4*c) + 9*a*sin(4*d*x + 4*c)^3 + 36*(a*sin...
```

Fricas [**A**]
time = 0.47, size = 162, normalized size = 0.71

$$\frac{\left(48\,Ba\cos\left(dx+c\right)^{3}+8\left(8\,A+15\,B\right)a\cos\left(dx+c\right)^{2}+2\left(88\,A+75\,B\right)a\cos\left(dx+c\right)+3\left(88\,A+75\,B\right)a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-3\left(\left(88\,A+75\,B\right)a\cos\left(dx+c\right)+\left(88\,A+75\,B\right)a\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)}{192\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/192*((48*B*a*cos(d*x + c)^3 + 8*(8*A + 15*B)*a*cos(d*x + c)^2 + 2*(88*A +
 75*B)*a*cos(d*x + c) + 3*(88*A + 75*B)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))*sin(d*x + c) - 3*((88*A + 75*B)*a*cos(d*x + c) + (88*A + 75*B)*
a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(
d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

3.175 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=180

$$\frac{a^{3/2}(14A+11B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}+\frac{a^2(14A+11B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(6A+7B}{12d\sqrt{}}$$

```
[Out] 1/8*a^(3/2)*(14*A+11*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d
+1/12*a^2*(6*A+7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/
8*a^2*(14*A+11*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/3*
a*B*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.26, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3055, 3060, 2849, 2853, 222}

$$\frac{a^{3/2}(14A+11B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(6A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(14A+11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(8*d) + (a^2*(14*A + 11*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[
a + a*Cos[c + d*x]]) + (a^2*(6*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(1
2*d*Sqrt[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c +
d*x]]*Sin[c + d*x])/(3*d)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,\sin(}{3d}$$

$$= \frac{a^2(6A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{a^2(14A+11B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{8d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^2(14A+11B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{8d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^{3/2}(14A+11B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+}}\right.}{8d}$$

Mathematica [A]
time = 0.71, size = 119, normalized size = 0.66

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}\,(14A+11B)\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\,(42A+37B+2(6A+11B)\cos(c+dx)+4B\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(14*A + 11*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(42*A + 37*B + 2*(6*A
+ 11*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(154) = 308$.
time = 0.36, size = 357, normalized size = 1.98

method	result
default	$\frac{a(-1+\cos(dx+c))^2\left(12A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^2(dx+c))\sin(dx+c)+54A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\cos(dx+c)\sin(dx+c)+8B\sin(dx+c)\sqrt{\frac{\cos}{1-}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/24/d*a*(-1+cos(d*x+c))^2*(12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+
c)^2*sin(d*x+c)+54*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+c
)+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+42*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+22*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*cos(d*x+c)^2+33*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*cos(d*x+c)+42*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))*cos(d*x+c)+33*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
cos(d*x+c))*cos(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c
)^4/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3023 vs. $2(154) = 308$.
time = 0.85, size = 3023, normalized size = 16.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
```

```
+ 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (4*(a*cos(3/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(
3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(3/4)*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) -
 (3*a*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1))) + 1) - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/
4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*...
```

Fricas [A]
time = 0.42, size = 144, normalized size = 0.80

$$\frac{\left(8\,Ba\cos\left(dx+c\right)^{2}+2\left(6\,A+11\,B\right)a\cos\left(dx+c\right)+3\left(14\,A+11\,B\right)a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-3\left(\left(14\,A+11\,B\right)a\cos\left(dx+c\right)+\left(14\,A+11\,B\right)a\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin\left(dx+c\right)}\right)}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/24*((8*B*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*cos(d*x + c) + 3*(14*A + 11*
B)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((14*A +
 11*B)*a*cos(d*x + c) + (14*A + 11*B)*a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
 + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

3.176 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=133

$$\frac{a^{3/2}(12A+7B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}+\frac{a^2(4A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{aB\sqrt{\cos(c+dx)}\dots}{}$$

```
[Out] 1/4*a^(3/2)*(12*A+7*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+
1/4*a^2*(4*A+5*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/2*
a*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3055, 3060, 2853, 222}

$$\frac{a^{3/2}(12A+7B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) + (a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a +
 a*Cos[c + d*x]]) + (a*B*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c
+ d*x])/(2*d)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
```

```
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{aB\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{2d}+\frac{1}{2}\int$$

$$= \frac{a^2(4A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{aB\sqrt{\cos(c+}}{}$$

$$= \frac{a^2(4A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{aB\sqrt{\cos(c+}}{}$$

$$= \frac{a^{3/2}(12A+7B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}+\frac{a^2(4A}{}$$

Mathematica [A]
time = 0.41, size = 101, normalized size = 0.76

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\,(12A+7B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\,(4A+7B+2B\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(12*A + 7*B)*ArcSin
[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(4*A + 7*B + 2*B*Cos[c +
d*x])*Sin[(c + d*x)/2]))/(8*d)

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(113) = 226$.
time = 0.34, size = 283, normalized size = 2.13

method	result
default	$-\frac{a(-1+\cos(dx+c))\left(4A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\cos(dx+c)\sin(dx+c)+4A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin(dx+c)+2B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ (\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)

[Out] -1/4/d*a*(-1+cos(d*x+c))*(4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*
sin(d*x+c)+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+2*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+7*B*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)+12*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+7*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+
c)^2/cos(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1884 vs. $2(113) = 226$.
time = 0.74, size = 1884, normalized size = 14.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")

[Out] 1/16*(4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),

```
cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A +
(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c)
+ a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2
((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
```

```
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - 1))*sqrt(a))*B)/d
```

Fricas [A]
time = 0.42, size = 125, normalized size = 0.94

$$\frac{(2\,Ba\cos(dx+c)+(4\,A+7\,B)a)\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-((12\,A+7\,B)a\cos(dx+c)+(12\,A+7\,B)a)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{4\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*((2*B*a*cos(d*x + c) + (4*A + 7*B)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))*sin(d*x + c) - ((12*A + 7*B)*a*cos(d*x + c) + (12*A + 7*B)*a)*sq
rt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
 c))))/(d*cos(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\cos(c+dx)+1))^{\frac{3}{2}}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)*(A + B*cos(c + d*x))/sqrt(cos(c + d*
x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,{(a+a\,\cos(c+d\,x))}^{3/2}}{\sqrt{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2), x
)
```

3.177 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{a^{3/2}(2A+3B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}-\frac{a^2(2A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a\cos(c+dx)}\ldots}{d\sqrt{\cos(c\ldots}}$$

```
[Out] a^(3/2)*(2*A+3*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d-a^2*(
2*A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+2*a*A*sin(d*x+c
)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3054, 3060, 2853, 222}

$$\frac{a^{3/2}(2A+3B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^(3/2)*(2*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d - (a^2*(2*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
```

```
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+2\int \frac{\sqrt{a+a\cos}}{}$$

$$= -\frac{a^2(2A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a}}{d}$$

$$= -\frac{a^2(2A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a}}{d}$$

$$= \frac{a^{3/2}(2A+3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}-\frac{a^2(2A}{}$$

Mathematica [A]
time = 0.35, size = 107, normalized size = 0.85

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\,(2A+3B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2(2A+B\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(2*A + 3*B)*ArcSin[
Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*A + B*Cos[c + d*x])*Sin
[(c + d*x)/2]))/(2*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(112) = 224$.
time = 0.33, size = 300, normalized size = 2.38

method	result
default	$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(2A\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+3B\cos(dx+c)\sqrt{\frac{\cos(dx-}{1+\cos(d}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(a*(1+cos(d*x+c)))^(1/2)*(2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*B*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c))+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+B*sin(d*x+c)*cos(
d*x+c)+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*A*sin(d*x+c))*a/(1+cos(d*x+c))/cos(d*x+
c)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1801 vs. $2(112) = 224$.
time = 0.71, size = 1801, normalized size = 14.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
```

x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*B + 2*(
(a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*

```
x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
qrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4))/d
```

Fricas [A]
time = 0.41, size = 135, normalized size = 1.07

$$\frac{(Ba\cos(dx+c)+2\,Aa)\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-((2\,A+3\,B)a\cos(dx+c)^2+(2\,A+3\,B)a\cos(dx+c))\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{d\cos(dx+c)^2+d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] ((B*a*cos(d*x + c) + 2*A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin
(d*x + c) - ((2*A + 3*B)*a*cos(d*x + c)^2 + (2*A + 3*B)*a*cos(d*x + c))*sqr
t(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
c))))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\cos(c+dx)+1))^{\frac{3}{2}}\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)*(A + B*cos(c + d*x))/cos(c + d*x)**(
3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2), x
)
```

3.178 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{2a^{3/2}B\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a\cos(c+dx)}\ \text{si}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2*a^(3/2)*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2/3*a^2*(4*
A+3*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/3*a*A*sin(d*x
+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A]

time = 0.20, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3054, 3059, 2853, 222}

$$\frac{2a^{3/2}B\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a^(3/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^2*(4*A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3
/2))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\sqrt{a+a\cos}}{}$$

$$= \frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos}}{3d\,\mathrm{c}}$$

$$= \frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos}}{3d\,\mathrm{c}}$$

$$= \frac{2a^{3/2}B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^2(4A}{3d\sqrt{\cos(c+dx}}$$

Mathematica [A]

time = 0.41, size = 106, normalized size = 0.85

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}\,B\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+2(A+(5A+3B)\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*B*ArcSin[Sqrt[2]*
Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(A + (5*A + 3*B)*Cos[c + d*x])*Sin
[(c + d*x)/2]))/(3*d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.33, size = 211, normalized size = 1.69

method	result
default	$-\frac{2a\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(-3B\cos(dx+c)\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)-3B\sin(dx+}{3d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*a*(a*(1+cos(d*x+c)))^(1/2)*(-3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c))-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+5*A*cos(d*x+c)^2+3*B*cos(d*
x+c)^2-4*A*cos(d*x+c)-3*B*cos(d*x+c)-A)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. 2(107) = 214.
time = 0.65, size = 1124, normalized size = 8.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/6*(3*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
```

```
d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)))*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4) + 8*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*
sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)))/d
```

Fricas [**A**]
time = 0.39, size = 133, normalized size = 1.06

$$\frac{2\left(\left(\left(5A+3B\right)a\cos\left(dx+c\right)+Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-3\left(Ba\cos\left(dx+c\right)^{3}+Ba\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)\right)}{3\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 2/3*(((5*A + 3*B)*a*cos(d*x + c) + A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c) - 3*(B*a*cos(d*x + c)^3 + B*a*cos(d*x + c)^2)*sqrt(a)
*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\cos\left(c+dx\right)\right)}{\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)*(A + B*cos(c + d*x))/cos(c + d*x)**(
5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2), x
)
```

3.179 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{2a^2(6A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(18A+25B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a\cos(c+dx)}\,\mathrm{s}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 2/15*a^2*(6*A+5*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+2/1
5*a^2*(18*A+25*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/5*
a*A*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)
```

Rubi [A]

time = 0.22, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3054, 3059, 2850}

$$\frac{2a^2(6A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(18A+25B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a^2*(6*A + 5*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a^2*(18*A + 25*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*
Cos[c + d*x]^(5/2))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
```

)

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{\sqrt{a+a\cos(}{}$$

$$= \frac{2a^2(6A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a\cos(}{5d\cos}$$

$$= \frac{2a^2(6A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(18A+}{15d\sqrt{\cos(c+dx}}$$

Mathematica [A]
time = 0.35, size = 80, normalized size = 0.60

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,(24A+25B+2(9A+5B)\cos(c+dx)+(18A+25B)\cos(2(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(24*A + 25*B + 2*(9*A + 5*B)*Cos[c + d*x] + (
18*A + 25*B)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.26, size = 87, normalized size = 0.65

method	result	s

default	$-\frac{2a(-1+\cos(dx+c))\left(18A\left(\cos^2(dx+c)\right)+25B\left(\cos^2(dx+c)\right)+9A\cos(dx+c)+5B\cos(dx+c)+3A\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}}{15d\sin(dx+c)\cos(dx+c)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*a*(-1+cos(d*x+c))*(18*A*cos(d*x+c)^2+25*B*cos(d*x+c)^2+9*A*cos(d*x+
c)+5*B*cos(d*x+c)+3*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(116) = 232$.
time = 0.55, size = 344, normalized size = 2.57

$$\frac{4\left(\frac{5\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)B}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}+\frac{3\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)A\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 4/15*(5*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3
/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5)*B/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(
d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)) + 3*(5*sqrt(2)*a^(3/2)*sin(d*x + c)
/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/
2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos
(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A]
time = 0.36, size = 88, normalized size = 0.66

$$\frac{2\left((18\,A+25\,B)a\cos\left(dx+c\right)^2+(9\,A+5\,B)a\cos\left(dx+c\right)+3\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*((18*A + 25*B)*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) + 3*A*a)*
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4
+ d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 3.09, size = 195, normalized size = 1.46

$$\frac{2\,a\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(48\,A\,\sin\left(c+d\,x\right)+50\,B\,\sin\left(c+d\,x\right)+36\,A\,\sin\left(2\,c+2\,d\,x\right)+66\,A\,\sin\left(3\,c+3\,d\,x\right)+18\,A\,\sin\left(4\,c+4\,d\,x\right)+18\,A\,\sin\left(5\,c+5\,d\,x\right)+20\,B\,\sin\left(2\,c+2\,d\,x\right)+75\,B\,\sin\left(3\,c+3\,d\,x\right)+10\,B\,\sin\left(4\,c+4\,d\,x\right)+25\,B\,\sin\left(5\,c+5\,d\,x\right)\right)}{15\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(5\,c+5\,d\,x\right)+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*a*(a*(cos(c + d*x) + 1))^(1/2)*(48*A*sin(c + d*x) + 50*B*sin(c + d*x) +
36*A*sin(2*c + 2*d*x) + 66*A*sin(3*c + 3*d*x) + 18*A*sin(4*c + 4*d*x) + 18*
A*sin(5*c + 5*d*x) + 20*B*sin(2*c + 2*d*x) + 75*B*sin(3*c + 3*d*x) + 10*B*s
in(4*c + 4*d*x) + 25*B*sin(5*c + 5*d*x)))/(15*d*cos(c + d*x)^(1/2)*(10*cos(
c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos(4*c + 4*d*x) + c
os(5*c + 5*d*x) + 6))
```

3.180 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=181

$$\frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(52A+63B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{4a^2(52A+63B)\sin(}{105d\sqrt{\cos(c+dx)}\ \sqrt{a+}}$$

```
[Out] 2/35*a^2*(8*A+7*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+2/1
05*a^2*(52*A+63*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+4/1
05*a^2*(52*A+63*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/7
*a*A*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

Rubi [A]
time = 0.28, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3054, 3059, 2851, 2850}

$$\frac{2a^2(52A+63B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*(8*A + 7*B)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqr
t[a + a*Cos[c + d*x]]) + (4*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[
c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\sqrt{a+a\cos(\cdots}{\cdots}$$

$$= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(\cdots}}{7d\cos\cdots}$$

$$= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(52A+\cdots}{105d\cos^{\frac{3}{2}}(c+d\cdots}$$

$$= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(52A+\cdots}{105d\cos^{\frac{3}{2}}(c+d\cdots}$$

Mathematica [A]
time = 0.57, size = 102, normalized size = 0.56

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,(82A+63B+3(78A+77B)\cos(c+dx)+(52A+63B)\cos(2(c+dx))+52A\cos(3(c+dx))+63B\cos(3(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{105d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(82*A + 63*B + 3*(78*A + 77*B)*Cos[c + d*x] +
 (52*A + 63*B)*Cos[2*(c + d*x)] + 52*A*Cos[3*(c + d*x)] + 63*B*Cos[3*(c + d
*x)])*Tan[(c + d*x)/2])/(105*d*Cos[c + d*x]^(7/2))
```

Maple [A]
time = 0.27, size = 109, normalized size = 0.60

method	result
default	$-\frac{2a(-1+\cos(dx+c))(104A(\cos^3(dx+c))+126B(\cos^3(dx+c))+52A(\cos^2(dx+c))+63B(\cos^2(dx+c))+39A\cos(dx+c)+21B\cos(dx}{105d\sin(dx+c)\cos(dx+c)^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(104*A*cos(d*x+c)^3+126*B*cos(d*x+c)^3+52*A*cos(
d*x+c)^2+63*B*cos(d*x+c)^2+39*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*(a*(1+cos(
d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(157) = 314$.
time = 0.57, size = 481, normalized size = 2.66

$$\frac{4\left(\frac{21\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)B\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}+\frac{\left(\frac{105\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{245\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{273\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{171\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{38\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)A\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 4/105*(21*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
+ (105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a^(3/2
)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos(d*x
```

```
+ c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 1)))/d
```

Fricas [A]
time = 0.36, size = 107, normalized size = 0.59

$$\frac{2\left(2\left(52A+63B\right)a\cos\left(dx+c\right)^{3}+\left(52A+63B\right)a\cos\left(dx+c\right)^{2}+3\left(13A+7B\right)a\cos\left(dx+c\right)+15Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(2*(52*A + 63*B)*a*cos(d*x + c)^3 + (52*A + 63*B)*a*cos(d*x + c)^2 +
3*(13*A + 7*B)*a*cos(d*x + c) + 15*A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [B]
time = 6.72, size = 236, normalized size = 1.30

$$\frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\left(-\frac{8\,a\,{\mathrm{e}}^{\frac{c\,7{}\mathrm{i}}{2}+\frac{d\,x\,7{}\mathrm{i}}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A+3\,B\right)}{3\,d}+\frac{16\,a\,{\mathrm{e}}^{\frac{c\,7{}\mathrm{i}}{2}+\frac{d\,x\,7{}\mathrm{i}}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\left(13\,A+12\,B\right)}{15\,d}+\frac{8\,a\,{\mathrm{e}}^{\frac{c\,7{}\mathrm{i}}{2}+\frac{d\,x\,7{}\mathrm{i}}{2}}\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)\,\left(52\,A+63\,B\right)}{105\,d}\right)}{6\,\sqrt{\cos\left(c+d\,x\right)}\,{\mathrm{e}}^{\frac{c\,7{}\mathrm{i}}{2}+\frac{d\,x\,7{}\mathrm{i}}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,\sqrt{\cos\left(c+d\,x\right)}\,{\mathrm{e}}^{\frac{c\,7{}\mathrm{i}}{2}+\frac{d\,x\,7{}\mathrm{i}}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+2\,\sqrt{\cos\left(c+d\,x\right)}\,{\mathrm{e}}^{\frac{c\,7{}\mathrm{i}}{2}+\frac{d\,x\,7{}\mathrm{i}}{2}}\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)+2\,\sqrt{\cos\left(c+d\,x\right)}\,{\mathrm{e}}^{\frac{c\,7{}\mathrm{i}}{2}+\frac{d\,x\,7{}\mathrm{i}}{2}}\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] ((a + a*cos(c + d*x))^(1/2)*((16*a*exp((c*7i)/2 + (d*x*7i)/2)*sin((3*c)/2 +
 (3*d*x)/2)*(13*A + 12*B))/(15*d) - (8*a*exp((c*7i)/2 + (d*x*7i)/2)*sin(c/2
 + (d*x)/2)*(2*A + 3*B))/(3*d) + (8*a*exp((c*7i)/2 + (d*x*7i)/2)*sin((7*c)/
2 + (7*d*x)/2)*(52*A + 63*B))/(105*d)))/(6*cos(c + d*x)^(1/2)*exp((c*7i)/2
+ (d*x*7i)/2)*cos(c/2 + (d*x)/2) + 6*cos(c + d*x)^(1/2)*exp((c*7i)/2 + (d*x
*7i)/2)*cos((3*c)/2 + (3*d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*7i)/2 + (d*x
*7i)/2)*cos((5*c)/2 + (5*d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*7i)/2 + (d*x
*7i)/2)*cos((7*c)/2 + (7*d*x)/2))
```

3.181 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(34A+39B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{8a^2(34A+39B)\sin(c+\ldots}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{co}\ldots}}$$

```
[Out] 2/63*a^2*(10*A+9*B)*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+2/
105*a^2*(34*A+39*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+8/
315*a^2*(34*A+39*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+16
/315*a^2*(34*A+39*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2
/9*a*A*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)
```

Rubi [A]
time = 0.32, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3054, 3059, 2851, 2850}

$$\frac{8a^2(34A+39B)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(34A+39B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a^2(34A+39B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x
]
```

```
[Out] (2*a^2*(10*A + 9*B)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a^2*(34*A + 39*B)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)*Sq
rt[a + a*Cos[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sin[c + d*x])/(315*d*Cos[c +
 d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a^2*(34*A + 39*B)*Sin[c + d*x])
/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Co
s[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\sqrt{a+a\cos}}{}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos}}{9d\cos}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(34A}{105d\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(34A}{105d\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(34A}{105d\cos^{\frac{5}{2}}(c+}$$

Mathematica [A]
time = 0.72, size = 124, normalized size = 0.54

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,(376A+351B+(374A+324B)\cos(c+dx)+11(34A+39B)\cos(2(c+dx))+68A\cos(3(c+dx))+78B\cos(3(c+dx))+68A\cos(4(c+dx))+78B\cos(4(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{315d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
1/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(376*A + 351*B + (374*A + 324*B)*Cos[c + d*x]
 + 11*(34*A + 39*B)*Cos[2*(c + d*x)] + 68*A*Cos[3*(c + d*x)] + 78*B*Cos[3*(
c + d*x)] + 68*A*Cos[4*(c + d*x)] + 78*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2]
)/(315*d*Cos[c + d*x]^(9/2))
```

Maple [A]
time = 0.40, size = 131, normalized size = 0.57

method	result
default	$-\frac{2a(-1+\cos(dx+c))(272A(\cos^4(dx+c))+312B(\cos^4(dx+c))+136A(\cos^3(dx+c))+156B(\cos^3(dx+c))+102A(\cos^2(dx+c))+117B(}{315d\sin(dx+c)\cos(dx+c)^{\frac{9}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(272*A*cos(d*x+c)^4+312*B*cos(d*x+c)^4+136*A*cos
(d*x+c)^3+156*B*cos(d*x+c)^3+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+85*A*cos
(d*x+c)+45*B*cos(d*x+c)+35*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c
)^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(198) = 396.
time = 0.58, size = 573, normalized size = 2.51

$$\frac{4\left(\frac{3\left(\frac{105\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{245\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{273\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{171\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{38\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)B\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}+\frac{\left(\frac{315\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{840\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1344\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1242\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{517\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{94\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)A\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}\right)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 4/315*(3*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)
*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*B*(sin
```

```
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + 1)) + (315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x +
c) + 1) - 840*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sq
rt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1242*sqrt(2)*a^(3/2)*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7 + 517*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9 - 94*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11
)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c
) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1
)))/d
```

Fricas [**A**]
time = 0.38, size = 126, normalized size = 0.55

$$\frac{2\,(8\,(34\,A+39\,B)a\cos\left(dx+c\right)^{4}+4\,(34\,A+39\,B)a\cos\left(dx+c\right)^{3}+3\,(34\,A+39\,B)a\cos\left(dx+c\right)^{2}+5\,(17\,A+9\,B)a\cos\left(dx+c\right)+35\,Aa)\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{315\,\left(d\cos\left(dx+c\right)^{6}+d\cos\left(dx+c\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(8*(34*A + 39*B)*a*cos(d*x + c)^4 + 4*(34*A + 39*B)*a*cos(d*x + c)^3
+ 3*(34*A + 39*B)*a*cos(d*x + c)^2 + 5*(17*A + 9*B)*a*cos(d*x + c) + 35*A*a
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^
6 + d*cos(d*x + c)^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

[Out] Timed out

Mupad [B]
time = 7.02, size = 289, normalized size = 1.27

$$\frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\left(-\frac{16\,B\,a\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{3\,d}+\frac{16\,a\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)\,\left(34\,A+39\,B\right)}{35\,d}+\frac{32\,a\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)\,\left(34\,A+39\,B\right)}{315\,d}+\frac{96\,a\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A+B\right)}{5\,d}\right)}{12\,\sqrt{\cos\left(c+d\,x\right)}\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\,\sqrt{\cos\left(c+d\,x\right)}\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+8\,\sqrt{\cos\left(c+d\,x\right)}\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)+2\,\sqrt{\cos\left(c+d\,x\right)}\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)+2\,\sqrt{\cos\left(c+d\,x\right)}\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(11/2),x
)
```

```
[Out] ((a + a*cos(c + d*x))^(1/2)*((16*a*exp((c*9i)/2 + (d*x*9i)/2)*sin((5*c)/2 +
 (5*d*x)/2)*(34*A + 39*B))/(35*d) - (16*B*a*exp((c*9i)/2 + (d*x*9i)/2)*sin(
(3*c)/2 + (3*d*x)/2))/(3*d) + (32*a*exp((c*9i)/2 + (d*x*9i)/2)*sin((9*c)/2
+ (9*d*x)/2)*(34*A + 39*B))/(315*d) + (96*a*exp((c*9i)/2 + (d*x*9i)/2)*sin(
c/2 + (d*x)/2)*(A + B))/(5*d)))/(12*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*
9i)/2)*cos(c/2 + (d*x)/2) + 8*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)
*cos((3*c)/2 + (3*d*x)/2) + 8*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)
*cos((5*c)/2 + (5*d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)
*cos((7*c)/2 + (7*d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)
*cos((9*c)/2 + (9*d*x)/2))
```

3.182 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=274

$$\frac{a^{5/2}(326A+283B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{128d}+\frac{a^3(326A+283B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{128d\sqrt{a+a\cos(c+dx)}}+\frac{a^3(326A$$

```
[Out] 1/128*a^(5/2)*(326*A+283*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2
))/d+1/5*a*B*cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+1/192*a^3
*(326*A+283*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/240*a
^3*(170*A+157*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/128
*a^3*(326*A+283*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/4
0*a^2*(10*A+13*B)*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.45, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3055, 3060, 2849, 2853, 222}

$$\frac{a^{5/2}(326A+283B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^3(170A+157B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{240d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(326A+283B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(326A+283B)\sin(c+dx)\sqrt{\cos(c+dx)}}{128d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(10A+13B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{40d}+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(128*d) + (a^3*(326*A + 283*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(128
*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(326*A + 283*B)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(170*A + 157*B)*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(10*A + 13*
B)*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d) + (a*B*
Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c}{5d}$$

$$= \frac{a^2(10A+13B)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c-}}{40d}$$

$$= \frac{a^3(170A+157B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^3(326A+283B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^3(326A+283B)\sqrt{\cos(c+dx)}\sin(c+dx}{128d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^3(326A+283B)\sqrt{\cos(c+dx)}\sin(c+dx}{128d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^{5/2}(326A+283B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+}}\right.}{128d}$$

Mathematica [A]
time = 2.04, size = 159, normalized size = 0.58

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(15\sqrt{2}(326A+283B)\text{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}(5810A+5521B+(3620A+3874B)\cos(c+dx)+4(230A+331B)\cos(2(c+dx))+120A\cos(3(c+dx))+348B\cos(3(c+dx))+48B\cos(4(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(326*A + 283*B
)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(5810*A + 5521*B
+ (3620*A + 3874*B)*Cos[c + d*x] + 4*(230*A + 331*B)*Cos[2*(c + d*x)] + 120
*A*Cos[3*(c + d*x)] + 348*B*Cos[3*(c + d*x)] + 48*B*Cos[4*(c + d*x)])*Sin[(
c + d*x)/2]))/(3840*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(236) = 472$.
time = 0.43, size = 503, normalized size = 1.84

method	result
default	$-\frac{a^2(-1+\cos(dx+c))^3\left(480A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^4(dx+c))\sin(dx+c)+2320A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^3(dx+c))\sin(dx+c)+384B\sin(d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/1920/d*a^2*(-1+cos(d*x+c))^3*(480*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*co
s(d*x+c)^4*sin(d*x+c)+2320*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^3
*sin(d*x+c)+384*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5
+5100*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*sin(d*x+c)+1392*B*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+8150*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+c)+2264*B*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+4890*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*
sin(d*x+c)+2830*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2
+4245*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+4890*A*arct
an(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+4245
*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+
c))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(
5/2)/sin(d*x+c)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 10042 vs. $2(236) = 472$.
time = 1.19, size = 10042, normalized size = 36.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/7680*((10*(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2
/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5
*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(3/4)*((135*a^2*sin(4/5*arctan2(sin(5*
d*x + 5*c), cos(5*d*x + 5*c))) + 88*a^2*sin(3/5*arctan2(sin(5*d*x + 5*c), c
os(5*d*x + 5*c))) + 135*a^2*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5
*c))))*cos(3/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))
, cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - (135*a^2*cos
(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 88*a^2*cos(3/5*arctan2(
sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 135*a^2*cos(1/5*arctan2(sin(5*d*x +
```

```
5*c), cos(5*d*x + 5*c))) - 88*a^2)*sin(3/2*arctan2(sin(2/5*arctan2(sin(5*d*
x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
 5*c))) + 1)))*sqrt(a) + 6*(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5
*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/
5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(8*(a^2*cos(2/5*a
rctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2*sin(5*d*x + 5*c) + a^2*sin(5*
d*x + 5*c)*sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*a^2*c
os(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin(5*d*x + 5*c) + a^2*
sin(5*d*x + 5*c))*cos(5/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d
*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - 5
*(35*a^2*sin(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 35*a^2*sin(
3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 40*a^2*sin(2/5*arctan2(s
in(5*d*x + 5*c), cos(5*d*x + 5*c))) - 248*a^2*sin(1/5*arctan2(sin(5*d*x + 5
*c), cos(5*d*x + 5*c))))*cos(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c),
cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) +
1)) - 8*(a^2*cos(5*d*x + 5*c) + (a^2*cos(5*d*x + 5*c) - a^2)*cos(2/5*arctan
2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + (a^2*cos(5*d*x + 5*c) - a^2)*sin
(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 - a^2 + 2*(a^2*cos(5*d*
x + 5*c) - a^2)*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))*sin(5
/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*ar
ctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + 5*(35*a^2*cos(4/5*arctan
2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 35*a^2*cos(3/5*arctan2(sin(5*d*x +
 5*c), cos(5*d*x + 5*c))) - 40*a^2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*
d*x + 5*c))) - 168*a^2*cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))
 + 208*a^2)*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5
*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)))*sqrt(a)
+ 4245*(a^2*arctan2(-(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^
2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arct
an2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/
5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x +
 5*c), cos(5*d*x + 5*c))) + 1))*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
 + 5*c))) - cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin(1/2*ar
ctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2
(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))), (cos(2/5*arctan2(sin(5*d*x +
5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4
)*(cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*cos(1/2*arctan2(sin
(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*
x + 5*c), cos(5*d*x + 5*c))) + 1)) + sin(1/5*arctan2(sin(5*d*x + 5*c), cos(
5*d*x + 5*c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
+ 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))) + 1)
- a^2*arctan2(-(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + si
n(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(si
n(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/5*arct
an2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c),
```

```
 cos(5*d*x + 5*c))) + 1))*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c
))) - cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin(1/2*arctan2(
sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5
*d*x + 5*c), cos(5*d*x + 5*c))) + 1))), (cos(2/5*arctan2(sin(5*d*x + 5*c),
cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))
^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos
(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*cos(1/2*arctan2(sin(2/5*a
rctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*
c), cos(5*d*x + 5*c))) + 1)) + sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
+ 5*c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)
)), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))) - 1) - a^2*
arctan2((cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*a
rctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 +...
```

Fricas [**A**]
time = 0.47, size = 194, normalized size = 0.71

$$\frac{\left(384\,Ba^2\cos\left(dx+c\right)^4+48\left(10\,A+29\,B\right)a^2\cos\left(dx+c\right)^3+8\left(230\,A+283\,B\right)a^2\cos\left(dx+c\right)^2+10\left(326\,A+283\,B\right)a^2\cos\left(dx+c\right)+15\left(326\,A+283\,B\right)a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-15\left(\left(326\,A+283\,B\right)a^2\cos\left(dx+c\right)+\left(326\,A+283\,B\right)a^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)}{1920\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/1920*((384*B*a^2*cos(d*x + c)^4 + 48*(10*A + 29*B)*a^2*cos(d*x + c)^3 + 8
*(230*A + 283*B)*a^2*cos(d*x + c)^2 + 10*(326*A + 283*B)*a^2*cos(d*x + c) +
 15*(326*A + 283*B)*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*
x + c) - 15*((326*A + 283*B)*a^2*cos(d*x + c) + (326*A + 283*B)*a^2)*sqrt(a
)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))
))/(d*cos(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

3.183 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=227

$$\frac{a^{5/2}(200A+163B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d}+\frac{a^3(200A+163B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}}+\frac{a^3(104A$$

```
[Out] 1/64*a^(5/2)*(200*A+163*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2)
)/d+1/4*a*B*cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+1/96*a^3*(
104*A+95*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/64*a^3*(
200*A+163*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/24*a^2*
(8*A+11*B)*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.39, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3055, 3060, 2849, 2853, 222}

$$\frac{a^{5/2}(200A+163B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(104A+95B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(200A+163B)\sin(c+dx)\sqrt{\cos(c+dx)}}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{24d}+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(64*d) + (a^3*(200*A + 163*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d
*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(104*A + 95*B)*Cos[c + d*x]^(3/2)*Sin[c +
 d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(8*A + 11*B)*Cos[c + d*x]^(3/
2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(24*d) + (a*B*Cos[c + d*x]^(3/2)*
(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\sin(}{4d}$$

$$= \frac{a^2(8A+11B)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}}{24d}$$

$$= \frac{a^3(104A+95B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^3(200A+163B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{64d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^3(200A+163B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{64d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^{5/2}(200A+163B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx}{\sqrt{a+a\cos(c+}}\right.}{64d}$$

Mathematica [A]
time = 1.28, size = 137, normalized size = 0.60

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}\,(200A+163B)\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\,(632A+581B+(272A+362B)\cos(c+dx)+4(8A+23B)\cos(2(c+dx))+12B\cos(3(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(200*A + 163*B)
*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(632*A + 581*B + (
272*A + 362*B)*Cos[c + d*x] + 4*(8*A + 23*B)*Cos[2*(c + d*x)] + 12*B*Cos[3*
(c + d*x)])*Sin[(c + d*x)/2]))/(384*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(195) = 390$.
time = 0.26, size = 431, normalized size = 1.90

method	result

default	$\frac{a^2(-1+\cos(dx+c))^2\left(64A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^3(dx+c))\sin(dx+c)+336A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^2(dx+c))\sin(dx+c)+48B\sin(dx+c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/192/d*a^2*(-1+cos(d*x+c))^2*(64*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d
*x+c)^3*sin(d*x+c)+336*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*sin
(d*x+c)+48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+872*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+c)+184*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+600*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)*sin(d*x+c)+326*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)^2+489*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+6
00*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*
x+c)+489*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*
cos(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)^4/(cos(d*x
+c)/(1+cos(d*x+c)))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 9415 vs. $2(195) = 390$.
time = 1.13, size = 9415, normalized size = 41.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(
3*d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
```

ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
c))) + 1)^(1/4)(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(
1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
))) - 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2
*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +

```
 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) - 1))*sqrt(a))*A + (10*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(
4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a^2*sin(4*d...
```

Fricas [A]
time = 0.45, size = 174, normalized size = 0.77

$$\frac{\left(48\,Ba^2\cos\left(dx+c\right)^3+8\left(8\,A+23\,B\right)a^2\cos\left(dx+c\right)^2+2\left(136\,A+163\,B\right)a^2\cos\left(dx+c\right)+3\left(200\,A+163\,B\right)a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-3\left(\left(200\,A+163\,B\right)a^2\cos\left(dx+c\right)+\left(200\,A+163\,B\right)a^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)}{192\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/192*((48*B*a^2*cos(d*x + c)^3 + 8*(8*A + 23*B)*a^2*cos(d*x + c)^2 + 2*(13
6*A + 163*B)*a^2*cos(d*x + c) + 3*(200*A + 163*B)*a^2)*sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((200*A + 163*B)*a^2*cos(d*x + c)
+ (200*A + 163*B)*a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos (c+d x)}\ (A+B\ \cos (c+d x))\ (a+a\ \cos (c+d x))^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

3.184 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=180

$$\frac{a^{5/2}(38A+25B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}+\frac{a^3(54A+49B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(2A+3B}{}$$

```
[Out] 1/8*a^(5/2)*(38*A+25*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d
+1/3*a*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+1/24*a^3*(54*
A+49*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a^2*(2*A+3
*B)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.34, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3055, 3060, 2853, 222}

$$\frac{a^{5/2}(38A+25B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^3(54A+49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(2A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}{4d}+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]])/(8*d) + (a^3*(54*A + 49*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt
[a + a*Cos[c + d*x]]) + (a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(4*d) + (a*B*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{aB\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d}+\frac{1}{3}\ldots$$

$$= \frac{a^2(2A+3B)\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}\,\sin(c+\ldots}{4d}$$

$$= \frac{a^3(54A+49B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(2A+3B\ldots}{}$$

$$= \frac{a^3(54A+49B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(2A+3B\ldots}{}$$

$$= \frac{a^{5/2}(38A+25B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}+\frac{a^3(54\ldots}{}$$

Mathematica [A]
time = 0.82, size = 121, normalized size = 0.67

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,\sec\left(\tfrac{1}{2}(c+dx)\right)\left(3\sqrt{2}\,(38A+25B)\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\,(66A+79B+2(6A+17B)\cos(c+dx)+4B\cos(2(c+dx)))\sin\left(\tfrac{1}{2}(c+dx)\right)\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(38*A + 25*B)*A
rcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(66*A + 79*B + 2*(6*
A + 17*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(154) = 308$.
time = 0.38, size = 357, normalized size = 1.98

method	result
default	$-\frac{a^2(-1+\cos(dx+c))\left(12A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^2(dx+c))\sin(dx+c)+78A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\cos(dx+c)\sin(dx+c)+8B\sin(dx+c)\sqrt{\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/24/d*a^2*(-1+cos(d*x+c))*(12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x
+c)^2*sin(d*x+c)+78*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+
c)+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+66*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+34*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)^2+75*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)+114*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos
(d*x+c))*cos(d*x+c)+75*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)/cos(d*x+c))*cos(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3071 vs. $2(154) = 308$.
time = 0.85, size = 3071, normalized size = 17.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
```

*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*
d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (4*
(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c)
 - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*
a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*

```
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqr
t(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(
1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(
sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) +
1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), co...
```

Fricas [**A**]
time = 0.43, size = 154, normalized size = 0.86

$$\frac{\left(8\,Ba^{2}\cos\left(dx+c\right)^{2}+2\,(6\,A+17\,B)a^{2}\cos\left(dx+c\right)+3\,(22\,A+25\,B)a^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-3\,\left((38\,A+25\,B)a^{2}\cos\left(dx+c\right)+(38\,A+25\,B)a^{2}\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)}{24\,\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/24*((8*B*a^2*cos(d*x + c)^2 + 2*(6*A + 17*B)*a^2*cos(d*x + c) + 3*(22*A +
 25*B)*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((
38*A + 25*B)*a^2*cos(d*x + c) + (38*A + 25*B)*a^2)*sqrt(a)*arctan(sqrt(a*co
s(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)
 + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+a\,\cos(c+d\,x)\right)^{5/2}}{\sqrt{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2), x
)
```

3.185 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{a^{5/2}(20A+19B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}-\frac{a^3(4A-9B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}-\frac{a^2(4A-B)\sqrt{}}{}$$

```
[Out] 1/4*a^(5/2)*(20*A+19*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d
+2*a*A*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-1/4*a^3*(4*A-9*
B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)-1/2*a^2*(4*A-B)*sin
(d*x+c)*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.33, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3054, 3055, 3060, 2853, 222}

$$\frac{a^{5/2}(20A+19B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(4A-9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(4*d) - (a^3*(4*A - 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a
+ a*Cos[c + d*x]]) - (a^2*(4*A - B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d
*x]]*Sin[c + d*x])/(2*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+a\cos(}{}$$

$$= -\frac{a^2(4A-B)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,\sin(c}{2d}$$

$$= -\frac{a^3(4A-9B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(4A-B}{}$$

$$= -\frac{a^3(4A-9B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(4A-B}{}$$

$$= \frac{a^{5/2}(20A+19B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} - \frac{a^3(4.}{}$$

Mathematica [A]
time = 0.72, size = 126, normalized size = 0.71

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\,(20A+19B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2(8A+B+(4A+11B)\cos(c+dx)+B\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(20*A + 19*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(8*A + B + (4*A + 11*B
)*Cos[c + d*x] + B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d*Sqrt[Cos[c + d
*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(154) = 308$.
time = 0.35, size = 336, normalized size = 1.89

method	result
default	$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(20A\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+2B\left(\cos^2(dx+c)\right)\sin(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(20*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*B*
cos(d*x+c)^2*sin(d*x+c)+19*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+4*A*cos(d*x+
c)*sin(d*x+c)+20*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+11*B*sin(d*x+c)*cos(d*x+c)+19*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/cos(d*x+c))+8*A*sin(d*x+c))*a^2/(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2080 vs. $2(154) = 308$.
time = 0.74, size = 2080, normalized size = 11.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*((2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*d*
x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2*a
rctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
```

```
s(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*B + 4*(2*
(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c)
- (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 8*(a
^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) -
(a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)))*sqrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4))/d
```

Fricas [A]

time = 0.41, size = 164, normalized size = 0.92

$$\frac{\left(2\,Ba^{2}\cos\left(dx+c\right)^{2}+\left(4\,A+11\,B\right)a^{2}\cos\left(dx+c\right)+8\,Aa^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\left(\left(20\,A+19\,B\right)a^{2}\cos\left(dx+c\right)^{2}+\left(20\,A+19\,B\right)a^{2}\cos\left(dx+c\right)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin\left(dx+c\right)}\right)}{4\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*((2*B*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 8*A*a^2)*sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - ((20*A + 19*B)*a^2*
cos(d*x + c)^2 + (20*A + 19*B)*a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^2
 + d*cos(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2), x
)
```

3.186 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=173

$$\frac{a^{5/2}(2A+5B)\text{ArcSin}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}-\frac{a^3(14A+3B)\sqrt{\cos(c+dx)}\ \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(2A+B)\sqrt{}}{d}$$

```
[Out] a^(5/2)*(2*A+5*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2/3*a
*A*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)-1/3*a^3*(14*A+3*B)*
sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+2*a^2*(2*A+B)*sin(d*x+
c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3054, 3060, 2853, 222}

$$\frac{a^{5/2}(2A+5B)\text{ArcSin}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3(14A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(2A+B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d - (a^3*(14*A + 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^2*(2*A + B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*
Sqrt[Cos[c + d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*
Cos[c + d*x]^(3/2))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+a\cos(c}{}$$

$$= \frac{2a^2(2A+B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aA(a+}{}$$

$$= -\frac{a^3(14A+3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(2A+B}{}$$

$$= -\frac{a^3(14A+3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(2A+B}{}$$

$$= \frac{a^{5/2}(2A+5B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^3(14A-}{}$$

Mathematica [A]

time = 0.75, size = 130, normalized size = 0.75

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}\,(2A+5B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+(4A+3B+4(8A+3B)\cos(c+dx)+3B\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(2*A + 5*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (4*A + 3*B + 4*(8*A + 3*
B)*Cos[c + d*x] + 3*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d*Cos[c + d*x
]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(151) = 302$.
time = 0.38, size = 484, normalized size = 2.80

method	result
default	$-\frac{\left(\sin^2(dx+c)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(6A\left(\cos^2(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+15B(\mathrm{co}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/3/d*sin(d*x+c)^2*(a*(1+cos(d*x+c)))^(1/2)*(6*A*cos(d*x+c)^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c))+15*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+12*A*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)/cos(d*x+c))+30*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+6*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))+15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*B*cos(d*x+c)^2*sin(d*x+c)+16*A*cos(d*x
+c)*sin(d*x+c)+6*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*a^2/(-1+cos(d*x+c)
)/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2370 vs. $2(151) = 302$.
time = 0.76, size = 2370, normalized size = 13.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(3*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqr
t(a) + 8*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)))*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4) + 2*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))*sin(2*d*x + 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x
 + 2*c) + 4*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x +
2*c)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x
 + 2*c) - a^2 + 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)))*sqrt(a) + 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2
+ 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
 cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + s
```

```
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin
(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x +
2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2
*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - 1))*sqrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A]
time = 0.42, size = 169, normalized size = 0.98

$$\frac{\left(3\,Ba^{2}\cos\left(dx+c\right)^{2}+2\,(8\,A+3\,B)a^{2}\cos\left(dx+c\right)+2\,Aa^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-3\left((2\,A+5\,B)a^{2}\cos\left(dx+c\right)^{3}+(2\,A+5\,B)a^{2}\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)}{3\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((3*B*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 2*A*a^2)*sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((2*A + 5*B)*a^2
*cos(d*x + c)^3 + (2*A + 5*B)*a^2*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^
3 + d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\cos(c + dx))\,(a + a\cos(c + dx))^{5/2}}{\cos(c + dx)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2), x
)
```

3.187 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2a^{5/2}B\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(8A+5B)\sqrt{a+a\,\text{co}}}{15d\cos^{\frac{3}{2}}(c}$$

```
[Out] 2*a^(5/2)*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2/5*a*A*(a+
a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/15*a^3*(32*A+35*B)*sin(
d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/15*a^2*(8*A+5*B)*sin(d*x
+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A]

time = 0.30, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3054, 3059, 2853, 222}

$$\frac{2a^{5/2}B\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a^(5/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^3*(32*A + 35*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d
*Cos[c + d*x]^(3/2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d
*Cos[c + d*x]^(5/2))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\cos(c$$

$$= \frac{2a^2(8A+5B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a$$

$$= \frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+5B)}{}$$

$$= \frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+5B)}{}$$

$$= \frac{2a^{5/2}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^3(32A+}{15d\sqrt{\cos(c+dx}}$$

Mathematica [A]
time = 0.81, size = 130, normalized size = 0.76

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\left(30\sqrt{2}\,B\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{5}{2}}(c+dx)+2(49A+40B+2(14A+5B)\cos(c+dx)+(43A+40B)\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{30d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(30*Sqrt[2]*B*ArcSin[Sqrt[
2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(49*A + 40*B + 2*(14*A + 5*B)*C
os[c + d*x] + (43*A + 40*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(30*d*Cos[
c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(148) = 296$.
time = 0.40, size = 306, normalized size = 1.78

method	result
default	$-\frac{2a^2\sqrt{a\,(1+\cos\,(dx+c))}\left(-15B\sin(dx+c)(\cos^2(dx+c))\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)-30B$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*a^2*(a*(1+cos(d*x+c)))^(1/2)*(-15*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)/cos(d*x+c))-30*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-15*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))+43*A*cos(d*x+c)^3+40*B*cos(d*x+c)^3-29*A*cos(
d*x+c)^2-35*B*cos(d*x+c)^2-11*A*cos(d*x+c)-5*B*cos(d*x+c)-3*A)/sin(d*x+c)/c
os(d*x+c)^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1548 vs. $2(148) = 296$.
time = 0.67, size = 1548, normalized size = 9.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/30*(5*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
```

```
)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2
*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3
*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)
 + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x +
 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*co
s(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqr
t(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
+ 16*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^(5/
2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2)))/d
```

Fricas [A]

time = 0.43, size = 161, normalized size = 0.94

$$\frac{2\left(\left((43\,A+40\,B)a^2\cos\left(dx+c\right)^2+(14\,A+5\,B)a^2\cos\left(dx+c\right)+3\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-15\left(Ba^2\cos\left(dx+c\right)^4+Ba^2\cos\left(dx+c\right)^3\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)\right)}{15\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(((43*A + 40*B)*a^2*cos(d*x + c)^2 + (14*A + 5*B)*a^2*cos(d*x + c) + 3
*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 15*(B*a^
2*cos(d*x + c)^4 + B*a^2*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
 + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^4 + d*cos
(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2), x
)
```

3.188 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=181

$$\frac{2a^3(10A+11B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a^3(230A+301B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(10A+7B)\sqrt{a+a\cos}}{35d\cos^{\frac{5}{2}}(c}$$

```
[Out] 2/7*a*A*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/15*a^3*(10*A
+11*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+2/105*a^3*(230*
A+301*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/35*a^2*(10*
A+7*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)
```

Rubi [A]
time = 0.34, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3054, 3059, 2850}

$$\frac{2a^3(10A+11B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(230A+301B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(10A+7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*a^3*(10*A + 11*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[
c + d*x]]) + (2*a^3*(230*A + 301*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(10*A + 7*B)*Sqrt[a + a*Cos[c + d*x]]*S
in[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*
Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
```

```
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\cos$$

$$= \frac{2a^2(10A+7B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA(}{}$$

$$= \frac{2a^3(10A+11B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(10A+7B)}{}$$

$$= \frac{2a^3(10A+11B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(230A}{105d\sqrt{\cos(c+}}$$

Mathematica [A]
time = 0.65, size = 104, normalized size = 0.57

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,(290A+196B+(930A+987B)\cos(c+dx)+2(115A+98B)\cos(2(c+dx))+230A\cos(3(c+dx))+301B\cos(3(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{210d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(290*A + 196*B + (930*A + 987*B)*Cos[c + d*
x] + 2*(115*A + 98*B)*Cos[2*(c + d*x)] + 230*A*Cos[3*(c + d*x)] + 301*B*Cos
[3*(c + d*x)])*Tan[(c + d*x)/2])/(210*d*Cos[c + d*x]^(7/2))
```

Maple [A]
time = 0.28, size = 111, normalized size = 0.61

method	result
default	$-\frac{2a^2(-1+\cos(dx+c))(230A(\cos^3(dx+c))+301B(\cos^3(dx+c))+115A(\cos^2(dx+c))+98B(\cos^2(dx+c))+60A\cos(dx+c)+21B\cos(dx}{105d\sin(dx+c)\cos(dx+c)^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*a^2*(-1+cos(d*x+c))*(230*A*cos(d*x+c)^3+301*B*cos(d*x+c)^3+115*A*c
os(d*x+c)^2+98*B*cos(d*x+c)^2+60*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*(a*(1+c
os(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(157) = 314$.
time = 0.56, size = 396, normalized size = 2.19

$$\frac{8\left(\frac{7\left(\frac{15\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{28\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)B}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}+\frac{5\left(\frac{21\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{56\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{36\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)A\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 8/105*(7*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7)*B/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d
*x + c) + 1) + 1)^(7/2)) + 5*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c)
 + 1) - 56*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)
*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1
)))/d
```

Fricas [A]
time = 0.38, size = 114, normalized size = 0.63

$$\frac{2\left((230A+301B)a^2\cos(dx+c)^3+(115A+98B)a^2\cos(dx+c)^2+3(20A+7B)a^2\cos(dx+c)+15Aa^2\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{105\left(d\cos(dx+c)^5+d\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*((230*A + 301*B)*a^2*cos(d*x + c)^3 + (115*A + 98*B)*a^2*cos(d*x + c)
^2 + 3*(20*A + 7*B)*a^2*cos(d*x + c) + 15*A*a^2)*sqrt(a*cos(d*x + c) + a)*s
qrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**B**]
time = 6.86, size = 551, normalized size = 3.04

$$\frac{\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,\left(\frac{a^2\,(230\,A+301\,B)\,2i}{105\,d}-\frac{a^2\,e^{c\,3i+d\,x\,3i}\,(10\,A+17\,B)\,2i}{3\,d}+\frac{a^2\,e^{c\,4i+d\,x\,4i}\,(10\,A+17\,B)\,2i}{3\,d}+\frac{a^2\,e^{c\,2i+d\,x\,2i}\,(100\,A+113\,B)\,2i}{15\,d}-\frac{a^2\,e^{c\,5i+d\,x\,5i}\,(100\,A+113\,B)\,2i}{15\,d}-\frac{a^2\,e^{c\,7i+d\,x\,7i}\,(230\,A+301\,B)\,2i}{105\,d}-\frac{B\,a^2\,e^{c\,1i+d\,x\,1i}\,2i}{d}+\frac{B\,a^2\,e^{c\,6i+d\,x\,6i}\,2i}{d}\right)}{\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+e^{c\,1i+d\,x\,1i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+3\,e^{c\,2i+d\,x\,2i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+3\,e^{c\,3i+d\,x\,3i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+3\,e^{c\,4i+d\,x\,4i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+3\,e^{c\,5i+d\,x\,5i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+e^{c\,6i+d\,x\,6i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+e^{c\,7i+d\,x\,7i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] ((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(230*A
 + 301*B)*2i)/(105*d) - (a^2*exp(c*3i + d*x*3i)*(10*A + 17*B)*2i)/(3*d) + (
a^2*exp(c*4i + d*x*4i)*(10*A + 17*B)*2i)/(3*d) + (a^2*exp(c*2i + d*x*2i)*(1
00*A + 113*B)*2i)/(15*d) - (a^2*exp(c*5i + d*x*5i)*(100*A + 113*B)*2i)/(15*
d) - (a^2*exp(c*7i + d*x*7i)*(230*A + 301*B)*2i)/(105*d) - (B*a^2*exp(c*1i
+ d*x*1i)*2i)/d + (B*a^2*exp(c*6i + d*x*6i)*2i)/d))/((exp(- c*1i - d*x*1i)/
2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*1i + d*x*1i)*(exp(- c*1i - d*x*1i)/
2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c*2i + d*x*2i)*(exp(- c*1i - d*x*1i
)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c*3i + d*x*3i)*(exp(- c*1i - d*x*
1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c*4i + d*x*4i)*(exp(- c*1i - d*
```

```
x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c*5i + d*x*5i)*(exp(- c*1i -
d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*6i + d*x*6i)*(exp(- c*1i -
d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*7i + d*x*7i)*(exp(- c*1i -
d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2))
```

3.189 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a^3(292A+345B)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{4a^3(292A+345B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 2/9*a*A*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)+2/315*a^3*(124
*A+135*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+2/315*a^3*(2
92*A+345*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+4/315*a^3*
(292*A+345*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/21*a^2
*(4*A+3*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

Rubi [A]
time = 0.40, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3054, 3059, 2851, 2850}

$$\frac{2a^3(292A+345B)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a^3(292A+345B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{21d\cos^{\frac{7}{2}}(c+dx)}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x
]
```

```
[Out] (2*a^3*(124*A + 135*B)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3
/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(4*A + 3*B)*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) + (2*a*A*(a + a*Co
s[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+a\cos(c}{}$$

$$= \frac{2a^2(4A+3B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aA(a}{}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(4A+3B)}{}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(292A}{315d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(292A}{315d\cos^{\frac{3}{2}}(c+}$$

Mathematica [A]
time = 0.91, size = 126, normalized size = 0.55

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,(1454A+1395B+(1396A+1215B)\cos(c+dx)+2(803A+870B)\cos(2(c+dx))+292A\cos(3(c+dx))+345B\cos(3(c+dx))+292A\cos(4(c+dx))+345B\cos(4(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{630d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
1/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(1454*A + 1395*B + (1396*A + 1215*B)*Cos[c
+ d*x] + 2*(803*A + 870*B)*Cos[2*(c + d*x)] + 292*A*Cos[3*(c + d*x)] + 345*
B*Cos[3*(c + d*x)] + 292*A*Cos[4*(c + d*x)] + 345*B*Cos[4*(c + d*x)])*Tan[(
c + d*x)/2])/(630*d*Cos[c + d*x]^(9/2))
```

Maple [A]
time = 0.34, size = 133, normalized size = 0.58

method	result
default	$-\frac{2a^2(-1+\cos(dx+c))(584A(\cos^4(dx+c))+690B(\cos^4(dx+c))+292A(\cos^3(dx+c))+345B(\cos^3(dx+c))+219A(\cos^2(dx+c))+180}{315d\sin(dx+c)\cos(dx+c)^{\frac{9}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(584*A*cos(d*x+c)^4+690*B*cos(d*x+c)^4+292*A*c
os(d*x+c)^3+345*B*cos(d*x+c)^3+219*A*cos(d*x+c)^2+180*B*cos(d*x+c)^2+130*A*
cos(d*x+c)+45*B*cos(d*x+c)+35*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*
x+c)^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(198) = 396$.
time = 0.56, size = 533, normalized size = 2.34

$$\frac{8\left(\frac{15\left(\frac{21\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{56\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{36\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)B\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}+\frac{\left(\frac{315\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{945\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1449\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1287\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{572\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{104\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)A\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 8/315*(15*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*
a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*B*(sin(d*x
```

```
 + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + (315*sqrt(2)*a
^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a^(5/2)*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 572*sqrt
(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*sqrt(2)*a^(5/2)*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A]
time = 0.38, size = 135, normalized size = 0.59

$$\frac{2\left(2\left(292A+345B\right)a^2\cos\left(dx+c\right)^4+\left(292A+345B\right)a^2\cos\left(dx+c\right)^3+3\left(73A+60B\right)a^2\cos\left(dx+c\right)^2+5\left(26A+9B\right)a^2\cos\left(dx+c\right)+35Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{315\left(d\cos\left(dx+c\right)^6+d\cos\left(dx+c\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(2*(292*A + 345*B)*a^2*cos(d*x + c)^4 + (292*A + 345*B)*a^2*cos(d*x +
 c)^3 + 3*(73*A + 60*B)*a^2*cos(d*x + c)^2 + 5*(26*A + 9*B)*a^2*cos(d*x + c
) + 35*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*c
os(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 8.24, size = 647, normalized size = 2.84

$$\frac{\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,\left(\frac{a^2\,(292\,A+345\,B)\,4i}{315\,d}-\frac{a^2\,e^{c\,3i+d\,x\,3i}\,(2\,A+5\,B)\,4i}{3\,d}+\frac{a^2\,e^{c\,6i+d\,x\,6i}\,(2\,A+5\,B)\,4i}{3\,d}+\frac{a^2\,e^{c\,4i+d\,x\,4i}\,(24\,A+25\,B)\,4i}{5\,d}-\frac{a^2\,e^{c\,5i+d\,x\,5i}\,(24\,A+25\,B)\,4i}{5\,d}+\frac{a^2\,e^{c\,2i+d\,x\,2i}\,(146\,A+155\,B)\,4i}{35\,d}-\frac{a^2\,e^{c\,7i+d\,x\,7i}\,(146\,A+155\,B)\,4i}{35\,d}-\frac{a^2\,e^{c\,9i+d\,x\,9i}\,(292\,A+345\,B)\,4i}{315\,d}\right)}{\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+e^{c\,1i+d\,x\,1i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+4\,e^{c\,2i+d\,x\,2i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+4\,e^{c\,3i+d\,x\,3i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+6\,e^{c\,4i+d\,x\,4i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+6\,e^{c\,5i+d\,x\,5i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+4\,e^{c\,6i+d\,x\,6i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+4\,e^{c\,7i+d\,x\,7i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+e^{c\,8i+d\,x\,8i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+e^{c\,9i+d\,x\,9i}\,\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2),x
)

[Out] ((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(292*A
 + 345*B)*4i)/(315*d) - (a^2*exp(c*3i + d*x*3i)*(2*A + 5*B)*4i)/(3*d) + (a^
2*exp(c*6i + d*x*6i)*(2*A + 5*B)*4i)/(3*d) + (a^2*exp(c*4i + d*x*4i)*(24*A
+ 25*B)*4i)/(5*d) - (a^2*exp(c*5i + d*x*5i)*(24*A + 25*B)*4i)/(5*d) + (a^2*
exp(c*2i + d*x*2i)*(146*A + 155*B)*4i)/(35*d) - (a^2*exp(c*7i + d*x*7i)*(14
6*A + 155*B)*4i)/(35*d) - (a^2*exp(c*9i + d*x*9i)*(292*A + 345*B)*4i)/(315*
d)))/((exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*1i + d*
x*1i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 4*exp(c*2i +
d*x*2i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 4*exp(c*3i
+ d*x*3i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 6*exp(c*4
i + d*x*4i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 6*exp(c
*5i + d*x*5i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 4*exp
(c*6i + d*x*6i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 4*e
xp(c*7i + d*x*7i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + e
xp(c*8i + d*x*8i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + e
xp(c*9i + d*x*9i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2))
```

3.190 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=275

$$\frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a^3(710A+803B)\sin(c+dx)}{1155d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{8a^3(710A+803B)\,\mathrm{si}}{3465d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+}}$$

```
[Out] 2/11*a*A*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(11/2)+2/693*a^3*(1
94*A+209*B)*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+2/1155*a^3
*(710*A+803*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+8/3465*
a^3*(710*A+803*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+16/3
465*a^3*(710*A+803*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+
2/99*a^2*(14*A+11*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)
```

Rubi [A]
time = 0.45, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3054, 3059, 2851, 2850}

$$\frac{8a^3(710A+803B)\sin(c+dx)}{3465d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(710A+803B)\sin(c+dx)}{1155d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a^3(710A+803B)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(14A+11B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{99d\cos^{\frac{9}{2}}(c+dx)}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2),x
]
```

```
[Out] (2*a^3*(194*A + 209*B)*Sin[c + d*x])/(693*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(710*A + 803*B)*Sin[c + d*x])/(1155*d*Cos[c + d*x]^(
5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sin[c + d*x])/(3465
*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a^3*(710*A + 803*B)*S
in[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/
2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(1
1/2))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
```

```
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+a\cos}{}$$

$$= \frac{2a^2(14A+11B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2aA(}{}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(14A+11B}{}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(710A}{1155d\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(710A}{1155d\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(710A}{1155d\cos^{\frac{5}{2}}(c+}$$

Mathematica [A]
time = 1.06, size = 147, normalized size = 0.53

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,(9070A+7678B+(25070A+24827B)\cos(c+dx)+(9230A+9284B)\cos(2(c+dx))+9230A\cos(3(c+dx))+10439B\cos(3(c+dx))+1420A\cos(4(c+dx))+1606B\cos(4(c+dx))+1420A\cos(5(c+dx))+1606B\cos(5(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{6930d\cos^{\frac{11}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
3/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(9070*A + 7678*B + (25070*A + 24827*B)*Cos[
c + d*x] + (9230*A + 9284*B)*Cos[2*(c + d*x)] + 9230*A*Cos[3*(c + d*x)] + 1
0439*B*Cos[3*(c + d*x)] + 1420*A*Cos[4*(c + d*x)] + 1606*B*Cos[4*(c + d*x)]
 + 1420*A*Cos[5*(c + d*x)] + 1606*B*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])/(69
30*d*Cos[c + d*x]^(11/2))
```

Maple [A]
time = 0.50, size = 155, normalized size = 0.56

method	result
default	$-\frac{2a^2(-1+\cos(dx+c))\left(5680A\left(\cos^5(dx+c)\right)+6424B\left(\cos^5(dx+c)\right)+2840A\left(\cos^4(dx+c)\right)+3212B\left(\cos^4(dx+c)\right)+2130A\left(\cos^3(dx+c)\right)+\right.}{3465d\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(5680*A*cos(d*x+c)^5+6424*B*cos(d*x+c)^5+2840
*A*cos(d*x+c)^4+3212*B*cos(d*x+c)^4+2130*A*cos(d*x+c)^3+2409*B*cos(d*x+c)^3
+1775*A*cos(d*x+c)^2+1430*B*cos(d*x+c)^2+1120*A*cos(d*x+c)+385*B*cos(d*x+c)
+315*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(11/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(239) = 478$.
time = 0.56, size = 626, normalized size = 2.28

$$\frac{8\left(\frac{11\left(\frac{315\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{945\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1449\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1287\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{572\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{104\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)B\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}+\frac{5\left(\frac{693\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2310\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4620\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5478\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3575\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{1300\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{200\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}\right)A\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] 8/3465*(11*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(
2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
104*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 1)) + 5*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1
) - 2310*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4620*sqrt(2)
*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5478*sqrt(2)*a^(5/2)*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 + 3575*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x
 + c) + 1)^9 - 1300*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 +
 200*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2
)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A]
time = 0.39, size = 156, normalized size = 0.57

$$\frac{2\,(8\,(710\,A+803\,B)a^2\cos(dx+c)^5+4\,(710\,A+803\,B)a^2\cos(dx+c)^4+3\,(710\,A+803\,B)a^2\cos(dx+c)^3+5\,(355\,A+286\,B)a^2\cos(dx+c)^2+35\,(32\,A+11\,B)a^2\cos(dx+c)+315\,Aa^2)\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{3465\,(d\cos(dx+c)^7+d\cos(dx+c)^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(8*(710*A + 803*B)*a^2*cos(d*x + c)^5 + 4*(710*A + 803*B)*a^2*cos(d*
x + c)^4 + 3*(710*A + 803*B)*a^2*cos(d*x + c)^3 + 5*(355*A + 286*B)*a^2*cos
(d*x + c)^2 + 35*(32*A + 11*B)*a^2*cos(d*x + c) + 315*A*a^2)*sqrt(a*cos(d*x
 + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^7 + d*cos(d*x +
c)^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 7.32, size = 773, normalized size = 2.81

$$\frac{\sqrt{a+a\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}\right)}\left(\frac{a^2\,\left(710\,A+803\,B\right)\,16\mathrm{i}}{3465\,d}-\frac{a^2\,\mathrm{e}^{c\,5\mathrm{i}+d\,x\,5\mathrm{i}}\,\left(30\,A+41\,B\right)\,8\mathrm{i}}{15\,d}+\frac{a^2\,\mathrm{e}^{c\,6\mathrm{i}+d\,x\,6\mathrm{i}}\,\left(30\,A+41\,B\right)\,8\mathrm{i}}{15\,d}+\frac{a^2\,\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,\left(160\,A+157\,B\right)\,8\mathrm{i}}{35\,d}-\frac{a^2\,\mathrm{e}^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,\left(160\,A+157\,B\right)\,8\mathrm{i}}{35\,d}+\frac{a^2\,\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,\left(710\,A+803\,B\right)\,8\mathrm{i}}{315\,d}-\frac{a^2\,\mathrm{e}^{c\,9\mathrm{i}+d\,x\,9\mathrm{i}}\,\left(710\,A+803\,B\right)\,8\mathrm{i}}{315\,d}-\frac{a^2\,\mathrm{e}^{c\,11\mathrm{i}+d\,x\,11\mathrm{i}}\,\left(710\,A+803\,B\right)\,16\mathrm{i}}{3465\,d}-\frac{B\,a^2\,\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,8\mathrm{i}}{3\,d}+\frac{B\,a^2\,\mathrm{e}^{c\,8\mathrm{i}+d\,x\,8\mathrm{i}}\,8\mathrm{i}}{3\,d}\right)}{\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+5\,\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+5\,\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+10\,\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+10\,\mathrm{e}^{c\,5\mathrm{i}+d\,x\,5\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+10\,\mathrm{e}^{c\,6\mathrm{i}+d\,x\,6\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+10\,\mathrm{e}^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+5\,\mathrm{e}^{c\,8\mathrm{i}+d\,x\,8\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+5\,\mathrm{e}^{c\,9\mathrm{i}+d\,x\,9\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+\mathrm{e}^{c\,10\mathrm{i}+d\,x\,10\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+\mathrm{e}^{c\,11\mathrm{i}+d\,x\,11\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(13/2),x
)
```

```
[Out] ((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(710*A
 + 803*B)*16i)/(3465*d) - (a^2*exp(c*5i + d*x*5i)*(30*A + 41*B)*8i)/(15*d)
+ (a^2*exp(c*6i + d*x*6i)*(30*A + 41*B)*8i)/(15*d) + (a^2*exp(c*4i + d*x*4i
)*(160*A + 157*B)*8i)/(35*d) - (a^2*exp(c*7i + d*x*7i)*(160*A + 157*B)*8i)/
(35*d) + (a^2*exp(c*2i + d*x*2i)*(710*A + 803*B)*8i)/(315*d) - (a^2*exp(c*9
i + d*x*9i)*(710*A + 803*B)*8i)/(315*d) - (a^2*exp(c*11i + d*x*11i)*(710*A
+ 803*B)*16i)/(3465*d) - (B*a^2*exp(c*3i + d*x*3i)*8i)/(3*d) + (B*a^2*exp(c
*8i + d*x*8i)*8i)/(3*d)))/((exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^
(1/2) + exp(c*1i + d*x*1i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^
(1/2) + 5*exp(c*2i + d*x*2i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2
```

```
)^(1/2) + 5*exp(c*3i + d*x*3i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)
/2)^(1/2) + 10*exp(c*4i + d*x*4i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*
1i)/2)^(1/2) + 10*exp(c*5i + d*x*5i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d
*x*1i)/2)^(1/2) + 10*exp(c*6i + d*x*6i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i
+ d*x*1i)/2)^(1/2) + 10*exp(c*7i + d*x*7i)*(exp(- c*1i - d*x*1i)/2 + exp(c*
1i + d*x*1i)/2)^(1/2) + 5*exp(c*8i + d*x*8i)*(exp(- c*1i - d*x*1i)/2 + exp(
c*1i + d*x*1i)/2)^(1/2) + 5*exp(c*9i + d*x*9i)*(exp(- c*1i - d*x*1i)/2 + ex
p(c*1i + d*x*1i)/2)^(1/2) + exp(c*10i + d*x*10i)*(exp(- c*1i - d*x*1i)/2 +
exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*11i + d*x*11i)*(exp(- c*1i - d*x*1i)/2
+ exp(c*1i + d*x*1i)/2)^(1/2))
```

3.191 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=190

$$-\frac{(4A-7B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] -1/4*(4*A-7*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)+
(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c
))^(1/2))*2^(1/2)/d/a^(1/2)+1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*
x+c))^(1/2)+1/4*(4*A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2
)
```

Rubi [A]
time = 0.38, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3062, 3061, 2861, 211, 2853, 222}

$$-\frac{(4A-7B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -1/4*((4*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/
(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((4*A - B)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (B*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
```

```
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3aB}{2}+\frac{1}{2}a(4A-B)\cos(c+\right.}{\sqrt{a+a\cos(c+dx)}}}{2a}$$

$$= \frac{(4A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(4A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(4A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(4A-7B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}\,d} + \frac{\sqrt{2}\,(A-B)\tan^{-1}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.10, size = 348, normalized size = 1.83

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sqrt{2}\,e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(-4Adx+7Bdx+i(4A-7B)\sinh^{-1}\left(e^{i(c+dx)}\right)-8i\sqrt{2}(A-B)\log\left(1+e^{i(c+dx)}\right)-4iA\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)+7iB\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)+8i\sqrt{2}A\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)-8i\sqrt{2}B\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)}{d\sqrt{1+e^{2i(c+dx)}}}+\frac{4\sqrt{\cos(c+dx)}\,(4A-B+2B\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)}{d}\right)}{8\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (Cos[(c + d*x)/2]*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x))]*(-4*A*d*x + 7*B*d*x + I*(4*A - 7*B)*ArcSinh[E^(I*(c +
d*x))] - (8*I)*Sqrt[2]*(A - B)*Log[1 + E^(I*(c + d*x))] - (4*I)*A*Log[1 + S
qrt[1 + E^((2*I)*(c + d*x))]] + (7*I)*B*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x)
)]] + (8*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(
c + d*x))]] - (8*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^
((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]) + (4*Sqrt[Cos[c + d
*x]]*(4*A - B + 2*B*Cos[c + d*x])*Sin[(c + d*x)/2])/d))/(8*Sqrt[a*(1 + Cos[
c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(159) = 318$.
time = 0.40, size = 347, normalized size = 1.83

method	result
default	$-\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,(-1+\cos(dx+c))^{3}\left(4A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\cos(dx+c)\sin(dx+c)+4A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(4*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+c)+4*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(3/2)*sin(d*x+c)+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)^2-4*A*2^(1/2)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+4*B*2^(1/2)*cos(d*x+
c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-4*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+7*B*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c))/sin(d*x+c)^6/(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a),
 x)
```

Fricas [A]
time = 2.36, size = 184, normalized size = 0.97

$$\frac{\left(2\,B\cos\left(dx+c\right)+4\,A-B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\left(\left(4\,A-7\,B\right)\cos\left(dx+c\right)+4\,A-7\,B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{4\,\sqrt{2}\,((A-B)a\cos(dx+c)+(A-B)a)\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)}{\sqrt{a}}}{4\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*((2*B*cos(d*x + c) + 4*A - B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c
))*sin(d*x + c) + ((4*A - 7*B)*cos(d*x + c) + 4*A - 7*B)*sqrt(a)*arctan(sqr
```

```
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 4*sqrt(2
)*((A - B)*a*cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) +
 a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) +
 a*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2), x
)
```

3.192 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{(2A-B)\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] (2*A-B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-(A-B)*a
rctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2
))*2^(1/2)/d/a^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3062, 3061, 2861, 211, 2853, 222}

$$\frac{(2A-B)\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((2*A - B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a
]*d) - (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (B*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\frac{aB}{2}+\frac{1}{2}a(2A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c}}}{a}$$

$$= \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{(2A-B)\int \frac{\sqrt{a+a\cos(c+dx}}{\sqrt{\cos(c+dx)}}}{2a}$$

$$= \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{(2a(A-B))\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,d\right.}{}$$

$$= \frac{(2A-B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\,(A-B)\tan^{-1}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.76, size = 333, normalized size = 2.36

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sqrt{2}\,e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(2Adx-Bdx-i(2A-B)\sinh^{-1}\left(e^{i(c+dx)}\right)+2i\sqrt{2}\,(A-B)\log\left(1+e^{i(c+dx)}\right)+2iA\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-iB\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-2i\sqrt{2}\,A\log\left(1-e^{i(c+dx)}+\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}\right)+2i\sqrt{2}\,B\log\left(1-e^{i(c+dx)}+\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}\right)\right)}{d\sqrt{1+e^{2i(c+dx)}}}+\frac{4B\sqrt{\cos(c+dx)}\,\sin\left(\frac{1}{2}(c+dx)\right)}{d}\right)}{2\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (Cos[(c + d*x)/2]*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x))]*(2*A*d*x - B*d*x - I*(2*A - B)*ArcSinh[E^(I*(c + d*x))
] + (2*I)*Sqrt[2]*(A - B)*Log[1 + E^(I*(c + d*x))] + (2*I)*A*Log[1 + Sqrt[1
 + E^((2*I)*(c + d*x))]] - I*B*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (2*
I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))
]] + (2*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c
 + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]) + (4*B*Sqrt[Cos[c + d*x]]*Si
n[(c + d*x)/2])/d))/(2*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A]
time = 0.29, size = 216, normalized size = 1.53

method	result

default	$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,(-1+\cos(dx+c))^{2}\left(A\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-B\right.}{d\sin(dx+c)^{4}\left(\frac{\cos(dx+}{1+\cos(dx}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(A*2^(1/2)*
arcsin((-1+cos(d*x+c))/sin(d*x+c))+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)-B*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+2*A*arctan(sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-B*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))/sin(d*x+c)^4/(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 1.40, size = 168, normalized size = 1.19

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}\,B\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\left(\left(2\,A-B\right)\cos\left(dx+c\right)+2\,A-B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)+\frac{\sqrt{2}\,((A-B)a\cos(dx+c)+(A-B)a)\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)}{\sqrt{a}}}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A - B)*co
s(d*x + c) + 2*A - B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) + sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a
)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{\cos(c + dx)}}{\sqrt{a(\cos(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/sqrt(a*(cos(c + d*x) + 1))
, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c + d\,x)}\,(A + B\,\cos(c + d\,x))}{\sqrt{a + a\,\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2), x
)
```

3.193 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=100

$$\frac{2B\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)+(A-B)*arcta
n(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2
^(1/2)/d/a^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3061, 2861, 211, 2853, 222}

$$\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) +
 (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx=(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx+\frac{B\int \frac{\sqrt{a}}{\sqrt{}}}{}$$

$$=-\frac{(2a(A-B))\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+a}}\right)}{d}$$

$$=\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{}{\sqrt{2}}\right.}{}$$

Mathematica [A]
time = 0.15, size = 82, normalized size = 0.82

$$\frac{2\left(\sqrt{2}\,B\,\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+(A-B)\mathrm{ArcTan}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]
),x]
```

```
[Out] (2*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + (A - B)*ArcTan[Sin[(c + d*
x)/2]/Sqrt[Cos[c + d*x]]])*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A]
time = 0.28, size = 149, normalized size = 1.49

method	result
default	$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(\sqrt{\cos}\left(dx+c\right)\right)\left(-1+\cos(dx+c)\right)\left(A\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-\right.}{d\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,a\sin(dx+c)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*(A*2^(1/2)*ar
csin((-1+cos(d*x+c))/sin(d*x+c))-B*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))-2*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))/(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)/a/sin(d*x+c)^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 1.14, size = 96, normalized size = 0.96

$$-\frac{\sqrt{2}\,(A-B)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)+2\,B\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -(sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*B*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a(\cos(c+dx)+1)}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*sqrt(cos(c + d*x)
)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2)), x
)
```

3.194 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=99

$$-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}$$

```
[Out] -(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))*2^(1/2)/d/a^(1/2)+2*A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x
+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3063, 12, 2861, 211}

$$\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2\int -\frac{a(A-}{2\sqrt{\cos(c+dx)}\sqrt{}}}{a}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + (-A+B)\int \frac{}{\sqrt{\cos(c+}}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{(2a(A-B))\text{Subst}\left(\int \frac{}{2a}\right.}{}$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.70, size = 203, normalized size = 2.05

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{1}{2}(c+dx)\right)\left(10B\cos(c+dx)-(A-B)\left(-\frac{5}{4}(1+4\cos(c+dx)+\cos(2(c+dx)))\csc^4\left(\frac{1}{2}(c+dx)\right)\left(1-\cos(c+dx)+\tanh^{-1}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cos(c+dx)\sqrt{2-2\sec(c+dx)}\right)+\frac{1}{2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)\tan(c+dx)\right)\right)}{5d\cos^{\frac{3}{2}}(c+dx)\sqrt{a(1+\cos(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]
),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*(10*B*Cos[c + d*x] - (A - B)*((-5*(1 +
 4*Cos[c + d*x] + Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^4*(1 - Cos[c + d*x] +
ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*S
ec[c + d*x]]))/4 + (Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c +
```

```
d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/2)))/(5*d*Cos[c + d*x]^(3/2)*Sqrt[a*
(1 + Cos[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(84) = 168$.
time = 0.30, size = 230, normalized size = 2.32

method	result
default	$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\cos(dx+c)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\cos(dx+c)}{da(1+\cos(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(a*(1+cos(d*x+c)))^(1/2)*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*c
os(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*2^(1/2)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+A*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+2*A*sin
(d*x+c))/a/(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 0.39, size = 143, normalized size = 1.44

$$\frac{2\sqrt{a\cos\left(dx+c\right)+a}\,A\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\frac{\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)}{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}}\right)}{\sqrt{a}}}{ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] (2*sqrt(a*cos(d*x + c) + a)*A*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(2)*((A
 - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*arctan(1/2*sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*
x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(3/
2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos(c + d\,x)}{\cos(c + d\,x)^{3/2}\,\sqrt{a + a\,\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2)), x
)
```

3.195 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=142

$$\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}-\frac{\ldots}{3d\sqrt{\ldots}}$$

```
[Out] (A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c
))^(1/2))*2^(1/2)/d/a^(1/2)+2/3*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*
x+c))^(1/2)-2/3*(A-3*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2
)
```

Rubi [A]
time = 0.21, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3063, 12, 2861, 211}

$$\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]]]])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 3*B)*Sin[c + d*x])/(3*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-3B)+aA\cos(}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(}}}{3a}$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-3B)\sin(c}{3d\sqrt{\cos(c+dx)}\sqrt{a+a}}$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-3B)\sin(c}{3d\sqrt{\cos(c+dx)}\sqrt{a+a}}$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-3B)\sin(c}{3d\sqrt{\cos(c+dx)}\sqrt{a+a}}$$

$$= \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.82, size = 627, normalized size = 4.42

$$\frac{4B\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d\sqrt{a(1+\cos(c+dx))}\left(1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{3/2}} + \frac{8B\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d\sqrt{a(1+\cos(c+dx))}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}} + \frac{2(A-B)\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-12\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right) - 12\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\left(4-7\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3\sin^4\left(\frac{c}{2}+\frac{dx}{2}\right)\right) + 7\sqrt{-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\left(1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\left(15-20\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)+8\sin^4\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\left(3-7\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sqrt{-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}} - 3\tanh^{-1}\left(\sqrt{-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\right)\left(1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\right)\right)}{63d\sqrt{a(1+\cos(c+dx))}\left(1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]
),x]
```

[Out] (4*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(3*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (8*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(3*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (2*(A - B)*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*(-12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7*Sin[c/2 + (d*x)/2]^2)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))] - 3*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*(1 - 2*Sin[c/2 + (d*x)/2]^2))))/(63*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(119) = 238$.
time = 0.35, size = 383, normalized size = 2.70

method	result
default	$\frac{\left(\sin^2(dx+c)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(3A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^2(dx+c)\right)\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-3B\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\mathrm{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3/d*sin(d*x+c)^2*(a*(1+cos(d*x+c)))^(1/2)*(3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+2*A*cos(d*x+c)*sin(d*x+c)-6*B*sin(d*x+c)*cos(d*x+c)-2*A*sin(d*x+c))/a/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 0.41, size = 163, normalized size = 1.15

$$-\frac{2\left(\left(A-3\,B\right)\cos\left(dx+c\right)-A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\frac{3\,\sqrt{2}\,\left((A-B)a\cos(dx+c)^{3}+(A-B)a\cos(dx+c)^{2}\right)\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)}{2\left(\cos(dx+c)^{2}+\cos(dx+c)\right)\sqrt{a}}\right)}{\sqrt{a}}}{3\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*(2*((A - 3*B)*cos(d*x + c) - A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))*sin(d*x + c) - 3*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*
x + c)^2)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*si
n(d*x + c)/((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x
 + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+B\cos\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(5/
2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2)), x
)
```

3.196 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=187

$$-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}-$$

```
[Out] -(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))*2^(1/2)/d/a^(1/2)+2/5*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d
*x+c))^(1/2)-2/15*(A-5*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1
/2)+2/15*(13*A-5*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3063, 12, 2861, 211}

$$-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*Cos[c
+ d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 5*B)*Sin[c + d*x])/(15*d*C
os[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*(13*A - 5*B)*Sin[c + d*x])
/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
```

```
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-5B)+2aA\cos(c\cdots}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c\cdots}}}{5a}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+\cdots}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,c\cdots}}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+\cdots}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,c\cdots}}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+\cdots}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,c\cdots}}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+\cdots}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,c\cdots}}$$

$$= -\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 7.94, size = 1728, normalized size = 9.24

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]
),x]
```

```
[Out] (4*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(5*d*Sqrt[a*(1 + Cos[c + d*x])]
*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) + (16*B*Cos[c/2 + (d*x)/2]*(Sin[c/2 +
(d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[c/2 + (d*x)/2])/Sqrt[1
 - 2*Sin[c/2 + (d*x)/2]^2]))/(15*d*Sqrt[a*(1 + Cos[c + d*x])]) - (2*(A - B)
*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*(4725*Sin[c/2 + (d*x)/2]^2 - 48825
*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x
)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2,
 Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10
- 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin
[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 51
8760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*S
in[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)
/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2
+ (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt
[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*
Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^
16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d
*x)/2]^4*HypergeometricPFQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2
]^2)))/(675*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)
*(-1 + 2*Sin[c/2 + (d*x)/2]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(158) = 316$.

time = 0.38, size = 519, normalized size = 2.78

method	result
default	$\frac{(\sin^4(dx+c))\sqrt{a(1+\cos(dx+c))}\left(15A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{2}\,(\cos^3(dx+c))\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-15B\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/15/d*sin(d*x+c)^4*(a*(1+cos(d*x+c)))^(1/2)*(15*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(5/2)*2^(1/2)*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(d*x+c))-15*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/s
in(d*x+c))+45*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*cos(d*x+c)^2*arcs
in((-1+cos(d*x+c))/sin(d*x+c))-45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/
2)*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))+45*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(5/2)*2^(1/2)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-45*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))+15*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arcsin((-1+cos(d
*x+c))/sin(d*x+c))-15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arcsin((-
1+cos(d*x+c))/sin(d*x+c))+26*A*cos(d*x+c)^2*sin(d*x+c)-10*B*cos(d*x+c)^2*si
n(d*x+c)-2*A*cos(d*x+c)*sin(d*x+c)+10*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c
))/a/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3/cos(d*x+c)^(5/2)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]

time = 0.42, size = 180, normalized size = 0.96

$$\frac{2\left((13A-5B)\cos(dx+c)^2-(A-5B)\cos(dx+c)+3A\right)\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-\frac{15\sqrt{2}\left((A-B)a\cos(dx+c)^4+(A-B)a\cos(dx+c)^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}}\right)}{\sqrt{a}}}{15\left(ad\cos(dx+c)^4+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(2*((13*A - 5*B)*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) + 3*A)*sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*((A - B)*a*
cos(d*x + c)^4 + (A - B)*a*cos(d*x + c)^3)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x + c)
)*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a*cos(c + d*x))^(1/2)), x
)
```

3.197 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=197

$$\frac{(2A-3B)\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d}-\frac{(5A-9B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+$$

```
[Out] (2*A-3*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d+1/2*(
A-B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-1/4*(5*A-9*B)*arc
tan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))
/a^(3/2)/d*2^(1/2)-1/2*(A-3*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x
+c))^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3056, 3062, 3061, 2861, 211, 2853, 222}

$$\frac{(2A-3B)\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(5A-9B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((2*A - 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3
/2)*d) - ((5*A - 9*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) - ((A - 3*B)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
```

```
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\,\left(\frac{3}{2}a(A-B)-a(A\right.}{\sqrt{a+a\cos(c+d}}}{2a^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A-3B)\sqrt{\cos(c+dx)}\,\sin(c}{2ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A-3B)\sqrt{\cos(c+dx)}\,\sin(c}{2ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A-3B)\sqrt{\cos(c+dx)}\,\sin(c}{2ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(2A-3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-9B)\tan^{-1}\left(\frac{}{\sqrt{}}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.25, size = 362, normalized size = 1.84

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(4Adx-6Bdx-2i(2A-3B)\sinh^{-1}\left(e^{i(c+dx)}\right)+i\sqrt{2}(5A-9B)\log\left(1+e^{i(c+dx)}\right)+4iA\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-6iB\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-5i\sqrt{2}A\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)+9i\sqrt{2}B\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)}{d\sqrt{1+e^{2i(c+dx)}}}+\frac{2\sqrt{\cos(c+dx)}\,(-A+3B+2B\cos(c+dx))\sec\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{d}\right)}{2(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(4*A*d*x - 6*B*d*x - (2*I)*(2*A - 3*B)*ArcSinh[E^(I*
(c + d*x))] + I*Sqrt[2]*(5*A - 9*B)*Log[1 + E^(I*(c + d*x))] + (4*I)*A*Log[
1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (6*I)*B*Log[1 + Sqrt[1 + E^((2*I)*(c +
 d*x))]] - (5*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2
*I)*(c + d*x))]] + (9*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1
 + E^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]) + (2*Sqrt[Cos[
c + d*x]]*(-A + 3*B + 2*B*Cos[c + d*x])*Sec[(c + d*x)/2]*Tan[(c + d*x)/2])/
d))/(2*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(166) = 332$.
time = 0.34, size = 379, normalized size = 1.92

method	result
default	$-\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,(-1+\cos(dx+c))^{3}\left(2A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^{2}(dx+c))+5A\sqrt{2}\,\sin(dx+c)\cos(dx+ \right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(2*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2+5*A*2^(1/2)*sin(d*x+c)*cos(d*x+
c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)^3-9*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(
d*x+c))-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+8*A*arctan(sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)-2*B*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-12*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)+6*B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c))/sin(d*x+c)^7/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/
a^2
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

<u>Fricas</u> [A]
time = 3.98, size = 237, normalized size = 1.20

$$\frac{\sqrt{2}\left((5A-9B)\cos(dx+c)^{2}+2(5A-9B)\cos(dx+c)+5A-9B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2(2B\cos(dx+c)-A+3B)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-4\left((2A-3B)\cos(dx+c)^{2}+2(2A-3B)\cos(dx+c)+2A-3B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{4\left(a^{2}d\cos(dx+c)^{2}+2a^{2}d\cos(dx+c)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5*A
 - 9*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
```

```
(sqrt(a)*sin(d*x + c))) + 2*(2*B*cos(d*x + c) - A + 3*B)*sqrt(a*cos(d*x + c
) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 4*((2*A - 3*B)*cos(d*x + c)^2 + 2*
(2*A - 3*B)*cos(d*x + c) + 2*A - 3*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) +
a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^
2*d*cos(d*x + c) + a^2*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x
)
```

3.198 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{2B\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d}+\frac{(A-5B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(A-B}{2d}$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d+1/4*(A-5*B)
*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1
/2))/a^(3/2)/d*2^(1/2)+1/2*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x
+c))^(3/2)
```

Rubi [A]
time = 0.25, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3056, 3061, 2861, 211, 2853, 222}

$$\frac{(A-5B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{2B\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
 ((A - 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(A-B)+2aB\cos}{\sqrt{\cos(c+dx)}\,\sqrt{a+a}}}{2a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(A-5B)\int \frac{}{\sqrt{\cos(c+d}}}{}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A-5B)\text{Subst}\left(\int \frac{1}{2a^2+a}\right.}{}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{(A-5B)\tan^{-1}\left(\frac{}{\sqrt{2}\,\sqrt{c}}\right.}{2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.65, size = 313, normalized size = 2.16

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(4Bdx-4iB\sinh^{-1}\left(e^{i(c+dx)}\right)-i\sqrt{2}(A-5B)\log\left(1+e^{i(c+dx)}\right)+4iB\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)+i\sqrt{2}A\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)-5i\sqrt{2}B\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)}{d\sqrt{1+e^{2i(c+dx)}}}+\frac{2(A-B)\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{d}\right)}{2(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(4*B*d*x - (4*I)*B*ArcSinh[E^(I*(c + d*x))] - I*Sqrt
[2]*(A - 5*B)*Log[1 + E^(I*(c + d*x))] + (4*I)*B*Log[1 + Sqrt[1 + E^((2*I)*
(c + d*x))]] + I*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2
*I)*(c + d*x))]] - (5*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1
 + E^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]) + (2*(A - B)*S
qrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Tan[(c + d*x)/2])/d))/(2*(a*(1 + Cos[c +
 d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(120) = 240$.
time = 0.30, size = 298, normalized size = 2.06

method	result

default	$-\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,(-1+\cos(dx+c))^{2}\left(2A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^{2}(dx+c))+A\sqrt{2}\,\sin(dx+c)\cos(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(2*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2+A*2^(1/2)*sin(d*x+c)*cos(d*x+c)
*arcsin((-1+cos(d*x+c))/sin(d*x+c))-5*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-8*B*arc
tan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos
(d*x+c)-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+2*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/sin(d*x+c)^5/(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [A]
time = 3.12, size = 203, normalized size = 1.40

$$-\frac{\sqrt{2}\left((A-5B)\cos(dx+c)^{2}+2(A-5B)\cos(dx+c)+A-5B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a\cos(dx+c)+a}\,(A-B)\sqrt{\cos(dx+c)}\sin(dx+c)+8\left(B\cos(dx+c)^{2}+2B\cos(dx+c)+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{4\left(a^{2}d\cos(dx+c)^{2}+2a^{2}d\cos(dx+c)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*
B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(A - B)*sqrt(cos(d*x + c))*
sin(d*x + c) + 8*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sqrt(a)*arctan(s
```

```
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{\cos(c + dx)}}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(3
/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c + d\,x)}\,\left(A + B\,\cos(c + d\,x)\right)}{\left(a + a\,\cos(c + d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x
)
```

3.199 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{(3A+B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

```
[Out] 1/4*(3*A+B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos
(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(
a+a*cos(d*x+c))^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3057, 12, 2861, 211}

$$\frac{(3A+B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((3*A + B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{a(3A\cdots}{2\sqrt{\cos(c+dx)}\,\sqrt{\cdots}}}{2\cdots}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A+B)\int \frac{\cdots}{\sqrt{\cos(c\cdots}}}{\cdots}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(3A+B)\mathrm{Subst}\left(\int \frac{\cdots}{2\cdots}\right.}{\cdots}$$

$$= \frac{(3A+B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.24, size = 212, normalized size = 1.98

$$\frac{i(3A+B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+\frac{1}{2}i(A-B)e^{-\frac{1}{2}i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}}{d\sqrt{1+e^{2i(c+dx)}}\,(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] (I*(3*A + B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d
*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])
]*Cos[(c + d*x)/2]^3 + ((I/2)*(A - B)*(-1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]])/E^((I/2)*(c + d*x)))/(
d*Sqrt[1 + E^((2*I)*(c + d*x))]*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(88) = 176$.
time = 0.30, size = 246, normalized size = 2.30

method	result
default	$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos(dx+c)\right)\left(-2A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^2(dx+c)\right)+3A\sqrt{2}\,\sin(dx+c)\cos(dx+c)\arcsin\left(\frac{-1+\cos(}{\sin(dx}\right.\right.}{4d\sqrt{\frac{\cos(dx+c}{1+\cos(dx+}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(-2*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)*cos(d*x+c)^2+3*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d
*x+c))/sin(d*x+c))+B*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/s
in(d*x+c))+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*B*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*cos(d*x+c)^2-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/a^2/cos(d*x+c)^(1/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

Fricas [A]
time = 0.40, size = 164, normalized size = 1.53

$$\frac{\sqrt{2}\left((3A+B)\cos\left(dx+c\right)^{2}+2\left(3A+B\right)\cos\left(dx+c\right)+3A+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos\left(dx+c\right)+a}\sqrt{a}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right)-2\sqrt{a\cos\left(dx+c\right)+a}\left(A-B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A + B
)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x
+ c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x
```

```
+ c) + a)*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 +
2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{(c + dx)}}{(a(\cos{(c + dx)} + 1))^{\frac{3}{2}}\sqrt{\cos{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
[Out] Integral((A + B*cos(c + d*x))/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(cos(c + d
*x))), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos{(c + d\,x)}}{\sqrt{\cos{(c + d\,x)}}\,(a + a\,\cos{(c + d\,x)})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2)),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2)), x
)
```

3.200 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{(7A-3B)\text{ArcTan}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{2}\ \sqrt{\cos(c+dx)}\ \sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\ a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\ (a+a\cos(c+dx))^{3/2}}+\frac{}{2}$$

```
[Out] -1/4*(7*A-3*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*
cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c)
)^(3/2)/cos(d*x+c)^(1/2)+1/2*(5*A-B)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*c
os(d*x+c))^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3057, 3063, 12, 2861, 211}

$$-\frac{(7A-3B)\text{ArcTan}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{2}\ \sqrt{\cos(c+dx)}\ \sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\ a^{3/2}d}+\frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\ \sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\ (a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] -1/2*((7*A - 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(
2*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((5*A - B)*Sin[c + d*x
])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(5A-B)-a(A-B}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,c}}}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\,\mathrm{si}}{2ad\sqrt{\cos(c+dx)}\,\sqrt{}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\,\mathrm{si}}{2ad\sqrt{\cos(c+dx)}\,\sqrt{}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\,\mathrm{si}}{2ad\sqrt{\cos(c+dx)}\,\sqrt{}}$$

$$= -\frac{(7A-3B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.56, size = 206, normalized size = 1.32

$$\frac{ie^{-\frac{1}{2}i(c+dx)}\left(\frac{2(-1+e^{i(c+dx)})(B(1+e^{2i(c+dx)})-A(5+8e^{i(c+dx)}+5e^{2i(c+dx)}))}{(1+e^{i(c+dx)})^2}-\sqrt{2}\,(7A-3B)\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)}{2d\sqrt{\cos(c+dx)}\,(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] ((I/2)*((2*(-1 + E^(I*(c + d*x)))*(B*(1 + E^((2*I)*(c + d*x))) - A*(5 + 8*E
^(I*(c + d*x)) + 5*E^((2*I)*(c + d*x)))))/(1 + E^(I*(c + d*x)))^2 - Sqrt[2]
*(7*A - 3*B)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(S
qrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])*Cos[(c + d*x)/2]^3)/(d*E^((I/2)*(c
+ d*x))*Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(131) = 262$.
time = 0.33, size = 299, normalized size = 1.92

method	result
default	$\frac{\sqrt{a\,(1+\cos(dx+c))}\,\left(7A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)-3B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(7*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)-3*B*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*cos(d*x+c)+7*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*2^(1/2)*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-10*A*cos(d*x+c)^2+2*B*cos(d*
x+c)^2+2*A*cos(d*x+c)-2*B*cos(d*x+c)+8*A)/a^2/sin(d*x+c)/(1+cos(d*x+c))/cos
(d*x+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

Fricas [A]
time = 0.41, size = 201, normalized size = 1.29

$$-\frac{\sqrt{2}\left((7A-3B)\cos\left(dx+c\right)^{3}+2\,(7A-3B)\cos\left(dx+c\right)^{2}+(7A-3B)\cos\left(dx+c\right)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right)-2\left((5A-B)\cos\left(dx+c\right)+4A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{4\left(a^{2}d\cos\left(dx+c\right)^{3}+2\,a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^3 + 2*(7*A - 3*B)*cos(d*x + c)^2 +
(7*A - 3*B)*cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) +
a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x +
c))) - 2*((5*A - B)*cos(d*x + c) + 4*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2
*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a*(cos(c + d*x) + 1))**(3/2)*cos(c + d*x)**
(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2)), x
)
```

3.201 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=203

$$\frac{(11A-7B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}+\frac{}{6a}$$

```
[Out] -1/2*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+1/4*(11*A-7
*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2))/a^(3/2)/d*2^(1/2)+1/6*(7*A-3*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+
a*cos(d*x+c))^(1/2)-1/6*(19*A-15*B)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*co
s(d*x+c))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3057, 3063, 12, 2861, 211}

$$\frac{(11A-7B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(7A-3B)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-\frac{(19A-15B)\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((11*A - 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*
d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((7*A - 3*B)*Sin[c + d*x
])/(6*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((19*A - 15*B)*Sin
[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
```

```
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(7A-3B)-2a(A-B}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\,c}}}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sin}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sin}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sin}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sin}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= \frac{(11A-7B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.25, size = 213, normalized size = 1.05

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{i(11A-7B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{d\sqrt{1+e^{2i(c+dx)}}}-\frac{(11A-15B+24(A-B)\cos(c+dx)+(19A-15B)\cos(2(c+dx)))\sec\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{6d\cos^{\frac{3}{2}}(c+dx)}\right)}{(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*((I*(11*A - 7*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)
*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1
 + E^((2*I)*(c + d*x))])])/(d*Sqrt[1 + E^((2*I)*(c + d*x))]) - ((11*A - 15*
B + 24*(A - B)*Cos[c + d*x] + (19*A - 15*B)*Cos[2*(c + d*x)])*Sec[(c + d*x)
/2]*Tan[(c + d*x)/2])/(6*d*Cos[c + d*x]^(3/2))))/(a*(1 + Cos[c + d*x]))^(3/
2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(172) = 344.
time = 0.39, size = 443, normalized size = 2.18

method	result
default	$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\sin(dx+c)\left(33A\sin(dx+c)\left(\cos^{2}(dx+c)\right)\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}-21B\sin(dx+c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/12/d*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*(33*A*sin(d*x+c)*cos(d*x+c)^2*2^
(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-
21*B*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)+66*A*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-42*B*sin(d*x+c)
*cos(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)+33*A*sin(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(c
os(d*x+c)/(1+cos(d*x+c)))^(3/2)-21*B*sin(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-38*A*cos(d*x+c)^3+30*B*co
s(d*x+c)^3+14*A*cos(d*x+c)^2-6*B*cos(d*x+c)^2+32*A*cos(d*x+c)-24*B*cos(d*x+
c)-8*A)/a^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

Fricas [A]
time = 0.41, size = 221, normalized size = 1.09

$$\frac{3\sqrt{2}\left(\left(11A-7B\right)\cos\left(dx+c\right)^{4}+2\left(11A-7B\right)\cos\left(dx+c\right)^{3}+\left(11A-7B\right)\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{a\cos\left(dx+c\right)+a}\sqrt{a}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right)-2\left(\left(19A-15B\right)\cos\left(dx+c\right)^{2}+12\left(A-B\right)\cos\left(dx+c\right)-4A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{12\left(a^{2}d\cos\left(dx+c\right)^{4}+2a^{2}d\cos\left(dx+c\right)^{3}+a^{2}d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^4 + 2*(11*A - 7*B)*cos(d*x + c)^
3 + (11*A - 7*B)*cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x
```

```
+ c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos
(d*x + c))) - 2*((19*A - 15*B)*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) - 4
*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*
x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(3/2)),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(3/2)), x
)
```

3.202 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=246

$$\frac{(2A-5B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d}-\frac{(43A-115B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}$$

```
[Out] (2*A-5*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d+1/4*(
A-B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/16*(7*A-15*B)*c
os(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)-1/32*(43*A-115*B)*arc
tan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))
/a^(5/2)/d*2^(1/2)-1/16*(11*A-35*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*
cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.53, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3056, 3062, 3061, 2861, 211, 2853, 222}

$$\frac{(2A-5B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(43A-115B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{(7A-15B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((2*A - 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5
/2)*d) - ((43*A - 115*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
+ d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c
 + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((7*A - 15*B
)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) - ((
11*A - 35*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c +
d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-a(A-5B)\text{co}\right.}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(7A-15B)\cos^{\frac{3}{2}}(c+dx)\sin(}{16ad(a+a\cos(c+dx))^{3}}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(7A-15B)\cos^{\frac{3}{2}}(c+dx)\sin(}{16ad(a+a\cos(c+dx))^{3}}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(7A-15B)\cos^{\frac{3}{2}}(c+dx)\sin(}{16ad(a+a\cos(c+dx))^{3}}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(7A-15B)\cos^{\frac{3}{2}}(c+dx)\sin(}{16ad(a+a\cos(c+dx))^{3}}$$

$$= \frac{(2A-5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-115B)\tan^{-1}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.49, size = 376, normalized size = 1.53

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(32Adx-80Bdx-16i(2A-5B)\sinh^{-1}\left(e^{i(c+dx)}\right)+i\sqrt{2}(43A-115B)\log\left(1+e^{i(c+dx)}\right)+32iA\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-80iB\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-43i\sqrt{2}A\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)+115i\sqrt{2}B\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)}{\sqrt{1+e^{2i(c+dx)}}}+\sqrt{\cos(c+dx)}(-11A+43B+(-15A+55B)\cos(c+dx)+8B\cos(2(c+dx)))\sec^3\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d(a(1+\cos(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(32*A*d*x - 80*B*d*x - (16*I)*(2*A - 5*B)*ArcSinh[E^
(I*(c + d*x))] + I*Sqrt[2]*(43*A - 115*B)*Log[1 + E^(I*(c + d*x))] + (32*I)
*A*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (80*I)*B*Log[1 + Sqrt[1 + E^((2
*I)*(c + d*x))]] - (43*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[
1 + E^((2*I)*(c + d*x))]] + (115*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d*x))] + Sqrt[
Cos[c + d*x]]*(-11*A + 43*B + (-15*A + 55*B)*Cos[c + d*x] + 8*B*Cos[2*(c +
d*x)])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/2]))/(8*d*(a*(1 + Cos[c + d*x]))^(5
/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(209) = 418$.

time = 0.38, size = 647, normalized size = 2.63

method	result
default	$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(-1+\cos(dx+c)\right)^{5}\left(\cos^{\frac{5}{2}}(dx+c)\right)\left(30A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^{3}(dx+c)\right)+22A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*cos(d*x+c)^(5/2)*(30*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^3+22*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*cos(d*x+c)^2+43*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(
d*x+c))/sin(d*x+c))-32*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4-115
*B*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-30*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)+64*A*cos(d*x+c)^2*sin(d*x+c)*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+43*A*2^(1/2)
*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-78*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-160*B*cos(d*x+c)^2*sin(d*x+c)*arctan(sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-115*B*2^(1/2)*sin(d*x
+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-22*A*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)+64*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(
d*x+c))*sin(d*x+c)*cos(d*x+c)+40*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c)^2-160*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)*sin(d*x+c)*cos(d*x+c)+70*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/
sin(d*x+c)^11/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/a^3
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [A]

time = 8.39, size = 302, normalized size = 1.23

$$\frac{\sqrt{2}\left((43A-115B)\cos\left(dx+c\right)^{3}+3(43A-115B)\cos\left(dx+c\right)^{2}+3(43A-115B)\cos\left(dx+c\right)+43A-115B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\sin(dx+c)}\right)+2\left(16B\cos\left(dx+c\right)^{2}-5(3A-11B)\cos\left(dx+c\right)-11A+35B\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-32\left((2A-5B)\cos\left(dx+c\right)^{3}+3(2A-5B)\cos\left(dx+c\right)^{2}+3(2A-5B)\cos\left(dx+c\right)+2A-5B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\sin(dx+c)}\right)}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^3 + 3*(43*A - 115*B)*cos(d*x + c
)^2 + 3*(43*A - 115*B)*cos(d*x + c) + 43*A - 115*B)*sqrt(a)*arctan(sqrt(2)*
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(16
*B*cos(d*x + c)^2 - 5*(3*A - 11*B)*cos(d*x + c) - 11*A + 35*B)*sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 32*((2*A - 5*B)*cos(d*x + c)
^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + 3*(2*A - 5*B)*cos(d*x + c) + 2*A - 5*B)
*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*
x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
+ c) + a^3*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x
)
```

3.203 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=194

$$\frac{2B\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d}+\frac{(3A-43B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A-$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d+1/4*(A-B)*c
os(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/32*(3*A-43*B)*arctan(
1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(
5/2)/d*2^(1/2)+1/16*(3*A-11*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x
+c))^(3/2)
```

Rubi [A]
time = 0.37, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3056, 3061, 2861, 211, 2853, 222}

$$\frac{(3A-43B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2B\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{(3A-11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
 ((3*A - 43*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((3*A - 11*B)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\,\left(\frac{3}{2}a(A-B)+4a\right.}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A-11B)\sqrt{\cos(c+dx)}\,\mathrm{s}}{16ad(a+a\cos(c+dx}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A-11B)\sqrt{\cos(c+dx)}\,\mathrm{s}}{16ad(a+a\cos(c+dx}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A-11B)\sqrt{\cos(c+dx)}\,\mathrm{s}}{16ad(a+a\cos(c+dx}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{(3A-43B)\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{c}}\right.}{16}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.76, size = 329, normalized size = 1.70

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(32Bdx-32iB\sinh^{-1}\left(e^{i(c+dx)}\right)-i\sqrt{2}(3A-43B)\log\left(1+e^{i(c+dx)}\right)+32iB\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)+3i\sqrt{2}A\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)-43i\sqrt{2}B\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)}{\sqrt{1+e^{2i(c+dx)}}}+\sqrt{\cos(c+dx)}(3A-11B+(7A-15B)\cos(c+dx))\sec^3\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d(a(1+\cos(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(32*B*d*x - (32*I)*B*ArcSinh[E^(I*(c + d*x))] - I*Sq
rt[2]*(3*A - 43*B)*Log[1 + E^(I*(c + d*x))] + (32*I)*B*Log[1 + Sqrt[1 + E^(
(2*I)*(c + d*x))]] + (3*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt
[1 + E^((2*I)*(c + d*x))]] - (43*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d*x))] + Sqrt[
Cos[c + d*x]]*(3*A - 11*B + (7*A - 15*B)*Cos[c + d*x])*Sec[(c + d*x)/2]^3*T
an[(c + d*x)/2]))/(8*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(163) = 326$.
time = 0.34, size = 515, normalized size = 2.65

method	result
default	$-\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,(-1+\cos(dx+c))^{4}\left(14A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^{3}(dx+c))+6A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^{2}(d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/32/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*(14*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^3+6*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)*cos(d*x+c)^2+3*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*
x+c))/sin(d*x+c))-43*B*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))-14*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)+3*A*2^(1/2
)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-30*B*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-64*B*cos(d*x+c)^2*sin(d*x+c)*arctan(sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-43*B*2^(1/2)*sin(d*x+
c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-6*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)+8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-64*B*arctan(
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x
+c)+22*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/sin(d*x+c)^9/(cos(d*
x+c)/(1+cos(d*x+c)))^(5/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [A]
time = 6.42, size = 267, normalized size = 1.38

$$-\frac{\sqrt{2}\left((3A-43B)\cos\left(dx+c\right)^{3}+3\left(3A-43B\right)\cos\left(dx+c\right)^{2}+3\left(3A-43B\right)\cos\left(dx+c\right)+3A-43B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\sin(dx+c)}\right)-2\left((7A-15B)\cos\left(dx+c\right)+3A-11B\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+64\left(B\cos\left(dx+c\right)^{3}+3B\cos\left(dx+c\right)^{2}+3B\cos\left(dx+c\right)+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\sin(dx+c)}\right)}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^2
 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((7*A - 15
*B)*cos(d*x + c) + 3*A - 11*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c) + 64*(B*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c)
 + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*s
in(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos
(d*x + c) + a^3*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3008 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x
)
```

3.204 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{(5A+3B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{(A+7\ldots}{16\ldots}$$

```
[Out] 1/32*(5*A+3*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*
cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+1/4*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/
d/(a+a*cos(d*x+c))^(5/2)+1/16*(A+7*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*
cos(d*x+c))^(3/2)
```

Rubi [A]

time = 0.24, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3056, 3057, 12, 2861, 211}

$$\frac{(5A+3B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((5*A + 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((A + 7*B)*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(A-B)+a(A+3B)\cos(}{\sqrt{\cos(c+dx)}\,(a+a\cos(}}{4a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B)\sqrt{\cos(c+dx)}}{16ad(a+a\cos(c+}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B)\sqrt{\cos(c+dx)}}{16ad(a+a\cos(c+}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B)\sqrt{\cos(c+dx)}}{16ad(a+a\cos(c+}$$

$$= \frac{(5A+3B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.52, size = 198, normalized size = 1.29

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{i(5A+3B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+\frac{1}{2}\sqrt{\cos(c+dx)}\,(5A+3B+(A+7B)\cos(c+dx))\sec^3\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{4d(a(1+\cos(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((I*(5*A + 3*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*
(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1
+ E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + (Sqrt[Cos[c + d*x
]]*(5*A + 3*B + (A + 7*B)*Cos[c + d*x])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/2]
)/2))/(4*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(129) = 258$.
time = 0.32, size = 413, normalized size = 2.68

method	result
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,(-1+\cos(dx+c))^3\left(2A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^3(dx+c))+10A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^2(d\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/32/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(2*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^3+10*A*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*cos(d*x+c)^2+5*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x
+c))/sin(d*x+c))+3*B*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))
/sin(d*x+c))-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)+5*A*2^(1/2)*s
in(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+14*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+3*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arcsin((
-1+cos(d*x+c))/sin(d*x+c))-10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-8*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*cos(d*x+c))/sin(d*x+c)^7/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [A]
time = 0.38, size = 215, normalized size = 1.40

$$\frac{\sqrt{2}\left((5A+3B)\cos(dx+c)^3+3(5A+3B)\cos(dx+c)^2+3(5A+3B)\cos(dx+c)+5A+3B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)+2\left((A+7B)\cos(dx+c)+5A+3B\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 +
3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*c
os(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
+ a*cos(d*x + c))) + 2*((A + 7*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*
cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{(a(\cos(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(5
/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x
)
```

3.205 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$\frac{(19A+5B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}-\frac{(9A$$

```
[Out] 1/32*(19*A+5*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)
/d/(a+a*cos(d*x+c))^(5/2)-1/16*(9*A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a
*cos(d*x+c))^(3/2)
```

Rubi [A]
time = 0.25, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3057, 12, 2861, 211}

$$\frac{(19A+5B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] ((19*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((9*A - B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(7A+B)-a(A-B)\,\mathrm{co}}{\sqrt{\cos(c+dx)}\,(a+a\,\mathrm{co}}}{4a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(9A-B)\sqrt{\cos(c+dx}}{16ad(a+a\cos(c}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(9A-B)\sqrt{\cos(c+dx}}{16ad(a+a\cos(c}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(9A-B)\sqrt{\cos(c+dx}}{16ad(a+a\cos(c}$$

$$= \frac{(19A+5B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.47, size = 200, normalized size = 1.28

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{i(19A+5B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{1+e^{2i(c+dx)}}}-\frac{1}{2}\sqrt{\cos(c+dx)}\,(13A-5B+(9A-B)\cos(c+dx))\sec^3\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{4d(a(1+\cos(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((I*(19*A + 5*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)
*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1
 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - (Sqrt[Cos[c + d*
x]]*(13*A - 5*B + (9*A - B)*Cos[c + d*x])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/
2])/2))/(4*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(131) = 262$.
time = 0.31, size = 413, normalized size = 2.65

method	result
default	$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos(dx+c)\right)^{2}\left(-18A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^{3}(dx+c)\right)+19A\sqrt{2}\,\left(\cos^{2}(dx+c)\right)\sin(dx+c)\,\text{arcsi}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(-18*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)*cos(d*x+c)^3+19*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((
-1+cos(d*x+c))/sin(d*x+c))-26*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c
)^2+5*B*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+
19*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+18*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)+5*B*2^(1/2)*sin(d*x+c)*cos(d*x
+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*cos(d*x+c)^3+26*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+8*B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)^2-10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/a^3/cos(d*x+c)^(1/2)/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

Fricas [A]
time = 0.38, size = 217, normalized size = 1.39

$$\frac{\sqrt{2}\left((19A+5B)\cos(dx+c)^{3}+3(19A+5B)\cos(dx+c)^{2}+3(19A+5B)\cos(dx+c)+19A+5B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right)-2\left((9A-B)\cos(dx+c)+13A-5B\right)\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{32\left(a^{3}d\cos(dx+c)^{3}+3a^{3}d\cos(dx+c)^{2}+3a^{3}d\cos(dx+c)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2
+ 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt
(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c
)^2 + a*cos(d*x + c))) - 2*((9*A - B)*cos(d*x + c) + 13*A - 5*B)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{5}{2}}\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a*(cos(c + d*x) + 1))**(5/2)*sqrt(cos(c + d
*x))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos(c + d\,x)}{\sqrt{\cos(c + d\,x)}\,(a + a\,\cos(c + d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2)), x
)
```

3.206 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{(75A-19B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}$$

```
[Out] -1/32*(75*A-19*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a
+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x
+c))^(5/2)/cos(d*x+c)^(1/2)-1/16*(13*A-5*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))
^(3/2)/cos(d*x+c)^(1/2)+1/16*(49*A-9*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(
a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3057, 3063, 12, 2861, 211}

$$-\frac{(75A-19B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(49A-9B)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}-\frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] -1/16*((75*A - 19*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x]
)/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) - ((13*A - 5*B)*Sin[c
 + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((49*A -
9*B)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
```

```
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(9A-B)-2a(A-B)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(}}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5}{16ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5}{16ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5}{16ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5}{16ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{(75A-19B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right.}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.62, size = 217, normalized size = 1.07

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-\frac{i(75A-19B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+\frac{(113A-9B+2(85A-13B)\cos(c+dx)+(49A-9B)\cos(2(c+dx)))\sec^3\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{4\sqrt{\cos(c+dx)}}\right)}{4d(a(1+\cos(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-I)*(75*A - 19*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((
2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + ((113*A - 9*
B + 2*(85*A - 13*B)*Cos[c + d*x] + (49*A - 9*B)*Cos[2*(c + d*x)])*Sec[(c +
d*x)/2]^3*Tan[(c + d*x)/2])/(4*Sqrt[Cos[c + d*x]])))/(4*d*(a*(1 + Cos[c + d
*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(172) = 344$.
time = 0.35, size = 443, normalized size = 2.18

method	result
default	$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,(-1+\cos(dx+c))\left(75A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-19$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(75*A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*
x+c))-19*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*2^(1/2
)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+150*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)-38*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*cos(d*x+c)+75*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*2^(1
/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-19*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-98*A*cos(d*x+c)^3+
18*B*cos(d*x+c)^3-72*A*cos(d*x+c)^2+8*B*cos(d*x+c)^2+106*A*cos(d*x+c)-26*B*
cos(d*x+c)+64*A)/a^3/sin(d*x+c)^3/(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.40, size = 248, normalized size = 1.22

$$-\frac{\sqrt{2}\left((75\,A-19\,B)\cos\left(dx+c\right)^4+3\,(75\,A-19\,B)\cos\left(dx+c\right)^3+3\,(75\,A-19\,B)\cos\left(dx+c\right)^2+(75\,A-19\,B)\cos\left(dx+c\right)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{a}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)-2\left((49\,A-9\,B)\cos\left(dx+c\right)^2+(85\,A-13\,B)\cos\left(dx+c\right)+32\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{32\left(a^3d\cos\left(dx+c\right)^4+3\,a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+a^3d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^4 + 3*(75*A - 19*B)*cos(d*x + c)
^3 + 3*(75*A - 19*B)*cos(d*x + c)^2 + (75*A - 19*B)*cos(d*x + c))*sqrt(a)*a
rctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d
```

```
*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((49*A - 9*B)*cos(d*x + c)
^2 + (85*A - 13*B)*cos(d*x + c) + 32*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2)),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2)), x
)
```

3.207 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=250

$$\frac{(163A-75B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}-1$$

```
[Out] -1/4*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2)-1/16*(17*A-
9*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+1/32*(163*A-75*
B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^
(1/2))/a^(5/2)/d*2^(1/2)+1/48*(95*A-39*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)
/(a+a*cos(d*x+c))^(1/2)-1/48*(299*A-147*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2
)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3057, 3063, 12, 2861, 211}

$$\frac{(163A-75B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(95A-39B)\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(299A-147B)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}-\frac{(17A-9B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] ((163*A - 75*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/
(4*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) - ((17*A - 9*B)*Sin[c +
 d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((95*A - 39
*B)*Sin[c + d*x])/(48*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) -
((299*A - 147*B)*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(11A-3B)-3a(A-B)\cos}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+}}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin}{16ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin}{16ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin}{16ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin}{16ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin}{16ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= \frac{(163A-75B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.86, size = 239, normalized size = 0.96

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{3i(163A-75B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{1+e^{2i(c+dx)}}} - \frac{(878A-510B+(1537A-825B)\cos(c+dx)+2(503A-255B)\cos(2(c+dx))+299A\cos(3(c+dx))-147B\cos(3(c+dx)))\sec^3\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{8\cos^{\frac{3}{2}}(c+dx)}\right)}{12d(a(1+\cos(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((3*I)*(163*A - 75*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^
((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*
Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - ((878*A -
510*B + (1537*A - 825*B)*Cos[c + d*x] + 2*(503*A - 255*B)*Cos[2*(c + d*x)]
+ 299*A*Cos[3*(c + d*x)] - 147*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^3*Tan[(
c + d*x)/2])/(8*Cos[c + d*x]^(3/2))))/(12*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(213) = 426$.
time = 0.29, size = 571, normalized size = 2.28

method	result
default	$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(489A\sin(dx+c)\sqrt{2}\,\left(\cos^{3}(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-225B\sin(dx+c)\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/96/d*(a*(1+cos(d*x+c)))^(1/2)*(489*A*sin(d*x+c)*2^(1/2)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-225*B*sin(d*x+c)*2^(1/2)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+1467*A*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-675*B*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1467*A*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-675*B*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+489*A*sin(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-225*B*sin(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-598*A*cos(d*x+c)^4+294*B*cos(d*x+c)^4-408*A*cos(d*x+c)^3+216*B*cos(d*x+c)^3+686*A*cos(d*x+c)^2-318*B*cos(d*x+c)^2+384*A*cos(d*x+c)-192*B*cos(d*x+c)-64*A)/a^3/sin(d*x+c)/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)

Fricas [A]
time = 0.42, size = 270, normalized size = 1.08

$$\frac{3\sqrt{2}\left((163A-75B)\cos(dx+c)^{5}+3(163A-75B)\cos(dx+c)^{4}+3(163A-75B)\cos(dx+c)^{3}+(163A-75B)\cos(dx+c)^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right)-2\left((299A-147B)\cos(dx+c)^{3}+(503A-255B)\cos(dx+c)^{2}+32(5A-3B)\cos(dx+c)-32A\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{96\left(a^{3}d\cos(dx+c)^{5}+3a^{3}d\cos(dx+c)^{4}+3a^{3}d\cos(dx+c)^{3}+a^{3}d\cos(dx+c)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^5 + 3*(163*A - 75*B)*cos(d*x +
 c)^4 + 3*(163*A - 75*B)*cos(d*x + c)^3 + (163*A - 75*B)*cos(d*x + c)^2)*sq
rt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((299*A - 147*B)*co
s(d*x + c)^3 + (503*A - 255*B)*cos(d*x + c)^2 + 32*(5*A - 3*B)*cos(d*x + c)
 - 32*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*c
os(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos
(d*x + c)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(5/2)), x
)
```

3.208 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=293

$$\frac{(2A-7B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d}-\frac{(177A-637B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d}$$

```
[Out] (2*A-7*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d+1/6*(
A-B)*cos(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)+1/16*(3*A-7*B)*co
s(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)+1/192*(79*A-259*B)*cos
(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)-1/128*(177*A-637*B)*a
rctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2
))/a^(7/2)/d*2^(1/2)-7/64*(7*A-27*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/d/(a+a
*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3056, 3062, 3061, 2861, 211, 2853, 222}

$$\frac{(2A-7B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{(177A-637B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{7(7A-27B)\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^3d\sqrt{a\cos(c+dx)+a}}+\frac{(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}+\frac{(3A-7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{16ad(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((2*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7
/2)*d) - ((177*A - 637*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
 + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Cos[
c + d*x]^(7/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((3*A - 7*B
)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) + ((
79*A - 259*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*
x])^(3/2)) - (7*(7*A - 27*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*a^3*d*Sqr
t[a + a*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7}{2}a(A-B)-a(A-7B)\right.}{(a+a\cos(c+dx))^{5/2}}}{6a^2}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(}{16ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(}{16ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(}{16ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(}{16ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(}{16ad(a+a\cos(c+dx))}$$

$$= \frac{(2A-7B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} - \frac{(177A-637B)\tan}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.66, size = 396, normalized size = 1.35

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{3\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(128Adx-448Bdx-64i(2A-7B)\sinh^{-1}\left(e^{i(c+dx)}\right)+i\sqrt{2}(177A-637B)\log\left(1+e^{i(c+dx)}\right)+128iA\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-448iB\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-177i\sqrt{2}A\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)+637i\sqrt{2}B\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)}{\sqrt{1+e^{2i(c+dx)}}}+\frac{1}{4}\sqrt{\cos(c+dx)}\left(-541A+2233B+(-724A+3172B)\cos(c+dx)+(-247A+1099B)\cos(2(c+dx))+96B\cos(3(c+dx))\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{48d(a(1+\cos(c+dx)))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*((3*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c +
 d*x)))/E^(I*(c + d*x))]*(128*A*d*x - 448*B*d*x - (64*I)*(2*A - 7*B)*ArcSin
h[E^(I*(c + d*x))] + I*Sqrt[2]*(177*A - 637*B)*Log[1 + E^(I*(c + d*x))] + (
128*I)*A*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (448*I)*B*Log[1 + Sqrt[1
+ E^((2*I)*(c + d*x))]] - (177*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[
```

```
2]*Sqrt[1 + E^((2*I)*(c + d*x))]] + (637*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x
)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d*x))]
 + (Sqrt[Cos[c + d*x]]*(-541*A + 2233*B + (-724*A + 3172*B)*Cos[c + d*x] +
(-247*A + 1099*B)*Cos[2*(c + d*x)] + 96*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2
]^5*Tan[(c + d*x)/2])/4))/(48*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(250) = 500$.
time = 0.41, size = 887, normalized size = 3.03

method	result	size
default	Expression too large to display	887

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^7*cos(d*x+c)^(7/2)*(494*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4+724*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)*cos(d*x+c)^3+531*A*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+
cos(d*x+c))/sin(d*x+c))-384*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^
5-1911*B*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
-200*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2+768*A*arctan(sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^3*sin(d*x+c)+10
62*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-181
4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4-2688*B*arctan(sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^3*sin(d*x+c)-3822
*B*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-724*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)+1536*A*cos(d*x+c)^2*sin(d*x+c
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+531*A*2^(
1/2)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-686*B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-5376*B*cos(d*x+c)^2*sin(d*x+c)*arct
an(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-1911*B*2^(1/2)*
sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-294*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)+768*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)+1750*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*cos(d*x+c)^2-2688*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)+1134*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*cos(d*x+c))/sin(d*x+c)^15/(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

Fricas [A]
time = 14.42, size = 368, normalized size = 1.26

$$\frac{3\sqrt{2}\left((177A-637B)\cos(dx+c)^4+4(177A-637B)\cos(dx+c)^3+6(177A-637B)\cos(dx+c)^2+4(177A-637B)\cos(dx+c)+177A-637B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2\left(192B\cos(dx+c)^3-(247A-1099B)\cos(dx+c)^2-2(181A-721B)\cos(dx+c)-147A+567B\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-384\left((2A-7B)\cos(dx+c)^4+4(2A-7B)\cos(dx+c)^3+6(2A-7B)\cos(dx+c)^2+4(2A-7B)\cos(dx+c)+2A-7B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((177*A - 637*B)*cos(d*x + c)^4 + 4*(177*A - 637*B)*cos(d*
x + c)^3 + 6*(177*A - 637*B)*cos(d*x + c)^2 + 4*(177*A - 637*B)*cos(d*x + c
) + 177*A - 637*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(192*B*cos(d*x + c)^3 - (247*A - 109
9*B)*cos(d*x + c)^2 - 2*(181*A - 721*B)*cos(d*x + c) - 147*A + 567*B)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 384*((2*A - 7*B)*cos(
d*x + c)^4 + 4*(2*A - 7*B)*cos(d*x + c)^3 + 6*(2*A - 7*B)*cos(d*x + c)^2 +
4*(2*A - 7*B)*cos(d*x + c) + 2*A - 7*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^{7/2}\,(A+B\,\cos(c+dx))}{(a+a\,\cos(c+dx))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),x)
[Out] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2), x
)
```

3.209 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=241

$$\frac{2B\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d}+\frac{(5A-177B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{(A$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d+1/6*(A-B)*c
os(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)+1/48*(5*A-17*B)*cos(d*x
+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)+1/128*(5*A-177*B)*arctan(1/
2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/
2)/d*2^(1/2)+1/64*(5*A-49*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x
+c))^(3/2)
```

Rubi [A]

time = 0.49, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3056, 3061, 2861, 211, 2853, 222}

$$\frac{(5A-177B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{2B\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}+\frac{(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}+\frac{(5A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{48ad(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7/2)*d) +
 ((5*A - 177*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Cos[c + d*x]^(
5/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((5*A - 17*B)*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) + ((5*A - 49*
B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
```

```
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)+6aB\cos(c\right.}{(a+a\cos(c+dx))^{5/2}}}{6a^2}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(5A-17B)\cos^{\frac{3}{2}}(c+dx)\sin}{48ad(a+a\cos(c+dx)}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(5A-17B)\cos^{\frac{3}{2}}(c+dx)\sin}{48ad(a+a\cos(c+dx)}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(5A-17B)\cos^{\frac{3}{2}}(c+dx)\sin}{48ad(a+a\cos(c+dx)}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(5A-17B)\cos^{\frac{3}{2}}(c+dx)\sin}{48ad(a+a\cos(c+dx)}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} + \frac{(5A-177B)\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{}}\right.}{64}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.17, size = 350, normalized size = 1.45

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{3\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(128Bdx-128iB\sinh^{-1}\left(e^{i(c+dx)}\right)-i\sqrt{2}(5A-177B)\log\left(1+e^{i(c+dx)}\right)+128iB\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)+5i\sqrt{2}A\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)-177i\sqrt{2}B\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)}{\sqrt{1+e^{2i(c+dx)}}}+\frac{1}{4}\sqrt{\cos(c+dx)}\left(97A-541B+4(25A-181B)\cos(c+dx)+(67A-247B)\cos(2(c+dx))\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{48d(a(1+\cos(c+dx)))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*((3*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c +
 d*x)))/E^(I*(c + d*x))]*(128*B*d*x - (128*I)*B*ArcSinh[E^(I*(c + d*x))] -
I*Sqrt[2]*(5*A - 177*B)*Log[1 + E^(I*(c + d*x))] + (128*I)*B*Log[1 + Sqrt[1
 + E^((2*I)*(c + d*x))]] + (5*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))]] - (177*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)
) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d*x))]
+ (Sqrt[Cos[c + d*x]]*(97*A - 541*B + 4*(25*A - 181*B)*Cos[c + d*x] + (67*A
 - 247*B)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2])/4))/(48*d*
(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 702 vs. $2(204) = 408$.
time = 0.37, size = 703, normalized size = 2.92

method	result
default	$-\frac{(-1+\cos(dx+c))^6 \left(\cos^{\frac{5}{2}}(dx+c)\right) \sqrt{a\,(1+\cos(dx+c))}\, \left(134A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}} (\cos^4(dx+c))+100A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}} (\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/384/d*(-1+cos(d*x+c))^6*cos(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*(134*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4+100*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)*cos(d*x+c)^3+15*A*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+c
os(d*x+c))/sin(d*x+c))-531*B*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos
(d*x+c))/sin(d*x+c))-104*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2+3
0*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-494*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4-768*B*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^3*sin(d*x+c)-1062*B*
2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-100*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)+15*A*2^(1/2)*sin(d*x+c)*cos(d*x+
c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-230*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)^3-1536*B*cos(d*x+c)^2*sin(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-531*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))-30*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+430*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-768*B*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)+294*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/sin(d*x+c)^13/(cos(d*x+c)/(1+cos
(d*x+c)))^(7/2)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

Fricas [A]
time = 8.27, size = 327, normalized size = 1.36

$$-\frac{3\sqrt{2}\left((5A-177B)\cos(dx+c)^4+4(5A-177B)\cos(dx+c)^3+6(5A-177B)\cos(dx+c)^2+4(5A-177B)\cos(dx+c)+5A-177B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\left((67A-247B)\cos(dx+c)^2+2(25A-181B)\cos(dx+c)+15A-147B\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)+768\left(B\cos(dx+c)^4+4B\cos(dx+c)^3+6B\cos(dx+c)^2+4B\cos(dx+c)+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
[Out] -1/384*(3*sqrt(2)*((5*A - 177*B)*cos(d*x + c)^4 + 4*(5*A - 177*B)*cos(d*x +
 c)^3 + 6*(5*A - 177*B)*cos(d*x + c)^2 + 4*(5*A - 177*B)*cos(d*x + c) + 5*A
 - 177*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)/(sqrt(a)*sin(d*x + c))) - 2*((67*A - 247*B)*cos(d*x + c)^2 + 2*(25*A - 18
1*B)*cos(d*x + c) + 15*A - 147*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c
))*sin(d*x + c) + 768*(B*cos(d*x + c)^4 + 4*B*cos(d*x + c)^3 + 6*B*cos(d*x
+ c)^2 + 4*B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos
(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),x)
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2), x
)
```

3.210 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=201

$$\frac{(7A+5B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}+\frac{(A-13B)\ldots}{48a\ldots}$$

```
[Out] 1/6*(A-B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)+1/128*(7*A+5
*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2))/a^(7/2)/d*2^(1/2)+1/48*(A-13*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+
a*cos(d*x+c))^(5/2)+1/192*(17*A+67*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+
a*cos(d*x+c))^(3/2)
```

Rubi [A]
time = 0.36, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3056, 3057, 12, 2861, 211}

$$\frac{(7A+5B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}+\frac{(A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{48ad(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((7*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((A - 13*B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) + ((17*A + 67*B)*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
```

```
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\,(\frac{3}{2}a(A-B)+a(A}{(a+a\cos(c+dx))^{5/2}}}{6a^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A-13B)\sqrt{\cos(c+dx)}\,\sin(}{48ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A-13B)\sqrt{\cos(c+dx)}\,\sin(}{48ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A-13B)\sqrt{\cos(c+dx)}\,\sin(}{48ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A-13B)\sqrt{\cos(c+dx)}\,\sin(}{48ad(a+a\cos(c+dx))}$$

$$= \frac{(7A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} + \frac{(A}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.30, size = 217, normalized size = 1.08

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{3i(7A+5B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+\frac{1}{8}\sqrt{\cos(c+dx)}\,(59A+97B+20(7A+5B)\cos(c+dx)+(17A+67B)\cos(2(c+dx)))\sec^5\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d(a(1+\cos(c+dx)))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((3*I)*(7*A + 5*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2
*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + (Sqrt[Cos[c +
 d*x]]*(59*A + 97*B + 20*(7*A + 5*B)*Cos[c + d*x] + (17*A + 67*B)*Cos[2*(c
+ d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2])/8))/(24*d*(a*(1 + Cos[c + d*x
]))^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(170) = 340.
time = 0.35, size = 549, normalized size = 2.73

method	result
default	$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,(-1+\cos(dx+c))^5\left(34A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^4(dx+c))+140A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^3(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/384/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*(34*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4+140*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*cos(d*x+c)^3+21*A*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos
(d*x+c))/sin(d*x+c))+15*B*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos(d*
x+c))/sin(d*x+c))+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2+42*A*2
^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+134*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+30*B*2^(1/2)*cos(d*x+c)^2*sin(d
*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-140*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)*cos(d*x+c)+21*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))-34*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+15*B*2^(1/2
)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-42*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)-70*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-
30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/sin(d*x+c)^11/(cos(d*x+c
)/(1+cos(d*x+c)))^(5/2)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

Fricas [A]
time = 0.39, size = 266, normalized size = 1.32

$$\frac{3\sqrt{2}\left((7A+5B)\cos\left(dx+c\right)^4+4(7A+5B)\cos\left(dx+c\right)^3+6(7A+5B)\cos\left(dx+c\right)^2+4(7A+5B)\cos\left(dx+c\right)+7A+5B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos\left(dx+c\right)+a}\sqrt{a}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)+2\left((17A+67B)\cos\left(dx+c\right)^2+10(7A+5B)\cos\left(dx+c\right)+21A+15B\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{384\left(a^4d\cos\left(dx+c\right)^4+4a^4d\cos\left(dx+c\right)^3+6a^4d\cos\left(dx+c\right)^2+4a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((7*A + 5*B)*cos(d*x + c)^4 + 4*(7*A + 5*B)*cos(d*x + c)^3
 + 6*(7*A + 5*B)*cos(d*x + c)^2 + 4*(7*A + 5*B)*cos(d*x + c) + 7*A + 5*B)*s
qrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c
))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*((17*A + 67*B)*cos
(d*x + c)^2 + 10*(7*A + 5*B)*cos(d*x + c) + 21*A + 15*B)*sqrt(a*cos(d*x + c
) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos
(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2), x
)
```

3.211 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=201

$$\frac{(13A+7B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}+\frac{(A+$$

```
[Out] 1/128*(13*A+7*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+
a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)+1/6*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2
)/d/(a+a*cos(d*x+c))^(7/2)+1/16*(A+3*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+
a*cos(d*x+c))^(5/2)-1/192*(5*A-17*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a
*cos(d*x+c))^(3/2)
```

Rubi [A]
time = 0.37, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3056, 3057, 12, 2861, 211}

$$\frac{(13A+7B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((13*A + 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((A + 3*B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) - ((5*A - 17*B)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{1}{2}a(A-B)+2a(A+2B)\cos}{\sqrt{\cos(c+dx)}\,(a+a\cos(}}{6a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A+3B)\sqrt{\cos(c+dx)}}{16ad(a+a\cos(c+}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A+3B)\sqrt{\cos(c+dx)}}{16ad(a+a\cos(c+}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A+3B)\sqrt{\cos(c+dx)}}{16ad(a+a\cos(c+}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A+3B)\sqrt{\cos(c+dx)}}{16ad(a+a\cos(c+}$$

$$= \frac{(13A+7B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.14, size = 215, normalized size = 1.07

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{3i(13A+7B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+\frac{1}{8}\sqrt{\cos(c+dx)}\,(73A+59B+4(A+35B)\cos(c+dx)+(-5A+17B)\cos(2(c+dx)))\sec^5\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d(a(1+\cos(c+dx)))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((3*I)*(13*A + 7*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((
2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + (Sqrt[Cos[c
+ d*x]]*(73*A + 59*B + 4*(A + 35*B)*Cos[c + d*x] + (-5*A + 17*B)*Cos[2*(c +
 d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2])/8))/(24*d*(a*(1 + Cos[c + d*x]
))^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(170) = 340$.
time = 0.36, size = 549, normalized size = 2.73

method	result
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,(-1+\cos(dx+c))^{4}\left(10A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^{4}(dx+c))-4A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^{3}(d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/384/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*(10*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4-4*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)*cos(d*x+c)^3-39*A*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos(d
*x+c))/sin(d*x+c))-21*B*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos(d*x+
c))/sin(d*x+c))-88*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2-78*A*2^
(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-34*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4-42*B*2^(1/2)*cos(d*x+c)^2*sin(d*x
+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)*cos(d*x+c)-39*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(
d*x+c))-106*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-21*B*2^(1/2)*s
in(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+78*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)+98*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+42*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/sin(d*x+c)^9/(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(7/2
), x)
```

Fricas [A]
time = 0.41, size = 264, normalized size = 1.31

$$\frac{3\sqrt{2}\left((13A+7B)\cos(dx+c)^{4}+4(13A+7B)\cos(dx+c)^{3}+6(13A+7B)\cos(dx+c)^{2}+4(13A+7B)\cos(dx+c)+13A+7B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right)-2\left((5A-17B)\cos(dx+c)^{2}-2(A+35B)\cos(dx+c)-39A-21B\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{384\left(a^{4}d\cos(dx+c)^{4}+4a^{4}d\cos(dx+c)^{3}+6a^{4}d\cos(dx+c)^{2}+4a^{4}d\cos(dx+c)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((13*A + 7*B)*cos(d*x + c)^4 + 4*(13*A + 7*B)*cos(d*x + c)
^3 + 6*(13*A + 7*B)*cos(d*x + c)^2 + 4*(13*A + 7*B)*cos(d*x + c) + 13*A + 7
*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*
x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((5*A - 17*B)
*cos(d*x + c)^2 - 2*(A + 35*B)*cos(d*x + c) - 39*A - 21*B)*sqrt(a*cos(d*x +
 c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*c
os(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2), x
)
```

3.212 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=203

$$\frac{(63A+13B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}-\frac{(5A$$

```
[Out] 1/128*(63*A+13*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a
+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)-1/6*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/
2)/d/(a+a*cos(d*x+c))^(7/2)-1/16*(5*A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a
+a*cos(d*x+c))^(5/2)-1/192*(103*A+5*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a
+a*cos(d*x+c))^(3/2)
```

Rubi [A]
time = 0.37, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3057, 12, 2861, 211}

$$\frac{(63A+13B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] ((63*A + 13*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - ((A - B)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) - ((5*A - B)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) - ((103*A + 5*B)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
```

```
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{1}{2}a(11A+B)-2a(A-B}{\sqrt{\cos(c+dx)}\,(a+a}}{6a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(5A-B)\sqrt{\cos(c+}}{16ad(a+a\cos(}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(5A-B)\sqrt{\cos(c+}}{16ad(a+a\cos(}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(5A-B)\sqrt{\cos(c+}}{16ad(a+a\cos(}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(5A-B)\sqrt{\cos(c+}}{16ad(a+a\cos(}$$

$$= \frac{(63A+13B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right.}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.19, size = 216, normalized size = 1.06

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{3i(63A+13B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{1+e^{2i(c+dx)}}}-\frac{1}{8}\sqrt{\cos(c+dx)}\,(493A-73B+(532A-4B)\cos(c+dx)+(103A+5B)\cos(2(c+dx)))\sec^5\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d(a(1+\cos(c+dx)))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((3*I)*(63*A + 13*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^(
(2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*S
qrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - (Sqrt[Cos[c
 + d*x]]*(493*A - 73*B + (532*A - 4*B)*Cos[c + d*x] + (103*A + 5*B)*Cos[2*(
c + d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2])/8))/(24*d*(a*(1 + Cos[c + d
*x]))^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(172) = 344$.
time = 0.34, size = 549, normalized size = 2.70

method	result
default	$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos(dx+c)\right)^{3}\left(206A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^{4}(dx+c)\right)+532A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^{3}(dx+c)\right)-189A\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(206*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*cos(d*x+c)^4+532*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(
d*x+c)^3-189*A*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d
*x+c))-39*B*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+
c))+184*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2-378*A*2^(1/2)*cos(
d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+10*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)^4-78*B*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin
((-1+cos(d*x+c))/sin(d*x+c))-532*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*
x+c)-189*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
-14*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-39*B*2^(1/2)*sin(d*x+c
)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-390*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)-74*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+78*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
a^4/cos(d*x+c)^(1/2)/sin(d*x+c)^7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sqrt(cos(d*x + c
))), x)
```

Fricas [A]
time = 0.43, size = 266, normalized size = 1.31

$$\frac{3\sqrt{2}\left((63A+13B)\cos(dx+c)^4+4(63A+13B)\cos(dx+c)^3+6(63A+13B)\cos(dx+c)^2+4(63A+13B)\cos(dx+c)+63A+13B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)-2\left((103A+5B)\cos(dx+c)^2+2(133A-B)\cos(dx+c)+195A-39B\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((63*A + 13*B)*cos(d*x + c)^4 + 4*(63*A + 13*B)*cos(d*x +
c)^3 + 6*(63*A + 13*B)*cos(d*x + c)^2 + 4*(63*A + 13*B)*cos(d*x + c) + 63*A
 + 13*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(c
os(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((103*A
+ 5*B)*cos(d*x + c)^2 + 2*(133*A - B)*cos(d*x + c) + 195*A - 39*B)*sqrt(a*c
os(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4
*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4
*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(7/2)), x
)
```

3.213 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{3(121A-21B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}$$

```
[Out] -3/128*(121*A-21*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/
(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)-1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d
*x+c))^(7/2)/cos(d*x+c)^(1/2)-1/48*(19*A-7*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c
))^(5/2)/cos(d*x+c)^(1/2)-1/192*(199*A-43*B)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+
c))^(3/2)/cos(d*x+c)^(1/2)+1/192*(691*A-103*B)*sin(d*x+c)/a^3/d/cos(d*x+c)^
(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.51, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3057, 3063, 12, 2861, 211}

$$-\frac{3(121A-21B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{(691A-103B)\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}-\frac{(199A-43B)\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}-\frac{(19A-7B)\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^{5/2}}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] (-3*(121*A - 21*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - ((A - B)*Sin[c + d*x
])/(6*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)) - ((19*A - 7*B)*Sin[
c + d*x])/(48*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) - ((199*A
- 43*B)*Sin[c + d*x])/(192*a^2*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3
/2)) + ((691*A - 103*B)*Sin[c + d*x])/(192*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{1}{2}a(13A-B)-3a(A-B}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(}}{6a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7}{48ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7}{48ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7}{48ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7}{48ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7}{48ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{3(121A-21B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx}}\right.}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.77, size = 240, normalized size = 0.96

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(-\frac{9i(121A-21B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+\frac{(5284A-532B+9(941A-121B)\cos(c+dx)+4(937A-133B)\cos(2(c+dx))+691A\cos(3(c+dx))-103B\cos(3(c+dx)))\sec^5\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{16\sqrt{\cos(c+dx)}}\right)}{24d(a(1+\cos(c+dx)))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((-9*I)*(121*A - 21*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E
^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]
*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + ((5284*A
- 532*B + 9*(941*A - 121*B)*Cos[c + d*x] + 4*(937*A - 133*B)*Cos[2*(c + d*x
)] + 691*A*Cos[3*(c + d*x)] - 103*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^5*Ta
n[(c + d*x)/2])/(16*Sqrt[Cos[c + d*x]])))/(24*d*(a*(1 + Cos[c + d*x]))^(7/2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(213) = 426$.

time = 0.38, size = 581, normalized size = 2.32

method	result
default	$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos(dx+c)\right)^{2}\left(-1089A\left(\cos^{3}(dx+c)\right)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(-1089*A*cos(d*x+c)^3*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))+189*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-3267*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)+567*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))-3267*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+567*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*cos(d*x+c)+1382*A*cos(d*x+c)^4-1089*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-206*B*cos(d*x+c
)^4+189*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*2^(1/2)*arcsin((-1+c
os(d*x+c))/sin(d*x+c))+2366*A*cos(d*x+c)^3-326*B*cos(d*x+c)^3-550*A*cos(d*x
+c)^2+142*B*cos(d*x+c)^2-2430*A*cos(d*x+c)+390*B*cos(d*x+c)-768*A)/a^4/sin(
d*x+c)^5/(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]

time = 0.42, size = 298, normalized size = 1.19

$$-\frac{9\sqrt{2}\left((121A-21B)\cos(dx+c)^{5}+4(121A-21B)\cos(dx+c)^{4}+6(121A-21B)\cos(dx+c)^{3}+4(121A-21B)\cos(dx+c)^{2}+(121A-21B)\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right)-2\left((691A-103B)\cos(dx+c)^{3}+2(937A-133B)\cos(dx+c)^{2}+39(41A-5B)\cos(dx+c)+384A\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{384\left(a^{4}d\cos(dx+c)^{5}+4a^{4}d\cos(dx+c)^{4}+6a^{4}d\cos(dx+c)^{3}+4a^{4}d\cos(dx+c)^{2}+a^{4}d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(9*sqrt(2)*((121*A - 21*B)*cos(d*x + c)^5 + 4*(121*A - 21*B)*cos(d*x
 + c)^4 + 6*(121*A - 21*B)*cos(d*x + c)^3 + 4*(121*A - 21*B)*cos(d*x + c)^2
 + (121*A - 21*B)*cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x +
 c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(
d*x + c))) - 2*((691*A - 103*B)*cos(d*x + c)^3 + 2*(937*A - 133*B)*cos(d*x
+ c)^2 + 39*(41*A - 5*B)*cos(d*x + c) + 384*A)*sqrt(a*cos(d*x + c) + a)*sqr
t(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(d*x + c)^
4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(7/2)), x
)
```

3.214 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=297

$$\frac{(1015A-363B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}-$$

```
[Out] -1/6*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2)-1/48*(23*A-
11*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2)-1/64*(109*A-41
*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+1/128*(1015*A-
363*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))/a^(7/2)/d*2^(1/2)+1/192*(579*A-199*B)*sin(d*x+c)/a^3/d/cos(d*x+c
)^(3/2)/(a+a*cos(d*x+c))^(1/2)-1/192*(1887*A-691*B)*sin(d*x+c)/a^3/d/cos(d*
x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.64, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3057, 3063, 12, 2861, 211}

$$\frac{(1015A-363B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{(579A-199B)\sin(c+dx)}{192a^3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(1887A-691B)\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(109A-41B)\sin(c+dx)}{64a^2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-\frac{(23A-11B)\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] ((1015*A - 363*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - ((A - B)*Sin[c + d*x]
)/(6*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)) - ((23*A - 11*B)*Sin[
c + d*x])/(48*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) - ((109*A
- 41*B)*Sin[c + d*x])/(64*a^2*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/
2)) + ((579*A - 199*B)*Sin[c + d*x])/(192*a^3*d*Cos[c + d*x]^(3/2)*Sqrt[a +
 a*Cos[c + d*x]]) - ((1887*A - 691*B)*Sin[c + d*x])/(192*a^3*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{3}{2}a(5A-B)-4a(A-B)\cos(}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx}}{6a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\,\mathrm{s}}{48ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\,\mathrm{s}}{48ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\,\mathrm{s}}{48ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\,\mathrm{s}}{48ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\,\mathrm{s}}{48ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\,\mathrm{s}}{48ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= \frac{(1015A-363B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right.}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.69, size = 262, normalized size = 0.88

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{3i(1015A-363B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{1+e^{2i(c+dx)}}}-\frac{(21641A-8469B+4(9415A-3579B)\cos(c+dx)+8(3069A-1145B)\cos(2(c+dx))+10164A\cos(3(c+dx))-3748B\cos(3(c+dx))+1887A\cos(4(c+dx))-691B\cos(4(c+dx)))\sec^5\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{32\cos^{\frac{3}{2}}(c+dx)}\right)}{24d(a(1+\cos(c+dx)))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((3*I)*(1015*A - 363*B)*E^((I/2)*(c + d*x))*Sqrt[(1 +
E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - ((21641*
A - 8469*B + 4*(9415*A - 3579*B)*Cos[c + d*x] + 8*(3069*A - 1145*B)*Cos[2*(
c + d*x)] + 10164*A*Cos[3*(c + d*x)] - 3748*B*Cos[3*(c + d*x)] + 1887*A*Cos
```

```
[4*(c + d*x)] - 691*B*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2]
)/(32*Cos[c + d*x]^(3/2))))/(24*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(254) = 508$.
time = 0.29, size = 715, normalized size = 2.41

method	result
default	$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,(-1+\cos(dx+c))\left(-3045A\left(\cos^4(dx+c)\right)\sin(dx+c)\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(-3045*A*cos(d*x+c)^4*sin
(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))+1089*B*cos(d*x+c)^4*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-12180*A*sin(d*x+c)*2^(1/2)*cos(d
*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)+4356*B*sin(d*x+c)*2^(1/2)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*
arcsin((-1+cos(d*x+c))/sin(d*x+c))-18270*A*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+6534*B
*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)-12180*A*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+4356*B*sin(d*x+c
)*cos(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(
d*x+c)))^(3/2)-3045*A*sin(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1089*B*sin(d*x+c)*2^(1/2)*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+3774*A*cos(d*x+c)^5-
1382*B*cos(d*x+c)^5+6390*A*cos(d*x+c)^4-2366*B*cos(d*x+c)^4-1662*A*cos(d*x+
c)^3+550*B*cos(d*x+c)^3-6710*A*cos(d*x+c)^2+2430*B*cos(d*x+c)^2-2048*A*cos(
d*x+c)+768*B*cos(d*x+c)+256*A)/a^4/sin(d*x+c)^3/(1+cos(d*x+c))^2/cos(d*x+c)
^(3/2)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.41, size = 319, normalized size = 1.07

$$\frac{3\sqrt{2}\left((1015A-363B)\cos(dx+c)^{6}+4(1015A-363B)\cos(dx+c)^{5}+6(1015A-363B)\cos(dx+c)^{4}+4(1015A-363B)\cos(dx+c)^{3}+(1015A-363B)\cos(dx+c)^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right)-2\left((1887A-691B)\cos(dx+c)^{4}+2(2541A-937B)\cos(dx+c)^{3}+39(109A-41B)\cos(dx+c)^{2}+128(7A-3B)\cos(dx+c)-128A\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{384\left(a^{4}d\cos(dx+c)^{6}+4a^{4}d\cos(dx+c)^{5}+6a^{4}d\cos(dx+c)^{4}+4a^{4}d\cos(dx+c)^{3}+a^{4}d\cos(dx+c)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((1015*A - 363*B)*cos(d*x + c)^6 + 4*(1015*A - 363*B)*cos(
d*x + c)^5 + 6*(1015*A - 363*B)*cos(d*x + c)^4 + 4*(1015*A - 363*B)*cos(d*x
 + c)^3 + (1015*A - 363*B)*cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(
a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)
^2 + a*cos(d*x + c))) - 2*((1887*A - 691*B)*cos(d*x + c)^4 + 2*(2541*A - 93
7*B)*cos(d*x + c)^3 + 39*(109*A - 41*B)*cos(d*x + c)^2 + 128*(7*A - 3*B)*co
s(d*x + c) - 128*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c
))/(a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4
+ 4*a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(7/2)), x
)
```

3.215 $\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{1}{8}(4aA+3bB)x+\frac{(Ab+aB)\sin(c+dx)}{d}+\frac{(4aA+3bB)\cos(c+dx)\sin(c+dx)}{8d}+\frac{bB\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 1/8*(4*A*a+3*B*b)*x+(A*b+B*a)*sin(d*x+c)/d+1/8*(4*A*a+3*B*b)*cos(d*x+c)*sin
(d*x+c)/d+1/4*b*B*cos(d*x+c)^3*sin(d*x+c)/d-1/3*(A*b+B*a)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3047, 3102, 2827, 2715, 8, 2713}

$$-\frac{(aB+Ab)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{(4aA+3bB)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4aA+3bB)+\frac{bB\sin(c+dx)\cos^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((4*a*A + 3*b*B)*x)/8 + ((A*b + a*B)*Sin[c + d*x])/d + ((4*a*A + 3*b*B)*Cos
[c + d*x]*Sin[c + d*x])/(8*d) + (b*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (
(A*b + a*B)*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx &= \int \cos^2(c+dx)\left(aA+(Ab+aB)\cos(c+dx)+\right.\\ &= \frac{bB\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^2(c+dx)\\ &= \frac{bB\cos^3(c+dx)\sin(c+dx)}{4d}+(Ab+aB)\int \cos^3\\ &= \frac{(4aA+3bB)\cos(c+dx)\sin(c+dx)}{8d}+\frac{bB\cos^3(}{}\\ &= \frac{1}{8}(4aA+3bB)x+\frac{(Ab+aB)\sin(c+dx)}{d}+\frac{(4a}{}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 91, normalized size = 0.87

$$\frac{48aAc+36bBc+48aAdx+36bBdx+96(Ab+aB)\sin(c+dx)-32(Ab+aB)\sin^3(c+dx)+24(aA+bB)\sin(2(c+dx))+3bB\sin(4(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (48*a*A*c + 36*b*B*c + 48*a*A*d*x + 36*b*B*d*x + 96*(A*b + a*B)*Sin[c + d*x
] - 32*(A*b + a*B)*Sin[c + d*x]^3 + 24*(a*A + b*B)*Sin[2*(c + d*x)] + 3*b*B
*Sin[4*(c + d*x)])/(96*d)
```

Maple [A]
time = 0.14, size = 107, normalized size = 1.02

method	result
derivativedivides	$\frac{Bb\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ab\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+\frac{aB\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+aA\left(\mathrm{s}\right.}{d}$
default	$\frac{Bb\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ab\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+\frac{aB\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+aA\left(\mathrm{s}\right.}{d}$
risch	$\frac{axA}{2}+\frac{3bBx}{8}+\frac{3\sin(dx+c)Ab}{4d}+\frac{3aB\sin(dx+c)}{4d}+\frac{Bb\sin(4dx+4c)}{32d}+\frac{\sin(3dx+3c)Ab}{12d}+\frac{\sin(3dx+3c)aB}{12d}+\mathrm{s}$
norman	$\left(\frac{aA}{2}+\frac{3Bb}{8}\right)x+\left(2aA+\frac{3Bb}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2aA+\frac{3Bb}{2}\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3aA+\frac{9Bb}{4}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{aA}{2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(B*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A
*b*(cos(d*x+c)^2+2)*sin(d*x+c)+1/3*a*B*(cos(d*x+c)^2+2)*sin(d*x+c)+a*A*(1/2
*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.26, size = 101, normalized size = 0.96

$$\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa-32\,(\sin(dx+c)^3-3\,\sin(dx+c))Ba-32\,(\sin(dx+c)^3-3\,\sin(dx+c))Ab+3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Bb}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a - 32*(sin(d*x + c)^3 - 3*sin(
d*x + c))*B*a - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b + 3*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b)/d
```

Fricas [A]
time = 0.42, size = 81, normalized size = 0.77

$$\frac{3\,(4\,Aa+3\,Bb)dx+\left(6\,Bb\cos(dx+c)^3+8\,(Ba+Ab)\cos(dx+c)^2+16\,Ba+16\,Ab+3\,(4\,Aa+3\,Bb)\cos(dx+c)\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(4*A*a + 3*B*b)*d*x + (6*B*b*cos(d*x + c)^3 + 8*(B*a + A*b)*cos(d*x
 + c)^2 + 16*B*a + 16*A*b + 3*(4*A*a + 3*B*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(97) = 194$.
time = 0.20, size = 252, normalized size = 2.40

$$\begin{cases} \frac{Aax\sin^2(c+dx)}{2} + \frac{Aax\cos^2(c+dx)}{2} + \frac{Aa\sin(c+dx)\cos(c+dx)}{2d} + \frac{2Ab\sin^3(c+dx)}{3d} + \frac{Ab\sin(c+dx)\cos^2(c+dx)}{d} + \frac{2Ba\sin^3(c+dx)}{3d} + \frac{Ba\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Bbx\sin^4(c+dx)}{8} + \frac{3Bbx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Bbx\cos^4(c+dx)}{8} + \frac{3Bb\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5Bb\sin(c+dx)\cos^3(c+dx)}{8d} & \text{for } d \neq 0 \\ x(A + B\cos(c))(a + b\cos(c))\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
d*x)*cos(c + d*x)/(2*d) + 2*A*b*sin(c + d*x)**3/(3*d) + A*b*sin(c + d*x)*co
s(c + d*x)**2/d + 2*B*a*sin(c + d*x)**3/(3*d) + B*a*sin(c + d*x)*cos(c + d*
x)**2/d + 3*B*b*x*sin(c + d*x)**4/8 + 3*B*b*x*sin(c + d*x)**2*cos(c + d*x)*
*2/4 + 3*B*b*x*cos(c + d*x)**4/8 + 3*B*b*sin(c + d*x)**3*cos(c + d*x)/(8*d)
 + 5*B*b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c))*(
a + b*cos(c))*cos(c)**2, True))
```

Giac [A]
time = 0.43, size = 89, normalized size = 0.85

$$\frac{1}{8}(4Aa + 3Bb)x + \frac{Bb\sin(4dx + 4c)}{32d} + \frac{(Ba + Ab)\sin(3dx + 3c)}{12d} + \frac{(Aa + Bb)\sin(2dx + 2c)}{4d} + \frac{3(Ba + Ab)\sin(dx + c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/8*(4*A*a + 3*B*b)*x + 1/32*B*b*sin(4*d*x + 4*c)/d + 1/12*(B*a + A*b)*sin(
3*d*x + 3*c)/d + 1/4*(A*a + B*b)*sin(2*d*x + 2*c)/d + 3/4*(B*a + A*b)*sin(d
*x + c)/d
```

Mupad [B]
time = 0.47, size = 117, normalized size = 1.11

$$\frac{A\,a\,x}{2} + \frac{3\,B\,b\,x}{8} + \frac{3\,A\,b\,\sin(c + d\,x)}{4\,d} + \frac{3\,B\,a\,\sin(c + d\,x)}{4\,d} + \frac{A\,a\,\sin(2\,c + 2\,d\,x)}{4\,d} + \frac{A\,b\,\sin(3\,c + 3\,d\,x)}{12\,d} + \frac{B\,a\,\sin(3\,c + 3\,d\,x)}{12\,d} + \frac{B\,b\,\sin(2\,c + 2\,d\,x)}{4\,d} + \frac{B\,b\,\sin(4\,c + 4\,d\,x)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

```
[Out] (A*a*x)/2 + (3*B*b*x)/8 + (3*A*b*sin(c + d*x))/(4*d) + (3*B*a*sin(c + d*x))
/(4*d) + (A*a*sin(2*c + 2*d*x))/(4*d) + (A*b*sin(3*c + 3*d*x))/(12*d) + (B*
a*sin(3*c + 3*d*x))/(12*d) + (B*b*sin(2*c + 2*d*x))/(4*d) + (B*b*sin(4*c +
4*d*x))/(32*d)
```

3.216 $\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=84

$$\frac{1}{2}(Ab+aB)x+\frac{(3aA+2bB)\sin(c+dx)}{3d}+\frac{(Ab+aB)\cos(c+dx)\sin(c+dx)}{2d}+\frac{bB\cos^2(c+dx)\sin(c+dx)}{3d}$$

```
[Out] 1/2*(A*b+B*a)*x+1/3*(3*A*a+2*B*b)*sin(d*x+c)/d+1/2*(A*b+B*a)*cos(d*x+c)*sin
(d*x+c)/d+1/3*b*B*cos(d*x+c)^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3047, 3102, 2813}

$$\frac{(3aA+2bB)\sin(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(aB+Ab)+\frac{bB\sin(c+dx)\cos^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((A*b + a*B)*x)/2 + ((3*a*A + 2*b*B)*Sin[c + d*x])/(3*d) + ((A*b + a*B)*Cos
[c + d*x]*Sin[c + d*x])/(2*d) + (b*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos(c+dx)\left(aA+(Ab+aB)\cos(c+dx)+bB\right.$$
$$= \frac{bB\cos^2(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}\int \cos(c+dx)(3$$
$$= \frac{1}{2}(Ab+aB)x+\frac{(3aA+2bB)\sin(c+dx)}{3d}+\frac{(Ab}{}$$

Mathematica [A]
time = 0.19, size = 75, normalized size = 0.89

$$\frac{6Abc+6aBc+6Abdx+6aBdx+3(4aA+3bB)\sin(c+dx)+3(Ab+aB)\sin(2(c+dx))+bB\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
[Out] (6*A*b*c + 6*a*B*c + 6*A*b*d*x + 6*a*B*d*x + 3*(4*a*A + 3*b*B)*Sin[c + d*x]
 + 3*(A*b + a*B)*Sin[2*(c + d*x)] + b*B*Sin[3*(c + d*x)])/(12*d)
```

Maple [A]
time = 0.11, size = 85, normalized size = 1.01

method	result
derivativedivides	$\frac{\frac{Bb\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+Ab\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aB\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aA\sin(dx+c)}{d}$
default	$\frac{\frac{Bb\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+Ab\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aB\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aA\sin(dx+c)}{d}$
risch	$\frac{xAb}{2}+\frac{aBx}{2}+\frac{\sin(dx+c)aA}{d}+\frac{3bB\sin(dx+c)}{4d}+\frac{Bb\sin(3dx+3c)}{12d}+\frac{\sin(2dx+2c)Ab}{4d}+\frac{aB\sin(2dx+2c)}{4d}$
norman	$\frac{\left(\frac{Ab}{2}+\frac{aB}{2}\right)x+\left(\frac{Ab}{2}+\frac{aB}{2}\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3Ab}{2}+\frac{3aB}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3Ab}{2}+\frac{3aB}{2}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{(2aA-Ab-}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*B*b*(cos(d*x+c)^2+2)*sin(d*x+c)+A*b*(1/2*sin(d*x+c)*cos(d*x+c)+1/2
*d*x+1/2*c)+a*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+a*A*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 79, normalized size = 0.94

$$\frac{3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba+3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ab-4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Bb+12\,Aa\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*A*b - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*b + 12*A*a*sin(d*x + c))
/d
```

Fricas [A]
time = 0.35, size = 60, normalized size = 0.71

$$\frac{3\left(Ba+Ab\right)dx+\left(2\,Bb\cos\left(dx+c\right)^{2}+6\,Aa+4\,Bb+3\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/6*(3*(B*a + A*b)*d*x + (2*B*b*cos(d*x + c)^2 + 6*A*a + 4*B*b + 3*(B*a + A
*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(76) = 152$.
time = 0.13, size = 168, normalized size = 2.00

$$\begin{cases} \frac{Aa\sin\left(c+dx\right)}{d}+\frac{Abx\sin^{2}\left(c+dx\right)}{2}+\frac{Abx\cos^{2}\left(c+dx\right)}{2}+\frac{Ab\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{Bax\sin^{2}\left(c+dx\right)}{2}+\frac{Bax\cos^{2}\left(c+dx\right)}{2}+\frac{Ba\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{2Bb\sin^{3}\left(c+dx\right)}{3d}+\frac{Bb\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d} & \text{for}\: d\neq 0 \\ x\left(A+B\cos\left(c\right)\right)\left(a+b\cos\left(c\right)\right)\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*sin(c + d*x)/d + A*b*x*sin(c + d*x)**2/2 + A*b*x*cos(c + d*x
)**2/2 + A*b*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*x*sin(c + d*x)**2/2 + B*
a*x*cos(c + d*x)**2/2 + B*a*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*b*sin(c +
 d*x)**3/(3*d) + B*b*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*c
os(c))*(a + b*cos(c))*cos(c), True))
```

Giac [A]
time = 0.41, size = 68, normalized size = 0.81

$$\frac{1}{2}\left(Ba+Ab\right)x+\frac{Bb\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{\left(Ba+Ab\right)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{\left(4\,Aa+3\,Bb\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(B*a + A*b)*x + 1/12*B*b*sin(3*d*x + 3*c)/d + 1/4*(B*a + A*b)*sin(2*d*x
 + 2*c)/d + 1/4*(4*A*a + 3*B*b)*sin(d*x + c)/d
```

Mupad [B]
time = 0.40, size = 84, normalized size = 1.00

$$\frac{A\,b\,x}{2}+\frac{B\,a\,x}{2}+\frac{A\,a\,\sin\left(c+d\,x\right)}{d}+\frac{3\,B\,b\,\sin\left(c+d\,x\right)}{4\,d}+\frac{A\,b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,b\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

```
[Out] (A*b*x)/2 + (B*a*x)/2 + (A*a*sin(c + d*x))/d + (3*B*b*sin(c + d*x))/(4*d) +
 (A*b*sin(2*c + 2*d*x))/(4*d) + (B*a*sin(2*c + 2*d*x))/(4*d) + (B*b*sin(3*c
 + 3*d*x))/(12*d)
```

3.217 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{1}{2}(2aA+bB)x+\frac{(Ab+aB)\sin(c+dx)}{d}+\frac{bB\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*(2*A*a+B*b)*x+(A*b+B*a)*sin(d*x+c)/d+1/2*b*B*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2813\}$

$$\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{1}{2}x(2aA+bB)+\frac{bB\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((2*a*A + b*B)*x)/2 + ((A*b + a*B)*Sin[c + d*x])/d + (b*B*Cos[c + d*x]*Sin[
c + d*x])/(2*d)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx=\frac{1}{2}(2aA+bB)x+\frac{(Ab+aB)\sin(c+dx)}{d}+\frac{bB\cos(c+dx)\,s}{2d}$$

Mathematica [A]
time = 0.11, size = 51, normalized size = 0.98

$$\frac{2bBc+4aAdx+2bBdx+4(Ab+aB)\sin(c+dx)+bB\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

[Out] (2*b*B*c + 4*a*A*d*x + 2*b*B*d*x + 4*(A*b + a*B)*Sin[c + d*x] + b*B*Sin[2*(c + d*x)])/(4*d)

Maple [A]
time = 0.08, size = 57, normalized size = 1.10

method	result
risch	$axA+\frac{bBx}{2}+\frac{\sin(dx+c)Ab}{d}+\frac{aB\sin(dx+c)}{d}+\frac{\sin(2dx+2c)Bb}{4d}$
derivativedivides	$\frac{Bb\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ab\sin(dx+c)+aB\sin(dx+c)+aA(dx+c)}{d}$
default	$\frac{Bb\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ab\sin(dx+c)+aB\sin(dx+c)+aA(dx+c)}{d}$
norman	$\frac{\left(aA+\frac{Bb}{2}\right)x+\left(aA+\frac{Bb}{2}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2aA+Bb)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{(2Ab+2aB-Bb)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2Ab+2aB+Bb}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(B*b*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+A*b*sin(d*x+c)+a*B*sin(d*x+c)+a*A*(d*x+c))

Maxima [A]
time = 0.27, size = 55, normalized size = 1.06

$$\frac{4\,(dx+c)Aa+(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Bb+4\,Ba\sin{(dx+c)}+4\,Ab\sin{(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/4*(4*(d*x + c)*A*a + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*b + 4*B*a*sin(d*x + c) + 4*A*b*sin(d*x + c))/d

Fricas [A]
time = 0.38, size = 42, normalized size = 0.81

$$\frac{(2\,Aa+Bb)dx+(Bb\cos{(dx+c)}+2\,Ba+2\,Ab)\sin{(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] 1/2*((2*A*a + B*b)*d*x + (B*b*cos(d*x + c) + 2*B*a + 2*A*b)*sin(d*x + c))/d

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(44) = 88$.
time = 0.08, size = 94, normalized size = 1.81

$$\begin{cases} Aax + \frac{Ab\sin(c+dx)}{d} + \frac{Ba\sin(c+dx)}{d} + \frac{Bbx\sin^2(c+dx)}{2} + \frac{Bbx\cos^2(c+dx)}{2} + \frac{Bb\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d \neq 0 \\ x(A + B\cos(c))(a + b\cos(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x + A*b*sin(c + d*x)/d + B*a*sin(c + d*x)/d + B*b*x*sin(c +
d*x)**2/2 + B*b*x*cos(c + d*x)**2/2 + B*b*sin(c + d*x)*cos(c + d*x)/(2*d),
Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c)), True))
```

Giac [A]
time = 0.44, size = 45, normalized size = 0.87

$$\frac{1}{2}(2Aa + Bb)x + \frac{Bb\sin(2dx + 2c)}{4d} + \frac{(Ba + Ab)\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a + B*b)*x + 1/4*B*b*sin(2*d*x + 2*c)/d + (B*a + A*b)*sin(d*x + c)
/d
```

Mupad [B]
time = 0.36, size = 50, normalized size = 0.96

$$A\,a\,x + \frac{B\,b\,x}{2} + \frac{A\,b\,\sin(c + d\,x)}{d} + \frac{B\,a\,\sin(c + d\,x)}{d} + \frac{B\,b\,\sin(2\,c + 2\,d\,x)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

```
[Out] A*a*x + (B*b*x)/2 + (A*b*sin(c + d*x))/d + (B*a*sin(c + d*x))/d + (B*b*sin(
2*c + 2*d*x))/(4*d)
```

3.218 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=35

$$(Ab+aB)x+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\sin(c+dx)}{d}$$

```
[Out] (A*b+B*a)*x+a*A*arctanh(sin(d*x+c))/d+b*B*sin(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3047, 3102, 2814, 3855}

$$x(aB+Ab)+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (A*b + a*B)*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*Sin[c + d*x])/d
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx = \int \left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+\right.$$
$$= \frac{bB\sin(c+dx)}{d} + \int (aA+(Ab+aB)\cos(c+d$$
$$= (Ab+aB)x + \frac{bB\sin(c+dx)}{d} + (aA)\int \sec(c+$$
$$= (Ab+aB)x + \frac{aA\tanh^{-1}(\sin(c+dx))}{d} + \frac{bB\sin}{}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 1.31

$$Abx+aBx+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\cos(dx)\sin(c)}{d}+\frac{bB\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
[Out] A*b*x + a*B*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*Cos[d*x]*Sin[c])/d + (
b*B*Cos[c]*Sin[d*x])/d
```

Maple [A]
time = 0.14, size = 48, normalized size = 1.37

method	result
derivativedivides	$\frac{aA\ln(\sec(dx+c)+\tan(dx+c))+aB(dx+c)+Ab(dx+c)+Bb\sin(dx+c)}{d}$
default	$\frac{aA\ln(\sec(dx+c)+\tan(dx+c))+aB(dx+c)+Ab(dx+c)+Bb\sin(dx+c)}{d}$
risch	$xAb+aBx-\frac{iBb\,e^{i(dx+c)}}{2d}+\frac{iBb\,e^{-i(dx+c)}}{2d}+\frac{aA\ln(e^{i(dx+c)}+i)}{d}-\frac{aA\ln(e^{i(dx+c)}-i)}{d}$
norman	$\frac{(Ab+aB)x+(Ab+aB)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2Ab+2aB)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2Bb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2Bb\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{aA}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*A*ln(sec(d*x+c)+tan(d*x+c))+a*B*(d*x+c)+A*b*(d*x+c)+B*b*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 47, normalized size = 1.34

$$\frac{(dx+c)Ba+(dx+c)Ab+Aa\log(\sec(dx+c)+\tan(dx+c))+Bb\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima
")
[Out] ((d*x + c)*B*a + (d*x + c)*A*b + A*a*log(sec(d*x + c) + tan(d*x + c)) + B*b
*sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 54, normalized size = 1.54

$$\frac{2(Ba+Ab)dx+Aa\log(\sin(dx+c)+1)-Aa\log(-\sin(dx+c)+1)+2Bb\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas
")
[Out] 1/2*(2*(B*a + A*b)*d*x + A*a*log(sin(d*x + c) + 1) - A*a*log(-sin(d*x + c)
+ 1) + 2*B*b*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(A+B\cos(c+dx))(a+b\cos(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x)
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(35) = 70.
time = 0.53, size = 79, normalized size = 2.26

$$\frac{Aa\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-Aa\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+(Ba+Ab)(dx+c)+\frac{2Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

```
[Out] (A*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - A*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (B*a + A*b)*(d*x + c) + 2*B*b*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

Mupad [B]
time = 0.48, size = 100, normalized size = 2.86

$$\frac{B\,b\,\sin\left(c+d\,x\right)}{d}+\frac{2\,A\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x),x)
```

```
[Out] (B*b*sin(c + d*x))/d + (2*A*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d + (2*A*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.219 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=35

$$bBx+\frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}$$

```
[Out] b*B*x+(A*b+B*a)*arctanh(sin(d*x+c))/d+a*A*tan(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3047, 3100, 2814, 3855}

$$\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+bBx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] b*B*x + ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (a*A*Tan[c + d*x])/d
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int (aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+$$
$$= \frac{aA\tan(c+dx)}{d}+\int (Ab+aB+bB\cos(c+dx$$
$$= bBx+\frac{aA\tan(c+dx)}{d}-(-Ab-aB)\int \sec(c$$
$$= bBx+\frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\,ta$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 1.23

$$bBx+\frac{Ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] b*B*x + (A*b*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*
A*Tan[c + d*x])/d
```

Maple [A]
time = 0.16, size = 57, normalized size = 1.63

method	result
derivativedivides	$\frac{aA\tan(dx+c)+aB\ln(\sec(dx+c)+\tan(dx+c))+Ab\ln(\sec(dx+c)+\tan(dx+c))+Bb(dx+c)}{d}$
default	$\frac{aA\tan(dx+c)+aB\ln(\sec(dx+c)+\tan(dx+c))+Ab\ln(\sec(dx+c)+\tan(dx+c))+Bb(dx+c)}{d}$
risch	$bBx+\frac{2iaA}{d(e^{2i(dx+c)}+1)}+\frac{\ln(e^{i(dx+c)}+i)Ab}{d}+\frac{a\ln(e^{i(dx+c)}+i)B}{d}-\frac{\ln(e^{i(dx+c)}-i)Ab}{d}-\frac{a\ln(e^{i(dx+c)}-i)B}{d}$
norman	$\frac{bBx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+bBx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-bBx-\frac{2aA\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4aA\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2aA\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-bBx\left(ta\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*A*tan(d*x+c)+a*B*ln(sec(d*x+c)+tan(d*x+c))+A*b*ln(sec(d*x+c)+tan(d*x
+c))+B*b*(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.
time = 0.29, size = 73, normalized size = 2.09

$$\frac{2\,(dx+c)Bb + Ba(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\left(dx+c\right)-1\right)) + Ab(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\left(dx+c\right)-1\right)) + 2\,Aa\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(d*x + c)*B*b + B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ A*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*A*a*tan(d*x + c))
/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(35) = 70$.
time = 0.38, size = 85, normalized size = 2.43

$$\frac{2\,Bbdx\cos\left(dx+c\right) + (Ba+Ab)\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right) - (Ba+Ab)\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right) + 2\,Aa\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fric
as")
```

```
[Out] 1/2*(2*B*b*d*x*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)*log(sin(d*x + c) + 1
) - (B*a + A*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*A*a*sin(d*x + c))/(
d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + B\cos\left(c + dx\right)\right)\left(a + b\cos\left(c + dx\right)\right)\sec^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(35) = 70$.
time = 0.44, size = 84, normalized size = 2.40

$$\frac{(dx+c)Bb + (Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - (Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac
")
```

```
[Out] ((d*x + c)*B*b + (B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (B*a + A*
b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B]
time = 0.48, size = 114, normalized size = 3.26

$$\frac{2\,B\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}-\frac{A\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2\mathrm{i}}{d}-\frac{B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] (2*B*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (B*a*atan((sin(c/2
+ (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/d - (A*b*atan((sin(c/2 + (d*x)/2)*1i
)/cos(c/2 + (d*x)/2))*2i)/d + (A*a*sin(c + d*x))/(d*cos(c + d*x))
```

3.220 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=61

$$\frac{(aA+2bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(Ab+aB)\tan(c+dx)}{d}+\frac{aA\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*(A*a+2*B*b)*arctanh(sin(d*x+c))/d+(A*b+B*a)*tan(d*x+c)/d+1/2*a*A*sec(d*
x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3047, 3100, 2827, 3852, 8, 3855}

$$\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(aA+2bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] ((a*A + 2*b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((A*b + a*B)*Tan[c + d*x])/d
+ (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int (aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+$$
$$= \frac{aA\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int (2(Ab+aB)$$
$$= \frac{aA\sec(c+dx)\tan(c+dx)}{2d} + (Ab+aB)\int \mathrm{sec}$$
$$= \frac{(aA+2bB)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{aA\sec(c+}{}$$
$$= \frac{(aA+2bB)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(Ab+aB)}{}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 1.23

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}+\frac{Ab\tan(c+dx)}{d}+\frac{aB\tan(c+dx)}{d}+\frac{aA\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/(2*d) + (b*B*ArcTanh[Sin[c + d*x]])/d + (A*b*Ta
n[c + d*x])/d + (a*B*Tan[c + d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Maple [A]
time = 0.18, size = 75, normalized size = 1.23

method	result
derivativedivides	$\frac{aA\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+aB\tan(dx+c)+Ab\tan(dx+c)+Bb\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{aA\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+aB\tan(dx+c)+Ab\tan(dx+c)+Bb\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$-\frac{i\left(Aa\,e^{3i(dx+c)}-2Ab\,e^{2i(dx+c)}-2Ba\,e^{2i(dx+c)}-aA\,e^{i(dx+c)}-2Ab-2aB\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{aA\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{\ln\left(e^{i(dx+c)}+i\right)Bb}{d}$
norman	$\frac{\frac{(aA-2Ab-2aB)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(aA+2Ab+2aB)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{(3aA-2Ab-2aB)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(3aA+2Ab+2aB)\left(\tan^3\left(\frac{dx}{2}+\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a*B*tan(
d*x+c)+A*b*tan(d*x+c)+B*b*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.28, size = 95, normalized size = 1.56

$$-\frac{Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-2\,Bb(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-4\,Ba\tan(dx+c)-4\,Ab\tan(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/4*(A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 2*B*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 4*B*a*tan(d*x + c) - 4*A*b*tan(d*x + c))/d
```

Fricas [A]
time = 0.39, size = 96, normalized size = 1.57

$$\frac{(Aa+2\,Bb)\cos(dx+c)^2\log(\sin(dx+c)+1)-(Aa+2\,Bb)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,(Aa+2\,(Ba+Ab)\cos(dx+c))\sin(dx+c)}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fric
as")
```

```
[Out] 1/4*((A*a + 2*B*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a + 2*B*b)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a + 2*(B*a + A*b)*cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))(a + b\cos(c + dx))\sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(57) = 114.
time = 0.46, size = 151, normalized size = 2.48

$$\frac{(Aa + 2\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (Aa + 2\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(Aa\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 2\,Ba\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 2\,Ab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + Aa\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 2\,Ba\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 2\,Ab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac
")
```

```
[Out] 1/2*((A*a + 2*B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + 2*B*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1
/2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c)
 + 2*B*a*tan(1/2*d*x + 1/2*c) + 2*A*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 1.27, size = 104, normalized size = 1.70

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\,(A\,a + 2\,A\,b + 2\,B\,a) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (2\,A\,b - A\,a + 2\,B\,a)}{d\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)} + \frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)\,(A\,a + 2\,B\,b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(A*a + 2*A*b + 2*B*a) - tan(c/2 + (d*x)/2)^3*(2*A*b - A
*a + 2*B*a))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (ata
nh(tan(c/2 + (d*x)/2))*(A*a + 2*B*b))/d
```

3.221 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=93

$$\frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2aA+3bB)\tan(c+dx)}{3d}+\frac{(Ab+aB)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aA\sec^2(c+$$

```
[Out] 1/2*(A*b+B*a)*arctanh(sin(d*x+c))/d+1/3*(2*A*a+3*B*b)*tan(d*x+c)/d+1/2*(A*b
+B*a)*sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3047, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{(2aA+3bB)\tan(c+dx)}{3d}+\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aB+Ab)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a*A + 3*b*B)*Tan[c + d*x])/
(3*d) + ((A*b + a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c + d*x]^2
*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= \int \left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+\right.\\ &= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (3(Ab+aB\\ &= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}+(Ab+aB)\int \mathrm{se}\\ &= \frac{(Ab+aB)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aA\sec^2(}{}\\ &= \frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2aA+3bB}{3}\end{aligned}$$

Mathematica [A]
time = 0.32, size = 67, normalized size = 0.72

$$\frac{3(Ab+aB)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(6aA+6bB+3(Ab+aB)\sec(c+dx)+2aA\tan^2(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (3*(A*b + a*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*a*A + 6*b*B + 3*(A*b
 + a*B)*Sec[c + d*x] + 2*a*A*Tan[c + d*x]^2))/(6*d)
```

Maple [A]
time = 0.22, size = 105, normalized size = 1.13

method	result
derivativedivides	$\frac{-aA\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+aB\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(s}{}\right.}{d}$
default	$\frac{-aA\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+aB\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(s}{}\right.}{d}$
risch	$-\frac{i\left(3Ab\,e^{5i(dx+c)}+3Ba\,e^{5i(dx+c)}-6Bb\,e^{4i(dx+c)}-12Aa\,e^{2i(dx+c)}-12Bb\,e^{2i(dx+c)}-3Ab\,e^{i(dx+c)}-3Ba\,e^{i(dx+c)}-4aA-6\right.}{3d\left(e^{2i(dx+c)}+1\right)^{3}}$
norman	$\frac{-\frac{4(aA-3Bb)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2(4aA-3Ab-3aB)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2(4aA+3Ab+3aB)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{(2aA-Ab-aB+2Bb)\left(\tan\right.}{d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a*B*(1/2*sec(d*x+c)*tan(d*x+c)
+1/2*ln(sec(d*x+c)+tan(d*x+c)))+A*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d
*x+c)+tan(d*x+c)))+B*b*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 127, normalized size = 1.37

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Aa-3\,Ba\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,Bb\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a - 3*B*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*A*b*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*B*b*tan(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 115, normalized size = 1.24

$$\frac{3\left(Ba+Ab\right)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\left(Ba+Ab\right)\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\left(2\,Aa+3\,Bb\right)\cos\left(dx+c\right)^{2}+2\,Aa+3\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(B*a + A*b)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a + A*b)*co
s(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(2*A*a + 3*B*b)*cos(d*x + c)^2 +
 2*A*a + 3*(B*a + A*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))(a + b\cos(c + dx))\sec^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(85) = 170.
time = 0.50, size = 210, normalized size = 2.26

$$\frac{3(Ba + Ab)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(Ba + Ab)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(6Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 3Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 3Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 6Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 4Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 12Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 6Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 3Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 3Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 6Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac
")
```

```
[Out] 1/6*(3*(B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a + A*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 - 3*B*a*tan
(1/2*d*x + 1/2*c)^5 - 3*A*b*tan(1/2*d*x + 1/2*c)^5 + 6*B*b*tan(1/2*d*x + 1/
2*c)^5 - 4*A*a*tan(1/2*d*x + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3 + 6*A
*a*tan(1/2*d*x + 1/2*c) + 3*B*a*tan(1/2*d*x + 1/2*c) + 3*A*b*tan(1/2*d*x +
1/2*c) + 6*B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 2.57, size = 145, normalized size = 1.56

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)(Ab + Ba)}{d} - \frac{(2Aa - Ab - Ba + 2Bb)\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(-\frac{4Aa}{3} - 4Bb\right)\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (2Aa + Ab + Ba + 2Bb)\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(A*b + B*a))/d - (tan(c/2 + (d*x)/2)*(2*A*a + A*
b + B*a + 2*B*b) - tan(c/2 + (d*x)/2)^3*((4*A*a)/3 + 4*B*b) + tan(c/2 + (d*
x)/2)^5*(2*A*a - A*b - B*a + 2*B*b))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2
 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.222 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=114

$$\frac{(3aA+4bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(Ab+aB)\tan(c+dx)}{d}+\frac{(3aA+4bB)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aA\sec^3(}{}$$

```
[Out] 1/8*(3*A*a+4*B*b)*arctanh(sin(d*x+c))/d+(A*b+B*a)*tan(d*x+c)/d+1/8*(3*A*a+4
*B*b)*sec(d*x+c)*tan(d*x+c)/d+1/4*a*A*sec(d*x+c)^3*tan(d*x+c)/d+1/3*(A*b+B*
a)*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3047, 3100, 2827, 3852, 3853, 3855}

$$\frac{(aB+Ab)\tan^3(c+dx)}{3d}+\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(3aA+4bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3aA+4bB)\tan(c+dx)\sec(c+dx)}{8d}+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] ((3*a*A + 4*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((A*b + a*B)*Tan[c + d*x])/
d + ((3*a*A + 4*b*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*A*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d) + ((A*b + a*B)*Tan[c + d*x]^3)/(3*d)
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
```

```
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx &= \int \left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+\right.\\ &= \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (4(Ab+aB\\ &= \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+(Ab+aB)\int \mathrm{se}\\ &= \frac{(3aA+4bB)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aA\sec}{}\\ &= \frac{(3aA+4bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(Ab+aB}{}\end{aligned}$$

Mathematica [A]
time = 0.69, size = 85, normalized size = 0.75

$$\frac{3(3aA+4bB)\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\left(9aA+12bB+8(Ab+aB)(2+\cos(2(c+dx)))\sec(c+dx)+6aA\sec^2(c+dx)\right)\tan(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*a*A + 4*b*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(9*a*A + 12*b*B + 8
*(A*b + a*B)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*a*A*Sec[c + d*x]^2)*Ta
n[c + d*x])/(24*d)
```

Maple [A]
time = 0.25, size = 131, normalized size = 1.15

method	result
derivativedivides	$\frac{aA\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-aB\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-Ab}{d}$
default	$\frac{aA\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-aB\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-Ab}{d}$
risch	$-\frac{i(9Aa\,e^{7i(dx+c)}+12Bb\,e^{7i(dx+c)}+33Aa\,e^{5i(dx+c)}+12Bb\,e^{5i(dx+c)}-48Ab\,e^{4i(dx+c)}-48Ba\,e^{4i(dx+c)}-33Aa\,e^{3i(dx+c)}-}{12d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{\frac{(5aA-8Ab-8aB+4Bb)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{(5aA+8Ab+8aB+4Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{(21aA-8Ab-8aB-12Bb)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{(21aA+}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*A*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))-a*B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-A*b*(-2/3-1/3*sec(d*x+c
)^2)*tan(d*x+c)+B*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)
)))
```

Maxima [A]
time = 0.27, size = 163, normalized size = 1.43

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ab-3\,Aa\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin(dx+c)+1\right)+3\log\left(\sin(dx+c)-1\right)\right)-12\,Bb\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)+1\right)+\log\left(\sin(dx+c)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*A*b - 3*A*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*B*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.36, size = 136, normalized size = 1.19

$$\frac{3\,(3\,Aa+4\,Bb)\cos(dx+c)^4\log\left(\sin(dx+c)+1\right)-3\,(3\,Aa+4\,Bb)\cos(dx+c)^4\log\left(-\sin(dx+c)+1\right)+2\left(16\,(Ba+Ab)\cos(dx+c)^3+3\,(3\,Aa+4\,Bb)\cos(dx+c)^2+6\,Aa+8\,(Ba+Ab)\cos(dx+c)\right)\sin(dx+c)}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(3*A*a + 4*B*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A*a + 4
*B*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(B*a + A*b)*cos(d*x + c
)^3 + 3*(3*A*a + 4*B*b)*cos(d*x + c)^2 + 6*A*a + 8*(B*a + A*b)*cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A+B\cos\left(c+dx\right)\right)\left(a+b\cos\left(c+dx\right)\right)\sec^{5}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(106) = 212.
time = 0.45, size = 304, normalized size = 2.67

$$\frac{3\left(3Aa+4Bb\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(3Aa+4Bb\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(15Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-24Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-24Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+12Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+9Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+40Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+40Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-12Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+9Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-40Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-40Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-12Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+15Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+24Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+24Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{4}}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac
")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a + 4*B
*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a*tan(1/2*d*x + 1/2*c)^7 -
 24*B*a*tan(1/2*d*x + 1/2*c)^7 - 24*A*b*tan(1/2*d*x + 1/2*c)^7 + 12*B*b*tan
(1/2*d*x + 1/2*c)^7 + 9*A*a*tan(1/2*d*x + 1/2*c)^5 + 40*B*a*tan(1/2*d*x + 1
/2*c)^5 + 40*A*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*b*tan(1/2*d*x + 1/2*c)^5 + 9
*A*a*tan(1/2*d*x + 1/2*c)^3 - 40*B*a*tan(1/2*d*x + 1/2*c)^3 - 40*A*b*tan(1/
2*d*x + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3 + 15*A*a*tan(1/2*d*x + 1/2
*c) + 24*B*a*tan(1/2*d*x + 1/2*c) + 24*A*b*tan(1/2*d*x + 1/2*c) + 12*B*b*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 3.86, size = 194, normalized size = 1.70

$$\frac{\left(\frac{5Aa}{4}-2Ab-2Ba+Bb\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+\left(\frac{3Aa}{4}+\frac{10Ab}{3}+\frac{10Ba}{3}-Bb\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+\left(\frac{3Aa}{4}-\frac{10Ab}{3}-\frac{10Ba}{3}-Bb\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+\left(\frac{5Aa}{4}+2Ab+2Ba+Bb\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{3Aa}{4}+Bb\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((5*A*a)/4 + 2*A*b + 2*B*a + B*b) + tan(c/2 + (d*x)/2)^
7*((5*A*a)/4 - 2*A*b - 2*B*a + B*b) - tan(c/2 + (d*x)/2)^3*((10*A*b)/3 - (3
*A*a)/4 + (10*B*a)/3 + B*b) + tan(c/2 + (d*x)/2)^5*((3*A*a)/4 + (10*A*b)/3
+ (10*B*a)/3 - B*b))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 -
4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (atanh(tan(c/2 + (d*x
)/2))*((3*A*a)/4 + B*b))/d
```

3.223 $\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=189

$$\frac{1}{8}\left(4a^2A+3Ab^2+6abB\right)x+\frac{\left(4b^2B+5a(2Ab+aB)\right)\sin(c+dx)}{5d}+\frac{\left(4a^2A+3Ab^2+6abB\right)\cos(c+dx)\sin(c$$

```
[Out] 1/8*(4*A*a^2+3*A*b^2+6*B*a*b)*x+1/5*(4*b^2*B+5*a*(2*A*b+B*a))*sin(d*x+c)/d+
1/8*(4*A*a^2+3*A*b^2+6*B*a*b)*cos(d*x+c)*sin(d*x+c)/d+1/20*b*(5*A*b+6*B*a)*
cos(d*x+c)^3*sin(d*x+c)/d+1/5*b*B*cos(d*x+c)^3*(a+b*cos(d*x+c))*sin(d*x+c)/
d-1/15*(4*b^2*B+5*a*(2*A*b+B*a))*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.20, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3069, 3102, 2827, 2715, 8, 2713}

$$\frac{\left(4a^2A+6abB+3Ab^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2A+6abB+3Ab^2\right)-\frac{\left(5a(aB+2Ab)+4b^2B\right)\sin^3(c+dx)}{15d}+\frac{\left(5a(aB+2Ab)+4b^2B\right)\sin(c+dx)}{5d}+\frac{b(6aB+5Ab)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*x)/8 + ((4*b^2*B + 5*a*(2*A*b + a*B))*Sin[c
+ d*x])/(5*d) + ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + (b*(5*A*b + 6*a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(20*d) + (b*B*Cos[c
+ d*x]^3*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d) - ((4*b^2*B + 5*a*(2*A*b
+ a*B))*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{bB\cos^3(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{5d}$$

$$= \frac{b(5Ab+6aB)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{bB\cos}{}$$

$$= \frac{b(5Ab+6aB)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{bB\cos}{}$$

$$= \frac{(4a^2A+3Ab^2+6abB)\cos(c+dx)\sin(c+dx)}{8d}$$

$$= \frac{1}{8}\left(4a^2A+3Ab^2+6abB\right)x + \frac{(4b^2B+5a(2Ab-}{5}$$

Mathematica [**A**]

time = 0.52, size = 146, normalized size = 0.77

$$\frac{60(4a^2A+3Ab^2+6abB)(c+dx)+60(12aAb+6a^2B+5b^2B)\sin(c+dx)+120(a^2A+Ab^2+2abB)\sin(2(c+dx))+10(8aAb+4a^2B+5b^2B)\sin(3(c+dx))+15b(Ab+2aB)\sin(4(c+dx))+6b^2B\sin(5(c+dx))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (60*(4*a^2*A + 3*A*b^2 + 6*a*b*B)*(c + d*x) + 60*(12*a*A*b + 6*a^2*B + 5*b^
2*B)*Sin[c + d*x] + 120*(a^2*A + A*b^2 + 2*a*b*B)*Sin[2*(c + d*x)] + 10*(8*
a*A*b + 4*a^2*B + 5*b^2*B)*Sin[3*(c + d*x)] + 15*b*(A*b + 2*a*B)*Sin[4*(c +
 d*x)] + 6*b^2*B*Sin[5*(c + d*x)])/(480*d)
```

Maple [A]
time = 0.17, size = 184, normalized size = 0.97

method	result
derivativedivides	$a^2A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{Ba^2\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+\frac{2Aab\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+2Bab\left(\frac{\left(\cos^3(dx+c)+\right.}{}\right.$
default	$a^2A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{Ba^2\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+\frac{2Aab\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+2Bab\left(\frac{\left(\cos^3(dx+c)+\right.}{}\right.$
risch	$\frac{a^2xA}{2}+\frac{3xAb^2}{8}+\frac{3xBab}{4}+\frac{3\sin(dx+c)Aab}{2d}+\frac{3\sin(dx+c)Ba^2}{4d}+\frac{5b^2B\sin(dx+c)}{8d}+\frac{Bb^2\sin(5dx+5c)}{80d}+$ sin
norman	$(\frac{1}{2}a^2A+\frac{3}{8}Ab^2+\frac{3}{4}Bab)x+(5a^2A+\frac{15}{4}Ab^2+\frac{15}{2}Bab)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(5a^2A+\frac{15}{4}Ab^2+\frac{15}{2}Bab)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{1}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+1/3*B*a^2*(cos(d*x+c)^
2+2)*sin(d*x+c)+2/3*A*a*b*(cos(d*x+c)^2+2)*sin(d*x+c)+2*B*a*b*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*b^2*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5*B*b^2*(8/3+cos(d*x+c)^4+4/3*cos
(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 176, normalized size = 0.93

$$\frac{120(2dx+2c+\sin(2dx+2c))Aa^2-160\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^2-320\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aab+30(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Bab+15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Ab^2+32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Bb^2}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 - 160*(sin(d*x + c)^3 - 3
*sin(d*x + c))*B*a^2 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b + 30*(12
*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a*b + 15*(12*d*x + 1
2*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*b^2 + 32*(3*sin(d*x + c)^5 -
 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*b^2)/d
```

Fricas [**A**]
time = 0.37, size = 142, normalized size = 0.75

$$\frac{15\,(4\,Aa^2+6\,Bab+3\,Ab^2)dx+\left(24\,Bb^2\cos\left(dx+c\right)^4+30\,(2\,Bab+Ab^2)\cos\left(dx+c\right)^3+80\,Ba^2+160\,Aab+64\,Bb^2+8\,(5\,Ba^2+10\,Aab+4\,Bb^2)\cos\left(dx+c\right)^2+15\,(4\,Aa^2+6\,Bab+3\,Ab^2)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*d*x + (24*B*b^2*cos(d*x + c)^4 + 30
*(2*B*a*b + A*b^2)*cos(d*x + c)^3 + 80*B*a^2 + 160*A*a*b + 64*B*b^2 + 8*(5*
B*a^2 + 10*A*a*b + 4*B*b^2)*cos(d*x + c)^2 + 15*(4*A*a^2 + 6*B*a*b + 3*A*b^
2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(184) = 368$.
time = 0.36, size = 459, normalized size = 2.43

$$\begin{cases} \frac{Aa^2x\sin^2(c+dx)}{2}+\frac{Aa^2x\cos^2(c+dx)}{2}+\frac{Aa^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{4Aab\sin^3(c+dx)}{3d}+\frac{2Aab\sin(c+dx)\cos^2(c+dx)}{d}+\frac{3Ab^2x\sin^4(c+dx)}{8}+\frac{3Ab^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3Ab^2x\cos^4(c+dx)}{8}+\frac{3Ab^2\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5Ab^2\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{2Ba^2\sin^3(c+dx)}{3d}+\frac{Ba^2\sin(c+dx)\cos^2(c+dx)}{d}+\frac{3Babx\sin^4(c+dx)}{4}+\frac{3Babx\sin^2(c+dx)\cos^2(c+dx)}{2}+\frac{3Babx\cos^4(c+dx)}{4}+\frac{3Bab\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{5Bab\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{8Bb^2\sin^5(c+dx)}{15d}+\frac{4Bb^2\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{Bb^2\sin(c+dx)\cos^4(c+dx)}{d} & \text{for}\ d\neq 0 \\ x(A+B\cos(c))\,(a+b\cos(c))^2\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**2/2 + A*a**2
*sin(c + d*x)*cos(c + d*x)/(2*d) + 4*A*a*b*sin(c + d*x)**3/(3*d) + 2*A*a*b*
sin(c + d*x)*cos(c + d*x)**2/d + 3*A*b**2*x*sin(c + d*x)**4/8 + 3*A*b**2*x*
sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*A*b**2*x*cos(c + d*x)**4/8 + 3*A*b**2
*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*A*b**2*sin(c + d*x)*cos(c + d*x)**3
/(8*d) + 2*B*a**2*sin(c + d*x)**3/(3*d) + B*a**2*sin(c + d*x)*cos(c + d*x)*
*2/d + 3*B*a*b*x*sin(c + d*x)**4/4 + 3*B*a*b*x*sin(c + d*x)**2*cos(c + d*x)
**2/2 + 3*B*a*b*x*cos(c + d*x)**4/4 + 3*B*a*b*sin(c + d*x)**3*cos(c + d*x)/
(4*d) + 5*B*a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 8*B*b**2*sin(c + d*x)*
*5/(15*d) + 4*B*b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + B*b**2*sin(c +
 d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))**2*cos
(c)**2, True))
```

Giac [**A**]
time = 0.46, size = 156, normalized size = 0.83

$$\frac{Bb^2\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{1}{8}\,(4\,Aa^2+6\,Bab+3\,Ab^2)x+\frac{(2\,Bab+Ab^2)\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{(4\,Ba^2+8\,Aab+5\,Bb^2)\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{(Aa^2+2\,Bab+Ab^2)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{(6\,Ba^2+12\,Aab+5\,Bb^2)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/80*B*b^2*sin(5*d*x + 5*c)/d + 1/8*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*x + 1/32*
(2*B*a*b + A*b^2)*sin(4*d*x + 4*c)/d + 1/48*(4*B*a^2 + 8*A*a*b + 5*B*b^2)*s
in(3*d*x + 3*c)/d + 1/4*(A*a^2 + 2*B*a*b + A*b^2)*sin(2*d*x + 2*c)/d + 1/8*
(6*B*a^2 + 12*A*a*b + 5*B*b^2)*sin(d*x + c)/d
```

Mupad [B]
time = 3.93, size = 307, normalized size = 1.62

$$\frac{x\left(A\,a^2+\frac{3\,B\,a\,b}{2}+\frac{3\,A\,b^2}{4}\right)}{2}+\frac{\left(2\,B\,a^2-\frac{5\,A\,b^2}{4}-A\,a^2+2\,B\,b^2+4\,A\,a\,b-\frac{5\,B\,a\,b}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{16\,B\,a^2}{3}-\frac{A\,b^2}{2}-2\,A\,a^2+\frac{8\,B\,b^2}{3}+\frac{32\,A\,a\,b}{3}-B\,a\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{20\,B\,a^2}{3}+\frac{40\,A\,a\,b}{3}+\frac{116\,B\,b^2}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(2\,A\,a^2+\frac{A\,b^2}{2}+\frac{16\,B\,a^2}{3}+\frac{8\,B\,b^2}{3}+\frac{32\,A\,a\,b}{3}+B\,a\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(A\,a^2+\frac{5\,A\,b^2}{4}+2\,B\,a^2+2\,B\,b^2+4\,A\,a\,b+\frac{5\,B\,a\,b}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (x*(A*a^2 + (3*A*b^2)/4 + (3*B*a*b)/2))/2 + (tan(c/2 + (d*x)/2)^5*((20*B*a^
2)/3 + (116*B*b^2)/15 + (40*A*a*b)/3) - tan(c/2 + (d*x)/2)^9*(A*a^2 + (5*A*
b^2)/4 - 2*B*a^2 - 2*B*b^2 - 4*A*a*b + (5*B*a*b)/2) + tan(c/2 + (d*x)/2)^3*
(2*A*a^2 + (A*b^2)/2 + (16*B*a^2)/3 + (8*B*b^2)/3 + (32*A*a*b)/3 + B*a*b) -
 tan(c/2 + (d*x)/2)^7*(2*A*a^2 + (A*b^2)/2 - (16*B*a^2)/3 - (8*B*b^2)/3 - (
32*A*a*b)/3 + B*a*b) + tan(c/2 + (d*x)/2)*(A*a^2 + (5*A*b^2)/4 + 2*B*a^2 +
2*B*b^2 + 4*A*a*b + (5*B*a*b)/2))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 +
 (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 +
(d*x)/2)^10 + 1))
```

3.224 $\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=170

$$\frac{1}{8}\left(8aAb+4a^2B+3b^2B\right)x+\frac{(4a^2Ab+4Ab^3-a^3B+8ab^2B)\sin(c+dx)}{6bd}+\frac{(8aAb-2a^2B+9b^2B)\cos(c+dx}{24d}$$

```
[Out] 1/8*(8*A*a*b+4*B*a^2+3*B*b^2)*x+1/6*(4*A*a^2*b+4*A*b^3-B*a^3+8*B*a*b^2)*sin
(d*x+c)/b/d+1/24*(8*A*a*b-2*B*a^2+9*B*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/12*(4*
A*b-B*a)*(a+b*cos(d*x+c))^2*sin(d*x+c)/b/d+1/4*B*(a+b*cos(d*x+c))^3*sin(d*x
+c)/b/d
```

Rubi [A]
time = 0.15, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3047, 3102, 2832, 2813}

$$\frac{(-2a^2B+8aAb+9b^2B)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x(4a^2B+8aAb+3b^2B)+\frac{(a^3(-B)+4a^2Ab+8ab^2B+4Ab^3)\sin(c+dx)}{6bd}+\frac{(4Ab-aB)\sin(c+dx)(a+b\cos(c+dx))^2}{12bd}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^3}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((8*a*A*b + 4*a^2*B + 3*b^2*B)*x)/8 + ((4*a^2*A*b + 4*A*b^3 - a^3*B + 8*a*b
^2*B)*Sin[c + d*x])/(6*b*d) + ((8*a*A*b - 2*a^2*B + 9*b^2*B)*Cos[c + d*x]*S
in[c + d*x])/(24*d) + ((4*A*b - a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
12*b*d) + (B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*b*d)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^2\left(A\cos(c+dx)+B\cos^2\right.$$
$$= \frac{B(a+b\cos(c+dx))^3\sin(c+dx)}{4bd}+\frac{\int(a+b\,\mathrm{co}}{}$$
$$= \frac{(4Ab-aB)(a+b\cos(c+dx))^2\sin(c+dx)}{12bd}+$$
$$= \frac{1}{8}\left(8aAb+4a^2B+3b^2B\right)x+\frac{(4a^2Ab+4Ab^3-}{}$$

Mathematica [A]
time = 0.51, size = 118, normalized size = 0.69

$$\frac{12(8aAb+4a^2B+3b^2B)(c+dx)+24(4a^2A+3Ab^2+6abB)\sin(c+dx)+24(2aAb+a^2B+b^2B)\sin(2(c+dx))+8b(Ab+2aB)\sin(3(c+dx))+3b^2B\sin(4(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (12*(8*a*A*b + 4*a^2*B + 3*b^2*B)*(c + d*x) + 24*(4*a^2*A + 3*A*b^2 + 6*a*b
*B)*Sin[c + d*x] + 24*(2*a*A*b + a^2*B + b^2*B)*Sin[2*(c + d*x)] + 8*b*(A*b
 + 2*a*B)*Sin[3*(c + d*x)] + 3*b^2*B*Sin[4*(c + d*x)])/(96*d)
```

Maple [A]
time = 0.13, size = 152, normalized size = 0.89

method	result

derivativedivides	$\frac{a^2A\sin(dx+c)+B\,a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2Aab\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{2Bab\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+}{d}$
default	$\frac{a^2A\sin(dx+c)+B\,a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2Aab\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{2Bab\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+}{d}$
risch	$xAab+\frac{a^2Bx}{2}+\frac{3b^2Bx}{8}+\frac{\sin(dx+c)a^2A}{d}+\frac{3\sin(dx+c)A\,b^2}{4d}+\frac{3\sin(dx+c)Bab}{2d}+\frac{B\,b^2\sin(4dx+4c)}{32d}+\frac{\sin(3d}{}$
norman	$\frac{(Aab+\frac{1}{2}B\,a^2+\frac{3}{8}B\,b^2)x+(Aab+\frac{1}{2}B\,a^2+\frac{3}{8}B\,b^2)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(4Aab+2B\,a^2+\frac{3}{2}B\,b^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(4Aab+2B}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*A*sin(d*x+c)+B*a^2*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+2*A*a
*b*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+2/3*B*a*b*(cos(d*x+c)^2+2)*sin
(d*x+c)+1/3*A*b^2*(cos(d*x+c)^2+2)*sin(d*x+c)+B*b^2*(1/4*(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.27, size = 142, normalized size = 0.84

$$\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2+48\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aab-64\,(\sin(dx+c)^3-3\,\sin(dx+c))Bab-32\,(\sin(dx+c)^3-3\,\sin(dx+c))Ab^2+3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Bb^2+96\,Aa^2\sin(dx+c)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 + 48*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a*b - 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a*b - 32*(sin(d*x
 + c)^3 - 3*sin(d*x + c))*A*b^2 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*s
in(2*d*x + 2*c))*B*b^2 + 96*A*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 114, normalized size = 0.67

$$\frac{3\,(4\,Ba^2+8\,Aab+3\,Bb^2)dx+\left(6\,Bb^2\cos(dx+c)^3+24\,Aa^2+32\,Bab+16\,Ab^2+8\,(2\,Bab+Ab^2)\cos(dx+c)^2+3\,(4\,Ba^2+8\,Aab+3\,Bb^2)\cos(dx+c)\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*d*x + (6*B*b^2*cos(d*x + c)^3 + 24*A*
a^2 + 32*B*a*b + 16*A*b^2 + 8*(2*B*a*b + A*b^2)*cos(d*x + c)^2 + 3*(4*B*a^2
 + 8*A*a*b + 3*B*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(162) = 324$.
time = 0.22, size = 338, normalized size = 1.99

$$\begin{cases} \frac{Aa^2\sin(c+dx)}{d} + Aabx\sin^2(c+dx) + Aabx\cos^2(c+dx) + \frac{Aab\sin(c+dx)\cos(c+dx)}{d} + \frac{2Ab^2\sin^3(c+dx)}{3d} + \frac{Ab^2\sin(c+dx)\cos^2(c+dx)}{d} + \frac{Ba^2x\sin^2(c+dx)}{2} + \frac{Ba^2x\cos^2(c+dx)}{2} + \frac{Ba^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{4Bab\sin^3(c+dx)}{3d} + \frac{2Bab\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Bb^2x\sin^4(c+dx)}{8} + \frac{3Bb^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Bb^2x\cos^4(c+dx)}{8} + \frac{3Bb^2\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5Bb^2\sin(c+dx)\cos^3(c+dx)}{8d} & \text{for } d \neq 0 \\ x(A + B\cos(c))(a + b\cos(c))^2\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*sin(c + d*x)/d + A*a*b*x*sin(c + d*x)**2 + A*a*b*x*cos(c
+ d*x)**2 + A*a*b*sin(c + d*x)*cos(c + d*x)/d + 2*A*b**2*sin(c + d*x)**3/(3
*d) + A*b**2*sin(c + d*x)*cos(c + d*x)**2/d + B*a**2*x*sin(c + d*x)**2/2 +
B*a**2*x*cos(c + d*x)**2/2 + B*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 4*B*a
*b*sin(c + d*x)**3/(3*d) + 2*B*a*b*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*b**
2*x*sin(c + d*x)**4/8 + 3*B*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B*
b**2*x*cos(c + d*x)**4/8 + 3*B*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*
B*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c))*(a
+ b*cos(c))**2*cos(c), True))
```

Giac [A]
time = 0.42, size = 124, normalized size = 0.73

$$\frac{Bb^2\sin(4\,dx+4\,c)}{32\,d} + \frac{1}{8}\left(4\,Ba^2+8\,Aab+3\,Bb^2\right)x + \frac{(2\,Bab+Ab^2)\sin(3\,dx+3\,c)}{12\,d} + \frac{(Ba^2+2\,Aab+Bb^2)\sin(2\,dx+2\,c)}{4\,d} + \frac{(4\,Aa^2+6\,Bab+3\,Ab^2)\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/32*B*b^2*sin(4*d*x + 4*c)/d + 1/8*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*x + 1/12*
(2*B*a*b + A*b^2)*sin(3*d*x + 3*c)/d + 1/4*(B*a^2 + 2*A*a*b + B*b^2)*sin(2*
d*x + 2*c)/d + 1/4*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*sin(d*x + c)/d
```

Mupad [B]
time = 0.51, size = 169, normalized size = 0.99

$$\frac{B\,a^2\,x}{2} + \frac{3\,B\,b^2\,x}{8} + \frac{A\,a^2\,\sin(c+d\,x)}{d} + \frac{3\,A\,b^2\,\sin(c+d\,x)}{4\,d} + A\,a\,b\,x + \frac{B\,a^2\,\sin(2\,c+2\,d\,x)}{4\,d} + \frac{A\,b^2\,\sin(3\,c+3\,d\,x)}{12\,d} + \frac{B\,b^2\,\sin(2\,c+2\,d\,x)}{4\,d} + \frac{B\,b^2\,\sin(4\,c+4\,d\,x)}{32\,d} + \frac{3\,B\,a\,b\,\sin(c+d\,x)}{2\,d} + \frac{A\,a\,b\,\sin(2\,c+2\,d\,x)}{2\,d} + \frac{B\,a\,b\,\sin(3\,c+3\,d\,x)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (B*a^2*x)/2 + (3*B*b^2*x)/8 + (A*a^2*sin(c + d*x))/d + (3*A*b^2*sin(c + d*x
))/(4*d) + A*a*b*x + (B*a^2*sin(2*c + 2*d*x))/(4*d) + (A*b^2*sin(3*c + 3*d*
x))/(12*d) + (B*b^2*sin(2*c + 2*d*x))/(4*d) + (B*b^2*sin(4*c + 4*d*x))/(32*
d) + (3*B*a*b*sin(c + d*x))/(2*d) + (A*a*b*sin(2*c + 2*d*x))/(2*d) + (B*a*b
*sin(3*c + 3*d*x))/(6*d)
```

3.225 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=107

$$\frac{1}{2}\left(2a^2A+Ab^2+2abB\right)x+\frac{2(3aAb+a^2B+b^2B)\sin(c+dx)}{3d}+\frac{b(3Ab+2aB)\cos(c+dx)\sin(c+dx)}{6d}+\frac{B(a-$$

```
[Out] 1/2*(2*A*a^2+A*b^2+2*B*a*b)*x+2/3*(3*A*a*b+B*a^2+B*b^2)*sin(d*x+c)/d+1/6*b*
(3*A*b+2*B*a)*cos(d*x+c)*sin(d*x+c)/d+1/3*B*(a+b*cos(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2832, 2813}

$$\frac{2(a^2B+3aAb+b^2B)\sin(c+dx)}{3d}+\frac{1}{2}x\left(2a^2A+2abB+Ab^2\right)+\frac{b(2aB+3Ab)\sin(c+dx)\cos(c+dx)}{6d}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((2*a^2*A + A*b^2 + 2*a*b*B)*x)/2 + (2*(3*a*A*b + a^2*B + b^2*B)*Sin[c + d*
x])/(3*d) + (b*(3*A*b + 2*a*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (B*(a + b
*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx &= \frac{B(a+b\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (a+b\cos(c+dx))\\ &= \frac{1}{2}\left(2a^2A+Ab^2+2abB\right)x+\frac{2(3aAb+a^2B+b^2B)\sin(c+d}{3d}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 90, normalized size = 0.84

$$\frac{6(2a^2A + Ab^2 + 2abB)(c + dx) + 3(8aAb + 4a^2B + 3b^2B)\sin(c + dx) + 3b(Ab + 2aB)\sin(2(c + dx)) + b^2B\sin(3(c + dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*(2*a^2*A + A*b^2 + 2*a*b*B)*(c + d*x) + 3*(8*a*A*b + 4*a^2*B + 3*b^2*B)*
Sin[c + d*x] + 3*b*(A*b + 2*a*B)*Sin[2*(c + d*x)] + b^2*B*Sin[3*(c + d*x)])
/(12*d)
```

Maple [A]
time = 0.10, size = 114, normalized size = 1.07

method	result
derivativedivides	$\frac{\frac{B b^2(\cos^2(dx+c)+2)\sin(dx+c)}{3}+A b^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2Bab\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2Aab\sin(dx+c}{d}$
default	$\frac{\frac{B b^2(\cos^2(dx+c)+2)\sin(dx+c)}{3}+A b^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2Bab\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2Aab\sin(dx+c}{d}$
risch	$a^2xA+\frac{xAb^2}{2}+xBab+\frac{2\sin(dx+c)Aab}{d}+\frac{\sin(dx+c)Ba^2}{d}+\frac{3b^2B\sin(dx+c)}{4d}+\frac{\sin(3dx+3c)Bb^2}{12d}+\sin($
norman	$(a^2A+\frac{1}{2}Ab^2+Bab)x+(a^2A+\frac{1}{2}Ab^2+Bab)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(3a^2A+\frac{3}{2}Ab^2+3Bab)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(3a^2A+\frac{3}{2}Ab^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*B*b^2*(cos(d*x+c)^2+2)*sin(d*x+c)+A*b^2*(1/2*sin(d*x+c)*cos(d*x+c)
+1/2*d*x+1/2*c)+2*B*a*b*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+2*A*a*b*s
in(d*x+c)+B*a^2*sin(d*x+c)+a^2*A*(d*x+c))
```

Maxima [A]
time = 0.26, size = 108, normalized size = 1.01

$$\frac{12(dx+c)Aa^2 + 6(2dx+2c+\sin(2dx+2c))Bab + 3(2dx+2c+\sin(2dx+2c))Ab^2 - 4(\sin(dx+c)^3 - 3\sin(dx+c))Bb^2 + 12Ba^2\sin(dx+c) + 24Aab\sin(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*A*a^2 + 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b + 3*(2*
d*x + 2*c + sin(2*d*x + 2*c))*A*b^2 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B
*b^2 + 12*B*a^2*sin(d*x + c) + 24*A*a*b*sin(d*x + c))/d
```

Fricas [A]
time = 0.40, size = 85, normalized size = 0.79

$$\frac{3\left(2\,Aa^{2}+2\,Bab+Ab^{2}\right)dx+\left(2\,Bb^{2}\cos\left(dx+c\right)^{2}+6\,Ba^{2}+12\,Aab+4\,Bb^{2}+3\left(2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*A*a^2 + 2*B*a*b + A*b^2)*d*x + (2*B*b^2*cos(d*x + c)^2 + 6*B*a^2
+ 12*A*a*b + 4*B*b^2 + 3*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A]
time = 0.14, size = 199, normalized size = 1.86

$$\begin{cases} Aa^{2}x+\frac{2Aab\sin\left(c+dx\right)}{d}+\frac{Ab^{2}x\sin^{2}\left(c+dx\right)}{2}+\frac{Ab^{2}x\cos^{2}\left(c+dx\right)}{2}+\frac{Ab^{2}\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{Ba^{2}\sin\left(c+dx\right)}{d}+Babx\sin^{2}\left(c+dx\right)+Babx\cos^{2}\left(c+dx\right)+\frac{Bab\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{2Bb^{2}\sin^{3}\left(c+dx\right)}{3d}+\frac{Bb^{2}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\left(A+B\cos\left(c\right)\right)\left(a+b\cos\left(c\right)\right)^{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x + 2*A*a*b*sin(c + d*x)/d + A*b**2*x*sin(c + d*x)**2/2 +
 A*b**2*x*cos(c + d*x)**2/2 + A*b**2*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*
*2*sin(c + d*x)/d + B*a*b*x*sin(c + d*x)**2 + B*a*b*x*cos(c + d*x)**2 + B*a
*b*sin(c + d*x)*cos(c + d*x)/d + 2*B*b**2*sin(c + d*x)**3/(3*d) + B*b**2*si
n(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))**
2, True))
```

Giac [A]
time = 0.43, size = 93, normalized size = 0.87

$$\frac{Bb^{2}\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{1}{2}\left(2\,Aa^{2}+2\,Bab+Ab^{2}\right)x+\frac{\left(2\,Bab+Ab^{2}\right)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{\left(4\,Ba^{2}+8\,Aab+3\,Bb^{2}\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*B*b^2*sin(3*d*x + 3*c)/d + 1/2*(2*A*a^2 + 2*B*a*b + A*b^2)*x + 1/4*(2*
B*a*b + A*b^2)*sin(2*d*x + 2*c)/d + 1/4*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*sin(d
*x + c)/d
```

Mupad [B]
time = 0.45, size = 115, normalized size = 1.07

$$A\,a^{2}\,x+\frac{A\,b^{2}\,x}{2}+\frac{B\,a^{2}\,\sin\left(c+d\,x\right)}{d}+\frac{3\,B\,b^{2}\,\sin\left(c+d\,x\right)}{4\,d}+B\,a\,b\,x+\frac{A\,b^{2}\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,b^{2}\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{2\,A\,a\,b\,\sin\left(c+d\,x\right)}{d}+\frac{B\,a\,b\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

```
[Out] A*a^2*x + (A*b^2*x)/2 + (B*a^2*sin(c + d*x))/d + (3*B*b^2*sin(c + d*x))/(4*
d) + B*a*b*x + (A*b^2*sin(2*c + 2*d*x))/(4*d) + (B*b^2*sin(3*c + 3*d*x))/(1
2*d) + (2*A*a*b*sin(c + d*x))/d + (B*a*b*sin(2*c + 2*d*x))/(2*d)
```

3.226 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=86

$$\frac{1}{2}\left(4aAb+2a^2B+b^2B\right)x+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(2Ab+3aB)\sin(c+dx)}{2d}+\frac{bB(a+b\cos(c+dx))\sin}{2d}$$

```
[Out] 1/2*(4*A*a*b+2*B*a^2+B*b^2)*x+a^2*A*arctanh(sin(d*x+c))/d+1/2*b*(2*A*b+3*B*
a)*sin(d*x+c)/d+1/2*b*B*(a+b*cos(d*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3069, 3102, 2814, 3855}

$$\frac{1}{2}x\left(2a^2B+4aAb+b^2B\right)+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(3aB+2Ab)\sin(c+dx)}{2d}+\frac{bB\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] ((4*a*A*b + 2*a^2*B + b^2*B)*x)/2 + (a^2*A*ArcTanh[Sin[c + d*x]])/d + (b*(2
*A*b + 3*a*B)*Sin[c + d*x])/(2*d) + (b*B*(a + b*Cos[c + d*x])*Sin[c + d*x])
/(2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{bB(a+b\cos(c+dx))\sin(c+dx)}{2d}+\frac{1}{2}\int \left(2a^2A\right.$$
$$= \frac{b(2Ab+3aB)\sin(c+dx)}{2d}+\frac{bB(a+b\cos(c+dx\right.}{2d}$$
$$= \frac{1}{2}\left(4aAb+2a^2B+b^2B\right)x+\frac{b(2Ab+3aB)\sin(c}{2d}$$
$$= \frac{1}{2}\left(4aAb+2a^2B+b^2B\right)x+\frac{a^2A\tanh^{-1}(\sin(c+}{d}$$

Mathematica [A]
time = 0.26, size = 120, normalized size = 1.40

$$\frac{2(4aAb+2a^2B+b^2B)(c+dx)-4a^2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4a^2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+4b(Ab+2aB)\sin(c+dx)+b^2B\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*(4*a*A*b + 2*a^2*B + b^2*B)*(c + d*x) - 4*a^2*A*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 4*a^2*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*b*(
A*b + 2*a*B)*Sin[c + d*x] + b^2*B*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.17, size = 94, normalized size = 1.09

method	result
derivativedivides	$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))+B\,a^2(dx+c)+2Aab(dx+c)+2Bab\sin(dx+c)+A\,b^2\sin(dx+c)+B\,b^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\right.}{d}$

default	$\frac{a^2 A \ln(\sec(dx+c)+\tan(dx+c))+B\,a^2(dx+c)+2Aab(dx+c)+2Bab\sin(dx+c)+A\,b^2\sin(dx+c)+B\,b^2\left(\frac{\sin(dx+c)\cos(dx+c}{2}\right.}{d}$
risch	$2xAab+a^2Bx+\frac{b^2Bx}{2}-\frac{i e^{i(dx+c)}A\,b^2}{2d}-\frac{i e^{i(dx+c)}Bab}{d}+\frac{i e^{-i(dx+c)}A\,b^2}{2d}+\frac{i e^{-i(dx+c)}Bab}{d}+\frac{a^2 A\ln(e^{i}}{}$
norman	$\frac{(2Aab+B\,a^2+\frac{1}{2}B\,b^2)x+(2Aab+B\,a^2+\frac{1}{2}B\,b^2)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(6Aab+3B\,a^2+\frac{3}{2}B\,b^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(6Aab+3.}{\left(1+\tan^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*A*ln(sec(d*x+c)+tan(d*x+c))+B*a^2*(d*x+c)+2*A*a*b*(d*x+c)+2*B*a*b*
sin(d*x+c)+A*b^2*sin(d*x+c)+B*b^2*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)
)
```

Maxima [A]
time = 0.28, size = 92, normalized size = 1.07

$$\frac{4\,(dx+c)Ba^2+8\,(dx+c)Aab+(2\,dx+2\,c+\sin(2\,dx+2\,c))Bb^2+4\,Aa^2\log(\sec(dx+c)+\tan(dx+c))+8\,Bab\sin(dx+c)+4\,Ab^2\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(4*(d*x + c)*B*a^2 + 8*(d*x + c)*A*a*b + (2*d*x + 2*c + sin(2*d*x + 2*c
))*B*b^2 + 4*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 8*B*a*b*sin(d*x + c)
+ 4*A*b^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 87, normalized size = 1.01

$$\frac{Aa^2\log(\sin(dx+c)+1)-Aa^2\log(-\sin(dx+c)+1)+(2\,Ba^2+4\,Aab+Bb^2)dx+(Bb^2\cos(dx+c)+4\,Bab+2\,Ab^2)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/2*(A*a^2*log(sin(d*x + c) + 1) - A*a^2*log(-sin(d*x + c) + 1) + (2*B*a^2
+ 4*A*a*b + B*b^2)*d*x + (B*b^2*cos(d*x + c) + 4*B*a*b + 2*A*b^2)*sin(d*x +
 c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\,(a+b\cos(c+dx))^2\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(80) = 160.
time = 0.45, size = 178, normalized size = 2.07

$$\frac{2\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\left(2\,Ba^2+4\,Aab+Bb^2\right)(dx+c)+\frac{2\left(4\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/2*(2*A*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*A*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (2*B*a^2 + 4*A*a*b + B*b^2)*(d*x + c) + 2*(4*B*a*b*tan(
1/2*d*x + 1/2*c)^3 + 2*A*b^2*tan(1/2*d*x + 1/2*c)^3 - B*b^2*tan(1/2*d*x + 1
/2*c)^3 + 4*B*a*b*tan(1/2*d*x + 1/2*c) + 2*A*b^2*tan(1/2*d*x + 1/2*c) + B*b
^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 0.69, size = 169, normalized size = 1.97

$$\frac{A\,b^2\,\sin\left(c+d\,x\right)}{d}+\frac{2\,A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{B\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{B\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{2\,B\,a\,b\,\sin\left(c+d\,x\right)}{d}+\frac{4\,A\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x),x)
```

```
[Out] (A*b^2*sin(c + d*x))/d + (2*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/d + (2*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (B*b^2*a
tan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (B*b^2*sin(2*c + 2*d*x))/(4
*d) + (2*B*a*b*sin(c + d*x))/d + (4*A*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2)))/d
```

3.227 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=60

$$b(Ab+2aB)x+\frac{a(2Ab+aB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2B\sin(c+dx)}{d}+\frac{a^2A\tan(c+dx)}{d}$$

```
[Out] b*(A*b+2*B*a)*x+a*(2*A*b+B*a)*arctanh(sin(d*x+c))/d+b^2*B*sin(d*x+c)/d+a^2*
A*tan(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3067, 3102, 2814, 3855}

$$\frac{a^2A\tan(c+dx)}{d}+\frac{a(aB+2Ab)\tanh^{-1}(\sin(c+dx))}{d}+bx(2aB+Ab)+\frac{b^2B\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] b*(A*b + 2*a*B)*x + (a*(2*A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (b^2*B*Sin[
c + d*x])/d + (a^2*A*Tan[c + d*x])/d
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{a^2A\tan(c+dx)}{d} - \int \left(-a(2Ab+aB)-b(Ab+\right.$$
$$= \frac{b^2B\sin(c+dx)}{d} + \frac{a^2A\tan(c+dx)}{d} - \int \left(-a(2\right.$$
$$= b(Ab+2aB)x + \frac{b^2B\sin(c+dx)}{d} + \frac{a^2A\tan(c+}{d}$$
$$= b(Ab+2aB)x + \frac{a(2Ab+aB)\tanh^{-1}(\sin(c+d}{d}$$

Mathematica [A]
time = 0.55, size = 109, normalized size = 1.82

$$\frac{b(Ab+2aB)(c+dx)-a(2Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+a(2Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+b^2B\sin(c+dx)+a^2A\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (b*(A*b + 2*a*B)*(c + d*x) - a*(2*A*b + a*B)*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + a*(2*A*b + a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^2
*B*Sin[c + d*x] + a^2*A*Tan[c + d*x])/d
```

Maple [A]
time = 0.19, size = 86, normalized size = 1.43

method	result
derivativedivides	$\frac{a^2A\tan(dx+c)+Ba^2\ln(\sec(dx+c)+\tan(dx+c))+2Aab\ln(\sec(dx+c)+\tan(dx+c))+2Bab(dx+c)+Ab^2(dx+c)+Bb^2\sin(}{d}$
default	$\frac{a^2A\tan(dx+c)+Ba^2\ln(\sec(dx+c)+\tan(dx+c))+2Aab\ln(\sec(dx+c)+\tan(dx+c))+2Bab(dx+c)+Ab^2(dx+c)+Bb^2\sin(}{d}$

risch	$xAb^2 + 2xBab - \frac{iBb^2 e^{i(dx+c)}}{2d} + \frac{iBb^2 e^{-i(dx+c)}}{2d} + \frac{2ia^2A}{d(e^{2i(dx+c)}+1)} + \frac{2a\ln(e^{i(dx+c)}+i)Ab}{d} + \frac{a^2\ln(e^{i(dx}{d}$
norman	$\frac{(-Ab^2-2Bab)x+(-2Ab^2-4Bab)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(Ab^2+2Bab)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2Ab^2+4Bab)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*tan(d*x+c)+B*a^2*ln(sec(d*x+c)+tan(d*x+c))+2*A*a*b*ln(sec(d*x+c)
+tan(d*x+c))+2*B*a*b*(d*x+c)+A*b^2*(d*x+c)+B*b^2*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 103, normalized size = 1.72

$$\frac{4(dx+c)Bab + 2(dx+c)Ab^2 + Ba^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2Aab(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2Bb^2\sin(dx+c) + 2Aa^2\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*(4*(d*x + c)*B*a*b + 2*(d*x + c)*A*b^2 + B*a^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 2*A*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*B*b^2*sin(d*x + c) + 2*A*a^2*tan(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 117, normalized size = 1.95

$$\frac{2(2Bab + Ab^2)dx\cos(dx+c) + (Ba^2 + 2Aab)\cos(dx+c)\log(\sin(dx+c)+1) - (Ba^2 + 2Aab)\cos(dx+c)\log(-\sin(dx+c)+1) + 2(Bb^2\cos(dx+c) + Aa^2)\sin(dx+c)}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*(2*(2*B*a*b + A*b^2)*d*x*cos(d*x + c) + (B*a^2 + 2*A*a*b)*cos(d*x + c)*
log(sin(d*x + c) + 1) - (B*a^2 + 2*A*a*b)*cos(d*x + c)*log(-sin(d*x + c) +
1) + 2*(B*b^2*cos(d*x + c) + A*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c+dx))(a + b\cos(c+dx))^2 \sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(60) = 120$.
time = 0.46, size = 152, normalized size = 2.53

$$\frac{(2\,Bab + Ab^2)(dx + c) + (Ba^2 + 2\,Aab)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (Ba^2 + 2\,Aab)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] ((2*B*a*b + A*b^2)*(d*x + c) + (B*a^2 + 2*A*a*b)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (B*a^2 + 2*A*a*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a^2*
tan(1/2*d*x + 1/2*c)^3 - B*b^2*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*x +
 1/2*c) + B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

Mupad [B]
time = 0.88, size = 169, normalized size = 2.82

$$\frac{A\,a^2\tan(c+d\,x)}{d} + \frac{2\,A\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{B\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d\,\cos\left(c+d\,x\right)} + \frac{4\,B\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2\mathrm{i}}{d} - \frac{A\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,4\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] (A*a^2*tan(c + d*x))/d + (2*A*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d - (B*a^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/d + (B*
b^2*sin(2*c + 2*d*x))/(2*d*cos(c + d*x)) - (A*a*b*atan((sin(c/2 + (d*x)/2)*
1i)/cos(c/2 + (d*x)/2))*4i)/d + (4*B*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/d
```

3.228 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=80

$$b^2Bx+\frac{(a^2A+2Ab^2+4abB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(2Ab+aB)\tan(c+dx)}{d}+\frac{a^2A\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] b^2*B*x+1/2*(A*a^2+2*A*b^2+4*B*a*b)*arctanh(sin(d*x+c))/d+a*(2*A*b+B*a)*tan
(d*x+c)/d+1/2*a^2*A*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3067, 3100, 2814, 3855}

$$\frac{(a^2A+4abB+2Ab^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2A\tan(c+dx)\sec(c+dx)}{2d}+\frac{a(aB+2Ab)\tan(c+dx)}{d}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] b^2*B*x + ((a^2*A + 2*A*b^2 + 4*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2
*A*b + a*B)*Tan[c + d*x])/d + (a^2*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{a^2A\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{2}\int (-2a(2Ab$$

$$= \frac{a(2Ab+aB)\tan(c+dx)}{d} + \frac{a^2A\sec(c+dx)\tan}{2d}$$

$$= b^2Bx + \frac{a(2Ab+aB)\tan(c+dx)}{d} + \frac{a^2A\sec(c}{}$$

$$= b^2Bx + \frac{(a^2A+2Ab^2+4abB)\tanh^{-1}(\sin(c+d}{2d}$$

Mathematica [A]
time = 0.31, size = 67, normalized size = 0.84

$$\frac{2b^2Bdx+(a^2A+2Ab^2+4abB)\tanh^{-1}(\sin(c+dx))+a(4Ab+2aB+aA\sec(c+dx))\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^2*B*d*x + (a^2*A + 2*A*b^2 + 4*a*b*B)*ArcTanh[Sin[c + d*x]] + a*(4*A*b
 + 2*a*B + a*A*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.22, size = 112, normalized size = 1.40

method	result
derivativedivides	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+B\,a^2\tan(dx+c)+2Aab\tan(dx+c)+2Bab\ln(\sec(dx+c)+\tan(dx+c))}{d}$

default	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+B\,a^2\tan(dx+c)+2Aab\tan(dx+c)+2Bab\ln(\sec(dx+c)+\tan(dx+c}{d}$
risch	$b^2Bx-\frac{ia\left(Aa\,e^{3i(dx+c)}-4Ab\,e^{2i(dx+c)}-2Ba\,e^{2i(dx+c)}-aA\,e^{i(dx+c)}-4Ab-2aB\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{a^2A\ln\left(e^{i(dx+c)}-i\right)}{2d}-\frac{\ln(e^{i}}{}$
norman	$\frac{b^2Bx+\frac{a(aA-4Ab-2aB)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a(aA+4Ab+2aB)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+b^2Bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b^2Bx\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+B*a^2*tan(d*x+c)+2*A*a*b*tan(d*x+c)+2*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+A*b^2*ln(sec(d*x+c)+tan(d*x+c))+B*b^2*(d*x+c))

Maxima [A]
time = 0.27, size = 140, normalized size = 1.75

$$\frac{4\,(dx+c)Bb^2-Aa^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+4\,Bab(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+2\,Ab^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+4\,Ba^2\tan\left(dx+c\right)+8\,Aab\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")

[Out] 1/4*(4*(d*x + c)*B*b^2 - A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*B*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*A*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a^2*tan(d*x + c) + 8*A*a*b*tan(d*x + c))/d

Fricas [A]
time = 0.39, size = 136, normalized size = 1.70

$$\frac{4\,Bb^2dx\cos\left(dx+c\right)^2+\left(Aa^2+4\,Bab+2\,Ab^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(Aa^2+4\,Bab+2\,Ab^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(Aa^2+2\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")

[Out] 1/4*(4*B*b^2*d*x*cos(d*x + c)^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a^2 + 4*B*a*b + 2*A*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a^2 + 2*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(A+B\cos\left(c+dx\right)\right)\left(a+b\cos\left(c+dx\right)\right)^2\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sec(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(76) = 152.

time = 0.52, size = 190, normalized size = 2.38

$$\frac{2\,(dx+c)Bb^2+(Aa^2+4\,Bab+2\,Ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa^2+4\,Bab+2\,Ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(d*x + c)*B*b^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x +
1/2*c) + 1)) - (A*a^2 + 4*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) + 2*(A*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*A
*a*b*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*x + 1/2*c) + 2*B*a^2*tan(1/2*
d*x + 1/2*c) + 4*A*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2
)/d
```

Mupad [B]

time = 0.98, size = 176, normalized size = 2.20

$$\frac{2\left(\frac{A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+A\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+B\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+2\,B\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\right)}{d}+\frac{\frac{B\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{2}+\frac{A\,a^2\,\sin\left(c+d\,x\right)}{2}+A\,a\,b\,\sin\left(2\,c+2\,d\,x\right)}{d\,\left(\frac{\cos\left(2\,c+2\,d\,x\right)}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^3,x)
```

```
[Out] (2*((A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + A*b^2*atanh(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + B*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)) + 2*B*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/d +
((B*a^2*sin(2*c + 2*d*x))/2 + (A*a^2*sin(c + d*x))/2 + A*a*b*sin(2*c + 2*d*
x))/(d*(cos(2*c + 2*d*x)/2 + 1/2))
```

3.229 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=116

$$\frac{(2aAb+a^2B+2b^2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2a^2A+3Ab^2+6abB)\tan(c+dx)}{3d}+\frac{a(2Ab+aB)\sec(c+dx)}{2d}$$

```
[Out] 1/2*(2*A*a*b+B*a^2+2*B*b^2)*arctanh(sin(d*x+c))/d+1/3*(2*A*a^2+3*A*b^2+6*B*
a*b)*tan(d*x+c)/d+1/2*a*(2*A*b+B*a)*sec(d*x+c)*tan(d*x+c)/d+1/3*a^2*A*sec(d
*x+c)^2*tan(d*x+c)/d
```

Rubi [A]

time = 0.18, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3067, 3100, 2827, 3852, 8, 3855}

$$\frac{(2a^2A+6abB+3Ab^2)\tan(c+dx)}{3d}+\frac{(a^2B+2aAb+2b^2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{a(aB+2Ab)\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((2*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a^2*A + 3*A
*b^2 + 6*a*b*B)*Tan[c + d*x])/(3*d) + (a*(2*A*b + a*B)*Sec[c + d*x]*Tan[c +
 d*x])/(2*d) + (a^2*A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
```

```
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{a^2A\sec^2(c+dx)\tan(c+dx)}{3d} - \frac{1}{3}\int (-3a(2Ab$$

$$= \frac{a(2Ab+aB)\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^2A\sec}{}$$

$$= \frac{a(2Ab+aB)\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^2A\sec}{}$$

$$= \frac{(2aAb+a^2B+2b^2B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a}{}$$

$$= \frac{(2aAb+a^2B+2b^2B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(2}{}$$

Mathematica [A]
time = 0.52, size = 92, normalized size = 0.79

$$\frac{3(2aAb+a^2B+2b^2B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3a(2Ab+aB)\sec(c+dx)+2(3a^2A+3Ab^2+6abB+a^2A\tan^2(c+dx))\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (3*(2*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*a*(2
*A*b + a*B)*Sec[c + d*x] + 2*(3*a^2*A + 3*A*b^2 + 6*a*b*B + a^2*A*Tan[c + d
*x]^2)))/(6*d)
```

Maple [A]

time = 0.24, size = 143, normalized size = 1.23

method	result
derivativedivides	$\frac{-a^2 A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+B a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2Aab\left(\frac{\sec(dx+c)\tan(dx+}{2}\right.}{d}$
default	$\frac{-a^2 A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+B a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2Aab\left(\frac{\sec(dx+c)\tan(dx+}{2}\right.}{d}$
risch	$-\frac{i\left(6Aab\,e^{5i(dx+c)}+3B\,a^2 e^{5i(dx+c)}-6A\,b^2 e^{4i(dx+c)}-12Bab\,e^{4i(dx+c)}-12A\,a^2 e^{2i(dx+c)}-12A\,b^2 e^{2i(dx+c)}-24Bab\,e^{2i(\right.}{3d\left(e^{2i(dx+c)}+1\right)^3}$
norman	$\frac{-\frac{2\left(2a^2A-2Aab-2A\,b^2-B\,a^2-4Bab\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(2a^2A-2Aab+2A\,b^2-B\,a^2+4Bab\right)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(2a^2A+2Aab-2\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-a^2*A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+B*a^2*(1/2*sec(d*x+c)*tan(d*
x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+2*A*a*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*
ln(sec(d*x+c)+tan(d*x+c)))+2*B*a*b*tan(d*x+c)+A*b^2*tan(d*x+c)+B*b^2*ln(sec
(d*x+c)+tan(d*x+c)))
```

Maxima [A]

time = 0.27, size = 172, normalized size = 1.48

$$\frac{4\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Aa^2-3\,Ba^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-6\,Aab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,Bb^2\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+24\,Bab\tan\left(dx+c\right)+12\,Ab^2\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 - 3*B*a^2*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 6*A*
a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 6*B*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
24*B*a*b*tan(d*x + c) + 12*A*b^2*tan(d*x + c))/d
```

Fricas [A]

time = 0.36, size = 150, normalized size = 1.29

$$\frac{3\left(Ba^2+2\,Aab+2\,Bb^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(Ba^2+2\,Aab+2\,Bb^2\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,Aa^2+2\left(2\,Aa^2+6\,Bab+3\,Ab^2\right)\cos\left(dx+c\right)^2+3\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) -
3*(B*a^2 + 2*A*a*b + 2*B*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*
A*a^2 + 2*(2*A*a^2 + 6*B*a*b + 3*A*b^2)*cos(d*x + c)^2 + 3*(B*a^2 + 2*A*a*b
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(108) = 216.
time = 0.45, size = 294, normalized size = 2.53

$$\frac{3\,(Ba^2+2\,Aab+2\,Bb^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Ba^2+2\,Aab+2\,Bb^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(
B*a^2 + 2*A*a*b + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^2*
tan(1/2*d*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b*tan(1/2*d
*x + 1/2*c)^5 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2
*c)^5 - 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 -
12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*ta
n(1/2*d*x + 1/2*c) + 6*A*a*b*tan(1/2*d*x + 1/2*c) + 12*B*a*b*tan(1/2*d*x +
1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [**B**]
time = 3.66, size = 227, normalized size = 1.96

$$\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{B\,a^2}{2}+A\,a\,b+B\,b^2\right)}{2\,B\,a^2+4\,A\,a\,b+4\,B\,b^2}\right)\,\left(B\,a^2+2\,A\,a\,b+2\,B\,b^2\right)}{d}-\frac{\left(2\,A\,a^2+2\,A\,b^2-B\,a^2-2\,A\,a\,b+4\,B\,a\,b\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(-\frac{4\,A\,a^2}{3}-8\,B\,a\,b-4\,A\,b^2\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(2\,A\,a^2+2\,A\,b^2+B\,a^2+2\,A\,a\,b+4\,B\,a\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^4,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((B*a^2)/2 + B*b^2 + A*a*b))/(2*B*a^2 + 4*B*b^
2 + 4*A*a*b))*(B*a^2 + 2*B*b^2 + 2*A*a*b))/d - (tan(c/2 + (d*x)/2)*(2*A*a^2
 + 2*A*b^2 + B*a^2 + 2*A*a*b + 4*B*a*b) - tan(c/2 + (d*x)/2)^3*((4*A*a^2)/3
 + 4*A*b^2 + 8*B*a*b) + tan(c/2 + (d*x)/2)^5*(2*A*a^2 + 2*A*b^2 - B*a^2 - 2
*A*a*b + 4*B*a*b))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + ta
n(c/2 + (d*x)/2)^6 - 1))
```

3.230 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=156

$$\frac{(3a^2A+4Ab^2+8abB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4aAb+2a^2B+3b^2B)\tan(c+dx)}{3d}+\frac{(3a^2A+4Ab^2+8abB)\,\mathrm{se}}{8d}$$

```
[Out] 1/8*(3*A*a^2+4*A*b^2+8*B*a*b)*arctanh(sin(d*x+c))/d+1/3*(4*A*a*b+2*B*a^2+3*
B*b^2)*tan(d*x+c)/d+1/8*(3*A*a^2+4*A*b^2+8*B*a*b)*sec(d*x+c)*tan(d*x+c)/d+1
/3*a*(2*A*b+B*a)*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a^2*A*sec(d*x+c)^3*tan(d*x+c
)/d
```

Rubi [A]

time = 0.20, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3067, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{(2a^2B+4aAb+3b^2B)\tan(c+dx)}{3d}+\frac{(3a^2A+8abB+4Ab^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3a^2A+8abB+4Ab^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{a^2A\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{a(aB+2Ab)\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a*A*b + 2
*a^2*B + 3*b^2*B)*Tan[c + d*x])/(3*d) + ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (a*(2*A*b + a*B)*Sec[c + d*x]^2*Tan[c + d*x]
)/(3*d) + (a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
```

```
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx &= \frac{a^2A\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{1}{4}\int (-4a(2Ab \\
&= \frac{a(2Ab+aB)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{a^2A\,\mathrm{se}}{} \\
&= \frac{a(2Ab+aB)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{a^2A\,\mathrm{se}}{} \\
&= \frac{(3a^2A+4Ab^2+8abB)\sec(c+dx)\tan(c+dx)}{8d} \\
&= \frac{(3a^2A+4Ab^2+8abB)\tanh^{-1}(\sin(c+dx))}{8d} + \cdots
\end{aligned}
$$

Mathematica [A]

time = 0.78, size = 120, normalized size = 0.77

$$\frac{3(3a^2A+4Ab^2+8abB)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(24(2aAb+a^2B+b^2B)+3(3a^2A+4Ab^2+8abB)\sec(c+dx)+6a^2A\sec^3(c+dx)+8a(2Ab+aB)\tan^2(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*(
2*a*A*b + a^2*B + b^2*B) + 3*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*Sec[c + d*x] + 6
*a^2*A*Sec[c + d*x]^3 + 8*a*(2*A*b + a*B)*Tan[c + d*x]^2))/(24*d)
```

Maple [A]

time = 0.30, size = 185, normalized size = 1.19

method	result
derivativedivides	$\frac{a^2A\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-Ba^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-\cdots}{}$
default	$\frac{a^2A\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-Ba^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-\cdots}{}$
norman	$\frac{\left(7a^2A-4A\,b^2-8Bab\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(27a^2A-32Aab+12A\,b^2-16B\,a^2+24Bab\right)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{\left(27a^2A+32Aab+12A\,b^2+16B\cdots\right.}{6d}$
risch	$-\frac{i\left(9A\,a^2e^{7i(dx+c)}+12A\,b^2e^{7i(dx+c)}+24Bab\,e^{7i(dx+c)}-24B\,b^2e^{6i(dx+c)}+33A\,a^2e^{5i(dx+c)}+12A\,b^2e^{5i(dx+c)}+24Bab\,e^{5i\cdots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c
)+tan(d*x+c)))-B*a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-2*A*a*b*(-2/3-1/3*s
ec(d*x+c)^2)*tan(d*x+c)+2*B*a*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c
)+tan(d*x+c)))+A*b^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c
)))+B*b^2*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 228, normalized size = 1.46

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{2}+32\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aab-3\,Aa^{2}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-24\,Bab\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,Ab^{2}\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+48\,Bb^{2}\tan\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 + 32*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a*b - 3*A*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 24*B*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 12*A*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*B*b^2*tan(d*x +
 c))/d
```

Fricas [A]
time = 0.37, size = 180, normalized size = 1.15

$$\frac{3\left(3\,Aa^{2}+8\,Bab+4\,Ab^{2}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,Aa^{2}+8\,Bab+4\,Ab^{2}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\left(2\,Ba^{2}+4\,Aab+3\,Bb^{2}\right)\cos\left(dx+c\right)^{3}+6\,Aa^{2}+3\left(3\,Aa^{2}+8\,Bab+4\,Ab^{2}\right)\cos\left(dx+c\right)^{2}+8\left(Ba^{2}+2\,Aab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
- 3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2
*(8*(2*B*a^2 + 4*A*a*b + 3*B*b^2)*cos(d*x + c)^3 + 6*A*a^2 + 3*(3*A*a^2 + 8
*B*a*b + 4*A*b^2)*cos(d*x + c)^2 + 8*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(146) = 292.
time = 0.48, size = 478, normalized size = 3.06

$$\frac{3\,(3\,Aa^2+8\,Bab+4\,Ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(3\,Aa^2+8\,Bab+4\,Ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\,\left(15\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+24\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+12\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+9\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+40\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+80\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-24\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+72\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-80\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+48\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*
A*a^2*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b*t
an(1/2*d*x + 1/2*c)^7 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^2*tan(1/2*
d*x + 1/2*c)^7 - 24*B*b^2*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^2*tan(1/2*d*x + 1/
2*c)^5 + 40*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 80*A*a*b*tan(1/2*d*x + 1/2*c)^5
- 24*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*B*
b^2*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^2*tan(
1/2*d*x + 1/2*c)^3 - 80*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x
 + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*B*b^2*tan(1/2*d*x + 1/2*
c)^3 + 15*A*a^2*tan(1/2*d*x + 1/2*c) + 24*B*a^2*tan(1/2*d*x + 1/2*c) + 48*A
*a*b*tan(1/2*d*x + 1/2*c) + 24*B*a*b*tan(1/2*d*x + 1/2*c) + 12*A*b^2*tan(1/
2*d*x + 1/2*c) + 24*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^4)/d
```

Mupad [B]
time = 3.87, size = 314, normalized size = 2.01

$$\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,A\,a^2}{8}+B\,a\,b+\frac{A\,b^2}{2}\right)}{\frac{3\,A\,a^2}{2}+4\,B\,a\,b+2\,A\,b^2}\right)\,\left(\frac{3\,A\,a^2}{4}+2\,B\,a\,b+A\,b^2\right)}{d}+\frac{\left(\frac{5\,A\,a^2}{4}+A\,b^2-2\,B\,a^2-2\,B\,b^2-4\,A\,a\,b+2\,B\,a\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{3\,A\,a^2}{4}-A\,b^2+\frac{10\,B\,a^2}{3}+6\,B\,b^2+\frac{20\,A\,a\,b}{3}-2\,B\,a\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{3\,A\,a^2}{4}-A\,b^2-\frac{10\,B\,a^2}{3}-6\,B\,b^2-\frac{20\,A\,a\,b}{3}-2\,B\,a\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{5\,A\,a^2}{4}+A\,b^2+2\,B\,a^2+2\,B\,b^2+4\,A\,a\,b+2\,B\,a\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((3*A*a^2)/8 + (A*b^2)/2 + B*a*b))/((3*A*a^2)/
2 + 2*A*b^2 + 4*B*a*b))*((3*A*a^2)/4 + A*b^2 + 2*B*a*b))/d + (tan(c/2 + (d*
x)/2)^7*((5*A*a^2)/4 + A*b^2 - 2*B*a^2 - 2*B*b^2 - 4*A*a*b + 2*B*a*b) - tan
(c/2 + (d*x)/2)^3*(A*b^2 - (3*A*a^2)/4 + (10*B*a^2)/3 + 6*B*b^2 + (20*A*a*b
)/3 + 2*B*a*b) + tan(c/2 + (d*x)/2)^5*((3*A*a^2)/4 - A*b^2 + (10*B*a^2)/3 +
 6*B*b^2 + (20*A*a*b)/3 - 2*B*a*b) + tan(c/2 + (d*x)/2)*((5*A*a^2)/4 + A*b^
2 + 2*B*a^2 + 2*B*b^2 + 4*A*a*b + 2*B*a*b))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*
tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.231 $\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=269

$$\frac{1}{16}\left(8a^3A+18aAb^2+18a^2bB+5b^3B\right)x+\frac{(15a^2Ab+4Ab^3+5a^3B+12ab^2B)\sin(c+dx)}{5d}+\frac{(8a^3A+18aAb$$

```
[Out] 1/16*(8*A*a^3+18*A*a*b^2+18*B*a^2*b+5*B*b^3)*x+1/5*(15*A*a^2*b+4*A*b^3+5*B*
a^3+12*B*a*b^2)*sin(d*x+c)/d+1/16*(8*A*a^3+18*A*a*b^2+18*B*a^2*b+5*B*b^3)*c
os(d*x+c)*sin(d*x+c)/d+1/24*b*(18*A*a*b+14*B*a^2+5*B*b^2)*cos(d*x+c)^3*sin(
d*x+c)/d+1/15*b^2*(3*A*b+4*B*a)*cos(d*x+c)^4*sin(d*x+c)/d+1/6*b*B*cos(d*x+c
)^3*(a+b*cos(d*x+c))^2*sin(d*x+c)/d-1/15*(15*A*a^2*b+4*A*b^3+5*B*a^3+12*B*a
*b^2)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.34, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3069, 3112, 3102, 2827, 2715, 8, 2713}

$$\frac{b(14a^2B+18aAb+5b^2B)\sin(c+dx)\cos^3(c+dx)}{24d}-\frac{(5a^3B+15a^2Ab+12ab^2B+4Ab^3)\sin^3(c+dx)}{15d}+\frac{(5a^3B+15a^2Ab+12ab^2B+4Ab^3)\sin(c+dx)}{5d}+\frac{(8a^3A+18a^2bB+18aAb^2+5b^3B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(8a^3A+18a^2bB+18aAb^2+5b^3B\right)+\frac{b^2(4aB+3Ab)\sin(c+dx)\cos^4(c+dx)}{15d}+\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((8*a^3*A + 18*a*A*b^2 + 18*a^2*b*B + 5*b^3*B)*x)/16 + ((15*a^2*A*b + 4*A*b
^3 + 5*a^3*B + 12*a*b^2*B)*Sin[c + d*x])/(5*d) + ((8*a^3*A + 18*a*A*b^2 + 1
8*a^2*b*B + 5*b^3*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (b*(18*a*A*b + 14*
a^2*B + 5*b^2*B)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*(3*A*b + 4*a*B)
*Cos[c + d*x]^4*Sin[c + d*x])/(15*d) + (b*B*Cos[c + d*x]^3*(a + b*Cos[c + d
*x])^2*Sin[c + d*x])/(6*d) - ((15*a^2*A*b + 4*A*b^3 + 5*a^3*B + 12*a*b^2*B)
*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{bB\cos^3(c+dx)(a+b\cos(c+dx))^2\sin(c+dx}{6d}$$
$$= \frac{b^2(3Ab+4aB)\cos^4(c+dx)\sin(c+dx)}{15d} + \frac{bB}{}$$
$$= \frac{b(18aAb+14a^2B+5b^2B)\cos^3(c+dx)\sin(c}{24d}$$
$$= \frac{b(18aAb+14a^2B+5b^2B)\cos^3(c+dx)\sin(c}{24d}$$
$$= \frac{(8a^3A+18aAb^2+18a^2bB+5b^3B)\cos(c+d}{16d}$$
$$= \frac{1}{16}\left(8a^3A+18aAb^2+18a^2bB+5b^3B\right)x + \frac{(1}{}$$

Mathematica [A]
time = 0.71, size = 289, normalized size = 1.07

$$\frac{480a^3Ac+1080aAb^2c+1080a^2bBc+300b^3Bc+480a^3Adx+1080aAb^2dx+1080a^2bBdx+300b^3Bdx+120(18a^2Ab+5Ab^3+6a^3B+15ab^2B)\sin(c+dx)+15(16a^3A+48aAb^2+48a^2bB+15b^3B)\sin(2(c+dx))+240a^2Ab\sin(3(c+dx))+100Ab^3\sin(3(c+dx))+80a^3B\sin(3(c+dx))+300ab^2B\sin(3(c+dx))+90aAb^2\sin(4(c+dx))+90a^2bB\sin(4(c+dx))+45b^3B\sin(4(c+dx))+12Ab^3\sin(5(c+dx))+36ab^2B\sin(5(c+dx))+5b^3B\sin(6(c+dx))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (480*a^3*A*c + 1080*a*A*b^2*c + 1080*a^2*b*B*c + 300*b^3*B*c + 480*a^3*A*d*
x + 1080*a*A*b^2*d*x + 1080*a^2*b*B*d*x + 300*b^3*B*d*x + 120*(18*a^2*A*b +
 5*A*b^3 + 6*a^3*B + 15*a*b^2*B)*Sin[c + d*x] + 15*(16*a^3*A + 48*a*A*b^2 +
 48*a^2*b*B + 15*b^3*B)*Sin[2*(c + d*x)] + 240*a^2*A*b*Sin[3*(c + d*x)] + 1
00*A*b^3*Sin[3*(c + d*x)] + 80*a^3*B*Sin[3*(c + d*x)] + 300*a*b^2*B*Sin[3*(
c + d*x)] + 90*a*A*b^2*Sin[4*(c + d*x)] + 90*a^2*b*B*Sin[4*(c + d*x)] + 45*
b^3*B*Sin[4*(c + d*x)] + 12*A*b^3*Sin[5*(c + d*x)] + 36*a*b^2*B*Sin[5*(c +
d*x)] + 5*b^3*B*Sin[6*(c + d*x)])/(960*d)
```

Maple [A]
time = 0.22, size = 270, normalized size = 1.00 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+1/3*a^3*B*(cos(d*x+c)^
2+2)*sin(d*x+c)+A*a^2*b*(cos(d*x+c)^2+2)*sin(d*x+c)+3*a^2*b*B*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*A*a*b^2*(1/4*(cos(d*x+c)^
3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3/5*B*a*b^2*(8/3+cos(d*x+c)^4+4
/3*cos(d*x+c)^2)*sin(d*x+c)+1/5*A*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*s
```

```
in(d*x+c)+b^3*B*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*
x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.27, size = 266, normalized size = 0.99

$$\frac{240 (2 dx + 2 c + \sin(2 dx + 2 c))Aa^3 - 320 (\sin(dx + c)^3 - 3 \sin(dx + c))Ba^3 - 960 (\sin(dx + c)^3 - 3 \sin(dx + c))Aa^2b + 90 (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c))Ba^2b + 90 (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c))Aab^2 + 192 (3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c))Bab^2 + 64 (3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c))Ab^3 - 5 (4 \sin(2 dx + 2 c)^3 - 60 dx - 60 c - 9 \sin(4 dx + 4 c) - 48 \sin(2 dx + 2 c))Bb^3}{960 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 - 320*(sin(d*x + c)^3 - 3
*sin(d*x + c))*B*a^3 - 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2*b + 90*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2*b + 90*(12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b^2 + 192*(3*sin(d*x +
 c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a*b^2 + 64*(3*sin(d*x + c)^5
 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*b^3 - 5*(4*sin(2*d*x + 2*c)^3 - 6
0*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*b^3)/d
```

Fricas [A]
time = 0.37, size = 211, normalized size = 0.78

$$\frac{15 (8 Aa^3 + 18 Ba^2b + 18 Aab^2 + 5 Bb^3)dx + (40 Bb^3 \cos(dx + c)^5 + 48 (3 Bab^2 + Ab^3) \cos(dx + c)^4 + 160 Ba^3 + 480 Aa^2b + 384 Bab^2 + 128 Ab^3 + 10 (18 Ba^2b + 18 Aab^2 + 5 Bb^3) \cos(dx + c)^3 + 16 (5 Ba^3 + 15 Aa^2b + 12 Bab^2 + 4 Ab^3) \cos(dx + c)^2 + 15 (8 Aa^3 + 18 Ba^2b + 18 Aab^2 + 5 Bb^3) \cos(dx + c)) \sin(dx + c)}{240 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 5*B*b^3)*d*x + (40*B*b^3*cos
(d*x + c)^5 + 48*(3*B*a*b^2 + A*b^3)*cos(d*x + c)^4 + 160*B*a^3 + 480*A*a^2
*b + 384*B*a*b^2 + 128*A*b^3 + 10*(18*B*a^2*b + 18*A*a*b^2 + 5*B*b^3)*cos(d
*x + c)^3 + 16*(5*B*a^3 + 15*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x + c)^2
 + 15*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 5*B*b^3)*cos(d*x + c))*sin(d*x +
 c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(277) = 554$.
time = 0.54, size = 721, normalized size = 2.68

$$\begin{cases} \frac{Aa^3x\sin^2(c+dx)}{2} + \frac{Aa^3x\cos^2(c+dx)}{2} + \frac{Aa^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{2Aa^2b\sin^3(c+dx)}{d} + \frac{3Aa^2b\sin(c+dx)\cos^2(c+dx)}{d} + \frac{9Aab^2x\sin^4(c+dx)}{8} + \frac{9Aab^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{9Aab^2x\cos^4(c+dx)}{8} + \frac{9Aab^2\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{15Aab^2\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{8Ab^3\sin^5(c+dx)}{15d} + \frac{4Ab^3\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{Ab^3\sin(c+dx)\cos^4(c+dx)}{d} + \frac{2Ba^3\sin^3(c+dx)}{3d} + \frac{Ba^3\sin(c+dx)\cos^2(c+dx)}{d} + \frac{9Ba^2bx\sin^4(c+dx)}{8} + \frac{9Ba^2bx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{9Ba^2bx\cos^4(c+dx)}{8} + \frac{9Ba^2b\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{15Ba^2b\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{8Bab^2\sin^5(c+dx)}{5d} + \frac{4Bab^2\sin^3(c+dx)\cos^2(c+dx)}{d} + \frac{3Bab^2\sin(c+dx)\cos^4(c+dx)}{d} + \frac{5Bb^3x\sin^6(c+dx)}{16} + \frac{15Bb^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15Bb^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5Bb^3x\cos^6(c+dx)}{16} + \frac{5Bb^3\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{5Bb^3\sin^3(c+dx)\cos^3(c+dx)}{6d} + \frac{11Bb^3\sin(c+dx)\cos^5(c+dx)}{16d} & \text{for}\ d \neq 0 \\ x(A + B\cos(c))(a + b\cos(c))^3\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*x*sin(c + d*x)**2/2 + A*a**3*x*cos(c + d*x)**2/2 + A*a**3
*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*a**2*b*sin(c + d*x)**3/d + 3*A*a**2*
```

```
b*sin(c + d*x)*cos(c + d*x)**2/d + 9*A*a*b**2*x*sin(c + d*x)**4/8 + 9*A*a*b
**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 9*A*a*b**2*x*cos(c + d*x)**4/8 +
9*A*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 15*A*a*b**2*sin(c + d*x)*co
s(c + d*x)**3/(8*d) + 8*A*b**3*sin(c + d*x)**5/(15*d) + 4*A*b**3*sin(c + d*
x)**3*cos(c + d*x)**2/(3*d) + A*b**3*sin(c + d*x)*cos(c + d*x)**4/d + 2*B*a
**3*sin(c + d*x)**3/(3*d) + B*a**3*sin(c + d*x)*cos(c + d*x)**2/d + 9*B*a**
2*b*x*sin(c + d*x)**4/8 + 9*B*a**2*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
9*B*a**2*b*x*cos(c + d*x)**4/8 + 9*B*a**2*b*sin(c + d*x)**3*cos(c + d*x)/(8
*d) + 15*B*a**2*b*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*B*a*b**2*sin(c + d
*x)**5/(5*d) + 4*B*a*b**2*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*B*a*b**2*si
n(c + d*x)*cos(c + d*x)**4/d + 5*B*b**3*x*sin(c + d*x)**6/16 + 15*B*b**3*x*
sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*B*b**3*x*sin(c + d*x)**2*cos(c + d*
x)**4/16 + 5*B*b**3*x*cos(c + d*x)**6/16 + 5*B*b**3*sin(c + d*x)**5*cos(c +
 d*x)/(16*d) + 5*B*b**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*B*b**3*s
in(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos
(c))**3*cos(c)**2, True))
```

Giac [A]
time = 0.45, size = 230, normalized size = 0.86

$$\frac{Bb^3\sin(6\,dx+6\,c)}{192\,d}+\frac{1}{16}\left(8\,Aa^3+18\,Ba^2b+18\,Aab^2+5\,Bb^3\right)x+\frac{(3\,Bab^2+Ab^3)\sin(5\,dx+5\,c)}{80\,d}+\frac{3\,(2\,Ba^2b+2\,Aab^2+Bb^3)\sin(4\,dx+4\,c)}{64\,d}+\frac{(4\,Ba^3+12\,Aa^2b+15\,Bab^2+5\,Ab^3)\sin(3\,dx+3\,c)}{48\,d}+\frac{(16\,Aa^3+48\,Ba^2b+48\,Aab^2+15\,Bb^3)\sin(2\,dx+2\,c)}{64\,d}+\frac{(6\,Ba^3+18\,Aa^2b+15\,Bab^2+5\,Ab^3)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*B*b^3*sin(6*d*x + 6*c)/d + 1/16*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 +
5*B*b^3)*x + 1/80*(3*B*a*b^2 + A*b^3)*sin(5*d*x + 5*c)/d + 3/64*(2*B*a^2*b
+ 2*A*a*b^2 + B*b^3)*sin(4*d*x + 4*c)/d + 1/48*(4*B*a^3 + 12*A*a^2*b + 15*B
*a*b^2 + 5*A*b^3)*sin(3*d*x + 3*c)/d + 1/64*(16*A*a^3 + 48*B*a^2*b + 48*A*a
*b^2 + 15*B*b^3)*sin(2*d*x + 2*c)/d + 1/8*(6*B*a^3 + 18*A*a^2*b + 15*B*a*b^
2 + 5*A*b^3)*sin(d*x + c)/d
```

Mupad [B]
time = 1.11, size = 352, normalized size = 1.31

$$\frac{A\,a^3\,x}{2}+\frac{5\,B\,b^3\,x}{16}+\frac{9\,A\,a\,b^2\,x}{8}+\frac{9\,B\,a^2\,b\,x}{8}+\frac{5\,A\,b^3\,\sin(c+d\,x)}{8\,d}+\frac{3\,B\,a^3\,\sin(c+d\,x)}{4\,d}+\frac{A\,a^3\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{5\,A\,b^3\,\sin(3\,c+3\,d\,x)}{48\,d}+\frac{B\,a^3\,\sin(3\,c+3\,d\,x)}{12\,d}+\frac{A\,b^3\,\sin(5\,c+5\,d\,x)}{80\,d}+\frac{15\,B\,b^3\,\sin(2\,c+2\,d\,x)}{64\,d}+\frac{3\,B\,b^3\,\sin(4\,c+4\,d\,x)}{64\,d}+\frac{B\,b^3\,\sin(6\,c+6\,d\,x)}{192\,d}+\frac{3\,A\,a\,b^2\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{A\,a^2\,b\,\sin(3\,c+3\,d\,x)}{4\,d}+\frac{3\,A\,a\,b^2\,\sin(4\,c+4\,d\,x)}{32\,d}+\frac{3\,B\,a^2\,b\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{5\,B\,a\,b^2\,\sin(3\,c+3\,d\,x)}{16\,d}+\frac{3\,B\,a^2\,b\,\sin(4\,c+4\,d\,x)}{32\,d}+\frac{3\,B\,a\,b^2\,\sin(5\,c+5\,d\,x)}{80\,d}+\frac{9\,A\,a^2\,b\,\sin(c+d\,x)}{4\,d}+\frac{15\,B\,a\,b^2\,\sin(c+d\,x)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

```
[Out] (A*a^3*x)/2 + (5*B*b^3*x)/16 + (9*A*a*b^2*x)/8 + (9*B*a^2*b*x)/8 + (5*A*b^3
*sin(c + d*x))/(8*d) + (3*B*a^3*sin(c + d*x))/(4*d) + (A*a^3*sin(2*c + 2*d*
x))/(4*d) + (5*A*b^3*sin(3*c + 3*d*x))/(48*d) + (B*a^3*sin(3*c + 3*d*x))/(1
2*d) + (A*b^3*sin(5*c + 5*d*x))/(80*d) + (15*B*b^3*sin(2*c + 2*d*x))/(64*d)
 + (3*B*b^3*sin(4*c + 4*d*x))/(64*d) + (B*b^3*sin(6*c + 6*d*x))/(192*d) + (
```

```
3*A*a*b^2*sin(2*c + 2*d*x))/(4*d) + (A*a^2*b*sin(3*c + 3*d*x))/(4*d) + (3*A
*a*b^2*sin(4*c + 4*d*x))/(32*d) + (3*B*a^2*b*sin(2*c + 2*d*x))/(4*d) + (5*B
*a*b^2*sin(3*c + 3*d*x))/(16*d) + (3*B*a^2*b*sin(4*c + 4*d*x))/(32*d) + (3*
B*a*b^2*sin(5*c + 5*d*x))/(80*d) + (9*A*a^2*b*sin(c + d*x))/(4*d) + (15*B*a
*b^2*sin(c + d*x))/(8*d)
```

3.232 $\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=243

$$\frac{1}{8}\left(12a^2Ab+3Ab^3+4a^3B+9ab^2B\right)x+\frac{\left(15a^3Ab+60aAb^3-3a^4B+52a^2b^2B+16b^4B\right)\sin(c+dx)}{30bd}+\frac{(30a^2$$

```
[Out] 1/8*(12*A*a^2*b+3*A*b^3+4*B*a^3+9*B*a*b^2)*x+1/30*(15*A*a^3*b+60*A*a*b^3-3*
B*a^4+52*B*a^2*b^2+16*B*b^4)*sin(d*x+c)/b/d+1/120*(30*A*a^2*b+45*A*b^3-6*B*
a^3+71*B*a*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/60*(15*A*a*b-3*B*a^2+16*B*b^2)*(a
+b*cos(d*x+c))^2*sin(d*x+c)/b/d+1/20*(5*A*b-B*a)*(a+b*cos(d*x+c))^3*sin(d*x
+c)/b/d+1/5*B*(a+b*cos(d*x+c))^4*sin(d*x+c)/b/d
```

Rubi [A]
time = 0.23, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3047, 3102, 2832, 2813}

$$\frac{(-3a^2B+15aAb+16b^2B)\sin(c+dx)(a+b\cos(c+dx))^2}{60bd}+\frac{(-6a^3B+30a^2Ab+71ab^2B+45Ab^3)\sin(c+dx)\cos(c+dx)}{120d}+\frac{1}{8}x\left(4a^3B+12a^2Ab+9ab^2B+3Ab^3\right)+\frac{(-3a^4B+15a^3Ab+52a^2b^2B+60aAb^3+16b^4B)\sin(c+dx)}{30bd}+\frac{(5Ab-aB)\sin(c+dx)(a+b\cos(c+dx))^3}{20bd}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*x)/8 + ((15*a^3*A*b + 60*a*A*
b^3 - 3*a^4*B + 52*a^2*b^2*B + 16*b^4*B)*Sin[c + d*x])/(30*b*d) + ((30*a^2*
A*b + 45*A*b^3 - 6*a^3*B + 71*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(120*d) +
 ((15*a*A*b - 3*a^2*B + 16*b^2*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*
b*d) + ((5*A*b - a*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(20*b*d) + (B*(a
 + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*b*d)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
```

```
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^3\left(A\cos(c+dx)+B\cos^2(c\right.$$
$$= \frac{B(a+b\cos(c+dx))^4\sin(c+dx)}{5bd}+\frac{\int(a+b\cos}{}$$
$$= \frac{(5Ab-aB)(a+b\cos(c+dx))^3\sin(c+dx)}{20bd}+\frac{B}{}$$
$$= \frac{(15aAb-3a^2B+16b^2B)\left(a+b\cos(c+dx)\right)^2\sin}{60bd}$$
$$= \frac{1}{8}\left(12a^2Ab+3Ab^3+4a^3B+9ab^2B\right)x+\frac{(15a^3A}{}$$

Mathematica [A]
time = 0.76, size = 176, normalized size = 0.72

$$\frac{60(12a^2Ab+3Ab^3+4a^3B+9ab^2B)(c+dx)+60(8a^3A+18aAb^2+18a^2bB+5b^3B)\sin(c+dx)+120(3a^2Ab+Ab^3+a^3B+3ab^2B)\sin(2(c+dx))+10b(12aAb+12a^2B+5b^2B)\sin(3(c+dx))+15b^2(Ab+3aB)\sin(4(c+dx))+6b^3B\sin(5(c+dx))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (60*(12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*(c + d*x) + 60*(8*a^3*A +
18*a*A*b^2 + 18*a^2*b*B + 5*b^3*B)*Sin[c + d*x] + 120*(3*a^2*A*b + A*b^3 +
a^3*B + 3*a*b^2*B)*Sin[2*(c + d*x)] + 10*b*(12*a*A*b + 12*a^2*B + 5*b^2*B)*
```

```
Sin[3*(c + d*x)] + 15*b^2*(A*b + 3*a*B)*Sin[4*(c + d*x)] + 6*b^3*B*Sin[5*(c
 + d*x)])/(480*d)
```

Maple [A]
time = 0.17, size = 227, normalized size = 0.93

method	result
derivativedivides	$\frac{A\,a^3\sin(dx+c)+a^3B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3A\,a^2b\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2bB\left(\cos^2(dx+c)+2\right)\sin(d}{}$
default	$\frac{A\,a^3\sin(dx+c)+a^3B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3A\,a^2b\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2bB\left(\cos^2(dx+c)+2\right)\sin(d}{}$
risch	$\frac{3xA\,a^2b}{2}+\frac{3xA\,b^3}{8}+\frac{a^3Bx}{2}+\frac{9xBa\,b^2}{8}+\frac{a^3A\sin(dx+c)}{d}+\frac{9\sin(dx+c)Aa\,b^2}{4d}+\frac{9\sin(dx+c)a^2bB}{4d}+\frac{5\sin(dx-}{8c}$
norman	$\frac{\left(\frac{3}{2}A\,a^2b+\frac{3}{8}A\,b^3+\frac{1}{2}a^3B+\frac{9}{8}Ba\,b^2\right)x+\left(15A\,a^2b+\frac{15}{4}A\,b^3+5a^3B+\frac{45}{4}Ba\,b^2\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(15A\,a^2b+\frac{15}{4}A\,b^3+5a^3B\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^3*sin(d*x+c)+a^3*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+3*A*a
^2*b*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+a^2*b*B*(cos(d*x+c)^2+2)*sin
(d*x+c)+A*a*b^2*(cos(d*x+c)^2+2)*sin(d*x+c)+3*B*a*b^2*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*
x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5*b^3*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 217, normalized size = 0.89

$$\frac{120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^3+360\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2b-480\,(\sin(dx+c)^3-3\,\sin(dx+c))Ba^2b-480\,(\sin(dx+c)^3-3\,\sin(dx+c))Aab^2+45\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Bab^2+15\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Ab^3+32\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))Bb^3+480\,Aa^3\sin(dx+c)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 + 360*(2*d*x + 2*c + sin(
2*d*x + 2*c))*A*a^2*b - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2*b - 480
*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b^2 + 45*(12*d*x + 12*c + sin(4*d*x
+ 4*c) + 8*sin(2*d*x + 2*c))*B*a*b^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c)
 + 8*sin(2*d*x + 2*c))*A*b^3 + 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 1
5*sin(d*x + c))*B*b^3 + 480*A*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 174, normalized size = 0.72

$$\frac{15\,(4\,Ba^3+12\,Aa^2b+9\,Bab^2+3\,Ab^3)dx+\left(24\,Bb^3\cos\left(dx+c\right)^4+120\,Aa^3+240\,Ba^2b+240\,Aab^2+64\,Bb^3+30\,(3\,Bab^2+Ab^3)\cos\left(dx+c\right)^3+8\,(15\,Ba^2b+15\,Aab^2+4\,Bb^3)\cos\left(dx+c\right)^2+15\,(4\,Ba^3+12\,Aa^2b+9\,Bab^2+3\,Ab^3)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/120*(15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*d*x + (24*B*b^3*cos(
d*x + c)^4 + 120*A*a^3 + 240*B*a^2*b + 240*A*a*b^2 + 64*B*b^3 + 30*(3*B*a*b
^2 + A*b^3)*cos(d*x + c)^3 + 8*(15*B*a^2*b + 15*A*a*b^2 + 4*B*b^3)*cos(d*x
+ c)^2 + 15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(241) = 482.
time = 0.38, size = 551, normalized size = 2.27

$$\begin{cases}\frac{Aa^3\sin(c+dx)}{d}+\frac{3Aa^2bx\sin^2(c+dx)}{2}+\frac{3Aa^2bx\cos^2(c+dx)}{2}+\frac{3Aa^2b\sin(c+dx)\cos(c+dx)}{2d}+\frac{2Aab^2\sin^3(c+dx)}{d}+\frac{3Aab^2\sin(c+dx)\cos^2(c+dx)}{d}+\frac{3Ab^3x\sin^4(c+dx)}{8}+\frac{3Ab^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3Ab^3x\cos^4(c+dx)}{8}+\frac{3Ab^3\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5Ab^3\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{Ba^3x\sin^2(c+dx)}{2}+\frac{Ba^3x\cos^2(c+dx)}{2}+\frac{Ba^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{2Ba^2b\sin^3(c+dx)}{d}+\frac{3Ba^2b\sin(c+dx)\cos^2(c+dx)}{d}+\frac{9Bab^2x\sin^4(c+dx)}{8}+\frac{9Bab^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{9Bab^2x\cos^4(c+dx)}{8}+\frac{9Bab^2\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{15Bab^2\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{8Bb^3\sin^5(c+dx)}{15d}+\frac{4Bb^3\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{Bb^3\sin(c+dx)\cos^4(c+dx)}{d} & \text{for}\ d\neq 0\\ x(A+B\cos(c))\,(a+b\cos(c))^3\cos(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*sin(c + d*x)/d + 3*A*a**2*b*x*sin(c + d*x)**2/2 + 3*A*a**
2*b*x*cos(c + d*x)**2/2 + 3*A*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*
a*b**2*sin(c + d*x)**3/d + 3*A*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*
b**3*x*sin(c + d*x)**4/8 + 3*A*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3
*A*b**3*x*cos(c + d*x)**4/8 + 3*A*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) +
 5*A*b**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**3*x*sin(c + d*x)**2/2 +
 B*a**3*x*cos(c + d*x)**2/2 + B*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*
a**2*b*sin(c + d*x)**3/d + 3*B*a**2*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*B*
a*b**2*x*sin(c + d*x)**4/8 + 9*B*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4
 + 9*B*a*b**2*x*cos(c + d*x)**4/8 + 9*B*a*b**2*sin(c + d*x)**3*cos(c + d*x)
/(8*d) + 15*B*a*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*B*b**3*sin(c +
d*x)**5/(15*d) + 4*B*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + B*b**3*si
n(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))**
3*cos(c), True))
```

Giac [A]
time = 0.45, size = 188, normalized size = 0.77

$$\frac{Bb^3\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{1}{8}\left(4\,Ba^3+12\,Aa^2b+9\,Bab^2+3\,Ab^3\right)x+\frac{(3\,Bab^2+Ab^3)\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{(12\,Ba^2b+12\,Aab^2+5\,Bb^3)\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{(Ba^3+3\,Aa^2b+3\,Bab^2+Ab^3)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{(8\,Aa^3+18\,Ba^2b+18\,Aab^2+5\,Bb^3)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/80*B*b^3*sin(5*d*x + 5*c)/d + 1/8*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A
*b^3)*x + 1/32*(3*B*a*b^2 + A*b^3)*sin(4*d*x + 4*c)/d + 1/48*(12*B*a^2*b +
12*A*a*b^2 + 5*B*b^3)*sin(3*d*x + 3*c)/d + 1/4*(B*a^3 + 3*A*a^2*b + 3*B*a*b
^2 + A*b^3)*sin(2*d*x + 2*c)/d + 1/8*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 5
*B*b^3)*sin(d*x + c)/d
```

Mupad [B]
time = 0.78, size = 277, normalized size = 1.14

$$\frac{3\,A\,b^3\,x}{8}+\frac{B\,a^3\,x}{2}+\frac{3\,A\,a^2\,b\,x}{2}+\frac{9\,B\,a\,b^2\,x}{8}+\frac{A\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{5\,B\,b^3\,\sin\left(c+d\,x\right)}{8\,d}+\frac{A\,b^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,b^3\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{5\,B\,b^3\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}+\frac{B\,b^3\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}+\frac{3\,A\,a^2\,b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,a\,b^2\,\sin\left(3\,c+3\,d\,x\right)}{4\,d}+\frac{3\,B\,a\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a^2\,b\,\sin\left(3\,c+3\,d\,x\right)}{4\,d}+\frac{3\,B\,a\,b^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{9\,A\,a\,b^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{9\,B\,a^2\,b\,\sin\left(c+d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

```
[Out] (3*A*b^3*x)/8 + (B*a^3*x)/2 + (3*A*a^2*b*x)/2 + (9*B*a*b^2*x)/8 + (A*a^3*si
n(c + d*x))/d + (5*B*b^3*sin(c + d*x))/(8*d) + (A*b^3*sin(2*c + 2*d*x))/(4*
d) + (B*a^3*sin(2*c + 2*d*x))/(4*d) + (A*b^3*sin(4*c + 4*d*x))/(32*d) + (5*
B*b^3*sin(3*c + 3*d*x))/(48*d) + (B*b^3*sin(5*c + 5*d*x))/(80*d) + (3*A*a^2
*b*sin(2*c + 2*d*x))/(4*d) + (A*a*b^2*sin(3*c + 3*d*x))/(4*d) + (3*B*a*b^2*
sin(2*c + 2*d*x))/(4*d) + (B*a^2*b*sin(3*c + 3*d*x))/(4*d) + (3*B*a*b^2*sin
(4*c + 4*d*x))/(32*d) + (9*A*a*b^2*sin(c + d*x))/(4*d) + (9*B*a^2*b*sin(c +
 d*x))/(4*d)
```

3.233 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=171

$$\frac{1}{8}\left(8a^3A+12aAb^2+12a^2bB+3b^3B\right)x+\frac{\left(16a^2Ab+4Ab^3+3a^3B+12ab^2B\right)\sin(c+dx)}{6d}+\frac{b(20aAb+6a^2B+$$

```
[Out] 1/8*(8*A*a^3+12*A*a*b^2+12*B*a^2*b+3*B*b^3)*x+1/6*(16*A*a^2*b+4*A*b^3+3*B*a
^3+12*B*a*b^2)*sin(d*x+c)/d+1/24*b*(20*A*a*b+6*B*a^2+9*B*b^2)*cos(d*x+c)*si
n(d*x+c)/d+1/12*(4*A*b+3*B*a)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/4*B*(a+b*co
s(d*x+c))^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2832, 2813}

$$\frac{b(6a^2B+20aAb+9b^2B)\sin(c+dx)\cos(c+dx)}{24d}+\frac{(3a^3B+16a^2Ab+12ab^2B+4Ab^3)\sin(c+dx)}{6d}+\frac{1}{8}x(8a^3A+12a^2bB+12aAb^2+3b^3B)+\frac{(3aB+4Ab)\sin(c+dx)(a+b\cos(c+dx))^2}{12d}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*x)/8 + ((16*a^2*A*b + 4*A*b^
3 + 3*a^3*B + 12*a*b^2*B)*Sin[c + d*x])/(6*d) + (b*(20*a*A*b + 6*a^2*B + 9*
b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*A*b + 3*a*B)*(a + b*Cos[c +
d*x])^2*Sin[c + d*x])/(12*d) + (B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d
)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{B(a+b\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{4}\int (a+b\cos(c+dx)$$

$$= \frac{(4Ab+3aB)(a+b\cos(c+dx))^2\sin(c+dx)}{12d} + \frac{B(a+b\,\mathrm{co}}{}$$

$$= \frac{1}{8}\left(8a^3A+12aAb^2+12a^2bB+3b^3B\right)x + \frac{(16a^2Ab+4Ab^3}{}$$

Mathematica [A]
time = 0.45, size = 140, normalized size = 0.82

$$\frac{12(8a^3A+12aAb^2+12a^2bB+3b^3B)(c+dx)+24(12a^2Ab+3Ab^3+4a^3B+9ab^2B)\sin(c+dx)+24b(3aAb+3a^2B+b^2B)\sin(2(c+dx))+8b^2(Ab+3aB)\sin(3(c+dx))+3b^3B\sin(4(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (12*(8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*(c + d*x) + 24*(12*a^2*A*
b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*Sin[c + d*x] + 24*b*(3*a*A*b + 3*a^2*B +
 b^2*B)*Sin[2*(c + d*x)] + 8*b^2*(A*b + 3*a*B)*Sin[3*(c + d*x)] + 3*b^3*B*S
in[4*(c + d*x)])/(96*d)
```

Maple [A]
time = 0.13, size = 180, normalized size = 1.05

method	result
derivativedivides	$\frac{b^3B\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ab^3\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+Bab^2\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{}$
default	$\frac{b^3B\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ab^3\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+Bab^2\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{}$
risch	$a^3xA+\frac{3xAab^2}{2}+\frac{3xa^2bB}{2}+\frac{3b^3Bx}{8}+\frac{3\sin(dx+c)Aa^2b}{d}+\frac{3\sin(dx+c)Ab^3}{4d}+\frac{a^3B\sin(dx+c)}{d}+\frac{9\sin(dx}{4}$
norman	$\frac{(Aa^3+\frac{3}{2}Aab^2+\frac{3}{2}a^2bB+\frac{3}{8}b^3B)x+(Aa^3+\frac{3}{2}Aab^2+\frac{3}{2}a^2bB+\frac{3}{8}b^3B)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(4Aa^3+6Aab^2+6a^2bB+\frac{3}{2}b^3B)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*A*b^3*(cos(d*x+c)^2+2)*sin(d*x+c)+B*a*b^2*(cos(d*x+c)^2+2)*sin(d*x+c)+3*A*
a*b^2*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*B*(1/2*sin(d*x+c)*c
```

```
os(d*x+c)+1/2*d*x+1/2*c)+3*A*a^2*b*sin(d*x+c)+a^3*B*sin(d*x+c)+A*a^3*(d*x+c
))
```

Maxima [A]
time = 0.28, size = 171, normalized size = 1.00

$$\frac{96\,(dx+c)Aa^3+72\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba^2b+72\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aab^2-96\,\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Bab^2-32\,\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Ab^3+3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))Bb^3+96\,Ba^3\sin\left(dx+c\right)+288\,Aa^2b\sin\left(dx+c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(96*(d*x + c)*A*a^3 + 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2*b + 72
*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b^2 - 96*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*B*a*b^2 - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b^3 + 3*(12*d*x + 12
*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b^3 + 96*B*a^3*sin(d*x + c) +
 288*A*a^2*b*sin(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 136, normalized size = 0.80

$$\frac{3\,(8\,Aa^3+12\,Ba^2b+12\,Aab^2+3\,Bb^3)dx+\left(6\,Bb^3\cos\left(dx+c\right)^3+24\,Ba^3+72\,Aa^2b+48\,Bab^2+16\,Ab^3+8\,(3\,Bab^2+Ab^3)\cos\left(dx+c\right)^2+9\,(4\,Ba^2b+4\,Aab^2+Bb^3)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*d*x + (6*B*b^3*cos(d*
x + c)^3 + 24*B*a^3 + 72*A*a^2*b + 48*B*a*b^2 + 16*A*b^3 + 8*(3*B*a*b^2 + A
*b^3)*cos(d*x + c)^2 + 9*(4*B*a^2*b + 4*A*a*b^2 + B*b^3)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(170) = 340.
time = 0.23, size = 386, normalized size = 2.26

$$\begin{cases} Aa^3x+\frac{3Aa^2b\sin\left(c+dx\right)}{d}+\frac{3Aab^2x\sin^2\left(c+dx\right)}{2}+\frac{3Aab^2x\cos^2\left(c+dx\right)}{2}+\frac{3Aab^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{2Ab^3\sin^3\left(c+dx\right)}{3d}+\frac{Ab^3\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{Ba^3\sin\left(c+dx\right)}{d}+\frac{3Ba^2bx\sin^2\left(c+dx\right)}{2}+\frac{3Ba^2bx\cos^2\left(c+dx\right)}{2}+\frac{3Ba^2b\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{2Bab^2\sin^3\left(c+dx\right)}{d}+\frac{3Bab^2\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{3Bb^3x\sin^4\left(c+dx\right)}{8}+\frac{3Bb^3x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{3Bb^3x\cos^4\left(c+dx\right)}{8}+\frac{3Bb^3\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}+\frac{5Bb^3\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d} & \text{for}\: d\neq 0 \\ x(A+B\cos\left(c\right))\left(a+b\cos\left(c\right)\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*x + 3*A*a**2*b*sin(c + d*x)/d + 3*A*a*b**2*x*sin(c + d*x)
**2/2 + 3*A*a*b**2*x*cos(c + d*x)**2/2 + 3*A*a*b**2*sin(c + d*x)*cos(c + d*
x)/(2*d) + 2*A*b**3*sin(c + d*x)**3/(3*d) + A*b**3*sin(c + d*x)*cos(c + d*x
)**2/d + B*a**3*sin(c + d*x)/d + 3*B*a**2*b*x*sin(c + d*x)**2/2 + 3*B*a**2*
b*x*cos(c + d*x)**2/2 + 3*B*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*a*
b**2*sin(c + d*x)**3/d + 3*B*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*b*
*3*x*sin(c + d*x)**4/8 + 3*B*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B
```

```
*b**3*x*cos(c + d*x)**4/8 + 3*B*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5
*B*b**3*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c))*(a
 + b*cos(c))**3, True))
```

Giac [A]
time = 0.46, size = 148, normalized size = 0.87

$$\frac{Bb^3\sin(4\,dx+4\,c)}{32\,d}+\frac{1}{8}\left(8\,Aa^3+12\,Ba^2b+12\,Aab^2+3\,Bb^3\right)x+\frac{(3\,Bab^2+Ab^3)\sin(3\,dx+3\,c)}{12\,d}+\frac{(3\,Ba^2b+3\,Aab^2+Bb^3)\sin(2\,dx+2\,c)}{4\,d}+\frac{(4\,Ba^3+12\,Aa^2b+9\,Bab^2+3\,Ab^3)\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*B*b^3*sin(4*d*x + 4*c)/d + 1/8*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*
B*b^3)*x + 1/12*(3*B*a*b^2 + A*b^3)*sin(3*d*x + 3*c)/d + 1/4*(3*B*a^2*b + 3
*A*a*b^2 + B*b^3)*sin(2*d*x + 2*c)/d + 1/4*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^
2 + 3*A*b^3)*sin(d*x + c)/d
```

Mupad [B]
time = 0.57, size = 202, normalized size = 1.18

$$A\,a^3\,x+\frac{3\,B\,b^3\,x}{8}+\frac{3\,A\,a\,b^2\,x}{2}+\frac{3\,B\,a^2\,b\,x}{2}+\frac{3\,A\,b^3\,\sin(c+d\,x)}{4\,d}+\frac{B\,a^3\,\sin(c+d\,x)}{d}+\frac{A\,b^3\,\sin(3\,c+3\,d\,x)}{12\,d}+\frac{B\,b^3\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{B\,b^3\,\sin(4\,c+4\,d\,x)}{32\,d}+\frac{3\,A\,a\,b^2\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{3\,B\,a^2\,b\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{B\,a\,b^2\,\sin(3\,c+3\,d\,x)}{4\,d}+\frac{3\,A\,a^2\,b\,\sin(c+d\,x)}{d}+\frac{9\,B\,a\,b^2\,\sin(c+d\,x)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

```
[Out] A*a^3*x + (3*B*b^3*x)/8 + (3*A*a*b^2*x)/2 + (3*B*a^2*b*x)/2 + (3*A*b^3*sin(
c + d*x))/(4*d) + (B*a^3*sin(c + d*x))/d + (A*b^3*sin(3*c + 3*d*x))/(12*d)
+ (B*b^3*sin(2*c + 2*d*x))/(4*d) + (B*b^3*sin(4*c + 4*d*x))/(32*d) + (3*A*a
*b^2*sin(2*c + 2*d*x))/(4*d) + (3*B*a^2*b*sin(2*c + 2*d*x))/(4*d) + (B*a*b^
2*sin(3*c + 3*d*x))/(4*d) + (3*A*a^2*b*sin(c + d*x))/d + (9*B*a*b^2*sin(c +
 d*x))/(4*d)
```

3.234 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=137

$$\frac{1}{2}\left(6a^2Ab+Ab^3+2a^3B+3ab^2B\right)x+\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(9aAb+8a^2B+2b^2B)\sin(c+dx)}{3d}+\frac{b^2(3A\ldots$$

```
[Out] 1/2*(6*A*a^2*b+A*b^3+2*B*a^3+3*B*a*b^2)*x+a^3*A*arctanh(sin(d*x+c))/d+1/3*b
*(9*A*a*b+8*B*a^2+2*B*b^2)*sin(d*x+c)/d+1/6*b^2*(3*A*b+5*B*a)*cos(d*x+c)*si
n(d*x+c)/d+1/3*b*B*(a+b*cos(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.22, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3069, 3112, 3102, 2814, 3855}

$$\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(8a^2B+9aAb+2b^2B)\sin(c+dx)}{3d}+\frac{1}{2}x\left(2a^3B+6a^2Ab+3ab^2B+Ab^3\right)+\frac{b^2(5aB+3Ab)\sin(c+dx)\cos(c+dx)}{6d}+\frac{bB\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] ((6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*x)/2 + (a^3*A*ArcTanh[Sin[c + d*
x]])/d + (b*(9*a*A*b + 8*a^2*B + 2*b^2*B)*Sin[c + d*x])/(3*d) + (b^2*(3*A*b
 + 5*a*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (b*B*(a + b*Cos[c + d*x])^2*Si
n[c + d*x])/(3*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{bB(a+b\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (a+$$

$$= \frac{b^2(3Ab+5aB)\cos(c+dx)\sin(c+dx)}{6d}+\frac{bB(a$$

$$= \frac{b(9aAb+8a^2B+2b^2B)\sin(c+dx)}{3d}+\frac{b^2(3Ab$$

$$= \frac{1}{2}\left(6a^2Ab+Ab^3+2a^3B+3ab^2B\right)x+\frac{b(9aAb$$

$$= \frac{1}{2}\left(6a^2Ab+Ab^3+2a^3B+3ab^2B\right)x+\frac{a^3A\tanh$$

Mathematica [A]
time = 0.44, size = 159, normalized size = 1.16

$$\frac{6(6a^2Ab+Ab^3+2a^3B+3ab^2B)(c+dx)-12a^3A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^3A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+9b(4aAb+4a^2B+b^2B)\sin(c+dx)+3b^2(Ab+3aB)\sin(2(c+dx))+b^3B\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (6*(6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*(c + d*x) - 12*a^3*A*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^3*A*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 9*b*(4*a*A*b + 4*a^2*B + b^2*B)*Sin[c + d*x] + 3*b^2*(A*b + 3*a*
B)*Sin[2*(c + d*x)] + b^3*B*Sin[3*(c + d*x)])/(12*d)
```

Maple [A]
time = 0.19, size = 151, normalized size = 1.10

method	result
derivativedivides	$\frac{A a^3 \ln(\sec(dx+c)+\tan(dx+c))+a^3 B(dx+c)+3A a^2 b(dx+c)+3a^2 bB \sin(dx+c)+3Aa b^2 \sin(dx+c)+3Ba b^2 \left(\frac{\sin(dx+c)\cos}{2}\right.}{d}$
default	$\frac{A a^3 \ln(\sec(dx+c)+\tan(dx+c))+a^3 B(dx+c)+3A a^2 b(dx+c)+3a^2 bB \sin(dx+c)+3Aa b^2 \sin(dx+c)+3Ba b^2 \left(\frac{\sin(dx+c)\cos}{2}\right.}{d}$
risch	$3xA a^2 b+\frac{xA b^3}{2}+a^3 Bx+\frac{3xBa b^2}{2}-\frac{3i e^{i(dx+c)} Aa b^2}{2d}-\frac{3i e^{i(dx+c)} a^2 bB}{2d}-\frac{3i e^{i(dx+c)} b^3 B}{8d}+\frac{3i e^{-i(dx+c)}}{2d}$
norman	$\frac{(3A a^2 b+\frac{1}{2}A b^3+a^3 B+\frac{3}{2}Ba b^2)x+(3A a^2 b+\frac{1}{2}A b^3+a^3 B+\frac{3}{2}Ba b^2)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(12A a^2 b+2A b^3+4a^3 B+6Ba b^2)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*B*(d*x+c)+3*A*a^2*b*(d*x+c)+3*a^2*
b*B*sin(d*x+c)+3*A*a*b^2*sin(d*x+c)+3*B*a*b^2*(1/2*sin(d*x+c)*cos(d*x+c)+1/
2*d*x+1/2*c)+A*b^3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+1/3*b^3*B*(cos
(d*x+c)^2+2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 145, normalized size = 1.06

$$\frac{12\,(dx+c)Ba^3+36\,(dx+c)Aa^2b+9\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Bab^2+3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ab^3-4\,(\sin(dx+c)^3-3\,\sin(dx+c))Bb^3+12\,Aa^3\log(\sec(dx+c)+\tan(dx+c))+36\,Ba^2b\sin(dx+c)+36\,Aab^2\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(12*(d*x + c)*B*a^3 + 36*(d*x + c)*A*a^2*b + 9*(2*d*x + 2*c + sin(2*d*
x + 2*c))*B*a*b^2 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^3 - 4*(sin(d*x +
 c)^3 - 3*sin(d*x + c))*B*b^3 + 12*A*a^3*log(sec(d*x + c) + tan(d*x + c)) +
 36*B*a^2*b*sin(d*x + c) + 36*A*a*b^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 131, normalized size = 0.96

$$\frac{3\,Aa^3\log(\sin(dx+c)+1)-3\,Aa^3\log(-\sin(dx+c)+1)+3\,(2\,Ba^3+6\,Aa^2b+3\,Bab^2+Ab^3)dx+\left(2\,Bb^3\cos(dx+c)^2+18\,Ba^2b+18\,Aab^2+4\,Bb^3+3\,(3\,Bab^2+Ab^3)\cos(dx+c)\right)\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*A*a^3*log(sin(d*x + c) + 1) - 3*A*a^3*log(-sin(d*x + c) + 1) + 3*(2*
B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*d*x + (2*B*b^3*cos(d*x + c)^2 + 18*B
*a^2*b + 18*A*a*b^2 + 4*B*b^3 + 3*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x
 + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\,(a+b\cos(c+dx))^3\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(129) = 258.
time = 0.48, size = 314, normalized size = 2.29

$$\frac{6\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\,(2\,Ba^3+6\,Aa^2b+3\,Bab^2+Ab^3)(dx+c)+\frac{2\left(18\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,Aab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9\,Bab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,Bb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+36\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,Aab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,Bb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,Aab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Bab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Bb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/6*(6*A*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*A*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*(d*x + c) +
 2*(18*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 -
 9*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^
3*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*A*a*b^2*t
an(1/2*d*x + 1/2*c)^3 + 4*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 18*B*a^2*b*tan(1/2
*d*x + 1/2*c) + 18*A*a*b^2*tan(1/2*d*x + 1/2*c) + 9*B*a*b^2*tan(1/2*d*x + 1
/2*c) + 3*A*b^3*tan(1/2*d*x + 1/2*c) + 6*B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 1.91, size = 1924, normalized size = 14.04

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x),x)

[Out] (tan(c/2 + (d*x)/2)*(A*b^3 + 2*B*b^3 + 6*A*a*b^2 + 3*B*a*b^2 + 6*B*a^2*b) +
 tan(c/2 + (d*x)/2)^3*((4*B*b^3)/3 + 12*A*a*b^2 + 12*B*a^2*b) + tan(c/2 + (
d*x)/2)^5*(2*B*b^3 - A*b^3 + 6*A*a*b^2 - 3*B*a*b^2 + 6*B*a^2*b))/(d*(3*tan(
c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (a
tan(((((A*b^3*1i)/2 + B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*(32*A*a^3 + 1
6*A*b^3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a*b^2) + tan(c/2 + (d*x)/2)*(32*A^2*
a^6 + 8*A^2*b^6 + 32*B^2*a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2 + 72*B^2*a^
2*b^4 + 96*B^2*a^4*b^2 + 48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B*a^3*b^3))*(
(A*b^3*1i)/2 + B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*1i - (((A*b^3*1i)/2
+ B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*(32*A*a^3 + 16*A*b^3 + 32*B*a^3 +
 96*A*a^2*b + 48*B*a*b^2) - tan(c/2 + (d*x)/2)*(32*A^2*a^6 + 8*A^2*b^6 + 32
*B^2*a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2 + 72*B^2*a^2*b^4 + 96*B^2*a^4*b
^2 + 48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B*a^3*b^3))*((A*b^3*1i)/2 + B*a^3
*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*1i)/((((A*b^3*1i)/2 + B*a^3*1i + A*a^2*b
*3i + (B*a*b^2*3i)/2)*(32*A*a^3 + 16*A*b^3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a
*b^2) + tan(c/2 + (d*x)/2)*(32*A^2*a^6 + 8*A^2*b^6 + 32*B^2*a^6 + 96*A^2*a^
2*b^4 + 288*A^2*a^4*b^2 + 72*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + 48*A*B*a*b^5 +
192*A*B*a^5*b + 320*A*B*a^3*b^3))*((A*b^3*1i)/2 + B*a^3*1i + A*a^2*b*3i + (
B*a*b^2*3i)/2) + (((A*b^3*1i)/2 + B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*(
32*A*a^3 + 16*A*b^3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a*b^2) - tan(c/2 + (d*x)
/2)*(32*A^2*a^6 + 8*A^2*b^6 + 32*B^2*a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2
 + 72*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + 48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B
*a^3*b^3))*((A*b^3*1i)/2 + B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2) + 64*A*B
^2*a^9 - 64*A^2*B*a^9 - 192*A^3*a^8*b + 16*A^3*a^3*b^6 + 192*A^3*a^5*b^4 -
32*A^3*a^6*b^3 + 576*A^3*a^7*b^2 + 384*A^2*B*a^8*b + 144*A*B^2*a^5*b^4 + 19
2*A*B^2*a^7*b^2 + 96*A^2*B*a^4*b^5 + 640*A^2*B*a^6*b^3 - 96*A^2*B*a^7*b^2))
*(A*b^3 + 2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2))/d - (A*a^3*atan((A*a^3*(tan(c/2
 + (d*x)/2)*(32*A^2*a^6 + 8*A^2*b^6 + 32*B^2*a^6 + 96*A^2*a^2*b^4 + 288*A^2
*a^4*b^2 + 72*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + 48*A*B*a*b^5 + 192*A*B*a^5*b +
 320*A*B*a^3*b^3) + A*a^3*(32*A*a^3 + 16*A*b^3 + 32*B*a^3 + 96*A*a^2*b + 48
*B*a*b^2))*1i + A*a^3*(tan(c/2 + (d*x)/2)*(32*A^2*a^6 + 8*A^2*b^6 + 32*B^2*
a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2 + 72*B^2*a^2*b^4 + 96*B^2*a^4*b^2 +
48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B*a^3*b^3) - A*a^3*(32*A*a^3 + 16*A*b^
3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a*b^2))*1i)/(64*A*B^2*a^9 - 64*A^2*B*a^9 -
 192*A^3*a^8*b + A*a^3*(tan(c/2 + (d*x)/2)*(32*A^2*a^6 + 8*A^2*b^6 + 32*B^2
*a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2 + 72*B^2*a^2*b^4 + 96*B^2*a^4*b^2 +
 48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B*a^3*b^3) + A*a^3*(32*A*a^3 + 16*A*b
^3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a*b^2)) - A*a^3*(tan(c/2 + (d*x)/2)*(32*A
^2*a^6 + 8*A^2*b^6 + 32*B^2*a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2 + 72*B^2
*a^2*b^4 + 96*B^2*a^4*b^2 + 48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B*a^3*b^3)
 - A*a^3*(32*A*a^3 + 16*A*b^3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a*b^2)) + 16*A
^3*a^3*b^6 + 192*A^3*a^5*b^4 - 32*A^3*a^6*b^3 + 576*A^3*a^7*b^2 + 384*A^2*B
*a^8*b + 144*A*B^2*a^5*b^4 + 192*A*B^2*a^7*b^2 + 96*A^2*B*a^4*b^5 + 640*A^2
```

```
*B*a^6*b^3 - 96*A^2*B*a^7*b^2))*2i)/d
```

3.235 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=131

$$\frac{1}{2}b(6aAb+6a^2B+b^2B)\,x+\frac{a^2(3Ab+aB)\tanh^{-1}(\sin(c+dx))}{d}-\frac{b(2a^2A-Ab^2-3abB)\sin(c+dx)}{d}-\frac{b^2(2aA$$

```
[Out] 1/2*b*(6*A*a*b+6*B*a^2+B*b^2)*x+a^2*(3*A*b+B*a)*arctanh(sin(d*x+c))/d-b*(2*
A*a^2-A*b^2-3*B*a*b)*sin(d*x+c)/d-1/2*b^2*(2*A*a-B*b)*cos(d*x+c)*sin(d*x+c)
/d+a*A*(a+b*cos(d*x+c))^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.22, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3068, 3112, 3102, 2814, 3855}

$$-\frac{b(2a^2A-3abB-Ab^2)\sin(c+dx)}{d}+\frac{1}{2}bx(6a^2B+6aAb+b^2B)+\frac{a^2(aB+3Ab)\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2(2aA-bB)\sin(c+dx)\cos(c+dx)}{2d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (b*(6*a*A*b + 6*a^2*B + b^2*B)*x)/2 + (a^2*(3*A*b + a*B)*ArcTanh[Sin[c + d*
x]])/d - (b*(2*a^2*A - A*b^2 - 3*a*b*B)*Sin[c + d*x])/d - (b^2*(2*a*A - b*B
)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a*A*(a + b*Cos[c + d*x])^2*Tan[c + d*
x])/d
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
```

```
0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^2\tan(c+dx)}{d} + \int (a+$$

$$= -\frac{b^2(2aA-bB)\cos(c+dx)\sin(c+dx)}{2d} + \frac{aA}{}$$

$$= -\frac{b(2a^2A-Ab^2-3abB)\sin(c+dx)}{d} - \frac{b^2(2a}{}$$

$$= \frac{1}{2}b\left(6aAb+6a^2B+b^2B\right)x - \frac{b(2a^2A-Ab^2-}{}$$

$$= \frac{1}{2}b\left(6aAb+6a^2B+b^2B\right)x + \frac{a^2(3Ab+aB)\,\mathrm{ta}}{}$$

Mathematica [A]
time = 0.73, size = 217, normalized size = 1.66

$$\frac{2b(6aAb+6a^2B+b^2B)(c+dx) - 4a^2(3Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4a^2(3Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{4a^3A\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{4a^3A\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)} + 4b^2(Ab+3aB)\sin(c+dx) + b^3B\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (2*b*(6*a*A*b + 6*a^2*B + b^2*B)*(c + d*x) - 4*a^2*(3*A*b + a*B)*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] + 4*a^2*(3*A*b + a*B)*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + (4*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]) + (4*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]) + 4*b^2*(A*b + 3*a*B)*Sin[c + d*x] + b^3*B*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.21, size = 132, normalized size = 1.01

method	result
derivativedivides	$\frac{A a^3 \tan(dx+c)+a^3 B \ln(\sec(dx+c)+\tan(dx+c))+3A a^2 b \ln(\sec(dx+c)+\tan(dx+c))+3a^2 bB(dx+c)+3Aa b^2(dx+c)+3Ba}{d}$
default	$\frac{A a^3 \tan(dx+c)+a^3 B \ln(\sec(dx+c)+\tan(dx+c))+3A a^2 b \ln(\sec(dx+c)+\tan(dx+c))+3a^2 bB(dx+c)+3Aa b^2(dx+c)+3Ba}{d}$
risch	$3xAa b^2+3x a^2 bB+\frac{b^3 Bx}{2}-\frac{i b^3 B \,\mathrm{e}^{2i(dx+c)}}{8d}-\frac{i\mathrm{e}^{i(dx+c)} A b^3}{2d}-\frac{3i\mathrm{e}^{i(dx+c)} Ba b^2}{2d}+\frac{i\mathrm{e}^{-i(dx+c)} A b^3}{2d}+3i\mathrm{e}$
norman	$\frac{(-3Aa b^2-3a^2 bB-\frac{1}{2}b^3 B)x+(-9Aa b^2-9a^2 bB-\frac{3}{2}b^3 B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(3Aa b^2+3a^2 bB+\frac{1}{2}b^3 B)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*tan(d*x+c)+a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3*A*a^2*b*ln(sec(d*x+
c)+tan(d*x+c))+3*a^2*b*B*(d*x+c)+3*A*a*b^2*(d*x+c)+3*B*a*b^2*sin(d*x+c)+A*b
^3*sin(d*x+c)+b^3*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.30, size = 144, normalized size = 1.10

$$\frac{12\,(dx+c)Ba^2b+12\,(dx+c)Aab^2+(2\,dx+2\,c+\sin(2\,dx+2\,c))Bb^3+2\,Ba^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+6\,Aa^2b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+12\,Bab^2\sin(dx+c)+4\,Ab^3\sin(dx+c)+4\,Aa^3\tan(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/4*(12*(d*x + c)*B*a^2*b + 12*(d*x + c)*A*a*b^2 + (2*d*x + 2*c + sin(2*d*x
 + 2*c))*B*b^3 + 2*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
6*A*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*B*a*b^2*sin(
d*x + c) + 4*A*b^3*sin(d*x + c) + 4*A*a^3*tan(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 152, normalized size = 1.16

$$\frac{(6\,Ba^2b+6\,Aab^2+Bb^3)dx\cos(dx+c)+(Ba^3+3\,Aa^2b)\cos(dx+c)\log(\sin(dx+c)+1)-(Ba^3+3\,Aa^2b)\cos(dx+c)\log(-\sin(dx+c)+1)+\left(Bb^3\cos(dx+c)^2+2\,Aa^3+2\,(3\,Bab^2+Ab^3)\cos(dx+c)\right)\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*((6*B*a^2*b + 6*A*a*b^2 + B*b^3)*d*x*cos(d*x + c) + (B*a^3 + 3*A*a^2*b)
*cos(d*x + c)*log(sin(d*x + c) + 1) - (B*a^3 + 3*A*a^2*b)*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (B*b^3*cos(d*x + c)^2 + 2*A*a^3 + 2*(3*B*a*b^2 + A*b^3
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\,(a+b\cos(c+dx))^3\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.49, size = 234, normalized size = 1.79

$$-\frac{\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-(6\,Ba^2b+6\,Aab^2+Bb^3)(dx+c)-2\,(Ba^3+3\,Aa^2b)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+2\,(Ba^3+3\,Aa^2b)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Bab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Bb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Bab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Bb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (6*B*a^2*
b + 6*A*a*b^2 + B*b^3)*(d*x + c) - 2*(B*a^3 + 3*A*a^2*b)*log(abs(tan(1/2*d*
x + 1/2*c) + 1)) + 2*(B*a^3 + 3*A*a^2*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 - 2*(6*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2*c)^3 - B
*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*B*a*b^2*tan(1/2*d*x + 1/2*c) + 2*A*b^3*tan(
1/2*d*x + 1/2*c) + B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^2)/d
```

Mupad [B]
time = 1.35, size = 236, normalized size = 1.80

$$\frac{B\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+6\,A\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+6\,B\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2\mathrm{i}-A\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,6\mathrm{i}}{d}+\frac{\frac{A\,b^3\,\sin(2\,c+2\,d\,x)}{2}+\frac{B\,b^3\,\sin(3\,c+3\,d\,x)}{8}+A\,a^3\,\sin\left(c+d\,x\right)+\frac{B\,b^3\,\sin(c+d\,x)}{8}+\frac{3\,B\,a\,b^2\,\sin(2\,c+2\,d\,x)}{2}}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] (B*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - B*a^3*atan((sin(c/2 +
(d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i + 6*A*a*b^2*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)) - A*a^2*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))
*6i + 6*B*a^2*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + ((A*b^3*si
n(2*c + 2*d*x))/2 + (B*b^3*sin(3*c + 3*d*x))/8 + A*a^3*sin(c + d*x) + (B*b^
3*sin(c + d*x))/8 + (3*B*a*b^2*sin(2*c + 2*d*x))/2)/(d*cos(c + d*x))
```

3.236 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=124

$$b^2(Ab+3aB)x+\frac{a(a^2A+6Ab^2+6abB)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2(aA-2bB)\sin(c+dx)}{2d}+\frac{a^2(2Ab+aB)\tan}{d}$$

```
[Out] b^2*(A*b+3*B*a)*x+1/2*a*(A*a^2+6*A*b^2+6*B*a*b)*arctanh(sin(d*x+c))/d-1/2*b
^2*(A*a-2*B*b)*sin(d*x+c)/d+a^2*(2*A*b+B*a)*tan(d*x+c)/d+1/2*a*A*(a+b*cos(d
*x+c))^2*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]

time = 0.22, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3068, 3110, 3102, 2814, 3855}

$$\frac{a(a^2A+6abB+6Ab^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(aB+2Ab)\tan(c+dx)}{d}-\frac{b^2(aA-2bB)\sin(c+dx)}{2d}+b^2x(3aB+Ab)+\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] b^2*(A*b + 3*a*B)*x + (a*(a^2*A + 6*A*b^2 + 6*a*b*B)*ArcTanh[Sin[c + d*x]])
/(2*d) - (b^2*(a*A - 2*b*B)*Sin[c + d*x])/(2*d) + (a^2*(2*A*b + a*B)*Tan[c
+ d*x])/d + (a*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}$$

$$= \frac{a^2(2Ab+aB)\tan(c+dx)}{d} + \frac{aA(a+b\cos(c+}{}$$

$$= -\frac{b^2(aA-2bB)\sin(c+dx)}{2d} + \frac{a^2(2Ab+aB)\,\mathrm{ta}}{d}$$

$$= b^2(Ab+3aB)x - \frac{b^2(aA-2bB)\sin(c+dx)}{2d} + $$

$$= b^2(Ab+3aB)x + \frac{a(a^2A+6Ab^2+6abB)\tanh^{-}}{2d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 277 vs. 2(124) = 248.

time = 2.20, size = 277, normalized size = 2.23

$$\frac{4b^2(Ab+3aB)(c+dx)-2a(a^2A+6Ab^2+6abB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2a(a^2A+6Ab^2+6abB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{a^3A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4a^2(3Ab+aB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-\frac{a^3A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4a^2(3Ab+aB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}+4b^3B\sin(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (4*b^2*(A*b + 3*a*B)*(c + d*x) - 2*a*(a^2*A + 6*A*b^2 + 6*a*b*B)*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] + 2*a*(a^2*A + 6*A*b^2 + 6*a*b*B)*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])^2 + (4*a^2*(3*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]) - (a^3*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a^2*(3*A*b
 + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 4*b^3*B*S
in[c + d*x])/(4*d)
```

Maple [A]
time = 0.25, size = 141, normalized size = 1.14

method	result
derivativedivides	$\frac{A\,a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^3B\tan(dx+c)+3A\,a^2b\tan(dx+c)+3a^2bB\ln(\sec(dx+c)+\tan(da}{d}$
default	$\frac{A\,a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^3B\tan(dx+c)+3A\,a^2b\tan(dx+c)+3a^2bB\ln(\sec(dx+c)+\tan(da}{d}$
risch	$xA\,b^3+3xBa\,b^2-\frac{i\mathrm{e}^{i(dx+c)}b^3B}{2d}+\frac{i\mathrm{e}^{-i(dx+c)}b^3B}{2d}-\frac{ia^2\left(Aa\,\mathrm{e}^{3i(dx+c)}-6Ab\,\mathrm{e}^{2i(dx+c)}-2Ba\,\mathrm{e}^{2i(dx+c)}-aA\,\mathrm{e}^{i}\right.}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}$
norman	$\frac{(A\,b^3+3Ba\,b^2)x+(-4A\,b^3-12Ba\,b^2)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-A\,b^3-3Ba\,b^2)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-A\,b^3-3Ba\,b^2)x\left(\tan^8\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^3*B*
tan(d*x+c)+3*A*a^2*b*tan(d*x+c)+3*a^2*b*B*ln(sec(d*x+c)+tan(d*x+c))+3*A*a*b
^2*ln(sec(d*x+c)+tan(d*x+c))+3*B*a*b^2*(d*x+c)+A*b^3*(d*x+c)+b^3*B*sin(d*x+
c))
```

Maxima [A]
time = 0.27, size = 169, normalized size = 1.36

$$\frac{12\,(dx+c)Bab^2+4\,(dx+c)Ab^3-Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,Ba^2b(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+6\,Aab^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+4\,Bb^3\sin\left(dx+c\right)+4\,Ba^3\tan\left(dx+c\right)+12\,Aa^2b\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(12*(d*x + c)*B*a*b^2 + 4*(d*x + c)*A*b^3 - A*a^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*B*a^2*
b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*A*a*b^2*(log(sin(d*x
+ c) + 1) - log(sin(d*x + c) - 1)) + 4*B*b^3*sin(d*x + c) + 4*B*a^3*tan(d*x
 + c) + 12*A*a^2*b*tan(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 167, normalized size = 1.35

$$\frac{4\left(3\,Bab^{2}+Ab^{3}\right)dx\cos\left(dx+c\right)^{2}+\left(Aa^{3}+6\,Ba^{2}b+6\,Aab^{2}\right)\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-\left(Aa^{3}+6\,Ba^{2}b+6\,Aab^{2}\right)\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,Bb^{3}\cos\left(dx+c\right)^{2}+Aa^{3}+2\left(Ba^{3}+3\,Aa^{2}b\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*(3*B*a*b^2 + A*b^3)*d*x*cos(d*x + c)^2 + (A*a^3 + 6*B*a^2*b + 6*A*a*
b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a^3 + 6*B*a^2*b + 6*A*a*b^2)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*B*b^3*cos(d*x + c)^2 + A*a^3
+ 2*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A+B\cos\left(c+dx\right)\right)\left(a+b\cos\left(c+dx\right)\right)^{3}\sec^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3*sec(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(118) = 236.
time = 0.49, size = 239, normalized size = 1.93

$$\frac{\frac{4\,Bb^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+2\left(3\,Bab^{2}+Ab^{3}\right)\left(dx+c\right)+\left(Aa^{3}+6\,Ba^{2}b+6\,Aab^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(Aa^{3}+6\,Ba^{2}b+6\,Aab^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6\,Aa^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Aa^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*B*b^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(3*B*a*b
^2 + A*b^3)*(d*x + c) + (A*a^3 + 6*B*a^2*b + 6*A*a*b^2)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - (A*a^3 + 6*B*a^2*b + 6*A*a*b^2)*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^3*tan(1/2*d*x + 1/2*c)^
3 - 6*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + A*a^3*tan(1/2*d*x + 1/2*c) + 2*B*a^3
```

```
*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^2)/d
```

Mupad [B]
time = 1.56, size = 249, normalized size = 2.01

$$\frac{\frac{B\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{2}+\frac{B\,b^3\,\sin\left(3\,c+3\,d\,x\right)}{4}+\frac{A\,a^3\,\sin\left(c+d\,x\right)}{2}+\frac{B\,b^3\,\sin\left(c+d\,x\right)}{4}+\frac{3\,A\,a^2\,b\,\sin\left(2\,c+2\,d\,x\right)}{2}}{d\,\left(\frac{\cos\left(2\,c+2\,d\,x\right)}{2}+\frac{1}{2}\right)}-\frac{2\,\left(\frac{A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,1{}\mathrm{i}}{2}-A\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+A\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3{}\mathrm{i}-3\,B\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+B\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3{}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^3,x)
```

```
[Out] ((B*a^3*sin(2*c + 2*d*x))/2 + (B*b^3*sin(3*c + 3*d*x))/4 + (A*a^3*sin(c + d
*x))/2 + (B*b^3*sin(c + d*x))/4 + (3*A*a^2*b*sin(2*c + 2*d*x))/2)/(d*(cos(2
*c + 2*d*x)/2 + 1/2)) - (2*((A*a^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (
d*x)/2))*1i)/2 - A*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + A*a*b^
2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i - 3*B*a*b^2*atan(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + B*a^2*b*atan((sin(c/2 + (d*x)/2)*1i)/c
os(c/2 + (d*x)/2))*3i))/d
```

3.237 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=145

$$b^3Bx+\frac{(3a^2Ab+2Ab^3+a^3B+6ab^2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(2a^2A+8Ab^2+9abB)\tan(c+dx)}{3d}+\frac{a^2(5Ab$$

```
[Out] b^3*B*x+1/2*(3*A*a^2*b+2*A*b^3+B*a^3+6*B*a*b^2)*arctanh(sin(d*x+c))/d+1/3*a
*(2*A*a^2+8*A*b^2+9*B*a*b)*tan(d*x+c)/d+1/6*a^2*(5*A*b+3*B*a)*sec(d*x+c)*ta
n(d*x+c)/d+1/3*a*A*(a+b*cos(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3068, 3110, 3100, 2814, 3855}

$$\frac{a(2a^2A+9abB+8Ab^2)\tan(c+dx)}{3d}+\frac{a^2(3aB+5Ab)\tan(c+dx)\sec(c+dx)}{6d}+\frac{(a^3B+3a^2Ab+6ab^2B+2Ab^3)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}+b^3Bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] b^3*B*x + ((3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*ArcTanh[Sin[c + d*x]])
/(2*d) + (a*(2*a^2*A + 8*A*b^2 + 9*a*b*B)*Tan[c + d*x])/(3*d) + (a^2*(5*A*b
 + 3*a*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (a*A*(a + b*Cos[c + d*x])^2*Se
c[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
```

```
0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+d}{3d}$$

$$= \frac{a^2(5Ab+3aB)\sec(c+dx)\tan(c+dx)}{6d} + \frac{aA}{}$$

$$= \frac{a(2a^2A+8Ab^2+9abB)\tan(c+dx)}{3d} + \frac{a^2(5A}{}$$

$$= b^3Bx + \frac{a(2a^2A+8Ab^2+9abB)\tan(c+dx)}{3d}$$

$$= b^3Bx + \frac{(3a^2Ab+2Ab^3+a^3B+6ab^2B)\tanh}{2d}$$

Mathematica [A]

time = 0.64, size = 108, normalized size = 0.74

$$\frac{6b^3Bdx + 3(3a^2Ab + 2Ab^3 + a^3B + 6ab^2B)\tanh^{-1}(\sin(c+dx)) + 3a(2a^2A + 6Ab^2 + 6abB + a(3Ab + aB)\sec(c+dx))\tan(c+dx) + 2a^3A\tan^3(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (6*b^3*B*d*x + 3*(3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*ArcTanh[Sin[c +
d*x]] + 3*a*(2*a^2*A + 6*A*b^2 + 6*a*b*B + a*(3*A*b + a*B)*Sec[c + d*x])*Ta
n[c + d*x] + 2*a^3*A*Tan[c + d*x]^3)/(6*d)
```

Maple [A]

time = 0.28, size = 180, normalized size = 1.24

method	result
derivativedivides	$\frac{-Aa^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a^3B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3Aa^2b\left(\frac{\sec(dx+c)\tan(dx+c}{2}\right.}{}$
default	$\frac{-Aa^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a^3B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3Aa^2b\left(\frac{\sec(dx+c)\tan(dx+c}{2}\right.}{}$
risch	$b^3Bx-\frac{ia\left(9Aab\,e^{5i(dx+c)}+3Ba^2e^{5i(dx+c)}-18Ab^2e^{4i(dx+c)}-18Bab\,e^{4i(dx+c)}-12Aa^2e^{2i(dx+c)}-36Ab^2e^{2i(dx+c)}-3\right.}{3d\left(e^{2i(dx+c)}+1\right)^3}$
norman	$\frac{b^3Bx\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b^3Bx\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b^3Bx-\frac{8a\left(a^2A-3Ab^2-3Bab\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\left(2a^2A-3Aab+6Ab^2-Ba^2\right.}{d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-A*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^3*B*(1/2*sec(d*x+c)*tan(d*
x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*A*a^2*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/
2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^2*b*B*tan(d*x+c)+3*A*a*b^2*tan(d*x+c)+3*B*
a*b^2*ln(sec(d*x+c)+tan(d*x+c))+A*b^3*ln(sec(d*x+c)+tan(d*x+c))+b^3*B*(d*x+
c))
```

Maxima [A]

time = 0.28, size = 216, normalized size = 1.49

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^3+12(dx+c)Bb^3-3Ba^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-9Aa^2b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+18Bab^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+6Ab^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+36Ba^2b\tan(dx+c)+36Aab^2\tan(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 12*(d*x + c)*B*b^3 - 3*B*
a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 9*A*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 18*B*a*b^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 6*A*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 36*B*a^2*b*tan(d*x + c) + 36*A*a*b^2*tan(d*x + c))/d
```

Fricas [**A**]
time = 0.38, size = 189, normalized size = 1.30

$$\frac{12\,Bb^{3}dx\cos\left(dx+c\right)^{3}+3\,\left(Ba^{3}+3\,Aa^{2}b+6\,Bab^{2}+2\,Ab^{3}\right)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,\left(Ba^{3}+3\,Aa^{2}b+6\,Bab^{2}+2\,Ab^{3}\right)\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(2\,Aa^{3}+2\,\left(2\,Aa^{3}+9\,Ba^{2}b+9\,Aab^{2}\right)\cos\left(dx+c\right)^{2}+3\,\left(Ba^{3}+3\,Aa^{2}b\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(12*B*b^3*d*x*cos(d*x + c)^3 + 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*
b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^
2 + 2*A*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*A*a^3 + 2*(2*A*a^
3 + 9*B*a^2*b + 9*A*a*b^2)*cos(d*x + c)^2 + 3*(B*a^3 + 3*A*a^2*b)*cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(137) = 274.
time = 0.61, size = 336, normalized size = 2.32

$$\frac{6\,\left(dx+c\right)Bb^{3}+3\,\left(Ba^{3}+3\,Aa^{2}b+6\,Bab^{2}+2\,Ab^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,\left(Ba^{3}+3\,Aa^{2}b+6\,Bab^{2}+2\,Ab^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,\left(6\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-9\,Aa^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+18\,Ba^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+18\,Aab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-4\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-36\,Ba^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-36\,Aab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Aa^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,Ba^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,Aab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(d*x + c)*B*b^3 + 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*log(ab
s(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*
B*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*B*a^2*
```

```
b*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^3*tan(
1/2*d*x + 1/2*c)^3 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*tan(1/2
*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c) + 3*B*a^3*tan(1/2*d*x + 1/2*
c) + 9*A*a^2*b*tan(1/2*d*x + 1/2*c) + 18*B*a^2*b*tan(1/2*d*x + 1/2*c) + 18*
A*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 1.95, size = 526, normalized size = 3.63

$$\frac{\frac{A\,a^3\,\sin(3\,c+3\,d\,x)}{6}+\frac{B\,a^3\,\sin(2\,c+2\,d\,x)}{4}+\frac{A\,a^3\,\sin(c+d\,x)}{2}+\frac{3\,A\,a\,b^2\,\sin(c+d\,x)}{4}+\frac{3\,B\,a^2\,b\,\sin(c+d\,x)}{4}-\frac{A\,b^3\,\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3{}\mathrm{i}}{2}-\frac{B\,a^3\,\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3{}\mathrm{i}}{4}+\frac{3\,B\,b^3\,\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{3\,A\,a^2\,b\,\sin(2\,c+2\,d\,x)}{4}+\frac{3\,A\,a\,b^2\,\sin(3\,c+3\,d\,x)}{4}+\frac{3\,B\,a^2\,b\,\sin(3\,c+3\,d\,x)}{4}-\frac{A\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos(3\,c+3\,d\,x)\,1{}\mathrm{i}}{2}-\frac{B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos(3\,c+3\,d\,x)\,1{}\mathrm{i}}{4}+\frac{B\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos(3\,c+3\,d\,x)}{2}-\frac{A\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos(3\,c+3\,d\,x)\,3{}\mathrm{i}}{4}-\frac{B\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos(3\,c+3\,d\,x)\,3{}\mathrm{i}}{2}-\frac{A\,a^2\,b\,\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,9{}\mathrm{i}}{4}-\frac{B\,a\,b^2\,\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,9{}\mathrm{i}}{2}}{d\,\left(\frac{3\,\cos(c+d\,x)}{4}+\frac{\cos(3\,c+3\,d\,x)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] ((A*a^3*sin(3*c + 3*d*x))/6 + (B*a^3*sin(2*c + 2*d*x))/4 + (A*a^3*sin(c + d
*x))/2 + (3*A*a*b^2*sin(c + d*x))/4 + (3*B*a^2*b*sin(c + d*x))/4 - (A*b^3*c
os(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/2 - (B*a^3
*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/4 + (3*B
*b^3*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (3*A*a^2
*b*sin(2*c + 2*d*x))/4 + (3*A*a*b^2*sin(3*c + 3*d*x))/4 + (3*B*a^2*b*sin(3*
c + 3*d*x))/4 - (A*b^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos
(3*c + 3*d*x)*1i)/2 - (B*a^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2
))*cos(3*c + 3*d*x)*1i)/4 + (B*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2))*cos(3*c + 3*d*x))/2 - (A*a^2*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (
d*x)/2))*cos(3*c + 3*d*x)*3i)/4 - (B*a*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos
(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*3i)/2 - (A*a^2*b*cos(c + d*x)*atan((sin(c
/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*9i)/4 - (B*a*b^2*cos(c + d*x)*atan((s
in(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*9i)/2)/(d*((3*cos(c + d*x))/4 + c
os(3*c + 3*d*x)/4))
```

3.238 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=188

$$\frac{(3a^3A+12aAb^2+12a^2bB+8b^3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(6a^2Ab+3Ab^3+2a^3B+9ab^2B)\tan(c+dx)}{3d}+\frac{a}{}$$

```
[Out] 1/8*(3*A*a^3+12*A*a*b^2+12*B*a^2*b+8*B*b^3)*arctanh(sin(d*x+c))/d+1/3*(6*A*
a^2*b+3*A*b^3+2*B*a^3+9*B*a*b^2)*tan(d*x+c)/d+1/8*a*(3*A*a^2+10*A*b^2+12*B*
a*b)*sec(d*x+c)*tan(d*x+c)/d+1/6*a^2*(3*A*b+2*B*a)*sec(d*x+c)^2*tan(d*x+c)/
d+1/4*a*A*(a+b*cos(d*x+c))^2*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A]

time = 0.30, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3068, 3110, 3100, 2827, 3852, 8, 3855}

$$\frac{a(3a^2A+12abB+10Ab^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{a^2(2aB+3Ab)\tan(c+dx)\sec^2(c+dx)}{6d}+\frac{(2a^3B+6a^2Ab+9ab^2B+3Ab^3)\tan(c+dx)}{3d}+\frac{(3a^3A+12a^2bB+12aAb^2+8b^3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] ((3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((6*a^2*A*b + 3*A*b^3 + 2*a^3*B + 9*a*b^2*B)*Tan[c + d*x])/(3*d) + (a*(3
*a^2*A + 10*A*b^2 + 12*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(3*A*
b + 2*a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a*A*(a + b*Cos[c + d*x])^2
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
```

```
1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx &= \frac{aA(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx}{4d} \\
&= \frac{a^2(3Ab+2aB)\sec^2(c+dx)\tan(c+dx)}{6d} + \frac{aA}{} \\
&= \frac{a(3a^2A+10Ab^2+12abB)\sec(c+dx)\tan(c+}{8d} \\
&= \frac{a(3a^2A+10Ab^2+12abB)\sec(c+dx)\tan(c+}{8d} \\
&= \frac{(3a^3A+12aAb^2+12a^2bB+8b^3B)\tanh^{-1}(\sin}{8d} \\
&= \frac{(3a^3A+12aAb^2+12a^2bB+8b^3B)\tanh^{-1}(\sin}{8d}
\end{aligned}
$$

Mathematica [A]
time = 0.88, size = 140, normalized size = 0.74

$$
\frac{3(3a^3A+12aAb^2+12a^2bB+8b^3B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(24(3a^2Ab+Ab^3+a^3B+3ab^2B)+9a(a^2A+4Ab^2+4abB)\sec(c+dx)+6a^3A\sec^3(c+dx)+8a^2(3Ab+aB)\tan^2(c+dx)\right)}{24d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(24*(3*a^2*A*b + A*b^3 + a^3*B + 3*a*b^2*B) + 9*a*(a^2*A + 4*A*b
^2 + 4*a*b*B)*Sec[c + d*x] + 6*a^3*A*Sec[c + d*x]^3 + 8*a^2*(3*A*b + a*B)*T
an[c + d*x]^2))/(24*d)
```

Maple [A]
time = 0.30, size = 223, normalized size = 1.19 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c
)+tan(d*x+c)))-a^3*B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-3*A*a^2*b*(-2/3-1/3
*sec(d*x+c)^2)*tan(d*x+c)+3*a^2*b*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d
*x+c)+tan(d*x+c)))+3*A*a*b^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+t
an(d*x+c)))+3*B*a*b^2*tan(d*x+c)+A*b^3*tan(d*x+c)+b^3*B*ln(sec(d*x+c)+tan(d
*x+c)))
```

Maxima [A]
time = 0.28, size = 273, normalized size = 1.45

$$
\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^3+48\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^2b-3Aa^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-36Ba^2b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-36Aab^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+24Bb^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+144Bab^2\tan(dx+c)+48Ab^3\tan(dx+c)}{48d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 + 48*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a^2*b - 3*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(
d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*
x + c) - 1)) - 36*B*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) - 36*A*a*b^2*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*B*b^3*(lo
g(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 144*B*a*b^2*tan(d*x + c) + 4
8*A*b^3*tan(d*x + c))/d
```

Fricas [A]
time = 0.40, size = 211, normalized size = 1.12

$$\frac{3\,(3\,Aa^3+12\,Ba^2b+12\,Aab^2+8\,Bb^3)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\,(3\,Aa^3+12\,Ba^2b+12\,Aab^2+8\,Bb^3)\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\,\left(6\,Aa^3+8\,(2\,Ba^3+6\,Aa^2b+9\,Bab^2+3\,Ab^3)\cos(dx+c)^3+9\,(Aa^3+4\,Ba^2b+4\,Aab^2)\cos(dx+c)^2+8\,(Ba^3+3\,Aa^2b)\cos(dx+c)\right)\sin(dx+c)}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*cos(d*x + c)^4*log(si
n(d*x + c) + 1) - 3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1) + 2*(6*A*a^3 + 8*(2*B*a^3 + 6*A*a^2*b + 9*B*a*
b^2 + 3*A*b^3)*cos(d*x + c)^3 + 9*(A*a^3 + 4*B*a^2*b + 4*A*a*b^2)*cos(d*x +
 c)^2 + 8*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(178) = 356.
time = 0.48, size = 586, normalized size = 3.12

$$\frac{3\,(3\,Aa^3+12\,Ba^2b+12\,Aab^2+8\,Bb^3)\log\left(\left|\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)+1\right|\right)-3\,(3\,Aa^3+12\,Ba^2b+12\,Aab^2+8\,Bb^3)\log\left(\left|\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)-1\right|\right)+\frac{2\,\left(15\,Aa^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7-24\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7-72\,Aa^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7+36\,Ba^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7+36\,Aab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7-72\,Bab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7-24\,Ab^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7+9\,Aa^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5+40\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5+120\,Aa^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-36\,Ba^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-36\,Aab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5+216\,Bab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5+72\,Ab^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5+9\,Aa^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-40\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-120\,Aa^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-36\,Ba^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-36\,Aab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-216\,Bab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-72\,Ab^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3+15\,Aa^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+24\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+72\,Aa^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+36\,Ba^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+36\,Aab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+72\,Bab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+24\,Ab^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)\right)}{\left(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) - 3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^3*tan
(1/2*d*x + 1/2*c)^7 - 72*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 36*B*a^2*b*tan(1/
2*d*x + 1/2*c)^7 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*B*a*b^2*tan(1/2*d
*x + 1/2*c)^7 - 24*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^3*tan(1/2*d*x + 1/2
*c)^5 + 40*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 120*A*a^2*b*tan(1/2*d*x + 1/2*c)^
5 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 +
 216*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A
*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^2*b
*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 216*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*A*b^3*tan(1/
2*d*x + 1/2*c)^3 + 15*A*a^3*tan(1/2*d*x + 1/2*c) + 24*B*a^3*tan(1/2*d*x + 1
/2*c) + 72*A*a^2*b*tan(1/2*d*x + 1/2*c) + 36*B*a^2*b*tan(1/2*d*x + 1/2*c) +
 36*A*a*b^2*tan(1/2*d*x + 1/2*c) + 72*B*a*b^2*tan(1/2*d*x + 1/2*c) + 24*A*b
^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 3.95, size = 395, normalized size = 2.10

$$\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3Aa^3}{8}+\frac{3Ba^2b}{2}+\frac{3Aab^2}{2}+Bb^3\right)}{\frac{3Aa^3}{2}+6Ba^2b+6Aab^2+4Bb^3}\right)\left(\frac{3Aa^3}{4}+3Ba^2b+3Aab^2+2Bb^3\right)}{d}-\frac{\left(2Ab^3-\frac{5Aa^3}{4}+2Ba^3-3Aab^2+6Aa^2b+6Bab^2-3Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(3Aab^2-6Ab^3-\frac{10Ba^3}{3}-\frac{3Aa^3}{4}-10Aa^2b-18Bab^2+3Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(6Ab^3-\frac{3Aa^3}{4}+\frac{10Ba^3}{3}+3Aab^2+10Aa^2b+18Bab^2+3Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(-\frac{5Aa^3}{4}-2Ab^3-2Ba^3-3Aab^2-6Aa^2b-6Bab^2-3Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((3*A*a^3)/8 + B*b^3 + (3*A*a*b^2)/2 + (3*B*a^
2*b)/2))/((3*A*a^3)/2 + 4*B*b^3 + 6*A*a*b^2 + 6*B*a^2*b))*((3*A*a^3)/4 + 2*
B*b^3 + 3*A*a*b^2 + 3*B*a^2*b))/d - (tan(c/2 + (d*x)/2)^7*(2*A*b^3 - (5*A*a
^3)/4 + 2*B*a^3 - 3*A*a*b^2 + 6*A*a^2*b + 6*B*a*b^2 - 3*B*a^2*b) + tan(c/2
+ (d*x)/2)^3*(6*A*b^3 - (3*A*a^3)/4 + (10*B*a^3)/3 + 3*A*a*b^2 + 10*A*a^2*b
 + 18*B*a*b^2 + 3*B*a^2*b) - tan(c/2 + (d*x)/2)^5*((3*A*a^3)/4 + 6*A*b^3 +
(10*B*a^3)/3 - 3*A*a*b^2 + 10*A*a^2*b + 18*B*a*b^2 - 3*B*a^2*b) - tan(c/2 +
 (d*x)/2)*((5*A*a^3)/4 + 2*A*b^3 + 2*B*a^3 + 3*A*a*b^2 + 6*A*a^2*b + 6*B*a*
b^2 + 3*B*a^2*b))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*t
an(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.239 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=236

$$\frac{(9a^2Ab+4Ab^3+3a^3B+12ab^2B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(8a^3A+30aAb^2+30a^2bB+15b^3B)\tan(c+dx)}{15d}+\cdots$$

```
[Out] 1/8*(9*A*a^2*b+4*A*b^3+3*B*a^3+12*B*a*b^2)*arctanh(sin(d*x+c))/d+1/15*(8*A*
a^3+30*A*a*b^2+30*B*a^2*b+15*B*b^3)*tan(d*x+c)/d+1/8*(9*A*a^2*b+4*A*b^3+3*B
*a^3+12*B*a*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/15*a*(4*A*a^2+12*A*b^2+15*B*a*b)
*sec(d*x+c)^2*tan(d*x+c)/d+1/20*a^2*(7*A*b+5*B*a)*sec(d*x+c)^3*tan(d*x+c)/d
+1/5*a*A*(a+b*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d
```

Rubi [A]
time = 0.32, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3068, 3110, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{a(4a^2A+15abB+12Ab^2)\tan(c+dx)\sec^2(c+dx)}{15d}+\frac{a^2(5aB+7Ab)\tan(c+dx)\sec^3(c+dx)}{20d}+\frac{(8a^3A+30a^2bB+30aAb^2+15b^3B)\tan(c+dx)}{15d}+\frac{(3a^3B+9a^2Ab+12ab^2B+4Ab^3)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3a^3B+9a^2Ab+12ab^2B+4Ab^3)\tan(c+dx)\sec(c+dx)}{8d}+\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] ((9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*ArcTanh[Sin[c + d*x]])/(8*d)
+ ((8*a^3*A + 30*a*A*b^2 + 30*a^2*b*B + 15*b^3*B)*Tan[c + d*x])/(15*d) + ((
9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d
) + (a*(4*a^2*A + 12*A*b^2 + 15*a*b*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d)
+ (a^2*(7*A*b + 5*a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*A*(a + b*Co
s[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
```

```
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx &= \frac{aA(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{5d} \\ &= \frac{a^2(7Ab+5aB)\sec^3(c+dx)\tan(c+dx)}{20d}+\frac{aA(}{} \\ &= \frac{a(4a^2A+12Ab^2+15abB)\sec^2(c+dx)\tan(c+}{15d} \\ &= \frac{a(4a^2A+12Ab^2+15abB)\sec^2(c+dx)\tan(c+}{15d} \\ &= \frac{(9a^2Ab+4Ab^3+3a^3B+12ab^2B)\sec(c+dx)\,\mathrm{t}}{8d} \\ &= \frac{(9a^2Ab+4Ab^3+3a^3B+12ab^2B)\tanh^{-1}(\sin(c}{8d}\end{aligned}$$

Mathematica [A]
time = 2.86, size = 181, normalized size = 0.77

$$\frac{15(9a^2Ab+4Ab^3+3a^3B+12ab^2B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(15(9a^2Ab+4Ab^3+3a^3B+12ab^2B)\sec(c+dx)+30a^2(3Ab+aB)\sec^3(c+dx)+8(15(a^3A+3aAb^2+3a^2bB+b^3B)+5a(2a^2A+3Ab^2+3abB)\tan^2(c+dx)+3a^3A\tan^4(c+dx))\right)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (15*(9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(15*(9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Sec[c + d*x] +
30*a^2*(3*A*b + a*B)*Sec[c + d*x]^3 + 8*(15*(a^3*A + 3*a*A*b^2 + 3*a^2*b*B
+ b^3*B) + 5*a*(2*a^2*A + 3*A*b^2 + 3*a*b*B)*Tan[c + d*x]^2 + 3*a^3*A*Tan[c
 + d*x]^4)))/(120*d)
```

Maple [A]
time = 0.24, size = 275, normalized size = 1.17

method	result
derivativedivides	$\frac{-A\,a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+a^3B\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx}{}\right.}{}$
default	$\frac{-A\,a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+a^3B\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx}{}\right.}{}$

risch	$-\frac{i\left(-240Aab^2-64A\,a^3-240a^2bB-60A\,b^3\mathrm{e}^{i(dx+c)}-45B\,a^3\mathrm{e}^{i(dx+c)}-320A\,a^3\mathrm{e}^{2i(dx+c)}-480b^3B\,\mathrm{e}^{2i(dx+c)}+60A\,b^3\mathrm{e}^{9i(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-A*a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+a^3*B*(-(
-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))
+3*A*a^2*b*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c
)+tan(d*x+c)))-3*a^2*b*B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-3*A*a*b^2*(-2/3
-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*B*a*b^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(s
ec(d*x+c)+tan(d*x+c)))+A*b^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+t
an(d*x+c)))+b^3*B*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 341, normalized size = 1.44

$$\frac{16\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)Aa^3+240\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Ba^2b+240\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Aab^2-15\,Ba^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-45\,Aa^2b\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-180\,Bab^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-60\,Ab^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+240\,Bb^3\tan\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
240*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2*b + 240*(tan(d*x + c)^3 + 3*tan
(d*x + c))*A*a*b^2 - 15*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d
*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
 + c) - 1)) - 45*A*a^2*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 180*B*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c
) + 1) + log(sin(d*x + c) - 1)) - 60*A*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*B*b^3*tan(d*x +
 c))/d
```

Fricas [A]
time = 0.38, size = 249, normalized size = 1.06

$$\frac{15\left(3\,Ba^3+9\,Aa^2b+12\,Bab^2+4\,Ab^3\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(3\,Ba^3+9\,Aa^2b+12\,Bab^2+4\,Ab^3\right)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\left(8\,Aa^3+30\,Ba^2b+30\,Aab^2+15\,Bb^3\right)\cos\left(dx+c\right)^4+24\,Aa^3+15\left(3\,Ba^3+9\,Aa^2b+12\,Bab^2+4\,Ab^3\right)\cos\left(dx+c\right)^3+8\left(4\,Aa^3+15\,Ba^2b+15\,Aab^2\right)\cos\left(dx+c\right)^2+30\left(Ba^3+3\,Aa^2b\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x + c)^5*log(s
in(d*x + c) + 1) - 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x
```

```
+ c)^5*log(-sin(d*x + c) + 1) + 2*(8*(8*A*a^3 + 30*B*a^2*b + 30*A*a*b^2 + 1
5*B*b^3)*cos(d*x + c)^4 + 24*A*a^3 + 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 +
 4*A*b^3)*cos(d*x + c)^3 + 8*(4*A*a^3 + 15*B*a^2*b + 15*A*a*b^2)*cos(d*x +
c)^2 + 30*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(224) = 448.
time = 0.51, size = 722, normalized size = 3.06

$$\frac{15\left(3Ba^3+9Aa^2b+12Bab^2+4Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(3Ba^3+9Aa^2b+12Bab^2+4Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\,(\text{[illegible]})}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*B*a^3*t
an(1/2*d*x + 1/2*c)^9 - 225*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*B*a^2*b*ta
n(1/2*d*x + 1/2*c)^9 + 360*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 180*B*a*b^2*tan
(1/2*d*x + 1/2*c)^9 - 60*A*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*B*b^3*tan(1/2*d
*x + 1/2*c)^9 - 160*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 30*B*a^3*tan(1/2*d*x + 1
/2*c)^7 + 90*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 960*B*a^2*b*tan(1/2*d*x + 1/2
*c)^7 - 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 360*B*a*b^2*tan(1/2*d*x + 1/2*
c)^7 + 120*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 480*B*b^3*tan(1/2*d*x + 1/2*c)^7
+ 464*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 +
1200*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 720*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 16
0*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 90*A*a^2
*b*tan(1/2*d*x + 1/2*c)^3 - 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*A*a*b^
2*tan(1/2*d*x + 1/2*c)^3 - 360*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*A*b^3*t
an(1/2*d*x + 1/2*c)^3 - 480*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^3*tan(1/
2*d*x + 1/2*c) + 75*B*a^3*tan(1/2*d*x + 1/2*c) + 225*A*a^2*b*tan(1/2*d*x +
1/2*c) + 360*B*a^2*b*tan(1/2*d*x + 1/2*c) + 360*A*a*b^2*tan(1/2*d*x + 1/2*c
) + 180*B*a*b^2*tan(1/2*d*x + 1/2*c) + 60*A*b^3*tan(1/2*d*x + 1/2*c) + 120*
B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 3.89, size = 470, normalized size = 1.99

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3Ba^3}{8}+\frac{9Aa^2b}{8}+\frac{3Bab^2}{2}+\frac{Ab^3}{2}\right)}{\frac{3Ba^3}{2}+\frac{9Aa^2b}{2}+6Bab^2+2Ab^3}\right)\left(\frac{3Ba^3}{4}+\frac{9Aa^2b}{4}+3Bab^2+Ab^3\right)}{d}-\frac{\left(2Aa^3-Ab^3-\frac{5Ba^3}{4}+2Bb^3+6Aab^2-\frac{15Aa^2b}{4}-3Bab^2+6Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(2Ab^3-\frac{8Aa^3}{3}+\frac{Ba^3}{2}-8Bb^3-16Aab^2+\frac{3Aa^2b}{2}+6Bab^2-16Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{116Aa^3}{15}+20Ba^2b+20Aab^2+12Bb^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\frac{8Aa^3}{3}-2Ab^3-\frac{Ba^3}{2}-8Bb^3-16Aab^2-\frac{3Aa^2b}{2}-6Bab^2-16Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(2Aa^3+Ab^3+\frac{5Ba^3}{4}+2Bb^3+6Aab^2+\frac{15Aa^2b}{4}+3Bab^2+6Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^6,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((A*b^3)/2 + (3*B*a^3)/8 + (9*A*a^2*b)/8 + (3*
B*a*b^2)/2))/(2*A*b^3 + (3*B*a^3)/2 + (9*A*a^2*b)/2 + 6*B*a*b^2))*(A*b^3 +
(3*B*a^3)/4 + (9*A*a^2*b)/4 + 3*B*a*b^2))/d - (tan(c/2 + (d*x)/2)*(2*A*a^3
+ A*b^3 + (5*B*a^3)/4 + 2*B*b^3 + 6*A*a*b^2 + (15*A*a^2*b)/4 + 3*B*a*b^2 +
6*B*a^2*b) + tan(c/2 + (d*x)/2)^5*((116*A*a^3)/15 + 12*B*b^3 + 20*A*a*b^2 +
 20*B*a^2*b) + tan(c/2 + (d*x)/2)^9*(2*A*a^3 - A*b^3 - (5*B*a^3)/4 + 2*B*b^
3 + 6*A*a*b^2 - (15*A*a^2*b)/4 - 3*B*a*b^2 + 6*B*a^2*b) - tan(c/2 + (d*x)/2
)^3*((8*A*a^3)/3 + 2*A*b^3 + (B*a^3)/2 + 8*B*b^3 + 16*A*a*b^2 + (3*A*a^2*b)
/2 + 6*B*a*b^2 + 16*B*a^2*b) - tan(c/2 + (d*x)/2)^7*((8*A*a^3)/3 - 2*A*b^3
- (B*a^3)/2 + 8*B*b^3 + 16*A*a*b^2 - (3*A*a^2*b)/2 - 6*B*a*b^2 + 16*B*a^2*b
))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x
)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.240 $\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=366

$$\frac{1}{16}\left(8a^4A+36a^2Ab^2+5Ab^4+24a^3bB+20ab^3B\right)x+\frac{\left(140a^3Ab+112aAb^3+35a^4B+168a^2b^2B+24b^4B\right)\sin}{35d}$$

```
[Out] 1/16*(8*A*a^4+36*A*a^2*b^2+5*A*b^4+24*B*a^3*b+20*B*a*b^3)*x+1/35*(140*A*a^3
*b+112*A*a*b^3+35*B*a^4+168*B*a^2*b^2+24*B*b^4)*sin(d*x+c)/d+1/16*(8*A*a^4+
36*A*a^2*b^2+5*A*b^4+24*B*a^3*b+20*B*a*b^3)*cos(d*x+c)*sin(d*x+c)/d+1/168*b
*(224*A*a^2*b+35*A*b^3+104*B*a^3+140*B*a*b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/1
05*b^2*(49*A*a*b+31*B*a^2+18*B*b^2)*cos(d*x+c)^4*sin(d*x+c)/d+1/42*b*(7*A*b
+10*B*a)*cos(d*x+c)^3*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/7*b*B*cos(d*x+c)^3*
(a+b*cos(d*x+c))^3*sin(d*x+c)/d-1/105*(140*A*a^3*b+112*A*a*b^3+35*B*a^4+168
*B*a^2*b^2+24*B*b^4)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.54, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3069, 3128, 3112, 3102, 2827, 2715, 8, 2713}

$$\frac{b^2(31a^2B+49aAb+18b^2B)\sin(c+dx)\cos^4(c+dx)}{105d}+\frac{b(104a^3B+224a^2Ab+140ab^2B+35Ab^3)\sin(c+dx)\cos^3(c+dx)}{168d}-\frac{(35a^4B+140a^3Ab+168a^2b^2B+112aAb^3+24b^4B)\sin^3(c+dx)}{105d}+\frac{(35a^4B+140a^3Ab+168a^2b^2B+112aAb^3+24b^4B)\sin(c+dx)}{35d}+\frac{(8a^4A+24a^3bB+36a^2Ab^2+20ab^3B+5Ab^4)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(8a^4A+24a^3bB+36a^2Ab^2+20ab^3B+5Ab^4\right)+\frac{b(10aB+7Ab)\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{42d}+\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((8*a^4*A + 36*a^2*A*b^2 + 5*A*b^4 + 24*a^3*b*B + 20*a*b^3*B)*x)/16 + ((140
*a^3*A*b + 112*a*A*b^3 + 35*a^4*B + 168*a^2*b^2*B + 24*b^4*B)*Sin[c + d*x])
/(35*d) + ((8*a^4*A + 36*a^2*A*b^2 + 5*A*b^4 + 24*a^3*b*B + 20*a*b^3*B)*Cos
[c + d*x]*Sin[c + d*x])/(16*d) + (b*(224*a^2*A*b + 35*A*b^3 + 104*a^3*B + 1
40*a*b^2*B)*Cos[c + d*x]^3*Sin[c + d*x])/(168*d) + (b^2*(49*a*A*b + 31*a^2*
B + 18*b^2*B)*Cos[c + d*x]^4*Sin[c + d*x])/(105*d) + (b*(7*A*b + 10*a*B)*Co
s[c + d*x]^3*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(42*d) + (b*B*Cos[c + d*x
]^3*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(7*d) - ((140*a^3*A*b + 112*a*A*b^
3 + 35*a^4*B + 168*a^2*b^2*B + 24*b^4*B)*Sin[c + d*x]^3)/(105*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
```

```
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{bB\cos^3(c+dx)(a+b\cos(c+dx))^3\sin(c+dx)}{7d}$$

$$= \frac{b(7Ab+10aB)\cos^3(c+dx)(a+b\cos(c+dx))}{42d}$$

$$= \frac{b^2(49aAb+31a^2B+18b^2B)\cos^4(c+dx)\sin(c}{105d}$$

$$= \frac{b(224a^2Ab+35Ab^3+104a^3B+140ab^2B)\cos^3}{168d}$$

$$= \frac{b(224a^2Ab+35Ab^3+104a^3B+140ab^2B)\cos^3}{168d}$$

$$= \frac{(8a^4A+36a^2Ab^2+5Ab^4+24a^3bB+20ab^3B)}{16d}$$

$$= \frac{1}{16}\left(8a^4A+36a^2Ab^2+5Ab^4+24a^3bB+20ab^3\right.$$

Mathematica [A]
time = 0.58, size = 408, normalized size = 1.11

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3360*a^4*A*c + 15120*a^2*A*b^2*c + 2100*A*b^4*c + 10080*a^3*b*B*c + 8400*a
*b^3*B*c + 3360*a^4*A*d*x + 15120*a^2*A*b^2*d*x + 2100*A*b^4*d*x + 10080*a^
```

```
3*b*B*d*x + 8400*a*b^3*B*d*x + 105*(192*a^3*A*b + 160*a*A*b^3 + 48*a^4*B +
240*a^2*b^2*B + 35*b^4*B)*Sin[c + d*x] + 105*(16*a^4*A + 96*a^2*A*b^2 + 15*
A*b^4 + 64*a^3*b*B + 60*a*b^3*B)*Sin[2*(c + d*x)] + 2240*a^3*A*b*Sin[3*(c +
 d*x)] + 2800*a*A*b^3*Sin[3*(c + d*x)] + 560*a^4*B*Sin[3*(c + d*x)] + 4200*
a^2*b^2*B*Sin[3*(c + d*x)] + 735*b^4*B*Sin[3*(c + d*x)] + 1260*a^2*A*b^2*Si
n[4*(c + d*x)] + 315*A*b^4*Sin[4*(c + d*x)] + 840*a^3*b*B*Sin[4*(c + d*x)]
+ 1260*a*b^3*B*Sin[4*(c + d*x)] + 336*a*A*b^3*Sin[5*(c + d*x)] + 504*a^2*b^
2*B*Sin[5*(c + d*x)] + 147*b^4*B*Sin[5*(c + d*x)] + 35*A*b^4*Sin[6*(c + d*x
)] + 140*a*b^3*B*Sin[6*(c + d*x)] + 15*b^4*B*Sin[7*(c + d*x)])/(6720*d)
```

Maple [**A**]
time = 0.28, size = 368, normalized size = 1.01 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+1/3*a^4*B*(cos(d*x+c)^
2+2)*sin(d*x+c)+4/3*A*a^3*b*(cos(d*x+c)^2+2)*sin(d*x+c)+4*B*a^3*b*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6*A*a^2*b^2*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6/5*B*a^2*b^2*(8/3+cos(d*
x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4/5*A*a*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d
*x+c)^2)*sin(d*x+c)+4*B*a*b^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(
d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+A*b^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)
^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+1/7*B*b^4*(16/5+cos(d*x+c)^
6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [**A**]
time = 0.28, size = 366, normalized size = 1.00

$$\frac{1680\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^4-2240\,(\sin(dx+c)^3-3\,\sin(dx+c))Ba^4-8960\,(\sin(dx+c)^3-3\,\sin(dx+c))Aa^3b+840\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Ba^3b+1260\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^2b^2+2688\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))Ba^2b^2+1792\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))Aab^3-140\,(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c))Bab^3-35\,(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c))Ab^4-192\,(5\,\sin(dx+c)^7-21\,\sin(dx+c)^5+35\,\sin(dx+c)^3-35\,\sin(dx+c))Bb^4}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/6720*(1680*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 - 2240*(sin(d*x + c)^3
- 3*sin(d*x + c))*B*a^4 - 8960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3*b +
840*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3*b + 1260*
(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2*b^2 + 2688*(3
*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^2*b^2 + 1792*(3*
sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a*b^3 - 140*(4*sin(
2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*
B*a*b^3 - 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 4
8*sin(2*d*x + 2*c))*A*b^4 - 192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*
sin(d*x + c)^3 - 35*sin(d*x + c))*B*b^4)/d
```

Fricas [A]
time = 0.42, size = 289, normalized size = 0.79

$$\frac{105\,(8\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+20\,Bab^3+5\,Ab^4)dx+(240\,Bb^4\cos(dx+c)^6+280\,(4\,Bab^3+Ab^4)\cos(dx+c)^5+1120\,Ba^4+4480\,Aa^3b+5376\,Ba^2b^2+3584\,Aab^3+768\,Bb^4+96\,(21\,Ba^2b^2+14\,Aab^3+3\,Bb^4)\cos(dx+c)^4+70\,(24\,Ba^3b+36\,Aa^2b^2+20\,Bab^3+5\,Ab^4)\cos(dx+c)^3+16\,(35\,Ba^4+140\,Aa^3b+168\,Ba^2b^2+112\,Aab^3+24\,Bb^4)\cos(dx+c)^2+105\,(8\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+20\,Bab^3+5\,Ab^4)\cos(dx+c))\sin(dx+c)}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1680*(105*(8*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*d*
x + (240*B*b^4*cos(d*x + c)^6 + 280*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^5 + 11
20*B*a^4 + 4480*A*a^3*b + 5376*B*a^2*b^2 + 3584*A*a*b^3 + 768*B*b^4 + 96*(2
1*B*a^2*b^2 + 14*A*a*b^3 + 3*B*b^4)*cos(d*x + c)^4 + 70*(24*B*a^3*b + 36*A*
a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*cos(d*x + c)^3 + 16*(35*B*a^4 + 140*A*a^3*b
 + 168*B*a^2*b^2 + 112*A*a*b^3 + 24*B*b^4)*cos(d*x + c)^2 + 105*(8*A*a^4 +
24*B*a^3*b + 36*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*cos(d*x + c))*sin(d*x + c
))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. 2(391) = 782.
time = 0.79, size = 1017, normalized size = 2.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**4*x*sin(c + d*x)**2/2 + A*a**4*x*cos(c + d*x)**2/2 + A*a**4
*sin(c + d*x)*cos(c + d*x)/(2*d) + 8*A*a**3*b*sin(c + d*x)**3/(3*d) + 4*A*a
**3*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*A*a**2*b**2*x*sin(c + d*x)**4/4 +
9*A*a**2*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 9*A*a**2*b**2*x*cos(c +
 d*x)**4/4 + 9*A*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 15*A*a**2*b
**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 32*A*a*b**3*sin(c + d*x)**5/(15*d)
 + 16*A*a*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 4*A*a*b**3*sin(c + d
*x)*cos(c + d*x)**4/d + 5*A*b**4*x*sin(c + d*x)**6/16 + 15*A*b**4*x*sin(c +
 d*x)**4*cos(c + d*x)**2/16 + 15*A*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/1
6 + 5*A*b**4*x*cos(c + d*x)**6/16 + 5*A*b**4*sin(c + d*x)**5*cos(c + d*x)/(
16*d) + 5*A*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*A*b**4*sin(c +
d*x)*cos(c + d*x)**5/(16*d) + 2*B*a**4*sin(c + d*x)**3/(3*d) + B*a**4*sin(c
 + d*x)*cos(c + d*x)**2/d + 3*B*a**3*b*x*sin(c + d*x)**4/2 + 3*B*a**3*b*x*s
in(c + d*x)**2*cos(c + d*x)**2 + 3*B*a**3*b*x*cos(c + d*x)**4/2 + 3*B*a**3*
b*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 5*B*a**3*b*sin(c + d*x)*cos(c + d*x)
**3/(2*d) + 16*B*a**2*b**2*sin(c + d*x)**5/(5*d) + 8*B*a**2*b**2*sin(c + d*
x)**3*cos(c + d*x)**2/d + 6*B*a**2*b**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*
B*a*b**3*x*sin(c + d*x)**6/4 + 15*B*a*b**3*x*sin(c + d*x)**4*cos(c + d*x)**
```

```
2/4 + 15*B*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/4 + 5*B*a*b**3*x*cos(c
+ d*x)**6/4 + 5*B*a*b**3*sin(c + d*x)**5*cos(c + d*x)/(4*d) + 10*B*a*b**3*s
in(c + d*x)**3*cos(c + d*x)**3/(3*d) + 11*B*a*b**3*sin(c + d*x)*cos(c + d*x
)**5/(4*d) + 16*B*b**4*sin(c + d*x)**7/(35*d) + 8*B*b**4*sin(c + d*x)**5*co
s(c + d*x)**2/(5*d) + 2*B*b**4*sin(c + d*x)**3*cos(c + d*x)**4/d + B*b**4*s
in(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))*
*4*cos(c)**2, True))
```

Giac [**A**]
time = 0.51, size = 313, normalized size = 0.86

$$\frac{Bb^4\sin(7\,dx+7\,c)}{448\,d}+\frac{1}{16}\left(8\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+20\,Bab^3+5\,Ab^4\right)x+\frac{(4\,Bab^3+Ab^4)\sin(6\,dx+6\,c)}{192\,d}+\frac{(24\,Ba^2b^2+16\,Aab^3+7\,Bb^4)\sin(5\,dx+5\,c)}{320\,d}+\frac{(8\,Ba^3b+12\,Aa^2b^2+12\,Bab^3+3\,Ab^4)\sin(4\,dx+4\,c)}{64\,d}+\frac{(16\,Ba^4+64\,Aa^3b+120\,Ba^2b^2+80\,Aab^3+21\,Bb^4)\sin(3\,dx+3\,c)}{192\,d}+\frac{(16\,Aa^4+64\,Ba^3b+96\,Aa^2b^2+60\,Bab^3+15\,Ab^4)\sin(2\,dx+2\,c)}{64\,d}+\frac{(48\,Ba^4+192\,Aa^3b+240\,Ba^2b^2+160\,Aab^3+35\,Bb^4)\sin(dx+c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/448*B*b^4*sin(7*d*x + 7*c)/d + 1/16*(8*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2
+ 20*B*a*b^3 + 5*A*b^4)*x + 1/192*(4*B*a*b^3 + A*b^4)*sin(6*d*x + 6*c)/d +
1/320*(24*B*a^2*b^2 + 16*A*a*b^3 + 7*B*b^4)*sin(5*d*x + 5*c)/d + 1/64*(8*B*
a^3*b + 12*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*sin(4*d*x + 4*c)/d + 1/192*(16
*B*a^4 + 64*A*a^3*b + 120*B*a^2*b^2 + 80*A*a*b^3 + 21*B*b^4)*sin(3*d*x + 3*
c)/d + 1/64*(16*A*a^4 + 64*B*a^3*b + 96*A*a^2*b^2 + 60*B*a*b^3 + 15*A*b^4)*
sin(2*d*x + 2*c)/d + 1/64*(48*B*a^4 + 192*A*a^3*b + 240*B*a^2*b^2 + 160*A*a
*b^3 + 35*B*b^4)*sin(d*x + c)/d
```

Mupad [**B**]
time = 2.64, size = 436, normalized size = 1.19

$$\frac{420\,A\,a^4\,\sin(2\,c+2\,d\,x)+\frac{1575\,A\,b^4\,\sin(2\,c+2\,d\,x)}{4}+140\,B\,a^4\,\sin(3\,c+3\,d\,x)+\frac{315\,A\,b^4\,\sin(4\,c+4\,d\,x)}{4}+\frac{35\,A\,b^4\,\sin(6\,c+6\,d\,x)}{4}+\frac{735\,B\,b^4\,\sin(3\,c+3\,d\,x)}{4}+\frac{147\,B\,b^4\,\sin(5\,c+5\,d\,x)}{4}+\frac{15\,B\,b^4\,\sin(7\,c+7\,d\,x)}{4}+1260\,B\,a^4\,\sin(c+d\,x)+\frac{3675\,B\,b^4\,\sin(c+d\,x)}{4}+4200\,A\,a\,b^3\,\sin(c+d\,x)+5040\,A\,a^3\,b\,\sin(c+d\,x)+840\,A\,a^4\,d\,x+525\,A\,b^4\,d\,x+700\,A\,a\,b^3\,\sin(3\,c+3\,d\,x)+560\,A\,a^3\,b\,\sin(3\,c+3\,d\,x)+84\,A\,a\,b^3\,\sin(5\,c+5\,d\,x)+1575\,B\,a\,b^3\,\sin(2\,c+2\,d\,x)+1680\,B\,a^3\,b\,\sin(2\,c+2\,d\,x)+315\,B\,a\,b^3\,\sin(4\,c+4\,d\,x)+210\,B\,a^3\,b\,\sin(4\,c+4\,d\,x)+35\,B\,a\,b^3\,\sin(6\,c+6\,d\,x)+6300\,B\,a^2\,b^2\,\sin(c+d\,x)+2520\,A\,a^2\,b^2\,\sin(2\,c+2\,d\,x)+315\,A\,a^2\,b^2\,\sin(4\,c+4\,d\,x)+1050\,B\,a^2\,b^2\,\sin(3\,c+3\,d\,x)+126\,B\,a^2\,b^2\,\sin(5\,c+5\,d\,x)+2100\,B\,a\,b^3\,d\,x+2520\,B\,a^3\,b\,d\,x+3780\,A\,a^2\,b^2\,d\,x}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4,x)
```

```
[Out] (420*A*a^4*sin(2*c + 2*d*x) + (1575*A*b^4*sin(2*c + 2*d*x))/4 + 140*B*a^4*s
in(3*c + 3*d*x) + (315*A*b^4*sin(4*c + 4*d*x))/4 + (35*A*b^4*sin(6*c + 6*d*
x))/4 + (735*B*b^4*sin(3*c + 3*d*x))/4 + (147*B*b^4*sin(5*c + 5*d*x))/4 + (
15*B*b^4*sin(7*c + 7*d*x))/4 + 1260*B*a^4*sin(c + d*x) + (3675*B*b^4*sin(c
+ d*x))/4 + 4200*A*a*b^3*sin(c + d*x) + 5040*A*a^3*b*sin(c + d*x) + 840*A*a
^4*d*x + 525*A*b^4*d*x + 700*A*a*b^3*sin(3*c + 3*d*x) + 560*A*a^3*b*sin(3*c
 + 3*d*x) + 84*A*a*b^3*sin(5*c + 5*d*x) + 1575*B*a*b^3*sin(2*c + 2*d*x) + 1
680*B*a^3*b*sin(2*c + 2*d*x) + 315*B*a*b^3*sin(4*c + 4*d*x) + 210*B*a^3*b*s
in(4*c + 4*d*x) + 35*B*a*b^3*sin(6*c + 6*d*x) + 6300*B*a^2*b^2*sin(c + d*x)
 + 2520*A*a^2*b^2*sin(2*c + 2*d*x) + 315*A*a^2*b^2*sin(4*c + 4*d*x) + 1050*
B*a^2*b^2*sin(3*c + 3*d*x) + 126*B*a^2*b^2*sin(5*c + 5*d*x) + 2100*B*a*b^3*
d*x + 2520*B*a^3*b*d*x + 3780*A*a^2*b^2*d*x)/(1680*d)
```

3.241 $\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=325

$$\frac{1}{16}\left(32a^3Ab+24aAb^3+8a^4B+36a^2b^2B+5b^4B\right)x+\frac{\left(24a^4Ab+224a^2Ab^3+32Ab^5-4a^5B+121a^3b^2B+12\right.}{60bd}$$

```
[Out] 1/16*(32*A*a^3*b+24*A*a*b^3+8*B*a^4+36*B*a^2*b^2+5*B*b^4)*x+1/60*(24*A*a^4*
b+224*A*a^2*b^3+32*A*b^5-4*B*a^5+121*B*a^3*b^2+128*B*a*b^4)*sin(d*x+c)/b/d+
1/240*(48*A*a^3*b+232*A*a*b^3-8*B*a^4+178*B*a^2*b^2+75*B*b^4)*cos(d*x+c)*si
n(d*x+c)/d+1/120*(24*A*a^2*b+32*A*b^3-4*B*a^3+53*B*a*b^2)*(a+b*cos(d*x+c))^
2*sin(d*x+c)/b/d+1/120*(24*A*a*b-4*B*a^2+25*B*b^2)*(a+b*cos(d*x+c))^3*sin(d
*x+c)/b/d+1/30*(6*A*b-B*a)*(a+b*cos(d*x+c))^4*sin(d*x+c)/b/d+1/6*B*(a+b*cos
(d*x+c))^5*sin(d*x+c)/b/d
```

Rubi [A]
time = 0.33, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3047, 3102, 2832, 2813}

$$\frac{(-4a^2B+24aAb+25b^2B)\sin(c+dx)(a+b\cos(c+dx))^3}{120bd}+\frac{(-4a^3B+24a^2Ab+53ab^2B+32Ab^3)\sin(c+dx)(a+b\cos(c+dx))^2}{120bd}+\frac{(-8a^4B+48a^3Ab+178a^2b^2B+232aAb^3+75b^4B)\sin(c+dx)\cos(c+dx)}{240d}+\frac{1}{16}x(8a^4B+32a^3Ab+36a^2b^2B+24aAb^3+5b^4B)+\frac{(-4a^5B+24a^4Ab+121a^3b^2B+224a^2Ab^3+128ab^4B+32Ab^5)\sin(c+dx)}{60bd}+\frac{(6Ab-aB)\sin(c+dx)(a+b\cos(c+dx))^4}{30bd}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B)*x)/16 + ((24*
a^4*A*b + 224*a^2*A*b^3 + 32*A*b^5 - 4*a^5*B + 121*a^3*b^2*B + 128*a*b^4*B)
*Sin[c + d*x])/(60*b*d) + ((48*a^3*A*b + 232*a*A*b^3 - 8*a^4*B + 178*a^2*b^
2*B + 75*b^4*B)*Cos[c + d*x]*Sin[c + d*x])/(240*d) + ((24*a^2*A*b + 32*A*b^
3 - 4*a^3*B + 53*a*b^2*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(120*b*d) +
((24*a*A*b - 4*a^2*B + 25*b^2*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(120*
b*d) + ((6*A*b - a*B)*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(30*b*d) + (B*(a
 + b*Cos[c + d*x])^5*Sin[c + d*x])/(6*b*d)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^4\left(A\cos(c+dx)+B\cos^2\right.$$

$$= \frac{B(a+b\cos(c+dx))^5\sin(c+dx)}{6bd} + \frac{\int(a+b\cos}{}$$

$$= \frac{(6Ab-aB)(a+b\cos(c+dx))^4\sin(c+dx)}{30bd} +$$

$$= \frac{\left(24aAb-4a^2B+25b^2B\right)(a+b\cos(c+dx))^3}{120bd}$$

$$= \frac{\left(24a^2Ab+32Ab^3-4a^3B+53ab^2B\right)(a+b\cos}{120bd}$$

$$= \frac{1}{16}\left(32a^3Ab+24aAb^3+8a^4B+36a^2b^2B+5b^4\right.$$

Mathematica [A]
time = 0.75, size = 333, normalized size = 1.02

$$\frac{1920a^3Abc+1440aAb^3c+480a^4Bc+2160a^2b^2Bc+300b^4Bc+1920a^3Abdx+1440aAb^3dx+480a^4Bdx+2160a^2b^2Bdx+300b^4Bdx+120\left(8a^4A+36a^2Ab^2+5Ab^4+24a^3bB+20ab^3B\right)\sin(c+dx)+15\left(64a^3Ab+64aAb^3+16a^4B+96a^2b^2B+15b^4B\right)\sin(2(c+dx))+480a^2Ab^2\sin(3(c+dx))+100Ab^4\sin(3(c+dx))+320a^3bB\sin(3(c+dx))+400ab^3B\sin(3(c+dx))+120aAb^3\sin(4(c+dx))+180a^2b^2B\sin(4(c+dx))+45b^4B\sin(4(c+dx))+12Ab^4\sin(5(c+dx))+48ab^3B\sin(5(c+dx))+5b^4B\sin(6(c+dx))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (1920*a^3*A*b*c + 1440*a*A*b^3*c + 480*a^4*B*c + 2160*a^2*b^2*B*c + 300*b^4
*B*c + 1920*a^3*A*b*d*x + 1440*a*A*b^3*d*x + 480*a^4*B*d*x + 2160*a^2*b^2*B
*d*x + 300*b^4*B*d*x + 120*(8*a^4*A + 36*a^2*A*b^2 + 5*A*b^4 + 24*a^3*b*B +
 20*a*b^3*B)*Sin[c + d*x] + 15*(64*a^3*A*b + 64*a*A*b^3 + 16*a^4*B + 96*a^2
*b^2*B + 15*b^4*B)*Sin[2*(c + d*x)] + 480*a^2*A*b^2*Sin[3*(c + d*x)] + 100*
A*b^4*Sin[3*(c + d*x)] + 320*a^3*b*B*Sin[3*(c + d*x)] + 400*a*b^3*B*Sin[3*(
c + d*x)] + 120*a*A*b^3*Sin[4*(c + d*x)] + 180*a^2*b^2*B*Sin[4*(c + d*x)] +
 45*b^4*B*Sin[4*(c + d*x)] + 12*A*b^4*Sin[5*(c + d*x)] + 48*a*b^3*B*Sin[5*(
c + d*x)] + 5*b^4*B*Sin[6*(c + d*x)])/(960*d)
```

Maple [A]
time = 0.22, size = 316, normalized size = 0.97 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^4*sin(d*x+c)+a^4*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+4*A*a
^3*b*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+4/3*B*a^3*b*(cos(d*x+c)^2+2)
*sin(d*x+c)+2*A*a^2*b^2*(cos(d*x+c)^2+2)*sin(d*x+c)+6*B*a^2*b^2*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*A*a*b^3*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/5*B*a*b^3*(8/3+cos(d*x+c)^4
+4/3*cos(d*x+c)^2)*sin(d*x+c)+1/5*A*b^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)
*sin(d*x+c)+B*b^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(
d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.29, size = 307, normalized size = 0.94

$$\frac{240\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba^4+960\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa^3b-1280\,\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Ba^3b-1920\,\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Aa^2b^2+180\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))Ba^2b^2+120\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))Aab^3+256\,\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)Bab^3+64\,\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)Ab^4-5\,\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)Bb^4+960\,Aa^4\sin\left(dx+c\right)}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/960*(240*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 + 960*(2*d*x + 2*c + sin(
2*d*x + 2*c))*A*a^3*b - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3*b - 19
20*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2*b^2 + 180*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2*b^2 + 120*(12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b^3 + 256*(3*sin(d*x + c)^5 - 10*sin(d*x
 + c)^3 + 15*sin(d*x + c))*B*a*b^3 + 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)
^3 + 15*sin(d*x + c))*A*b^4 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*s
in(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*b^4 + 960*A*a^4*sin(d*x + c))/d
```

Fricas [A]
time = 0.39, size = 243, normalized size = 0.75

$$\frac{15\,(8\,Ba^4+32\,Aa^3b+36\,Ba^2b^2+24\,Aab^3+5\,Bb^4)dx+\left(40\,Bb^4\cos\left(dx+c\right)^5+240\,Aa^4+640\,Ba^3b+960\,Aa^2b^2+512\,Bab^3+128\,Ab^4+48\,(4\,Bab^3+Ab^4)\cos\left(dx+c\right)^4+10\,(36\,Ba^2b^2+24\,Aab^3+5\,Bb^4)\cos\left(dx+c\right)^3+32\,(10\,Ba^3b+15\,Aa^2b^2+8\,Bab^3+2\,Ab^4)\cos\left(dx+c\right)^2+15\,(8\,Ba^4+32\,Aa^3b+36\,Ba^2b^2+24\,Aab^3+5\,Bb^4)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/240*(15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*d*x
+ (40*B*b^4*cos(d*x + c)^5 + 240*A*a^4 + 640*B*a^3*b + 960*A*a^2*b^2 + 512*
B*a*b^3 + 128*A*b^4 + 48*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 10*(36*B*a^2*
b^2 + 24*A*a*b^3 + 5*B*b^4)*cos(d*x + c)^3 + 32*(10*B*a^3*b + 15*A*a^2*b^2
+ 8*B*a*b^3 + 2*A*b^4)*cos(d*x + c)^2 + 15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2
*b^2 + 24*A*a*b^3 + 5*B*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(335) = 670$.
time = 0.56, size = 811, normalized size = 2.50

$$\begin{cases} \frac{Aa^4\sin(c+dx)}{d} + 2Aa^3bx\sin^2(c+dx) + 2Aa^3bx\cos^2(c+dx) + \frac{2Aa^3b\sin(c+dx)\cos(c+dx)}{d} + \frac{4Aa^2b^2\sin^3(c+dx)}{d} + \frac{6Aa^2b^2\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Aab^3x\sin^4(c+dx)}{2} + 3Aab^3x\sin^2(c+dx)\cos^2(c+dx) + \frac{3Aab^3x\cos^4(c+dx)}{2} + \frac{3Aab^3\sin^3(c+dx)\cos(c+dx)}{2d} + \frac{5Aab^3\sin(c+dx)\cos^3(c+dx)}{2d} + \frac{8Ab^4\sin^5(c+dx)}{15d} + \frac{4Ab^4\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{Ab^4\sin(c+dx)\cos^4(c+dx)}{d} + \frac{Ba^4x\sin^2(c+dx)}{2} + \frac{Ba^4x\cos^2(c+dx)}{2} + \frac{Ba^4\sin(c+dx)\cos(c+dx)}{2d} + \frac{8Ba^3b\sin^3(c+dx)}{3d} + \frac{4Ba^3b\sin(c+dx)\cos^2(c+dx)}{d} + \frac{9Ba^2b^2x\sin^4(c+dx)}{4} + \frac{9Ba^2b^2x\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{9Ba^2b^2x\cos^4(c+dx)}{4} + \frac{9Ba^2b^2\sin^3(c+dx)\cos(c+dx)}{4d} + \frac{15Ba^2b^2\sin(c+dx)\cos^3(c+dx)}{4d} + \frac{32Bab^3\sin^5(c+dx)}{15d} + \frac{16Bab^3\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{4Bab^3\sin(c+dx)\cos^4(c+dx)}{d} + \frac{5Bb^4x\sin^6(c+dx)}{16} + \frac{15Bb^4x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15Bb^4x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5Bb^4x\cos^6(c+dx)}{16} + \frac{5Bb^4\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{5Bb^4\sin^3(c+dx)\cos^3(c+dx)}{6d} + \frac{11Bb^4\sin(c+dx)\cos^5(c+dx)}{16d} & \text{for}\: d \neq 0 \\ x(A + B\cos(c))(a + b\cos(c))^4\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**4*sin(c + d*x)/d + 2*A*a**3*b*x*sin(c + d*x)**2 + 2*A*a**3*
b*x*cos(c + d*x)**2 + 2*A*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*A*a**2*b**
2*sin(c + d*x)**3/d + 6*A*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a*
b**3*x*sin(c + d*x)**4/2 + 3*A*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3
*A*a*b**3*x*cos(c + d*x)**4/2 + 3*A*a*b**3*sin(c + d*x)**3*cos(c + d*x)/(2*
d) + 5*A*a*b**3*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 8*A*b**4*sin(c + d*x)*
*5/(15*d) + 4*A*b**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + A*b**4*sin(c +
 d*x)*cos(c + d*x)**4/d + B*a**4*x*sin(c + d*x)**2/2 + B*a**4*x*cos(c + d*x
)**2/2 + B*a**4*sin(c + d*x)*cos(c + d*x)/(2*d) + 8*B*a**3*b*sin(c + d*x)**
3/(3*d) + 4*B*a**3*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*B*a**2*b**2*x*sin(c
 + d*x)**4/4 + 9*B*a**2*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 9*B*a**2
*b**2*x*cos(c + d*x)**4/4 + 9*B*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d
) + 15*B*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 32*B*a*b**3*sin(c +
 d*x)**5/(15*d) + 16*B*a*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 4*B*a
*b**3*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*b**4*x*sin(c + d*x)**6/16 + 15*B
*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*B*b**4*x*sin(c + d*x)**2*co
s(c + d*x)**4/16 + 5*B*b**4*x*cos(c + d*x)**6/16 + 5*B*b**4*sin(c + d*x)**5
*cos(c + d*x)/(16*d) + 5*B*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*
B*b**4*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(A + B*cos(c))*(a
 + b*cos(c))**4*cos(c), True))
```

Giac [A]
time = 0.47, size = 263, normalized size = 0.81

$$\frac{Bb^4\sin(6\,dx+6\,c)}{192\,d} + \frac{1}{16}\left(8\,Ba^4 + 32\,Aa^3b + 36\,Ba^2b^2 + 24\,Aab^3 + 5\,Bb^4\right)x + \frac{(4\,Bab^3 + Ab^4)\sin(5\,dx+5\,c)}{80\,d} + \frac{(12\,Ba^2b^2 + 8\,Aab^3 + 3\,Bb^4)\sin(4\,dx+4\,c)}{64\,d} + \frac{(16\,Ba^3b + 24\,Aa^2b^2 + 20\,Bab^3 + 5\,Ab^4)\sin(3\,dx+3\,c)}{48\,d} + \frac{(16\,Ba^4 + 64\,Aa^3b + 96\,Ba^2b^2 + 64\,Aab^3 + 15\,Bb^4)\sin(2\,dx+2\,c)}{64\,d} + \frac{(8\,Aa^4 + 24\,Ba^3b + 36\,Aa^2b^2 + 20\,Bab^3 + 5\,Ab^4)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/192*B*b^4*sin(6*d*x + 6*c)/d + 1/16*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2
+ 24*A*a*b^3 + 5*B*b^4)*x + 1/80*(4*B*a*b^3 + A*b^4)*sin(5*d*x + 5*c)/d + 1
/64*(12*B*a^2*b^2 + 8*A*a*b^3 + 3*B*b^4)*sin(4*d*x + 4*c)/d + 1/48*(16*B*a^
3*b + 24*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*sin(3*d*x + 3*c)/d + 1/64*(16*B*
a^4 + 64*A*a^3*b + 96*B*a^2*b^2 + 64*A*a*b^3 + 15*B*b^4)*sin(2*d*x + 2*c)/d
 + 1/8*(8*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*sin(d*x
 + c)/d
```

Mupad [B]
time = 1.37, size = 403, normalized size = 1.24

$$\frac{B\,a^4\,x}{2}+\frac{5\,B\,b^4\,x}{16}+\frac{3\,A\,a\,b^3\,x}{2}+2\,A\,a^3\,b\,x+\frac{A\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{5\,A\,b^4\,\sin\left(c+d\,x\right)}{8\,d}+\frac{9\,B\,a^2\,b^2\,x}{4}+\frac{B\,a^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{5\,A\,b^4\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}+\frac{A\,b^4\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}+\frac{15\,B\,b^4\,\sin\left(2\,c+2\,d\,x\right)}{64\,d}+\frac{3\,B\,b^4\,\sin\left(4\,c+4\,d\,x\right)}{64\,d}+\frac{B\,b^4\,\sin\left(6\,c+6\,d\,x\right)}{192\,d}+\frac{A\,a\,b^3\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{A\,a^3\,b\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{A\,a\,b^3\,\sin\left(4\,c+4\,d\,x\right)}{8\,d}+\frac{9\,A\,a^2\,b^2\,\sin\left(c+d\,x\right)}{2\,d}+\frac{5\,B\,a\,b^3\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{B\,a^3\,b\,\sin\left(3\,c+3\,d\,x\right)}{3\,d}+\frac{B\,a\,b^3\,\sin\left(5\,c+5\,d\,x\right)}{20\,d}+\frac{A\,a^2\,b^2\,\sin\left(3\,c+3\,d\,x\right)}{2\,d}+\frac{3\,B\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{3\,B\,a^2\,b^2\,\sin\left(4\,c+4\,d\,x\right)}{16\,d}+\frac{5\,B\,a\,b^3\,\sin\left(c+d\,x\right)}{2\,d}+\frac{3\,B\,a^3\,b\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4,x)
```

```
[Out] (B*a^4*x)/2 + (5*B*b^4*x)/16 + (3*A*a*b^3*x)/2 + 2*A*a^3*b*x + (A*a^4*sin(c
 + d*x))/d + (5*A*b^4*sin(c + d*x))/(8*d) + (9*B*a^2*b^2*x)/4 + (B*a^4*sin(
2*c + 2*d*x))/(4*d) + (5*A*b^4*sin(3*c + 3*d*x))/(48*d) + (A*b^4*sin(5*c +
5*d*x))/(80*d) + (15*B*b^4*sin(2*c + 2*d*x))/(64*d) + (3*B*b^4*sin(4*c + 4*
d*x))/(64*d) + (B*b^4*sin(6*c + 6*d*x))/(192*d) + (A*a*b^3*sin(2*c + 2*d*x)
)/d + (A*a^3*b*sin(2*c + 2*d*x))/d + (A*a*b^3*sin(4*c + 4*d*x))/(8*d) + (9*
A*a^2*b^2*sin(c + d*x))/(2*d) + (5*B*a*b^3*sin(3*c + 3*d*x))/(12*d) + (B*a^
3*b*sin(3*c + 3*d*x))/(3*d) + (B*a*b^3*sin(5*c + 5*d*x))/(20*d) + (A*a^2*b^
2*sin(3*c + 3*d*x))/(2*d) + (3*B*a^2*b^2*sin(2*c + 2*d*x))/(2*d) + (3*B*a^2
*b^2*sin(4*c + 4*d*x))/(16*d) + (5*B*a*b^3*sin(c + d*x))/(2*d) + (3*B*a^3*b
*sin(c + d*x))/d
```

3.242 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=241

$$\frac{1}{8}\left(8a^4A+24a^2Ab^2+3Ab^4+16a^3bB+12ab^3B\right)x+\frac{(95a^3Ab+80aAb^3+12a^4B+112a^2b^2B+16b^4B)\sin(\ldots}{30d}$$

```
[Out] 1/8*(8*A*a^4+24*A*a^2*b^2+3*A*b^4+16*B*a^3*b+12*B*a*b^3)*x+1/30*(95*A*a^3*b
+80*A*a*b^3+12*B*a^4+112*B*a^2*b^2+16*B*b^4)*sin(d*x+c)/d+1/120*b*(130*A*a^
2*b+45*A*b^3+24*B*a^3+116*B*a*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/60*(35*A*a*b+1
2*B*a^2+16*B*b^2)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/20*(5*A*b+4*B*a)*(a+b*c
os(d*x+c))^3*sin(d*x+c)/d+1/5*B*(a+b*cos(d*x+c))^4*sin(d*x+c)/d
```

Rubi [A]
time = 0.22, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2832, 2813}

$$\frac{(12a^2B+35aAb+16b^2B)\sin(c+dx)(a+b\cos(c+dx))^2}{60d}+\frac{b(24a^3B+130a^2Ab+116ab^2B+45Ab^3)\sin(c+dx)\cos(c+dx)}{120d}+\frac{(12a^4B+95a^3Ab+112a^2b^2B+80aAb^3+16b^4B)\sin(c+dx)}{30d}+\frac{1}{8}x\left(8a^4A+16a^3bB+24a^2Ab^2+12ab^3B+3Ab^4\right)+\frac{(4aB+5Ab)\sin(c+dx)(a+b\cos(c+dx))^3}{20d}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((8*a^4*A + 24*a^2*A*b^2 + 3*A*b^4 + 16*a^3*b*B + 12*a*b^3*B)*x)/8 + ((95*a
^3*A*b + 80*a*A*b^3 + 12*a^4*B + 112*a^2*b^2*B + 16*b^4*B)*Sin[c + d*x])/(3
0*d) + (b*(130*a^2*A*b + 45*A*b^3 + 24*a^3*B + 116*a*b^2*B)*Cos[c + d*x]*Si
n[c + d*x])/(120*d) + ((35*a*A*b + 12*a^2*B + 16*b^2*B)*(a + b*Cos[c + d*x]
)^2*Sin[c + d*x])/(60*d) + ((5*A*b + 4*a*B)*(a + b*Cos[c + d*x])^3*Sin[c +
d*x])/(20*d) + (B*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*d)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{B(a+b\cos(c+dx))^4\sin(c+dx)}{5d} + \frac{1}{5}\int (a+b\cos(c+dx))$$

$$= \frac{(5Ab+4aB)(a+b\cos(c+dx))^3\sin(c+dx)}{20d} + \frac{B(a+b\cos(}{}$$

$$= \frac{\left(35aAb+12a^2B+16b^2B\right)(a+b\cos(c+dx))^2\sin(c+dx)}{60d}$$

$$= \frac{1}{8}\left(8a^4A+24a^2Ab^2+3Ab^4+16a^3bB+12ab^3B\right)x + \frac{(95a^3A}{}$$

Mathematica [**A**]
time = 0.43, size = 263, normalized size = 1.09

$$\frac{480a^4Ac+1440a^2Ab^2c+180Ab^4c+960a^3bBc+720ab^3Bc+480a^4Adx+1440a^2Ab^2dx+180Ab^4dx+960a^3bBdx+720ab^3Bdx+60(32a^3Ab+24aAb^3+8a^4B+36a^2b^2B+5b^4B)\sin(c+dx)+120b(6a^2Ab+Ab^3+4a^3B+4ab^2B)\sin(2(c+dx))+160aAb^3\sin(3(c+dx))+240a^2b^2B\sin(3(c+dx))+50b^4B\sin(3(c+dx))+15Ab^4\sin(4(c+dx))+60ab^3B\sin(4(c+dx))+6b^4B\sin(5(c+dx))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (480*a^4*A*c + 1440*a^2*A*b^2*c + 180*A*b^4*c + 960*a^3*b*B*c + 720*a*b^3*B
*c + 480*a^4*A*d*x + 1440*a^2*A*b^2*d*x + 180*A*b^4*d*x + 960*a^3*b*B*d*x +
 720*a*b^3*B*d*x + 60*(32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5
*b^4*B)*Sin[c + d*x] + 120*b*(6*a^2*A*b + A*b^3 + 4*a^3*B + 4*a*b^2*B)*Sin[
2*(c + d*x)] + 160*a*A*b^3*Sin[3*(c + d*x)] + 240*a^2*b^2*B*Sin[3*(c + d*x)
] + 50*b^4*B*Sin[3*(c + d*x)] + 15*A*b^4*Sin[4*(c + d*x)] + 60*a*b^3*B*Sin[
4*(c + d*x)] + 6*b^4*B*Sin[5*(c + d*x)])/(480*d)
```

Maple [**A**]
time = 0.17, size = 258, normalized size = 1.07 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*B*b^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*b^4*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*B*a*b^3*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*A*a*b^3*(cos(d*x+c)^2
+2)*sin(d*x+c)+2*B*a^2*b^2*(cos(d*x+c)^2+2)*sin(d*x+c)+6*A*a^2*b^2*(1/2*sin
(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+4*B*a^3*b*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*
d*x+1/2*c)+4*A*a^3*b*sin(d*x+c)+a^4*B*sin(d*x+c)+A*a^4*(d*x+c))
```

Maxima [**A**]
time = 0.28, size = 246, normalized size = 1.02

$$\frac{480\,(dx+c)Aa^4+480\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^3b+720\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2b^2-960\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^2b^2-640\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Aab^3+60\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Bab^3+15\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Ab^4+32\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c)\right)Bb^4+480\,Ba^4\sin(dx+c)+1920\,Aa^3b\sin(dx+c)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(480*(d*x + c)*A*a^4 + 480*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3*b +
 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2*b^2 - 960*(sin(d*x + c)^3 - 3*s
in(d*x + c))*B*a^2*b^2 - 640*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b^3 + 60
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a*b^3 + 15*(12*d
*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*b^4 + 32*(3*sin(d*x +
c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*b^4 + 480*B*a^4*sin(d*x + c)
+ 1920*A*a^3*b*sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 197, normalized size = 0.82

$$\frac{15\,(8\,Aa^4+16\,Ba^3b+24\,Aa^2b^2+12\,Bab^3+3\,Ab^4)dx+(24\,Bb^4\cos(dx+c)^4+120\,Ba^4+480\,Aa^3b+480\,Ba^2b^2+320\,Aab^3+64\,Bb^4+30\,(4\,Bab^3+Ab^4)\cos(dx+c)^3+16\,(15\,Ba^2b^2+10\,Aab^3+2\,Bb^4)\cos(dx+c)^2+15\,(16\,Ba^3b+24\,Aa^2b^2+12\,Bab^3+3\,Ab^4)\cos(dx+c))\sin(dx+c)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*d*x
+ (24*B*b^4*cos(d*x + c)^4 + 120*B*a^4 + 480*A*a^3*b + 480*B*a^2*b^2 + 320*
A*a*b^3 + 64*B*b^4 + 30*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^3 + 16*(15*B*a^2*b
^2 + 10*A*a*b^3 + 2*B*b^4)*cos(d*x + c)^2 + 15*(16*B*a^3*b + 24*A*a^2*b^2 +
 12*B*a*b^3 + 3*A*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(248) = 496.
time = 0.37, size = 580, normalized size = 2.41

$$\begin{cases} Aa^4x+\frac{4Aa^3b\sin(c+dx)}{d}+3Aa^2b^2x\sin^2(c+dx)+3Aa^2b^2x\cos^2(c+dx)+\frac{3Aa^2b^2\sin(c+dx)\cos(c+dx)}{d}+\frac{8Aab^3\sin^3(c+dx)}{3d}+\frac{4Aab^3\sin(c+dx)\cos^2(c+dx)}{d}+\frac{3Ab^4x\sin^4(c+dx)}{8}+\frac{3Ab^4x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3Ab^4x\cos^4(c+dx)}{8}+\frac{3Ab^4\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5Ab^4\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{Ba^4\sin(c+dx)}{d}+2Ba^3bx\sin^2(c+dx)+2Ba^3bx\cos^2(c+dx)+\frac{2Ba^3b\sin(c+dx)\cos(c+dx)}{d}+\frac{4Ba^2b^2\sin^3(c+dx)}{d}+\frac{6Ba^2b^2\sin(c+dx)\cos^2(c+dx)}{d}+\frac{3Bab^3x\sin^4(c+dx)}{2}+3Bab^3x\sin^2(c+dx)\cos^2(c+dx)+\frac{3Bab^3x\cos^4(c+dx)}{2}+\frac{3Bab^3\sin^3(c+dx)\cos(c+dx)}{2d}+\frac{5Bab^3\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{8Bb^4\sin^5(c+dx)}{15d}+\frac{4Bb^4\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{Bb^4\sin(c+dx)\cos^4(c+dx)}{d} & \text{for } d\neq 0 \\ x(A+B\cos(c))\,(a+b\cos(c))^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**4*x + 4*A*a**3*b*sin(c + d*x)/d + 3*A*a**2*b**2*x*sin(c + d
*x)**2 + 3*A*a**2*b**2*x*cos(c + d*x)**2 + 3*A*a**2*b**2*sin(c + d*x)*cos(c
 + d*x)/d + 8*A*a*b**3*sin(c + d*x)**3/(3*d) + 4*A*a*b**3*sin(c + d*x)*cos(
c + d*x)**2/d + 3*A*b**4*x*sin(c + d*x)**4/8 + 3*A*b**4*x*sin(c + d*x)**2*c
os(c + d*x)**2/4 + 3*A*b**4*x*cos(c + d*x)**4/8 + 3*A*b**4*sin(c + d*x)**3*
cos(c + d*x)/(8*d) + 5*A*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**4*s
in(c + d*x)/d + 2*B*a**3*b*x*sin(c + d*x)**2 + 2*B*a**3*b*x*cos(c + d*x)**2
 + 2*B*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*B*a**2*b**2*sin(c + d*x)**3/d
 + 6*B*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a*b**3*x*sin(c + d*x)
**4/2 + 3*B*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3*B*a*b**3*x*cos(c +
 d*x)**4/2 + 3*B*a*b**3*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 5*B*a*b**3*sin
```

```
(c + d*x)*cos(c + d*x)**3/(2*d) + 8*B*b**4*sin(c + d*x)**5/(15*d) + 4*B*b**
4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + B*b**4*sin(c + d*x)*cos(c + d*x)*
*4/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))**4, True))
```

Giac [A]
time = 0.45, size = 212, normalized size = 0.88

$$\frac{Bb^4\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{8}\left(8\,Aa^4+16\,Ba^3b+24\,Aa^2b^2+12\,Bab^3+3\,Ab^4\right)x+\frac{(4\,Bab^3+Ab^4)\sin(4\,dx+4\,c)}{32\,d}+\frac{(24\,Ba^2b^2+16\,Aab^3+5\,Bb^4)\sin(3\,dx+3\,c)}{48\,d}+\frac{(4\,Ba^3b+6\,Aa^2b^2+4\,Bab^3+Ab^4)\sin(2\,dx+2\,c)}{4\,d}+\frac{(8\,Ba^4+32\,Aa^3b+36\,Ba^2b^2+24\,Aab^3+5\,Bb^4)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/80*B*b^4*sin(5*d*x + 5*c)/d + 1/8*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 +
12*B*a*b^3 + 3*A*b^4)*x + 1/32*(4*B*a*b^3 + A*b^4)*sin(4*d*x + 4*c)/d + 1/4
8*(24*B*a^2*b^2 + 16*A*a*b^3 + 5*B*b^4)*sin(3*d*x + 3*c)/d + 1/4*(4*B*a^3*b
 + 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*sin(2*d*x + 2*c)/d + 1/8*(8*B*a^4 + 32*
A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*sin(d*x + c)/d
```

Mupad [B]
time = 0.88, size = 307, normalized size = 1.27

$$A\,a^4\,x+\frac{3\,A\,b^4\,x}{8}+\frac{3\,B\,a\,b^3\,x}{2}+2\,B\,a^3\,b\,x+\frac{B\,a^4\,\sin(c+d\,x)}{d}+\frac{5\,B\,b^4\,\sin(c+d\,x)}{8\,d}+3\,A\,a^2\,b^2\,x+\frac{A\,b^4\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{A\,b^4\,\sin(4\,c+4\,d\,x)}{32\,d}+\frac{5\,B\,b^4\,\sin(3\,c+3\,d\,x)}{48\,d}+\frac{B\,b^4\,\sin(5\,c+5\,d\,x)}{80\,d}+\frac{A\,a\,b^3\,\sin(3\,c+3\,d\,x)}{3\,d}+\frac{B\,a\,b^3\,\sin(2\,c+2\,d\,x)}{d}+\frac{B\,a^3\,b\,\sin(2\,c+2\,d\,x)}{d}+\frac{B\,a\,b^3\,\sin(4\,c+4\,d\,x)}{8\,d}+\frac{9\,B\,a^2\,b^2\,\sin(c+d\,x)}{2\,d}+\frac{3\,A\,a^2\,b^2\,\sin(2\,c+2\,d\,x)}{2\,d}+\frac{B\,a^2\,b^2\,\sin(3\,c+3\,d\,x)}{2\,d}+\frac{3\,A\,a\,b^3\,\sin(c+d\,x)}{d}+\frac{4\,A\,a^3\,b\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4,x)
```

```
[Out] A*a^4*x + (3*A*b^4*x)/8 + (3*B*a*b^3*x)/2 + 2*B*a^3*b*x + (B*a^4*sin(c + d*
x))/d + (5*B*b^4*sin(c + d*x))/(8*d) + 3*A*a^2*b^2*x + (A*b^4*sin(2*c + 2*d
*x))/(4*d) + (A*b^4*sin(4*c + 4*d*x))/(32*d) + (5*B*b^4*sin(3*c + 3*d*x))/(
48*d) + (B*b^4*sin(5*c + 5*d*x))/(80*d) + (A*a*b^3*sin(3*c + 3*d*x))/(3*d)
+ (B*a*b^3*sin(2*c + 2*d*x))/d + (B*a^3*b*sin(2*c + 2*d*x))/d + (B*a*b^3*si
n(4*c + 4*d*x))/(8*d) + (9*B*a^2*b^2*sin(c + d*x))/(2*d) + (3*A*a^2*b^2*sin
(2*c + 2*d*x))/(2*d) + (B*a^2*b^2*sin(3*c + 3*d*x))/(2*d) + (3*A*a*b^3*sin(
c + d*x))/d + (4*A*a^3*b*sin(c + d*x))/d
```

3.243 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=200

$$\frac{1}{8}\left(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B\right)x+\frac{a^4A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(34a^2Ab+4Ab^3+19a^3B+\ldots}{6d}$$

```
[Out] 1/8*(32*A*a^3*b+16*A*a*b^3+8*B*a^4+24*B*a^2*b^2+3*B*b^4)*x+a^4*A*arctanh(si
n(d*x+c))/d+1/6*b*(34*A*a^2*b+4*A*b^3+19*B*a^3+16*B*a*b^2)*sin(d*x+c)/d+1/2
4*b^2*(32*A*a*b+26*B*a^2+9*B*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/12*b*(4*A*b+7*B
*a)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/4*b*B*(a+b*cos(d*x+c))^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.36, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3069, 3128, 3112, 3102, 2814, 3855}

$$\frac{a^4A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2(26a^2B+32aAb+9b^2B)\sin(c+dx)\cos(c+dx)}{24d}+\frac{b(19a^3B+34a^2Ab+16ab^2B+4Ab^3)\sin(c+dx)}{6d}+\frac{1}{8}x\left(8a^4B+32a^3Ab+24a^2b^2B+16aAb^3+3b^4B\right)+\frac{b(7aB+4Ab)\sin(c+dx)(a+b\cos(c+dx))^2}{12d}+\frac{bB\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] ((32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4*B)*x)/8 + (a^4*A
*ArcTanh[Sin[c + d*x]])/d + (b*(34*a^2*A*b + 4*A*b^3 + 19*a^3*B + 16*a*b^2*
B)*Sin[c + d*x])/(6*d) + (b^2*(32*a*A*b + 26*a^2*B + 9*b^2*B)*Cos[c + d*x]*
Sin[c + d*x])/(24*d) + (b*(4*A*b + 7*a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*
x])/(12*d) + (b*B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
```

```
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{bB(a+b\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{4}\int (a+$$

$$= \frac{b(4Ab+7aB)(a+b\cos(c+dx))^2\sin(c+dx)}{12d}$$

$$= \frac{b^2(32aAb+26a^2B+9b^2B)\cos(c+dx)\sin(c+}{24d}$$

$$= \frac{b(34a^2Ab+4Ab^3+19a^3B+16ab^2B)\sin(c+}{6d}$$

$$= \frac{1}{8}(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B$$

$$= \frac{1}{8}(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B$$

Mathematica [A]

time = 0.41, size = 210, normalized size = 1.05

$$\frac{12(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B)(c+dx)-96a^4A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+96a^4A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+24b(24a^2Ab+3Ab^3+16a^3B+12ab^2B)\sin(c+dx)+24b^2(4aAb+6a^2B+b^2B)\sin(2(c+dx))+8b^3(Ab+4aB)\sin(3(c+dx))+3b^4B\sin(4(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (12*(32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4*B)*(c + d*x)
- 96*a^4*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 96*a^4*A*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 24*b*(24*a^2*A*b + 3*A*b^3 + 16*a^3*B + 12*a
*b^2*B)*Sin[c + d*x] + 24*b^2*(4*a*A*b + 6*a^2*B + b^2*B)*Sin[2*(c + d*x)]
+ 8*b^3*(A*b + 4*a*B)*Sin[3*(c + d*x)] + 3*b^4*B*Sin[4*(c + d*x)])/(96*d)
```

Maple [A]

time = 0.22, size = 218, normalized size = 1.09

method	result
derivativedivides	$\frac{A\,a^4\ln(\sec(dx+c)+\tan(dx+c))+a^4B(dx+c)+4A\,a^3b(dx+c)+4B\,a^3b\sin(dx+c)+6A\,a^2b^2\sin(dx+c)+6B\,a^2b^2\left(\frac{\sin(dx+}{}\right.}{}$
default	$\frac{A\,a^4\ln(\sec(dx+c)+\tan(dx+c))+a^4B(dx+c)+4A\,a^3b(dx+c)+4B\,a^3b\sin(dx+c)+6A\,a^2b^2\sin(dx+c)+6B\,a^2b^2\left(\frac{\sin(dx+}{}\right.}{}$
risch	$4xA\,a^3b+2xAa\,b^3+3xB\,a^2b^2+a^4Bx+\frac{3b^4Bx}{8}-\frac{2i\mathrm{e}^{i(dx+c)}B\,a^3b}{d}-\frac{3i\mathrm{e}^{i(dx+c)}A\,a^2b^2}{d}-\frac{3i\mathrm{e}^{i(dx+}}{2}$

norman	$\frac{(4Aa^3b+2Aab^3+3Ba^2b^2+a^4B+\frac{3}{8}Bb^4)x+(4Aa^3b+2Aab^3+3Ba^2b^2+a^4B+\frac{3}{8}Bb^4)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20Aa^3b+10A}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*B*(d*x+c)+4*A*a^3*b*(d*x+c)+4*B*a^
3*b*sin(d*x+c)+6*A*a^2*b^2*sin(d*x+c)+6*B*a^2*b^2*(1/2*sin(d*x+c)*cos(d*x+c
)+1/2*d*x+1/2*c)+4*A*a*b^3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+4/3*B*
a*b^3*(cos(d*x+c)^2+2)*sin(d*x+c)+1/3*A*b^4*(cos(d*x+c)^2+2)*sin(d*x+c)+B*b
^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.27, size = 208, normalized size = 1.04

$$\frac{96\,(dx+c)Ba^4+384\,(dx+c)Aa^3b+144\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2b^2+96\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aab^3-128\,\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Bab^3-32\,\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ab^4+3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Bb^4+96\,Aa^4\log(\sec(dx+c)+\tan(dx+c))+384\,Ba^3b\sin(dx+c)+576\,Aa^2b^2\sin(dx+c)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(96*(d*x + c)*B*a^4 + 384*(d*x + c)*A*a^3*b + 144*(2*d*x + 2*c + sin(2
*d*x + 2*c))*B*a^2*b^2 + 96*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b^3 - 128*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a*b^3 - 32*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*A*b^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b
^4 + 96*A*a^4*log(sec(d*x + c) + tan(d*x + c)) + 384*B*a^3*b*sin(d*x + c) +
 576*A*a^2*b^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.40, size = 183, normalized size = 0.92

$$\frac{12\,Aa^4\log(\sin(dx+c)+1)-12\,Aa^4\log(-\sin(dx+c)+1)+3\,(8\,Ba^4+32\,Aa^3b+24\,Ba^2b^2+16\,Aab^3+3\,Bb^4)dx+\left(6\,Bb^4\cos(dx+c)^3+96\,Ba^3b+144\,Aa^2b^2+64\,Bab^3+16\,Ab^4+8\,(4\,Bab^3+Ab^4)\cos(dx+c)^2+3\,(24\,Ba^2b^2+16\,Aab^3+3\,Bb^4)\cos(dx+c)\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/24*(12*A*a^4*log(sin(d*x + c) + 1) - 12*A*a^4*log(-sin(d*x + c) + 1) + 3*
(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*d*x + (6*B*b^4
*cos(d*x + c)^3 + 96*B*a^3*b + 144*A*a^2*b^2 + 64*B*a*b^3 + 16*A*b^4 + 8*(4
*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 3*(24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*
cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))\,(a + b\cos(c + dx))^4 \sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c),x)
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**4*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(190) = 380.
time = 0.51, size = 603, normalized size = 3.02

$$\frac{24\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-24\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+3\,(8\,Ba^4+32\,Aa^3b+24\,Ba^2b^2+16\,Aab^3+3\,Bb^4)(dx+c)+\frac{2\left(96\,Ba^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+144\,Aa^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-72\,Ba^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-48\,Aab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+96\,Bab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+24\,Ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-15\,Bb^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+288\,Ba^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+432\,Aa^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-72\,Ba^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-48\,Aab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+160\,Bab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+40\,Ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+9\,Bb^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+288\,Ba^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+432\,Aa^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+72\,Ba^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+48\,Aab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+160\,Bab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+40\,Ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9\,Bb^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+96\,Ba^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+144\,Aa^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+72\,Ba^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+48\,Aab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+96\,Bab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+24\,Ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+15\,Bb^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
[Out] 1/24*(24*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*A*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3
+ 3*B*b^4)*(d*x + c) + 2*(96*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 144*A*a^2*b^2
*tan(1/2*d*x + 1/2*c)^7 - 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b^3*
tan(1/2*d*x + 1/2*c)^7 + 96*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*A*b^4*tan(1
/2*d*x + 1/2*c)^7 - 15*B*b^4*tan(1/2*d*x + 1/2*c)^7 + 288*B*a^3*b*tan(1/2*d
*x + 1/2*c)^5 + 432*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 72*B*a^2*b^2*tan(1/2
*d*x + 1/2*c)^5 - 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 160*B*a*b^3*tan(1/2*d
*x + 1/2*c)^5 + 40*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 9*B*b^4*tan(1/2*d*x + 1/2
*c)^5 + 288*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 432*A*a^2*b^2*tan(1/2*d*x + 1/
2*c)^3 + 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 48*A*a*b^3*tan(1/2*d*x + 1/2
*c)^3 + 160*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 40*A*b^4*tan(1/2*d*x + 1/2*c)^
3 - 9*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 96*B*a^3*b*tan(1/2*d*x + 1/2*c) + 144*
A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 48*A*a
*b^3*tan(1/2*d*x + 1/2*c) + 96*B*a*b^3*tan(1/2*d*x + 1/2*c) + 24*A*b^4*tan(
1/2*d*x + 1/2*c) + 15*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 +
 1)^4)/d
```

Mupad [B]
time = 1.42, size = 369, normalized size = 1.84

$$\frac{3\,A\,b^4\,\sin(c+d\,x)}{4\,d}+\frac{2\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,B\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}+\frac{A\,b^4\,\sin(3\,c+3\,d\,x)}{12\,d}+\frac{B\,b^4\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{B\,b^4\,\sin(4\,c+4\,d\,x)}{32\,d}+\frac{4\,A\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{8\,A\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a\,b^3\,\sin(2\,c+2\,d\,x)}{d}+\frac{6\,A\,a^2\,b^2\,\sin(c+d\,x)}{d}+\frac{B\,a\,b^3\,\sin(3\,c+3\,d\,x)}{3\,d}+\frac{6\,B\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,B\,a^2\,b^2\,\sin(2\,c+2\,d\,x)}{2\,d}+\frac{3\,B\,a\,b^3\,\sin(c+d\,x)}{d}+\frac{4\,B\,a^3\,b\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x),x)
```

```
[Out] (3*A*b^4*sin(c + d*x))/(4*d) + (2*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/d + (2*B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3
*B*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*d) + (A*b^4*sin(3*c
+ 3*d*x))/(12*d) + (B*b^4*sin(2*c + 2*d*x))/(4*d) + (B*b^4*sin(4*c + 4*d*x)
)/(32*d) + (4*A*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*A
*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a*b^3*sin(2*c +
2*d*x))/d + (6*A*a^2*b^2*sin(c + d*x))/d + (B*a*b^3*sin(3*c + 3*d*x))/(3*d)
 + (6*B*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*B*a^2*b
^2*sin(2*c + 2*d*x))/(2*d) + (3*B*a*b^3*sin(c + d*x))/d + (4*B*a^3*b*sin(c
+ d*x))/d
```

3.244 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=195

$$\frac{1}{2}b\left(12a^2Ab+Ab^3+8a^3B+4ab^2B\right)x+\frac{a^3(4Ab+aB)\tanh^{-1}(\sin(c+dx))}{d}-\frac{b(6a^3A-12aAb^2-17a^2bB-}{3d}$$

```
[Out] 1/2*b*(12*A*a^2*b+A*b^3+8*B*a^3+4*B*a*b^2)*x+a^3*(4*A*b+B*a)*arctanh(sin(d*
x+c))/d-1/3*b*(6*A*a^3-12*A*a*b^2-17*B*a^2*b-2*B*b^3)*sin(d*x+c)/d-1/6*b^2*
(6*A*a^2-3*A*b^2-8*B*a*b)*cos(d*x+c)*sin(d*x+c)/d-1/3*b*(3*A*a-B*b)*(a+b*co
s(d*x+c))^2*sin(d*x+c)/d+a*A*(a+b*cos(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.37, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3068, 3128, 3112, 3102, 2814, 3855}

$$\frac{a^3(aB+4Ab)\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2(6a^2A-8abB-3Ab^2)\sin(c+dx)\cos(c+dx)}{6d}-\frac{b(6a^3A-17a^2bB-12aAb^2-2b^3B)\sin(c+dx)}{3d}+\frac{1}{2}bx\left(8a^3B+12a^2Ab+4ab^2B+Ab^3\right)-\frac{b(3aA-bB)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^3}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (b*(12*a^2*A*b + A*b^3 + 8*a^3*B + 4*a*b^2*B)*x)/2 + (a^3*(4*A*b + a*B)*Arc
Tanh[Sin[c + d*x]])/d - (b*(6*a^3*A - 12*a*A*b^2 - 17*a^2*b*B - 2*b^3*B)*Si
n[c + d*x])/(3*d) - (b^2*(6*a^2*A - 3*A*b^2 - 8*a*b*B)*Cos[c + d*x]*Sin[c +
 d*x])/(6*d) - (b*(3*a*A - b*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
+ (a*A*(a + b*Cos[c + d*x])^3*Tan[c + d*x])/d
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\tan(c+dx)}{d} + \int (a+$$

$$= -\frac{b(3aA-bB)(a+b\cos(c+dx))^2\sin(c+dx)}{3d}$$

$$= -\frac{b^2(6a^2A-3Ab^2-8abB)\cos(c+dx)\sin(c+}{6d}$$

$$= -\frac{b(6a^3A-12aAb^2-17a^2bB-2b^3B)\sin(c+}{3d}$$

$$= \frac{1}{2}b\left(12a^2Ab+Ab^3+8a^3B+4ab^2B\right)x - \frac{b(6a^3}{}$$

$$= \frac{1}{2}b\left(12a^2Ab+Ab^3+8a^3B+4ab^2B\right)x + \frac{a^3(4A}{}$$

Mathematica [A]

time = 0.71, size = 257, normalized size = 1.32

$$\frac{6b(12a^2Ab+Ab^3+8a^3B+4ab^2B)(c+dx)-12a^3(4Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^3(4Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{12a^4A\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{12a^4A\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}+3b^2(16aAb+24a^2B+3b^2B)\sin(c+dx)+3b^3(Ab+4aB)\sin(2(c+dx))+b^4B\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (6*b*(12*a^2*A*b + A*b^3 + 8*a^3*B + 4*a*b^2*B)*(c + d*x) - 12*a^3*(4*A*b +
 a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^3*(4*A*b + a*B)*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*a^4*A*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]) + (12*a^4*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]) + 3*b^2*(16*a*A*b + 24*a^2*B + 3*b^2*B)*Sin[c + d*x]
+ 3*b^3*(A*b + 4*a*B)*Sin[2*(c + d*x)] + b^4*B*Sin[3*(c + d*x)])/(12*d)
```

Maple [A]

time = 0.25, size = 189, normalized size = 0.97

method	result
derivativedivides	$\frac{A\,a^4\tan(dx+c)+a^4B\ln(\sec(dx+c)+\tan(dx+c))+4A\,a^3b\ln(\sec(dx+c)+\tan(dx+c))+4B\,a^3b(dx+c)+6A\,a^2b^2(dx+c)+}{}$
default	$\frac{A\,a^4\tan(dx+c)+a^4B\ln(\sec(dx+c)+\tan(dx+c))+4A\,a^3b\ln(\sec(dx+c)+\tan(dx+c))+4B\,a^3b(dx+c)+6A\,a^2b^2(dx+c)+}{}$
risch	$6xA\,a^2b^2+\frac{xA\,b^4}{2}+4xB\,a^3b+2xBa\,b^3+\frac{2i\mathrm{e}^{-i(dx+c)}Aa\,b^3}{d}+\frac{i\mathrm{e}^{-2i(dx+c)}A\,b^4}{8d}-\frac{3i\mathrm{e}^{i(dx+c)}B\,b^4}{8d}-$

norman	$\frac{\left(-6A\,a^2b^2-\frac{1}{2}A\,b^4-4B\,a^3b-2Ba\,b^3\right)x+\left(-30A\,a^2b^2-\frac{5}{2}A\,b^4-20B\,a^3b-10Ba\,b^3\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(6A\,a^2b^2+\frac{1}{2}A\,b^4+4\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*tan(d*x+c)+a^4*B*ln(sec(d*x+c)+tan(d*x+c))+4*A*a^3*b*ln(sec(d*x+
c)+tan(d*x+c))+4*B*a^3*b*(d*x+c)+6*A*a^2*b^2*(d*x+c)+6*B*a^2*b^2*sin(d*x+c)
+4*A*a*b^3*sin(d*x+c)+4*B*a*b^3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+A
*b^4*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+1/3*B*b^4*(cos(d*x+c)^2+2)*s
in(d*x+c))
```

Maxima [A]
time = 0.29, size = 197, normalized size = 1.01

$$\frac{48\,(dx+c)Ba^3b+72\,(dx+c)Aa^2b^2+12\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Bab^3+3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ab^4-4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Bb^4+6\,Ba^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+24\,Aa^3b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+72\,Ba^2b^2\sin(dx+c)+48\,Aab^3\sin(dx+c)+12\,Aa^4\tan(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/12*(48*(d*x + c)*B*a^3*b + 72*(d*x + c)*A*a^2*b^2 + 12*(2*d*x + 2*c + sin
(2*d*x + 2*c))*B*a*b^3 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^4 - 4*(sin(
d*x + c)^3 - 3*sin(d*x + c))*B*b^4 + 6*B*a^4*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 24*A*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1)) + 72*B*a^2*b^2*sin(d*x + c) + 48*A*a*b^3*sin(d*x + c) + 12*A*a^4*tan(d*
x + c))/d
```

Fricas [A]
time = 0.41, size = 196, normalized size = 1.01

$$\frac{3\,(8\,Ba^3b+12\,Aa^2b^2+4\,Bab^3+Ab^4)dx\cos(dx+c)+3\,(Ba^4+4\,Aa^3b)\cos(dx+c)\log(\sin(dx+c)+1)-3\,(Ba^4+4\,Aa^3b)\cos(dx+c)\log(-\sin(dx+c)+1)+\left(2\,Bb^4\cos(dx+c)^3+6\,Aa^4+3\,(4\,Bab^3+Ab^4)\cos(dx+c)^2+4\,(9\,Ba^2b^2+6\,Aab^3+Bb^4)\cos(dx+c)\right)\sin(dx+c)}{6\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(8*B*a^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*d*x*cos(d*x + c) + 3*
(B*a^4 + 4*A*a^3*b)*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*(B*a^4 + 4*A*a^3
*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + (2*B*b^4*cos(d*x + c)^3 + 6*A*a^4
 + 3*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 4*(9*B*a^2*b^2 + 6*A*a*b^3 + B*b^
4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 371, normalized size = 1.90

$$-\frac{\frac{12\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}-3\,\left(8\,Ba^{3}b+12\,Aa^{2}b^{2}+4\,Bab^{3}+Ab^{4}\right)(dx+c)-6\,\left(Ba^{4}+4\,Aa^{3}b\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+6\,\left(Ba^{4}+4\,Aa^{3}b\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,\left(36\,Ba^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+24\,Aab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-12\,Bab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ab^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,Bb^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+72\,Ba^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+48\,Aab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,Bb^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+36\,Ba^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,Aab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,Bab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ab^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Bb^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(8*B*a
^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c) - 6*(B*a^4 + 4*A*a^3*b)*
log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6*(B*a^4 + 4*A*a^3*b)*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(36*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 24*A*a*b^3*ta
n(1/2*d*x + 1/2*c)^5 - 12*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^4*tan(1/2*
d*x + 1/2*c)^5 + 6*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 72*B*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*B*b^4*tan(1/2*d*x + 1/2*
c)^3 + 36*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 24*A*a*b^3*tan(1/2*d*x + 1/2*c)
+ 12*B*a*b^3*tan(1/2*d*x + 1/2*c) + 3*A*b^4*tan(1/2*d*x + 1/2*c) + 6*B*b^4*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [**B**]
time = 2.27, size = 2522, normalized size = 12.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(2*A*a^4 + A*b^4 + 2*B*b^4 + 12*B*a^2*b^2 + 8*A*a*b^3 +
 4*B*a*b^3) + tan(c/2 + (d*x)/2)^7*(2*A*a^4 + A*b^4 - 2*B*b^4 - 12*B*a^2*b^
2 - 8*A*a*b^3 + 4*B*a*b^3) + tan(c/2 + (d*x)/2)^3*(6*A*a^4 - A*b^4 - (2*B*b
^4)/3 + 12*B*a^2*b^2 + 8*A*a*b^3 - 4*B*a*b^3) - tan(c/2 + (d*x)/2)^5*(A*b^4
 - 6*A*a^4 - (2*B*b^4)/3 + 12*B*a^2*b^2 + 8*A*a*b^3 + 4*B*a*b^3))/(d*(2*tan
(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 1)) - (
atan(((B*a^4 + 4*A*a^3*b)*((B*a^4 + 4*A*a^3*b)*(16*A*b^4 + 32*B*a^4 + 192*A
```

*a^2*b^2 + 128*A*a^3*b + 64*B*a*b^3 + 128*B*a^3*b) + tan(c/2 + (d*x)/2)*(8*
A^2*b^8 + 32*B^2*a^8 + 192*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*b^2
 + 128*B^2*a^2*b^6 + 512*B^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B*a*b^7 + 256
*A*B*a^7*b + 896*A*B*a^3*b^5 + 1536*A*B*a^5*b^3))*1i - (B*a^4 + 4*A*a^3*b)*
((B*a^4 + 4*A*a^3*b)*(16*A*b^4 + 32*B*a^4 + 192*A*a^2*b^2 + 128*A*a^3*b + 6
4*B*a*b^3 + 128*B*a^3*b) - tan(c/2 + (d*x)/2)*(8*A^2*b^8 + 32*B^2*a^8 + 192
*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*b^2 + 128*B^2*a^2*b^6 + 512*B
^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B*a*b^7 + 256*A*B*a^7*b + 896*A*B*a^3*b
^5 + 1536*A*B*a^5*b^3))*1i)/((B*a^4 + 4*A*a^3*b)*((B*a^4 + 4*A*a^3*b)*(16*A
*b^4 + 32*B*a^4 + 192*A*a^2*b^2 + 128*A*a^3*b + 64*B*a*b^3 + 128*B*a^3*b) +
 tan(c/2 + (d*x)/2)*(8*A^2*b^8 + 32*B^2*a^8 + 192*A^2*a^2*b^6 + 1152*A^2*a^
4*b^4 + 512*A^2*a^6*b^2 + 128*B^2*a^2*b^6 + 512*B^2*a^4*b^4 + 512*B^2*a^6*b
^2 + 64*A*B*a*b^7 + 256*A*B*a^7*b + 896*A*B*a^3*b^5 + 1536*A*B*a^5*b^3)) +
(B*a^4 + 4*A*a^3*b)*((B*a^4 + 4*A*a^3*b)*(16*A*b^4 + 32*B*a^4 + 192*A*a^2*b
^2 + 128*A*a^3*b + 64*B*a*b^3 + 128*B*a^3*b) - tan(c/2 + (d*x)/2)*(8*A^2*b^
8 + 32*B^2*a^8 + 192*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*b^2 + 128
*B^2*a^2*b^6 + 512*B^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B*a*b^7 + 256*A*B*a
^7*b + 896*A*B*a^3*b^5 + 1536*A*B*a^5*b^3)) - 256*B^3*a^11*b + 64*A^3*a^3*b
^9 + 1536*A^3*a^5*b^7 - 512*A^3*a^6*b^6 + 9216*A^3*a^7*b^5 - 6144*A^3*a^8*b
^4 + 256*B^3*a^6*b^6 + 1024*B^3*a^8*b^4 - 128*B^3*a^9*b^3 + 1024*B^3*a^10*b
^2 + 1152*A*B^2*a^5*b^7 + 5888*A*B^2*a^7*b^5 - 1056*A*B^2*a^8*b^4 + 7168*A*
B^2*a^9*b^3 - 2432*A*B^2*a^10*b^2 + 528*A^2*B*a^4*b^8 + 7552*A^2*B*a^6*b^6
- 2304*A^2*B*a^7*b^5 + 14592*A^2*B*a^8*b^4 - 7168*A^2*B*a^9*b^3))*(B*a^4*2i
 + A*a^3*b*8i))/d - (b*atan(((b*(tan(c/2 + (d*x)/2)*(8*A^2*b^8 + 32*B^2*a^8
 + 192*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*b^2 + 128*B^2*a^2*b^6 +
 512*B^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B*a*b^7 + 256*A*B*a^7*b + 896*A*B
*a^3*b^5 + 1536*A*B*a^5*b^3) - (b*(A*b^3 + 8*B*a^3 + 12*A*a^2*b + 4*B*a*b^2
)*(16*A*b^4 + 32*B*a^4 + 192*A*a^2*b^2 + 128*A*a^3*b + 64*B*a*b^3 + 128*B*a
^3*b)*1i)/2)*(A*b^3 + 8*B*a^3 + 12*A*a^2*b + 4*B*a*b^2))/2 + (b*(tan(c/2 +
(d*x)/2)*(8*A^2*b^8 + 32*B^2*a^8 + 192*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512
*A^2*a^6*b^2 + 128*B^2*a^2*b^6 + 512*B^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B
*a*b^7 + 256*A*B*a^7*b + 896*A*B*a^3*b^5 + 1536*A*B*a^5*b^3) + (b*(A*b^3 +
8*B*a^3 + 12*A*a^2*b + 4*B*a*b^2)*(16*A*b^4 + 32*B*a^4 + 192*A*a^2*b^2 + 12
8*A*a^3*b + 64*B*a*b^3 + 128*B*a^3*b)*1i)/2)*(A*b^3 + 8*B*a^3 + 12*A*a^2*b
+ 4*B*a*b^2))/2)/(64*A^3*a^3*b^9 - 256*B^3*a^11*b + 1536*A^3*a^5*b^7 - 512*
A^3*a^6*b^6 + 9216*A^3*a^7*b^5 - 6144*A^3*a^8*b^4 + 256*B^3*a^6*b^6 + 1024*
B^3*a^8*b^4 - 128*B^3*a^9*b^3 + 1024*B^3*a^10*b^2 - (b*(tan(c/2 + (d*x)/2)*
(8*A^2*b^8 + 32*B^2*a^8 + 192*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*
b^2 + 128*B^2*a^2*b^6 + 512*B^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B*a*b^7 +
256*A*B*a^7*b + 896*A*B*a^3*b^5 + 1536*A*B*a^5*b^3) - (b*(A*b^3 + 8*B*a^3 +
 12*A*a^2*b + 4*B*a*b^2)*(16*A*b^4 + 32*B*a^4 + 192*A*a^2*b^2 + 128*A*a^3*b
 + 64*B*a*b^3 + 128*B*a^3*b)*1i)/2)*(A*b^3 + 8*B*a^3 + 12*A*a^2*b + 4*B*a*b
^2)*1i)/2 + (b*(tan(c/2 + (d*x)/2)*(8*A^2*b^8 + 32*B^2*a^8 + 192*A^2*a^2*b^
6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*b^2 + 128*B^2*a^2*b^6 + 512*B^2*a^4*b^4
+ 512*B^2*a^6*b^2 + 64*A*B*a*b^7 + 256*A*B*a^7*b + 896*A*B*a^3*b^5 + 1536*A

```
*B*a^5*b^3) + (b*(A*b^3 + 8*B*a^3 + 12*A*a^2*b + 4*B*a*b^2)*(16*A*b^4 + 32*
B*a^4 + 192*A*a^2*b^2 + 128*A*a^3*b + 64*B*a*b^3 + 128*B*a^3*b)*1i)/2)*(A*b
^3 + 8*B*a^3 + 12*A*a^2*b + 4*B*a*b^2)*1i)/2 + 1152*A*B^2*a^5*b^7 + 5888*A*
B^2*a^7*b^5 - 1056*A*B^2*a^8*b^4 + 7168*A*B^2*a^9*b^3 - 2432*A*B^2*a^10*b^2
 + 528*A^2*B*a^4*b^8 + 7552*A^2*B*a^6*b^6 - 2304*A^2*B*a^7*b^5 + 14592*A^2*
B*a^8*b^4 - 7168*A^2*B*a^9*b^3))*(A*b^3 + 8*B*a^3 + 12*A*a^2*b + 4*B*a*b^2)
)/d
```

3.245 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=209

$$\frac{1}{2}b^2\left(8aAb+12a^2B+b^2B\right)x+\frac{a^2(a^2A+12Ab^2+8abB)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b(13a^2Ab-2Ab^3+4a^3B-8a}{2d}$$

```
[Out] 1/2*b^2*(8*A*a*b+12*B*a^2+B*b^2)*x+1/2*a^2*(A*a^2+12*A*b^2+8*B*a*b)*arctanh
(sin(d*x+c))/d-1/2*b*(13*A*a^2*b-2*A*b^3+4*B*a^3-8*B*a*b^2)*sin(d*x+c)/d-1/
2*b^2*(6*A*a*b+2*B*a^2-B*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/2*a*(5*A*b+2*B*a)*(
a+b*cos(d*x+c))^2*tan(d*x+c)/d+1/2*a*A*(a+b*cos(d*x+c))^3*sec(d*x+c)*tan(d*
x+c)/d
```

Rubi [A]
time = 0.40, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3068, 3126, 3112, 3102, 2814, 3855}

$$\frac{a^2(a^2A+8abB+12Ab^2)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2(2a^2B+6aAb-b^2B)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}b^2x(12a^2B+8aAb+b^2B)-\frac{b(4a^3B+13a^2Ab-8ab^2B-2Ab^3)\sin(c+dx)}{2d}+\frac{a(2aB+5Ab)\tan(c+dx)(a+b\cos(c+dx))^2}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^3}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (b^2*(8*a*A*b + 12*a^2*B + b^2*B)*x)/2 + (a^2*(a^2*A + 12*A*b^2 + 8*a*b*B)*
ArcTanh[Sin[c + d*x]])/(2*d) - (b*(13*a^2*A*b - 2*A*b^3 + 4*a^3*B - 8*a*b^2
*B)*Sin[c + d*x])/(2*d) - (b^2*(6*a*A*b + 2*a^2*B - b^2*B)*Cos[c + d*x]*Sin
[c + d*x])/(2*d) + (a*(5*A*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/
(2*d) + (a*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
```

```
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d}$$
$$= \frac{a(5Ab+2aB)(a+b\cos(c+dx))^2\tan(c+dx)}{2d}$$
$$= -\frac{b^2(6aAb+2a^2B-b^2B)\cos(c+dx)\sin(c+d}{2d}$$
$$= -\frac{b(13a^2Ab-2Ab^3+4a^3B-8ab^2B)\sin(c+dx}{2d}$$
$$= \frac{1}{2}b^2\left(8aAb+12a^2B+b^2B\right)x - \frac{b(13a^2Ab-2A}{}$$
$$= \frac{1}{2}b^2\left(8aAb+12a^2B+b^2B\right)x + \frac{a^2(a^2A+12Ab^2}{}$$

Mathematica [A]
time = 1.60, size = 310, normalized size = 1.48

$$\frac{2b^2(8aAb+12a^2B+b^2B)(c+dx)-2a^2(a^2A+12Ab^2+8abB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2a^2(a^2A+12Ab^2+8abB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{a^4A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4a^3(4Ab+aB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-\frac{a^4A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4a^3(4Ab+aB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}+4b^3(Ab+4aB)\sin(c+dx)+b^4B\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^2*(8*a*A*b + 12*a^2*B + b^2*B)*(c + d*x) - 2*a^2*(a^2*A + 12*A*b^2 + 8
*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*(a^2*A + 12*A*b^2
+ 8*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^4*A)/(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^2 + (4*a^3*(4*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]) - (a^4*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2 + (4*a^3*(4*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]) + 4*b^3*(A*b + 4*a*B)*Sin[c + d*x] + b^4*B*Sin[2*(c + d*x)])/(4*d
)
```

Maple [A]
time = 0.29, size = 187, normalized size = 0.89 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^4*B*
tan(d*x+c)+4*A*a^3*b*tan(d*x+c)+4*B*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6*A*a^2
*b^2*ln(sec(d*x+c)+tan(d*x+c))+6*B*a^2*b^2*(d*x+c)+4*A*a*b^3*(d*x+c)+4*B*a*
b^3*sin(d*x+c)+A*b^4*sin(d*x+c)+B*b^4*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/
2*c))
```

Maxima [A]
time = 0.28, size = 209, normalized size = 1.00

$$\frac{24\,(dx+c)Ba^2b^2+16\,(dx+c)Aab^3+(2\,dx+2\,c+\sin(2\,dx+2\,c))Bb^4-Aa^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+8\,Ba^3b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+12\,Aa^2b^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+16\,Bab^3\sin(dx+c)+4\,Ab^4\sin(dx+c)+4\,Ba^4\tan(dx+c)+16\,Aa^3b\tan(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(24*(d*x + c)*B*a^2*b^2 + 16*(d*x + c)*A*a*b^3 + (2*d*x + 2*c + sin(2*d
*x + 2*c))*B*b^4 - A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 8*B*a^3*b*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 12*A*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x +
c) - 1)) + 16*B*a*b^3*sin(d*x + c) + 4*A*b^4*sin(d*x + c) + 4*B*a^4*tan(d*x
 + c) + 16*A*a^3*b*tan(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 202, normalized size = 0.97

$$\frac{2\,(12\,Ba^2b^2+8\,Aab^3+Bb^4)dx\cos(dx+c)^2+(Aa^4+8\,Ba^3b+12\,Aa^2b^2)\cos(dx+c)^2\log(\sin(dx+c)+1)-(Aa^4+8\,Ba^3b+12\,Aa^2b^2)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,(Bb^4\cos(dx+c)^3+Aa^4+2\,(4\,Bab^3+Ab^4)\cos(dx+c)^2+2\,(Ba^4+4\,Aa^3b)\cos(dx+c))\sin(dx+c)}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(2*(12*B*a^2*b^2 + 8*A*a*b^3 + B*b^4)*d*x*cos(d*x + c)^2 + (A*a^4 + 8*B
*a^3*b + 12*A*a^2*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a^4 + 8*B*
a^3*b + 12*A*a^2*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(B*b^4*cos(
d*x + c)^3 + A*a^4 + 2*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 2*(B*a^4 + 4*A*
a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(197) = 394.
time = 0.54, size = 526, normalized size = 2.52

$$\frac{(12\,Ba^2b^2+8\,Aab^3+Bb^4)(dx+c)+(Aa^4+8\,Ba^3b+12\,Aa^2b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa^4+8\,Ba^3b+12\,Aa^2b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-2\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-8\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+8\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+2\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-8\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-8\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*((12*B*a^2*b^2 + 8*A*a*b^3 + B*b^4)*(d*x + c) + (A*a^4 + 8*B*a^3*b + 12
*A*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a^4 + 8*B*a^3*b + 12*A*
a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^4*tan(1/2*d*x + 1/2*c)
^7 - 2*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 8*
B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 2*A*b^4*tan(1/2*d*x + 1/2*c)^7 - B*b^4*tan
(1/2*d*x + 1/2*c)^7 + 3*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 2*B*a^4*tan(1/2*d*x
+ 1/2*c)^5 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 8*B*a*b^3*tan(1/2*d*x + 1/2
*c)^5 - 2*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 3
*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 8*A*a^3*b*
tan(1/2*d*x + 1/2*c)^3 - 8*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*A*b^4*tan(1/2
*d*x + 1/2*c)^3 - 3*B*b^4*tan(1/2*d*x + 1/2*c)^3 + A*a^4*tan(1/2*d*x + 1/2*
c) + 2*B*a^4*tan(1/2*d*x + 1/2*c) + 8*A*a^3*b*tan(1/2*d*x + 1/2*c) + 8*B*a*
b^3*tan(1/2*d*x + 1/2*c) + 2*A*b^4*tan(1/2*d*x + 1/2*c) + B*b^4*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

Mupad [B]
time = 2.31, size = 330, normalized size = 1.58

$$\frac{2\left(\frac{A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{B\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+4\,A\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+4\,B\,a^3\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+6\,A\,a^2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+6\,B\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\right)}{d}+\frac{\frac{B\,a^4\,\sin(2\,c+2\,d\,x)}{2}+\frac{A\,b^4\,\sin(3\,c+3\,d\,x)}{4}+\frac{B\,b^4\,\sin(2\,c+2\,d\,x)}{8}+\frac{B\,b^4\,\sin(4\,c+4\,d\,x)}{16}+\frac{A\,a^4\,\sin(c+d\,x)}{2}+\frac{A\,b^4\,\sin(c+d\,x)}{4}+B\,a\,b^3\,\sin\left(c+d\,x\right)+2\,A\,a^3\,b\,\sin\left(2\,c+2\,d\,x\right)+B\,a\,b^3\,\sin\left(3\,c+3\,d\,x\right)}{d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^3,x)
```

```
[Out] (2*((A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (B*b^4*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + 4*A*a*b^3*atan(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)) + 4*B*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)) + 6*A*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 6*B*a^2*b^2
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/d + ((B*a^4*sin(2*c + 2*d*x)
)/2 + (A*b^4*sin(3*c + 3*d*x))/4 + (B*b^4*sin(2*c + 2*d*x))/8 + (B*b^4*sin(
4*c + 4*d*x))/16 + (A*a^4*sin(c + d*x))/2 + (A*b^4*sin(c + d*x))/4 + B*a*b^
3*sin(c + d*x) + 2*A*a^3*b*sin(2*c + 2*d*x) + B*a*b^3*sin(3*c + 3*d*x))/(d*
(cos(2*c + 2*d*x)/2 + 1/2))
```

3.246 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=198

$$b^3(Ab+4aB)x+\frac{a(4a^2Ab+8Ab^3+a^3B+12ab^2B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2(8aAb+3a^2B-6b^2B)\sin(c+d}{6d}$$

```
[Out] b^3*(A*b+4*B*a)*x+1/2*a*(4*A*a^2*b+8*A*b^3+B*a^3+12*B*a*b^2)*arctanh(sin(d*
x+c))/d-1/6*b^2*(8*A*a*b+3*B*a^2-6*B*b^2)*sin(d*x+c)/d+1/3*a^2*(2*A*a^2+9*A
*b^2+9*B*a*b)*tan(d*x+c)/d+1/2*a*(2*A*b+B*a)*(a+b*cos(d*x+c))^2*sec(d*x+c)*
tan(d*x+c)/d+1/3*a*A*(a+b*cos(d*x+c))^3*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.38, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$,
Rules used = {3068, 3126, 3110, 3102, 2814, 3855}

$$-\frac{b^2(3a^2B+8aAb-6b^2B)\sin(c+dx)}{6d}+\frac{a^2(2a^2A+9abB+9Ab^2)\tan(c+dx)}{3d}+\frac{a(a^3B+4a^2Ab+12ab^2B+8Ab^3)\tanh^{-1}(\sin(c+dx))}{2d}+b^3x(4aB+Ab)+\frac{a(aB+2Ab)\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] b^3*(A*b + 4*a*B)*x + (a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b^2*B)*ArcTanh
[Sin[c + d*x]])/(2*d) - (b^2*(8*a*A*b + 3*a^2*B - 6*b^2*B)*Sin[c + d*x])/(6
*d) + (a^2*(2*a^2*A + 9*A*b^2 + 9*a*b*B)*Tan[c + d*x])/(3*d) + (a*(2*A*b +
a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*(a + b*
Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= \frac{aA(a+b\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx}{3d} \\
&= \frac{a(2Ab+aB)(a+b\cos(c+dx))^2\sec(c+dx)\,\mathrm{t}}{2d} \\
&= \frac{a^2(2a^2A+9Ab^2+9abB)\tan(c+dx)}{3d}+\frac{a(2A}{} \\
&= -\frac{b^2(8aAb+3a^2B-6b^2B)\sin(c+dx)}{6d}+\frac{a^2(2}{} \\
&= b^3(Ab+4aB)x-\frac{b^2(8aAb+3a^2B-6b^2B)\sin}{6d} \\
&= b^3(Ab+4aB)x+\frac{a(4a^2Ab+8Ab^3+a^3B+12}{2d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 415 vs. $2(198) = 396$.
time = 3.77, size = 415, normalized size = 2.10

$$
\frac{12b^3(Ab+4aB)(c+dx)-6a(4a^2Ab+8Ab^3+a^3B+12ab^2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+6a(4a^2Ab+8Ab^3+a^3B+12ab^2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{a^3(12Ab+a(A+3B))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{2a^4A\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{8a^2(a^2A+9Ab^2+6abB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{2a^4A\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{a^3(12Ab+a(A+3B))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{8a^2(a^2A+9Ab^2+6abB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}+12b^4B\sin(c+dx)}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (12*b^3*(A*b + 4*a*B)*(c + d*x) - 6*a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b
^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*a*(4*a^2*A*b + 8*A*b^3 +
 a^3*B + 12*a*b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*(12*A*
b + a*(A + 3*B)))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*a^4*A*Sin[(c
 + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (8*a^2*(a^2*A + 9*A*b
^2 + 6*a*b*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (2*
a^4*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - (a^3*(12*
A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (8*a^2*(a^2*A
 + 9*A*b^2 + 6*a*b*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]) + 12*b^4*B*Sin[c + d*x])/(12*d)
```

Maple [A]
time = 0.33, size = 209, normalized size = 1.06 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-A*a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^4*B*(1/2*sec(d*x+c)*tan(d*
x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+4*A*a^3*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/
```

```
2*ln(sec(d*x+c)+tan(d*x+c)))+4*B*a^3*b*tan(d*x+c)+6*A*a^2*b^2*tan(d*x+c)+6*
B*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+4*A*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+4*B
*a*b^3*(d*x+c)+A*b^4*(d*x+c)+B*b^4*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 245, normalized size = 1.24

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{4}+48\left(dx+c\right)Bab^{3}+12\left(dx+c\right)Ab^{4}-3\,Ba^{4}\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,Aa^{3}b\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+36\,Ba^{2}b^{2}(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+24\,Aab^{3}(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+12\,Bb^{4}\sin\left(dx+c\right)+48\,Ba^{3}b\tan\left(dx+c\right)+72\,Aa^{2}b^{2}\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 48*(d*x + c)*B*a*b^3 + 12
*(d*x + c)*A*b^4 - 3*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*A*a^3*b*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 36*B*a^2*b^
2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 24*A*a*b^3*(log(sin(d*x
 + c) + 1) - log(sin(d*x + c) - 1)) + 12*B*b^4*sin(d*x + c) + 48*B*a^3*b*ta
n(d*x + c) + 72*A*a^2*b^2*tan(d*x + c))/d
```

Fricas [A]
time = 0.39, size = 219, normalized size = 1.11

$$\frac{12\left(4\,Bab^{3}+Ab^{4}\right)dx\cos\left(dx+c\right)^{3}+3\left(Ba^{4}+4\,Aa^{3}b+12\,Ba^{2}b^{2}+8\,Aab^{3}\right)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\left(Ba^{4}+4\,Aa^{3}b+12\,Ba^{2}b^{2}+8\,Aab^{3}\right)\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(6\,Bb^{4}\cos\left(dx+c\right)^{3}+2\,Aa^{4}+4\left(Aa^{4}+6\,Ba^{3}b+9\,Aa^{2}b^{2}\right)\cos\left(dx+c\right)^{2}+3\left(Ba^{4}+4\,Aa^{3}b\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(12*(4*B*a*b^3 + A*b^4)*d*x*cos(d*x + c)^3 + 3*(B*a^4 + 4*A*a^3*b + 12
*B*a^2*b^2 + 8*A*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^4 + 4
*A*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1)
+ 2*(6*B*b^4*cos(d*x + c)^3 + 2*A*a^4 + 4*(A*a^4 + 6*B*a^3*b + 9*A*a^2*b^2)
*cos(d*x + c)^2 + 3*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(188) = 376.

time = 0.50, size = 387, normalized size = 1.95

$$\frac{\frac{12\,Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+6\,(4\,Bab^3+Ab^4)(dx+c)+3\,(Ba^4+4\,Aa^3b+12\,Ba^2b^2+8\,Aab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Ba^4+4\,Aa^3b+12\,Ba^2b^2+8\,Aab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+24\,Ba^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+36\,Aa^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-48\,Ba^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72\,Aa^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,Ba^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,Aa^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(12*B*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(4*B*a*
b^3 + A*b^4)*(d*x + c) + 3*(B*a^4 + 4*A*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*l
og(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a^4 + 4*A*a^3*b + 12*B*a^2*b^2 + 8
*A*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^4*tan(1/2*d*x + 1/2
*c)^5 - 3*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^3*b*tan(1/2*d*x + 1/2*c)^5
+ 24*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 -
 4*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*A*
a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^4*tan(1/2*d*x + 1/2*c) + 3*B*a^4*tan
(1/2*d*x + 1/2*c) + 12*A*a^3*b*tan(1/2*d*x + 1/2*c) + 24*B*a^3*b*tan(1/2*d*
x + 1/2*c) + 36*A*a^2*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^3)/d
```

Mupad [**B**]

time = 2.83, size = 636, normalized size = 3.21

$$\frac{\frac{A\,a^4\sin(3\,c+3\,d\,x)}{6}+\frac{B\,a^4\sin(2\,c+2\,d\,x)}{4}+\frac{B\,b^4\sin(2\,c+2\,d\,x)}{4}+\frac{B\,b^4\sin(4\,c+4\,d\,x)}{8}+\frac{A\,a^4\sin(c+d\,x)}{2}+B\,a^3\,b\sin(c+d\,x)+\frac{3\,A\,b^4\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+A\,a^3\,b\sin(2\,c+2\,d\,x)+\frac{3\,A\,a^2\,b^2\sin(c+d\,x)}{2}+B\,a^3\,b\sin(3\,c+3\,d\,x)+\frac{A\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)}{2}+\frac{3\,A\,a^2\,b^2\sin(3\,c+3\,d\,x)}{2}+2\,B\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)+6\,B\,a\,b^3\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-\frac{B\,a^4\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3{}\mathrm{i}}{4}-\frac{B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)\,1{}\mathrm{i}}{4}-A\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)\,2{}\mathrm{i}-A\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)\,1{}\mathrm{i}-B\,a^2\,b^2\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,9{}\mathrm{i}-B\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)\,3{}\mathrm{i}-A\,a\,b^3\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,6{}\mathrm{i}-A\,a^3\,b\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3{}\mathrm{i}}{d\,\left(\frac{3\cos(c+d\,x)}{4}+\frac{\cos(3\,c+3\,d\,x)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^4,x)
```

```
[Out] ((A*a^4*sin(3*c + 3*d*x))/6 + (B*a^4*sin(2*c + 2*d*x))/4 + (B*b^4*sin(2*c +
 2*d*x))/4 + (B*b^4*sin(4*c + 4*d*x))/8 + (A*a^4*sin(c + d*x))/2 + B*a^3*b*
sin(c + d*x) + (3*A*b^4*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/2 - (B*a^4*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)
/2))*3i)/4 + A*a^3*b*sin(2*c + 2*d*x) + (3*A*a^2*b^2*sin(c + d*x))/2 + B*a^
3*b*sin(3*c + 3*d*x) + (A*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*c
os(3*c + 3*d*x))/2 - (B*a^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2)
)*cos(3*c + 3*d*x)*1i)/4 + (3*A*a^2*b^2*sin(3*c + 3*d*x))/2 - A*a*b^3*atan(
(sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*2i - A*a^3*b*a
tan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*1i + 2*B*a
*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x) - B*a^2*b
^2*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*9i - B*a^2
*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*3i -
 A*a*b^3*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*6i -
 A*a^3*b*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i +
 6*B*a*b^3*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*((3
*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4))
```

3.247 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=216

$$b^4Bx+\frac{(3a^4A+24a^2Ab^2+8Ab^4+16a^3bB+32ab^3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(16a^2Ab+19Ab^3+4a^3B+34}{6d}$$

```
[Out] b^4*B*x+1/8*(3*A*a^4+24*A*a^2*b^2+8*A*b^4+16*B*a^3*b+32*B*a*b^3)*arctanh(si
n(d*x+c))/d+1/6*a*(16*A*a^2*b+19*A*b^3+4*B*a^3+34*B*a*b^2)*tan(d*x+c)/d+1/2
4*a^2*(9*A*a^2+26*A*b^2+32*B*a*b)*sec(d*x+c)*tan(d*x+c)/d+1/12*a*(7*A*b+4*B
*a)*(a+b*cos(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a*A*(a+b*cos(d*x+c))^3
*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.38, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3068, 3126, 3110, 3100, 2814, 3855}

$$\frac{a^2(9a^2A+32abB+26Ab^2)\tan(c+dx)\sec(c+dx)}{24d}+\frac{a(4a^3B+16a^2Ab+34ab^2B+19Ab^3)\tan(c+dx)}{6d}+\frac{(3a^4A+16a^3bB+24a^2Ab^2+32ab^3B+8Ab^4)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4aB+7Ab)\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{12d}+\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}+b^4Bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] b^4*B*x + ((3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B)*Arc
Tanh[Sin[c + d*x]])/(8*d) + (a*(16*a^2*A*b + 19*A*b^3 + 4*a^3*B + 34*a*b^2*
B)*Tan[c + d*x])/(6*d) + (a^2*(9*a^2*A + 26*A*b^2 + 32*a*b*B)*Sec[c + d*x]*
Tan[c + d*x])/(24*d) + (a*(7*A*b + 4*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*
x]^2*Tan[c + d*x])/(12*d) + (a*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[
c + d*x])/(4*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
```

```
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx &= \frac{aA(a+b\cos(c+dx))^3\sec^3(c+dx)\tan(c+dx)}{4d} \\
&= \frac{a(7Ab+4aB)(a+b\cos(c+dx))^2\sec^2(c+dx)}{12d} \\
&= \frac{a^2(9a^2A+26Ab^2+32abB)\sec(c+dx)\tan(c+}{24d} \\
&= \frac{a(16a^2Ab+19Ab^3+4a^3B+34ab^2B)\tan(c+}{6d} \\
&= b^4Bx+\frac{a(16a^2Ab+19Ab^3+4a^3B+34ab^2B)\,t}{6d} \\
&= b^4Bx+\frac{(3a^4A+24a^2Ab^2+8Ab^4+16a^3bB+3}{8d}
\end{aligned}
$$

Mathematica [A]
time = 0.72, size = 160, normalized size = 0.74

$$
\frac{24b^4Bdx+3(3a^4A+24a^2Ab^2+8Ab^4+16a^3bB+32ab^3B)\tanh^{-1}(\sin(c+dx))+3a(8(4a^2Ab+4Ab^3+a^3B+6ab^2B)+a(3a^2A+24Ab^2+16abB)\sec(c+dx)+2a^3A\sec^3(c+dx))\tan(c+dx)+8a^3(4Ab+aB)\tan^3(c+dx)}{24d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (24*b^4*B*d*x + 3*(3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3
*B)*ArcTanh[Sin[c + d*x]] + 3*a*(8*(4*a^2*A*b + 4*A*b^3 + a^3*B + 6*a*b^2*B
) + a*(3*a^2*A + 24*A*b^2 + 16*a*b*B)*Sec[c + d*x] + 2*a^3*A*Sec[c + d*x]^3
)*Tan[c + d*x] + 8*a^3*(4*A*b + a*B)*Tan[c + d*x]^3)/(24*d)
```

Maple [A]
time = 0.24, size = 260, normalized size = 1.20

method	result
derivativedivides	$\frac{A\,a^4\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^4B\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-}{}$
default	$\frac{A\,a^4\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^4B\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-}{}$
risch	$b^4Bx-\frac{ia\left(-144Ba\,b^2-64A\,a^2b-96A\,b^3-16a^3B-72Aa\,b^2e^{i(dx+c)}-48B\,a^2b\,e^{i(dx+c)}+72Aa\,b^2e^{7i(dx+c)}+48B\,a^2b\,e^{7i}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c
)+tan(d*x+c)))-a^4*B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-4*A*a^3*b*(-2/3-1/3
*sec(d*x+c)^2)*tan(d*x+c)+4*B*a^3*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d
*x+c)+tan(d*x+c)))+6*A*a^2*b^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)
+tan(d*x+c)))+6*B*a^2*b^2*tan(d*x+c)+4*A*a*b^3*tan(d*x+c)+4*B*a*b^3*ln(sec(
d*x+c)+tan(d*x+c))+A*b^4*ln(sec(d*x+c)+tan(d*x+c))+B*b^4*(d*x+c))
```

Maxima [A]
time = 0.28, size = 317, normalized size = 1.47

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{4}+64\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{3}b+48\left(dx+c\right)Bb^{4}-3\,Aa^{4}\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-48\,Ba^{3}b\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-72\,Aa^{2}b^{2}\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+96\,Bab^{3}\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+24\,Ab^{4}\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+288\,Ba^{2}b^{2}\tan\left(dx+c\right)+192\,Aab^{3}\tan\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^4 + 64*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a^3*b + 48*(d*x + c)*B*b^4 - 3*A*a^4*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 48*B*a^3*b*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 72*A*a^2*b^2*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 96*B*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 24*
A*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 288*B*a^2*b^2*tan(d
*x + c) + 192*A*a*b^3*tan(d*x + c))/d
```

Fricas [A]
time = 0.41, size = 250, normalized size = 1.16

$$\frac{48\,Bb^{4}dx\cos\left(dx+c\right)^{4}+3\left(3\,Aa^{4}+16\,Ba^{3}b+24\,Aa^{2}b^{2}+32\,Bab^{3}+8\,Ab^{4}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,Aa^{4}+16\,Ba^{3}b+24\,Aa^{2}b^{2}+32\,Bab^{3}+8\,Ab^{4}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(6\,Aa^{4}+16\left(Ba^{4}+4\,Aa^{3}b+9\,Ba^{2}b^{2}+6\,Aab^{3}\right)\cos\left(dx+c\right)^{3}+3\left(3\,Aa^{4}+16\,Ba^{3}b+24\,Aa^{2}b^{2}\right)\cos\left(dx+c\right)^{2}+8\left(Ba^{4}+4\,Aa^{3}b\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(48*B*b^4*d*x*cos(d*x + c)^4 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2
+ 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A*a^4 +
 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^4*log(-sin(
d*x + c) + 1) + 2*(6*A*a^4 + 16*(B*a^4 + 4*A*a^3*b + 9*B*a^2*b^2 + 6*A*a*b^
```

```
3)*cos(d*x + c)^3 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2)*cos(d*x + c)^2
+ 8*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 635 vs. 2(206) = 412.
time = 0.50, size = 635, normalized size = 2.94

$$\frac{24(dx+c)Bb^4+3(3Aa^4+16Ba^3b+24Aa^2b^2+32Bab^3+8Ab^4)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3(3Aa^4+16Ba^3b+24Aa^2b^2+32Bab^3+8Ab^4)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(15Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-24Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-96Aa^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+48Ba^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+72Aa^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-144Ba^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-96Aab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+9Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+40Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+160Aa^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-48Ba^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-72Aa^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+432Ba^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+288Aab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+9Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-40Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-160Aa^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-48Ba^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-72Aa^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-432Ba^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-288Aab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+24Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+96Aa^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+48Ba^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+72Aa^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+144Ba^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+96Aab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(24*(d*x + c)*B*b^4 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a*
b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a^4 + 16*B*a^3*b
 + 24*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
+ 2*(15*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 96
*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*A*
a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 96*
A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 40*B*a^4*
tan(1/2*d*x + 1/2*c)^5 + 160*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 48*B*a^3*b*ta
n(1/2*d*x + 1/2*c)^5 - 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 432*B*a^2*b^2*
tan(1/2*d*x + 1/2*c)^5 + 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^4*tan(1
/2*d*x + 1/2*c)^3 - 40*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*A*a^3*b*tan(1/2*d
*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*b^2*tan(1/2*d*
x + 1/2*c)^3 - 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*A*a*b^3*tan(1/2*d
*x + 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x + 1/2*c) + 24*B*a^4*tan(1/2*d*x + 1/2*
c) + 96*A*a^3*b*tan(1/2*d*x + 1/2*c) + 48*B*a^3*b*tan(1/2*d*x + 1/2*c) + 72
*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 96*A
*a*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 2.98, size = 1969, normalized size = 9.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^5,x)

[Out] ((27*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 + 9*A*b^4*atanh(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (9*A*a^4*sin(3*c + 3*d*x))/8 + 4*B
*a^4*sin(2*c + 2*d*x) + B*a^4*sin(4*c + 4*d*x) + 9*B*b^4*atan((9*A^2*a^8*si
n(c/2 + (d*x)/2) + 64*A^2*b^8*sin(c/2 + (d*x)/2) + 64*B^2*b^8*sin(c/2 + (d*
x)/2) + 384*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624*A^2*a^4*b^4*sin(c/2 + (d*x
)/2) + 144*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 1024*B^2*a^2*b^6*sin(c/2 + (d*x
)/2) + 1024*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 256*B^2*a^6*b^2*sin(c/2 + (d*x
)/2) + 1792*A*B*a^3*b^5*sin(c/2 + (d*x)/2) + 960*A*B*a^5*b^3*sin(c/2 + (d*x
)/2) + 512*A*B*a*b^7*sin(c/2 + (d*x)/2) + 96*A*B*a^7*b*sin(c/2 + (d*x)/2))/
(cos(c/2 + (d*x)/2)*(9*A^2*a^8 + 64*A^2*b^8 + 64*B^2*b^8 + 384*A^2*a^2*b^6
+ 624*A^2*a^4*b^4 + 144*A^2*a^6*b^2 + 1024*B^2*a^2*b^6 + 1024*B^2*a^4*b^4 +
 256*B^2*a^6*b^2 + 512*A*B*a*b^7 + 96*A*B*a^7*b + 1792*A*B*a^3*b^5 + 960*A*
B*a^5*b^3))) + (33*A*a^4*sin(c + d*x))/8 + 12*B*b^4*cos(2*c + 2*d*x)*atan((
9*A^2*a^8*sin(c/2 + (d*x)/2) + 64*A^2*b^8*sin(c/2 + (d*x)/2) + 64*B^2*b^8*s
in(c/2 + (d*x)/2) + 384*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624*A^2*a^4*b^4*si
n(c/2 + (d*x)/2) + 144*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 1024*B^2*a^2*b^6*si
n(c/2 + (d*x)/2) + 1024*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 256*B^2*a^6*b^2*si
n(c/2 + (d*x)/2) + 1792*A*B*a^3*b^5*sin(c/2 + (d*x)/2) + 960*A*B*a^5*b^3*si
n(c/2 + (d*x)/2) + 512*A*B*a*b^7*sin(c/2 + (d*x)/2) + 96*A*B*a^7*b*sin(c/2
+ (d*x)/2))/(cos(c/2 + (d*x)/2)*(9*A^2*a^8 + 64*A^2*b^8 + 64*B^2*b^8 + 384*
A^2*a^2*b^6 + 624*A^2*a^4*b^4 + 144*A^2*a^6*b^2 + 1024*B^2*a^2*b^6 + 1024*B
^2*a^4*b^4 + 256*B^2*a^6*b^2 + 512*A*B*a*b^7 + 96*A*B*a^7*b + 1792*A*B*a^3*
b^5 + 960*A*B*a^5*b^3))) + 3*B*b^4*cos(4*c + 4*d*x)*atan((9*A^2*a^8*sin(c/2
 + (d*x)/2) + 64*A^2*b^8*sin(c/2 + (d*x)/2) + 64*B^2*b^8*sin(c/2 + (d*x)/2)
 + 384*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624*A^2*a^4*b^4*sin(c/2 + (d*x)/2)
+ 144*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 1024*B^2*a^2*b^6*sin(c/2 + (d*x)/2)
+ 1024*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 256*B^2*a^6*b^2*sin(c/2 + (d*x)/2)
+ 1792*A*B*a^3*b^5*sin(c/2 + (d*x)/2) + 960*A*B*a^5*b^3*sin(c/2 + (d*x)/2)
+ 512*A*B*a*b^7*sin(c/2 + (d*x)/2) + 96*A*B*a^7*b*sin(c/2 + (d*x)/2))/(cos(
c/2 + (d*x)/2)*(9*A^2*a^8 + 64*A^2*b^8 + 64*B^2*b^8 + 384*A^2*a^2*b^6 + 624
*A^2*a^4*b^4 + 144*A^2*a^6*b^2 + 1024*B^2*a^2*b^6 + 1024*B^2*a^4*b^4 + 256*
B^2*a^6*b^2 + 512*A*B*a*b^7 + 96*A*B*a^7*b + 1792*A*B*a^3*b^5 + 960*A*B*a^5
*b^3))) + 6*B*a^3*b*sin(c + d*x) + 36*B*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2)) + 18*B*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) +
 12*A*a*b^3*sin(2*c + 2*d*x) + 16*A*a^3*b*sin(2*c + 2*d*x) + 6*A*a*b^3*sin(
4*c + 4*d*x) + 4*A*a^3*b*sin(4*c + 4*d*x) + 9*A*a^2*b^2*sin(c + d*x) + 6*B*
a^3*b*sin(3*c + 3*d*x) + (9*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2))*cos(2*c + 2*d*x))/2 + (9*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2))*cos(4*c + 4*d*x))/8 + 27*A*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)) + 12*A*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c +
 2*d*x) + 3*A*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(4*c + 4*
d*x) + 9*A*a^2*b^2*sin(3*c + 3*d*x) + 18*B*a^2*b^2*sin(2*c + 2*d*x) + 9*B*a
^2*b^2*sin(4*c + 4*d*x) + 48*B*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
```

```
x)/2))*cos(2*c + 2*d*x) + 24*B*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*cos(2*c + 2*d*x) + 12*B*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*cos(4*c + 4*d*x) + 6*B*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2))*cos(4*c + 4*d*x) + 36*A*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2))*cos(2*c + 2*d*x) + 9*A*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*cos(4*c + 4*d*x))/(12*d*(cos(2*c + 2*d*x)/2 + cos(4*c + 4*d*x)/8 +
 3/8))
```

3.248 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=267

$$\frac{(12a^3Ab+16aAb^3+3a^4B+24a^2b^2B+8b^4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(8a^4A+60a^2Ab^2+15Ab^4+40a^3bB}{15d}$$

```
[Out] 1/8*(12*A*a^3*b+16*A*a*b^3+3*B*a^4+24*B*a^2*b^2+8*B*b^4)*arctanh(sin(d*x+c)
)/d+1/15*(8*A*a^4+60*A*a^2*b^2+15*A*b^4+40*B*a^3*b+60*B*a*b^3)*tan(d*x+c)/d
+1/40*a*(60*A*a^2*b+56*A*b^3+15*B*a^3+110*B*a*b^2)*sec(d*x+c)*tan(d*x+c)/d+
1/30*a^2*(8*A*a^2+18*A*b^2+25*B*a*b)*sec(d*x+c)^2*tan(d*x+c)/d+1/20*a*(8*A*
b+5*B*a)*(a+b*cos(d*x+c))^2*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a*A*(a+b*cos(d*x+
c))^3*sec(d*x+c)^4*tan(d*x+c)/d
```

Rubi [A]

time = 0.47, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3068, 3126, 3110, 3100, 2827, 3852, 8, 3855}

$$\frac{a^2(8a^2A+25abB+18Ab^2)\tan(c+dx)\sec^2(c+dx)}{30d}+\frac{a(15a^3B+60a^2Ab+110ab^2B+56Ab^3)\tan(c+dx)\sec(c+dx)}{40d}+\frac{(8a^4A+40a^3bB+60a^2Ab^2+60ab^3B+15Ab^4)\tan(c+dx)}{15d}+\frac{(3a^4B+12a^3Ab+24a^2b^2B+16aAb^3+8b^4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(5aB+8Ab)\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{20d}+\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] ((12*a^3*A*b + 16*a*A*b^3 + 3*a^4*B + 24*a^2*b^2*B + 8*b^4*B)*ArcTanh[Sin[c
 + d*x]])/(8*d) + ((8*a^4*A + 60*a^2*A*b^2 + 15*A*b^4 + 40*a^3*b*B + 60*a*b
^3*B)*Tan[c + d*x])/(15*d) + (a*(60*a^2*A*b + 56*A*b^3 + 15*a^3*B + 110*a*b
^2*B)*Sec[c + d*x]*Tan[c + d*x])/(40*d) + (a^2*(8*a^2*A + 18*A*b^2 + 25*a*b
*B)*Sec[c + d*x]^2*Tan[c + d*x])/(30*d) + (a*(8*A*b + 5*a*B)*(a + b*Cos[c +
 d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*A*(a + b*Cos[c + d*x])^3*
Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
```

```
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx &= \frac{aA(a+b\cos(c+dx))^3\sec^4(c+dx)\tan(c+d}{5d}\\ &= \frac{a(8Ab+5aB)(a+b\cos(c+dx))^2\sec^3(c+dx}{20d}\\ &= \frac{a^2(8a^2A+18Ab^2+25abB)\sec^2(c+dx)\tan(c}{30d}\\ &= \frac{a(60a^2Ab+56Ab^3+15a^3B+110ab^2B)\sec(c}{40d}\\ &= \frac{a(60a^2Ab+56Ab^3+15a^3B+110ab^2B)\sec(c}{40d}\\ &= \frac{(12a^3Ab+16aAb^3+3a^4B+24a^2b^2B+8b^4B}{8d}\\ &= \frac{(12a^3Ab+16aAb^3+3a^4B+24a^2b^2B+8b^4B}{8d}\end{aligned}$$

Mathematica [A]
time = 4.38, size = 198, normalized size = 0.74

$$\frac{15(12a^3Ab+16aAb^3+3a^4B+24a^2b^2B+8b^4B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(120(a^4A+6a^2Ab^2+Ab^4+4a^3bB+4ab^3B)+15a(12a^2Ab+16Ab^3+3a^3B+24ab^2B)\sec(c+dx)+30a^3(4Ab+aB)\sec^3(c+dx)+80a^2(a^2A+3Ab^2+2abB)\tan^2(c+dx)+24a^4A\tan^4(c+dx)\right)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (15*(12*a^3*A*b + 16*a*A*b^3 + 3*a^4*B + 24*a^2*b^2*B + 8*b^4*B)*ArcTanh[Si
n[c + d*x]] + Tan[c + d*x]*(120*(a^4*A + 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B +
4*a*b^3*B) + 15*a*(12*a^2*A*b + 16*A*b^3 + 3*a^3*B + 24*a*b^2*B)*Sec[c + d*
```

```
x] + 30*a^3*(4*A*b + a*B)*Sec[c + d*x]^3 + 80*a^2*(a^2*A + 3*A*b^2 + 2*a*b*
B)*Tan[c + d*x]^2 + 24*a^4*A*Tan[c + d*x]^4))/(120*d)
```

Maple [A]
time = 0.26, size = 313, normalized size = 1.17 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-A*a^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+a^4*B*(-(
-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))
+4*A*a^3*b*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c
)+tan(d*x+c)))-4*B*a^3*b*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-6*A*a^2*b^2*(-2
/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+6*B*a^2*b^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*
ln(sec(d*x+c)+tan(d*x+c)))+4*A*a*b^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(
d*x+c)+tan(d*x+c)))+4*B*a*b^3*tan(d*x+c)+A*b^4*tan(d*x+c)+B*b^4*ln(sec(d*x+
c)+tan(d*x+c)))
```

Maxima [A]
time = 0.30, size = 386, normalized size = 1.45

$$\frac{16\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)Aa^{4}+320\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Ba^{3}b+480\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Aa^{2}b^{2}-15\,Ba^{4}\left(\frac{2\left(3\sin(dx+c)^{3}-5\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\sin(dx+c)^{2}+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-60\,Aa^{3}b\left(\frac{2\left(3\sin(dx+c)^{3}-5\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\sin(dx+c)^{2}+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-360\,Ba^{2}b^{2}\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-240\,Aab^{3}\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+120\,Bb^{4}\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+960\,Bab^{3}\tan\left(dx+c\right)+240\,Ab^{4}\tan\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
320*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3*b + 480*(tan(d*x + c)^3 + 3*tan
(d*x + c))*A*a^2*b^2 - 15*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d
*x + c) - 1)) - 60*A*a^3*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x
+ c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x +
c) - 1)) - 360*B*a^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) - 240*A*a*b^3*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120*B*b^4*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 960*B*a*b^3*tan(d*x + c) +
240*A*b^4*tan(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 281, normalized size = 1.05

$$\frac{15\left(3\,Ba^{4}+12\,Aa^{3}b+24\,Ba^{2}b^{2}+16\,Aab^{3}+8\,Bb^{4}\right)\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\left(3\,Ba^{4}+12\,Aa^{3}b+24\,Ba^{2}b^{2}+16\,Aab^{3}+8\,Bb^{4}\right)\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(24\,Aa^{4}+8\left(8\,Aa^{4}+40\,Ba^{3}b+60\,Aa^{2}b^{2}+60\,Bab^{3}+15\,Ab^{4}\right)\cos\left(dx+c\right)^{4}+15\left(3\,Ba^{4}+12\,Aa^{3}b+24\,Ba^{2}b^{2}+16\,Aab^{3}\right)\cos\left(dx+c\right)^{3}+16\left(2\,Aa^{4}+10\,Ba^{3}b+15\,Aa^{2}b^{2}\right)\cos\left(dx+c\right)^{2}+30\left(Ba^{4}+4\,Aa^{3}b\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*cos(
d*x + c)^5*log(sin(d*x + c) + 1) - 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2
+ 16*A*a*b^3 + 8*B*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(24*A*a^4
 + 8*(8*A*a^4 + 40*B*a^3*b + 60*A*a^2*b^2 + 60*B*a*b^3 + 15*A*b^4)*cos(d*x
+ c)^4 + 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3)*cos(d*x + c)
^3 + 16*(2*A*a^4 + 10*B*a^3*b + 15*A*a^2*b^2)*cos(d*x + c)^2 + 30*(B*a^4 +
4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 850 vs. 2(255) = 510.
time = 0.52, size = 850, normalized size = 3.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*log(
abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 +
16*A*a*b^3 + 8*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^4*tan
(1/2*d*x + 1/2*c)^9 - 75*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 300*A*a^3*b*tan(1/2
*d*x + 1/2*c)^9 + 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 720*A*a^2*b^2*tan(1/
2*d*x + 1/2*c)^9 - 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 240*A*a*b^3*tan(1
/2*d*x + 1/2*c)^9 + 480*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*A*b^4*tan(1/2*
d*x + 1/2*c)^9 - 160*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 30*B*a^4*tan(1/2*d*x +
1/2*c)^7 + 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 1280*B*a^3*b*tan(1/2*d*x +
1/2*c)^7 - 1920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*B*a^2*b^2*tan(1/2*d*
x + 1/2*c)^7 + 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 1920*B*a*b^3*tan(1/2*d*
x + 1/2*c)^7 - 480*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 464*A*a^4*tan(1/2*d*x + 1
/2*c)^5 + 1600*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 2400*A*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^5 + 2880*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 720*A*b^4*tan(1/2*d*x +
1/2*c)^5 - 160*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^4*tan(1/2*d*x + 1/2*c)
```

```
^3 - 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1280*B*a^3*b*tan(1/2*d*x + 1/2*c)
^3 - 1920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 720*B*a^2*b^2*tan(1/2*d*x + 1/
2*c)^3 - 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 1920*B*a*b^3*tan(1/2*d*x + 1/
2*c)^3 - 480*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^4*tan(1/2*d*x + 1/2*c)
+ 75*B*a^4*tan(1/2*d*x + 1/2*c) + 300*A*a^3*b*tan(1/2*d*x + 1/2*c) + 480*B*
a^3*b*tan(1/2*d*x + 1/2*c) + 720*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 360*B*a^2
*b^2*tan(1/2*d*x + 1/2*c) + 240*A*a*b^3*tan(1/2*d*x + 1/2*c) + 480*B*a*b^3*
tan(1/2*d*x + 1/2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c
)^2 - 1)^5)/d
```

Mupad [B]
time = 3.88, size = 555, normalized size = 2.08

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3Ba^4}{8}+\frac{3Aa^3b}{2}+3Ba^2b^2+2Aab^3+Bb^4\right)}{\frac{3Ba^4}{2}+6Aa^3b+12Ba^2b^2+8Aab^3+4Bb^4}\right)\left(\frac{3Ba^4}{4}+3Aa^3b+6Ba^2b^2+4Aab^3+2Bb^4\right)}{d}-\frac{\left(2Aa^4+2Ab^4-\frac{5Ba^4}{4}+12Aa^2b^2-6Ba^2b^2-4Aab^3-5Aa^3b+8Bab^3+8Ba^3b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(\frac{Ba^4}{2}-8Ab^4-\frac{8Aa^4}{3}-32Aa^2b^2+12Ba^2b^2+8Aab^3+2Aa^3b-32Bab^3-\frac{64Ba^3b}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{116Aa^4}{15}+\frac{80Ba^3b}{3}+40Aa^2b^2+48Bab^3+12Ab^4\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\frac{8Aa^4}{3}-8Ab^4-\frac{Ba^4}{2}-32Aa^2b^2-12Ba^2b^2-8Aab^3-2Aa^3b-32Bab^3-\frac{64Ba^3b}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(2Aa^4+2Ab^4+\frac{5Ba^4}{4}+12Aa^2b^2+6Ba^2b^2+4Aab^3+5Aa^3b+8Bab^3+8Ba^3b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^6,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((3*B*a^4)/8 + B*b^4 + 3*B*a^2*b^2 + 2*A*a*b^3
 + (3*A*a^3*b)/2))/((3*B*a^4)/2 + 4*B*b^4 + 12*B*a^2*b^2 + 8*A*a*b^3 + 6*A*
a^3*b))*((3*B*a^4)/4 + 2*B*b^4 + 6*B*a^2*b^2 + 4*A*a*b^3 + 3*A*a^3*b))/d -
(tan(c/2 + (d*x)/2)*(2*A*a^4 + 2*A*b^4 + (5*B*a^4)/4 + 12*A*a^2*b^2 + 6*B*a
^2*b^2 + 4*A*a*b^3 + 5*A*a^3*b + 8*B*a*b^3 + 8*B*a^3*b) + tan(c/2 + (d*x)/2
)^5*((116*A*a^4)/15 + 12*A*b^4 + 40*A*a^2*b^2 + 48*B*a*b^3 + (80*B*a^3*b)/3
) + tan(c/2 + (d*x)/2)^9*(2*A*a^4 + 2*A*b^4 - (5*B*a^4)/4 + 12*A*a^2*b^2 -
6*B*a^2*b^2 - 4*A*a*b^3 - 5*A*a^3*b + 8*B*a*b^3 + 8*B*a^3*b) - tan(c/2 + (d
*x)/2)^3*((8*A*a^4)/3 + 8*A*b^4 + (B*a^4)/2 + 32*A*a^2*b^2 + 12*B*a^2*b^2 +
 8*A*a*b^3 + 2*A*a^3*b + 32*B*a*b^3 + (64*B*a^3*b)/3) - tan(c/2 + (d*x)/2)^
7*((8*A*a^4)/3 + 8*A*b^4 - (B*a^4)/2 + 32*A*a^2*b^2 - 12*B*a^2*b^2 - 8*A*a*
b^3 - 2*A*a^3*b + 32*B*a*b^3 + (64*B*a^3*b)/3))/(d*(5*tan(c/2 + (d*x)/2)^2
- 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^
8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.249 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$

Optimal. Leaf size=324

$$\frac{(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(32a^3Ab+40aAb^3+8a^4B+60a^2b^2B}{15d}$$

```
[Out] 1/16*(5*A*a^4+36*A*a^2*b^2+8*A*b^4+24*B*a^3*b+32*B*a*b^3)*arctanh(sin(d*x+c
))/d+1/15*(32*A*a^3*b+40*A*a*b^3+8*B*a^4+60*B*a^2*b^2+15*B*b^4)*tan(d*x+c)/
d+1/16*(5*A*a^4+36*A*a^2*b^2+8*A*b^4+24*B*a^3*b+32*B*a*b^3)*sec(d*x+c)*tan(
d*x+c)/d+1/15*a*(16*A*a^2*b+13*A*b^3+4*B*a^3+27*B*a*b^2)*sec(d*x+c)^2*tan(d
*x+c)/d+1/120*a^2*(25*A*a^2+48*A*b^2+72*B*a*b)*sec(d*x+c)^3*tan(d*x+c)/d+1/
10*a*(3*A*b+2*B*a)*(a+b*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d+1/6*a*A*(a+
b*cos(d*x+c))^3*sec(d*x+c)^5*tan(d*x+c)/d
```

Rubi [A]
time = 0.51, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3068, 3126, 3110, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{a^2(25a^2A+72abB+48Ab^2)\tan(c+dx)\sec^3(c+dx)}{120d}+\frac{a(4a^3B+16a^2Ab+27ab^2B+13Ab^3)\tan(c+dx)\sec^2(c+dx)}{15d}+\frac{(8a^4B+32a^3Ab+60a^2b^2B+40aAb^3+15b^4B)\tan(c+dx)}{15d}+\frac{(5a^4A+24a^3bB+36a^2Ab^2+32ab^3B+8Ab^4)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(5a^4A+24a^3bB+36a^2Ab^2+32ab^3B+8Ab^4)\tan(c+dx)\sec(c+dx)}{16d}+\frac{a(2aB+3Ab)\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{10d}+\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

```
[Out] ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*ArcTanh[Sin[c
 + d*x]])/(16*d) + ((32*a^3*A*b + 40*a*A*b^3 + 8*a^4*B + 60*a^2*b^2*B + 15*
b^4*B)*Tan[c + d*x])/(15*d) + ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b
*B + 32*a*b^3*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a*(16*a^2*A*b + 13*A*
b^3 + 4*a^3*B + 27*a*b^2*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d) + (a^2*(25*
a^2*A + 48*A*b^2 + 72*a*b*B)*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + (a*(3*A
*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(10*d) + (a
*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
```

```
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx &= \frac{aA(a+b\cos(c+dx))^3\sec^5(c+dx)\tan(c+d\\
&= \frac{a(3Ab+2aB)(a+b\cos(c+dx))^2\sec^4(c+dx}{10d}\\
&= \frac{a^2(25a^2A+48Ab^2+72abB)\sec^3(c+dx)\tan(}{120d}\\
&= \frac{a(16a^2Ab+13Ab^3+4a^3B+27ab^2B)\sec^2(c+}{15d}\\
&= \frac{a(16a^2Ab+13Ab^3+4a^3B+27ab^2B)\sec^2(c+}{15d}\\
&= \frac{(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B}{16d}\\
&= \frac{(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B}{16d}\end{aligned}$$

(The first line's denominator is $6d$.)

Mathematica [A]
time = 2.84, size = 244, normalized size = 0.75

$$\frac{15(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(240(4a^3Ab+4aAb^3+a^4B+6a^2b^2B+b^4B)+15(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B)\sec(c+dx)+10a^2(5a^2A+36Ab^2+24abB)\sec^3(c+dx)+40a^4A\sec^5(c+dx)+160a(4a^2Ab+2Ab^3+a^3B+3ab^2B)\tan^2(c+dx)+48a^3(4Ab+aB)\tan^4(c+dx)\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

```
[Out] (15*(5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*ArcTanh[Si
n[c + d*x]] + Tan[c + d*x]*(240*(4*a^3*A*b + 4*a*A*b^3 + a^4*B + 6*a^2*b^2*
B + b^4*B) + 15*(5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B
)*Sec[c + d*x] + 10*a^2*(5*a^2*A + 36*A*b^2 + 24*a*b*B)*Sec[c + d*x]^3 + 40
*a^4*A*Sec[c + d*x]^5 + 160*a*(4*a^2*A*b + 2*A*b^3 + a^3*B + 3*a*b^2*B)*Tan
[c + d*x]^2 + 48*a^3*(4*A*b + a*B)*Tan[c + d*x]^4))/(240*d)
```

Maple [A]
time = 0.29, size = 375, normalized size = 1.16

method	result
derivativedivides	$\frac{A a^4\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-a^4 B\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c\right.}{5}\right.}{}$
default	$\frac{A a^4\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-a^4 B\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c\right.}{5}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+
c)+5/16*ln(sec(d*x+c)+tan(d*x+c)))-a^4*B*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d
*x+c)^2)*tan(d*x+c)-4*A*a^3*b*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*ta
n(d*x+c)+4*B*a^3*b*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(s
ec(d*x+c)+tan(d*x+c)))+6*A*a^2*b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan
(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-6*B*a^2*b^2*(-2/3-1/3*sec(d*x+c)^2)*
tan(d*x+c)-4*A*a*b^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*B*a*b^3*(1/2*sec(
d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+A*b^4*(1/2*sec(d*x+c)*tan(
d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+B*b^4*tan(d*x+c))
```

Maxima [A]
time = 0.29, size = 474, normalized size = 1.46

$$\frac{32\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ba^4+128\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^3b+960\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2b^2+640\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aab^3-5Aa^4\left(\frac{2\left(15\sin(dx+c)^5-40\sin(dx+c)^3+33\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)\right)-120Ba^3b\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-180Aa^2b^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-480Bab^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-120Ab^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+480Bb^4\tan(dx+c)}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 +
128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3*b + 960*
(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2*b^2 + 640*(tan(d*x + c)^3 + 3*tan(d
*x + c))*A*a*b^3 - 5*A*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*s
in(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 1
5*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 120*B*a^3*b*(2*(3*sin
(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*l
og(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 180*A*a^2*b^2*(2*(3*sin(d
*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 480*B*a*b^3*(2*sin(d*x + c)
/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12
0*A*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) + 480*B*b^4*tan(d*x + c))/d
```

Fricas [**A**]
time = 0.41, size = 327, normalized size = 1.01

$$\frac{15\,(5\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+32\,Bab^3+8\,Ab^4)\cos(dx+c)^6\log(\sin(dx+c)+1)-15\,(5\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+32\,Bab^3+8\,Ab^4)\cos(dx+c)^6\log(-\sin(dx+c)+1)+2\,(16\,(8\,Ba^4+32\,Aa^3b+60\,Ba^2b^2+40\,Aab^3+15\,Bb^4)\cos(dx+c)^5+40\,Aa^4+15\,(5\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+32\,Bab^3+8\,Ab^4)\cos(dx+c)^4+32\,(2\,Ba^4+8\,Aa^3b+15\,Ba^2b^2+10\,Aab^3)\cos(dx+c)^3+10\,(5\,Aa^4+24\,Ba^3b+36\,Aa^2b^2)\cos(dx+c)^2+48\,(Ba^4+4\,Aa^3b)\cos(dx+c))\sin(dx+c)}{480\,d\cos(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(
d*x + c)^6*log(sin(d*x + c) + 1) - 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2
+ 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(8*B*
a^4 + 32*A*a^3*b + 60*B*a^2*b^2 + 40*A*a*b^3 + 15*B*b^4)*cos(d*x + c)^5 + 4
0*A*a^4 + 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*c
os(d*x + c)^4 + 32*(2*B*a^4 + 8*A*a^3*b + 15*B*a^2*b^2 + 10*A*a*b^3)*cos(d*
x + c)^3 + 10*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2)*cos(d*x + c)^2 + 48*(B*
a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**7,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1186 vs. 2(310) = 620.
time = 0.55, size = 1186, normalized size = 3.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*log(
abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 +
32*B*a*b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(165*A*a^4*tan
(1/2*d*x + 1/2*c)^11 - 240*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*A*a^3*b*tan(
1/2*d*x + 1/2*c)^11 + 600*B*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*A*a^2*b^2*t
an(1/2*d*x + 1/2*c)^11 - 1440*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*A*a*b
^3*tan(1/2*d*x + 1/2*c)^11 + 480*B*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*A*b^
4*tan(1/2*d*x + 1/2*c)^11 - 240*B*b^4*tan(1/2*d*x + 1/2*c)^11 + 25*A*a^4*ta
n(1/2*d*x + 1/2*c)^9 + 560*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 2240*A*a^3*b*tan(
1/2*d*x + 1/2*c)^9 - 840*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 1260*A*a^2*b^2*ta
n(1/2*d*x + 1/2*c)^9 + 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 3520*A*a*b^3
*tan(1/2*d*x + 1/2*c)^9 - 1440*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 360*A*b^4*t
an(1/2*d*x + 1/2*c)^9 + 1200*B*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*A*a^4*tan(1
/2*d*x + 1/2*c)^7 - 1248*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 4992*A*a^3*b*tan(1/
2*d*x + 1/2*c)^7 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*A*a^2*b^2*tan(1
/2*d*x + 1/2*c)^7 - 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*A*a*b^3*ta
n(1/2*d*x + 1/2*c)^7 + 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*A*b^4*tan(1
/2*d*x + 1/2*c)^7 - 2400*B*b^4*tan(1/2*d*x + 1/2*c)^7 + 450*A*a^4*tan(1/2*d
*x + 1/2*c)^5 + 1248*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 4992*A*a^3*b*tan(1/2*d*
x + 1/2*c)^5 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 360*A*a^2*b^2*tan(1/2*d
*x + 1/2*c)^5 + 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 5760*A*a*b^3*tan(1/
2*d*x + 1/2*c)^5 + 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 240*A*b^4*tan(1/2*d
*x + 1/2*c)^5 + 2400*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 25*A*a^4*tan(1/2*d*x +
1/2*c)^3 - 560*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*A*a^3*b*tan(1/2*d*x + 1/
2*c)^3 - 840*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1260*A*a^2*b^2*tan(1/2*d*x +
1/2*c)^3 - 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*A*a*b^3*tan(1/2*d*x
 + 1/2*c)^3 - 1440*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 360*A*b^4*tan(1/2*d*x +
 1/2*c)^3 - 1200*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 165*A*a^4*tan(1/2*d*x + 1/2
*c) + 240*B*a^4*tan(1/2*d*x + 1/2*c) + 960*A*a^3*b*tan(1/2*d*x + 1/2*c) + 6
00*B*a^3*b*tan(1/2*d*x + 1/2*c) + 900*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 1440
*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 960*A*a*b^3*tan(1/2*d*x + 1/2*c) + 480*B*
a*b^3*tan(1/2*d*x + 1/2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*c) + 240*B*b^4*tan
(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B]
time = 3.75, size = 706, normalized size = 2.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^7,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((5*A*a^4)/16 + (A*b^4)/2 + (9*A*a^2*b^2)/4 +
2*B*a*b^3 + (3*B*a^3*b)/2))/((5*A*a^4)/4 + 2*A*b^4 + 9*A*a^2*b^2 + 8*B*a*b^
3 + 6*B*a^3*b))*((5*A*a^4)/8 + A*b^4 + (9*A*a^2*b^2)/2 + 4*B*a*b^3 + 3*B*a^
3*b))/d + (tan(c/2 + (d*x)/2)*((11*A*a^4)/8 + A*b^4 + 2*B*a^4 + 2*B*b^4 + (
15*A*a^2*b^2)/2 + 12*B*a^2*b^2 + 8*A*a*b^3 + 8*A*a^3*b + 4*B*a*b^3 + 5*B*a^
3*b) + tan(c/2 + (d*x)/2)^11*((11*A*a^4)/8 + A*b^4 - 2*B*a^4 - 2*B*b^4 + (1
5*A*a^2*b^2)/2 - 12*B*a^2*b^2 - 8*A*a*b^3 - 8*A*a^3*b + 4*B*a*b^3 + 5*B*a^3
*b) - tan(c/2 + (d*x)/2)^3*(3*A*b^4 - (5*A*a^4)/24 + (14*B*a^4)/3 + 10*B*b^
4 + (21*A*a^2*b^2)/2 + 44*B*a^2*b^2 + (88*A*a*b^3)/3 + (56*A*a^3*b)/3 + 12*
B*a*b^3 + 7*B*a^3*b) + tan(c/2 + (d*x)/2)^9*((5*A*a^4)/24 - 3*A*b^4 + (14*B
*a^4)/3 + 10*B*b^4 - (21*A*a^2*b^2)/2 + 44*B*a^2*b^2 + (88*A*a*b^3)/3 + (56
*A*a^3*b)/3 - 12*B*a*b^3 - 7*B*a^3*b) + tan(c/2 + (d*x)/2)^5*((15*A*a^4)/4
+ 2*A*b^4 + (52*B*a^4)/5 + 20*B*b^4 + 3*A*a^2*b^2 + 72*B*a^2*b^2 + 48*A*a*b
^3 + (208*A*a^3*b)/5 + 8*B*a*b^3 + 2*B*a^3*b) + tan(c/2 + (d*x)/2)^7*((15*A
*a^4)/4 + 2*A*b^4 - (52*B*a^4)/5 - 20*B*b^4 + 3*A*a^2*b^2 - 72*B*a^2*b^2 -
48*A*a*b^3 - (208*A*a^3*b)/5 + 8*B*a*b^3 + 2*B*a^3*b))/(d*(15*tan(c/2 + (d*
x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (
d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.250 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{(2a^2+b^2)(Ab-aB)x}{2b^4}-\frac{2a^3(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^4\sqrt{a+b}\,d}-\frac{(3aAb-3a^2B-2b^2B)\sin(c+dx)}{3b^3d}+$$

```
[Out] 1/2*(2*a^2+b^2)*(A*b-B*a)*x/b^4-1/3*(3*A*a*b-3*B*a^2-2*B*b^2)*sin(d*x+c)/b^
3/d+1/2*(A*b-B*a)*cos(d*x+c)*sin(d*x+c)/b^2/d+1/3*B*cos(d*x+c)^2*sin(d*x+c)
/b/d-2*a^3*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^4
/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3069, 3128, 3102, 2814, 2738, 211}

$$-\frac{2a^3(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\,\sqrt{a+b}}+\frac{x(2a^2+b^2)(Ab-aB)}{2b^4}-\frac{(-3a^2B+3aAb-2b^2B)\sin(c+dx)}{3b^3d}+\frac{(Ab-aB)\sin(c+dx)\cos(c+dx)}{2b^2d}+\frac{B\sin(c+dx)\cos^2(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*a^2 + b^2)*(A*b - a*B)*x)/(2*b^4) - (2*a^3*(A*b - a*B)*ArcTan[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) - ((3*a
*A*b - 3*a^2*B - 2*b^2*B)*Sin[c + d*x])/(3*b^3*d) + ((A*b - a*B)*Cos[c + d*
x]*Sin[c + d*x])/(2*b^2*d) + (B*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{B\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{\cos(c+dx)(2aB+2bB\cos(c+dx)+3(Ab-aB)\text{co}}{a+b\cos(c+dx)}}{3b}$$

$$= \frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{B\cos^2(c+dx)\sin(c+dx)}{3bd} +$$

$$= -\frac{(3aAb-3a^2B-2b^2B)\sin(c+dx)}{3b^3d} + \frac{(Ab-aB)\cos(c+dx)\sin(}{2b^2d}$$

$$= \frac{(2a^2+b^2)(Ab-aB)x}{2b^4} - \frac{(3aAb-3a^2B-2b^2B)\sin(c+dx)}{3b^3d} + \frac{(A}{}$$

$$= \frac{(2a^2+b^2)(Ab-aB)x}{2b^4} - \frac{(3aAb-3a^2B-2b^2B)\sin(c+dx)}{3b^3d} + \frac{(A}{}$$

$$= \frac{(2a^2+b^2)(Ab-aB)x}{2b^4} - \frac{2a^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right.}{\sqrt{a-b}\,b^4\sqrt{a+b}\,d}$$

Mathematica [A]
time = 0.52, size = 152, normalized size = 0.85

$$\frac{6(2a^2+b^2)(Ab-aB)(c+dx) - \frac{24a^3(-Ab+aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + 3b(-4aAb+4a^2B+3b^2B)\sin(c+dx) + 3b^2(Ab-aB)\sin(2(c+dx)) + b^3B\sin(3(c+dx))}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (6*(2*a^2 + b^2)*(A*b - a*B)*(c + d*x) - (24*a^3*(-(A*b) + a*B)*ArcTanh[((a
 - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 3*b*(-4*a*A*b
 + 4*a^2*B + 3*b^2*B)*Sin[c + d*x] + 3*b^2*(A*b - a*B)*Sin[2*(c + d*x)] + b
^3*B*Sin[3*(c + d*x)])/(12*b^4*d)
```

Maple [A]
time = 0.29, size = 240, normalized size = 1.35

method	result
derivativedivides	$-\frac{\frac{2a^3(Ab-aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b^4\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(-Aa\,b^2-\frac{1}{2}A\,b^3+a^2bB+\frac{1}{2}Ba\,b^2+b^3B\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2Aa\,b^2+2a^2b\right.}{\left(1+\tan\right.}}{}$

default	$-\frac{2a^3(Ab-aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b^4\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(-Aa\,b^2-\frac{1}{2}A\,b^3+a^2bB+\frac{1}{2}Ba\,b^2+b^3B\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2Aa\,b^2+2a^2\right.\right.}{\left(1+\mathrm{ta}\right.}$
risch	$\frac{xA\,a^2}{b^3}+\frac{xA}{2b}-\frac{x\,a^3B}{b^4}-\frac{aBx}{2b^2}-\frac{ie^{i(dx+c)}B\,a^2}{2b^3d}+\frac{ie^{i(dx+c)}Aa}{2b^2d}-\frac{3ie^{i(dx+c)}B}{8bd}+\frac{ie^{-i(dx+c)}B\,a^2}{2b^3d}+\frac{3ie^{-i(dx}}{8bd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^3*(A*b-B*a)/b^4/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-
b)/((a-b)*(a+b))^(1/2))+2/b^4*(((-A*a*b^2-1/2*A*b^3+a^2*b*B+1/2*B*a*b^2+b^3
*B)*tan(1/2*d*x+1/2*c)^5+(-2*A*a*b^2+2*a^2*b*B+2/3*b^3*B)*tan(1/2*d*x+1/2*c
)^3+(-A*a*b^2+a^2*b*B+b^3*B+1/2*A*b^3-1/2*B*a*b^2)*tan(1/2*d*x+1/2*c))/(1+t
an(1/2*d*x+1/2*c)^2)^3+1/2*(2*A*a^2*b+A*b^3-2*B*a^3-B*a*b^2)*arctan(tan(1/2
*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.41, size = 541, normalized size = 3.04

$$\left[-\frac{3\left(2Ba^5-2Aa^4b-Ba^3b^2+Aa^2b^3-Bab^4+Ab^5\right)dx-3\left(Ba^4-Aa^3b\right)\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)-\left(6Ba^4b-6Aa^3b^2-2Ba^2b^3+6Aab^4-4Bb^5+2\left(Ba^2b^3-Bb^5\right)\cos(dx+c)^2-3\left(Ba^3b^2-Aa^2b^3-Bab^4+Ab^5\right)\cos(dx+c)\right)\sin(dx+c)}{6\left(a^2b^4-b^6\right)d},-\frac{3\left(2Ba^5-2Aa^4b-Ba^3b^2+Aa^2b^3-Bab^4+Ab^5\right)dx-6\left(Ba^4-Aa^3b\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)-\left(6Ba^4b-6Aa^3b^2-2Ba^2b^3+6Aab^4-4Bb^5+2\left(Ba^2b^3-Bb^5\right)\cos(dx+c)^2-3\left(Ba^3b^2-Aa^2b^3-Bab^4+Ab^5\right)\cos(dx+c)\right)\sin(dx+c)}{6\left(a^2b^4-b^6\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [-1/6*(3*(2*B*a^5 - 2*A*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*d*
x - 3*(B*a^4 - A*a^3*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
 - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*B*a^4
```

```
*b - 6*A*a^3*b^2 - 2*B*a^2*b^3 + 6*A*a*b^4 - 4*B*b^5 + 2*(B*a^2*b^3 - B*b^5
)*cos(d*x + c)^2 - 3*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c)
)*sin(d*x + c))/((a^2*b^4 - b^6)*d), -1/6*(3*(2*B*a^5 - 2*A*a^4*b - B*a^3*b
^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*d*x - 6*(B*a^4 - A*a^3*b)*sqrt(a^2 - b^2)
*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*B*a^4*b
- 6*A*a^3*b^2 - 2*B*a^2*b^3 + 6*A*a*b^4 - 4*B*b^5 + 2*(B*a^2*b^3 - B*b^5)*c
os(d*x + c)^2 - 3*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c))*s
in(d*x + c))/((a^2*b^4 - b^6)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(161) = 322.
time = 0.46, size = 360, normalized size = 2.02

$$-\frac{\frac{3\,(2\,Ba^3-2\,Aa^2b+Bab^2-Ab^3)(dx+c)}{b^4}+\frac{12\,(Ba^4-Aa^3b)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}-\frac{2\left(6\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3b^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(3*(2*B*a^3 - 2*A*a^2*b + B*a*b^2 - A*b^3)*(d*x + c)/b^4 + 12*(B*a^4 -
 A*a^3*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 -
b^2)*b^4) - 2*(6*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b*tan(1/2*d*x + 1/2*c
)^5 + 3*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*B
*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*A*a*b*ta
n(1/2*d*x + 1/2*c)^3 + 4*B*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*tan(1/2*d*x
 + 1/2*c) - 6*A*a*b*tan(1/2*d*x + 1/2*c) - 3*B*a*b*tan(1/2*d*x + 1/2*c) + 3
*A*b^2*tan(1/2*d*x + 1/2*c) + 6*B*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
 1/2*c)^2 + 1)^3*b^3))/d
```

Mupad [B]
time = 5.09, size = 2500, normalized size = 14.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)

[Out] ((tan(c/2 + (d*x)/2)*(A*b^2 + 2*B*a^2 + 2*B*b^2 - 2*A*a*b - B*a*b))/b^3 + (
tan(c/2 + (d*x)/2)^5*(2*B*a^2 - A*b^2 + 2*B*b^2 - 2*A*a*b + B*a*b))/b^3 + (
4*tan(c/2 + (d*x)/2)^3*(3*B*a^2 + B*b^2 - 3*A*a*b))/(3*b^3))/(d*(3*tan(c/2
+ (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (atan(
(((2*a^2 + b^2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3
*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5
 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^
3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 -
2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^
5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 + ((2*a^2 + b^2)
*(A*b - B*a)*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*
B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*
b^12))/b^9 - (tan(c/2 + (d*x)/2)*(2*a^2 + b^2)*(A*b - B*a)*(8*a*b^10 - 16*a
^2*b^9 + 8*a^3*b^8)*4i)/b^10)*1i)/(2*b^4)))/(2*b^4) + ((2*a^2 + b^2)*(A*b -
 B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^
8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16
*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5
- 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a
^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 3
2*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 - ((2*a^2 + b^2)*(A*b - B*a)*((8*(2*A*
b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b
^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 + (tan(c/2
+ (d*x)/2)*(2*a^2 + b^2)*(A*b - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*4i
)/b^10)*1i)/(2*b^4)))/(2*b^4))/((16*(4*B^3*a^11 - 6*B^3*a^10*b + A^3*a^3*b^
8 - 2*A^3*a^4*b^7 + 5*A^3*a^5*b^6 - 6*A^3*a^6*b^5 + 6*A^3*a^7*b^4 - 4*A^3*a
^8*b^3 - B^3*a^6*b^5 + 2*B^3*a^7*b^4 - 5*B^3*a^8*b^3 + 6*B^3*a^9*b^2 - 12*A
*B^2*a^10*b + 3*A*B^2*a^5*b^6 - 6*A*B^2*a^6*b^5 + 15*A*B^2*a^7*b^4 - 18*A*B
^2*a^8*b^3 + 18*A*B^2*a^9*b^2 - 3*A^2*B*a^4*b^7 + 6*A^2*B*a^5*b^6 - 15*A^2*
B*a^6*b^5 + 18*A^2*B*a^7*b^4 - 18*A^2*B*a^8*b^3 + 12*A^2*B*a^9*b^2))/b^9 -
((2*a^2 + b^2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*
A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5
- 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3
*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2
*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5
 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 + ((2*a^2 + b^2)*
(A*b - B*a)*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B
*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b
^12))/b^9 - (tan(c/2 + (d*x)/2)*(2*a^2 + b^2)*(A*b - B*a)*(8*a*b^10 - 16*a^
2*b^9 + 8*a^3*b^8)*4i)/b^10)*1i)/(2*b^4))*1i)/(2*b^4) + ((2*a^2 + b^2)*(A*b
 - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*
a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 +
16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^
5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B

*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 - ((2*a^2 + b^2)*(A*b - B*a)*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 + (tan(c/2 + (d*x)/2)*(2*a^2 + b^2)*(A*b - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*4i)/b^10)*1i)/(2*b^4))*1i)/(2*b^4)))*(2*a^2 + b^2)*(A*b - B*a))/(b^4*d) + (a^3*atan(((a^3*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 + (a^3*(-(a + b)*(a - b))^(1/2)*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 - (8*a^3*tan(c/2 + (d*x)/2)*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8))/(b^6*(b^6 - a^2*b^4)))*(A*b - B*a))/(b^6 - a^2*b^4))*1i)/(b^6 - a^2*b^4) + (a^3*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 - (a^3*(-(a + b)*(a - b))^(1/2)*((8*(2*A*b^13 + 2*A*a^2*b^1...

3.251 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=134

$$-\frac{(2aAb-2a^2B-b^2B)\,x}{2b^3}+\frac{2a^2(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d}+\frac{(Ab-aB)\sin(c+dx)}{b^2d}+\frac{B\cos(c$$

```
[Out] -1/2*(2*A*a*b-2*B*a^2-B*b^2)*x/b^3+(A*b-B*a)*sin(d*x+c)/b^2/d+1/2*B*cos(d*x
+c)*sin(d*x+c)/b/d+2*a^2*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a
+b)^(1/2))/b^3/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3069, 3102, 2814, 2738, 211}

$$\frac{2a^2(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\,\sqrt{a+b}}-\frac{x(-2a^2B+2aAb-b^2B)}{2b^3}+\frac{(Ab-aB)\sin(c+dx)}{b^2d}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] -1/2*((2*a*A*b - 2*a^2*B - b^2*B)*x)/b^3 + (2*a^2*(A*b - a*B)*ArcTan[(Sqrt[
a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) + ((
A*b - a*B)*Sin[c + d*x])/(b^2*d) + (B*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{B\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{aB+bB\cos(c+dx)+2(Ab-aB)\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2b}$$

$$= \frac{(Ab-aB)\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{abB-(2aAb-}{a-}}{}$$

$$= -\frac{(2aAb-2a^2B-b^2B)\,x}{2b^3} + \frac{(Ab-aB)\sin(c+dx)}{b^2d} + \frac{B\cos(c+d}{2}$$

$$= -\frac{(2aAb-2a^2B-b^2B)\,x}{2b^3} + \frac{(Ab-aB)\sin(c+dx)}{b^2d} + \frac{B\cos(c+d}{2}$$

$$= -\frac{(2aAb-2a^2B-b^2B)\,x}{2b^3} + \frac{2a^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(c-}{\sqrt{a+b}}\right.}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d}$$

Mathematica [A]
time = 0.35, size = 121, normalized size = 0.90

$$\frac{2(-2aAb+2a^2B+b^2B)(c+dx)+\frac{8a^2(-Ab+aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+4b(Ab-aB)\sin(c+dx)+b^2B\sin(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(-2*a*A*b + 2*a^2*B + b^2*B)*(c + d*x) + (8*a^2*(-(A*b) + a*B)*ArcTanh[(
(a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 4*b*(A*b -
a*B)*Sin[c + d*x] + b^2*B*Sin[2*(c + d*x)])/(4*b^3*d)
```

Maple [A]
time = 0.23, size = 169, normalized size = 1.26

method	result
derivativedivides	$\frac{\frac{2a^2(Ab-aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b^3\sqrt{(a-b)(a+b)}}-\frac{2\left(\frac{\left(-Ab^2+Bab+\frac{1}{2}Bb^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-Ab^2+Bab-\frac{1}{2}Bb^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.}{b^3}}{d}$
default	$\frac{\frac{2a^2(Ab-aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b^3\sqrt{(a-b)(a+b)}}-\frac{2\left(\frac{\left(-Ab^2+Bab+\frac{1}{2}Bb^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-Ab^2+Bab-\frac{1}{2}Bb^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.}{b^3}}{d}$
risch	$-\frac{xAa}{b^2}+\frac{xBa^2}{b^3}+\frac{Bx}{2b}-\frac{ie^{i(dx+c)}A}{2bd}+\frac{ie^{i(dx+c)}aB}{2b^2d}+\frac{ie^{-i(dx+c)}A}{2bd}-\frac{ie^{-i(dx+c)}aB}{2b^2d}-\frac{a^2\ln\left(e^{i(dx+c)}+\frac{ia^2}{}\right.}{\sqrt{-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^2*(A*b-B*a)/b^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b
)/((a-b)*(a+b))^(1/2))-2/b^3*(((-A*b^2+B*a*b+1/2*B*b^2)*tan(1/2*d*x+1/2*c)^
3+(-A*b^2+B*a*b-1/2*B*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1
/2*(2*A*a*b-2*B*a^2-B*b^2)*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.41, size = 426, normalized size = 3.18

$$\left[\frac{(2\,Ba^4-2\,Aa^3b-Ba^2b^2+2\,Aab^3-Bb^4)dx+(Ba^3-Aa^2b)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-(2\,Ba^3b-2\,Aa^2b^2-2\,Bab^3+2\,Ab^4-(Ba^2b^2-Bb^4)\cos(dx+c))\sin(dx+c)}{2\,(a^2b^3-b^5)d}, \frac{(2\,Ba^4-2\,Aa^3b-Ba^2b^2+2\,Aab^3-Bb^4)dx-2\,(Ba^3-Aa^2b)\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\,\sin(dx+c)}\right)-(2\,Ba^3b-2\,Aa^2b^2-2\,Bab^3+2\,Ab^4-(Ba^2b^2-Bb^4)\cos(dx+c))\sin(dx+c)}{2\,(a^2b^3-b^5)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/2*((2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b^4)*d*x + (B*a^3 -
A*a^2*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
 c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (2*B*a^3*b - 2*A*a^2*b^
2 - 2*B*a*b^3 + 2*A*b^4 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))*sin(d*x + c))/(
(a^2*b^3 - b^5)*d), 1/2*((2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b
^4)*d*x - 2*(B*a^3 - A*a^2*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/
(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*
A*b^4 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d)
]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 227, normalized size = 1.69

$$\frac{\frac{(2\,Ba^2-2\,Aab+Bb^2)(dx+c)}{b^3}+\frac{4\,(Ba^3-Aa^2b)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^3}-\frac{2\left(2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((2*B*a^2 - 2*A*a*b + B*b^2)*(d*x + c)/b^3 + 4*(B*a^3 - A*a^2*b)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*
c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^3) - 2*(2
*B*a*tan(1/2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 + B*b*tan(1/2*d*
x + 1/2*c)^3 + 2*B*a*tan(1/2*d*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c) - B*
b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

Mupad [B]
time = 4.00, size = 2500, normalized size = 18.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(2*A*b - 2*B*a + B*b))/b^2 - (tan(c/2 + (d*x)/2)^3*(2*
B*a - 2*A*b + B*b))/b^2)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4
+ 1)) - (atan((((((8*(2*B*b^10 + 8*A*a^2*b^8 - 4*A*a^3*b^7 + 2*B*a^2*b^8 -
 6*B*a^3*b^7 + 4*B*a^4*b^6 - 4*A*a*b^9 - 2*B*a*b^9))/b^6 - (4*tan(c/2 + (d*
x)/2)*(B*a^2*2i + B*b^2*1i - A*a*b*2i)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/
b^7)*(B*a^2*2i + B*b^2*1i - A*a*b*2i))/(2*b^3) - (8*tan(c/2 + (d*x)/2)*(8*B
^2*a^7 - B^2*b^7 + 3*B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A^2*a^3*
b^4 - 16*A^2*a^4*b^3 + 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b^4 - 16*
B^2*a^4*b^3 + 16*B^2*a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B*a^2*b^5
+ 20*A*B*a^3*b^4 - 28*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4)*(B*a^2*2i + B*b^2
*1i - A*a*b*2i)*1i)/(2*b^3) - (((((8*(2*B*b^10 + 8*A*a^2*b^8 - 4*A*a^3*b^7
+ 2*B*a^2*b^8 - 6*B*a^3*b^7 + 4*B*a^4*b^6 - 4*A*a*b^9 - 2*B*a*b^9))/b^6 + (
4*tan(c/2 + (d*x)/2)*(B*a^2*2i + B*b^2*1i - A*a*b*2i)*(8*a*b^8 - 16*a^2*b^7
 + 8*a^3*b^6))/b^7)*(B*a^2*2i + B*b^2*1i - A*a*b*2i))/(2*b^3) + (8*tan(c/2
+ (d*x)/2)*(8*B^2*a^7 - B^2*b^7 + 3*B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^
5 + 12*A^2*a^3*b^4 - 16*A^2*a^4*b^3 + 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^
2*a^3*b^4 - 16*B^2*a^4*b^3 + 16*B^2*a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b -
12*A*B*a^2*b^5 + 20*A*B*a^3*b^4 - 28*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4)*(B
*a^2*2i + B*b^2*1i - A*a*b*2i)*1i)/(2*b^3))/((16*(4*B^3*a^8 - 6*B^3*a^7*b +
 4*A^3*a^4*b^4 - 4*A^3*a^5*b^3 - B^3*a^3*b^5 + 2*B^3*a^4*b^4 - 5*B^3*a^5*b^
3 + 6*B^3*a^6*b^2 - 12*A*B^2*a^7*b + A*B^2*a^2*b^6 - 2*A*B^2*a^3*b^5 + 9*A*
B^2*a^4*b^4 - 12*A*B^2*a^5*b^3 + 16*A*B^2*a^6*b^2 - 4*A^2*B*a^3*b^5 + 6*A^2
*B*a^4*b^4 - 14*A^2*B*a^5*b^3 + 12*A^2*B*a^6*b^2))/b^6 + (((((8*(2*B*b^10 +
 8*A*a^2*b^8 - 4*A*a^3*b^7 + 2*B*a^2*b^8 - 6*B*a^3*b^7 + 4*B*a^4*b^6 - 4*A*
a*b^9 - 2*B*a*b^9))/b^6 - (4*tan(c/2 + (d*x)/2)*(B*a^2*2i + B*b^2*1i - A*a*
b*2i)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/b^7)*(B*a^2*2i + B*b^2*1i - A*a*b
*2i))/(2*b^3) - (8*tan(c/2 + (d*x)/2)*(8*B^2*a^7 - B^2*b^7 + 3*B^2*a*b^6 -
16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A^2*a^3*b^4 - 16*A^2*a^4*b^3 + 8*A^2*a^5*
b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b^4 - 16*B^2*a^4*b^3 + 16*B^2*a^5*b^2 + 4*
A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B*a^2*b^5 + 20*A*B*a^3*b^4 - 28*A*B*a^4*b^3
 + 32*A*B*a^5*b^2))/b^4)*(B*a^2*2i + B*b^2*1i - A*a*b*2i))/(2*b^3) + (((((8
*(2*B*b^10 + 8*A*a^2*b^8 - 4*A*a^3*b^7 + 2*B*a^2*b^8 - 6*B*a^3*b^7 + 4*B*a^
4*b^6 - 4*A*a*b^9 - 2*B*a*b^9))/b^6 + (4*tan(c/2 + (d*x)/2)*(B*a^2*2i + B*b
^2*1i - A*a*b*2i)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/b^7)*(B*a^2*2i + B*b^
2*1i - A*a*b*2i))/(2*b^3) + (8*tan(c/2 + (d*x)/2)*(8*B^2*a^7 - B^2*b^7 + 3*
B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A^2*a^3*b^4 - 16*A^2*a^4*b^3
+ 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b^4 - 16*B^2*a^4*b^3 + 16*B^2*
```

```
a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B*a^2*b^5 + 20*A*B*a^3*b^4 - 28
*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4)*(B*a^2*2i + B*b^2*1i - A*a*b*2i))/(2*b
^3)))*(B*a^2*2i + B*b^2*1i - A*a*b*2i)*1i)/(b^3*d) + (a^2*atan(((a^2*(-(a +
 b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(8*B^2*a^7 - B^2*b^7
+ 3*B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A^2*a^3*b^4 - 16*A^2*a^4*
b^3 + 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b^4 - 16*B^2*a^4*b^3 + 16*
B^2*a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B*a^2*b^5 + 20*A*B*a^3*b^4
- 28*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4 + (a^2*(-(a + b)*(a - b))^(1/2)*(A*
b - B*a)*((8*(2*B*b^10 + 8*A*a^2*b^8 - 4*A*a^3*b^7 + 2*B*a^2*b^8 - 6*B*a^3*
b^7 + 4*B*a^4*b^6 - 4*A*a*b^9 - 2*B*a*b^9))/b^6 + (8*a^2*tan(c/2 + (d*x)/2)
*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/(
b^4*(b^5 - a^2*b^3))))/(b^5 - a^2*b^3))*1i)/(b^5 - a^2*b^3) + (a^2*(-(a + b
)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(8*B^2*a^7 - B^2*b^7 +
3*B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A^2*a^3*b^4 - 16*A^2*a^4*b^
3 + 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b^4 - 16*B^2*a^4*b^3 + 16*B^
2*a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B*a^2*b^5 + 20*A*B*a^3*b^4 -
28*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4 - (a^2*(-(a + b)*(a - b))^(1/2)*(A*b
- B*a)*((8*(2*B*b^10 + 8*A*a^2*b^8 - 4*A*a^3*b^7 + 2*B*a^2*b^8 - 6*B*a^3*b^
7 + 4*B*a^4*b^6 - 4*A*a*b^9 - 2*B*a*b^9))/b^6 - (8*a^2*tan(c/2 + (d*x)/2)*(
-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/(b^
4*(b^5 - a^2*b^3))))/(b^5 - a^2*b^3))*1i)/(b^5 - a^2*b^3))/((16*(4*B^3*a^8
- 6*B^3*a^7*b + 4*A^3*a^4*b^4 - 4*A^3*a^5*b^3 - B^3*a^3*b^5 + 2*B^3*a^4*b^4
 - 5*B^3*a^5*b^3 + 6*B^3*a^6*b^2 - 12*A*B^2*a^7*b + A*B^2*a^2*b^6 - 2*A*B^2
*a^3*b^5 + 9*A*B^2*a^4*b^4 - 12*A*B^2*a^5*b^3 + 16*A*B^2*a^6*b^2 - 4*A^2*B*
a^3*b^5 + 6*A^2*B*a^4*b^4 - 14*A^2*B*a^5*b^3 + 12*A^2*B*a^6*b^2))/b^6 + (a^
2*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(8*B^2*a^7 -
B^2*b^7 + 3*B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A^2*a^3*b^4 - 16*
A^2*a^4*b^3 + 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b^4 - 16*B^2*a^4*b
^3 + 16*B^2*a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B*a^2*b^5 + 20*A*B*
a^3*b^4 - 28*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4...
```

3.252 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=89

$$\frac{(Ab-aB)x}{b^2}-\frac{2a(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d}+\frac{B\sin(c+dx)}{bd}$$

```
[Out] (A*b-B*a)*x/b^2+B*sin(d*x+c)/b/d-2*a*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d
*x+1/2*c)/(a+b)^(1/2))/b^2/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3047, 3102, 12, 2814, 2738, 211}

$$-\frac{2a(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\,\sqrt{a+b}}+\frac{x(Ab-aB)}{b^2}+\frac{B\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 - (2*a*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (B*Sin[c + d*x])/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx &= \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx\\ &= \frac{B\sin(c+dx)}{bd}+\frac{\int \frac{(Ab-aB)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}\\ &= \frac{B\sin(c+dx)}{bd}+\frac{(Ab-aB)\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}\\ &= \frac{(Ab-aB)x}{b^2}+\frac{B\sin(c+dx)}{bd}-\frac{(a(Ab-aB))\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2}\\ &= \frac{(Ab-aB)x}{b^2}+\frac{B\sin(c+dx)}{bd}-\frac{(2a(Ab-aB))\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\right.}{b^2d}\\ &= \frac{(Ab-aB)x}{b^2}-\frac{2a(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d}+\frac{B\sin(}{b}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 85, normalized size = 0.96

$$\frac{(Ab-aB)(c+dx)-\frac{2a(-Ab+aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+bB\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*(c + d*x) - (2*a*(-(A*b) + a*B)*ArcTanh[((a - b)*Tan[(c + d*x)
/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + b*B*Sin[c + d*x])/(b^2*d)
```

Maple [A]
time = 0.20, size = 110, normalized size = 1.24

method	result
derivativedivides	$\frac{-\frac{2a(Ab-aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}+\frac{\frac{2Bb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2(Ab-aB)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}}{d}$
default	$\frac{-\frac{2a(Ab-aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}+\frac{\frac{2Bb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2(Ab-aB)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}}{d}$
risch	$\frac{xA}{b}-\frac{aBx}{b^2}-\frac{i\mathrm{e}^{i(dx+c)}B}{2bd}+\frac{i\mathrm{e}^{-i(dx+c)}B}{2bd}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+a\sqrt{-a^2+b^2}}{\sqrt{-a^2+b^2}\,b}\right)A}{\sqrt{-a^2+b^2}\,db}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+\right.}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a*(A*b-B*a)/b^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)
/((a-b)*(a+b))^(1/2))+2/b^2*(B*b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2
)+(A*b-B*a)*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.42, size = 322, normalized size = 3.62

$$\left[-\frac{2\,(Ba^3-Aa^2b-Bab^2+Ab^3)dx-(Ba^2-Aab)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-2\,(Ba^2b-Bb^3)\sin(dx+c)}{2\,(a^2b^2-b^4)d},\ -\frac{(Ba^3-Aa^2b-Bab^2+Ab^3)dx-(Ba^2-Aab)\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\,\sin(dx+c)}\right)-(Ba^2b-Bb^3)\sin(dx+c)}{(a^2b^2-b^4)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] [-1/2*(2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a^2 - A*a*b)*sqrt(-a^
2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a
^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*
b^2 - b^4)*d), -((B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a^2 - A*a*b)*
sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))
) - (B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. $2(76) = 152$.
time = 63.54, size = 3225, normalized size = 36.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cos(c)), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (A*d*x*ta
n(c/2 + d*x/2)**3/(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + A*d*x*
tan(c/2 + d*x/2)/(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + A*tan(c
/2 + d*x/2)**2/(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + A/(b*d*ta
n(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + B*d*x*tan(c/2 + d*x/2)**3/(b*d*
tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + B*d*x*tan(c/2 + d*x/2)/(b*d*t
an(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + 3*B*tan(c/2 + d*x/2)**2/(b*d*t
an(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + B/(b*d*tan(c/2 + d*x/2)**3 + b
*d*tan(c/2 + d*x/2)), Eq(a, -b)), ((A*sin(c + d*x)/d + B*x*sin(c + d*x)**2/
2 + B*x*cos(c + d*x)**2/2 + B*sin(c + d*x)*cos(c + d*x)/(2*d))/a, Eq(b, 0))
, (x*(A + B*cos(c))*cos(c)/(a + b*cos(c)), Eq(d, 0)), (A*d*x*tan(c/2 + d*x/
2)**2/(b*d*tan(c/2 + d*x/2)**2 + b*d) + A*d*x/(b*d*tan(c/2 + d*x/2)**2 + b*
d) - A*tan(c/2 + d*x/2)**3/(b*d*tan(c/2 + d*x/2)**2 + b*d) - A*tan(c/2 + d*
x/2)/(b*d*tan(c/2 + d*x/2)**2 + b*d) - B*d*x*tan(c/2 + d*x/2)**2/(b*d*tan(c
/2 + d*x/2)**2 + b*d) - B*d*x/(b*d*tan(c/2 + d*x/2)**2 + b*d) + B*tan(c/2 +
 d*x/2)**3/(b*d*tan(c/2 + d*x/2)**2 + b*d) + 3*B*tan(c/2 + d*x/2)/(b*d*tan(
c/2 + d*x/2)**2 + b*d), Eq(a, b)), (A*a*b*d*x*sqrt(-a/(a - b) - b/(a - b))*
tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)
**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a
 - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) + A*a*b*d
*x*sqrt(-a/(a - b) - b/(a - b))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(
c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a
 - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b)
)) - A*a*b*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))*tan(c/2 +
```

```
d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b*
*2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan
(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - A*a*b*log(-sqrt(-
a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b**2*d*sqrt(-a/(a - b) - b/(a
 - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d
*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b)
- b/(a - b))) + A*a*b*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))*
tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)
**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a
 - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) + A*a*b*l
og(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b**2*d*sqrt(-a/(a -
b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b))
 - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a
/(a - b) - b/(a - b))) - A*b**2*d*x*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 +
d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b*
*2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan
(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - A*b**2*d*x*sqrt(-
a/(a - b) - b/(a - b))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x
/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b
/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - B*a*
*2*d*x*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(-a/(
a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a -
 b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqr
t(-a/(a - b) - b/(a - b))) - B*a**2*d*x*sqrt(-a/(a - b) - b/(a - b))/(a*b**
2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a
- b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2
 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) + B*a**2*log(-sqrt(-a/(a - b) - b/(
a - b)) + tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b) -
 b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b
**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a
- b) - b/(a - b))) + B*a**2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d
*x/2))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*
d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/
2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - B*a**2*log(sqrt(-a/(
a - b) - b/(a - b)) + tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(
-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/
(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d
*sqrt(-a/(a - b) - b/(a - b))) - B*a**2*log(sqrt(-a/(a - b) - b/(a - b)) +
tan(c/2 + d*x/2))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**
2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a -
 b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) + B*a*b*d*x
*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b)
 - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sq...
```

Giac [A]

time = 0.48, size = 142, normalized size = 1.60

$$-\frac{\frac{(Ba-Ab)(dx+c)}{b^2}-\frac{2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}+\frac{2\,(Ba^2-Aab)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -((B*a - A*b)*(d*x + c)/b^2 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 + 1)*b) + 2*(B*a^2 - A*a*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
 + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^
2 - b^2)))/(sqrt(a^2 - b^2)*b^2))/d
```

Mupad [B]
time = 1.12, size = 541, normalized size = 6.08

$$\frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,(a^2-b^2)}-\frac{2\,A\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,(a^2-b^2)}-\frac{B\,b\,\sin(c+d\,x)}{d\,(a^2-b^2)}+\frac{2\,A\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d\,(a^2-b^2)}-\frac{2\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d\,(a^2-b^2)}+\frac{A\,a\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d\,\sqrt{b^2-a^2}}-\frac{A\,a\,\ln\left(\frac{b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d\,\sqrt{b^2-a^2}}-\frac{B\,a^2\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d\,\sqrt{b^2-a^2}}+\frac{B\,a^2\,\ln\left(\frac{b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d\,\sqrt{b^2-a^2}}+\frac{B\,a^2\,\sin(c+d\,x)}{b\,d\,(a^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] (2*B*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (2*A*
b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (B*b*sin(c
 + d*x))/(d*(a^2 - b^2)) + (2*A*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2)))/(b*d*(a^2 - b^2)) - (2*B*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/(b^2*d*(a^2 - b^2)) + (A*a*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*
x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b*d*(b^
2 - a^2)^(1/2)) - (A*a*log((b*sin(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2) + c
os(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b*d*(b^2 - a^2)^
(1/2)) - (B*a^2*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b^2*d*(b^2 - a^2)^(1/2)
) + (B*a^2*log((b*sin(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2) + cos(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b^2*d*(b^2 - a^2)^(1/2)) + (
B*a^2*sin(c + d*x))/(b*d*(a^2 - b^2))
```

3.253 $\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{Bx}{b}+\frac{2(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}$$

```
[Out] B*x/b+2*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b/d/(a
-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2814, 2738, 211}

$$\frac{2(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\,\sqrt{a+b}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*x)/b + (2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{Bx}{b} - \frac{(-Ab+aB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b}$$
$$= \frac{Bx}{b} + \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$
$$= \frac{Bx}{b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}$$

Mathematica [A]
time = 0.14, size = 68, normalized size = 1.01

$$\frac{B(c+dx) + \frac{2(-Ab+aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*(c + d*x) + (2*(-(A*b) + a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a
^2 + b^2]])/Sqrt[-a^2 + b^2])/(b*d)
```

Maple [A]
time = 0.16, size = 73, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{2(Ab-aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}+\frac{2B\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$
default	$\frac{\frac{2(Ab-aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}+\frac{2B\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$
risch	$\frac{Bx}{b}-\frac{\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)A}{\sqrt{-a^2+b^2}\,d}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)aB}{\sqrt{-a^2+b^2}\,db}+\frac{\ln\left(e^{i(dx+c)}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(A*b-B*a)/b/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-
b)*(a+b))^(1/2))+2*B/b*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.38, size = 242, normalized size = 3.61

$$\left[\frac{2\,(Ba^2 - Bb^2)dx + (Ba - Ab)\sqrt{-a^2 + b^2}\,\log\left(\frac{2\,ab\cos(dx+c) + (2\,a^2 - b^2)\cos(dx+c)^2 + 2\,\sqrt{-a^2 + b^2}\,(a\cos(dx+c) + b)\sin(dx+c) - a^2 + 2\,b^2}{b^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + a^2}\right)}{2\,(a^2b - b^3)d}, \frac{(Ba^2 - Bb^2)dx - (Ba - Ab)\sqrt{a^2 - b^2}\,\arctan\left(-\frac{a\cos(dx+c) + b}{\sqrt{a^2 - b^2}\,\sin(dx+c)}\right)}{(a^2b - b^3)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(B*a^2 - B*b^2)*d*x + (B*a - A*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d
*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
 + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
+ a^2)))/((a^2*b - b^3)*d), ((B*a^2 - B*b^2)*d*x - (B*a - A*b)*sqrt(a^2 - b
^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))))/((a^2*b -
 b^3)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(56) = 112$.
time = 12.77, size = 524, normalized size = 7.82

$$\begin{cases} \frac{\tilde{\infty} x(A + B\cos(c))}{\cos(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{A\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{bd} + \frac{Bx}{b} - \frac{B\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{bd} & \text{for } a = b \\ \frac{A}{bd\tan\left(\frac{c}{2} + \frac{dx}{2}\right)} + \frac{Bx}{b} + \frac{B}{bd\tan\left(\frac{c}{2} + \frac{dx}{2}\right)} & \text{for } a = -b \\ \frac{Ax + \frac{B\sin(c+dx)}{d}}{a} & \text{for } b = 0 \\ \frac{x(A + B\cos(c))}{a + b\cos(c)} & \text{for } d = 0 \\ \frac{Ab\log\left(-\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} - b^2d\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} - \frac{Ab\log\left(\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} - b^2d\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} + \frac{Badx\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}}{abd\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} - b^2d\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} - \frac{Ba\log\left(-\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} - b^2d\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} + \frac{Ba\log\left(\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} - b^2d\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} - \frac{Bbdx\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}}{abd\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} - b^2d\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cos(c))/cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (A
*tan(c/2 + d*x/2)/(b*d) + B*x/b - B*tan(c/2 + d*x/2)/(b*d), Eq(a, b)), (A/(
```

```
b*d*tan(c/2 + d*x/2)) + B*x/b + B/(b*d*tan(c/2 + d*x/2)), Eq(a, -b)), ((A*x
 + B*sin(c + d*x)/d)/a, Eq(b, 0)), (x*(A + B*cos(c))/(a + b*cos(c)), Eq(d,
0)), (A*b*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqrt
(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) - A*b*log(s
qrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqrt(-a/(a - b) - b/
(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) + B*a*d*x*sqrt(-a/(a - b) -
 b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) -
b/(a - b))) - B*a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*
b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) + B
*a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqrt(-a/(a -
 b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) - B*b*d*x*sqrt(-a/(
a - b) - b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a
 - b) - b/(a - b))), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(58) = 116$.
time = 0.47, size = 296, normalized size = 4.42

$$-\frac{\frac{\left(\sqrt{a^2-b^2}\,B(2a-b)|a-b|-\sqrt{a^2-b^2}\,Ab|a-b|-\sqrt{a^2-b^2}\,A|a-b||b|+\sqrt{a^2-b^2}\,B|a-b||b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{\frac{2a+\sqrt{-4(a+b)(a-b)+4a^2}}{a-b}}}\right)\right)}{(a^2-2ab+b^2)b^2+(a^3-2a^2b+ab^2)|b|}+\frac{(2Ba-Ab-Bb+A|b|-B|b|)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{\frac{2a-\sqrt{-4(a+b)(a-b)+4a^2}}{a-b}}}\right)\right)}{b^2-a|b|}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -((sqrt(a^2 - b^2)*B*(2*a - b)*abs(a - b) - sqrt(a^2 - b^2)*A*b*abs(a - b)
- sqrt(a^2 - b^2)*A*abs(a - b)*abs(b) + sqrt(a^2 - b^2)*B*abs(a - b)*abs(b)
)*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*
c)/sqrt((2*a + sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/((a^2 - 2*a*b +
 b^2)*b^2 + (a^3 - 2*a^2*b + a*b^2)*abs(b)) + (2*B*a - A*b - B*b + A*abs(b)
 - B*abs(b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2
*d*x + 1/2*c)/sqrt((2*a - sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/(b^2
 - a*abs(b)))/d
```

Mupad [**B**]
time = 1.72, size = 344, normalized size = 5.13

$$\frac{a\left(B\ln\left(\frac{b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sqrt{-(a+b)\,(a-b)}-B\ln\left(\frac{a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sqrt{b^2-a^2}\right)-Ab\ln\left(\frac{b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sqrt{-(a+b)\,(a-b)}+Ab\ln\left(\frac{a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sqrt{b^2-a^2}}{b\,d\,(a^2-b^2)}+\frac{2B\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x)),x)
```

```
[Out] (a*(B*log((b*sin(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2))*(-(a + b)*(a - b))^(1/2) - B*log((a
*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)
^(1/2))/cos(c/2 + (d*x)/2))*(b^2 - a^2)^(1/2)) - A*b*log((b*sin(c/2 + (d*x)
```

```
/2) - a*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2
+ (d*x)/2))*(-(a + b)*(a - b))^(1/2) + A*b*log((a*sin(c/2 + (d*x)/2) - b*si
n(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)
)*(b^2 - a^2)^(1/2))/(b*d*(a^2 - b^2)) + (2*B*atan(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2)))/(b*d)
```

3.254 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=76

$$-\frac{2(Ab-aB)\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}\,d}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}$$

```
[Out] A*arctanh(sin(d*x+c))/a/d-2*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)
/(a+b)^(1/2))/a/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3080, 3855, 2738, 211}

$$\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2(Ab-aB)\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt
[a - b]*Sqrt[a + b]*d) + (A*ArcTanh[Sin[c + d*x]])/(a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\int \sec(c+dx)\,dx}{a} + \frac{(-Ab+aB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{ad} - \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \right.}{ad}$$

$$= -\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}\,d} + \frac{A\tanh^{-1}(\sin(c+d}{ad}$$

Mathematica [A]
time = 0.18, size = 112, normalized size = 1.47

$$\frac{\frac{2(Ab-aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + A\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*(A*b - a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[
-a^2 + b^2] + A*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]))/(a*d)
```

Maple [A]
time = 0.38, size = 92, normalized size = 1.21

method	result
derivativedivides	$\frac{\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}+\frac{2(-Ab+aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}}{d}$
default	$\frac{\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}+\frac{2(-Ab+aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}}{d}$

risch	$-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+a\sqrt{-a^2+b^2}}{\sqrt{-a^2+b^2}\,b}\right)Ab}{\sqrt{-a^2+b^2}\,da}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+a\sqrt{-a^2+b^2}}{\sqrt{-a^2+b^2}\,b}\right)B}{\sqrt{-a^2+b^2}\,d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia}{}\right.}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A/a*ln(tan(1/2*d*x+1/2*c)+1)+2/a*(-A*b+B*a)/((a-b)*(a+b))^(1/2)*arctan
(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-A/a*ln(tan(1/2*d*x+1/2*c)-1)
)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.74, size = 304, normalized size = 4.00

$$\left[\frac{(Ba-Ab)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+(Aa^2-Ab^2)\log\left(\sin\left(dx+c\right)+1\right)-(Aa^2-Ab^2)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,(a^3-ab^2)d},\frac{2\,(Ba-Ab)\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\,\sin(dx+c)}\right)+(Aa^2-Ab^2)\log\left(\sin\left(dx+c\right)+1\right)-(Aa^2-Ab^2)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,(a^3-ab^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] [1/2*((B*a - A*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*
cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (A*a^2 - A*b^2
)*log(sin(d*x + c) + 1) - (A*a^2 - A*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a
*b^2)*d), 1/2*(2*(B*a - A*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(
sqrt(a^2 - b^2)*sin(d*x + c))) + (A*a^2 - A*b^2)*log(sin(d*x + c) + 1) - (A
*a^2 - A*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

Giac [A]
time = 0.48, size = 127, normalized size = 1.67

$$\frac{\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)(Ba-Ab)}{\sqrt{a^2-b^2}\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] (A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - A*log(abs(tan(1/2*d*x + 1/2*c) -
1))/a + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(B*a - A*b)/(s
qrt(a^2 - b^2)*a))/d
```

Mupad [B]
time = 1.60, size = 342, normalized size = 4.50

$$\frac{2\,A\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}+\frac{b\,\left(A\,\ln\left(\frac{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{-\left(a+b\right)\,\left(a-b\right)}-A\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{b^2-a^2}\right)-B\,a\,\ln\left(\frac{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{-\left(a+b\right)\,\left(a-b\right)}+B\,a\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{b^2-a^2}}{a\,d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))),x)
```

```
[Out] (2*A*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) + (b*(A*log((a*cos
(c/2 + (d*x)/2) + b*cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/
2))/cos(c/2 + (d*x)/2))*(-(a + b)*(a - b))^(1/2) - A*log((a*sin(c/2 + (d*x)
/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2
+ (d*x)/2))*(b^2 - a^2)^(1/2)) - B*a*log((a*cos(c/2 + (d*x)/2) + b*cos(c/2
+ (d*x)/2) + sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2))*(-(a
 + b)*(a - b))^(1/2) + B*a*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2)
 + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2))*(b^2 - a^2)^(1
/2))/(a*d*(a^2 - b^2))
```

3.255 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=99

$$\frac{2b(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}\,d}-\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c+dx)}{ad}$$

```
[Out] -(A*b-B*a)*arctanh(sin(d*x+c))/a^2/d+2*b*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1
/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/d/(a-b)^(1/2)/(a+b)^(1/2)+A*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.13, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3079, 12, 2826, 3855, 2738, 211}

$$\frac{2b(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*b*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*S
qrt[a - b]*Sqrt[a + b]*d) - ((A*b - a*B)*ArcTanh[Sin[c + d*x]])/(a^2*d) + (
A*Tan[c + d*x])/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{A\tan(c+dx)}{ad}+\frac{\int \frac{(-Ab+aB)\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}\\
&= \frac{A\tan(c+dx)}{ad}+\frac{(-Ab+aB)\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}\\
&= \frac{A\tan(c+dx)}{ad}-\frac{(Ab-aB)\int \sec(c+dx)\,dx}{a^2}+\frac{(b(Ab-aB))\int \frac{}{a}}{a^2}\\
&= -\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c+dx)}{ad}+\frac{(2b(Ab-a}{}\\
&= \frac{2b(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}\,d}-\frac{(Ab-aB)\tanh^{-1}(}{a^2d}
\end{aligned}$$

Mathematica [A]

time = 0.61, size = 129, normalized size = 1.30

$$\frac{-\frac{2b(Ab-aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+(Ab-aB)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+aA\tan(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-2*b*(A*b - a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sq
rt[-a^2 + b^2] + (A*b - a*B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + a*A*Tan[c + d*x])/(a^2*d)
```

Maple [A]
time = 0.44, size = 144, normalized size = 1.45

method	result
derivativedivides	$\frac{-\frac{A}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-Ab+aB)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}+\frac{2b(Ab-aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{a^2\sqrt{(a-b)(a+b)}}-\frac{A}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+}{d}$
default	$\frac{-\frac{A}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-Ab+aB)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}+\frac{2b(Ab-aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{a^2\sqrt{(a-b)(a+b)}}-\frac{A}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+}{d}$
risch	$\frac{2iA}{da\left(e^{2i(dx+c)}+1\right)}-\frac{b^2\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)A}{\sqrt{-a^2+b^2}\,da^2}+\frac{b\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)B}{\sqrt{-a^2+b^2}\,da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-A/a/(tan(1/2*d*x+1/2*c)+1)+1/a^2*(-A*b+B*a)*ln(tan(1/2*d*x+1/2*c)+1)+
2*b*(A*b-B*a)/a^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b
)*(a+b))^(1/2))-A/a/(tan(1/2*d*x+1/2*c)-1)+(A*b-B*a)/a^2*ln(tan(1/2*d*x+1/2
*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(89) = 178$.
time = 0.48, size = 460, normalized size = 4.65

$$\left[\frac{(Bab-Ab^2)\sqrt{-a^2+b^2}\cos(dx+c)\log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2+2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)+(Ba^3-Aa^2b-Bab^2+Ab^3)\cos(dx+c)\log(\sin(dx+c)+1)-(Ba^3-Aa^2b-Bab^2+Ab^3)\cos(dx+c)\log(-\sin(dx+c)+1)+2(Aa^3-Aab^2)\sin(dx+c)}{2(a^4-a^2b^2)d\cos(dx+c)}, -\frac{2(Bab-Ab^2)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)\cos(dx+c)-(Ba^3-Aa^2b-Bab^2+Ab^3)\cos(dx+c)\log(\sin(dx+c)+1)+(Ba^3-Aa^2b-Bab^2+Ab^3)\cos(dx+c)\log(-\sin(dx+c)+1)-2(Aa^3-Aab^2)\sin(dx+c)}{2(a^4-a^2b^2)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/2*((B*a*b - A*b^2)*sqrt(-a^2 + b^2)*cos(d*x + c)*log((2*a*b*cos(d*x + c)
 + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*s
in(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))
 + (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) -
 (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) +
2*(A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*x + c)), -1/2*(2
*(B*a*b - A*b^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b
^2)*sin(d*x + c)))*cos(d*x + c) - (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d
*x + c)*log(sin(d*x + c) + 1) + (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x
 + c)*log(-sin(d*x + c) + 1) - 2*(A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^
2*b^2)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x)), x)
```

Giac [A]
time = 0.45, size = 175, normalized size = 1.77

$$\frac{\frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a}+\frac{2(Bab-Ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] ((B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - (B*a - A*b)*log(abs(t
an(1/2*d*x + 1/2*c) - 1))/a^2 - 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/
2*c)^2 - 1)*a) + 2*(B*a*b - A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2
*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(
a^2 - b^2)))/(sqrt(a^2 - b^2)*a^2))/d
```

Mupad [B]
time = 1.99, size = 675, normalized size = 6.82

$$\frac{A\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2{}\mathrm{i}}{d\,\left(a^2-b^2\right)}-\frac{B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2{}\mathrm{i}}{d\,\left(a^2-b^2\right)}+\frac{A\,a\,\tan\left(c+d\,x\right)}{d\,\left(a^2-b^2\right)}-\frac{A\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2{}\mathrm{i}}{a^2\,d\,\left(a^2-b^2\right)}+\frac{B\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2{}\mathrm{i}}{a\,d\,\left(a^2-b^2\right)}-\frac{A\,b^2\,\tan\left(c+d\,x\right)}{a\,d\,\left(a^2-b^2\right)}-\frac{B\,b\,\mathrm{atan}\left(\frac{\left(a^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}+2\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(b^2-a^2\right)}^{3/2}-2\,b^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}+3\,a^2\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}-a^3\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}-a^4\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a\,b^2-a^3\right)}^2}\right)\,\sqrt{-\left(a+b\right)\,\left(a-b\right)}\,2{}\mathrm{i}}{a\,d\,\left(a^2-b^2\right)}+\frac{A\,b^2\,\mathrm{atan}\left(\frac{\left(a^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}+2\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(b^2-a^2\right)}^{3/2}-2\,b^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}+3\,a^2\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}-a^3\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}-a^4\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a\,b^2-a^3\right)}^2}\right)\,\sqrt{-\left(a+b\right)\,\left(a-b\right)}\,2{}\mathrm{i}}{a^2\,d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))),x)
```

```
[Out] (A*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/(d*(a^2 - b^2)) -
 (B*a*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/(d*(a^2 - b^2))
+ (A*a*tan(c + d*x))/(d*(a^2 - b^2)) - (A*b^3*atan((sin(c/2 + (d*x)/2)*1i)/
cos(c/2 + (d*x)/2))*2i)/(a^2*d*(a^2 - b^2)) + (B*b^2*atan((sin(c/2 + (d*x)/
2)*1i)/cos(c/2 + (d*x)/2))*2i)/(a*d*(a^2 - b^2)) - (A*b^2*tan(c + d*x))/(a*
d*(a^2 - b^2)) - (B*b*atan(((a^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 2*b
^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 2*b^5*sin(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2) + 3*a^2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^3*b^2*sin(c/
2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^4*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)
)*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)^2))*(-(a + b)*(a - b))^(1/2)*2i)/(a
*d*(a^2 - b^2)) + (A*b^2*atan(((a^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) +
2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 2*b^5*sin(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2) + 3*a^2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^3*b^2*sin
(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^4*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1
/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)^2))*(-(a + b)*(a - b))^(1/2)*2i)
/(a^2*d*(a^2 - b^2))
```

3.256 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=143

$$-\frac{2b^2(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}\,d}+\frac{(a^2A+2Ab^2-2abB)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(Ab-aB)\tan}{a^2d}$$

```
[Out] 1/2*(A*a^2+2*A*b^2-2*B*a*b)*arctanh(sin(d*x+c))/a^3/d-2*b^2*(A*b-B*a)*arcta
n((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/d/(a-b)^(1/2)/(a+b)^(1/2)
-(A*b-B*a)*tan(d*x+c)/a^2/d+1/2*A*sec(d*x+c)*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.31, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3079, 3134, 3080, 3855, 2738, 211}

$$-\frac{2b^2(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\,\sqrt{a+b}}-\frac{(Ab-aB)\tan(c+dx)}{a^2d}+\frac{(a^2A-2abB+2Ab^2)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{A\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*b^2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
3*Sqrt[a - b]*Sqrt[a + b]*d) + ((a^2*A + 2*A*b^2 - 2*a*b*B)*ArcTanh[Sin[c +
 d*x]])/(2*a^3*d) - ((A*b - a*B)*Tan[c + d*x])/(a^2*d) + (A*Sec[c + d*x]*Ta
n[c + d*x])/(2*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
```

```
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{(-2(Ab-aB)+aA\cos(c+dx)+Ab\cos^2(c+dx)}{a+b\cos(c+dx)}}{2a}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{a^2d} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int (a^2A+}{}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{a^2d} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(b^2(Ab}{}$$

$$= \frac{(a^2A+2Ab^2-2abB)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{(Ab-aB)\tan(c+}{a^2d}$$

$$= -\frac{2b^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}\,d} + \frac{(a^2A+2Ab^2-2}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 300 vs. $2(143) = 286$.

time = 1.76, size = 300, normalized size = 2.10

$$\frac{\frac{8b^2(Ab-aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - 2(a^2A+2Ab^2-2abB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2(a^2A+2Ab^2-2abB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{a^2A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{4a(-Ab+aB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} - \frac{a^2A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{4a(-Ab+aB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((8*b^2*(A*b - a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/S
qrt[-a^2 + b^2] - 2*(a^2*A + 2*A*b^2 - 2*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]] + 2*(a^2*A + 2*A*b^2 - 2*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + (a^2*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*a*(-(A*b
) + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (a^2*A)/
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a*(-(A*b) + a*B)*Sin[(c + d*x)
/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(4*a^3*d)
```

Maple [**A**]

time = 0.54, size = 229, normalized size = 1.60

method	result
derivativedivides	$\frac{-\frac{A}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-aA-2Ab+2aB}{2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2A+2A\,b^2-2Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^3}-\frac{2b^2(Ab-aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)}}\right)}{a^3\sqrt{(a-b)(a+}}}{d}$

default	$\frac{-\frac{A}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{-aA-2Ab+2aB}{2a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^{2}A+2A\,b^{2}-2Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^{3}}-\frac{2b^{2}(Ab-aB)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots}{\sqrt{(a-b)\,(\cdots}}\right.}{a^{3}\sqrt{(a-b)\,(a+b)}\cdots}}{d}$ …
risch	$-\frac{i\left(Aa\,e^{3i(dx+c)}+2Ab\,e^{2i(dx+c)}-2Ba\,e^{2i(dx+c)}-aA\,e^{i(dx+c)}+2Ab-2aB\right)}{a^{2}d\left(e^{2i(dx+c)}+1\right)^{2}}+\frac{A\ln\left(e^{i(dx+c)}+i\right)}{2da}+\frac{\ln\left(e^{i(dx+c)}+i\right)A\,b^{2}}{a^{3}d}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*A/a/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-A*a-2*A*b+2*B*a)/a^2/(tan(1/2*
d*x+1/2*c)+1)+1/2*(A*a^2+2*A*b^2-2*B*a*b)/a^3*ln(tan(1/2*d*x+1/2*c)+1)-2*b^
2*(A*b-B*a)/a^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*
(a+b))^(1/2))+1/2*A/a/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-A*a-2*A*b+2*B*a)/a^2/(
tan(1/2*d*x+1/2*c)-1)+1/2/a^3*(-A*a^2-2*A*b^2+2*B*a*b)*ln(tan(1/2*d*x+1/2*c
)-1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: ValueError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(129) = 258$.
time = 2.81, size = 589, normalized size = 4.12

$$\left[\frac{2\,(Bab^{2}-Ab^{3})\sqrt{-a^{2}+b^{2}}\cos(dx+c)^{2}\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^{2}-b^{2})\cos(dx+c)^{2}-2\,\sqrt{-a^{2}+b^{2}}\,(a\cos(dx+c)+b)\sin(dx+c)-a^{2}+2\,b^{2}}{b^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+a^{2}}\right)+(Aa^{4}-2\,Ba^{3}b+Aa^{2}b^{2}+2\,Bab^{3}-2\,Ab^{4})\cos(dx+c)^{2}\log(\sin(dx+c)+1)-(Aa^{4}-2\,Ba^{3}b+Aa^{2}b^{2}+2\,Bab^{3}-2\,Ab^{4})\cos(dx+c)^{2}\log(-\sin(dx+c)+1)+2\,(Aa^{4}-Aa^{2}b^{2}+2\,(Ba^{4}-Aa^{3}b-Ba^{2}b^{2}+Aab^{3})\cos(dx+c))\sin(dx+c)}{4\,(a^{5}-a^{3}b^{2})d\cos(dx+c)^{2}},\ \frac{4\,(Bab^{2}-Ab^{3})\sqrt{a^{2}-b^{2}}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^{2}-b^{2}}\sin(dx+c)}\right)\cos(dx+c)^{2}+(Aa^{4}-2\,Ba^{3}b+Aa^{2}b^{2}+2\,Bab^{3}-2\,Ab^{4})\cos(dx+c)^{2}\log(\sin(dx+c)+1)-(Aa^{4}-2\,Ba^{3}b+Aa^{2}b^{2}+2\,Bab^{3}-2\,Ab^{4})\cos(dx+c)^{2}\log(-\sin(dx+c)+1)+2\,(Aa^{4}-Aa^{2}b^{2}+2\,(Ba^{4}-Aa^{3}b-Ba^{2}b^{2}+Aab^{3})\cos(dx+c))\sin(dx+c)}{4\,(a^{5}-a^{3}b^{2})d\cos(dx+c)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/4*(2*(B*a*b^2 - A*b^3)*sqrt(-a^2 + b^2)*cos(d*x + c)^2*log((2*a*b*cos(d*
x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
+ b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
```

```
 a^2)) + (A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*cos(d*x + c)
^2*log(sin(d*x + c) + 1) - (A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A
*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a^4 - A*a^2*b^2 + 2*(B*a
^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3
*b^2)*d*cos(d*x + c)^2), 1/4*(4*(B*a*b^2 - A*b^3)*sqrt(a^2 - b^2)*arctan(-(
a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 + (A*a^4
 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - (A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*cos(d*x +
 c)^2*log(-sin(d*x + c) + 1) + 2*(A*a^4 - A*a^2*b^2 + 2*(B*a^4 - A*a^3*b -
B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x
 + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{a + b\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(129) = 258.
time = 0.47, size = 269, normalized size = 1.88

$$\frac{\frac{(Aa^2-2Bab+2Ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{(Aa^2-2Bab+2Ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}-\frac{4(Bab^2-Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^3}+\frac{2\left(Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((A*a^2 - 2*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - (
A*a^2 - 2*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 4*(B*a*
b^2 - A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2
 - b^2)*a^3) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3
 + 2*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + 2*B*a*tan(1/2*
d*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*
a^2))/d
```

Mupad [B]
time = 4.21, size = 2500, normalized size = 17.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))),x)
```

```
[Out] (B*a*sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*b*
sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*a*sin(c
 + d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*a*atan((sin(c/2
+ (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2
 + 1/2)) + (B*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(d*(a^
2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*b^2*atan((sin(c/2 + (d*x)/2)*1i)/
cos(c/2 + (d*x)/2))*1i)/(2*a*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A
*b^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(a^3*d*(a^2 - b^2
)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2
 + (d*x)/2))*1i)/(a^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*b^3*si
n(2*c + 2*d*x))/(2*a^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b^2*s
in(2*c + 2*d*x))/(2*a*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*a*atan
((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(2*d*(a^2
 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B*b*atan((sin(c/2 + (d*x)/2)*1i)/cos
(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 +
1/2)) - (A*b^2*sin(c + d*x))/(2*a*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2))
 + (A*b^3*atan(((A^2*a^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*A^2*b^7*s
in(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*A^2*b^9*sin(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2) - A^2*a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*A^2*a^2*b^7*
sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*A^2*a^4*b^5*sin(c/2 + (d*x)/2)*(b^
2 - a^2)^(1/2) - 3*A^2*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*A^2
*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 2*A^2*a^7*b^2*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2) + 8*B^2*a^2*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2
) - 8*B^2*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 12*B^2*a^4*b^5*sin
(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4*B^2*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 -
 a^2)^(1/2) - 4*B^2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 4*B^2*a^
7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 16*A*B*a*b^6*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(3/2) + 16*A*B*a*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4
*A*B*a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 20*A*B*a^3*b^6*sin(c/2 +
(d*x)/2)*(b^2 - a^2)^(1/2) + 4*A*B*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(
1/2) + 4*A*B*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))*1i)/(cos(c/2 + (
d*x)/2)*(a*b^2 - a^3)*(A^2*a^7 - 3*A^2*a^3*b^4 + 2*A^2*a^5*b^2 - 4*B^2*a^3*
b^4 + 4*B^2*a^5*b^2 - 4*A*B*a^6*b + 4*A*B*a^2*b^5)))*(-(a + b)*(a - b))^(1/
2)*1i)/(a^3*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*b^2*atan((sin(c/
2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(2*a*d*(a^2 - b^2
)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*b^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2
 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(a^3*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 +
1/2)) - (B*b^2*atan(((A^2*a^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*A^2*
b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*A^2*b^9*sin(c/2 + (d*x)/2)*(b^
2 - a^2)^(1/2) - A^2*a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*A^2*a^2
*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*A^2*a^4*b^5*sin(c/2 + (d*x)/2
```

)*(b^2 - a^2)^(1/2) - 3*A^2*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*A^2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 2*A^2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*B^2*a^2*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*B^2*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 12*B^2*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4*B^2*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4*B^2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 4*B^2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 16*A*B*a*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 16*A*B*a*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4*A*B*a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 20*A*B*a^3*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 4*A*B*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 4*A*B*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(A^2*a^7 - 3*A^2*a^3*b^4 + 2*A^2*a^5*b^2 - 4*B^2*a^3*b^4 + 4*B^2*a^5*b^2 - 4*A*B*a^6*b + 4*A*B*a^2*b^5)))*(-(a + b)*(a - b))^(1/2)*1i)/(a^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(a^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*b^3*atan(((A^2*a^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*A^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*A^2*b^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - A^2*a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*A^2*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*A^2*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 3*A^2*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*A^2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 2*A^2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*B^2*a^2*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*B^2*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 12*B^2*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4*B^2*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4*B^2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 4*B^2*a^7*b^2*sin(c/2...

3.257 $\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{2b^3(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}\,d}-\frac{(a^2+2b^2)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(2a^2A+3Ab^2-3\ldots}{3a\ldots}$$

```
[Out] -1/2*(a^2+2*b^2)*(A*b-B*a)*arctanh(sin(d*x+c))/a^4/d+2*b^3*(A*b-B*a)*arctan
((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/d/(a-b)^(1/2)/(a+b)^(1/2)+
1/3*(2*A*a^2+3*A*b^2-3*B*a*b)*tan(d*x+c)/a^3/d-1/2*(A*b-B*a)*sec(d*x+c)*tan
(d*x+c)/a^2/d+1/3*A*sec(d*x+c)^2*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.50, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3079, 3134, 3080, 3855, 2738, 211}

$$\frac{2b^3(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{(Ab-aB)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{(a^2+2b^2)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(2a^2A-3abB+3Ab^2)\tan(c+dx)}{3a^3d}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*b^3*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4
*Sqrt[a - b]*Sqrt[a + b]*d) - ((a^2 + 2*b^2)*(A*b - a*B)*ArcTanh[Sin[c + d*
x]])/(2*a^4*d) + ((2*a^2*A + 3*A*b^2 - 3*a*b*B)*Tan[c + d*x])/(3*a^3*d) - (
(A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + (A*Sec[c + d*x]^2*Tan[c
+ d*x])/(3*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
```

```
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{(-3(Ab-aB)+2aA\cos(c+dx)+2Ab\cos^2(c+dx}{a+b\cos(c+dx)}}{3a}$$

$$= -\frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{A\sec^2(c+dx)\tan(c+dx)}{3ad}$$

$$= \frac{(2a^2A+3Ab^2-3abB)\tan(c+dx)}{3a^3d} - \frac{(Ab-aB)\sec(c+dx)\tan(}{2a^2d}$$

$$= \frac{(2a^2A+3Ab^2-3abB)\tan(c+dx)}{3a^3d} - \frac{(Ab-aB)\sec(c+dx)\tan(}{2a^2d}$$

$$= -\frac{(a^2+2b^2)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2a^4d} + \frac{(2a^2A+3Ab^2-3a}{3a^3}$$

$$= \frac{2b^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}\,d} - \frac{(a^2+2b^2)(Ab-aB}{2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 422 vs. $2(187) = 374$.
time = 2.34, size = 422, normalized size = 2.26

$$\frac{\frac{24b^3(-Ab+aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - 6(a^2+2b^2)(-Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 6(a^2+2b^2)(-Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{a^2(-3Ab+a(A+3B))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{2a^3A\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{4a(2a^2A+3Ab^2-3abB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{2a^3A\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - \frac{a^2(-3Ab+a(A+3B))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{4a(2a^2A+3Ab^2-3abB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((24*b^3*(-(A*b) + a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])/Sqrt[-a^2 + b^2] - 6*(a^2 + 2*b^2)*(-(A*b) + a*B)*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] + 6*(a^2 + 2*b^2)*(-(A*b) + a*B)*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + (a^2*(-3*A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^2 + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])^3 + (4*a*(2*a^2*A + 3*A*b^2 - 3*a*b*B)*Sin[(c + d*x)/2])/(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]) + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^3 - (a^2*(-3*A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 + (4*a*(2*a^2*A + 3*A*b^2 - 3*a*b*B)*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]))/(12*a^4*d)
```

Maple [A]
time = 0.62, size = 335, normalized size = 1.79

method	result

derivativedivides	$\frac{-\frac{A}{3a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{-aA-Ab+aB}{2a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{\left(-A\,a^{2}b-2A\,b^{3}+a^{3}B+2B a\,b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^{4}}-\frac{2a^{2}A+Aab+2A\,b^{2}-B\,a^{2}}{2a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{}$
default	$\frac{-\frac{A}{3a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{-aA-Ab+aB}{2a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{\left(-A\,a^{2}b-2A\,b^{3}+a^{3}B+2B a\,b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^{4}}-\frac{2a^{2}A+Aab+2A\,b^{2}-B\,a^{2}}{2a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{}$
risch	$\frac{i\left(3Aab\,e^{5i(dx+c)}-3B\,a^{2}e^{5i(dx+c)}+6A\,b^{2}e^{4i(dx+c)}-6Bab\,e^{4i(dx+c)}+12A\,a^{2}e^{2i(dx+c)}+12A\,b^{2}e^{2i(dx+c)}-12Bab\,e^{2i(dx+\right.}{3d\,a^{3}\left(e^{2i(dx+c)}+1\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*A/a/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-A*a-A*b+B*a)/a^2/(tan(1/2*d*x+
1/2*c)+1)^2+1/2/a^4*(-A*a^2*b-2*A*b^3+B*a^3+2*B*a*b^2)*ln(tan(1/2*d*x+1/2*c
)+1)-1/2*(2*A*a^2+A*a*b+2*A*b^2-B*a^2-2*B*a*b)/a^3/(tan(1/2*d*x+1/2*c)+1)+2
*b^3*(A*b-B*a)/a^4/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-
b)*(a+b))^(1/2))-1/3*A/a/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(A*a+A*b-B*a)/a^2/(ta
n(1/2*d*x+1/2*c)-1)^2+1/2*(A*a^2*b+2*A*b^3-B*a^3-2*B*a*b^2)/a^4*ln(tan(1/2*
d*x+1/2*c)-1)-1/2*(2*A*a^2+A*a*b+2*A*b^2-B*a^2-2*B*a*b)/a^3/(tan(1/2*d*x+1/
2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.82, size = 729, normalized size = 3.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/12*(6*(B*a*b^3 - A*b^4)*sqrt(-a^2 + b^2)*cos(d*x + c)^3*log((2*a*b*cos(d
*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
 + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
+ a^2)) + 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)
*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*
a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2
*A*a^5 - 2*A*a^3*b^2 + 2*(2*A*a^5 - 3*B*a^4*b + A*a^3*b^2 + 3*B*a^2*b^3 - 3
*A*a*b^4)*cos(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3*b^2 + A*a^2*b^3)*cos(
d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x + c)^3), -1/12*(12*(B*a*
b^3 - A*b^4)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*
sin(d*x + c)))*cos(d*x + c)^3 - 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3
- 2*B*a*b^4 + 2*A*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*(B*a^5 - A*
a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) - 2*(2*A*a^5 - 2*A*a^3*b^2 + 2*(2*A*a^5 - 3*B*a^4*b + A*a^3
*b^2 + 3*B*a^2*b^3 - 3*A*a*b^4)*cos(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3
*b^2 + A*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x +
c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^4(c + dx)}{a + b\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**4/(a + b*cos(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(170) = 340.
time = 0.48, size = 412, normalized size = 2.20

$$\frac{\frac{3\,(Ba^3-Aa^2b+2\,Bab^2-2\,Ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{3\,(Ba^3-Aa^2b+2\,Bab^2-2\,Ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{12\,(Bab^3-Ab^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{2\left(6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(B*a^3 - A*a^2*b + 2*B*a*b^2 - 2*A*b^3)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/a^4 - 3*(B*a^3 - A*a^2*b + 2*B*a*b^2 - 2*A*b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^4 + 12*(B*a*b^3 - A*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
```

```
)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)
^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*
a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^2*tan(1
/2*d*x + 1/2*c)^3 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c) -
3*A*a*b*tan(1/2*d*x + 1/2*c) - 6*B*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1
/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^3))/d
```

Mupad [B]
time = 4.90, size = 2500, normalized size = 13.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^4*(a + b*cos(c + d*x))),x)
```

```
[Out] (atan(((((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*
a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 +
3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b
^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B
*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4
+ 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 + (((8*(2*B*a^13 - 4*A*a^8*b^5 + 6*A
*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3 + 2*B*a
^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 - (4*tan(c/2 + (d*x)/2)*(8*a^10*b +
 8*a^8*b^3 - 16*a^9*b^2)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/a^10)*(2*
A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/(2*a^4))*(2*A*b^3 - B*a^3 + A*a^2*b -
 2*B*a*b^2)*1i)/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16
*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5
 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3
*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 1
6*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^
5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 - (((8*(2*B*a^13
- 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6
*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 + (4*tan(c/2 + (
d*x)/2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*
B*a*b^2))/a^10)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/(2*a^4))*(2*A*b^3
- B*a^3 + A*a^2*b - 2*B*a*b^2)*1i)/(2*a^4))/((16*(4*A^3*b^11 - 6*A^3*a*b^10
 + 6*A^3*a^2*b^9 - 5*A^3*a^3*b^8 + 2*A^3*a^4*b^7 - A^3*a^5*b^6 - 4*B^3*a^3*
b^8 + 6*B^3*a^4*b^7 - 6*B^3*a^5*b^6 + 5*B^3*a^6*b^5 - 2*B^3*a^7*b^4 + B^3*a
^8*b^3 - 12*A^2*B*a*b^10 + 12*A*B^2*a^2*b^9 - 18*A*B^2*a^3*b^8 + 18*A*B^2*a
^4*b^7 - 15*A*B^2*a^5*b^6 + 6*A*B^2*a^6*b^5 - 3*A*B^2*a^7*b^4 + 18*A^2*B*a^
2*b^9 - 18*A^2*B*a^3*b^8 + 15*A^2*B*a^4*b^7 - 6*A^2*B*a^5*b^6 + 3*A^2*B*a^6
*b^5))/a^9 + (((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 +
3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*
b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2
```

*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 +
2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^
5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 + (((8*(2*B*a^13 - 4*A*a^8*b^5
+ 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3 +
2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 - (4*tan(c/2 + (d*x)/2)*(8*a^
10*b + 8*a^8*b^3 - 16*a^9*b^2)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/a^1
0)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/(2*a^4))*(2*A*b^3 - B*a^3 + A*a
^2*b - 2*B*a*b^2))/(2*a^4) - (((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 -
16*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*
b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*
a^3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2
- 16*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4
*b^5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 - (((8*(2*B*a^
13 - 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4
- 6*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 + (4*tan(c/2
+ (d*x)/2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2)*(2*A*b^3 - B*a^3 + A*a^2*b -
2*B*a*b^2))/a^10)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/(2*a^4))*(2*A*b
^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/(2*a^4)))*(2*A*b^3 - B*a^3 + A*a^2*b - 2
*B*a*b^2)*1i)/(a^4*d) - ((tan(c/2 + (d*x)/2)*(2*A*a^2 + 2*A*b^2 + B*a^2 - A
*a*b - 2*B*a*b))/a^3 + (tan(c/2 + (d*x)/2)^5*(2*A*a^2 + 2*A*b^2 - B*a^2 + A
*a*b - 2*B*a*b))/a^3 - (4*tan(c/2 + (d*x)/2)^3*(A*a^2 + 3*A*b^2 - 3*B*a*b))
/(3*a^3))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (
d*x)/2)^6 - 1)) + (b^3*atan(((b^3*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*
tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*a^8*b + 16*A
^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^
3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*
a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B*a^8*b + 32*
A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4 + 14*A*B*a^6
*b^3 - 6*A*B*a^7*b^2))/a^6 + (b^3*(-(a + b)*(a - b))^(1/2)*((8*(2*B*a^13 -
4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6*B
*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 - (8*b^3*tan(c/2 +
(d*x)/2)*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a^10*b + 8*a^8*b^3 - 16*a
^9*b^2))/(a^6*(a^6 - a^4*b^2)))*(A*b - B*a))/(a^6 - a^4*b^2))*1i)/(a^6 - a^
4*b^2) + (b^3*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(
8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*
a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*
B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6
*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B*a^8...

3.258 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=263

$$-\frac{(4aAb-6a^2B-b^2B)\,x}{2b^4}+\frac{2a^2(2a^2Ab-3Ab^3-3a^3B+4ab^2B)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^4(a+b)^{3/2}d}+\frac{(2a^2Ab-$$

```
[Out] -1/2*(4*A*a*b-6*B*a^2-B*b^2)*x/b^4+2*a^2*(2*A*a^2*b-3*A*b^3-3*B*a^3+4*B*a*b
^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^4/(a+b
)^(3/2)/d+(2*A*a^2*b-A*b^3-3*B*a^3+2*B*a*b^2)*sin(d*x+c)/b^3/(a^2-b^2)/d-1/
2*(2*A*a*b-3*B*a^2+B*b^2)*cos(d*x+c)*sin(d*x+c)/b^2/(a^2-b^2)/d+a*(A*b-B*a)
*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.44, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3068, 3128, 3102, 2814, 2738, 211}

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\,(a^2-b^2)\,(a+b\cos(c+dx))}-\frac{(-3a^2B+2aAb+b^2B)\sin(c+dx)\cos(c+dx)}{2b^2d\,(a^2-b^2)}-\frac{x(-6a^2B+4aAb-b^2B)}{2b^4}+\frac{2a^2(-3a^3B+2a^2Ab+4ab^2B-3Ab^3)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{(-3a^3B+2a^2Ab+2ab^2B-Ab^3)\sin(c+dx)}{b^3d\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -1/2*((4*a*A*b - 6*a^2*B - b^2*B)*x)/b^4 + (2*a^2*(2*a^2*A*b - 3*A*b^3 - 3*
a^3*B + 4*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(3/2)*b^4*(a + b)^(3/2)*d) + ((2*a^2*A*b - A*b^3 - 3*a^3*B + 2*a*b^2*B
)*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) - ((2*a*A*b - 3*a^2*B + b^2*B)*Cos[c +
d*x]*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^2*Si
n[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\,(a^2-b^2)\,d(a+b\cos(c+dx))} - \int \frac{\cos(c+dx)(-2a(Ab-aB)+b(Ab \cdots}{\cdots}$$

$$= -\frac{(2aAb-3a^2B+b^2B)\cos(c+dx)\sin(c+dx)}{2b^2\,(a^2-b^2)\,d} + \frac{a(Ab-aB)\,\mathrm{co}\cdots}{b\,(a^2-b^2)\,\cdots}$$

$$= \frac{(2a^2Ab-Ab^3-3a^3B+2ab^2B)\sin(c+dx)}{b^3\,(a^2-b^2)\,d} - \frac{(2aAb-3a^2B+\cdots}{2b\cdots}$$

$$= -\frac{(4aAb-6a^2B-b^2B)\,x}{2b^4} + \frac{(2a^2Ab-Ab^3-3a^3B+2ab^2B)\sin(\cdots}{b^3\,(a^2-b^2)\,d}$$

$$= -\frac{(4aAb-6a^2B-b^2B)\,x}{2b^4} + \frac{(2a^2Ab-Ab^3-3a^3B+2ab^2B)\sin(\cdots}{b^3\,(a^2-b^2)\,d}$$

$$= -\frac{(4aAb-6a^2B-b^2B)\,x}{2b^4} + \frac{2a^2(2a^2Ab-3Ab^3-3a^3B+4ab^2B\cdots}{(a-b)^{3/2}b^4(a\cdots}$$

Mathematica [A]
time = 1.13, size = 184, normalized size = 0.70

$$\frac{2(-4aAb+6a^2B+b^2B)\,(c+dx)-\frac{8a^2(-2a^2Ab+3Ab^3+3a^3B-4ab^2B)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}+4b(Ab-2aB)\sin(c+dx)+\frac{4a^3b(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}+b^2B\sin(2(c+dx))}{4b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*(-4*a*A*b + 6*a^2*B + b^2*B)*(c + d*x) - (8*a^2*(-2*a^2*A*b + 3*A*b^3 +
3*a^3*B - 4*a*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/
(-a^2 + b^2)^(3/2) + 4*b*(A*b - 2*a*B)*Sin[c + d*x] + (4*a^3*b*(A*b - a*B)*
Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])) + b^2*B*Sin[2*(c + d*x
)])/(4*b^4*d)
```

Maple [A]
time = 0.41, size = 267, normalized size = 1.02

method	result
derivativedivides	$\frac{2a^2\left(\frac{a(Ab-aB)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(2A\,a^2b-3A\,b^3-3a^3B+4Ba\,b^2\right)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)\,(a+b)}}\cdots\right.}{(a-b)(a+b)\sqrt{(a-b)\,(a+b)}}\cdots\right.}{b^4}\cdots$

default	$\frac{2a^2\left(\frac{a(Ab-aB)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{(2A\,a^2b-3A\,b^3-3a^3B+4Ba\,b^2)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^4}$
risch	$-\frac{2xAa}{b^3}+\frac{3xB\,a^2}{b^4}+\frac{Bx}{2b^2}+\frac{iB\,\mathrm{e}^{-2i(dx+c)}}{8b^2d}+\frac{i\mathrm{e}^{i(dx+c)}aB}{b^3d}-\frac{iB\,\mathrm{e}^{2i(dx+c)}}{8b^2d}-\frac{i\mathrm{e}^{i(dx+c)}A}{2b^2d}+\frac{i\mathrm{e}^{-i(dx+c)}A}{2b^2d}-\frac{}{b^4(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2*a^2/b^4*(a*(A*b-B*a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1
/2*c)^2-b*tan(1/2*d*x+1/2*c)^2+a+b)+(2*A*a^2*b-3*A*b^3-3*B*a^3+4*B*a*b^2)/(
a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)
)^(1/2)))-2/b^4*(((-A*b^2+2*B*a*b+1/2*B*b^2)*tan(1/2*d*x+1/2*c)^3+(-A*b^2+2
*B*a*b-1/2*B*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(4*A*a
*b-6*B*a^2-B*b^2)*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

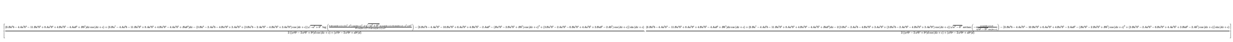
Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.43, size = 965, normalized size = 3.67



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*((6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 -
 4*A*a*b^6 + B*b^7)*d*x*cos(d*x + c) + (6*B*a^7 - 4*A*a^6*b - 11*B*a^5*b^2
```

```
+ 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*d*x - (3*B*a^6 - 2*A*a
^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3 + (3*B*a^5*b - 2*A*a^4*b^2 - 4*B*a^3*b^3 +
 3*A*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a
^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
+ c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*B
*a^6*b - 4*A*a^5*b^2 - 10*B*a^4*b^3 + 6*A*a^3*b^4 + 4*B*a^2*b^5 - 2*A*a*b^6
 - (B*a^4*b^3 - 2*B*a^2*b^5 + B*b^7)*cos(d*x + c)^2 + (3*B*a^5*b^2 - 2*A*a^
4*b^3 - 6*B*a^3*b^4 + 4*A*a^2*b^5 + 3*B*a*b^6 - 2*A*b^7)*cos(d*x + c))*sin(
d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^
6 + a*b^8)*d), 1/2*((6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 +
 4*B*a^2*b^5 - 4*A*a*b^6 + B*b^7)*d*x*cos(d*x + c) + (6*B*a^7 - 4*A*a^6*b -
 11*B*a^5*b^2 + 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*d*x - 2*
(3*B*a^6 - 2*A*a^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3 + (3*B*a^5*b - 2*A*a^4*b^2
 - 4*B*a^3*b^3 + 3*A*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(
d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*B*a^6*b - 4*A*a^5*b^2 -
10*B*a^4*b^3 + 6*A*a^3*b^4 + 4*B*a^2*b^5 - 2*A*a*b^6 - (B*a^4*b^3 - 2*B*a^2
*b^5 + B*b^7)*cos(d*x + c)^2 + (3*B*a^5*b^2 - 2*A*a^4*b^3 - 6*B*a^3*b^4 + 4
*A*a^2*b^5 + 3*B*a*b^6 - 2*A*b^7)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2
*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 338, normalized size = 1.29

$$\frac{\frac{4\left(3\,Ba^5-2\,Aa^4b-4\,Ba^3b^2+3\,Aa^2b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{a^2-b^2}}-\frac{4\left(Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(a^2b^3-b^5\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}+\frac{\left(6\,Ba^2-4\,Aab+Bb^2\right)\left(dx+c\right)}{b^4}-\frac{2\left(4\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*(3*B*a^5 - 2*A*a^4*b - 4*B*a^3*b^2 + 3*A*a^2*b^3)*(pi*floor(1/2*(d*x
 + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) - 4*(B
*a^4*tan(1/2*d*x + 1/2*c) - A*a^3*b*tan(1/2*d*x + 1/2*c))/((a^2*b^3 - b^5)*
(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + (6*B*a^2 -
 4*A*a*b + B*b^2)*(d*x + c)/b^4 - 2*(4*B*a*tan(1/2*d*x + 1/2*c)^3 - 2*A*b*t
```

```
an(1/2*d*x + 1/2*c)^3 + B*b*tan(1/2*d*x + 1/2*c)^3 + 4*B*a*tan(1/2*d*x + 1/
2*c) - 2*A*b*tan(1/2*d*x + 1/2*c) - B*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
 + 1/2*c)^2 + 1)^2*b^3))/d
```

Mupad [B]
time = 9.21, size = 2500, normalized size = 9.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (a^2*atan(((a^2*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*B^2
*a^10 + B^2*b^10 - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3
*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 +
32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*
a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9
- 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*
a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8*b^2))/(a*b^8 + b^9
 - a^2*b^7 - a^3*b^6) + (a^2*((8*(2*B*b^15 + 12*A*a^2*b^13 + 12*A*a^3*b^12
- 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6*b^9 + 6*B*a^2*b^13 - 16*B*a^3*b^12
 - 14*B*a^4*b^11 + 28*B*a^5*b^10 + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^14)
)/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (8*a^2*tan(c/2 + (d*x)/2)*(-(a + b
)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2)*(8*a*b^13
- 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8
+ b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(-(a
+ b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2))/(b^10
- 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*
b^2)*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + (a^2*(-(a + b)^3*(a - b
)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 + B^2*b^10 - 2*B^2*a*b^9 - 7
2*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5
*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 -
20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2
*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 -
40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*
B*a^7*b^3 + 96*A*B*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a^2*((8*(
2*B*b^15 + 12*A*a^2*b^13 + 12*A*a^3*b^12 - 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8
*A*a^6*b^9 + 6*B*a^2*b^13 - 16*B*a^3*b^12 - 14*B*a^4*b^11 + 28*B*a^5*b^10 +
 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^14))/(a*b^11 + b^12 - a^2*b^10 - a^3*
b^9) + (8*a^2*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*
B*a^3 - 2*A*a^2*b - 4*B*a*b^2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^
4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 -
 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 +
 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)
)*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2)*1i)/(b^10 - 3*a^2*b^8 + 3*a^4
```

*b^6 - a^6*b^4))/((16*(108*B^3*a^11 - 54*B^3*a^10*b - 48*A^3*a^4*b^7 - 24*A
^3*a^5*b^6 + 80*A^3*a^6*b^5 + 16*A^3*a^7*b^4 - 32*A^3*a^8*b^3 + 4*B^3*a^3*b
^8 - 4*B^3*a^4*b^7 + 41*B^3*a^5*b^6 - 9*B^3*a^6*b^5 + 63*B^3*a^7*b^4 + 81*B
^3*a^8*b^3 - 216*B^3*a^9*b^2 - 216*A*B^2*a^10*b - 3*A*B^2*a^2*b^9 + 3*A*B^2
*a^3*b^8 - 63*A*B^2*a^4*b^7 + 15*A*B^2*a^5*b^6 - 186*A*B^2*a^6*b^5 - 162*A*
B^2*a^7*b^4 + 468*A*B^2*a^8*b^3 + 108*A*B^2*a^9*b^2 + 24*A^2*B*a^3*b^8 - 6*
A^2*B*a^4*b^7 + 168*A^2*B*a^5*b^6 + 108*A^2*B*a^6*b^5 - 336*A^2*B*a^7*b^4 -
 72*A^2*B*a^8*b^3 + 144*A^2*B*a^9*b^2))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9
) - (a^2*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 +
 B^2*b^10 - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 +
20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*
a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5
 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*
B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5
 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8*b^2))/(a*b^8 + b^9 - a^2*
b^7 - a^3*b^6) + (a^2*((8*(2*B*b^15 + 12*A*a^2*b^13 + 12*A*a^3*b^12 - 20*A*
a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6*b^9 + 6*B*a^2*b^13 - 16*B*a^3*b^12 - 14*B
*a^4*b^11 + 28*B*a^5*b^10 + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^14))/(a*b^
11 + b^12 - a^2*b^10 - a^3*b^9) - (8*a^2*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a
- b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2)*(8*a*b^13 - 8*a^2
*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 -
 a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(-(a + b)^3*
(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2))/(b^10 - 3*a^2
*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2))/(
b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + (a^2*(-(a + b)^3*(a - b)^3)^(1/2)
*((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 + B^2*b^10 - 2*B^2*a*b^9 - 72*B^2*a^9*
b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*
A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3
*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 -
 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3
*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3
+ 96*A*B*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a^2*((8*(2*B*b^15 +
 12*A*a^2*b^13 + 12*A*a^3*b^12 - 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6*b^9
 + 6*B*a^2*b^13 - 16*B*a^3*b^12 - 14*B*a^4*b^11...

3.259 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2} dx$

Optimal. Leaf size=155

$$\frac{(Ab-2aB)x}{b^3}-\frac{2a(a^2Ab-2Ab^3-2a^3B+3ab^2B)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d}+\frac{B\sin(c+dx)}{b^2d}-\frac{a^2(Ab}{b^2\,(a^2-b}$$

```
[Out] (A*b-2*B*a)*x/b^3-2*a*(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^2)*arctan((a-b)^(1/2
)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^3/(a+b)^(3/2)/d+B*sin(d*x+c
)/b^2/d-a^2*(A*b-B*a)*sin(d*x+c)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.30, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3067, 3102, 2814, 2738, 211}

$$-\frac{a^2(Ab-aB)\sin(c+dx)}{b^2d\,(a^2-b^2)\,(a+b\cos(c+dx))}-\frac{2a(-2a^3B+a^2Ab+3ab^2B-2Ab^3)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{x(Ab-2aB)}{b^3}+\frac{B\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*x)/b^3 - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a*b^2*B)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^
(3/2)*d) + (B*Sin[c + d*x])/(b^2*d) - (a^2*(A*b - a*B)*Sin[c + d*x])/(b^2*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = -\frac{a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{ab(Ab-aB)+\left(a^2-b^2\right)(Ab-aB)\cos}{a+b\cos(c}}{b^2\left(a^2\right.}$$

$$= \frac{B\sin(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{ab^2(Ab-aB)+}{}}{}$$

$$= \frac{(Ab-2aB)x}{b^3} + \frac{B\sin(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))}$$

$$= \frac{(Ab-2aB)x}{b^3} + \frac{B\sin(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{2a\left(a^2Ab-2Ab^3-2a^3B+3ab^2B\right)\tan^{-1}\left(\frac{\sqrt{a-}}{}\right.}{(a-b)^{3/2}b^3(a+b)^{3/2}d}$$

Mathematica [A]

time = 0.88, size = 147, normalized size = 0.95

$$\frac{(Ab-2aB)(c+dx)+\frac{2a\left(-a^2Ab+2Ab^3+2a^3B-3ab^2B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{3/2}}+bB\sin(c+dx)+\frac{a^2b(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*(c + d*x) + (2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*B
)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2)
+ b*B*Sin[c + d*x] + (a^2*b*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(
a + b*Cos[c + d*x])))/(b^3*d)
```

Maple [A]
time = 0.34, size = 205, normalized size = 1.32

method	result
derivativedivides	$-\frac{\frac{2a\left(\frac{a(Ab-aB)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}-b^{2}\right)\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(A\,a^{2}b-2A\,b^{3}-2a^{3}B+3Ba\,b^{2}\right)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)\,(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)\,(a+b)}}\right)}{b^{3}}}{d}$
default	$-\frac{\frac{2a\left(\frac{a(Ab-aB)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}-b^{2}\right)\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(A\,a^{2}b-2A\,b^{3}-2a^{3}B+3Ba\,b^{2}\right)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)\,(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)\,(a+b)}}\right)}{b^{3}}}{d}$
risch	$\frac{xA}{b^{2}}-\frac{2xaB}{b^{3}}-\frac{iB\,\mathrm{e}^{i(dx+c)}}{2b^{2}d}+\frac{iB\,\mathrm{e}^{-i(dx+c)}}{2b^{2}d}+\frac{2ia^{2}(-Ab+aB)\left(a\,\mathrm{e}^{i(dx+c)}+b\right)}{b^{3}(a^{2}-b^{2})d\left(b\,\mathrm{e}^{2i(dx+c)}+2a\,\mathrm{e}^{i(dx+c)}+b\right)}-\frac{a^{3}\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^{2}-ib^{2}}{b\sqrt{}}\right.}{\sqrt{-a^{2}+b^{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-2*a/b^3*(a*(A*b-B*a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/
2*c)^2-b*tan(1/2*d*x+1/2*c)^2+a+b)+(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^2)/(a-b
)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(
1/2)))+2/b^3*(B*b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+(A*b-2*B*a)*a
rctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(148) = 296$.
time = 0.44, size = 788, normalized size = 5.08

$$\left[-\frac{2\left(2Ba^5b - Aa^4b^2 - 4Ba^3b^3 + 2Aa^2b^4 + 2Bab^5 - Ab^6\right)dx\cos(dx+c) + 2\left(2Ba^6 - Aa^5b - 4Ba^4b^2 + 2Aa^3b^3 + 2Ba^2b^4 - Aab^5\right)dx + \left(2Ba^5 - Aa^4b - 3Ba^3b^2 + 2Aa^2b^3 + \left(2Ba^4b - Aa^3b^2 - 3Ba^2b^3 + 2Aab^4\right)\cos(dx+c)\right)\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c) + (2a^2-b^2)\cos(dx+c)^2 + 2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c) - a^2 + 2b^2}{b^2\cos(dx+c)^2 + 2ab\cos(dx+c) + a^2}\right) - 2\left(2Ba^5b - Aa^4b^2 - 3Ba^3b^3 + Aa^2b^4 + Bab^5 + \left(Ba^4b^2 - 2Ba^2b^4 + Bb^6\right)\cos(dx+c)\right)\sin(dx+c)}{2\left(\left(a^4b^4 - 2a^2b^6 + b^8\right)d\cos(dx+c) + \left(a^5b^3 - 2a^3b^5 + ab^7\right)d\right)}, -\frac{\left(2Ba^5b - Aa^4b^2 - 4Ba^3b^3 + 2Aa^2b^4 + 2Bab^5 - Ab^6\right)dx\cos(dx+c) + \left(2Ba^6 - Aa^5b - 4Ba^4b^2 + 2Aa^3b^3 + 2Ba^2b^4 - Aab^5\right)dx - \left(2Ba^5 - Aa^4b - 3Ba^3b^2 + 2Aa^2b^3 + \left(2Ba^4b - Aa^3b^2 - 3Ba^2b^3 + 2Aab^4\right)\cos(dx+c)\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right) - \left(2Ba^5b - Aa^4b^2 - 3Ba^3b^3 + Aa^2b^4 + Bab^5 + \left(Ba^4b^2 - 2Ba^2b^4 + Bb^6\right)\cos(dx+c)\right)\sin(dx+c)}{\left(a^4b^4 - 2a^2b^6 + b^8\right)d\cos(dx+c) + \left(a^5b^3 - 2a^3b^5 + ab^7\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(2*(2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A
*b^6)*d*x*cos(d*x + c) + 2*(2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 +
 2*B*a^2*b^4 - A*a*b^5)*d*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^
3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(-a
^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-
a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3
+ A*a^2*b^4 + B*a*b^5 + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6)*cos(d*x + c))*sin
(d*x + c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b
^5 + a*b^7)*d), -((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*
a*b^5 - A*b^6)*d*x*cos(d*x + c) + (2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^
3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*d*x - (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A
*a^2*b^3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*
sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))
) - (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b^5 + (B*a^4*b^2
 - 2*B*a^2*b^4 + B*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^4 - 2*a^2*b^6 +
 b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1116 vs. $2(148) = 296$.
time = 0.53, size = 1116, normalized size = 7.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] ((4*B*a^6*b^2 - 2*A*a^5*b^3 - 2*B*a^5*b^3 + A*a^4*b^4 - 9*B*a^4*b^4 + 5*A*a^3*b^5 + 4*B*a^3*b^5 - 2*A*a^2*b^6 + 5*B*a^2*b^6 - 3*A*a*b^7 - 2*B*a*b^7 + A*b^8 + 2*B*a^3*abs(-a^2*b^3 + b^5) - A*a^2*b*abs(-a^2*b^3 + b^5) - B*a^2*b*abs(-a^2*b^3 + b^5) + A*a*b^2*abs(-a^2*b^3 + b^5) - 2*B*a*b^2*abs(-a^2*b^3 + b^5) + A*b^3*abs(-a^2*b^3 + b^5))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*a^3*b^2 - 2*a*b^4 + sqrt(-4*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*(a^3*b^2 - a^2*b^3 - a*b^4 + b^5) + 4*(a^3*b^2 - a*b^4)^2))/(a^3*b^2 - a^2*b^3 - a*b^4 + b^5))))/(a^3*b^2*abs(-a^2*b^3 + b^5) - a*b^4*abs(-a^2*b^3 + b^5) + (a^2*b^3 - b^5)^2) + ((a^2*b - a*b^2 - b^3)*sqrt(a^2 - b^2)*A*abs(-a^2*b^3 + b^5)*abs(-a + b) - (2*a^3 - a^2*b - 2*a*b^2)*sqrt(a^2 - b^2)*B*abs(-a^2*b^3 + b^5)*abs(-a + b) - (2*a^5*b^3 - a^4*b^4 - 5*a^3*b^5 + 2*a^2*b^6 + 3*a*b^7 - b^8)*sqrt(a^2 - b^2)*A*abs(-a + b) + (4*a^6*b^2 - 2*a^5*b^3 - 9*a^4*b^4 + 4*a^3*b^5 + 5*a^2*b^6 - 2*a*b^7)*sqrt(a^2 - b^2)*B*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*a^3*b^2 - 2*a*b^4 - sqrt(-4*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*(a^3*b^2 - a^2*b^3 - a*b^4 + b^5) + 4*(a^3*b^2 - a*b^4)^2))/(a^3*b^2 - a^2*b^3 - a*b^4 + b^5))))/((a^2*b^3 - b^5)^2*(a^2 - 2*a*b + b^2) - (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*abs(-a^2*b^3 + b^5)) + 2*(2*B*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d*x + 1/2*c) - A*a^2*b*tan(1/2*d*x + 1/2*c) + B*a^2*b*tan(1/2*d*x + 1/2*c) - B*a*b^2*tan(1/2*d*x + 1/2*c) - B*b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2*b^2 - b^4)))/d

Mupad [B]
time = 5.17, size = 2500, normalized size = 16.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)

[Out] (log(tan(c/2 + (d*x)/2) + 1i)*(A*b - 2*B*a)*1i)/(b^3*d) - ((2*tan(c/2 + (d*x)/2)^3*(A*a^2*b - B*b^3 - 2*B*a^3 + B*a*b^2 + B*a^2*b))/(b^2*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)*(B*b^3 - 2*B*a^3 + A*a^2*b + B*a*b^2 - B*a^2*b))/(b^2*(a + b)*(a - b)))/(d*(a + b + tan(c/2 + (d*x)/2)^4*(a - b) + 2*a*tan(c/2 + (d*x)/2)^2)) - (log(tan(c/2 + (d*x)/2) - 1i)*(A*b*1i - B*a*2i))/(b^3*d) - (a*atan(((a*((32*tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B

^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8
*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b
^7 - a^2*b^5 - a^3*b^4) + (a*((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^
5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6
+ 2*A*a*b^11 + 2*B*a*b^11))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*a*tan(c
/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3
*B*a*b^2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^
6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a
^6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a
*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(-(a + b)^3*(a - b)^3)^(1/2
)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^
5 - a^6*b^3) + (a*((32*tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^
7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5
*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B
^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8
*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b
^7 - a^2*b^5 - a^3*b^4) - (a*((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^
5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6
+ 2*A*a*b^11 + 2*B*a*b^11))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a*tan(c
/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3
*B*a*b^2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^
6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a
^6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a
*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(-(a + b)^3*(a - b)^3)^(1/2
)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^
5 - a^6*b^3))/((64*(8*B^3*a^8 - 2*A^3*a*b^7 - 4*B^3*a^7*b - 2*A^3*a^2*b^6 +
3*A^3*a^3*b^5 + A^3*a^4*b^4 - A^3*a^5*b^3 + 12*B^3*a^4*b^4 + 6*B^3*a^5*b^3
- 20*B^3*a^6*b^2 - 12*A*B^2*a^7*b - 20*A*B^2*a^3*b^5 - 13*A*B^2*a^4*b^4 +
32*A*B^2*a^5*b^3 + 8*A*B^2*a^6*b^2 + 11*A^2*B*a^2*b^6 + 9*A^2*B*a^3*b^5 - 1
7*A^2*B*a^4*b^4 - 5*A^2*B*a^5*b^3 + 6*A^2*B*a^6*b^2))/(a*b^8 + b^9 - a^2*b^
7 - a^3*b^6) - (a*((32*tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^
7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5
*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B
^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8
*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b
^7 - a^2*b^5 - a^3*b^4) + (a*((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^
5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6
+ 2*A*a*b^11 + 2*B*a*b^11))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*a*tan(c
/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3
*B*a*b^2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^
6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a
^6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a
*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(-(a + b)^3*(a - b)^3)^(1/2
)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 -
a^6*b^3) + (a*((32*tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 -

```
8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^
3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*
a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*
B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b^7
- a^2*b^5 - a^3*b^4) - (a*((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^5*b
^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2
*A*a*b^11 + 2*B*a*b^11))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a*tan(c/2
+ (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*
a*b^2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b
^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*
b^3)))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^
2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(-(a + b)^3*(a - b)^3)^(1/2)*(
2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^
6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 ...
```

3.260 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=122

$$\frac{Bx}{b^2}-\frac{2(Ab^3+a^3B-2ab^2B)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d}+\frac{a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d(a+b\cos(c+dx))}$$

```
[Out] B*x/b^2-2*(A*b^3+B*a^3-2*B*a*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+
b)^(1/2))/(a-b)^(3/2)/b^2/(a+b)^(3/2)/d+a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/
d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3047, 3100, 2814, 2738, 211}

$$-\frac{2(a^3B-2ab^2B+Ab^3)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(Ab-aB)\sin(c+dx)}{bd\,(a^2-b^2)\,(a+b\cos(c+dx))}+\frac{Bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*x)/b^2 - (2*(A*b^3 + a^3*B - 2*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) + (a*(A*b - a*B)*Si
n[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{b(Ab-aB)-\left(a^2-b^2\right)B\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{Bx}{b^2} + \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(Ab^3+a\left(a^2-2b^2\right)B\right)\int \frac{\cdots}{a+\cdots}}{b^2\left(a^2-b^2\right)}$$

$$= \frac{Bx}{b^2} + \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(2\left(Ab^3+a\left(a^2-2b^2\right)B\right)\right)\cdots}{\cdots}$$

$$= \frac{Bx}{b^2} - \frac{2\left(Ab^3+a^3B-2ab^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d} + \frac{a\cdots}{b\left(a^2\cdots\right.}$$

Mathematica [A]
time = 0.59, size = 119, normalized size = 0.98

$$\frac{B(c+dx) - \frac{2\left(Ab^3+a\left(a^2-2b^2\right)B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{3/2}} + \frac{ab(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*(c + d*x) - (2*(A*b^3 + a*(a^2 - 2*b^2)*B)*ArcTanh[((a - b)*Tan[(c + d*x
)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + (a*b*(A*b - a*B)*Sin[c + d*x]
)/((a - b)*(a + b)*(a + b*Cos[c + d*x]))))/(b^2*d)
```

Maple [A]
time = 0.26, size = 161, normalized size = 1.32

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{a(Ab-aB)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(Ab^3+a^3B-2Bab^2\right)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^2}+\frac{2B\cdots}{}}{d}$
default	$\frac{-\frac{2\left(-\frac{a(Ab-aB)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(Ab^3+a^3B-2Bab^2\right)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^2}+\frac{2B\cdots}{}}{d}$
risch	$\frac{Bx}{b^2}-\frac{2ia(Ab-aB)\left(a\,e^{i(dx+c)}+b\right)}{b^2(-a^2+b^2)d\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}-\frac{b\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+a\sqrt{-a^2+b^2}}{\sqrt{-a^2+b^2}\,b}\right)A}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}-\frac{\ln\left(e^{i(dx+c)}\cdots\right.}{\sqrt{-\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^2*(-a*(A*b-B*a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2
*c)^2-b*tan(1/2*d*x+1/2*c)^2+a+b)+(A*b^3+B*a^3-2*B*a*b^2)/(a-b)/(a+b)/((a-b
)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))+2*B/b^
2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(114) = 228$.
time = 0.43, size = 552, normalized size = 4.52

$$\left[\frac{2\,(Ba^4b-2\,Ba^2b^3+Bb^5)dx\cos(dx+c)+2\,(Ba^5-2\,Ba^3b^2+Bab^4)dx-(Ba^4-2\,Ba^2b^2+Aab^3+(Ba^3b-2\,Bab^3+Ab^4)\cos(dx+c))\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-2\,(Ba^4b-Aa^3b^2-Ba^2b^3+Aab^4)\sin(dx+c)}{2\,((a^4b^3-2\,a^2b^5+b^7)d\cos(dx+c)+(a^5b^2-2\,a^3b^4+ab^6)d)},\ \frac{(Ba^4b-2\,Ba^2b^3+Bb^5)dx\cos(dx+c)+(Ba^5-2\,Ba^3b^2+Bab^4)dx-(Ba^4-2\,Ba^2b^2+Aab^3+(Ba^3b-2\,Bab^3+Ab^4)\cos(dx+c))\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)-(Ba^4b-Aa^3b^2-Ba^2b^3+Aab^4)\sin(dx+c)}{(a^4b^3-2\,a^2b^5+b^7)d\cos(dx+c)+(a^5b^2-2\,a^3b^4+ab^6)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*(2*(B*a^4*b - 2*B*a^2*b^3 + B*b^5)*d*x*cos(d*x + c) + 2*(B*a^5 - 2*B*a
^3*b^2 + B*a*b^4)*d*x - (B*a^4 - 2*B*a^2*b^2 + A*a*b^3 + (B*a^3*b - 2*B*a*b
^3 + A*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2
 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x +
c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(B*a
^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/((a^4*b^3 - 2*a^2*b^5
 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d), ((B*a^4*b - 2*B*
a^2*b^3 + B*b^5)*d*x*cos(d*x + c) + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*d*x - (
B*a^4 - 2*B*a^2*b^2 + A*a*b^3 + (B*a^3*b - 2*B*a*b^3 + A*b^4)*cos(d*x + c))
*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)
)) - (B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/((a^4*b^3 -
2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 199, normalized size = 1.63

$$\frac{\frac{2\,(Ba^3-2\,Bab^2+Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^2-b^4)\sqrt{a^2-b^2}}+\frac{(dx+c)B}{b^2}-\frac{2\,(Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right))}{(a^2b-b^3)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] (2*(B*a^3 - 2*B*a*b^2 + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2
```

```
- b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) + (d*x + c)*B/b^2 - 2*(B*a^2*tan
(1/2*d*x + 1/2*c) - A*a*b*tan(1/2*d*x + 1/2*c))/((a^2*b - b^3)*(a*tan(1/2*d
*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)))/d
```

Mupad [B]
time = 7.79, size = 2500, normalized size = 20.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*B*atan(((B*((B*((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 -
 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a
^3*b^3) - (B*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^
6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))*1i
)/b^2 + (32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5
 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^
5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))/b^2 - (B*((B*((32*(
A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4
 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (B*tan(c/2 + (
d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^
4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))*1i)/b^2 - (32*tan(c/2 + (d
*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a
^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b
^4 + b^5 - a^2*b^3 - a^3*b^2)))/b^2)/((64*(B^3*a^5 - A*B^2*b^5 + A^2*B*b^5
+ 2*B^3*a*b^4 - B^3*a^4*b + 2*B^3*a^2*b^3 - 3*B^3*a^3*b^2 - 3*A*B^2*a*b^4 +
 A*B^2*a^2*b^3 + A*B^2*a^3*b^2))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (B*((B
*((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*
a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (B*tan(
c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2
*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))*1i)/b^2 + (32*tan(c
/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b +
3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3
))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))*1i)/b^2 + (B*((B*((32*(A*a^2*b^7 - B*
b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2
*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (B*tan(c/2 + (d*x)/2)*(2*a*b
^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(
a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))*1i)/b^2 - (32*tan(c/2 + (d*x)/2)*(A^2*b^
6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2
*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2
*b^3 - a^3*b^2))*1i)/b^2)))/(b^2*d) + (atan(((((32*tan(c/2 + (d*x)/2)*(A^2*
b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B
^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a
^2*b^3 - a^3*b^2) + (((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^
```

```
7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4
- a^3*b^3) - (32*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B
*a^3 - 2*B*a*b^2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5
- 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b
^4 - a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2))/(
b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3
 + B*a^3 - 2*B*a*b^2)*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (((32*t
an(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*
b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3
*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (((32*(A*a^2*b^7 - B*b^9 - A*b^9
 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/
(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a -
 b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 +
 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^
8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3
+ B*a^3 - 2*B*a*b^2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(-(a + b)^3*
(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2)*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b
^4 - a^6*b^2))/((64*(B^3*a^5 - A*B^2*b^5 + A^2*B*b^5 + 2*B^3*a*b^4 - B^3*a^
4*b + 2*B^3*a^2*b^3 - 3*B^3*a^3*b^2 - 3*A*B^2*a*b^4 + A*B^2*a^2*b^3 + A*B^2
*a^3*b^2))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (((32*tan(c/2 + (d*x)/2)*(A^
2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4
*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 -
 a^2*b^3 - a^3*b^2) + (((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*
b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^
4 - a^3*b^3) - (32*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 +
 B*a^3 - 2*B*a*b^2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^
5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4
*b^4 - a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2))
/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b
^3 + B*a^3 - 2*B*a*b^2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) - (((32*ta
n(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b
 + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*
b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (((32*(A*a^2*b^7 - B*b^9 - A*b^9
- A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b...
```

3.261 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{2(aA-bB)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))}$$

```
[Out] 2*(A*a-B*b)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/
(a+b)^(3/2)/d-(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2833, 12, 2738, 211}

$$\frac{2(aA-bB)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(3/2)*(a + b)^(3/2)*d) - ((A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*
Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= -\frac{(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))}+\frac{\int \frac{-aA+bB}{a+b\cos(c+dx)}\,dx}{-a^2+b^2} \\
&= -\frac{(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))}+\frac{(aA-bB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2-b^2} \\
&= -\frac{(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))}+\frac{(2(aA-bB))\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\right.\right.}{(a^2-b^2)\,d} \\
&= \frac{2(aA-bB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))}
\end{aligned}
$$

Mathematica [A]
time = 0.39, size = 97, normalized size = 0.97

$$
\frac{\frac{2(aA-bB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}+\frac{(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*(a*A - b*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2
 + b^2)^(3/2) + ((-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c
 + d*x])))/d
```

Maple [A]
time = 0.20, size = 128, normalized size = 1.28

method	result

derivativedivides	$\frac{-\frac{2(Ab-aB)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}-b^{2}\right)\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{2(aA-Bb)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{-\frac{2(Ab-aB)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}-b^{2}\right)\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{2(aA-Bb)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$
risch	$\frac{2i(Ab-aB)\left(a\,e^{i(dx+c)}+b\right)}{b(-a^{2}+b^{2})d\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}-\frac{a\ln\left(e^{i(dx+c)}+\frac{ia^{2}-ib^{2}+a\sqrt{-a^{2}+b^{2}}}{b\sqrt{-a^{2}+b^{2}}}\right)A}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)d}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^{2}-ib^{2}+}{b\sqrt{}}\right.}{\sqrt{-a^{2}+b^{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(A*b-B*a)/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*ta
n(1/2*d*x+1/2*c)^2+a+b)+2*(A*a-B*b)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(
tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.37, size = 379, normalized size = 3.79

$$\left[-\frac{\left(Aa^{2}-Bab+\left(Aab-Bb^{2}\right)\cos\left(dx+c\right)\right)\sqrt{-a^{2}+b^{2}}\log\left(\frac{2\,ab\cos\left(dx+c\right)+\left(2\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+2\,\sqrt{-a^{2}+b^{2}}\left(a\cos\left(dx+c\right)+b\right)\sin\left(dx+c\right)-a^{2}+2\,b^{2}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}}\right)-2\left(Ba^{3}-Aa^{2}b-Bab^{2}+Ab^{3}\right)\sin\left(dx+c\right)}{2\left(\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)d\cos\left(dx+c\right)+\left(a^{5}-2\,a^{3}b^{2}+ab^{4}\right)d\right)},\ \frac{\left(Aa^{2}-Bab+\left(Aab-Bb^{2}\right)\cos\left(dx+c\right)\right)\sqrt{a^{2}-b^{2}}\arctan\left(-\frac{a\cos\left(dx+c\right)+b}{\sqrt{a^{2}-b^{2}}\sin\left(dx+c\right)}\right)+\left(Ba^{3}-Aa^{2}b-Bab^{2}+Ab^{3}\right)\sin\left(dx+c\right)}{\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)d\cos\left(dx+c\right)+\left(a^{5}-2\,a^{3}b^{2}+ab^{4}\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((A*a^2 - B*a*b + (A*a*b - B*b^2)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(
(2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*
cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*c
os(d*x + c) + a^2)) - 2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*sin(d*x + c))/(
```

```
(a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), ((
A*a^2 - B*a*b + (A*a*b - B*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*co
s(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (B*a^3 - A*a^2*b - B*a*b^
2 + A*b^3)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 -
 2*a^3*b^2 + a*b^4)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

Giac [A]
time = 0.48, size = 159, normalized size = 1.59

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)(Aa-Bb)}{(a^2-b^2)^{\frac{3}{2}}}-\frac{Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)(a^2-b^2)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="giac")
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(A*a - B*b)/(a^2 -
 b^2)^(3/2) - (B*a*tan(1/2*d*x + 1/2*c) - A*b*tan(1/2*d*x + 1/2*c))/((a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2 - b^2)))/d
```

Mupad [B]
time = 0.73, size = 113, normalized size = 1.13

$$\frac{2\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a-2\,b)}{2\,\sqrt{a+b}\,\sqrt{a-b}}\right)\,(A\,a-B\,b)}{d\,{(a+b)}^{3/2}\,{(a-b)}^{3/2}}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A\,b-B\,a)}{d\,(a+b)\,(a-b)\,\left((a-b)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^2,x)
[Out] (2*atan((tan(c/2 + (d*x)/2)*(2*a - 2*b))/(2*(a + b)^(1/2)*(a - b)^(1/2)))*(
A*a - B*b))/(d*(a + b)^(3/2)*(a - b)^(3/2)) - (2*tan(c/2 + (d*x)/2)*(A*b -
B*a))/(d*(a + b)*(a - b)*(a + b + tan(c/2 + (d*x)/2)^2*(a - b)))
```

3.262 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$-\frac{2(2a^2Ab-Ab^3-a^3B)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{b(Ab-aB)\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\cos(c+dx))}$$

```
[Out] -2*(2*A*a^2*b-A*b^3-B*a^3)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2
))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+A*arctanh(sin(d*x+c))/a^2/d+b*(A*b-B*a)*si
n(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3079, 3080, 3855, 2738, 211}

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\,(a^2-b^2)\,(a+b\cos(c+dx))}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{2(a^3(-B)+2a^2Ab-Ab^3)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*(2*a^2*A*b - A*b^3 - a^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (A*ArcTanh[Sin[c + d*x]])/(a
^2*d) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
```

```
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(A\left(a^2-b^2\right)-a(Ab-aB)\cos(c+dx)\right)\sec(c-}{a+b\cos(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{A\int \sec(c+dx)\,dx}{a^2} - \frac{\left(2a^2Ab-\right.}{}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(2\left(2a^2\right.\right.}{}$$

$$= -\frac{2\left(2a^2Ab-Ab^3-a^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{A\tanh^{-1}(}{}$$

Mathematica [A]
time = 0.67, size = 191, normalized size = 1.44

$$\frac{\cos(c+dx)(B+A\sec(c+dx))\left(\frac{2\left(-2a^2Ab+Ab^3+a^3B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{3/2}} - A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{ab(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}\right)}{a^2d(A+B\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*((2*(-2*a^2*A*b + A*b^3 + a^3*B)*ArcTanh
[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - A*Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + (a*b*(A*b - a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x]
))))/(a^2*d*(A + B*Cos[c + d*x]))
```

Maple [A]
time = 0.57, size = 182, normalized size = 1.37

method	result
derivativedivides	$\frac{\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{2\left(-\frac{a(Ab-aB)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(2Aa^2b-Ab^3-a^3B\right)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\cdots\right.}{\sqrt{(a-b)\cdots}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+\cdots}}\right)}{a^2}}{d}$
default	$\frac{\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{2\left(-\frac{a(Ab-aB)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(2Aa^2b-Ab^3-a^3B\right)\arctan\left(\frac{\tan\left(\frac{dx}{2}+\cdots\right.}{\sqrt{(a-b)\cdots}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+\cdots}}\right)}{a^2}}{d}$
risch	$-\frac{2i(Ab-aB)\left(a\,e^{i(dx+c)}+b\right)}{\left(-a^2+b^2\right)da\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}-\frac{2\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)Ab}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{\ln\left(e^{i(dx+c)}-\frac{ia^2-\cdots}{\cdots}\right.}{\sqrt{-a^2+\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A/a^2*ln(tan(1/2*d*x+1/2*c)+1)-2/a^2*(-a*(A*b-B*a)*b/(a^2-b^2)*tan(1/2
*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2+a+b)+(2*A*a^2*b-
A*b^3-B*a^3)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b
)/((a-b)*(a+b))^(1/2)))-A/a^2*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(123) = 246$.
time = 2.96, size = 684, normalized size = 5.14

$$\left[\frac{\left(Ba^4 - 2\,Aa^3b + Aab^3 + \left(Ba^3b - 2\,Aa^2b^2 + Ab^4\right)\cos\left(dx + c\right)\right)\sqrt{-a^2 + b^2}\log\left(\frac{2\,ab\cos(dx+c) + (2\,a^2-b^2)\cos(dx+c)^2 - 2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c) - a^2 + 2\,b^2}{b^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + a^2}\right) + \left(Aa^5 - 2\,Aa^3b^2 + Aab^4 + \left(Aa^4b - 2\,Aa^2b^3 + Ab^5\right)\cos\left(dx + c\right)\right)\log\left(\sin\left(dx + c\right) + 1\right) - \left(Aa^5 - 2\,Aa^3b^2 + Aab^4 + \left(Aa^4b - 2\,Aa^2b^3 + Ab^5\right)\cos\left(dx + c\right)\right)\log\left(-\sin\left(dx + c\right) + 1\right) - 2\left(Ba^4b - Aa^3b^2 - Ba^2b^3 + Aab^4\right)\sin\left(dx + c\right)}{2\left(\left(a^6b - 2\,a^4b^3 + a^2b^5\right)d\cos\left(dx + c\right) + \left(a^7 - 2\,a^5b^2 + a^3b^4\right)d\right)}, \frac{2\left(Ba^4 - 2\,Aa^3b + Aab^3 + \left(Ba^3b - 2\,Aa^2b^2 + Ab^4\right)\cos\left(dx + c\right)\right)\sqrt{a^2 - b^2}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right) + \left(Aa^5 - 2\,Aa^3b^2 + Aab^4 + \left(Aa^4b - 2\,Aa^2b^3 + Ab^5\right)\cos\left(dx + c\right)\right)\log\left(\sin\left(dx + c\right) + 1\right) - \left(Aa^5 - 2\,Aa^3b^2 + Aab^4 + \left(Aa^4b - 2\,Aa^2b^3 + Ab^5\right)\cos\left(dx + c\right)\right)\log\left(-\sin\left(dx + c\right) + 1\right) - 2\left(Ba^4b - Aa^3b^2 - Ba^2b^3 + Aab^4\right)\sin\left(dx + c\right)}{2\left(\left(a^6b - 2\,a^4b^3 + a^2b^5\right)d\cos\left(dx + c\right) + \left(a^7 - 2\,a^5b^2 + a^3b^4\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((B*a^4 - 2*A*a^3*b + A*a*b^3 + (B*a^3*b - 2*A*a^2*b^2 + A*b^4)*cos(d*
x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (A*a^5 - 2*A*a^3*b^2 + A
*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1
) - (A*a^5 - 2*A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*
x + c))*log(-sin(d*x + c) + 1) - 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b
^4)*sin(d*x + c))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*
a^5*b^2 + a^3*b^4)*d), 1/2*(2*(B*a^4 - 2*A*a^3*b + A*a*b^3 + (B*a^3*b - 2*A
*a^2*b^2 + A*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b
)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (A*a^5 - 2*A*a^3*b^2 + A*a*b^4 + (A*a^4
*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) - (A*a^5 - 2*
A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(-si
n(d*x + c) + 1) - 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c
))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b
^4)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**2, x)
```

Giac [A]
time = 0.48, size = 223, normalized size = 1.68

$$\frac{\frac{2\,(Ba^3 - 2\,Aa^2b + Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a - 2\,b) + \arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\sqrt{a^2 - b^2}}\right)\right)}{(a^4 - a^2b^2)\sqrt{a^2 - b^2}} + \frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right)}{a^2} - \frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right)}{a^2} - \frac{2\,\left(Bab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - Ab^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{(a^3 - ab^2)\left(a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + a + b\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] (2*(B*a^3 - 2*A*a^2*b + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 -
b^2)))/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) + A*log(abs(tan(1/2*d*x + 1/2*c) +
 1))/a^2 - A*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 - 2*(B*a*b*tan(1/2*d*x
+ 1/2*c) - A*b^2*tan(1/2*d*x + 1/2*c))/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*
c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)))/d
```

Mupad [B]
time = 7.81, size = 2500, normalized size = 18.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] - (A*atan(((A*((A*((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2
 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a
^4*b^2) - (32*A*tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6
*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2))))/a^
2 - (32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2
*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b +
2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))*1i)/a^2 - (A*((A*((32*(A
*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2
+ 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*A*tan(c/2 +
 (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*
b^2))/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2))))/a^2 + (32*tan(c/2 + (d*x)/2
)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^
4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b +
a^5 - a^2*b^3 - a^3*b^2))*1i)/a^2)/((A*((A*((32*(A*a^4*b^5 - B*a^9 - A*a^9
- 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b))/(
a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (32*A*tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^
4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^5 -
 a^2*b^3 - a^3*b^2))))/a^2 - (32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 +
B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2
*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))
)/a^2 - (64*(A^3*b^5 + A*B^2*a^5 - A^2*B*a^5 - A^3*a*b^4 + 2*A^3*a^4*b - 3*
A^3*a^2*b^3 + 2*A^3*a^3*b^2 - 3*A^2*B*a^4*b + A^2*B*a^2*b^3 + A^2*B*a^3*b^2
))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (A*((A*((32*(A*a^4*b^5 - B*a^9 - A*a
^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b)
)/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*A*tan(c/2 + (d*x)/2)*(2*a^9*b - 2
*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^
```

5 - a^2*b^3 - a^3*b^2))))/a^2 + (32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))/a^2))*2i)/(a^2*d) - (atan((((-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) + (((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(A*b^3 + B*a^3 - 2*A*a^2*b)*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) + ((-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) - (((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (32*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(A*b^3 + B*a^3 - 2*A*a^2*b)*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))/((64*(A^3*b^5 + A*B^2*a^5 - A^2*B*a^5 - A^3*a*b^4 + 2*A^3*a^4*b - 3*A^3*a^2*b^3 + 2*A^3*a^3*b^2 - 3*A^2*B*a^4*b + A^2*B*a^2*b^3 + A^2*B*a^3*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - ((-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) + (((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(A*b^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) + ((-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) - (((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a...

3.263 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$\frac{2b(3a^2Ab-2Ab^3-2a^3B+ab^2B)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d}-\frac{(2Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{(a^2A$$

```
[Out] 2*b*(3*A*a^2*b-2*A*b^3-2*B*a^3+B*a*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*
c)/(a+b)^(1/2))/a^3/(a-b)^(3/2)/(a+b)^(3/2)/d-(2*A*b-B*a)*arctanh(sin(d*x+c
))/a^3/d+(A*a^2-2*A*b^2+B*a*b)*tan(d*x+c)/a^2/(a^2-b^2)/d+b*(A*b-B*a)*tan(d
*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A]

time = 0.45, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3079, 3134, 3080, 3855, 2738, 211}

$$-\frac{(2Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{(a^2A+abB-2Ab^2)\tan(c+dx)}{a^2d(a^2-b^2)}+\frac{b(Ab-aB)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{2b(-2a^3B+3a^2Ab+ab^2B-2Ab^3)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*b*(3*a^2*A*b - 2*A*b^3 - 2*a^3*B + a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c +
 d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2*A*b - a*B
)*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((a^2*A - 2*A*b^2 + a*b*B)*Tan[c + d*x])
/(a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b
*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
```

```
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{a\,(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{\int \frac{(a^2A-2Ab^2+abB-a(Ab-aB)\cos(c+}{a+b\,\mathrm{co}}}{a\,(a^2}$$

$$= \frac{(a^2A-2Ab^2+abB)\tan(c+dx)}{a^2\,(a^2-b^2)\,d} + \frac{b(Ab-aB)\tan(c+dx)}{a\,(a^2-b^2)\,d(a+b\cos(c+dx}$$

$$= \frac{(a^2A-2Ab^2+abB)\tan(c+dx)}{a^2\,(a^2-b^2)\,d} + \frac{b(Ab-aB)\tan(c+dx)}{a\,(a^2-b^2)\,d(a+b\cos(c+dx}$$

$$= -\frac{(2Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{(a^2A-2Ab^2+abB)\tan(c}{a^2\,(a^2-b^2)\,d}$$

$$= \frac{2b(3a^2Ab-2Ab^3-2a^3B+ab^2B)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d}$$

Mathematica [A]
time = 2.06, size = 240, normalized size = 1.27

$$\frac{-\frac{2b(-3a^2Ab+2Ab^3+2a^3B-ab^2B)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} + 2Ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - aB\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2Ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + aB\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{ab^2(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + aA\tan(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((-2*b*(-3*a^2*A*b + 2*A*b^3 + 2*a^3*B - a*b^2*B)*ArcTanh[((a - b)*Tan[(c +
 d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + 2*A*b*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] - a*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*A*
b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a*B*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + (a*b^2*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a +
b*Cos[c + d*x])) + a*A*Tan[c + d*x])/(a^3*d)
```

Maple [A]
time = 0.76, size = 241, normalized size = 1.28

method	result
derivativedivides	$\frac{-\frac{A}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} + \frac{(-2Ab+aB)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3} + \frac{2b\left(-\frac{a(Ab-aB)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)} + \frac{(3A\,a^2b}{}\right.}{a^3}}{d}$

default	$\frac{-\frac{A}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-2Ab+aB)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3}+\frac{2b\left(-\frac{a(Ab-aB)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(3A\,a^2b-\cdots\right.}{\cdots}\right.}{a^3}}{d}$
risch	$\frac{2i\left(-Aa\,b^2\mathrm{e}^{3i(dx+c)}+B\,a^2b\,\mathrm{e}^{3i(dx+c)}+A\,a^2b\,\mathrm{e}^{2i(dx+c)}-2A\,b^3\mathrm{e}^{2i(dx+c)}+Ba\,b^2\mathrm{e}^{2i(dx+c)}+2A\,a^3\mathrm{e}^{i(dx+c)}-3Aa\,b^2\mathrm{e}^{i(dx+c)}+\cdots\right.}{d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)\left(a^2-b^2\right)\left(b\,\mathrm{e}^{2i(dx+c)}+2a\,\mathrm{e}^{i(dx+c)}+b\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-A/a^2/(tan(1/2*d*x+1/2*c)+1)+1/a^3*(-2*A*b+B*a)*ln(tan(1/2*d*x+1/2*c)
+1)+2*b/a^3*(-a*(A*b-B*a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2
*c)^2-b*tan(1/2*d*x+1/2*c)^2+a+b)+(3*A*a^2*b-2*A*b^3-2*B*a^3+B*a*b^2)/(a-b)
/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1
/2)))-A/a^2/(tan(1/2*d*x+1/2*c)-1)+(2*A*b-B*a)/a^3*ln(tan(1/2*d*x+1/2*c)-1)
)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(180) = 360$.
time = 7.07, size = 1088, normalized size = 5.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

[Out] [-1/2*(((2*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + 2*A*b^5)*cos(d*x + c)^2 + (2*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(A*a^6 - 2*A*a^4*b^2 + A*a^2*b^4 + (A*a^5*b + B*a^4*b^2 - 3*A*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c)), -1/2*(2*((2*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + 2*A*b^5)*cos(d*x + c)^2 + (2*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(A*a^6 - 2*A*a^4*b^2 + A*a^2*b^4 + (A*a^5*b + B*a^4*b^2 - 3*A*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c))]

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)

[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**2, x)

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(180) = 360.
time = 0.49, size = 404, normalized size = 2.14

$$\frac{\frac{2\,(2\,Ba^3b-3\,Aa^2b^2-Bab^3+2\,Ab^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^5-a^3b^2)\sqrt{a^2-b^2}}-\frac{2\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Bab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Aab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Bab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)(a^4-a^2b^2)}+\frac{(Ba-2\,Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{(Ba-2\,Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="giac")

```
[Out] (2*(2*B*a^3*b - 3*A*a^2*b^2 - B*a*b^3 + 2*A*b^4)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) - 2*(A*a^3*tan
(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - A*a*b^2*tan(1/2*d*x
+ 1/2*c)^3 - B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2*c)^
3 + A*a^3*tan(1/2*d*x + 1/2*c) + A*a^2*b*tan(1/2*d*x + 1/2*c) - A*a*b^2*tan
(1/2*d*x + 1/2*c) + B*a*b^2*tan(1/2*d*x + 1/2*c) - 2*A*b^3*tan(1/2*d*x + 1/
2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d
*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)) + (B*a - 2*A*b)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1))/a^3 - (B*a - 2*A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3)
/d
```

Mupad [B]
time = 8.52, size = 2500, normalized size = 13.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^2),x)
```

```
[Out] (atan(((((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*
a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4
*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^
3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*
b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 - a^4*b
^3 - a^5*b^2) + (((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4 - 3*A
*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*a^11*b
 + 2*B*a^11*b))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*tan(c/2 + (d*x)/2)*
(2*A*b - B*a)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2
*a^10*b^2))/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2)))*(2*A*b - B*a))/a^3)*(2
*A*b - B*a)*1i)/a^3 + (((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2
*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*
A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4
 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^
6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*
b + a^7 - a^4*b^3 - a^5*b^2) - (((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*
A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b
^2 + 2*A*a^11*b + 2*B*a^11*b))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*tan(
c/2 + (d*x)/2)*(2*A*b - B*a)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4
- 4*a^9*b^3 - 2*a^10*b^2))/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2)))*(2*A*b
- B*a))/a^3)*(2*A*b - B*a)*1i)/a^3)/((64*(8*A^3*b^8 - 4*A^3*a*b^7 - 2*B^3*a
^7*b - 20*A^3*a^2*b^6 + 6*A^3*a^3*b^5 + 12*A^3*a^4*b^4 - B^3*a^3*b^5 + B^3*
a^4*b^4 + 3*B^3*a^5*b^3 - 2*B^3*a^6*b^2 - 12*A^2*B*a*b^7 + 6*A*B^2*a^2*b^6
- 5*A*B^2*a^3*b^5 - 17*A*B^2*a^4*b^4 + 9*A*B^2*a^5*b^3 + 11*A*B^2*a^6*b^2 +
 8*A^2*B*a^2*b^6 + 32*A^2*B*a^3*b^5 - 13*A^2*B*a^4*b^4 - 20*A^2*B*a^5*b^3))
```

/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A ^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B* a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (((32*(A*a^7*b^5 - 2*A*a^6*b^ 6 - B*a^12 + 5*A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a^9 *b^3 + B*a^10*b^2 + 2*A*a^11*b + 2*B*a^11*b))/(a^8*b + a^9 - a^6*b^3 - a^7* b^2) + (32*tan(c/2 + (d*x)/2)*(2*A*b - B*a)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b ^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5 *b^2)))*(2*A*b - B*a))/a^3)*(2*A*b - B*a))/a^3 - (((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a ^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2* B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5* b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b ^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*a^11*b + 2*B*a^11*b))/(a^8*b + a^9 - a^ 6*b^3 - a^7*b^2) - (32*tan(c/2 + (d*x)/2)*(2*A*b - B*a)*(2*a^11*b - 2*a^6*b ^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/(a^3*(a^6*b + a^7 - a ^4*b^3 - a^5*b^2)))*(2*A*b - B*a))/a^3)*(2*A*b - B*a))/a^3))*(2*A*b - B*a)* 2i)/(a^3*d) - ((2*tan(c/2 + (d*x)/2)^3*(A*a*b^2 - 2*A*b^3 - A*a^3 + A*a^2*b + B*a*b^2))/(a^2*(a + b)*(a - b)) - (2*tan(c/2 + (d*x)/2)*(A*a^3 - 2*A*b^3 - A*a*b^2 + A*a^2*b + B*a*b^2))/(a^2*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^4*(a - b) - 2*b*tan(c/2 + (d*x)/2)^2)) + (b*atan(((b*((32*tan(c /2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2 *b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B ^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b ^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (b *((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a ^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*a^11*b + 2*B*a^11*b))/ (a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*b*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*(2*a^11*b - 2*a^6* b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4* b^3 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^ 5*b^4 - 3*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a ^2*b - B*a*b^2)*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) + (b*((32*tan(c /2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2 *b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B ^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*...

3.264 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=270

$$-\frac{2b^2(4a^2Ab-3Ab^3-3a^3B+2ab^2B)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d}+\frac{(a^2A+6Ab^2-4abB)\tanh^{-1}(\sin(c+}{2a^4d}$$

```
[Out] -2*b^2*(4*A*a^2*b-3*A*b^3-3*B*a^3+2*B*a*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x
+1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*(A*a^2+6*A*b^2-4*B*a
*b)*arctanh(sin(d*x+c))/a^4/d-(2*A*a^2*b-3*A*b^3-B*a^3+2*B*a*b^2)*tan(d*x+c
)/a^3/(a^2-b^2)/d+1/2*(A*a^2-3*A*b^2+2*B*a*b)*sec(d*x+c)*tan(d*x+c)/a^2/(a^
2-b^2)/d+b*(A*b-B*a)*sec(d*x+c)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.65, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3079, 3134, 3080, 3855, 2738, 211}

$$\frac{(a^2A+2abB-3Ab^2)\tan(c+dx)\sec(c+dx)}{2a^2d\,(a^2-b^2)}+\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\,(a^2-b^2)\,(a+b\cos(c+dx))}+\frac{(a^2A-4abB+6Ab^2)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{(a^3(-B)+2a^2Ab+2ab^2B-3Ab^3)\tan(c+dx)}{a^3d\,(a^2-b^2)}-\frac{2b^2(-3a^3B+4a^2Ab+2ab^2B-3Ab^3)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^2*(4*a^2*A*b - 3*A*b^3 - 3*a^3*B + 2*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((a^2*A
+ 6*A*b^2 - 4*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - ((2*a^2*A*b - 3*A*b
^3 - a^3*B + 2*a*b^2*B)*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2*A - 3*A*b
^2 + 2*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b*(A*b -
a*B)*Sec[c + d*x]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2A-3Ab^2+2abB-a(Ab-aB)\cos\ldots\right.}{a\ldots}}{\ldots}$$

$$= \frac{\left(a^2A-3Ab^2+2abB\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sec(c\ldots}{a\left(a^2-b^2\right)d(a\ldots}$$

$$= -\frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2A-3Ab^2+2\ldots\right.}{2\ldots}$$

$$= -\frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2A-3Ab^2+2\ldots\right.}{2\ldots}$$

$$= \frac{\left(a^2A+6Ab^2-4abB\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{\left(2a^2Ab-3Ab^3-a^3\ldots\right.}{a^3(a\ldots}$$

$$= -\frac{2b^2\left(4a^2Ab-3Ab^3-3a^3B+2ab^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\ldots\right.}{a^4(a-b)^{3/2}(a+b)^{3/2}d}$$

Mathematica [A]
time = 6.29, size = 438, normalized size = 1.62

$$-\frac{2b^2\left(-4a^2Ab+3Ab^3+3a^3B-2ab^2B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{a^4\left(a^2-b^2\right)\sqrt{-a^2+b^2}\,d} + \frac{\left(-a^2A-6Ab^2+4abB\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\left(a^2A+6Ab^2-4abB\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{A}{4a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{A}{4a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{-2Ab\sin\left(\frac{1}{2}(c+dx)\right)+aB\sin\left(\frac{1}{2}(c+dx)\right)}{a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{-2Ab\sin\left(\frac{1}{2}(c+dx)\right)+aB\sin\left(\frac{1}{2}(c+dx)\right)}{a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{Ab^4\sin(c+dx)-ab^3B\sin(c+dx)}{a^3(a-b)(a+b)d(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^2*(-4*a^2*A*b + 3*A*b^3 + 3*a^3*B - 2*a*b^2*B)*ArcTanh[((a - b)*Tan[(
c + d*x)/2])/Sqrt[-a^2 + b^2]])/(a^4*(a^2 - b^2)*Sqrt[-a^2 + b^2]*d) + ((-(
a^2*A) - 6*A*b^2 + 4*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2*a^
4*d) + ((a^2*A + 6*A*b^2 - 4*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
])/(2*a^4*d) + A/(4*a^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - A/(4*a
^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (-2*A*b*Sin[(c + d*x)/2] +
a*B*Sin[(c + d*x)/2])/(a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (-2*A
*b*Sin[(c + d*x)/2] + a*B*Sin[(c + d*x)/2])/(a^3*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])) + (A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x])/(a^3*(a - b)*(
a + b)*d*(a + b*Cos[c + d*x]))
```

Maple [A]
time = 0.87, size = 326, normalized size = 1.21

method	result

derivativedivides	$\frac{-\frac{A}{2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-aA-4Ab+2aB}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2A+6A\,b^2-4Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^4}-\frac{2b^2\left(-\frac{a(Ab-aB)b\,t}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}\right.}{}}{}$
default	$\frac{-\frac{A}{2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-aA-4Ab+2aB}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2A+6A\,b^2-4Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^4}-\frac{2b^2\left(-\frac{a(Ab-aB)b\,t}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-1/2*A/a^2/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-A*a-4*A*b+2*B*a)/a^3/(tan(1/
2*d*x+1/2*c)+1)+1/2*(A*a^2+6*A*b^2-4*B*a*b)/a^4*ln(tan(1/2*d*x+1/2*c)+1)-2*
b^2/a^4*(-a*(A*b-B*a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^
2-b*tan(1/2*d*x+1/2*c)^2+a+b)+(4*A*a^2*b-3*A*b^3-3*B*a^3+2*B*a*b^2)/(a-b)/(
a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2
)))+1/2*A/a^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-A*a-4*A*b+2*B*a)/a^3/(tan(1/2*
d*x+1/2*c)-1)+1/2/a^4*(-A*a^2-6*A*b^2+4*B*a*b)*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(256) = 512$.
time = 12.30, size = 1329, normalized size = 4.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(2*((3*B*a^3*b^3 - 4*A*a^2*b^4 - 2*B*a*b^5 + 3*A*b^6)*cos(d*x + c)^3
+ (3*B*a^4*b^2 - 4*A*a^3*b^3 - 2*B*a^2*b^4 + 3*A*a*b^5)*cos(d*x + c)^2)*sqr
t(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sq
rt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*
x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - ((A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4*b
^3 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^3 + (A*
a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a^2*b^5 +
6*A*a*b^6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((A*a^6*b - 4*B*a^5*b^2
+ 4*A*a^4*b^3 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x +
 c)^3 + (A*a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B
*a^2*b^5 + 6*A*a*b^6)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(A*a^7 - 2
*A*a^5*b^2 + A*a^3*b^4 + 2*(B*a^6*b - 2*A*a^5*b^2 - 3*B*a^4*b^3 + 5*A*a^3*b
^4 + 2*B*a^2*b^5 - 3*A*a*b^6)*cos(d*x + c)^2 + (2*B*a^7 - 3*A*a^6*b - 4*B*a
^5*b^2 + 6*A*a^4*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c))*sin(d*x + c
))/((a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(d*x + c)^3 + (a^9 - 2*a^7*b^2 + a^5
*b^4)*d*cos(d*x + c)^2), 1/4*(4*((3*B*a^3*b^3 - 4*A*a^2*b^4 - 2*B*a*b^5 + 3
*A*b^6)*cos(d*x + c)^3 + (3*B*a^4*b^2 - 4*A*a^3*b^3 - 2*B*a^2*b^4 + 3*A*a*b
^5)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2
- b^2)*sin(d*x + c))) + ((A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4*b^3 + 8*B*a^3*b^4
 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^3 + (A*a^7 - 4*B*a^6*b
+ 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a^2*b^5 + 6*A*a*b^6)*cos(d
*x + c)^2)*log(sin(d*x + c) + 1) - ((A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4*b^3 +
8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^3 + (A*a^7 -
 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a^2*b^5 + 6*A*a
*b^6)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(A*a^7 - 2*A*a^5*b^2 + A*a
^3*b^4 + 2*(B*a^6*b - 2*A*a^5*b^2 - 3*B*a^4*b^3 + 5*A*a^3*b^4 + 2*B*a^2*b^5
 - 3*A*a*b^6)*cos(d*x + c)^2 + (2*B*a^7 - 3*A*a^6*b - 4*B*a^5*b^2 + 6*A*a^4
*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b - 2*a
^6*b^3 + a^4*b^5)*d*cos(d*x + c)^3 + (a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x
+ c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{(a + b\cos(c + dx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x))**2, x)
```

Giac [A]
time = 0.48, size = 378, normalized size = 1.40

$$-\frac{\frac{4\left(3 Ba^3b^2-4 Aa^2b^3-2 Bab^4+3 Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-a^4b^2)\sqrt{a^2-b^2}}+\frac{4\left(Bab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{(a^5-a^3b^2)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)}-\frac{(Aa^2-4Bab+6Ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}+\frac{(Aa^2-4Bab+6Ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}-\frac{2\left(Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*(3*B*a^3*b^2 - 4*A*a^2*b^3 - 2*B*a*b^4 + 3*A*b^5)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(
1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*sqrt(a^2 - b^2)) + 4*(
B*a*b^3*tan(1/2*d*x + 1/2*c) - A*b^4*tan(1/2*d*x + 1/2*c))/((a^5 - a^3*b^2)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - (A*a^2 -
4*B*a*b + 6*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 + (A*a^2 - 4*B*a*
b + 6*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 2*(A*a*tan(1/2*d*x +
1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 4*A*b*tan(1/2*d*x + 1/2*c)^3 + A*
a*tan(1/2*d*x + 1/2*c) + 2*B*a*tan(1/2*d*x + 1/2*c) - 4*A*b*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

Mupad [B]
time = 9.28, size = 2500, normalized size = 9.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^2),x)
```

```
[Out] (atan(-((((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 - 72*A^2*a*b^9 - 2*
A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5
*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 -
32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*
a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 17
6*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*
a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (((8*(2*A*a^
15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^11*b^4 - 16*A*a^12
*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a^11*b^4 + 12*B*a^1
2*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) -
 (4*tan(c/2 + (d*x)/2)*(A*a^2 + 6*A*b^2 - 4*B*a*b)*(8*a^13*b - 8*a^8*b^6 +
8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^
6*b^3 - a^7*b^2)))*(A*a^2 + 6*A*b^2 - 4*B*a*b))/(2*a^4))*(A*a^2 + 6*A*b^2 -
 4*B*a*b)*1i)/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 - 7
2*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*
b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 3
2*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a
```

^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*
A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a
^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2
) + (((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^11
*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a^1
1*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))/(a^11*b + a^12 - a^9*b
^3 - a^10*b^2) + (4*tan(c/2 + (d*x)/2)*(A*a^2 + 6*A*b^2 - 4*B*a*b)*(8*a^13*
b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(
a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*(A*a^2 + 6*A*b^2 - 4*B*a*b))/(2*a^4))*(A
*a^2 + 6*A*b^2 - 4*B*a*b)*1i)/(2*a^4))/((16*(108*A^3*b^11 - 54*A^3*a*b^10 -
216*A^3*a^2*b^9 + 81*A^3*a^3*b^8 + 63*A^3*a^4*b^7 - 9*A^3*a^5*b^6 + 41*A^3
*a^6*b^5 - 4*A^3*a^7*b^4 + 4*A^3*a^8*b^3 - 32*B^3*a^3*b^8 + 16*B^3*a^4*b^7
+ 80*B^3*a^5*b^6 - 24*B^3*a^6*b^5 - 48*B^3*a^7*b^4 - 216*A^2*B*a*b^10 + 144
*A*B^2*a^2*b^9 - 72*A*B^2*a^3*b^8 - 336*A*B^2*a^4*b^7 + 108*A*B^2*a^5*b^6 +
168*A*B^2*a^6*b^5 - 6*A*B^2*a^7*b^4 + 24*A*B^2*a^8*b^3 + 108*A^2*B*a^2*b^9
+ 468*A^2*B*a^3*b^8 - 162*A^2*B*a^4*b^7 - 186*A^2*B*a^5*b^6 + 15*A^2*B*a^6
*b^5 - 63*A^2*B*a^7*b^4 + 3*A^2*B*a^8*b^3 - 3*A^2*B*a^9*b^2))/(a^11*b + a^1
2 - a^9*b^3 - a^10*b^2) - (((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 -
72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^
4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 +
32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2
*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 9
6*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B
*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b
^2) - (((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^
11*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a
^11*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))/(a^11*b + a^12 - a^9
*b^3 - a^10*b^2) - (4*tan(c/2 + (d*x)/2)*(A*a^2 + 6*A*b^2 - 4*B*a*b)*(8*a^1
3*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4
*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*(A*a^2 + 6*A*b^2 - 4*B*a*b))/(2*a^4))*
(A*a^2 + 6*A*b^2 - 4*B*a*b))/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(A^2*a^10 +
72*A^2*b^10 - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^
7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*
A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5
*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*
A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5
*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^
6*b^3 - a^7*b^2) + (((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*
b^5 - 14*A*a^11*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10
*b^5 - 20*B*a^11*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))/(a^11*b
+ a^12 - a^9*b^3 - a^10*b^2) + (4*tan(c/2 + (d*x)/2)*(A*a^2 + 6*A*b^2 - 4*
B*a*b)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^
12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*(A*a^2 + 6*A*b^2 - 4*B*a*
b))/(2*a^4))*(A*a^2 + 6*A*b^2 - 4*B*a*b))/(2*a^4)))*(A*a^2 + 6*A*b^2 - 4*B*
a*b)*1i)/(a^4*d) - ((tan(c/2 + (d*x)/2)^5*(A*a^4 + 6*A*b^4 - 2*B*a^4 - 5*A*

```
a^2*b^2 + 2*B*a^2*b^2 - 3*A*a*b^3 + 3*A*a^3*b - 4*B*a*b^3 + 2*B*a^3*b))/((a
^3*b - a^4)*(a + b)) + (tan(c/2 + (d*x)/2)*(A*a^4 + 6*A*b^4 + 2*B*a^4 - 5*A
*a^2*b^2 - 2*B*a^2*b^2 + 3*A*a*b^3 - 3*A*a^3*b - 4*B*a*b^3 + 2*B*a^3*b))/((
a^3*b - a^4)*(a + b)) + (2*tan(c/2 + (d*x)/2)^3*(A*a^4 - 6*A*b^4 + 3*A*a^2*
b^2 + 4*B*a*b^3 - 2*B*a^3*b))/(a*(a^2*b - a^3)*...
```

3.265 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=398

$$-\frac{(6aAb-12a^2B-b^2B)\,x}{2b^5}+\frac{a^2(6a^4Ab-15a^2Ab^3+12Ab^5-12a^5B+29a^3b^2B-20ab^4B)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\cdots}{\sqrt{a}\cdots}\right.}{(a-b)^{5/2}b^5(a+b)^{5/2}d}$$

```
[Out] -1/2*(6*A*a*b-12*B*a^2-B*b^2)*x/b^5+a^2*(6*A*a^4*b-15*A*a^2*b^3+12*A*b^5-12
*B*a^5+29*B*a^3*b^2-20*B*a*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))/(a-b)^(5/2)/b^5/(a+b)^(5/2)/d+1/2*(6*A*a^4*b-11*A*a^2*b^3+2*A*b^5-1
2*B*a^5+21*B*a^3*b^2-6*B*a*b^4)*sin(d*x+c)/b^4/(a^2-b^2)^2/d-1/2*(3*A*a^3*b
-6*A*a*b^3-6*B*a^4+10*B*a^2*b^2-B*b^4)*cos(d*x+c)*sin(d*x+c)/b^3/(a^2-b^2)^
2/d+1/2*a*(A*b-B*a)*cos(d*x+c)^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^
2+1/2*a*(2*A*a^2*b-5*A*b^3-4*B*a^3+7*B*a*b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/(
a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A]

time = 1.14, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3068, 3126, 3128, 3102, 2814, 2738, 211}

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{x(-12a^2B+6aAb-b^2B)}{2b^5}+\frac{a(-4a^3B+2a^2Ab+7ab^2B-5Ab^3)\sin(c+dx)\cos^2(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{(-6a^4B+3a^3Ab+10a^2b^2B-6aAb^3-b^4B)\sin(c+dx)\cos(c+dx)}{2b^3d\left(a^2-b^2\right)^2}+\frac{a^2(-12a^5B+6a^4Ab+29a^3b^2B-15a^2Ab^3-20ab^4B+12Ab^5)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{5/2}(a+b)^{5/2}}+\frac{(-12a^5B+6a^4Ab+21a^3b^2B-11a^2Ab^3-6ab^4B+2Ab^5)\sin(c+dx)}{2b^4d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -1/2*((6*a*A*b - 12*a^2*B - b^2*B)*x)/b^5 + (a^2*(6*a^4*A*b - 15*a^2*A*b^3
+ 12*A*b^5 - 12*a^5*B + 29*a^3*b^2*B - 20*a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*d) + ((6*a^4*A
*b - 11*a^2*A*b^3 + 2*A*b^5 - 12*a^5*B + 21*a^3*b^2*B - 6*a*b^4*B)*Sin[c +
d*x])/(2*b^4*(a^2 - b^2)^2*d) - ((3*a^3*A*b - 6*a*A*b^3 - 6*a^4*B + 10*a^2*
b^2*B - b^4*B)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d) + (a*(A*b
 - a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^2) + (a*(2*a^2*A*b - 5*A*b^3 - 4*a^3*B + 7*a*b^2*B)*Cos[c + d*x]^2*Sin[c
+ d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{\int \cos^2(c+dx)\left(-3a(Ab-aB)+2b(Ab \ldots\right.}{\ldots}$$

$$= \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\cos(c+dx))^2} + \frac{a(2a^2Ab-5Ab^3-4a^3B+\ldots}{2b^2\,(a^2-b^2)^2\,\ldots}$$

$$= -\frac{(3a^3Ab-6aAb^3-6a^4B+10a^2b^2B-b^4B)\cos(c+dx)\sin(c+d\ldots}{2b^3\,(a^2-b^2)^2\,d}$$

$$= \frac{(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6ab^4B)\sin(c+d\ldots}{2b^4\,(a^2-b^2)^2\,d}$$

$$= -\frac{(6aAb-12a^2B-b^2B)\,x}{2b^5} + \frac{(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B\ldots}{2b^4\,(a^2-\ldots}$$

$$= -\frac{(6aAb-12a^2B-b^2B)\,x}{2b^5} + \frac{(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B\ldots}{2b^4\,(a^2-\ldots}$$

$$= -\frac{(6aAb-12a^2B-b^2B)\,x}{2b^5} + \frac{a^2(6a^4Ab-15a^2Ab^3+12Ab^5-12a\ldots}{(a\ldots}$$

Mathematica [A]
time = 3.59, size = 734, normalized size = 1.84

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((16*a^2*(-6*a^4*A*b + 15*a^2*A*b^3 - 12*A*b^5 + 12*a^5*B - 29*a^3*b^2*B +
20*a*b^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b
^2)^(5/2) + (-48*a^7*A*b*c + 72*a^5*A*b^3*c - 24*a*A*b^7*c + 96*a^8*B*c - 1
36*a^6*b^2*B*c - 12*a^4*b^4*B*c + 48*a^2*b^6*B*c + 4*b^8*B*c - 48*a^7*A*b*d
*x + 72*a^5*A*b^3*d*x - 24*a*A*b^7*d*x + 96*a^8*B*d*x - 136*a^6*b^2*B*d*x -
 12*a^4*b^4*B*d*x + 48*a^2*b^6*B*d*x + 4*b^8*B*d*x + 16*a*b*(a^2 - b^2)^2*(
-6*a*A*b + 12*a^2*B + b^2*B)*(c + d*x)*Cos[c + d*x] + 4*(-(a^2*b) + b^3)^2*
(-6*a*A*b + 12*a^2*B + b^2*B)*(c + d*x)*Cos[2*(c + d*x)] + 48*a^6*A*b^2*Sin
[c + d*x] - 84*a^4*A*b^4*Sin[c + d*x] + 8*a^2*A*b^6*Sin[c + d*x] + 4*A*b^8*
Sin[c + d*x] - 96*a^7*b*B*Sin[c + d*x] + 160*a^5*b^3*B*Sin[c + d*x] - 32*a^
3*b^5*B*Sin[c + d*x] - 8*a*b^7*B*Sin[c + d*x] + 36*a^5*A*b^3*Sin[2*(c + d*x
)] - 64*a^3*A*b^5*Sin[2*(c + d*x)] + 16*a*A*b^7*Sin[2*(c + d*x)] - 72*a^6*b
^2*B*Sin[2*(c + d*x)] + 130*a^4*b^4*B*Sin[2*(c + d*x)] - 48*a^2*b^6*B*Sin[2
*(c + d*x)] + 2*b^8*B*Sin[2*(c + d*x)] + 4*a^4*A*b^4*Sin[3*(c + d*x)] - 8*a
^2*A*b^6*Sin[3*(c + d*x)] + 4*A*b^8*Sin[3*(c + d*x)] - 8*a^5*b^3*B*Sin[3*(c
 + d*x)] + 16*a^3*b^5*B*Sin[3*(c + d*x)] - 8*a*b^7*B*Sin[3*(c + d*x)] + a^4
*b^4*B*Sin[4*(c + d*x)] - 2*a^2*b^6*B*Sin[4*(c + d*x)] + b^8*B*Sin[4*(c + d
*x)])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2))/(16*b^5*d)
```

Maple [A]
time = 0.76, size = 402, normalized size = 1.01

method	result
derivativedivides	$\frac{2a^2\left(\frac{\frac{\left(4A\,a^2b-Aa\,b^2-8A\,b^3-6a^3B+a^2bB+10Ba\,b^2\right)ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{ba\left(4A\,a^2b+Aa\,b^2-8A\,b^3-6a^3B-a^2bB+10Ba\,b^2\right)\tan\left(\cdots\right.}{2(a+b)(a-b)^2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}\cdots\right.}{b^5}$
default	$\frac{2a^2\left(\frac{\frac{\left(4A\,a^2b-Aa\,b^2-8A\,b^3-6a^3B+a^2bB+10Ba\,b^2\right)ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{ba\left(4A\,a^2b+Aa\,b^2-8A\,b^3-6a^3B-a^2bB+10Ba\,b^2\right)\tan\left(\cdots\right.}{2(a+b)(a-b)^2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}\cdots\right.}{b^5}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2*a^2/b^5*((1/2*(4*A*a^2*b-A*a*b^2-8*A*b^3-6*B*a^3+B*a^2*b+10*B*a*b^2)
*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*b*a*(4*A*a^2*b+A*a*b^2-
8*A*b^3-6*B*a^3-B*a^2*b+10*B*a*b^2)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(a*ta
n(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(6*A*a^4*b-15*A*a^2*b^
```

```
3+12*A*b^5-12*B*a^5+29*B*a^3*b^2-20*B*a*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+
b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))-2/b^5*(((-A
*b^2+3*B*a*b+1/2*B*b^2)*tan(1/2*d*x+1/2*c)^3+(-A*b^2+3*B*a*b-1/2*B*b^2)*tan
(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(6*A*a*b-12*B*a^2-B*b^2)*ar
ctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 871 vs. 2(378) = 756.
time = 0.50, size = 1812, normalized size = 4.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(2*(12*B*a^8*b^2 - 6*A*a^7*b^3 - 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^
4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b^8 + 6*A*a*b^9 - B*b^10)*d*x*cos(d*x + c)^2
 + 4*(12*B*a^9*b - 6*A*a^8*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5
 - 18*A*a^4*b^6 - 9*B*a^3*b^7 + 6*A*a^2*b^8 - B*a*b^9)*d*x*cos(d*x + c) + 2
*(12*B*a^10 - 6*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A
*a^5*b^5 - 9*B*a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*d*x + (12*B*a^9 - 6*A*a^8
*b - 29*B*a^7*b^2 + 15*A*a^6*b^3 + 20*B*a^5*b^4 - 12*A*a^4*b^5 + (12*B*a^7*
b^2 - 6*A*a^6*b^3 - 29*B*a^5*b^4 + 15*A*a^4*b^5 + 20*B*a^3*b^6 - 12*A*a^2*b
^7)*cos(d*x + c)^2 + 2*(12*B*a^8*b - 6*A*a^7*b^2 - 29*B*a^6*b^3 + 15*A*a^5*
b^4 + 20*B*a^4*b^5 - 12*A*a^3*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*
b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d
*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + a^2)) - 2*(12*B*a^9*b - 6*A*a^8*b^2 - 33*B*a^7*b^3 + 17*A*a^6*b^4
+ 27*B*a^5*b^5 - 13*A*a^4*b^6 - 6*B*a^3*b^7 + 2*A*a^2*b^8 - (B*a^6*b^4 - 3*
B*a^4*b^6 + 3*B*a^2*b^8 - B*b^10)*cos(d*x + c)^3 + 2*(2*B*a^7*b^3 - A*a^6*b
^4 - 6*B*a^5*b^5 + 3*A*a^4*b^6 + 6*B*a^3*b^7 - 3*A*a^2*b^8 - 2*B*a*b^9 + A*
```

```
b^10)*cos(d*x + c)^2 + (18*B*a^8*b^2 - 9*A*a^7*b^3 - 50*B*a^6*b^4 + 25*A*a^
5*b^5 + 43*B*a^4*b^6 - 20*A*a^3*b^7 - 11*B*a^2*b^8 + 4*A*a*b^9)*cos(d*x + c
))*sin(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^
2 + 2*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5
 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d), 1/2*((12*B*a^8*b^2 - 6*A*a^7*b^3 -
 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b^8 +
6*A*a*b^9 - B*b^10)*d*x*cos(d*x + c)^2 + 2*(12*B*a^9*b - 6*A*a^8*b^2 - 35*B
*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B*a^3*b^7 + 6*A*a
^2*b^8 - B*a*b^9)*d*x*cos(d*x + c) + (12*B*a^10 - 6*A*a^9*b - 35*B*a^8*b^2
+ 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 - 9*B*a^4*b^6 + 6*A*a^3*b^7 -
B*a^2*b^8)*d*x - (12*B*a^9 - 6*A*a^8*b - 29*B*a^7*b^2 + 15*A*a^6*b^3 + 20*B
*a^5*b^4 - 12*A*a^4*b^5 + (12*B*a^7*b^2 - 6*A*a^6*b^3 - 29*B*a^5*b^4 + 15*A
*a^4*b^5 + 20*B*a^3*b^6 - 12*A*a^2*b^7)*cos(d*x + c)^2 + 2*(12*B*a^8*b - 6*
A*a^7*b^2 - 29*B*a^6*b^3 + 15*A*a^5*b^4 + 20*B*a^4*b^5 - 12*A*a^3*b^6)*cos(
d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin
(d*x + c))) - (12*B*a^9*b - 6*A*a^8*b^2 - 33*B*a^7*b^3 + 17*A*a^6*b^4 + 27*
B*a^5*b^5 - 13*A*a^4*b^6 - 6*B*a^3*b^7 + 2*A*a^2*b^8 - (B*a^6*b^4 - 3*B*a^4
*b^6 + 3*B*a^2*b^8 - B*b^10)*cos(d*x + c)^3 + 2*(2*B*a^7*b^3 - A*a^6*b^4 -
6*B*a^5*b^5 + 3*A*a^4*b^6 + 6*B*a^3*b^7 - 3*A*a^2*b^8 - 2*B*a*b^9 + A*b^10)
*cos(d*x + c)^2 + (18*B*a^8*b^2 - 9*A*a^7*b^3 - 50*B*a^6*b^4 + 25*A*a^5*b^5
 + 43*B*a^4*b^6 - 20*A*a^3*b^7 - 11*B*a^2*b^8 + 4*A*a*b^9)*cos(d*x + c))*si
n(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + 2
*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5 - 3*
a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2712 vs. 2(378) = 756.
time = 0.70, size = 2712, normalized size = 6.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*((3*(2*a^5*b - a^4*b^2 - 4*a^3*b^3 + 2*a^2*b^4 + 2*a*b^5)*sqrt(a^2 - b^
2)*A*abs(a^4*b^5 - 2*a^2*b^7 + b^9)*abs(-a + b) - (12*a^6 - 6*a^5*b - 23*a^
4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 - a*b^5 + b^6)*sqrt(a^2 - b^2)*B*abs(a^4*b^
5 - 2*a^2*b^7 + b^9)*abs(-a + b) + 3*(4*a^10*b^5 - 2*a^9*b^6 - 17*a^8*b^7 +
 8*a^7*b^8 + 28*a^6*b^9 - 12*a^5*b^10 - 21*a^4*b^11 + 8*a^3*b^12 + 6*a^2*b^
13 - 2*a*b^14)*sqrt(a^2 - b^2)*A*abs(-a + b) - (24*a^11*b^4 - 12*a^10*b^5 -
 100*a^9*b^6 + 47*a^8*b^7 + 158*a^7*b^8 - 68*a^6*b^9 - 111*a^5*b^10 + 42*a^
4*b^11 + 28*a^3*b^12 - 8*a^2*b^13 + a*b^14 - b^15)*sqrt(a^2 - b^2)*B*abs(-a
 + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*tan(1/2*d*x + 1/2*c)/sq
rt((4*a^5*b^4 - 8*a^3*b^6 + 4*a*b^8 + sqrt(-16*(a^5*b^4 + a^4*b^5 - 2*a^3*b
^6 - 2*a^2*b^7 + a*b^8 + b^9)*(a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 +
a*b^8 - b^9) + 16*(a^5*b^4 - 2*a^3*b^6 + a*b^8)^2))/(a^5*b^4 - a^4*b^5 - 2*
a^3*b^6 + 2*a^2*b^7 + a*b^8 - b^9))))/((a^4*b^5 - 2*a^2*b^7 + b^9)^2*(a^2 -
 2*a*b + b^2) + (a^7*b^4 - 2*a^6*b^5 - a^5*b^6 + 4*a^4*b^7 - a^3*b^8 - 2*a^
2*b^9 + a*b^10)*abs(a^4*b^5 - 2*a^2*b^7 + b^9)) + (24*B*a^11*b^4 - 12*A*a^1
0*b^5 - 12*B*a^10*b^5 + 6*A*a^9*b^6 - 100*B*a^9*b^6 + 51*A*a^8*b^7 + 47*B*a
^8*b^7 - 24*A*a^7*b^8 + 158*B*a^7*b^8 - 84*A*a^6*b^9 - 68*B*a^6*b^9 + 36*A*
a^5*b^10 - 111*B*a^5*b^10 + 63*A*a^4*b^11 + 42*B*a^4*b^11 - 24*A*a^3*b^12 +
 28*B*a^3*b^12 - 18*A*a^2*b^13 - 8*B*a^2*b^13 + 6*A*a*b^14 + B*a*b^14 - B*b
^15 - 12*B*a^6*abs(a^4*b^5 - 2*a^2*b^7 + b^9) + 6*A*a^5*b*abs(a^4*b^5 - 2*a
^2*b^7 + b^9) + 6*B*a^5*b*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - 3*A*a^4*b^2*abs(
a^4*b^5 - 2*a^2*b^7 + b^9) + 23*B*a^4*b^2*abs(a^4*b^5 - 2*a^2*b^7 + b^9) -
12*A*a^3*b^3*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - 10*B*a^3*b^3*abs(a^4*b^5 - 2*
a^2*b^7 + b^9) + 6*A*a^2*b^4*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - 10*B*a^2*b^4*
abs(a^4*b^5 - 2*a^2*b^7 + b^9) + 6*A*a*b^5*abs(a^4*b^5 - 2*a^2*b^7 + b^9) +
 B*a*b^5*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - B*b^6*abs(a^4*b^5 - 2*a^2*b^7 + b
^9))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*tan(1/2*d*x + 1/2*c)/sqrt
((4*a^5*b^4 - 8*a^3*b^6 + 4*a*b^8 - sqrt(-16*(a^5*b^4 + a^4*b^5 - 2*a^3*b^6
 - 2*a^2*b^7 + a*b^8 + b^9)*(a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 + a*
b^8 - b^9) + 16*(a^5*b^4 - 2*a^3*b^6 + a*b^8)^2))/(a^5*b^4 - a^4*b^5 - 2*a^
3*b^6 + 2*a^2*b^7 + a*b^8 - b^9))))/(a^5*b^4*abs(a^4*b^5 - 2*a^2*b^7 + b^9)
 - 2*a^3*b^6*abs(a^4*b^5 - 2*a^2*b^7 + b^9) + a*b^8*abs(a^4*b^5 - 2*a^2*b^7
 + b^9) - (a^4*b^5 - 2*a^2*b^7 + b^9)^2) - 2*(12*B*a^7*tan(1/2*d*x + 1/2*c)
^7 - 6*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 18*B*a^6*b*tan(1/2*d*x + 1/2*c)^7 +
 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 17*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 +
 9*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 -
 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 2*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 +
 2*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 13*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 +
 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 4*B*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 2*A*
b^7*tan(1/2*d*x + 1/2*c)^7 + B*b^7*tan(1/2*d*x + 1/2*c)^7 + 36*B*a^7*tan(1/
2*d*x + 1/2*c)^5 - 18*A*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 18*B*a^6*b*tan(1/2*d
*x + 1/2*c)^5 + 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 67*B*a^5*b^2*tan(1/2*d
*x + 1/2*c)^5 + 35*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 29*B*a^4*b^3*tan(1/2*
d*x + 1/2*c)^5 - 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 26*B*a^3*b^4*tan(1/2
*d*x + 1/2*c)^5 - 10*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 5*B*a^2*b^5*tan(1/2
```

```
*d*x + 1/2*c)^5 + 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 4*B*a*b^6*tan(1/2*d*x
+ 1/2*c)^5 + 2*A*b^7*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^7*tan(1/2*d*x + 1/2*c)^
5 + 36*B*a^7*tan(1/2*d*x + 1/2*c)^3 - 18*A*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 1
8*B*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 67*
B*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 35*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29
*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 2
6*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 10*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 +
5*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^3 - 4*B
*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^7*tan(1/2*d*x + 1/2*c)^3 + 3*B*b^7*ta
n(1/2*d*x + 1/2*c)^3 + 12*B*a^7*tan(1/2*d*x + 1/2*c) - 6*A*a^6*b*tan(1/2*d*
x + 1/2*c) + 18*B*a^6*b*tan(1/2*d*x + 1/2*c) - 9*A*a^5*b^2*tan(1/2*d*x + 1/
2*c) - 17*B*a^5*b^2*tan(1/2*d*x + 1/2*c) + 9*A*a^4*b^3*tan(1/2*d*x + 1/2*c)
 - 33*B*a^4*b^3*tan(1/2*d*x + 1/2*c) + 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c) -
2*B*a^3*b^4*tan(1/2*d*x + 1/2*c) + 2*A*a^2*b^5*tan(1/2*d*x + 1/2*c) + 13*B*
a^2*b^5*tan(1/2*d*x + 1/2*c) - 4*A*a*b^6*tan(1/2*d*x + 1/2*c) + 4*B*a*b^6*t
an(1/2*d*x + 1/2*c) - 2*A*b^7*tan(1/2*d*x + 1/2*c) - B*b^7*tan(1/2*d*x + 1/
2*c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*
x + 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

Mupad [B]
time = 12.01, size = 2500, normalized size = 6.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(3*B*b^7 - 36*B*a^7 - 2*A*b^7 + 10*A*a^2*b^5 + 16*A*
a^3*b^4 - 35*A*a^4*b^3 - 9*A*a^5*b^2 + 5*B*a^2*b^5 - 26*B*a^3*b^4 - 29*B*a^
4*b^3 + 67*B*a^5*b^2 - 4*A*a*b^6 + 18*A*a^6*b + 4*B*a*b^6 + 18*B*a^6*b))/((
a + b)^2*(b^6 - 2*a*b^5 + a^2*b^4)) - (tan(c/2 + (d*x)/2)^3*(2*A*b^7 + 36*B
*a^7 + 3*B*b^7 - 10*A*a^2*b^5 + 16*A*a^3*b^4 + 35*A*a^4*b^3 - 9*A*a^5*b^2 +
 5*B*a^2*b^5 + 26*B*a^3*b^4 - 29*B*a^4*b^3 - 67*B*a^5*b^2 - 4*A*a*b^6 - 18*
A*a^6*b - 4*B*a*b^6 + 18*B*a^6*b))/((a + b)^2*(b^6 - 2*a*b^5 + a^2*b^4)) +
(tan(c/2 + (d*x)/2)^7*(B*b^6 - 12*B*a^6 - 2*A*b^6 + 4*A*a^2*b^4 - 12*A*a^3*
b^3 - 3*A*a^4*b^2 - 8*B*a^2*b^4 - 10*B*a^3*b^3 + 23*B*a^4*b^2 + 2*A*a*b^5 +
 6*A*a^5*b + 5*B*a*b^5 + 6*B*a^5*b))/((a*b^4 - b^5)*(a + b)^2) + (tan(c/2 +
 (d*x)/2)*(2*A*b^6 - 12*B*a^6 + B*b^6 - 4*A*a^2*b^4 - 12*A*a^3*b^3 + 3*A*a^
4*b^2 - 8*B*a^2*b^4 + 10*B*a^3*b^3 + 23*B*a^4*b^2 + 2*A*a*b^5 + 6*A*a^5*b -
 5*B*a*b^5 - 6*B*a^5*b))/((a + b)*(b^6 - 2*a*b^5 + a^2*b^4)))/(d*(2*a*b + t
an(c/2 + (d*x)/2)^4*(6*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^2*(4*a*b + 4*a^2)
- tan(c/2 + (d*x)/2)^6*(4*a*b - 4*a^2) + tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b
+ b^2) + a^2 + b^2)) + (atan(((((8*tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B^2*b
^14 - 2*B^2*a*b^13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11 + 3
6*A^2*a^4*b^10 + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*
```

A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72*A^2
*a^12*b^2 + 21*B^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B^2*a
^5*b^9 + 18*B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*
b^5 + 1538*B^2*a^10*b^4 + 1104*B^2*a^11*b^3 - 1104*B^2*a^12*b^2 - 12*A*B*a*
b^13 - 288*A*B*a^13*b + 24*A*B*a^2*b^12 - 108*A*B*a^3*b^11 + 192*A*B*a^4*b^
10 - 72*A*B*a^5*b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6
 - 1650*A*B*a^9*b^5 - 1128*A*B*a^10*b^4 + 1128*A*B*a^11*b^3 + 288*A*B*a^12*
b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 -
a^6*b^9 - a^7*b^8) + (((4*(4*B*b^21 + 48*A*a^2*b^19 + 72*A*a^3*b^18 - 156*A
*a^4*b^17 - 84*A*a^5*b^16 + 192*A*a^6*b^15 + 48*A*a^7*b^14 - 108*A*a^8*b^13
 - 12*A*a^9*b^12 + 24*A*a^10*b^11 + 28*B*a^2*b^19 - 80*B*a^3*b^18 - 120*B*a
^4*b^17 + 276*B*a^5*b^16 + 164*B*a^6*b^15 - 360*B*a^7*b^14 - 100*B*a^8*b^13
 + 212*B*a^9*b^12 + 24*B*a^10*b^11 - 48*B*a^11*b^10 - 24*A*a*b^20))/(a*b^18
 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^
7*b^12) - (4*tan(c/2 + (d*x)/2)*(B*a^2*12i + B*b^2*1i - A*a*b*6i)*(8*a*b^19
 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*
a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/(b^5*(a*b^14 + b^15 - 3
*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(B*
a^2*12i + B*b^2*1i - A*a*b*6i))/(2*b^5))*(B*a^2*12i + B*b^2*1i - A*a*b*6i)*
1i)/(2*b^5) + (((8*tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B^2*b^14 - 2*B^2*a*b^
13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11 + 36*A^2*a^4*b^10 +
 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 28
8*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72*A^2*a^12*b^2 + 21*B
^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B^2*a^5*b^9 + 18*B^2*
a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a
^10*b^4 + 1104*B^2*a^11*b^3 - 1104*B^2*a^12*b^2 - 12*A*B*a*b^13 - 288*A*B*a
^13*b + 24*A*B*a^2*b^12 - 108*A*B*a^3*b^11 + 192*A*B*a^4*b^10 - 72*A*B*a^5*
b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6 - 1650*A*B*a^9*
b^5 - 1128*A*B*a^10*b^4 + 1128*A*B*a^11*b^3 + 288*A*B*a^12*b^2))/(a*b^14 +
b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^
8) - (((4*(4*B*b^21 + 48*A*a^2*b^19 + 72*A*a^3*b^18 - 156*A*a^4*b^17 - 84*A
*a^5*b^16 + 192*A*a^6*b^15 + 48*A*a^7*b^14 - 108*A*a^8*b^13 - 12*A*a^9*b^12
 + 24*A*a^10*b^11 + 28*B*a^2*b^19 - 80*B*a^3*b^18 - 120*B*a^4*b^17 + 276*B*
a^5*b^16 + 164*B*a^6*b^15 - 360*B*a^7*b^14 - 100*B*a^8*b^13 + 212*B*a^9*b^1
2 + 24*B*a^10*b^11 - 48*B*a^11*b^10 - 24*A*a*b^20))/(a*b^18 + b^19 - 3*a^2*
b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) + (4*tan
(c/2 + (d*x)/2)*(B*a^2*12i + B*b^2*1i - A*a*b*6i)*(8*a*b^19 - 8*a^2*b^18 -
32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^
8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/(b^5*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^
3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(B*a^2*12i + B*b^2*
1i - A*a*b*6i))/(2*b^5))*(B*a^2*12i + B*b^2*1i - A*a*b*6i)*1i)/(2*b^5))/((8
*(1728*B^3*a^15 - 864*B^3*a^14*b - 432*A^3*a^4*b^11 - 432*A^3*a^5*b^10 + 14
04*A^3*a^6*b^9 + 756*A^3*a^7*b^8 - 1728*A^3*a^8*b^7 - 486*A^3*a^9*b^6 + 972
*A^3*a^10*b^5 + 108*A^3*a^11*b^4 - 216*A^3*a^12*b^3 + 20*B^3*a^3*b^12 - 20*
B^3*a^4*b^11 + 411*B^3*a^5*b^10 - 11*B^3*a^6*b^9 + 1314*B^3*a^7*b^8 + 2326*

```
B^3*a^8*b^7 - 7829*B^3*a^9*b^6 - 4770*B^3*a^10*b^5 + 11700*B^3*a^11*b^4 + 3
456*B^3*a^12*b^3 - 7344*B^3*a^13*b^2 - 2592*A*B^2*a^14*b - 12*A*B^2*a^2*b^1
3 + 12*A*B^2*a^3*b^12 - 489*A*B^2*a^4*b^11 + 9*A*B^2*a^5*b^10 - 2892*A*B^2*
a^6*b^9 - 3972*A*B^2*a^7*b^8 + 13347*A*B^2*a^8*...
```

3.266 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=280

$$\frac{(Ab-3aB)x}{b^4}-\frac{a(2a^4Ab-5a^2Ab^3+6Ab^5-6a^5B+15a^3b^2B-12ab^4B)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}d}-\frac{(a\cdots}{}$$

```
[Out] (A*b-3*B*a)*x/b^4-a*(2*A*a^4*b-5*A*a^2*b^3+6*A*b^5-6*B*a^5+15*B*a^3*b^2-12*
B*a*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^4
/(a+b)^(5/2)/d-1/2*(A*a*b-3*B*a^2+2*B*b^2)*sin(d*x+c)/b^3/(a^2-b^2)/d+1/2*a
*(A*b-B*a)*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/2*a^2
*(A*a^2*b-4*A*b^3-3*B*a^3+6*B*a*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*cos(
d*x+c))
```

Rubi [A]
time = 0.81, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3068, 3110, 3102, 2814, 2738, 211}

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\left(-3a^2B+aAb+2b^2B\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)}-\frac{a^2\left(-3a^3B+a^2Ab+6ab^2B-4Ab^3\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{a\left(-6a^5B+2a^4Ab+15a^3b^2B-5a^2Ab^3-12ab^4B+6Ab^5\right)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{x(Ab-3aB)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((A*b - 3*a*B)*x)/b^4 - (a*(2*a^4*A*b - 5*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 1
5*a^3*b^2*B - 12*a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) - ((a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c
 + d*x])/(2*b^3*(a^2 - b^2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x]
)/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(a^2*A*b - 4*A*b^3 - 3*
a^3*B + 6*a*b^2*B)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]
))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{\int \cos(c+dx)\left(-2a(Ab-aB)+2b(Ab-\right.}{}$$

$$= \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{a^2(a^2Ab-4Ab^3-3a^3B+}{2b^3\,(a^2-b^2)^2\,d(a+b}$$

$$= -\frac{(aAb-3a^2B+2b^2B)\sin(c+dx)}{2b^3\,(a^2-b^2)\,d} + \frac{a(Ab-aB)\cos^2(c+dx)\sin}{2b\,(a^2-b^2)\,d(a+b\cos(c}$$

$$= \frac{(Ab-3aB)x}{b^4} - \frac{(aAb-3a^2B+2b^2B)\sin(c+dx)}{2b^3\,(a^2-b^2)\,d} + \frac{a(Ab-aB)}{2b\,(a^2-b^2)}$$

$$= \frac{(Ab-3aB)x}{b^4} - \frac{(aAb-3a^2B+2b^2B)\sin(c+dx)}{2b^3\,(a^2-b^2)\,d} + \frac{a(Ab-aB)}{2b\,(a^2-b^2)}$$

$$= \frac{(Ab-3aB)x}{b^4} - \frac{a(2a^4Ab-5a^2Ab^3+6Ab^5-6a^5B+15a^3b^2B-1}{(a-b)^{5/2}b^4(a+b}$$

Mathematica [A]
time = 2.22, size = 232, normalized size = 0.83

$$\frac{2(Ab-3aB)(c+dx) - \frac{2a\left(-2a^4Ab+5a^2Ab^3-6Ab^5+6a^5B-15a^3b^2B+12ab^4B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + 2bB\sin(c+dx) + \frac{a^3b(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2} + \frac{a^2b(-3a^2Ab+6Ab^3+5a^3B-8ab^2B)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*(A*b - 3*a*B)*(c + d*x) - (2*a*(-2*a^4*A*b + 5*a^2*A*b^3 - 6*A*b^5 + 6*a
^5*B - 15*a^3*b^2*B + 12*a*b^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]])/(-a^2 + b^2)^(5/2) + 2*b*B*Sin[c + d*x] + (a^3*b*(A*b - a*B)*S
in[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a^2*b*(-3*a^2*A*b
+ 6*A*b^3 + 5*a^3*B - 8*a*b^2*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*
Cos[c + d*x])))/(2*b^4*d)
```

Maple [A]
time = 0.66, size = 341, normalized size = 1.22

method	result

derivativedivides	$-\frac{2a\left(\frac{\frac{\left(2A\,a^{2}b-Aa\,b^{2}-6A\,b^{3}-4a^{3}B+a^{2}bB+8Ba\,b^{2}\right)ab\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{ba\left(2A\,a^{2}b+Aa\,b^{2}-6A\,b^{3}-4a^{3}B-a^{2}bB+8Ba\,b^{2}\right)\tan\left(\frac{dx}{2}\right.}{2(a+b)(a-b)^{2}}}{\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^{2}}\right.}{b^{4}}$
default	$-\frac{2a\left(\frac{\frac{\left(2A\,a^{2}b-Aa\,b^{2}-6A\,b^{3}-4a^{3}B+a^{2}bB+8Ba\,b^{2}\right)ab\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{ba\left(2A\,a^{2}b+Aa\,b^{2}-6A\,b^{3}-4a^{3}B-a^{2}bB+8Ba\,b^{2}\right)\tan\left(\frac{dx}{2}\right.}{2(a+b)(a-b)^{2}}}{\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^{2}}\right.}{b^{4}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-2*a/b^4*((1/2*(2*A*a^2*b-A*a*b^2-6*A*b^3-4*B*a^3+B*a^2*b+8*B*a*b^2)*a
*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*b*a*(2*A*a^2*b+A*a*b^2-6*
A*b^3-4*B*a^3-B*a^2*b+8*B*a*b^2)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(a*tan(1
/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(2*A*a^4*b-5*A*a^2*b^3+6*
A*b^5-6*B*a^5+15*B*a^3*b^2-12*B*a*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1
/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))+2/b^4*(B*b*tan(1/
2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+(A*b-3*B*a)*arctan(tan(1/2*d*x+1/2*c)
)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(268) = 536$.
time = 0.46, size = 1561, normalized size = 5.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(4*(3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6
 - 3*A*a^2*b^7 - 3*B*a*b^8 + A*b^9)*d*x*cos(d*x + c)^2 + 8*(3*B*a^8*b - A*a
^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^
7 + A*a*b^8)*d*x*cos(d*x + c) + 4*(3*B*a^9 - A*a^8*b - 9*B*a^7*b^2 + 3*A*a^
6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7)*d*x - (6*B*a^8
 - 2*A*a^7*b - 15*B*a^6*b^2 + 5*A*a^5*b^3 + 12*B*a^4*b^4 - 6*A*a^3*b^5 + (6
*B*a^6*b^2 - 2*A*a^5*b^3 - 15*B*a^4*b^4 + 5*A*a^3*b^5 + 12*B*a^2*b^6 - 6*A*
a*b^7)*cos(d*x + c)^2 + 2*(6*B*a^7*b - 2*A*a^6*b^2 - 15*B*a^5*b^3 + 5*A*a^4
*b^4 + 12*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*
b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d
*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + a^2)) - 2*(6*B*a^8*b - 2*A*a^7*b^2 - 17*B*a^6*b^3 + 7*A*a^5*b^4 +
13*B*a^4*b^5 - 5*A*a^3*b^6 - 2*B*a^2*b^7 + 2*(B*a^6*b^3 - 3*B*a^4*b^5 + 3*B
*a^2*b^7 - B*b^9)*cos(d*x + c)^2 + (9*B*a^7*b^2 - 3*A*a^6*b^3 - 25*B*a^5*b^
4 + 9*A*a^4*b^5 + 20*B*a^3*b^6 - 6*A*a^2*b^7 - 4*B*a*b^8)*cos(d*x + c))*sin
(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*
(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c) + (a^8*b^4 - 3*a^
6*b^6 + 3*a^4*b^8 - a^2*b^10)*d), -1/2*(2*(3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^
5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^2*b^7 - 3*B*a*b^8 + A*b^9)*d*x*co
s(d*x + c)^2 + 4*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9*B*a
^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*d*x*cos(d*x + c) + 2*(3*B*a^9
 - A*a^8*b - 9*B*a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^
3*b^6 + A*a^2*b^7)*d*x - (6*B*a^8 - 2*A*a^7*b - 15*B*a^6*b^2 + 5*A*a^5*b^3
+ 12*B*a^4*b^4 - 6*A*a^3*b^5 + (6*B*a^6*b^2 - 2*A*a^5*b^3 - 15*B*a^4*b^4 +
5*A*a^3*b^5 + 12*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x + c)^2 + 2*(6*B*a^7*b - 2*A
*a^6*b^2 - 15*B*a^5*b^3 + 5*A*a^4*b^4 + 12*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x
 + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*
x + c))) - (6*B*a^8*b - 2*A*a^7*b^2 - 17*B*a^6*b^3 + 7*A*a^5*b^4 + 13*B*a^4
*b^5 - 5*A*a^3*b^6 - 2*B*a^2*b^7 + 2*(B*a^6*b^3 - 3*B*a^4*b^5 + 3*B*a^2*b^7
 - B*b^9)*cos(d*x + c)^2 + (9*B*a^7*b^2 - 3*A*a^6*b^3 - 25*B*a^5*b^4 + 9*A*
a^4*b^5 + 20*B*a^3*b^6 - 6*A*a^2*b^7 - 4*B*a*b^8)*cos(d*x + c))*sin(d*x + c
))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^7*b^5
 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c) + (a^8*b^4 - 3*a^6*b^6 +
3*a^4*b^8 - a^2*b^10)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(268) = 536.
time = 0.54, size = 543, normalized size = 1.94

$$-\frac{\frac{(6Ba^6-2Aa^5b-15Ba^4b^2+5Aa^3b^3+12Ba^2b^4-6Aab^5)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^4b^4-2a^2b^6+b^8)\sqrt{a^2-b^2}}-\frac{4Ba^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2Aa^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5Ba^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3Aa^4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7Ba^4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+5Aa^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8Ba^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6Aa^2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4Ba^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2Aa^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5Ba^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3Aa^4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-7Ba^4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5Aa^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8Ba^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6Aa^2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{(a^4b^3-2a^2b^5+b^7)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)^2}+\frac{(3Ba-Ab)(dx+c)}{b^4}-\frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -((6*B*a^6 - 2*A*a^5*b - 15*B*a^4*b^2 + 5*A*a^3*b^3 + 12*B*a^2*b^4 - 6*A*a*
b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2
*b^6 + b^8)*sqrt(a^2 - b^2)) - (4*B*a^6*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^5*b*
tan(1/2*d*x + 1/2*c)^3 - 5*B*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^4*b^2*tan
(1/2*d*x + 1/2*c)^3 - 7*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^3*b^3*tan(
1/2*d*x + 1/2*c)^3 + 8*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^2*b^4*tan(1
/2*d*x + 1/2*c)^3 + 4*B*a^6*tan(1/2*d*x + 1/2*c) - 2*A*a^5*b*tan(1/2*d*x +
1/2*c) + 5*B*a^5*b*tan(1/2*d*x + 1/2*c) - 3*A*a^4*b^2*tan(1/2*d*x + 1/2*c)
- 7*B*a^4*b^2*tan(1/2*d*x + 1/2*c) + 5*A*a^3*b^3*tan(1/2*d*x + 1/2*c) - 8*B
*a^3*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^3
 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 +
a + b)^2) + (3*B*a - A*b)*(d*x + c)/b^4 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/
2*d*x + 1/2*c)^2 + 1)*b^3))/d
```

Mupad [B]
time = 7.66, size = 2500, normalized size = 8.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(6*B*a^5 - 2*B*b^5 + 6*A*a^2*b^3 + A*a^3*b^2 + 4*B*a
^2*b^3 - 12*B*a^3*b^2 - 2*A*a^4*b + 2*B*a*b^4 - 3*B*a^4*b))/((a*b^3 - b^4)*
(a + b)^2) + (tan(c/2 + (d*x)/2)*(6*B*a^5 + 2*B*b^5 + 6*A*a^2*b^3 - A*a^3*b
^2 - 4*B*a^2*b^3 - 12*B*a^3*b^2 - 2*A*a^4*b + 2*B*a*b^4 + 3*B*a^4*b))/((a +
 b)*(b^5 - 2*a*b^4 + a^2*b^3)) + (2*tan(c/2 + (d*x)/2)^3*(6*B*a^6 - 2*B*b^6
 + 5*A*a^3*b^3 + 6*B*a^2*b^4 - 13*B*a^4*b^2 - 2*A*a^5*b))/(b*(a*b^2 - b^3)*
(a + b)^2*(a - b)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a*b + 3*a^2 - b^2)
+ tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - tan(c/2 + (d*x)/2)
^4*(2*a*b - 3*a^2 + b^2))) + (log(tan(c/2 + (d*x)/2) + 1i)*(A*b - 3*B*a)*1i
```

)/(b^4*d) - (log(tan(c/2 + (d*x)/2) - 1i)*(A*b*1i - B*a*3i))/(b^4*d) - (a*a
tan(((a*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^12 + 72*B^2*a^12 - 8*A^2*a*b^11 - 7
2*B^2*a^11*b + 24*A^2*a^2*b^10 + 32*A^2*a^3*b^9 - 52*A^2*a^4*b^8 - 48*A^2*a
^5*b^7 + 57*A^2*a^6*b^6 + 32*A^2*a^7*b^5 - 32*A^2*a^8*b^4 - 8*A^2*a^9*b^3 +
 8*A^2*a^10*b^2 + 36*B^2*a^2*b^10 - 72*B^2*a^3*b^9 + 36*B^2*a^4*b^8 + 288*B
^2*a^5*b^7 - 288*B^2*a^6*b^6 - 432*B^2*a^7*b^5 + 441*B^2*a^8*b^4 + 288*B^2*
a^9*b^3 - 288*B^2*a^10*b^2 - 24*A*B*a*b^11 - 48*A*B*a^11*b + 48*A*B*a^2*b^1
0 - 72*A*B*a^3*b^9 - 192*A*B*a^4*b^8 + 252*A*B*a^5*b^7 + 288*A*B*a^6*b^6 -
318*A*B*a^7*b^5 - 192*A*B*a^8*b^4 + 192*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a*
b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a
^7*b^6) + (a*((8*(4*A*b^18 - 8*A*a^2*b^16 + 34*A*a^3*b^15 + 6*A*a^4*b^14 -
36*A*a^5*b^13 - 4*A*a^6*b^12 + 18*A*a^7*b^11 + 2*A*a^8*b^10 - 4*A*a^9*b^9 +
 24*B*a^2*b^16 + 36*B*a^3*b^15 - 78*B*a^4*b^14 - 42*B*a^5*b^13 + 96*B*a^6*b
^12 + 24*B*a^7*b^11 - 54*B*a^8*b^10 - 6*B*a^9*b^9 + 12*B*a^10*b^8 - 12*A*a*
b^17 - 12*B*a*b^17))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12
+ 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a
 - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b
- 12*B*a*b^4)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b
^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((
b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^1
2 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*
b^6)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B
*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 1
0*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 -
 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4)*1i)/(2*(b^1
4 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)) + (a*((8
*tan(c/2 + (d*x)/2)*(4*A^2*b^12 + 72*B^2*a^12 - 8*A^2*a*b^11 - 72*B^2*a^11*
b + 24*A^2*a^2*b^10 + 32*A^2*a^3*b^9 - 52*A^2*a^4*b^8 - 48*A^2*a^5*b^7 + 57
*A^2*a^6*b^6 + 32*A^2*a^7*b^5 - 32*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 8*A^2*a^10
*b^2 + 36*B^2*a^2*b^10 - 72*B^2*a^3*b^9 + 36*B^2*a^4*b^8 + 288*B^2*a^5*b^7
- 288*B^2*a^6*b^6 - 432*B^2*a^7*b^5 + 441*B^2*a^8*b^4 + 288*B^2*a^9*b^3 - 2
88*B^2*a^10*b^2 - 24*A*B*a*b^11 - 48*A*B*a^11*b + 48*A*B*a^2*b^10 - 72*A*B*
a^3*b^9 - 192*A*B*a^4*b^8 + 252*A*B*a^5*b^7 + 288*A*B*a^6*b^6 - 318*A*B*a^7
*b^5 - 192*A*B*a^8*b^4 + 192*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a*b^12 + b^13
 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (
a*((8*(4*A*b^18 - 8*A*a^2*b^16 + 34*A*a^3*b^15 + 6*A*a^4*b^14 - 36*A*a^5*b^
13 - 4*A*a^6*b^12 + 18*A*a^7*b^11 + 2*A*a^8*b^10 - 4*A*a^9*b^9 + 24*B*a^2*b
^16 + 36*B*a^3*b^15 - 78*B*a^4*b^14 - 42*B*a^5*b^13 + 96*B*a^6*b^12 + 24*B*
a^7*b^11 - 54*B*a^8*b^10 - 6*B*a^9*b^9 + 12*B*a^10*b^8 - 12*A*a*b^17 - 12*B
*a*b^17))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^1
1 - a^6*b^10 - a^7*b^9) + (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1
/2)*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^
4)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^
6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^14 - 5*a^
2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12 + b^13 -

```
3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*(-(a
 + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 +
2*A*a^4*b - 12*B*a*b^4))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 +
 5*a^8*b^6 - a^10*b^4)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 -
5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4)*1i)/(2*(b^14 - 5*a^2*b
^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))/((16*(108*B^3*a^12
 - 12*A^3*a*b^11 - 54*B^3*a^11*b - 24*A^3*a^2*b^10 + 34*A^3*a^3*b^9 + 26*A^
3*a^4*b^8 - 36*A^3*a^5*b^7 - 13*A^3*a^6*b^6 + 18*A^3*a^7*b^5 + 2*A^3*a^8*b^
4 - 4*A^3*a^9*b^3 + 216*B^3*a^4*b^8 + 216*B^3*a^5*b^7 - 702*B^3*a^6*b^6 - 3
78*B^3*a^7*b^5 + 864*B^3*a^8*b^4 + 243*B^3*a^9*b^3 - 486*B^3*a^10*b^2 - 108
*A*B^2*a^11*b - 252*A*B^2*a^3*b^9 - 324*A*B^2*a^4*b^8 + 774*A*B^2*a^5*b^7 +
 486*A*B^2*a^6*b^6 - 900*A*B^2*a^7*b^5 - 279*A*B^2*a^8*b^4 + 486*A*B^2*a^9*
b^3 + 54*A*B^2*a^10*b^2 + 96*A^2*B*a^2*b^10 + 1...
```

3.267 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=211

$$\frac{Bx}{b^3}+\frac{(a^2Ab^3+2Ab^5-2a^5B+5a^3b^2B-6ab^4B)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d}-\frac{a^2(Ab-aB)\sin(c+}{2b^2\,(a^2-b^2)\,d(a+b\cos(c}$$

```
[Out] B*x/b^3+(A*a^2*b^3+2*A*b^5-2*B*a^5+5*B*a^3*b^2-6*B*a*b^4)*arctan((a-b)^(1/2
)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^3/(a+b)^(5/2)/d-1/2*a^2*(A*
b-B*a)*sin(d*x+c)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*a*(A*a^2*b-4*A*b^3
-3*B*a^3+6*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.38, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3067, 3100, 2814, 2738, 211}

$$-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d\,(a^2-b^2)\,(a+b\cos(c+dx))^2}+\frac{a(-3a^3B+a^2Ab+6ab^2B-4Ab^3)\sin(c+dx)}{2b^2d\,(a^2-b^2)^2\,(a+b\cos(c+dx))}+\frac{(-2a^5B+5a^3b^2B+a^2Ab^3-6ab^4B+2Ab^5)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{Bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (B*x)/b^3 + ((a^2*A*b^3 + 2*A*b^5 - 2*a^5*B + 5*a^3*b^2*B - 6*a*b^4*B)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^
(5/2)*d) - (a^2*(A*b - a*B)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) + (a*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sin[c + d*x])/(2
*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2(a^2-b^2)\,d(a+b\cos(c+dx))^2} + \frac{\int \frac{2ab(Ab-aB)+(a^2-2b^2)(Ab-aB}{(a+b}}{2b^2}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2(a^2-b^2)\,d(a+b\cos(c+dx))^2} + \frac{a(a^2Ab-4Ab^3-3a^3B+}{2b^2(a^2-b^2)^2\,d(a+}$$

$$= \frac{Bx}{b^3} - \frac{a^2(Ab-aB)\sin(c+dx)}{2b^2(a^2-b^2)\,d(a+b\cos(c+dx))^2} + \frac{a(a^2Ab-4Ab^3-3a^3}{2b^2(a^2-b^2)^2\,d(}$$

$$= \frac{Bx}{b^3} - \frac{a^2(Ab-aB)\sin(c+dx)}{2b^2(a^2-b^2)\,d(a+b\cos(c+dx))^2} + \frac{a(a^2Ab-4Ab^3-3a^3}{2b^2(a^2-b^2)^2\,d(}$$

$$= \frac{Bx}{b^3} + \frac{(a^2Ab^3+2Ab^5-2a^5B+5a^3b^2B-6ab^4B)\tan^{-1}\left(\frac{\sqrt{a-}}{\sqrt{}}\right.}{(a-b)^{5/2}b^3(a+b)^{5/2}d}$$

Mathematica [A]
time = 1.45, size = 204, normalized size = 0.97

$$\frac{2B(c+dx)+\frac{2(-a^2Ab^3-2Ab^5+2a^5B-5a^3b^2B+6ab^4B)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}+\frac{a^2b(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}+\frac{ab(a^2Ab-4Ab^3-3a^3B+6ab^2B)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*B*(c + d*x) + (2*(-(a^2*A*b^3) - 2*A*b^5 + 2*a^5*B - 5*a^3*b^2*B + 6*a*b
^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5
/2) + (a^2*b*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d
*x])^2) + (a*b*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sin[c + d*x])/((a
- b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*b^3*d)
```

Maple [A]
time = 0.55, size = 282, normalized size = 1.34 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2/b^3*((-1/2*(A*a*b^2+4*A*b^3+2*B*a^3-B*a^2*b-6*B*a*b^2)*a*b/(a-b)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*b*a*(A*a*b^2-4*A*b^3-2*B*a^3-B*a^2*b+
6*B*a*b^2)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(
1/2*d*x+1/2*c)^2+a+b)^2+1/2*(A*a^2*b^3+2*A*b^5-2*B*a^5+5*B*a^3*b^2-6*B*a*b^
4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/
((a-b)*(a+b))^(1/2)))+2*B/b^3*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(201) = 402$.
time = 0.42, size = 1152, normalized size = 5.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(4*(B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8)*d*x*cos(d*x + c)^2
+ 8*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*d*x*cos(d*x + c) + 4*(B
*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*d*x + (2*B*a^7 - 5*B*a^5*b^2
- A*a^4*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5 + (2*B*a^5*b^2 - 5*B*a^3*b^4 - A*a^
2*b^5 + 6*B*a*b^6 - 2*A*b^7)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 5*B*a^4*b^3 -
A*a^3*b^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*
a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos
(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(
d*x + c) + a^2)) - 2*(2*B*a^7*b - 7*B*a^5*b^3 + 3*A*a^4*b^4 + 5*B*a^3*b^5 -
 3*A*a^2*b^6 + (3*B*a^6*b^2 - A*a^5*b^3 - 9*B*a^4*b^4 + 5*A*a^3*b^5 + 6*B*a
^2*b^6 - 4*A*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^6*b^5 - 3*a^4*b^7 + 3*a
^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^
10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d), 1/2*(2
*(B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8)*d*x*cos(d*x + c)^2 + 4*(B*
a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*d*x*cos(d*x + c) + 2*(B*a^8 -
3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*d*x - (2*B*a^7 - 5*B*a^5*b^2 - A*a^4
*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5 + (2*B*a^5*b^2 - 5*B*a^3*b^4 - A*a^2*b^5 +
 6*B*a*b^6 - 2*A*b^7)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 5*B*a^4*b^3 - A*a^3*b
^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(
d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*B*a^7*b - 7*B*a^5*b^3 +
3*A*a^4*b^4 + 5*B*a^3*b^5 - 3*A*a^2*b^6 + (3*B*a^6*b^2 - A*a^5*b^3 - 9*B*a^
4*b^4 + 5*A*a^3*b^5 + 6*B*a^2*b^6 - 4*A*a*b^7)*cos(d*x + c))*sin(d*x + c))/
((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3
*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^
4*b^7 - a^2*b^9)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(201) = 402.
time = 0.48, size = 455, normalized size = 2.16

$$-\frac{\frac{\left(2 \, B a^{5} - 5 \, B a^{3} b^{2} - A a^{2} b^{3} + 6 \, B a b^{4} - 2 \, A b^{5}\right)\left(\pi \left\lfloor \frac{d x + c}{2 \, \pi} + \frac{1}{2} \right\rfloor \mathrm{sgn}\left(2 \, a - 2 \, b\right) + \arctan\left(\frac{a \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)}{\sqrt{a^{2} - b^{2}}}\right)\right)}{\left(a^{4} b^{3} - 2 \, a^{2} b^{5} + b^{7}\right) \sqrt{a^{2} - b^{2}}} - \frac{(d x + c) B}{b^{3}} + \frac{2 \, B a^{5} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 3 \, B a^{4} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} + A a^{3} b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 5 \, B a^{3} b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} + 3 \, A a^{2} b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} + 6 \, B a^{2} b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 4 \, A a b^{4} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} + 2 \, B a^{5} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 3 \, B a^{4} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - A a^{3} b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 5 \, B a^{3} b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 3 \, A a^{2} b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 6 \, B a^{2} b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 4 \, A a b^{4} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)}{\left(a^{4} b^{2} - 2 \, a^{2} b^{4} + b^{6}\right) \left(a \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + a + b\right)^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -((2*B*a^5 - 5*B*a^3*b^2 - A*a^2*b^3 + 6*B*a*b^4 - 2*A*b^5)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(
1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 -
 b^2)) - (d*x + c)*B/b^3 + (2*B*a^5*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^4*b*tan(
1/2*d*x + 1/2*c)^3 + A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^3*b^2*tan(1/2
*d*x + 1/2*c)^3 + 3*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*b^3*tan(1/2*
d*x + 1/2*c)^3 - 4*A*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^5*tan(1/2*d*x + 1
/2*c) + 3*B*a^4*b*tan(1/2*d*x + 1/2*c) - A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5
*B*a^3*b^2*tan(1/2*d*x + 1/2*c) + 3*A*a^2*b^3*tan(1/2*d*x + 1/2*c) - 6*B*a^
2*b^3*tan(1/2*d*x + 1/2*c) + 4*A*a*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*
a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b
)^2))/d
```

Mupad [B]
time = 9.95, size = 2500, normalized size = 11.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] (2*B*atan(-((B*((B*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^12 +
 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11 -
36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*
A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b
^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (B*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a
^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^
9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)*8i)/(b^3*(a*b^10 + b^11 - 3*a^2*b^
9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*1i)/b^3 + (8*t
an(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 - 8*B
^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 -
52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*
a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(a
*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^
7*b^4)))/b^3 - (B*((B*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^1
2 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11
 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 -
 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^
4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (B*tan(c/2 + (d*x)/2)*(8*a*b^15 -
8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7
*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)*8i)/(b^3*(a*b^10 + b^11 - 3*a^2
```

*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*1i)/b^3 - (
8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 -
8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7
 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B
^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3))
/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 -
 a^7*b^4)))/b^3)/((B*((B*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*
b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b
^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^
6 - 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3
*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (B*tan(c/2 + (d*x)/2)*(8*a*b^15
 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*
a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)*8i)/(b^3*(a*b^10 + b^11 - 3*
a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*1i)/b^3
 + (8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9
 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*
b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 3
2*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^
3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^
5 - a^7*b^4))*1i)/b^3 - (16*(4*B^3*a^9 - 4*A*B^2*b^9 + 4*A^2*B*b^9 + 12*B^3
*a*b^8 - 2*B^3*a^8*b + 24*B^3*a^2*b^7 - 34*B^3*a^3*b^6 - 26*B^3*a^4*b^5 + 3
6*B^3*a^5*b^4 + 13*B^3*a^6*b^3 - 18*B^3*a^7*b^2 - 20*A*B^2*a*b^8 + 6*A*B^2*
a^2*b^7 + 2*A*B^2*a^3*b^6 + 2*A*B^2*a^5*b^4 - 2*A*B^2*a^6*b^3 - 2*A*B^2*a^7
*b^2 + 4*A^2*B*a^2*b^7 + A^2*B*a^4*b^5))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^
3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (B*((B*((8*(4*A*b^15
 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*
a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*
B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12
 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^
6) + (B*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^
12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^
10*b^6)*8i)/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5
*b^6 - a^6*b^5 - a^7*b^4)))*1i)/b^3 - (8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8
*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^
4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 +
 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^
3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*
b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3)))/(b^3*d) - ((ta
n(c/2 + (d*x)/2)^3*(2*B*a^4 + A*a^2*b^2 - 6*B*a^2*b^2 + 4*A*a*b^3 - B*a^3*b
))/((a*b^2 - b^3)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*B*a^4 - A*a^2*b^2 - 6
*B*a^2*b^2 + 4*A*a*b^3 + B*a^3*b))/((a + b)*(b^4 - 2*a*b^3 + a^2*b^2)))/(d*
(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 -
 2*a*b + b^2) + a^2 + b^2)) + (atan((((-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(
c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 - 8*B^2*
a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*

```
B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8
*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5...
```

3.268 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$-\frac{(3aAb-a^2B-2b^2B)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}+\frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{(a^2Ab+2Ab^3}{2b\left(a^2-b\right.}$$

```
[Out] -(3*A*a*b-B*a^2-2*B*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))
/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*co
s(d*x+c))^2+1/2*(A*a^2*b+2*A*b^3+B*a^3-4*B*a*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/
d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.20, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3047, 3100, 2833, 12, 2738, 211}

$$-\frac{(a^2(-B)+3aAb-2b^2B)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{(a^3B+a^2Ab-4ab^2B+2Ab^3)\sin(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -(((3*a*A*b - a^2*B - 2*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) + (a*(A*b - a*B)*Sin[c + d*x])/(2*
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((a^2*A*b + 2*A*b^3 + a^3*B - 4*a
*b^2*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{2b(Ab-aB)-\left(aAb+a^2B-2b^2B\right)\cos(c+}{(a+b\cos(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)}{2b\left(a^2-b^2\right)^2d(a+b\cos(c}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)}{2b\left(a^2-b^2\right)^2d(a+b\cos(c}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)}{2b\left(a^2-b^2\right)^2d(a+b\cos(c}$$

$$= -\frac{\left(3aAb-a^2B-2b^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a(Ab}{2b\left(a^2-b^2\right.}$$

Mathematica [A]
time = 0.90, size = 172, normalized size = 0.96

$$\frac{-\frac{2(-3aAb+a^2B+2b^2B)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}+\frac{a(Ab-aB)\sin(c+dx)}{(a-b)b(a+b)(a+b\cos(c+dx))^2}+\frac{(a^2Ab+2Ab^3+a^3B-4ab^2B)\sin(c+dx)}{(a-b)^2b(a+b)^2(a+b\cos(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(-3*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]])/(-a^2 + b^2)^(5/2) + (a*(A*b - a*B)*Sin[c + d*x])/((a - b)*b*(
a + b)*(a + b*Cos[c + d*x])^2) + ((a^2*A*b + 2*A*b^3 + a^3*B - 4*a*b^2*B)*S
in[c + d*x])/((a - b)^2*b*(a + b)^2*(a + b*Cos[c + d*x])))/(2*d)
```

Maple [A]
time = 0.41, size = 234, normalized size = 1.30

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(2a^2A+Aab+2A\,b^2-B\,a^2-4Bab\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(2a^2A-Aab+2A\,b^2+B\,a^2-4Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}-\frac{\left(3Aab-B\,a^2-2B\,b\right.}{\left(a^4-2a^2b^2\right.}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(2a^2A+Aab+2A\,b^2-B\,a^2-4Bab\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(2a^2A-Aab+2A\,b^2+B\,a^2-4Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}-\frac{\left(3Aab-B\,a^2-2B\,b\right.}{\left(a^4-2a^2b^2\right.}}{d}$
risch	$\frac{i(3Aa\,b^4\mathrm{e}^{3i(dx+c)}+2B\,a^4b\,\mathrm{e}^{3i(dx+c)}-5B\,a^2b^3\mathrm{e}^{3i(dx+c)}+2A\,a^4b\,\mathrm{e}^{2i(dx+c)}+5A\,a^2b^3\mathrm{e}^{2i(dx+c)}+2A\,b^5\mathrm{e}^{2i(dx+c)}+2B\,a^5\mathrm{e}^2}{b^2(a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(2*A*a^2+A*a*b+2*A*b^2-B*a^2-4*B*a*b)/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3-1/2*(2*A*a^2-A*a*b+2*A*b^2+B*a^2-4*B*a*b)/(a+b)/(a^2-2*
a*b+b^2)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2
+a+b)^2-(3*A*a*b-B*a^2-2*B*b^2)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arc
tan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(165) = 330$.
time = 0.41, size = 740, normalized size = 4.11

$$\left[-\frac{\left(Ba^4-3\,Aa^3b+2\,Ba^2b^2+\left(Ba^2b^2-3\,Aab^3+2\,Bb^4\right)\cos\left(dx+c\right)^2+2\left(Ba^3b-3\,Aa^2b^2+2\,Bab^3\right)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2+2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-2\left(2\,Aa^5-3\,Ba^4b-Aa^3b^2+3\,Ba^2b^3-Aab^4+\left(Ba^5+Aa^4b-5\,Ba^3b^2+Aa^2b^3+4\,Bab^4-2\,Ab^5\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4\left(\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)d\cos\left(dx+c\right)^2+2\left(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7\right)d\cos\left(dx+c\right)+\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)d\right)},\ \frac{\left(Ba^4-3\,Aa^3b+2\,Ba^2b^2+\left(Ba^2b^2-3\,Aab^3+2\,Bb^4\right)\cos\left(dx+c\right)^2+2\left(Ba^3b-3\,Aa^2b^2+2\,Bab^3\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)+\left(2\,Aa^5-3\,Ba^4b-Aa^3b^2+3\,Ba^2b^3-Aab^4+\left(Ba^5+Aa^4b-5\,Ba^3b^2+Aa^2b^3+4\,Bab^4-2\,Ab^5\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2\left(\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)d\cos\left(dx+c\right)^2+2\left(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7\right)d\cos\left(dx+c\right)+\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] [-1/4*((B*a^4 - 3*A*a^3*b + 2*B*a^2*b^2 + (B*a^2*b^2 - 3*A*a*b^3 + 2*B*b^4)
*cos(d*x + c)^2 + 2*(B*a^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqrt(
-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*A*a^5 - 3*B*a^4*b - A*a^3*b^2 +
3*B*a^2*b^3 - A*a*b^4 + (B*a^5 + A*a^4*b - 5*B*a^3*b^2 + A*a^2*b^3 + 4*B*a*
b^4 - 2*A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos
(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), 1/2*((B*a^4 - 3*A*a
^3*b + 2*B*a^2*b^2 + (B*a^2*b^2 - 3*A*a*b^3 + 2*B*b^4)*cos(d*x + c)^2 + 2*(
B*a^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a
*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (2*A*a^5 - 3*B*a^4*b -
 A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 + (B*a^5 + A*a^4*b - 5*B*a^3*b^2 + A*a^2
*b^3 + 4*B*a*b^4 - 2*A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(165) = 330.
time = 0.48, size = 391, normalized size = 2.17

$$\frac{\frac{(Ba^2-3Aab+2Bb^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{2Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{(a^4-2a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] ((B*a^2 - 3*A*a*b + 2*B*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*
b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^
2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (2*A*a^3*tan(1/2*d*x + 1/2
*c)^3 - B*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*B
*a^2*b*tan(1/2*d*x + 1/2*c)^3 + A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*B*a*b^2*
tan(1/2*d*x + 1/2*c)^3 - 2*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*A*a^3*tan(1/2*d
*x + 1/2*c) + B*a^3*tan(1/2*d*x + 1/2*c) + A*a^2*b*tan(1/2*d*x + 1/2*c) - 3
*B*a^2*b*tan(1/2*d*x + 1/2*c) + A*a*b^2*tan(1/2*d*x + 1/2*c) - 4*B*a*b^2*ta
n(1/2*d*x + 1/2*c) + 2*A*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

Mupad [B]
time = 3.74, size = 248, normalized size = 1.38

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2Aa^2+2Ab^2-Ba^2+Aab-4Bab)}{(a+b)^2(a-b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2Aa^2+2Ab^2+Ba^2-Aab-4Bab)}{(a+b)(a^2-2ab+b^2)}}{d\left(2ab+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2a^2-2b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(a^2-2ab+b^2)+a^2+b^2\right)}+\frac{\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a-2b)(a^2-2ab+b^2)}{2\sqrt{a+b}\,(a-b)^{5/2}}\right)(Ba^2-3Aab+2Bb^2)}{d\,(a+b)^{5/2}(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(2*A*a^2 + 2*A*b^2 - B*a^2 + A*a*b - 4*B*a*b))/((a +
 b)^2*(a - b)) + (tan(c/2 + (d*x)/2)*(2*A*a^2 + 2*A*b^2 + B*a^2 - A*a*b - 4
*B*a*b))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2
*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (a
tan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2)*(
a - b)^(5/2)))*(B*a^2 + 2*B*b^2 - 3*A*a*b))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.269 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=164

$$\frac{(2a^2A+Ab^2-3abB)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{(Ab-aB)\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}-\frac{(3aAb-a^2B-2b^2}{2\left(a^2-b^2\right)^2d(a+}$$

```
[Out] (2*A*a^2+A*b^2-3*B*a*b)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/
(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x
+c))^2-1/2*(3*A*a*b-B*a^2-2*B*b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c)
)
```

Rubi [A]
time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2833, 12, 2738, 211}

$$\frac{(2a^2A-3abB+Ab^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{(Ab-aB)\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
 b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((A*b - a*B)*Sin[c + d*x])/(2*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*a*A*b - a^2*B - 2*b^2*B)*Sin[c + d*x
])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{(Ab-aB)\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{\int \frac{-2(aA-bB)+(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2(a^2-b^2)}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{(3aAb-a^2B-2b^2B)\sin(c+dx)}{2(a^2-b^2)^2\,d(a+b\cos(c+dx))} + \frac{\int 2a \cdots}{\cdots}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{(3aAb-a^2B-2b^2B)\sin(c+dx)}{2(a^2-b^2)^2\,d(a+b\cos(c+dx))} + \frac{(2a \cdots}{\cdots}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{(3aAb-a^2B-2b^2B)\sin(c+dx)}{2(a^2-b^2)^2\,d(a+b\cos(c+dx))} + \frac{(2a \cdots}{\cdots}$$

$$= \frac{(2a^2A+Ab^2-3abB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{(Ab-aB)\sin(c+ \cdots}{2(a^2-b^2)\,d(a+b\cos(\cdots}$$

Mathematica [A]
time = 0.70, size = 157, normalized size = 0.96

$$\frac{-\frac{2(2a^2A+Ab^2-3abB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2} + \frac{(-3aAb+a^2B+2b^2B)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a
^2 + b^2]])/(-a^2 + b^2)^(5/2) + ((-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a
+ b)*(a + b*Cos[c + d*x])^2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sin[c + d*x])/
((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*d)
```

Maple [A]
time = 0.36, size = 232, normalized size = 1.41

method	result
derivativedivides	$\frac{\frac{-\frac{(4Aab+A\,b^2-2B\,a^2-Bab-2B\,b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^2+2ab+b^2)}-\frac{(4Aab-A\,b^2-2B\,a^2+Bab-2B\,b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^2-2ab+b^2)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{(2a^2A+A\,b^2-3Bab)\arctan}{(a^4-2a^2b^2+b^4)\sqrt{}}}{d}$
default	$\frac{\frac{-\frac{(4Aab+A\,b^2-2B\,a^2-Bab-2B\,b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^2+2ab+b^2)}-\frac{(4Aab-A\,b^2-2B\,a^2+Bab-2B\,b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^2-2ab+b^2)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{(2a^2A+A\,b^2-3Bab)\arctan}{(a^4-2a^2b^2+b^4)\sqrt{}}}{d}$
risch	$\frac{i(-2A\,a^2b^2e^{3i(dx+c)}-A\,b^4e^{3i(dx+c)}+3Ba\,b^3e^{3i(dx+c)}-6A\,a^3b\,e^{2i(dx+c)}-3Aa\,b^3e^{2i(dx+c)}+2B\,a^4e^{2i(dx+c)}+5B\,a^2b^2e^{2i(}}{b(a^2-b^2)^2d(b\,e^{2i(dx+c)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3-1/2*(4*A*a*b-A*b^2-2*B*a^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2*a
*b+b^2)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2+
a+b)^2+(2*A*a^2+A*b^2-3*B*a*b)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arct
an(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(150) = 300$.
time = 0.40, size = 742, normalized size = 4.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*((2*A*a^4 - 3*B*a^3*b + A*a^2*b^2 + (2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)
*cos(d*x + c)^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(
-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*B*a^5 - 4*A*a^4*b - B*a^3*b^2 +
5*A*a^2*b^3 - B*a*b^4 - A*b^5 + (B*a^4*b - 3*A*a^3*b^2 + B*a^2*b^3 + 3*A*a*
b^4 - 2*B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos
(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), 1/2*((2*A*a^4 - 3*B
*a^3*b + A*a^2*b^2 + (2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 2*(
2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a
*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (2*B*a^5 - 4*A*a^4*b -
 B*a^3*b^2 + 5*A*a^2*b^3 - B*a*b^4 - A*b^5 + (B*a^4*b - 3*A*a^3*b^2 + B*a^2
*b^3 + 3*A*a*b^4 - 2*B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(150) = 300.
time = 0.46, size = 390, normalized size = 2.38

$$\frac{\frac{(2Aa^2-3Bab+Ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{2Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2Bb^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2Bb^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{(a^4-2a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*A*a^2 - 3*B*a*b + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*
b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^
2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (2*B*a^3*tan(1/2*d*x + 1/2
*c)^3 - 4*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 +
 3*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + A*b^3*
tan(1/2*d*x + 1/2*c)^3 - 2*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d
*x + 1/2*c) - 4*A*a^2*b*tan(1/2*d*x + 1/2*c) + B*a^2*b*tan(1/2*d*x + 1/2*c)
 - 3*A*a*b^2*tan(1/2*d*x + 1/2*c) + B*a*b^2*tan(1/2*d*x + 1/2*c) + A*b^3*ta
n(1/2*d*x + 1/2*c) + 2*B*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

Mupad [B]
time = 3.54, size = 248, normalized size = 1.51

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,B\,a^2-A\,b^2+2\,B\,b^2-4\,A\,a\,b+B\,a\,b\right)}{{\left(a+b\right)}^2\,\left(a-b\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A\,b^2+2\,B\,a^2+2\,B\,b^2-4\,A\,a\,b-B\,a\,b\right)}{\left(a+b\right)\,\left(a^2-2\,a\,b+b^2\right)}}{d\,\left(2\,a\,b+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-2\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^2-2\,a\,b+b^2\right)+a^2+b^2\right)}+\frac{\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^2-2\,a\,b+b^2\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{5/2}}\right)\,\left(2\,A\,a^2-3\,B\,a\,b+A\,b^2\right)}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(2*B*a^2 - A*b^2 + 2*B*b^2 - 4*A*a*b + B*a*b))/((a +
 b)^2*(a - b)) + (tan(c/2 + (d*x)/2)*(A*b^2 + 2*B*a^2 + 2*B*b^2 - 4*A*a*b -
 B*a*b))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2
*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (a
tan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2)*(
a - b)^(5/2)))*(2*A*a^2 + A*b^2 - 3*B*a*b))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.270 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=214

$$-\frac{(6a^4Ab-5a^2Ab^3+2Ab^5-2a^5B-a^3b^2B)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{}{2a\,(a}$$

```
[Out] -(6*A*a^4*b-5*A*a^2*b^3+2*A*b^5-2*B*a^5-B*a^3*b^2)*arctan((a-b)^(1/2)*tan(1
/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+A*arctanh(sin(d*x+
c))/a^3/d+1/2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*b
*(5*A*a^2*b-2*A*b^3-3*B*a^3)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.48, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3079, 3134, 3080, 3855, 2738, 211}

$$\frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\,(a^2-b^2)\,(a+b\cos(c+dx))^2}+\frac{b(-3a^3B+5a^2Ab-2Ab^3)\sin(c+dx)}{2a^2d\,(a^2-b^2)^2\,(a+b\cos(c+dx))}-\frac{(-2a^5B+6a^4Ab-a^3b^2B-5a^2Ab^3+2Ab^5)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -(((6*a^4*A*b - 5*a^2*A*b^3 + 2*A*b^5 - 2*a^5*B - a^3*b^2*B)*ArcTan[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d))
+ (A*ArcTanh[Sin[c + d*x]])/(a^3*d) + (b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^
2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b*(5*a^2*A*b - 2*A*b^3 - 3*a^3*B)*Sin
[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
```

```
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{(2A(a^2-b^2)-2a(Ab-aB)\cos(c+dx)}{(a+b\cos(}}{2a\,(a^2}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b(5a^2Ab-2Ab^3-3a^3B)\sin(}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b(5a^2Ab-2Ab^3-3a^3B)\sin(}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b(5}{2a}$$

$$= -\frac{(6a^4Ab-5a^2Ab^3+2Ab^5-2a^5B-a^3b^2B)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(}{\sqrt{a+b}}\right.}{a^3(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A]
time = 1.39, size = 269, normalized size = 1.26

$$\frac{\cos(c+dx)(B+A\sec(c+dx))\left(-\frac{2(-6a^4Ab+5a^2Ab^3-2Ab^5+2a^5B+a^3b^2B)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}-2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{a^2b(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}+\frac{ab(5a^2Ab-2Ab^3-3a^3B)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}\right)}{2a^3d(A+B\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*((-2*(-6*a^4*A*b + 5*a^2*A*b^3 - 2*A*b^5
 + 2*a^5*B + a^3*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])/(-a^2 + b^2)^(5/2) - 2*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*A*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^2*b*(A*b - a*B)*Sin[c + d*x])
/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a*b*(5*a^2*A*b - 2*A*b^3 - 3*a
^3*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x]))))/(2*a^3*d*(
A + B*Cos[c + d*x]))
```

Maple [A]
time = 0.99, size = 302, normalized size = 1.41 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A/a^3*ln(tan(1/2*d*x+1/2*c)+1)-2/a^3*((-1/2*(6*A*a^2*b+A*a*b^2-2*A*b^3
-4*B*a^3-B*a^2*b)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*b*a*(6
*A*a^2*b-A*a*b^2-2*A*b^3-4*B*a^3+B*a^2*b)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))
/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(6*A*a^4*b-5*A*a
```

```
^2*b^3+2*A*b^5-2*B*a^5-B*a^3*b^2)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*a
rctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))-A/a^3*ln(tan(1/2*d*x+1
/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(199) = 398$.
time = 12.44, size = 1400, normalized size = 6.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] [1/4*((2*B*a^7 - 6*A*a^6*b + B*a^5*b^2 + 5*A*a^4*b^3 - 2*A*a^2*b^5 + (2*B*a
^5*b^2 - 6*A*a^4*b^3 + B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7)*cos(d*x + c)^2 +
2*(2*B*a^6*b - 6*A*a^5*b^2 + B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*x +
 c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^
2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(A*a^8 - 3*A*a^6*b^2 + 3*
A*a^4*b^4 - A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*cos
(d*x + c)^2 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*cos(d*x + c
))*log(sin(d*x + c) + 1) - 2*(A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6
 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*cos(d*x + c)^2 + 2*(A*a^
7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*cos(d*x + c))*log(-sin(d*x + c)
+ 1) - 2*(4*B*a^7*b - 6*A*a^6*b^2 - 5*B*a^5*b^3 + 9*A*a^4*b^4 + B*a^3*b^5 -
 3*A*a^2*b^6 + (3*B*a^6*b^2 - 5*A*a^5*b^3 - 3*B*a^4*b^4 + 7*A*a^3*b^5 - 2*A
*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3
*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos
(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d), 1/2*((2*B*a^7 - 6*
A*a^6*b + B*a^5*b^2 + 5*A*a^4*b^3 - 2*A*a^2*b^5 + (2*B*a^5*b^2 - 6*A*a^4*b^
3 + B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 6*A*
```

```
a^5*b^2 + B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2
)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (A*a^8 - 3
*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b
^6 - A*b^8)*cos(d*x + c)^2 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b
^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - (A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^
4 - A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*cos(d*x + c
)^2 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*cos(d*x + c))*log(-
sin(d*x + c) + 1) - (4*B*a^7*b - 6*A*a^6*b^2 - 5*B*a^5*b^3 + 9*A*a^4*b^4 +
B*a^3*b^5 - 3*A*a^2*b^6 + (3*B*a^6*b^2 - 5*A*a^5*b^3 - 3*B*a^4*b^4 + 7*A*a^
3*b^5 - 2*A*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3*a^
5*b^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4
*b^7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(199) = 398.
time = 0.49, size = 481, normalized size = 2.25

$$\frac{\frac{(2Ba^5 - 6Aa^4b + Ba^3b^2 + 5Aa^2b^3 - 2Ab^5)\left(\pi\left\lfloor\frac{dx+c}{2\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(2a - 2b) + \arctan\left(\frac{a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{a^2 - b^2}}\right)\right)}{(a^7 - 2a^5b^2 + a^3b^4)\sqrt{a^2 - b^2}} + \frac{A\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right)}{a^3} - \frac{A\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)}{a^3} - \frac{4Ba^4b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 6Aa^3b^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 3Ba^3b^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 5Aa^2b^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - Ba^2b^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 3Aab^4\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 2Ab^5\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 4Ba^4b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 6Aa^3b^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 3Ba^3b^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 5Aa^2b^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - Ba^2b^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 3Aab^4\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 2Ab^5\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{(a^6 - 2a^4b^2 + a^2b^4)\left(a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a + b\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] ((2*B*a^5 - 6*A*a^4*b + B*a^3*b^2 + 5*A*a^2*b^3 - 2*A*b^5)*(pi*floor(1/2*(d
*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 -
b^2)) + A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - A*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^3 - (4*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^3*b^2*tan(1/2*
d*x + 1/2*c)^3 - 3*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^2*b^3*tan(1/2*d
*x + 1/2*c)^3 - B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 3*A*a*b^4*tan(1/2*d*x +
1/2*c)^3 - 2*A*b^5*tan(1/2*d*x + 1/2*c)^3 + 4*B*a^4*b*tan(1/2*d*x + 1/2*c)
- 6*A*a^3*b^2*tan(1/2*d*x + 1/2*c) + 3*B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*A
*a^2*b^3*tan(1/2*d*x + 1/2*c) - B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*A*a*b^4*
tan(1/2*d*x + 1/2*c) + 2*A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^
2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

Mupad [B]
time = 9.63, size = 2500, normalized size = 11.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(2*A*b^4 - 6*A*a^2*b^2 + B*a^2*b^2 - A*a*b^3 + 4*B*a
^3*b))/((a^2*b - a^3)*(a + b)^2) - (tan(c/2 + (d*x)/2)*(2*A*b^4 - 6*A*a^2*b
^2 - B*a^2*b^2 + A*a*b^3 + 4*B*a^3*b))/((a + b)*(a^4 - 2*a^3*b + a^2*b^2)))
/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a
^2 - 2*a*b + b^2) + a^2 + b^2)) - (A*atan(((A*((8*tan(c/2 + (d*x)/2)*(4*A^2
*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^
8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*
A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b -
 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3))/(a^10*b + a^11 - a^4*b^7 -
 a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) + (A*((8*(4*A*a^1
5 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*
A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3 - 8*A*a^13*b^2 - 2*B*a^8*b^7 + 2*
B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 - 12*A*a^14*b - 4*B*a^14*b))/(a^12*
b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*
b^2) + (8*A*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*
b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 -
8*a^14*b^2))/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^
4 - 3*a^8*b^3 - 3*a^9*b^2))))/a^3)*1i)/a^3 + (A*((8*tan(c/2 + (d*x)/2)*(4*A
^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*
b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 3
2*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b
 - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3))/(a^10*b + a^11 - a^4*b^7
 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) - (A*((8*(4*A*a
^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9*b^6 - 3
6*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3 - 8*A*a^13*b^2 - 2*B*a^8*b^7 +
2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 - 12*A*a^14*b - 4*B*a^14*b))/(a^1
2*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^1
1*b^2) - (8*A*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^
8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3
- 8*a^14*b^2))/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*
b^4 - 3*a^8*b^3 - 3*a^9*b^2))))/a^3)*1i)/a^3)/((16*(4*A^3*b^9 + 4*A*B^2*a^9
 - 4*A^2*B*a^9 - 2*A^3*a*b^8 + 12*A^3*a^8*b - 18*A^3*a^2*b^7 + 13*A^3*a^3*b
^6 + 36*A^3*a^4*b^5 - 26*A^3*a^5*b^4 - 34*A^3*a^6*b^3 + 24*A^3*a^7*b^2 - 20
*A^2*B*a^8*b + A*B^2*a^5*b^4 + 4*A*B^2*a^7*b^2 - 2*A^2*B*a^2*b^7 - 2*A^2*B*
a^3*b^6 + 2*A^2*B*a^4*b^5 + 2*A^2*B*a^6*b^3 + 6*A^2*B*a^7*b^2))/(a^12*b + a
^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)
+ (A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a
```

```
*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*
A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^
4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^
7*b^3))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*
b^3 - 3*a^9*b^2) + (A*((8*(4*A*a^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8
+ 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3
 - 8*A*a^13*b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 -
 12*A*a^14*b - 4*B*a^14*b))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5
+ 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (8*A*tan(c/2 + (d*x)/2)*(8*a^15*b
- 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*
b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/(a^3*(a^10*b + a^11 - a^4*b^
7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))))/a^3))/a^3 -
 (A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*
b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A
^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4
 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7
*b^3))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b
^3 - 3*a^9*b^2) - (A*((8*(4*A*a^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 +
 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3
- 8*A*a^13*b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 -
12*A*a^14*b - 4*B*a^14*b))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 +
 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (8*A*tan(c/2 + (d*x)/2)*(8*a^15*b -
 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b
^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/(a^3*(a^10*b + a^11 - a^4*b^7
 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))))/a^3))/a^3))*
2i)/(a^3*d) - (atan((((-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(
4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a
^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4
+ 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^
9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7...
```

3.271 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=299

$$\frac{b(12a^4Ab-15a^2Ab^3+6Ab^5-6a^5B+5a^3b^2B-2ab^4B)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d}-\frac{(3Ab-aB)\tanh^{-}}{a^4d}$$

```
[Out] b*(12*A*a^4*b-15*A*a^2*b^3+6*A*b^5-6*B*a^5+5*B*a^3*b^2-2*B*a*b^4)*arctan((a
-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(5/2)/(a+b)^(5/2)/d-(3*
A*b-B*a)*arctanh(sin(d*x+c))/a^4/d+1/2*(2*A*a^4-11*A*a^2*b^2+6*A*b^4+5*B*a^
3*b-2*B*a*b^3)*tan(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b*(A*b-B*a)*tan(d*x+c)/a/(a
^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*b*(6*A*a^2*b-3*A*b^3-4*B*a^3+B*a*b^2)*tan(
d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A]

time = 1.14, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3079, 3134, 3080, 3855, 2738, 211}

$$-\frac{(3Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{b(Ab-aB)\tan(c+dx)}{2ad\,(a^2-b^2)\,(a+b\cos(c+dx))^2}+\frac{b(-4a^3B+6a^2Ab+ab^2B-3Ab^3)\tan(c+dx)}{2a^2d\,(a^2-b^2)^2\,(a+b\cos(c+dx))}+\frac{(2a^4A+5a^3bB-11a^2Ab^2-2ab^3B+6Ab^4)\tan(c+dx)}{2a^3d\,(a^2-b^2)^2}+\frac{b(-6a^5B+12a^4Ab+5a^3b^2B-15a^2Ab^3-2ab^4B+6Ab^5)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 5*a^3*b^2*B - 2*a*b^4*B
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a
 + b)^(5/2)*d) - ((3*A*b - a*B)*ArcTanh[Sin[c + d*x]])/(a^4*d) + ((2*a^4*A
- 11*a^2*A*b^2 + 6*A*b^4 + 5*a^3*b*B - 2*a*b^3*B)*Tan[c + d*x])/(2*a^3*(a^2
 - b^2)^2*d) + (b*(A*b - a*B)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) + (b*(6*a^2*A*b - 3*A*b^3 - 4*a^3*B + a*b^2*B)*Tan[c + d*x])/(2
*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{2a(a^2-b^2)\,d(a+b\cos(c+dx))^2} + \frac{\int \frac{(2a^2A-3Ab^2+abB-2a(Ab-aB)\cos(}{(a+b}}{2a}$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{2a(a^2-b^2)\,d(a+b\cos(c+dx))^2} + \frac{b(6a^2Ab-3Ab^3-4a^3B+ab^2}{2a^2(a^2-b^2)^2\,d(a+b\,\mathrm{co}}$$

$$= \frac{(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B)\tan(c+dx)}{2a^3(a^2-b^2)^2\,d} + \frac{b(}{2a(a^2}$$

$$= \frac{(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B)\tan(c+dx)}{2a^3(a^2-b^2)^2\,d} + \frac{b(}{2a(a^2}$$

$$= -\frac{(3Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{(2a^4A-11a^2Ab^2+6Ab^4+5}{2a^3(a^2-}$$

$$= \frac{b(12a^4Ab-15a^2Ab^3+6Ab^5-6a^5B+5a^3b^2B-2ab^4B)\tan^{-1}\left(\frac{\sqrt{}}{-}\right.}{a^4(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A]
time = 6.06, size = 352, normalized size = 1.18

$$\frac{-\frac{2b(12a^4Ab-15a^2Ab^3+6Ab^5-6a^5B+5a^3b^2B-2ab^4B)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + 2(3Ab-aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2(-3Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{2aA\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{2aA\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{a^2b^2(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2} + \frac{ab^2(-7a^2Ab+4Ab^3+5a^3B-2ab^2B)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 5*a^3*b^2*B - 2*a*b
^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5
/2) + 2*(3*A*b - a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(-3*A*b
+ a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a*A*Sin[(c + d*x)/2])/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (2*a*A*Sin[(c + d*x)/2])/(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]) + (a^2*b^2*(-(A*b) + a*B)*Sin[c + d*x])/((a - b
)*(a + b)*(a + b*Cos[c + d*x])^2) + (a*b^2*(-7*a^2*A*b + 4*A*b^3 + 5*a^3*B
- 2*a*b^2*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*a
^4*d)
```

Maple [A]
time = 1.19, size = 376, normalized size = 1.26

method	result

derivativedivides	$-\frac{A}{a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-3Ab+aB)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{4}}+\frac{2b\left(\frac{-\frac{\left(8A\,a^{2}b+Aa\,b^{2}-4A\,b^{3}-6a^{3}B-a^{2}bB+2Ba\,b^{2}\right)ab\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}}{\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\right.\right.}\right.}{}$
default	$-\frac{A}{a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-3Ab+aB)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{4}}+\frac{2b\left(\frac{-\frac{\left(8A\,a^{2}b+Aa\,b^{2}-4A\,b^{3}-6a^{3}B-a^{2}bB+2Ba\,b^{2}\right)ab\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}}{\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\right.\right.}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-A/a^3/(tan(1/2*d*x+1/2*c)+1)+1/a^4*(-3*A*b+B*a)*ln(tan(1/2*d*x+1/2*c)
+1)+2*b/a^4*((-1/2*(8*A*a^2*b+A*a*b^2-4*A*b^3-6*B*a^3-B*a^2*b+2*B*a*b^2)*a*
b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*b*a*(8*A*a^2*b-A*a*b^2-4*A
*b^3-6*B*a^3+B*a^2*b+2*B*a*b^2)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(a*tan(1/
2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(12*A*a^4*b-15*A*a^2*b^3+6
*A*b^5-6*B*a^5+5*B*a^3*b^2-2*B*a*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/
2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))-A/a^3/(tan(1/2*d*x
+1/2*c)-1)+(3*A*b-B*a)/a^4*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(284) = 568$.
time = 26.82, size = 2100, normalized size = 7.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(((6*B*a^5*b^3 - 12*A*a^4*b^4 - 5*B*a^3*b^5 + 15*A*a^2*b^6 + 2*B*a*b^7
 - 6*A*b^8)*cos(d*x + c)^3 + 2*(6*B*a^6*b^2 - 12*A*a^5*b^3 - 5*B*a^4*b^4 +
15*A*a^3*b^5 + 2*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x + c)^2 + (6*B*a^7*b - 12*A*
a^6*b^2 - 5*B*a^5*b^3 + 15*A*a^4*b^4 + 2*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x +
 c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^
2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*((B*a^7*b^2 - 3*A*a^6*b^3
 - 3*B*a^5*b^4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^8 + 3*A*b^
9)*cos(d*x + c)^3 + 2*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b^4 +
3*B*a^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*cos(d*x + c)^2 + (B*a^9
- 3*A*a^8*b - 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 - B*a^3
*b^6 + 3*A*a^2*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - 2*((B*a^7*b^2 - 3
*A*a^6*b^3 - 3*B*a^5*b^4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^
8 + 3*A*b^9)*cos(d*x + c)^3 + 2*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*
a^5*b^4 + 3*B*a^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*cos(d*x + c)^2
 + (B*a^9 - 3*A*a^8*b - 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b
^5 - B*a^3*b^6 + 3*A*a^2*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(2*A
*a^9 - 6*A*a^7*b^2 + 6*A*a^5*b^4 - 2*A*a^3*b^6 + (2*A*a^7*b^2 + 5*B*a^6*b^3
 - 13*A*a^5*b^4 - 7*B*a^4*b^5 + 17*A*a^3*b^6 + 2*B*a^2*b^7 - 6*A*a*b^8)*cos
(d*x + c)^2 + (4*A*a^8*b + 6*B*a^7*b^2 - 20*A*a^6*b^3 - 9*B*a^5*b^4 + 25*A*
a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^10*b^2
 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d*cos(d*x + c)^3 + 2*(a^11*b - 3*a^9*b^
3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c)^2 + (a^12 - 3*a^10*b^2 + 3*a^8*b^4
- a^6*b^6)*d*cos(d*x + c)), -1/2*(((6*B*a^5*b^3 - 12*A*a^4*b^4 - 5*B*a^3*b^
5 + 15*A*a^2*b^6 + 2*B*a*b^7 - 6*A*b^8)*cos(d*x + c)^3 + 2*(6*B*a^6*b^2 - 1
2*A*a^5*b^3 - 5*B*a^4*b^4 + 15*A*a^3*b^5 + 2*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x
 + c)^2 + (6*B*a^7*b - 12*A*a^6*b^2 - 5*B*a^5*b^3 + 15*A*a^4*b^4 + 2*B*a^3*
b^5 - 6*A*a^2*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) +
b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((B*a^7*b^2 - 3*A*a^6*b^3 - 3*B*a^5*b^
4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^8 + 3*A*b^9)*cos(d*x +
c)^3 + 2*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b^4 + 3*B*a^4*b^5 -
 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*cos(d*x + c)^2 + (B*a^9 - 3*A*a^8*b -
 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 - B*a^3*b^6 + 3*A*a^
2*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((B*a^7*b^2 - 3*A*a^6*b^3 - 3*
B*a^5*b^4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^8 + 3*A*b^9)*co
s(d*x + c)^3 + 2*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b^4 + 3*B*a
^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*cos(d*x + c)^2 + (B*a^9 - 3*A
*a^8*b - 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 - B*a^3*b^6
+ 3*A*a^2*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) - (2*A*a^9 - 6*A*a^7*b^
2 + 6*A*a^5*b^4 - 2*A*a^3*b^6 + (2*A*a^7*b^2 + 5*B*a^6*b^3 - 13*A*a^5*b^4 -
```

```
7*B*a^4*b^5 + 17*A*a^3*b^6 + 2*B*a^2*b^7 - 6*A*a*b^8)*cos(d*x + c)^2 + (4*
A*a^8*b + 6*B*a^7*b^2 - 20*A*a^6*b^3 - 9*B*a^5*b^4 + 25*A*a^4*b^5 + 3*B*a^3
*b^6 - 9*A*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^10*b^2 - 3*a^8*b^4 + 3*
a^6*b^6 - a^4*b^8)*d*cos(d*x + c)^3 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a
^5*b^7)*d*cos(d*x + c)^2 + (a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(
d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 574 vs. 2(284) = 568.
time = 0.52, size = 574, normalized size = 1.92

$$\frac{\frac{(6Ba^5b - 12Aa^4b^2 - 5Ba^3b^3 + 15Aa^2b^4 + 2Bab^5 - 6Ab^6)\left(\pi\left\lfloor\frac{dx+c}{2\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(-2a + 2b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{a^2 - b^2}}\right)\right)}{(a^8 - 2a^6b^2 + a^4b^4)\sqrt{a^2 - b^2}} + \frac{6Ba^4b^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 8Aa^3b^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 5Ba^3b^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 7Aa^2b^4\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 3Ba^2b^4\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 5Aab^5\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 2Bab^5\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 4Ab^6\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 6Ba^4b^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 8Aa^3b^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 5Ba^3b^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 7Aa^2b^4\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 3Ba^2b^4\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 5Aab^5\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 2Bab^5\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 4Ab^6\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{(a^7 - 2a^5b^2 + a^3b^4)\left(a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a + b\right)^2} + \frac{(Ba - 3Ab)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right)}{a^4} - \frac{(Ba - 3Ab)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)}{a^4} - \frac{2A\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] ((6*B*a^5*b - 12*A*a^4*b^2 - 5*B*a^3*b^3 + 15*A*a^2*b^4 + 2*B*a*b^5 - 6*A*b
^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^8 - 2*a^6*b^2
+ a^4*b^4)*sqrt(a^2 - b^2)) + (6*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*A*a^3
*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 7*A*a^2*
b^4*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 + 5*A*a*b^5
*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*A*b^6*tan(1/
2*d*x + 1/2*c)^3 + 6*B*a^4*b^2*tan(1/2*d*x + 1/2*c) - 8*A*a^3*b^3*tan(1/2*d
*x + 1/2*c) + 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c) - 7*A*a^2*b^4*tan(1/2*d*x +
1/2*c) - 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c) + 5*A*a*b^5*tan(1/2*d*x + 1/2*c)
- 2*B*a*b^5*tan(1/2*d*x + 1/2*c) + 4*A*b^6*tan(1/2*d*x + 1/2*c))/((a^7 - 2*
a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a
 + b)^2) + (B*a - 3*A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - (B*a - 3*
A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 2*A*tan(1/2*d*x + 1/2*c)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)*a^3))/d
```

Mupad [B]
time = 12.91, size = 2500, normalized size = 8.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^3),x)

[Out] ((tan(c/2 + (d*x)/2)^5*(6*A*b^5 - 2*A*a^5 - 12*A*a^2*b^3 + 4*A*a^3*b^2 + B*a^2*b^3 + 6*B*a^3*b^2 - 3*A*a*b^4 + 2*A*a^4*b - 2*B*a*b^4))/((a^3*b - a^4)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*A*a^5 + 6*A*b^5 - 12*A*a^2*b^3 - 4*A*a^3*b^2 - B*a^2*b^3 + 6*B*a^3*b^2 + 3*A*a*b^4 + 2*A*a^4*b - 2*B*a*b^4))/((a + b)*(a^5 - 2*a^4*b + a^3*b^2)) - (2*tan(c/2 + (d*x)/2)^3*(2*A*a^6 - 6*A*b^6 + 13*A*a^2*b^4 - 6*A*a^4*b^2 - 5*B*a^3*b^3 + 2*B*a*b^5))/(a*(a^2*b - a^3)*(a + b)^2*(a - b)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2*(2*a*b - a^2 + 3*b^2) - tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - tan(c/2 + (d*x)/2)^4*(2*a*b + a^2 - 3*b^2))) + (atan(((((8*tan(c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 288*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2*a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24*B^2*a^10*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^3*b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (((8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16*b^2 - 4*B*a^9*b^9 + 2*B*a^10*b^8 + 18*B*a^11*b^7 - 4*B*a^12*b^6 - 36*B*a^13*b^5 + 6*B*a^14*b^4 + 34*B*a^15*b^3 - 8*B*a^16*b^2 - 12*A*a^17*b - 12*B*a^17*b))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) + (8*tan(c/2 + (d*x)/2)*(3*A*b - B*a)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*(3*A*b - B*a))/a^4)*(3*A*b - B*a)*1i)/a^4 + (((8*tan(c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 288*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2*a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24*B^2*a^10*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^3*b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (((8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16*b^2 - 4*B*a^9*b^9 + 2*B*a^10*b^8 + 18*B*a^11*b^7 - 4*B*a^12*b^6 - 36*B*a^13*b^5 + 6*B*a^14*b^4 + 34*B*a^15*b^3 - 8*B*a^16*b^2 - 12*A*a^17*b - 12*B*a^17*b))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a

^13*b^3 - 3*a^14*b^2) - (8*tan(c/2 + (d*x)/2)*(3*A*b - B*a)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*(3*A*b - B*a))/a^4)*(3*A*b - B*a)*1i)/a^4)/((16*(108*A^3*b^12 - 54*A^3*a*b^11 - 12*B^3*a^11*b - 486*A^3*a^2*b^10 + 243*A^3*a^3*b^9 + 864*A^3*a^4*b^8 - 378*A^3*a^5*b^7 - 702*A^3*a^6*b^6 + 216*A^3*a^7*b^5 + 216*A^3*a^8*b^4 - 4*B^3*a^3*b^9 + 2*B^3*a^4*b^8 + 18*B^3*a^5*b^7 - 13*B^3*a^6*b^6 - 36*B^3*a^7*b^5 + 26*B^3*a^8*b^4 + 34*B^3*a^9*b^3 - 24*B^3*a^10*b^2 - 108*A^2*B*a*b^11 + 36*A*B^2*a^2*b^10 - 18*A*B^2*a^3*b^9 - 162*A*B^2*a^4*b^8 + 105*A*B^2*a^5*b^7 + 312*A*B^2*a^6*b^6 - 198*A*B^2*a^7*b^5 - 282*A*B^2*a^8*b^4 + 156*A*B^2*a^9*b^3 + 96*A*B^2*a^10*b^2 + 54*A^2*B*a^2*b^10 + 486*A^2*B*a^3*b^9 - 279*A^2*B*a^4*b^8 - 900*A^2*B*a^5*b^7 + 486*A^2*B*a^6*b^6 + 774*A^2*B*a^7*b^5 - 324*A^2*B*a^8*b^4 - 252*A^2*B*a^9*b^3))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (((8*tan(c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 288*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2*a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24*B^2*a^10*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^3*b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (((8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^...

3.272 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=402

$$-\frac{b^2(20a^4Ab-29a^2Ab^3+12Ab^5-12a^5B+15a^3b^2B-6ab^4B)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d}+\frac{(a^2A+12A\cdots}{}$$

```
[Out] -b^2*(20*A*a^4*b-29*A*a^2*b^3+12*A*b^5-12*B*a^5+15*B*a^3*b^2-6*B*a*b^4)*arc
tan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(5/2)/(a+b)^(5/2)
/d+1/2*(A*a^2+12*A*b^2-6*B*a*b)*arctanh(sin(d*x+c))/a^5/d-1/2*(6*A*a^4*b-21
*A*a^2*b^3+12*A*b^5-2*B*a^5+11*B*a^3*b^2-6*B*a*b^4)*tan(d*x+c)/a^4/(a^2-b^2
)^2/d+1/2*(A*a^4-10*A*a^2*b^2+6*A*b^4+6*B*a^3*b-3*B*a*b^3)*sec(d*x+c)*tan(d
*x+c)/a^3/(a^2-b^2)^2/d+1/2*b*(A*b-B*a)*sec(d*x+c)*tan(d*x+c)/a/(a^2-b^2)/d
/(a+b*cos(d*x+c))^2+1/2*b*(7*A*a^2*b-4*A*b^3-5*B*a^3+2*B*a*b^2)*sec(d*x+c)*
tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A]

time = 1.45, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3079, 3134, 3080, 3855, 2738, 211}

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{(a^2A-6abB+12Ab^2)\tanh^{-1}(\sin(c+dx))}{2a^5d}+\frac{b(-5a^3B+7a^2Ab+2ab^2B-4Ab^3)\tan(c+dx)\sec(c+dx)}{2a^2d(a^2-b^2)^2(a+b\cos(c+dx))}+\frac{(a^4A+6a^3bB-10a^2Ab^2-3ab^3B+6Ab^4)\tan(c+dx)\sec(c+dx)}{2a^3d(a^2-b^2)^2}-\frac{b^2(-12a^5B+20a^4Ab+15a^3b^2B-29a^2Ab^3-6ab^4B+12Ab^5)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{(-2a^5B+6a^4Ab+11a^3b^2B-21a^2Ab^3-6ab^4B+12Ab^5)\tan(c+dx)}{2a^4d(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((b^2*(20*a^4*A*b - 29*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B - 6*
a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(
5/2)*(a + b)^(5/2)*d)) + ((a^2*A + 12*A*b^2 - 6*a*b*B)*ArcTanh[Sin[c + d*x]
])/(2*a^5*d) - ((6*a^4*A*b - 21*a^2*A*b^3 + 12*A*b^5 - 2*a^5*B + 11*a^3*b^2
*B - 6*a*b^4*B)*Tan[c + d*x])/(2*a^4*(a^2 - b^2)^2*d) + ((a^4*A - 10*a^2*A*
b^2 + 6*A*b^4 + 6*a^3*b*B - 3*a*b^3*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*(a
^2 - b^2)^2*d) + (b*(A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*(a^2 - b^2)
*d*(a + b*Cos[c + d*x])^2) + (b*(7*a^2*A*b - 4*A*b^3 - 5*a^3*B + 2*a*b^2*B)
*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2\left(a^2A-2Ab^2+abB\right)-2a(Ab-aB}{(}}{}$$

$$= \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b(7a^2Ab-4Ab^3-5a^3B+}{2a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{(a^4A-10a^2Ab^2+6Ab^4+6a^3bB-3ab^3B)\sec(c+dx)\tan(c+dx}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+11a^3b^2B-6ab^4B)\tan(c+}{2a^4\left(a^2-b^2\right)^2 d}$$

$$= -\frac{(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+11a^3b^2B-6ab^4B)\tan(c+}{2a^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{(a^2A+12Ab^2-6abB)\tanh^{-1}(\sin(c+dx))}{2a^5d} - \frac{(6a^4Ab-21a^2Ab^3}{}$$

$$= -\frac{b^2(20a^4Ab-29a^2Ab^3+12Ab^5-12a^5B+15a^3b^2B-6ab^4B)\tan}{a^5(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A]
time = 3.08, size = 507, normalized size = 1.26

$$\frac{\frac{16b^2(20a^4Ab-29a^2Ab^3+12Ab^5-12a^5B+15a^3b^2B-6ab^4B)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} - 8(a^2A+12Ab^2-6abB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 8(a^2A+12Ab^2-6abB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{2a(4a^7A-30a^5Ab^2+68a^3Ab^4-36aAb^6+8a^6bB-32a^4b^3B+18a^2b^5B+(-16a^6Ab+14a^4Ab^3+47a^2Ab^5-36Ab^7+8a^7B-10a^5b^2B-25a^3b^4B+18ab^6B)\cos(c+dx)+2ab(-11a^4Ab+32a^2Ab^3-18Ab^5+4a^5B-16a^3b^2B+9ab^4B)\cos(2(c+dx))-6a^4Ab^3\cos(3(c+dx))+21a^2Ab^5\cos(3(c+dx))-12Ab^7\cos(3(c+dx))+2a^5b^2B\cos(3(c+dx))-11a^3b^4B\cos(3(c+dx))+6ab^6B\cos(3(c+dx)))\sec(c+dx)\tan(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2}}{16a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((16*b^2*(20*a^4*A*b - 29*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B -
6*a*b^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^
2)^(5/2) - 8*(a^2*A + 12*A*b^2 - 6*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 8*(a^2*A + 12*A*b^2 - 6*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]] + (2*a*(4*a^7*A - 30*a^5*A*b^2 + 68*a^3*A*b^4 - 36*a*A*b^6 + 8*a^6*b
*B - 32*a^4*b^3*B + 18*a^2*b^5*B + (-16*a^6*A*b + 14*a^4*A*b^3 + 47*a^2*A*b
^5 - 36*A*b^7 + 8*a^7*B - 10*a^5*b^2*B - 25*a^3*b^4*B + 18*a*b^6*B)*Cos[c +
 d*x] + 2*a*b*(-11*a^4*A*b + 32*a^2*A*b^3 - 18*A*b^5 + 4*a^5*B - 16*a^3*b^2
*B + 9*a*b^4*B)*Cos[2*(c + d*x)] - 6*a^4*A*b^3*Cos[3*(c + d*x)] + 21*a^2*A*
b^5*Cos[3*(c + d*x)] - 12*A*b^7*Cos[3*(c + d*x)] + 2*a^5*b^2*B*Cos[3*(c + d
```

```
*x)] - 11*a^3*b^4*B*Cos[3*(c + d*x)] + 6*a*b^6*B*Cos[3*(c + d*x)])*Sec[c +
d*x]*Tan[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2))/(16*a^5*d)
```

Maple [A]
time = 1.42, size = 460, normalized size = 1.14

method	result
derivativedivides	$-\frac{A}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-aA-6Ab+2aB}{2a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2A+12A\,b^2-6Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^5}-\frac{2b^2\left(-\frac{\left(10A\,a^2b+Aa\,b^2-6A\,b^3-8\cdots\right.}{2(a\cdots}\cdots\right.}{\cdots}$
default	$-\frac{A}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-aA-6Ab+2aB}{2a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2A+12A\,b^2-6Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^5}-\frac{2b^2\left(-\frac{\left(10A\,a^2b+Aa\,b^2-6A\,b^3-8\cdots\right.}{2(a\cdots}\cdots\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-1/2*A/a^3/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-A*a-6*A*b+2*B*a)/a^4/(tan(1/
2*d*x+1/2*c)+1)+1/2*(A*a^2+12*A*b^2-6*B*a*b)/a^5*ln(tan(1/2*d*x+1/2*c)+1)-2
*b^2/a^5*((-1/2*(10*A*a^2*b+A*a*b^2-6*A*b^3-8*B*a^3-B*a^2*b+4*B*a*b^2)*a*b/
(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*b*a*(10*A*a^2*b-A*a*b^2-6*A*
b^3-8*B*a^3+B*a^2*b+4*B*a*b^2)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(a*tan(1/2
*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(20*A*a^4*b-29*A*a^2*b^3+12
*A*b^5-12*B*a^5+15*B*a^3*b^2-6*B*a*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(
1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))+1/2*A/a^3/(tan(1
/2*d*x+1/2*c)-1)^2-1/2*(-A*a-6*A*b+2*B*a)/a^4/(tan(1/2*d*x+1/2*c)-1)+1/2/a^
5*(-A*a^2-12*A*b^2+6*B*a*b)*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(382) = 764$.
time = 40.78, size = 2416, normalized size = 6.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(((12*B*a^5*b^4 - 20*A*a^4*b^5 - 15*B*a^3*b^6 + 29*A*a^2*b^7 + 6*B*a*b
^8 - 12*A*b^9)*cos(d*x + c)^4 + 2*(12*B*a^6*b^3 - 20*A*a^5*b^4 - 15*B*a^4*b
^5 + 29*A*a^3*b^6 + 6*B*a^2*b^7 - 12*A*a*b^8)*cos(d*x + c)^3 + (12*B*a^7*b^
2 - 20*A*a^6*b^3 - 15*B*a^5*b^4 + 29*A*a^4*b^5 + 6*B*a^3*b^6 - 12*A*a^2*b^7
)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*
cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + ((A*a^8*b^2 -
6*B*a^7*b^3 + 9*A*a^6*b^4 + 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*B*a^3*b^7 + 35
*A*a^2*b^8 + 6*B*a*b^9 - 12*A*b^10)*cos(d*x + c)^4 + 2*(A*a^9*b - 6*B*a^8*b
^2 + 9*A*a^7*b^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 18*B*a^4*b^6 + 35*A*a^3*b^
7 + 6*B*a^2*b^8 - 12*A*a*b^9)*cos(d*x + c)^3 + (A*a^10 - 6*B*a^9*b + 9*A*a^
8*b^2 + 18*B*a^7*b^3 - 33*A*a^6*b^4 - 18*B*a^5*b^5 + 35*A*a^4*b^6 + 6*B*a^3
*b^7 - 12*A*a^2*b^8)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((A*a^8*b^2 -
6*B*a^7*b^3 + 9*A*a^6*b^4 + 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*B*a^3*b^7 + 35
*A*a^2*b^8 + 6*B*a*b^9 - 12*A*b^10)*cos(d*x + c)^4 + 2*(A*a^9*b - 6*B*a^8*b
^2 + 9*A*a^7*b^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 18*B*a^4*b^6 + 35*A*a^3*b^
7 + 6*B*a^2*b^8 - 12*A*a*b^9)*cos(d*x + c)^3 + (A*a^10 - 6*B*a^9*b + 9*A*a^
8*b^2 + 18*B*a^7*b^3 - 33*A*a^6*b^4 - 18*B*a^5*b^5 + 35*A*a^4*b^6 + 6*B*a^3
*b^7 - 12*A*a^2*b^8)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(A*a^10 - 3
*A*a^8*b^2 + 3*A*a^6*b^4 - A*a^4*b^6 + (2*B*a^8*b^2 - 6*A*a^7*b^3 - 13*B*a^
6*b^4 + 27*A*a^5*b^5 + 17*B*a^4*b^6 - 33*A*a^3*b^7 - 6*B*a^2*b^8 + 12*A*a*b
^9)*cos(d*x + c)^3 + (4*B*a^9*b - 11*A*a^8*b^2 - 20*B*a^7*b^3 + 43*A*a^6*b^
4 + 25*B*a^5*b^5 - 50*A*a^4*b^6 - 9*B*a^3*b^7 + 18*A*a^2*b^8)*cos(d*x + c)^
2 + 2*(B*a^10 - 2*A*a^9*b - 3*B*a^8*b^2 + 6*A*a^7*b^3 + 3*B*a^6*b^4 - 6*A*a
^5*b^5 - B*a^4*b^6 + 2*A*a^3*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11*b^2 -
3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d*cos(d*x + c)^4 + 2*(a^12*b - 3*a^10*b^3
+ 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c)^3 + (a^13 - 3*a^11*b^2 + 3*a^9*b^4 -
a^7*b^6)*d*cos(d*x + c)^2), 1/4*(2*((12*B*a^5*b^4 - 20*A*a^4*b^5 - 15*B*a^3
*b^6 + 29*A*a^2*b^7 + 6*B*a*b^8 - 12*A*b^9)*cos(d*x + c)^4 + 2*(12*B*a^6*b^
3 - 20*A*a^5*b^4 - 15*B*a^4*b^5 + 29*A*a^3*b^6 + 6*B*a^2*b^7 - 12*A*a*b^8)*
```

```
cos(d*x + c)^3 + (12*B*a^7*b^2 - 20*A*a^6*b^3 - 15*B*a^5*b^4 + 29*A*a^4*b^5
 + 6*B*a^3*b^6 - 12*A*a^2*b^7)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*c
os(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + ((A*a^8*b^2 - 6*B*a^7*b^
3 + 9*A*a^6*b^4 + 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*B*a^3*b^7 + 35*A*a^2*b^8
 + 6*B*a*b^9 - 12*A*b^10)*cos(d*x + c)^4 + 2*(A*a^9*b - 6*B*a^8*b^2 + 9*A*a
^7*b^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 18*B*a^4*b^6 + 35*A*a^3*b^7 + 6*B*a^
2*b^8 - 12*A*a*b^9)*cos(d*x + c)^3 + (A*a^10 - 6*B*a^9*b + 9*A*a^8*b^2 + 18
*B*a^7*b^3 - 33*A*a^6*b^4 - 18*B*a^5*b^5 + 35*A*a^4*b^6 + 6*B*a^3*b^7 - 12*
A*a^2*b^8)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((A*a^8*b^2 - 6*B*a^7*b^
3 + 9*A*a^6*b^4 + 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*B*a^3*b^7 + 35*A*a^2*b^8
 + 6*B*a*b^9 - 12*A*b^10)*cos(d*x + c)^4 + 2*(A*a^9*b - 6*B*a^8*b^2 + 9*A*a
^7*b^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 18*B*a^4*b^6 + 35*A*a^3*b^7 + 6*B*a^
2*b^8 - 12*A*a*b^9)*cos(d*x + c)^3 + (A*a^10 - 6*B*a^9*b + 9*A*a^8*b^2 + 18
*B*a^7*b^3 - 33*A*a^6*b^4 - 18*B*a^5*b^5 + 35*A*a^4*b^6 + 6*B*a^3*b^7 - 12*
A*a^2*b^8)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(A*a^10 - 3*A*a^8*b^2
 + 3*A*a^6*b^4 - A*a^4*b^6 + (2*B*a^8*b^2 - 6*A*a^7*b^3 - 13*B*a^6*b^4 + 27
*A*a^5*b^5 + 17*B*a^4*b^6 - 33*A*a^3*b^7 - 6*B*a^2*b^8 + 12*A*a*b^9)*cos(d*
x + c)^3 + (4*B*a^9*b - 11*A*a^8*b^2 - 20*B*a^7*b^3 + 43*A*a^6*b^4 + 25*B*a
^5*b^5 - 50*A*a^4*b^6 - 9*B*a^3*b^7 + 18*A*a^2*b^8)*cos(d*x + c)^2 + 2*(B*a
^10 - 2*A*a^9*b - 3*B*a^8*b^2 + 6*A*a^7*b^3 + 3*B*a^6*b^4 - 6*A*a^5*b^5 - B
*a^4*b^6 + 2*A*a^3*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11*b^2 - 3*a^9*b^4
+ 3*a^7*b^6 - a^5*b^8)*d*cos(d*x + c)^4 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^
5 - a^6*b^7)*d*cos(d*x + c)^3 + (a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d
*cos(d*x + c)^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{(a + b\cos(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x))**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. $2(382) = 764$.
time = 0.54, size = 1395, normalized size = 3.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/2*(2*(12*B*a^5*b^2 - 20*A*a^4*b^3 - 15*B*a^3*b^4 + 29*A*a^2*b^5 + 6*B*a*
b^6 - 12*A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(
-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^9
- 2*a^7*b^2 + a^5*b^4)*sqrt(a^2 - b^2)) - 2*(A*a^7*tan(1/2*d*x + 1/2*c)^7 -
 2*B*a^7*tan(1/2*d*x + 1/2*c)^7 + 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 4*B*a^
6*b*tan(1/2*d*x + 1/2*c)^7 - 13*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 2*B*a^5*
b^2*tan(1/2*d*x + 1/2*c)^7 - 2*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 16*B*a^4*
b^3*tan(1/2*d*x + 1/2*c)^7 + 33*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^3*
b^4*tan(1/2*d*x + 1/2*c)^7 - 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^2*
b^5*tan(1/2*d*x + 1/2*c)^7 - 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 6*B*a*b^6*
tan(1/2*d*x + 1/2*c)^7 + 12*A*b^7*tan(1/2*d*x + 1/2*c)^7 + 3*A*a^7*tan(1/2*
d*x + 1/2*c)^5 - 2*B*a^7*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^6*b*tan(1/2*d*x + 1
/2*c)^5 - 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 5*A*a^5*b^2*tan(1/2*d*x + 1/2*
c)^5 + 10*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 26*A*a^4*b^3*tan(1/2*d*x + 1/2
*c)^5 + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 29*A*a^3*b^4*tan(1/2*d*x + 1/
2*c)^5 - 35*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 67*A*a^2*b^5*tan(1/2*d*x + 1
/2*c)^5 - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^6*tan(1/2*d*x + 1/2
*c)^5 + 18*B*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*A*b^7*tan(1/2*d*x + 1/2*c)^5
 + 3*A*a^7*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^
6*b*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^5*b^2
*tan(1/2*d*x + 1/2*c)^3 - 10*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 26*A*a^4*b^
3*tan(1/2*d*x + 1/2*c)^3 + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*A*a^3*b
^4*tan(1/2*d*x + 1/2*c)^3 + 35*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 67*A*a^2*
b^5*tan(1/2*d*x + 1/2*c)^3 - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*A*a*b^
6*tan(1/2*d*x + 1/2*c)^3 - 18*B*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^7*tan
(1/2*d*x + 1/2*c)^3 + A*a^7*tan(1/2*d*x + 1/2*c) + 2*B*a^7*tan(1/2*d*x + 1/
2*c) - 4*A*a^6*b*tan(1/2*d*x + 1/2*c) + 4*B*a^6*b*tan(1/2*d*x + 1/2*c) - 13
*A*a^5*b^2*tan(1/2*d*x + 1/2*c) - 2*B*a^5*b^2*tan(1/2*d*x + 1/2*c) + 2*A*a^
4*b^3*tan(1/2*d*x + 1/2*c) - 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c) + 33*A*a^3*b
^4*tan(1/2*d*x + 1/2*c) - 9*B*a^3*b^4*tan(1/2*d*x + 1/2*c) + 17*A*a^2*b^5*t
an(1/2*d*x + 1/2*c) + 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c) - 18*A*a*b^6*tan(1/2
*d*x + 1/2*c) + 6*B*a*b^6*tan(1/2*d*x + 1/2*c) - 12*A*b^7*tan(1/2*d*x + 1/2
*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x
 + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) - (A*a^2 - 6*B*a*b + 1
2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 + (A*a^2 - 6*B*a*b + 12*A*b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5)/d
```

Mupad [B]
time = 12.56, size = 2500, normalized size = 6.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^3),x)
```

[Out] ((tan(c/2 + (d*x)/2)^3*(3*A*a^7 + 36*A*b^7 + 2*B*a^7 - 67*A*a^2*b^5 - 29*A*a^3*b^4 + 26*A*a^4*b^3 + 5*A*a^5*b^2 - 9*B*a^2*b^5 + 35*B*a^3*b^4 + 16*B*a^4*b^3 - 10*B*a^5*b^2 + 18*A*a*b^6 - 4*A*a^6*b - 18*B*a*b^6 - 4*B*a^6*b))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2)) + (tan(c/2 + (d*x)/2)^5*(3*A*a^7 - 36*A*b^7 - 2*B*a^7 + 67*A*a^2*b^5 - 29*A*a^3*b^4 - 26*A*a^4*b^3 + 5*A*a^5*b^2 - 9*B*a^2*b^5 - 35*B*a^3*b^4 + 16*B*a^4*b^3 + 10*B*a^5*b^2 + 18*A*a*b^6 + 4*A*a^6*b + 18*B*a*b^6 - 4*B*a^6*b))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2)) - (tan(c/2 + (d*x)/2)^7*(A*a^6 - 12*A*b^6 - 2*B*a^6 + 23*A*a^2*b^4 - 10*A*a^3*b^3 - 8*A*a^4*b^2 - 3*B*a^2*b^4 - 12*B*a^3*b^3 + 4*B*a^4*b^2 + 6*A*a*b^5 + 5*A*a^5*b + 6*B*a*b^5 + 2*B*a^5*b))/((a^4*b - a^5)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(A*a^6 - 12*A*b^6 + 2*B*a^6 + 23*A*a^2*b^4 + 10*A*a^3*b^3 - 8*A*a^4*b^2 + 3*B*a^2*b^4 - 12*B*a^3*b^3 - 4*B*a^4*b^2 - 6*A*a*b^5 - 5*A*a^5*b + 6*B*a*b^5 + 2*B*a^5*b))/((a + b)*(a^6 - 2*a^5*b + a^4*b^2)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^4*(2*a^2 - 6*b^2) - tan(c/2 + (d*x)/2)^2*(4*a*b + 4*b^2) + tan(c/2 + (d*x)/2)^6*(4*a*b - 4*b^2) + tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) - (atan(((((8*tan(c/2 + (d*x)/2)*(A^2*a^14 + 288*A^2*b^14 - 288*A^2*a*b^13 - 2*A^2*a^13*b - 1104*A^2*a^2*b^12 + 1104*A^2*a^3*b^11 + 1538*A^2*a^4*b^10 - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^10*b^4 - 40*A^2*a^11*b^3 + 21*A^2*a^12*b^2 + 72*B^2*a^2*b^12 - 72*B^2*a^3*b^11 - 288*B^2*a^4*b^10 + 288*B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B^2*a^10*b^4 - 72*B^2*a^11*b^3 + 36*B^2*a^12*b^2 - 288*A*B*a*b^13 - 12*A*B*a^13*b + 288*A*B*a^2*b^12 + 1128*A*B*a^3*b^11 - 1128*A*B*a^4*b^10 - 1650*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^5 + 192*A*B*a^10*b^4 - 108*A*B*a^11*b^3 + 24*A*B*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) - (((4*(4*A*a^21 - 48*A*a^10*b^11 + 24*A*a^11*b^10 + 212*A*a^12*b^9 - 100*A*a^13*b^8 - 360*A*a^14*b^7 + 164*A*a^15*b^6 + 276*A*a^16*b^5 - 120*A*a^17*b^4 - 80*A*a^18*b^3 + 28*A*a^19*b^2 + 24*B*a^11*b^10 - 12*B*a^12*b^9 - 108*B*a^13*b^8 + 48*B*a^14*b^7 + 192*B*a^15*b^6 - 84*B*a^16*b^5 - 156*B*a^17*b^4 + 72*B*a^18*b^3 + 48*B*a^19*b^2 - 24*B*a^20*b))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4*tan(c/2 + (d*x)/2)*(A*a^2 + 12*A*b^2 - 6*B*a*b)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(A*a^2 + 12*A*b^2 - 6*B*a*b))/(2*a^5))*(A*a^2 + 12*A*b^2 - 6*B*a*b)*1i)/(2*a^5) + (((8*tan(c/2 + (d*x)/2)*(A^2*a^14 + 288*A^2*b^14 - 288*A^2*a*b^13 - 2*A^2*a^13*b - 1104*A^2*a^2*b^12 + 1104*A^2*a^3*b^11 + 1538*A^2*a^4*b^10 - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^10*b^4 - 40*A^2*a^11*b^3 + 21*A^2*a^12*b^2 + 72*B^2*a^2*b^12 - 72*B^2*a^3*b^11 - 288*B^2*a^4*b^10 + 288*B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B^2*a^10*b^4 - 72*B^2*a^11*b^3 + 36*B^2*a^12*b^2 - 288*A*B*a*b^13 - 12*A*B*a^13*b + 288*A*B*a^2*b^12 + 1128*A*B*a^3*b^11 - 1128*A*B*a^4*b^10 - 1650*A*B*a^5*b^9 + 1632

```
*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^5 + 192*A*
B*a^10*b^4 - 108*A*B*a^11*b^3 + 24*A*B*a^12*b^2))/(a^14*b + a^15 - a^8*b^7
- a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) + (((4*(4*A*
a^21 - 48*A*a^10*b^11 + 24*A*a^11*b^10 + 212*A*a^12*b^9 - 100*A*a^13*b^8 -
360*A*a^14*b^7 + 164*A*a^15*b^6 + 276*A*a^16*b^5 - 120*A*a^17*b^4 - 80*A*a^
18*b^3 + 28*A*a^19*b^2 + 24*B*a^11*b^10 - 12*B*a^12*b^9 - 108*B*a^13*b^8 +
48*B*a^14*b^7 + 192*B*a^15*b^6 - 84*B*a^16*b^5 - 156*B*a^17*b^4 + 72*B*a^18
*b^3 + 48*B*a^19*b^2 - 24*B*a^20*b))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 +
 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) + (4*tan(c/2 + (d*x)/2)
*(A*a^2 + 12*A*b^2 - 6*B*a*b)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^1
2*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3
 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^
11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(A*a^2 + 12*A*b^2 - 6*B*a*b))/(2*a^5))*
(A*a^2 + 12*A*b^2 - 6*B*a*b)*1i)/(2*a^5))/((8*(1728*A^3*b^15 - 864*A^3*a*b^
14 - 7344*A^3*a^2*b^13 + 3456*A^3*a^3*b^12 + 11700*A^3*a^4*b^11 - 4770*A^3*
a^5*b^10 - 7829*A^3*a^6*b^9 + 2326*A^3*a^7*b^8 + 1314*A^3*a^8*b^7 - 11*A^3*
a^9*b^6 + 411*A^3*a^10*b^5 - 20*A^3*a^11*b^4 + 20*A^3*a^12*b^3 - 216*B^3*a^
3*b^12 + 108*B^3*a^4*b^11 + 972*B^3*a^5*b^10 - 486*B^3*a^6*b^9 - 1728*B^3*a
^7*b^8 + 756*B^3*a^8*b^7 + 1404*B^3*a^9*b^6 - 432*B^3*a^10*b^5 - 432*B^3*a^
11*b^4 - 2592*A^2*B*a*b^14 + 1296*A*B^2*a^2*b^13 - 648*A*B^2*a^3*b^12 - 572
4*A*B^2*a^4*b^11 + 2808*A*B^2*a^5*b^10 + 9828*A*B^2*a^6*b^9 - 4203*A*B^2*a^
7*b^8 - 7524*A*B^2*a^8*b^7 + 2268*A*B^2*a^9*b^6...
```

3.273 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=409

$$\frac{(Ab-4aB)x}{b^5}-\frac{a(2a^6Ab-7a^4Ab^3+8a^2Ab^5-8Ab^7-8a^7B+28a^5b^2B-35a^3b^4B+20ab^6B)\,\mathrm{ArcTan}\left(\sqrt{a}\right.}{(a-b)^{7/2}b^5(a+b)^{7/2}d}$$

```
[Out] (A*b-4*B*a)*x/b^5-a*(2*A*a^6*b-7*A*a^4*b^3+8*A*a^2*b^5-8*A*b^7-8*B*a^7+28*B
*a^5*b^2-35*B*a^3*b^4+20*B*a*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+
b)^(1/2))/(a-b)^(7/2)/b^5/(a+b)^(7/2)/d-1/6*(3*A*a^3*b-8*A*a*b^3-12*B*a^4+2
3*B*a^2*b^2-6*B*b^4)*sin(d*x+c)/b^4/(a^2-b^2)^2/d+1/3*a*(A*b-B*a)*cos(d*x+c
)^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*a*(A*a^2*b-6*A*b^3-4*B*
a^3+9*B*a*b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2
-1/2*a^2*(A*a^4*b-2*A*a^2*b^3+6*A*b^5-4*B*a^5+11*B*a^3*b^2-12*B*a*b^4)*sin(
d*x+c)/b^4/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 3.29, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3068, 3126, 3110, 3102, 2814, 2738, 211}

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\,(a^2-b^2)\,(a+b\cos(c+dx))^3}+\frac{a(-4a^3B+a^2Ab+9ab^2B-6Ab^3)\sin(c+dx)\cos^2(c+dx)}{6b^2d\,(a^2-b^2)^2\,(a+b\cos(c+dx))^2}-\frac{(-12a^4B+3a^3Ab+23a^2b^2B-8aAb^3-6b^4B)\sin(c+dx)}{6b^4d\,(a^2-b^2)^2}-\frac{a^2(-4a^5B+a^4Ab+11a^3b^2B-2a^2Ab^3-12ab^4B+6Ab^5)\sin(c+dx)}{2b^4d\,(a^2-b^2)^3\,(a+b\cos(c+dx))}-\frac{a(-8a^7B+2a^6Ab+28a^5b^2B-7a^4Ab^3-35a^3b^4B+8a^2Ab^5+20ab^6B-8Ab^7)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{x(Ab-4aB)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((A*b - 4*a*B)*x)/b^5 - (a*(2*a^6*A*b - 7*a^4*A*b^3 + 8*a^2*A*b^5 - 8*A*b^7
 - 8*a^7*B + 28*a^5*b^2*B - 35*a^3*b^4*B + 20*a*b^6*B)*ArcTan[(Sqrt[a - b]*
Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5*(a + b)^(7/2)*d) - ((3*a
^3*A*b - 8*a*A*b^3 - 12*a^4*B + 23*a^2*b^2*B - 6*b^4*B)*Sin[c + d*x])/(6*b^
4*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^3) + (a*(a^2*A*b - 6*A*b^3 - 4*a^3*B + 9*a*b^
2*B)*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x
])^2) - (a^2*(a^4*A*b - 2*a^2*A*b^3 + 6*A*b^5 - 4*a^5*B + 11*a^3*b^2*B - 12
*a*b^4*B)*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(-3a(Ab-aB)+3b(}{}}{}$$

$$= \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+}{6b^2\,(a^2-b^2)^2}$$

$$= \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+}{6b^2\,(a^2-b^2)^2}$$

$$= -\frac{(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B)\sin(c+dx)}{6b^4\,(a^2-b^2)^2\,d} + \frac{a(}{3}$$

$$= \frac{(Ab-4aB)x}{b^5} - \frac{(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B)\sin}{6b^4\,(a^2-b^2)^2\,d}$$

$$= \frac{(Ab-4aB)x}{b^5} - \frac{(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B)\sin}{6b^4\,(a^2-b^2)^2\,d}$$

$$= \frac{(Ab-4aB)x}{b^5} - \frac{a(2a^6Ab-7a^4Ab^3+8a^2Ab^5-8Ab^7-8a^7B+2}{(a-b}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1155 vs. $2(409) = 818$.
time = 6.56, size = 1155, normalized size = 2.82

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]

[Out] ((24*a*(-2*a^6*A*b + 7*a^4*A*b^3 - 8*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 28*a^5
*b^2*B + 35*a^3*b^4*B - 20*a*b^6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt
[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - (-24*a^9*A*b*c + 36*a^7*A*b^3*c + 36*a^
5*A*b^5*c - 84*a^3*A*b^7*c + 36*a*A*b^9*c + 96*a^10*B*c - 144*a^8*b^2*B*c -
144*a^6*b^4*B*c + 336*a^4*b^6*B*c - 144*a^2*b^8*B*c - 24*a^9*A*b*d*x + 36*
a^7*A*b^3*d*x + 36*a^5*A*b^5*d*x - 84*a^3*A*b^7*d*x + 36*a*A*b^9*d*x + 96*a
^10*B*d*x - 144*a^8*b^2*B*d*x - 144*a^6*b^4*B*d*x + 336*a^4*b^6*B*d*x - 144
*a^2*b^8*B*d*x + 18*b*(-a^2 + b^2)^3*(4*a^2 + b^2)*(A*b - 4*a*B)*(c + d*x)*
Cos[c + d*x] + 36*a*b^2*(a^2 - b^2)^3*(-(A*b) + 4*a*B)*(c + d*x)*Cos[2*(c +
d*x)] - 6*a^6*A*b^4*c*Cos[3*(c + d*x)] + 18*a^4*A*b^6*c*Cos[3*(c + d*x)] -
18*a^2*A*b^8*c*Cos[3*(c + d*x)] + 6*A*b^10*c*Cos[3*(c + d*x)] + 24*a^7*b^3
*B*c*Cos[3*(c + d*x)] - 72*a^5*b^5*B*c*Cos[3*(c + d*x)] + 72*a^3*b^7*B*c*Co
s[3*(c + d*x)] - 24*a*b^9*B*c*Cos[3*(c + d*x)] - 6*a^6*A*b^4*d*x*Cos[3*(c +
d*x)] + 18*a^4*A*b^6*d*x*Cos[3*(c + d*x)] - 18*a^2*A*b^8*d*x*Cos[3*(c + d*
x)] + 6*A*b^10*d*x*Cos[3*(c + d*x)] + 24*a^7*b^3*B*d*x*Cos[3*(c + d*x)] - 7
2*a^5*b^5*B*d*x*Cos[3*(c + d*x)] + 72*a^3*b^7*B*d*x*Cos[3*(c + d*x)] - 24*a
*b^9*B*d*x*Cos[3*(c + d*x)] + 24*a^8*A*b^2*Sin[c + d*x] - 57*a^6*A*b^4*Sin[
c + d*x] + 72*a^4*A*b^6*Sin[c + d*x] + 36*a^2*A*b^8*Sin[c + d*x] - 96*a^9*b
*B*Sin[c + d*x] + 228*a^7*b^3*B*Sin[c + d*x] - 135*a^5*b^5*B*Sin[c + d*x] -
90*a^3*b^7*B*Sin[c + d*x] + 18*a*b^9*B*Sin[c + d*x] + 30*a^7*A*b^3*Sin[2*(
c + d*x)] - 90*a^5*A*b^5*Sin[2*(c + d*x)] + 120*a^3*A*b^7*Sin[2*(c + d*x)]
- 120*a^8*b^2*B*Sin[2*(c + d*x)] + 336*a^6*b^4*B*Sin[2*(c + d*x)] - 300*a^4
*b^6*B*Sin[2*(c + d*x)] + 18*a^2*b^8*B*Sin[2*(c + d*x)] + 6*b^10*B*Sin[2*(c
+ d*x)] + 11*a^6*A*b^4*Sin[3*(c + d*x)] - 32*a^4*A*b^6*Sin[3*(c + d*x)] +
36*a^2*A*b^8*Sin[3*(c + d*x)] - 44*a^7*b^3*B*Sin[3*(c + d*x)] + 125*a^5*b^5
*B*Sin[3*(c + d*x)] - 114*a^3*b^7*B*Sin[3*(c + d*x)] + 18*a*b^9*B*Sin[3*(c
+ d*x)] - 3*a^6*b^4*B*Sin[4*(c + d*x)] + 9*a^4*b^6*B*Sin[4*(c + d*x)] - 9*a
^2*b^8*B*Sin[4*(c + d*x)] + 3*b^10*B*Sin[4*(c + d*x)])/((a^2 - b^2)^3*(a +
b*Cos[c + d*x])^3))/(24*b^5*d)

Maple [A]

time = 1.10, size = 552, normalized size = 1.35

method	result
derivativedivides	$-\frac{2a\left(\frac{\left(2A a^4b-A a^3b^2-6A a^2b^3+4Aab^4+12Ab^5-6Ba^5+2Ba^4b+18Ba^3b^2-5Ba^2b^3-20Bab^4\right)ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}+\frac{2\left(3A a^4b-11A\right.}{}}{}\right.}{}$

default	$-\frac{2a\left(\frac{\left(2A\,a^{4}b-A\,a^{3}b^{2}-6A\,a^{2}b^{3}+4Aa\,b^{4}+12A\,b^{5}-6B\,a^{5}+2B\,a^{4}b+18B\,a^{3}b^{2}-5B\,a^{2}b^{3}-20Ba\,b^{4}\right)ab\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{3}+3a^{2}b+3b^{2}a+b^{3}\right)}+\frac{2\left(3A\,a^{4}b-1\right.}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-2*a/b^5*((1/2*(2*A*a^4*b-A*a^3*b^2-6*A*a^2*b^3+4*A*a*b^4+12*A*b^5-6*B
*a^5+2*B*a^4*b+18*B*a^3*b^2-5*B*a^2*b^3-20*B*a*b^4)*a*b/(a-b)/(a^3+3*a^2*b+
3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(3*A*a^4*b-11*A*a^2*b^3+18*A*b^5-9*B*
a^5+29*B*a^3*b^2-30*B*a*b^4)*a*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3+1/2*(2*A*a^4*b+A*a^3*b^2-6*A*a^2*b^3-4*A*a*b^4+12*A*b^5-6*B*a^5-
2*B*a^4*b+18*B*a^3*b^2+5*B*a^2*b^3-20*B*a*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b
^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2+
a+b)^3+1/2*(2*A*a^6*b-7*A*a^4*b^3+8*A*a^2*b^5-8*A*b^7-8*B*a^7+28*B*a^5*b^2-
35*B*a^3*b^4+20*B*a*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*
arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))+2/b^5*(B*b*tan(1/2*d*
x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+(A*b-4*B*a)*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1249 vs. $2(395) = 790$.
time = 0.60, size = 2567, normalized size = 6.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")

[Out] [-1/12*(12*(4*B*a^9*b^3 - A*a^8*b^4 - 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5
*b^7 - 6*A*a^4*b^8 - 16*B*a^3*b^9 + 4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12)*d*x
*cos(d*x + c)^3 + 36*(4*B*a^10*b^2 - A*a^9*b^3 - 16*B*a^8*b^4 + 4*A*a^7*b^5
 + 24*B*a^6*b^6 - 6*A*a^5*b^7 - 16*B*a^4*b^8 + 4*A*a^3*b^9 + 4*B*a^2*b^10 -
 A*a*b^11)*d*x*cos(d*x + c)^2 + 36*(4*B*a^11*b - A*a^10*b^2 - 16*B*a^9*b^3
+ 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^5*b^7 + 4*A*a^4*b^8 + 4
*B*a^3*b^9 - A*a^2*b^10)*d*x*cos(d*x + c) + 12*(4*B*a^12 - A*a^11*b - 16*B*
a^10*b^2 + 4*A*a^9*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6 + 4*A*a^
5*b^7 + 4*B*a^4*b^8 - A*a^3*b^9)*d*x - 3*(8*B*a^11 - 2*A*a^10*b - 28*B*a^9*
b^2 + 7*A*a^8*b^3 + 35*B*a^7*b^4 - 8*A*a^6*b^5 - 20*B*a^5*b^6 + 8*A*a^4*b^7
 + (8*B*a^8*b^3 - 2*A*a^7*b^4 - 28*B*a^6*b^5 + 7*A*a^5*b^6 + 35*B*a^4*b^7 -
 8*A*a^3*b^8 - 20*B*a^2*b^9 + 8*A*a*b^10)*cos(d*x + c)^3 + 3*(8*B*a^9*b^2 -
 2*A*a^8*b^3 - 28*B*a^7*b^4 + 7*A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20
*B*a^3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + 3*(8*B*a^10*b - 2*A*a^9*b^2 - 28
*B*a^8*b^3 + 7*A*a^7*b^4 + 35*B*a^6*b^5 - 8*A*a^5*b^6 - 20*B*a^4*b^7 + 8*A*
a^3*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
- a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(24*B*a
^11*b - 6*A*a^10*b^2 - 92*B*a^9*b^3 + 23*A*a^8*b^4 + 133*B*a^7*b^5 - 43*A*a
^6*b^6 - 71*B*a^5*b^7 + 26*A*a^4*b^8 + 6*B*a^3*b^9 + 6*(B*a^8*b^4 - 4*B*a^6
*b^6 + 6*B*a^4*b^8 - 4*B*a^2*b^10 + B*b^12)*cos(d*x + c)^3 + (44*B*a^9*b^3
- 11*A*a^8*b^4 - 169*B*a^7*b^5 + 43*A*a^6*b^6 + 239*B*a^5*b^7 - 68*A*a^4*b^
8 - 132*B*a^3*b^9 + 36*A*a^2*b^10 + 18*B*a*b^11)*cos(d*x + c)^2 + 3*(20*B*a
^10*b^2 - 5*A*a^9*b^3 - 77*B*a^8*b^4 + 20*A*a^7*b^5 + 110*B*a^6*b^6 - 35*A*
a^5*b^7 - 59*B*a^4*b^8 + 20*A*a^3*b^9 + 6*B*a^2*b^10)*cos(d*x + c))*sin(d*x
 + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^16)*d*cos(d*x +
 c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^15)*d*cos(d*
x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14)*d*
cos(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d
), -1/6*(6*(4*B*a^9*b^3 - A*a^8*b^4 - 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5
*b^7 - 6*A*a^4*b^8 - 16*B*a^3*b^9 + 4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12)*d*x
*cos(d*x + c)^3 + 18*(4*B*a^10*b^2 - A*a^9*b^3 - 16*B*a^8*b^4 + 4*A*a^7*b^5
 + 24*B*a^6*b^6 - 6*A*a^5*b^7 - 16*B*a^4*b^8 + 4*A*a^3*b^9 + 4*B*a^2*b^10 -
 A*a*b^11)*d*x*cos(d*x + c)^2 + 18*(4*B*a^11*b - A*a^10*b^2 - 16*B*a^9*b^3
+ 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^5*b^7 + 4*A*a^4*b^8 + 4
*B*a^3*b^9 - A*a^2*b^10)*d*x*cos(d*x + c) + 6*(4*B*a^12 - A*a^11*b - 16*B*a
^10*b^2 + 4*A*a^9*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6 + 4*A*a^5
*b^7 + 4*B*a^4*b^8 - A*a^3*b^9)*d*x - 3*(8*B*a^11 - 2*A*a^10*b - 28*B*a^9*b
^2 + 7*A*a^8*b^3 + 35*B*a^7*b^4 - 8*A*a^6*b^5 - 20*B*a^5*b^6 + 8*A*a^4*b^7
+ (8*B*a^8*b^3 - 2*A*a^7*b^4 - 28*B*a^6*b^5 + 7*A*a^5*b^6 + 35*B*a^4*b^7 -
8*A*a^3*b^8 - 20*B*a^2*b^9 + 8*A*a*b^10)*cos(d*x + c)^3 + 3*(8*B*a^9*b^2 -
2*A*a^8*b^3 - 28*B*a^7*b^4 + 7*A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*
```

```
B*a^3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + 3*(8*B*a^10*b - 2*A*a^9*b^2 - 28*
B*a^8*b^3 + 7*A*a^7*b^4 + 35*B*a^6*b^5 - 8*A*a^5*b^6 - 20*B*a^4*b^7 + 8*A*a
^3*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^
2 - b^2)*sin(d*x + c))) - (24*B*a^11*b - 6*A*a^10*b^2 - 92*B*a^9*b^3 + 23*A
*a^8*b^4 + 133*B*a^7*b^5 - 43*A*a^6*b^6 - 71*B*a^5*b^7 + 26*A*a^4*b^8 + 6*B
*a^3*b^9 + 6*(B*a^8*b^4 - 4*B*a^6*b^6 + 6*B*a^4*b^8 - 4*B*a^2*b^10 + B*b^12
)*cos(d*x + c)^3 + (44*B*a^9*b^3 - 11*A*a^8*b^4 - 169*B*a^7*b^5 + 43*A*a^6*
b^6 + 239*B*a^5*b^7 - 68*A*a^4*b^8 - 132*B*a^3*b^9 + 36*A*a^2*b^10 + 18*B*a
*b^11)*cos(d*x + c)^2 + 3*(20*B*a^10*b^2 - 5*A*a^9*b^3 - 77*B*a^8*b^4 + 20*
A*a^7*b^5 + 110*B*a^6*b^6 - 35*A*a^5*b^7 - 59*B*a^4*b^8 + 20*A*a^3*b^9 + 6*
B*a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12
 - 4*a^2*b^14 + b^16)*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^1
1 - 4*a^3*b^13 + a*b^15)*d*cos(d*x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6
*b^10 - 4*a^4*b^12 + a^2*b^14)*d*cos(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a
^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 966 vs. 2(395) = 790.
time = 0.50, size = 966, normalized size = 2.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(8*B*a^8 - 2*A*a^7*b - 28*B*a^6*b^2 + 7*A*a^5*b^3 + 35*B*a^4*b^4 -
8*A*a^3*b^5 - 20*B*a^2*b^6 + 8*A*a*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*s
gn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(a^2 - b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*sqrt(a^2 - b^2)
) - (18*B*a^9*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^8*b*tan(1/2*d*x + 1/2*c)^5 - 4
2*B*a^8*b*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 - 24
*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 11
7*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 45*A*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 -
24*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 -
105*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 60*A*a^3*b^6*tan(1/2*d*x + 1/2*c)^5
```

```
+ 60*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^5
 + 36*B*a^9*tan(1/2*d*x + 1/2*c)^3 - 12*A*a^8*b*tan(1/2*d*x + 1/2*c)^3 - 15
2*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 + 56*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^3 +
236*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 - 116*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^3
 - 120*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 72*A*a^2*b^7*tan(1/2*d*x + 1/2*c)
^3 + 18*B*a^9*tan(1/2*d*x + 1/2*c) - 6*A*a^8*b*tan(1/2*d*x + 1/2*c) + 42*B*
a^8*b*tan(1/2*d*x + 1/2*c) - 15*A*a^7*b^2*tan(1/2*d*x + 1/2*c) - 24*B*a^7*b
^2*tan(1/2*d*x + 1/2*c) + 6*A*a^6*b^3*tan(1/2*d*x + 1/2*c) - 117*B*a^6*b^3*
tan(1/2*d*x + 1/2*c) + 45*A*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*B*a^5*b^4*tan
(1/2*d*x + 1/2*c) + 6*A*a^4*b^5*tan(1/2*d*x + 1/2*c) + 105*B*a^4*b^5*tan(1/
2*d*x + 1/2*c) - 60*A*a^3*b^6*tan(1/2*d*x + 1/2*c) + 60*B*a^3*b^6*tan(1/2*d
*x + 1/2*c) - 36*A*a^2*b^7*tan(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b^6 + 3*
a^2*b^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a +
b)^3) + 3*(4*B*a - A*b)*(d*x + c)/b^5 - 6*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*
d*x + 1/2*c)^2 + 1)*b^4))/d
```

Mupad [B]
time = 12.51, size = 2500, normalized size = 6.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^4,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1i)*(A*b - 4*B*a)*1i)/(b^5*d) - ((tan(c/2 + (d*x)
/2)^7*(12*A*a^2*b^5 - 2*B*b^7 - 8*B*a^7 + 4*A*a^3*b^4 - 6*A*a^4*b^3 - A*a^5
*b^2 + 6*B*a^2*b^5 - 26*B*a^3*b^4 - 11*B*a^4*b^3 + 24*B*a^5*b^2 + 2*A*a^6*b
 + 2*B*a*b^6 + 4*B*a^6*b))/(b^4*(a + b)^3*(a - b)) - (tan(c/2 + (d*x)/2)^3*
(72*B*a^8 + 18*B*b^8 + 36*A*a^2*b^6 - 96*A*a^3*b^5 - 14*A*a^4*b^4 + 59*A*a^
5*b^3 + 3*A*a^6*b^2 - 72*B*a^2*b^6 - 60*B*a^3*b^5 + 273*B*a^4*b^4 + 47*B*a^
5*b^3 - 236*B*a^6*b^2 - 18*A*a^7*b - 12*B*a^7*b))/(3*b^4*(a + b)^2*(a - b)^
3) - (tan(c/2 + (d*x)/2)^5*(72*B*a^8 + 18*B*b^8 - 36*A*a^2*b^6 - 96*A*a^3*b
^5 + 14*A*a^4*b^4 + 59*A*a^5*b^3 - 3*A*a^6*b^2 - 72*B*a^2*b^6 + 60*B*a^3*b^
5 + 273*B*a^4*b^4 - 47*B*a^5*b^3 - 236*B*a^6*b^2 - 18*A*a^7*b + 12*B*a^7*b)
)/(3*b^4*(a + b)^3*(a - b)^2) + (tan(c/2 + (d*x)/2)*(2*B*b^7 - 8*B*a^7 + 12
*A*a^2*b^5 - 4*A*a^3*b^4 - 6*A*a^4*b^3 + A*a^5*b^2 - 6*B*a^2*b^5 - 26*B*a^3
*b^4 + 11*B*a^4*b^3 + 24*B*a^5*b^2 + 2*A*a^6*b + 2*B*a*b^6 - 4*B*a^6*b))/(b
^4*(a + b)*(a - b)^3))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2 + (d*x)/2)^4*(6*a*b^
2 - 6*a^3) + tan(c/2 + (d*x)/2)^2*(6*a^2*b + 4*a^3 - 2*b^3) + tan(c/2 + (d*
x)/2)^6*(4*a^3 - 6*a^2*b + 2*b^3) + a^3 + b^3 + tan(c/2 + (d*x)/2)^8*(3*a*b
^2 - 3*a^2*b + a^3 - b^3))) - (log(tan(c/2 + (d*x)/2) - 1i)*(A*b*1i - B*a*4
i))/(b^5*d) - (a*atan(((a*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^16 + 128*B^2*a^16
 - 8*A^2*a*b^15 - 128*B^2*a^15*b + 44*A^2*a^2*b^14 + 48*A^2*a^3*b^13 - 92*A
^2*a^4*b^12 - 120*A^2*a^5*b^11 + 156*A^2*a^6*b^10 + 160*A^2*a^7*b^9 - 164*A
^2*a^8*b^8 - 120*A^2*a^9*b^7 + 117*A^2*a^10*b^6 + 48*A^2*a^11*b^5 - 48*A^2*
```

a^12*b^4 - 8*A^2*a^13*b^3 + 8*A^2*a^14*b^2 + 64*B^2*a^2*b^14 - 128*B^2*a^3*
b^13 + 80*B^2*a^4*b^12 + 768*B^2*a^5*b^11 - 824*B^2*a^6*b^10 - 1920*B^2*a^7
*b^9 + 2025*B^2*a^8*b^8 + 2560*B^2*a^9*b^7 - 2600*B^2*a^10*b^6 - 1920*B^2*a
^11*b^5 + 1920*B^2*a^12*b^4 + 768*B^2*a^13*b^3 - 768*B^2*a^14*b^2 - 32*A*B*
a*b^15 - 64*A*B*a^15*b + 64*A*B*a^2*b^14 - 160*A*B*a^3*b^13 - 384*A*B*a^4*b
^12 + 592*A*B*a^5*b^11 + 960*A*B*a^6*b^10 - 1128*A*B*a^7*b^9 - 1280*A*B*a^8
*b^8 + 1306*A*B*a^9*b^7 + 960*A*B*a^10*b^6 - 948*A*B*a^11*b^5 - 384*A*B*a^1
2*b^4 + 384*A*B*a^13*b^3 + 64*A*B*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 -
5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*
b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) + (a*(-(a + b)^7*(a - b)^7)^(1/2)*
((8*(4*A*b^24 - 12*A*a^2*b^22 + 64*A*a^3*b^21 + 20*A*a^4*b^20 - 110*A*a^5*b
^19 - 30*A*a^6*b^18 + 110*A*a^7*b^17 + 30*A*a^8*b^16 - 70*A*a^9*b^15 - 14*A
*a^10*b^14 + 26*A*a^11*b^13 + 2*A*a^12*b^12 - 4*A*a^13*b^11 + 40*B*a^2*b^22
 + 72*B*a^3*b^21 - 190*B*a^4*b^20 - 146*B*a^5*b^19 + 386*B*a^6*b^18 + 174*B
*a^7*b^17 - 434*B*a^8*b^16 - 126*B*a^9*b^15 + 286*B*a^10*b^14 + 50*B*a^11*b
^13 - 104*B*a^12*b^12 - 8*B*a^13*b^11 + 16*B*a^14*b^10 - 16*A*a*b^23 - 16*B
*a*b^23))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b
^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^1
1*b^12) - (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8
*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*
b - 20*B*a*b^6)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^
5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^
10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11 - 8*a^14*b^10))/((b^19
- 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^
12*b^7 - a^14*b^5)*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 +
 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b
^9 - a^11*b^8)))*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*
b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6))/(2*(b^19 - 7*a^2*b^17 + 21*a^
4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)))
(-(a + b)^7(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^
3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6)*1i)/(2*(b^19 - 7*
a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b
^7 - a^14*b^5)) + (a*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^16 + 128*B^2*a^16 - 8*
A^2*a*b^15 - 128*B^2*a^15*b + 44*A^2*a^2*b^14 + 48*A^2*a^3*b^13 - 92*A^2*a^
4*b^12 - 120*A^2*a^5*b^11 + 156*A^2*a^6*b^10 + 160*A^2*a^7*b^9 - 164*A^2*a^
8*b^8 - 120*A^2*a^9*b^7 + 117*A^2*a^10*b^6 + 48*A^2*a^11*b^5 - 48*A^2*a^12*
b^4 - 8*A^2*a^13*b^3 + 8*A^2*a^14*b^2 + 64*B^2*a^2*b^14 - 128*B^2*a^3*b^13
+ 80*B^2*a^4*b^12 + 768*B^2*a^5*b^11 - 824*B^2*a^6*b^10 - 1920*B^2*a^7*b^9
+ 2025*B^2*a^8*b^8 + 2560*B^2*a^9*b^7 - 2600*B^2*a^10*b^6 - 1920*B^2*a^11*b
^5 + 1920*B^2*a^12*b^4 + 768*B^2*a^13*b^3 - 768*B^2*a^14*b^2 - 32*A*B*a*b^1
5 - 64*A*B*a^15*b + 64*A*B*a^2*b^14 - 160*A*B*a^3*b^13 - 384*A*B*a^4*b^12 +
 592*A*B*a^5*b^11 + 960*A*B*a^6*b^10 - 1128*A*B*a^7*b^9 - 1280*A*B*a^8*b^8
+ 1306*A*B*a^9*b^7 + 960*A*B*a^10*b^6 - 948*A*B*a^11*b^5 - 384*A*B*a^12*b^4
 + 384*A*B*a^13*b^3 + 64*A*B*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3
*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11

```
+ 5*a^9*b^10 - a^10*b^9 - a^11*b^8) - (a*(-(a +...
```

3.274 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=301

$$\frac{Bx}{b^4}-\frac{(3a^2Ab^5+2Ab^7+2a^7B-7a^5b^2B+8a^3b^4B-8ab^6B)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^4(a+b)^{7/2}d}+\frac{a(Ab-aB)\,\mathrm{c}}{3b\,(a^2-b^2)}$$

```
[Out] B*x/b^4-(3*A*a^2*b^5+2*A*b^7+2*B*a^7-7*B*a^5*b^2+8*B*a^3*b^4-8*B*a*b^6)*arc
tan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^4/(a+b)^(7/2)
/d+1/3*a*(A*b-B*a)*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^3
+1/6*a^2*(5*A*b^3+3*B*a^3-8*B*a*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*cos(
d*x+c))^2-1/6*a*(A*a^2*b^3-16*A*b^5+9*B*a^5-28*B*a^3*b^2+34*B*a*b^4)*sin(d*
x+c)/b^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.78, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3068, 3110, 3100, 2814, 2738, 211}

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\,(a^2-b^2)\,(a+b\cos(c+dx))^3}+\frac{a^2(3a^3B-8ab^2B+5Ab^3)\sin(c+dx)}{6b^3d\,(a^2-b^2)^2\,(a+b\cos(c+dx))^2}-\frac{a(9a^5B-28a^3b^2B+a^2Ab^3+34ab^4B-16Ab^5)\sin(c+dx)}{6b^3d\,(a^2-b^2)^3\,(a+b\cos(c+dx))}-\frac{(2a^7B-7a^5b^2B+8a^3b^4B+3a^2Ab^5-8ab^6B+2Ab^7)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{Bx}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (B*x)/b^4 - ((3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B - 7*a^5*b^2*B + 8*a^3*b^4*B -
 8*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7
/2)*b^4*(a + b)^(7/2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (a^2*(5*A*b^3 + 3*a^3*B - 8*a*b^2*
B)*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - (a*(a^2*A
*b^3 - 16*A*b^5 + 9*a^5*B - 28*a^3*b^2*B + 34*a*b^4*B)*Sin[c + d*x])/(6*b^3
*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)(-2a(Ab-aB)+3b(A}{(a}}{3}$$

$$= \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{a^2(5Ab^3+3a^3B-8ab^2B}{6b^3\,(a^2-b^2)^2\,d(a+b\,\text{co}}$$

$$= \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{a^2(5Ab^3+3a^3B-8ab^2B}{6b^3\,(a^2-b^2)^2\,d(a+b\,\text{co}}$$

$$= \frac{Bx}{b^4} + \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{a^2(5Ab^3+3a^3B-}{6b^3\,(a^2-b^2)^2\,d(a}$$

$$= \frac{Bx}{b^4} + \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{a^2(5Ab^3+3a^3B-}{6b^3\,(a^2-b^2)^2\,d(a}$$

$$= \frac{Bx}{b^4} - \frac{(3a^2Ab^5+2Ab^7+2a^7B-7a^5b^2B+8a^3b^4B-8ab^6B)\tan}{(a-b)^{7/2}b^4(a+b)^{7/2}d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 717 vs. 2(301) = 602.
time = 3.32, size = 717, normalized size = 2.38

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-24*(3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B - 7*a^5*b^2*B + 8*a^3*b^4*B - 8*a*b^
6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/
2) + (24*a^9*B*c - 36*a^7*b^2*B*c - 36*a^5*b^4*B*c + 84*a^3*b^6*B*c - 36*a*
b^8*B*c + 24*a^9*B*d*x - 36*a^7*b^2*B*d*x - 36*a^5*b^4*B*d*x + 84*a^3*b^6*B
*d*x - 36*a*b^8*B*d*x + 18*b*(a^2 - b^2)^3*(4*a^2 + b^2)*B*(c + d*x)*Cos[c
+ d*x] + 36*a*b^2*(a^2 - b^2)^3*B*(c + d*x)*Cos[2*(c + d*x)] + 6*a^6*b^3*B*
c*Cos[3*(c + d*x)] - 18*a^4*b^5*B*c*Cos[3*(c + d*x)] + 18*a^2*b^7*B*c*Cos[3
*(c + d*x)] - 6*b^9*B*c*Cos[3*(c + d*x)] + 6*a^6*b^3*B*d*x*Cos[3*(c + d*x)]
 - 18*a^4*b^5*B*d*x*Cos[3*(c + d*x)] + 18*a^2*b^7*B*d*x*Cos[3*(c + d*x)] -
6*b^9*B*d*x*Cos[3*(c + d*x)] + 18*a^5*A*b^4*Sin[c + d*x] + 39*a^3*A*b^6*Sin
[c + d*x] + 18*a*A*b^8*Sin[c + d*x] - 24*a^8*b*B*Sin[c + d*x] + 57*a^6*b^3*
B*Sin[c + d*x] - 72*a^4*b^5*B*Sin[c + d*x] - 36*a^2*b^7*B*Sin[c + d*x] + 6*
a^4*A*b^5*Sin[2*(c + d*x)] + 54*a^2*A*b^7*Sin[2*(c + d*x)] - 30*a^7*b^2*B*S
in[2*(c + d*x)] + 90*a^5*b^4*B*Sin[2*(c + d*x)] - 120*a^3*b^6*B*Sin[2*(c +
```

```
d*x)] + 2*a^5*A*b^4*Sin[3*(c + d*x)] - 5*a^3*A*b^6*Sin[3*(c + d*x)] + 18*a*
A*b^8*Sin[3*(c + d*x)] - 11*a^6*b^3*B*Sin[3*(c + d*x)] + 32*a^4*b^5*B*Sin[3
*(c + d*x)] - 36*a^2*b^7*B*Sin[3*(c + d*x)])/((a^2 - b^2)^3*(a + b*Cos[c +
d*x])^3))/(24*b^4*d)
```

Maple [A]
time = 0.90, size = 459, normalized size = 1.52

method	result
derivativedivides	$-\frac{2\left(\frac{-\frac{\left(2A\,a^{2}b^{3}+3Aa\,b^{4}+6A\,b^{5}-2B\,a^{5}+B\,a^{4}b+6B\,a^{3}b^{2}-4B\,a^{2}b^{3}-12Ba\,b^{4}\right)ab\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{3}+3a^{2}b+3b^{2}a+b^{3}\right)}-\frac{2\left(A\,a^{2}b^{3}+9A\,b^{5}-3B\,a^{5}+11B\,a^{3}b^{2}\cdots\right.}{3\left(a^{2}+2ab+b^{2}\right)\left(\cdots\right.}}{\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\cdots\right.}\right.}{\cdots}$
default	$-\frac{2\left(\frac{-\frac{\left(2A\,a^{2}b^{3}+3Aa\,b^{4}+6A\,b^{5}-2B\,a^{5}+B\,a^{4}b+6B\,a^{3}b^{2}-4B\,a^{2}b^{3}-12Ba\,b^{4}\right)ab\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{3}+3a^{2}b+3b^{2}a+b^{3}\right)}-\frac{2\left(A\,a^{2}b^{3}+9A\,b^{5}-3B\,a^{5}+11B\,a^{3}b^{2}\cdots\right.}{3\left(a^{2}+2ab+b^{2}\right)\left(\cdots\right.}}{\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\cdots\right.}\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-2/b^4*((-1/2*(2*A*a^2*b^3+3*A*a*b^4+6*A*b^5-2*B*a^5+B*a^4*b+6*B*a^3*b
^2-4*B*a^2*b^3-12*B*a*b^4)*a*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+
1/2*c)^5-2/3*(A*a^2*b^3+9*A*b^5-3*B*a^5+11*B*a^3*b^2-18*B*a*b^4)*a*b/(a^2+2
*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(2*A*a^2*b^3-3*A*a*b^4+6
*A*b^5-2*B*a^5-B*a^4*b+6*B*a^3*b^2+4*B*a^2*b^3-12*B*a*b^4)*a*b/(a+b)/(a^3-3
*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d
*x+1/2*c)^2+a+b)^3+1/2*(3*A*a^2*b^5+2*A*b^7+2*B*a^7-7*B*a^5*b^2+8*B*a^3*b^4
-8*B*a*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/
2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))+2*B/b^4*arctan(tan(1/2*d*x+1/2*c))
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(286) = 572$.
time = 0.53, size = 1857, normalized size = 6.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(12*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11)*d*
x*cos(d*x + c)^3 + 36*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8
+ B*a*b^10)*d*x*cos(d*x + c)^2 + 36*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^5 -
 4*B*a^4*b^7 + B*a^2*b^9)*d*x*cos(d*x + c) + 12*(B*a^11 - 4*B*a^9*b^2 + 6*B
*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*d*x + 3*(2*B*a^10 - 7*B*a^8*b^2 + 8*B*a
^6*b^4 + 3*A*a^5*b^5 - 8*B*a^4*b^6 + 2*A*a^3*b^7 + (2*B*a^7*b^3 - 7*B*a^5*b
^5 + 8*B*a^3*b^7 + 3*A*a^2*b^8 - 8*B*a*b^9 + 2*A*b^10)*cos(d*x + c)^3 + 3*(
2*B*a^8*b^2 - 7*B*a^6*b^4 + 8*B*a^4*b^6 + 3*A*a^3*b^7 - 8*B*a^2*b^8 + 2*A*a
*b^9)*cos(d*x + c)^2 + 3*(2*B*a^9*b - 7*B*a^7*b^3 + 8*B*a^5*b^5 + 3*A*a^4*b
^6 - 8*B*a^3*b^7 + 2*A*a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*c
os(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x
+ c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x +
 c) + a^2)) - 2*(6*B*a^10*b - 23*B*a^8*b^3 - 4*A*a^7*b^4 + 43*B*a^6*b^5 - 7
*A*a^5*b^6 - 26*B*a^4*b^7 + 11*A*a^3*b^8 + (11*B*a^8*b^3 - 2*A*a^7*b^4 - 43
*B*a^6*b^5 + 7*A*a^5*b^6 + 68*B*a^4*b^7 - 23*A*a^3*b^8 - 36*B*a^2*b^9 + 18*
A*a*b^10)*cos(d*x + c)^2 + 3*(5*B*a^9*b^2 - 20*B*a^7*b^4 - A*a^6*b^5 + 35*B
*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^3*b^8 + 9*A*a^2*b^9)*cos(d*x + c))*sin(d*x
+ c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d*cos(d*x + c
)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x
+ c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos
(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 + a^3*b^12)*d),
1/6*(6*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11)*d*x*c
os(d*x + c)^3 + 18*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8 + B
*a*b^10)*d*x*cos(d*x + c)^2 + 18*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^5 - 4*
B*a^4*b^7 + B*a^2*b^9)*d*x*cos(d*x + c) + 6*(B*a^11 - 4*B*a^9*b^2 + 6*B*a^7
*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*d*x - 3*(2*B*a^10 - 7*B*a^8*b^2 + 8*B*a^6*b
^4 + 3*A*a^5*b^5 - 8*B*a^4*b^6 + 2*A*a^3*b^7 + (2*B*a^7*b^3 - 7*B*a^5*b^5 +
 8*B*a^3*b^7 + 3*A*a^2*b^8 - 8*B*a*b^9 + 2*A*b^10)*cos(d*x + c)^3 + 3*(2*B*
a^8*b^2 - 7*B*a^6*b^4 + 8*B*a^4*b^6 + 3*A*a^3*b^7 - 8*B*a^2*b^8 + 2*A*a*b^9
)*cos(d*x + c)^2 + 3*(2*B*a^9*b - 7*B*a^7*b^3 + 8*B*a^5*b^5 + 3*A*a^4*b^6 -
```

```
 8*B*a^3*b^7 + 2*A*a^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*
x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*B*a^10*b - 23*B*a^8*b^3 -
4*A*a^7*b^4 + 43*B*a^6*b^5 - 7*A*a^5*b^6 - 26*B*a^4*b^7 + 11*A*a^3*b^8 + (1
1*B*a^8*b^3 - 2*A*a^7*b^4 - 43*B*a^6*b^5 + 7*A*a^5*b^6 + 68*B*a^4*b^7 - 23*
A*a^3*b^8 - 36*B*a^2*b^9 + 18*A*a*b^10)*cos(d*x + c)^2 + 3*(5*B*a^9*b^2 - 2
0*B*a^7*b^4 - A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^3*b^8 + 9*A*a
^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*
a^2*b^13 + b^15)*d*cos(d*x + c)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4
*a^3*b^12 + a*b^14)*d*cos(d*x + c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9
- 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8
 - 4*a^5*b^10 + a^3*b^12)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(286) = 572.
time = 0.49, size = 813, normalized size = 2.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(2*B*a^7 - 7*B*a^5*b^2 + 8*B*a^3*b^4 + 3*A*a^2*b^5 - 8*B*a*b^6 + 2*A
*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^4 - 3*a^
4*b^6 + 3*a^2*b^8 - b^10)*sqrt(a^2 - b^2)) + 3*(d*x + c)*B/b^4 - (6*B*a^8*t
an(1/2*d*x + 1/2*c)^5 - 15*B*a^7*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^6*b^2*tan
(1/2*d*x + 1/2*c)^5 - 6*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 45*B*a^5*b^3*tan
(1/2*d*x + 1/2*c)^5 + 3*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^4*b^4*tan(
1/2*d*x + 1/2*c)^5 - 6*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*B*a^3*b^5*tan(
1/2*d*x + 1/2*c)^5 + 27*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b^6*tan
(1/2*d*x + 1/2*c)^5 - 18*A*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^8*tan(1/2*
d*x + 1/2*c)^3 - 56*B*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^5*b^3*tan(1/2*
d*x + 1/2*c)^3 + 116*B*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 32*A*a^3*b^5*tan(1/
2*d*x + 1/2*c)^3 - 72*B*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*A*a*b^7*tan(1/2
*d*x + 1/2*c)^3 + 6*B*a^8*tan(1/2*d*x + 1/2*c) + 15*B*a^7*b*tan(1/2*d*x + 1
```

```
/2*c) - 6*B*a^6*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^5*b^3*tan(1/2*d*x + 1/2*c)
 - 45*B*a^5*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6
*B*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6*A*a^3*b^5*tan(1/2*d*x + 1/2*c) + 60*B*a
^3*b^5*tan(1/2*d*x + 1/2*c) - 27*A*a^2*b^6*tan(1/2*d*x + 1/2*c) + 36*B*a^2*
b^6*tan(1/2*d*x + 1/2*c) - 18*A*a*b^7*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c
)^2 + a + b)^3))/d
```

Mupad [B]
time = 12.58, size = 2500, normalized size = 8.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^4,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(3*A*a^2*b^4 - 2*B*a^6 + 2*A*a^3*b^3 - 12*B*a^2*b^4
- 4*B*a^3*b^3 + 6*B*a^4*b^2 + 6*A*a*b^5 + B*a^5*b))/((a*b^3 - b^4)*(a + b)^
3) - (tan(c/2 + (d*x)/2)*(2*B*a^6 + 3*A*a^2*b^4 - 2*A*a^3*b^3 + 12*B*a^2*b^
4 - 4*B*a^3*b^3 - 6*B*a^4*b^2 - 6*A*a*b^5 + B*a^5*b))/((a + b)*(3*a*b^5 - b
^6 - 3*a^2*b^4 + a^3*b^3)) + (4*tan(c/2 + (d*x)/2)^3*(A*a^3*b^3 - 3*B*a^6 -
 18*B*a^2*b^4 + 11*B*a^4*b^2 + 9*A*a*b^5))/(3*(a + b)^2*(b^5 - 2*a*b^4 + a^
2*b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*
b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b +
 a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (2*B*
atan(((B*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B
^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^
12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8
+ 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^10*b^4 +
48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*
B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/(a*b^16 + b^17 - 5*a^2*b^15 -
 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8
*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) + (B*((8*(4*A*b^21 + 4*B*b^21 - 6*A
*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*a^6*b^15 - 14
*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64*B*a^3*b^18 +
 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^14 + 30*B*a^8
*b^13 - 70*B*a^9*b^12 - 14*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*a^12*b^9 - 4*
B*a^13*b^8 - 4*A*a*b^20 - 16*B*a*b^20))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3
*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12
+ 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (B*tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*
a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^
7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^
12*b^10 + 8*a^13*b^9 - 8*a^14*b^8)*8i)/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5
*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b
^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*1i)/b^4))/b^4 + (B*((8*tan(c/2 + (d
```

x)/2)(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^10*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) - (B*((8*(4*A*b^21 + 4*B*b^21 - 6*A*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*a^6*b^15 - 14*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64*B*a^3*b^18 + 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^14 + 30*B*a^8*b^13 - 70*B*a^9*b^12 - 14*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*a^12*b^9 - 4*B*a^13*b^8 - 4*A*a*b^20 - 16*B*a*b^20))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (B*tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8)*8i)/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*1i)/b^4))/b^4)/((16*(4*B^3*a^13 - 4*A*B^2*b^13 + 4*A^2*B*b^13 + 16*B^3*a*b^12 - 2*B^3*a^12*b + 48*B^3*a^2*b^11 - 64*B^3*a^3*b^10 - 64*B^3*a^4*b^9 + 110*B^3*a^5*b^8 + 66*B^3*a^6*b^7 - 110*B^3*a^7*b^6 - 34*B^3*a^8*b^5 + 70*B^3*a^9*b^4 + 11*B^3*a^10*b^3 - 26*B^3*a^11*b^2 - 28*A*B^2*a*b^12 + 6*A*B^2*a^2*b^11 - 22*A*B^2*a^3*b^10 + 6*A*B^2*a^4*b^9 + 14*A*B^2*a^5*b^8 - 14*A*B^2*a^6*b^7 - 20*A*B^2*a^7*b^6 + 6*A*B^2*a^8*b^5 + 6*A*B^2*a^9*b^4 + 12*A^2*B*a^2*b^11 + 9*A^2*B*a^4*b^9))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (B*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^10*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) + (B*((8*(4*A*b^21 + 4*B*b^21 - 6*A*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*a^6*b^15 - 14*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64*B*a^3*b^18 + 20*B*a^4*b^...

3.275 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=274

$$\frac{(a^3A+4aAb^2-3a^2bB-2b^3B)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}+\frac{a(a^2A\cdots}{6\cdots}$$

```
[Out] (A*a^3+4*A*a*b^2-3*B*a^2*b-2*B*b^3)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(
a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/3*a^2*(A*b-B*a)*sin(d*x+c)/b^2/(a^2
-b^2)/d/(a+b*cos(d*x+c))^3+1/6*a*(A*a^2*b-6*A*b^3-4*B*a^3+9*B*a*b^2)*sin(d*
x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*(A*a^4*b-10*A*a^2*b^3-6*A*b^5
+2*B*a^5-5*B*a^3*b^2+18*B*a*b^4)*sin(d*x+c)/b^2/(a^2-b^2)^3/d/(a+b*cos(d*x+
c))
```

Rubi [A]
time = 0.41, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3067, 3100, 2833, 12, 2738, 211}

$$-\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{(a^3A-3a^2bB+4aAb^2-2b^3B)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a(-4a^3B+a^2Ab+9ab^2B-6Ab^3)\sin(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{(2a^5B+a^4Ab-5a^3b^2B-10a^2Ab^3+18ab^4B-6Ab^5)\sin(c+dx)}{6b^2d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (a^2*(A*b - a*B)*Sin[
c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (a*(a^2*A*b - 6*A*
b^3 - 4*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[
c + d*x])^2) + ((a^4*A*b - 10*a^2*A*b^3 - 6*A*b^5 + 2*a^5*B - 5*a^3*b^2*B +
 18*a*b^4*B)*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{\int \frac{3ab(Ab-aB)+(a^2-3b^2)(Ab-aB\cdots}{(a+b\cdots}}{3b^2\cdots}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+\cdots}{6b^2(a^2-b^2)^2\,d(a+b\cdots}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+\cdots}{6b^2(a^2-b^2)^2\,d(a+b\cdots}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+\cdots}{6b^2(a^2-b^2)^2\,d(a+b\cdots}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+\cdots}{6b^2(a^2-b^2)^2\,d(a+b\cdots}$$

$$= \frac{(a^3A+4aAb^2-3a^2bB-2b^3B)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \cdots$$

Mathematica [A]

time = 1.35, size = 251, normalized size = 0.92

$$\frac{-\frac{24(a^3A+4aAb^2-3a^2bB-2b^3B)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{2(-25a^4Ab-14a^2Ab^3-6Ab^5+10a^5B+17a^3b^2B+18ab^4B+6a(a^4A-9a^2Ab^2-2Ab^4+a^3bB+9ab^3B)\cos(c+dx)+(a^4Ab-10a^2Ab^3-6Ab^5+2a^5B-5a^3b^2B+18ab^4B)\cos(2(c+dx)))\sin(c+dx)}{(a+b\cos(c+dx))^3}}{24(a^2-b^2)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-24*(a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTanh[((a - b)*Tan[(c + d
*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(-25*a^4*A*b - 14*a^2*A*b^
3 - 6*A*b^5 + 10*a^5*B + 17*a^3*b^2*B + 18*a*b^4*B + 6*a*(a^4*A - 9*a^2*A*b
^2 - 2*A*b^4 + a^3*b*B + 9*a*b^3*B)*Cos[c + d*x] + (a^4*A*b - 10*a^2*A*b^3
- 6*A*b^5 + 2*a^5*B - 5*a^3*b^2*B + 18*a*b^4*B)*Cos[2*(c + d*x)])*Sin[c + d
*x])/(a + b*Cos[c + d*x])^3)/(24*(a^2 - b^2)^3*d)
```

Maple [A]

time = 0.71, size = 371, normalized size = 1.35

method	result
derivativedivides	$\frac{-\frac{(Aa^3+6Aa^2b+2Aab^2+2Ab^3-2a^3B-3a^2bB-6Bab^2)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^3+3a^2b+3b^2a+b^3)}-\frac{4(7Aa^2b+3Ab^3-a^3B-9Bab^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2-2ab+b^2)(a^2+2ab+b^2)}+\cdots}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^3}$

default	$\frac{-\frac{\left(A\,a^{3}+6A\,a^{2}b+2Aa\,b^{2}+2A\,b^{3}-2a^{3}B-3a^{2}bB-6Ba\,b^{2}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)\left(a^{3}+3a^{2}b+3b^{2}a+b^{3}\right)}-\frac{4\left(7A\,a^{2}b+3A\,b^{3}-a^{3}B-9Ba\,b^{2}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^{2}-2ab+b^{2}\right)\left(a^{2}+2ab+b^{2}\right)}+\frac{\left(A\,a^{3}\right.}{}}{\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^{3}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2*(-1/2*(A*a^3+6*A*a^2*b+2*A*a*b^2+2*A*b^3-2*B*a^3-3*B*a^2*b-6*B*a*b^2
)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(7*A*a^2*b+3*A*b
^3-B*a^3-9*B*a*b^2)/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/
2*(A*a^3-6*A*a^2*b+2*A*a*b^2-2*A*b^3+2*B*a^3-3*B*a^2*b+6*B*a*b^2)/(a+b)/(a^
3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/
2*d*x+1/2*c)^2+a+b)^3+(A*a^3+4*A*a*b^2-3*B*a^2*b-2*B*b^3)/(a^6-3*a^4*b^2+3*
a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(260) = 520.
time = 0.47, size = 1220, normalized size = 4.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [-1/12*(3*(A*a^6 - 3*B*a^5*b + 4*A*a^4*b^2 - 2*B*a^3*b^3 + (A*a^3*b^3 - 3*B
*a^2*b^4 + 4*A*a*b^5 - 2*B*b^6)*cos(d*x + c)^3 + 3*(A*a^4*b^2 - 3*B*a^3*b^3
```

```
 + 4*A*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c)^2 + 3*(A*a^5*b - 3*B*a^4*b^2 + 4*A
*a^3*b^3 - 2*B*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x +
 c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b
)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^
2)) - 2*(4*B*a^7 - 13*A*a^6*b + 7*B*a^5*b^2 + 11*A*a^4*b^3 - 11*B*a^3*b^4 +
 2*A*a^2*b^5 + (2*B*a^7 + A*a^6*b - 7*B*a^5*b^2 - 11*A*a^4*b^3 + 23*B*a^3*b
^4 + 4*A*a^2*b^5 - 18*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^2 + 3*(A*a^7 + B*a^6*
b - 10*A*a^5*b^2 + 8*B*a^4*b^3 + 7*A*a^3*b^4 - 9*B*a^2*b^5 + 2*A*a*b^6)*cos
(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^
11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b
^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2
*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)
*d), 1/6*(3*(A*a^6 - 3*B*a^5*b + 4*A*a^4*b^2 - 2*B*a^3*b^3 + (A*a^3*b^3 - 3
*B*a^2*b^4 + 4*A*a*b^5 - 2*B*b^6)*cos(d*x + c)^3 + 3*(A*a^4*b^2 - 3*B*a^3*b
^3 + 4*A*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c)^2 + 3*(A*a^5*b - 3*B*a^4*b^2 + 4
*A*a^3*b^3 - 2*B*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x
+ c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (4*B*a^7 - 13*A*a^6*b + 7*B*a^5
*b^2 + 11*A*a^4*b^3 - 11*B*a^3*b^4 + 2*A*a^2*b^5 + (2*B*a^7 + A*a^6*b - 7*B
*a^5*b^2 - 11*A*a^4*b^3 + 23*B*a^3*b^4 + 4*A*a^2*b^5 - 18*B*a*b^6 + 6*A*b^7
)*cos(d*x + c)^2 + 3*(A*a^7 + B*a^6*b - 10*A*a^5*b^2 + 8*B*a^4*b^3 + 7*A*a^
3*b^4 - 9*B*a^2*b^5 + 2*A*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*
a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a
^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a
^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^
2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. 2(260) = 520.
time = 0.50, size = 689, normalized size = 2.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(A*a^3 - 3*B*a^2*b + 4*A*a*b^2 - 2*B*b^3)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 -
 b^2)) + (3*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^5*tan(1/2*d*x + 1/2*c)^5 +
 12*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 3*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*
A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*
A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*
A*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 18*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^
5*tan(1/2*d*x + 1/2*c)^5 - 4*B*a^5*tan(1/2*d*x + 1/2*c)^3 + 28*A*a^4*b*tan(
1/2*d*x + 1/2*c)^3 - 32*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*A*a^2*b^3*tan
(1/2*d*x + 1/2*c)^3 + 36*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^5*tan(1/2*
d*x + 1/2*c)^3 - 3*A*a^5*tan(1/2*d*x + 1/2*c) - 6*B*a^5*tan(1/2*d*x + 1/2*c
) + 12*A*a^4*b*tan(1/2*d*x + 1/2*c) - 3*B*a^4*b*tan(1/2*d*x + 1/2*c) + 27*A
*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*B*a^3*b^2*tan(1/2*d*x + 1/2*c) + 12*A*a^2
*b^3*tan(1/2*d*x + 1/2*c) - 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a*b^4*t
an(1/2*d*x + 1/2*c) - 18*B*a*b^4*tan(1/2*d*x + 1/2*c) + 6*A*b^5*tan(1/2*d*x
 + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

Mupad [B]
time = 4.15, size = 440, normalized size = 1.61

$$\frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}}\right)\,\left(A\,a^3-3\,B\,a^2\,b+4\,A\,a\,b^2-2\,B\,b^3\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}-\frac{\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(-B\,a^3+7\,A\,a^2\,b-9\,B\,a\,b^2+3\,A\,b^3\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(A\,a^3+2\,A\,b^3-2\,B\,a^3+2\,A\,a\,b^2+6\,A\,a^2\,b-6\,B\,a\,b^2-3\,B\,a^2\,b\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A\,a^3-2\,A\,b^3+2\,B\,a^3+2\,A\,a\,b^2-6\,A\,a^2\,b+6\,B\,a\,b^2-3\,B\,a^2\,b\right)}{\left(a+b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}}{d\,\left(3\,a\,b^2-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)+3\,a^2\,b+a^3+b^3+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^4,x)
```

```
[Out] (atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(
a + b)^(1/2)*(a - b)^(7/2)))*(A*a^3 - 2*B*b^3 + 4*A*a*b^2 - 3*B*a^2*b))/(d*
(a + b)^(7/2)*(a - b)^(7/2)) - ((4*tan(c/2 + (d*x)/2)^3*(3*A*b^3 - B*a^3 +
7*A*a^2*b - 9*B*a*b^2))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x
)/2)^5*(A*a^3 + 2*A*b^3 - 2*B*a^3 + 2*A*a*b^2 + 6*A*a^2*b - 6*B*a*b^2 - 3*B
*a^2*b))/((a + b)^3*(a - b)) - (tan(c/2 + (d*x)/2)*(A*a^3 - 2*A*b^3 + 2*B*a
^3 + 2*A*a*b^2 - 6*A*a^2*b + 6*B*a*b^2 - 3*B*a^2*b))/((a + b)*(3*a*b^2 - 3*
a^2*b + a^3 - b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b
- 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3)
 + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^
3)))
```

3.276 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=263

$$-\frac{(4a^2Ab+Ab^3-a^3B-4ab^2B)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}+\frac{a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}+\frac{(2a^2A}{6}$$

```
[Out] -(4*A*a^2*b+A*b^3-B*a^3-4*B*a*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a
+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d+1/3*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)
/d/(a+b*cos(d*x+c))^3+1/6*(2*A*a^2*b+3*A*b^3+B*a^3-6*B*a*b^2)*sin(d*x+c)/b/
(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*(2*A*a^3*b+13*A*a*b^3+B*a^4-10*B*a^2*b
^2-6*B*b^4)*sin(d*x+c)/b/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.36, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3047, 3100, 2833, 12, 2738, 211}

$$\frac{a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\left(a^3(-B)+4a^2Ab-4ab^2B+Ab^3\right)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(a^3B+2a^2Ab-6ab^2B+3Ab^3\right)\sin(c+dx)}{6bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{\left(a^4B+2a^3Ab-10a^2b^2B+13aAb^3-6b^4B\right)\sin(c+dx)}{6bd\left(a^2-b^2\right)^3(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -(((4*a^2*A*b + A*b^3 - a^3*B - 4*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) + (a*(A*b - a*B)*Sin[c
 + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((2*a^2*A*b + 3*A*b^3
 + a^3*B - 6*a*b^2*B)*Sin[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x
])^2) + ((2*a^3*A*b + 13*a*A*b^3 + a^4*B - 10*a^2*b^2*B - 6*b^4*B)*Sin[c +
d*x])/(6*b*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^3} - \frac{\int \frac{3b(Ab-aB)-(2aAb+a^2B-3b^2B)\cos}{(a+b\cos(c+dx))^3}}{3b(a^2-b^2)}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2}{6b(a^2-b^2)^2 d(a+b\cos(}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2}{6b(a^2-b^2)^2 d(a+b\cos(}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2}{6b(a^2-b^2)^2 d(a+b\cos(}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2}{6b(a^2-b^2)^2 d(a+b\cos(}$$

$$= -\frac{(4a^2Ab+Ab^3-a^3B-4ab^2B)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + $$

Mathematica [A]
time = 1.19, size = 252, normalized size = 0.96

$$\frac{-\frac{24(-4a^2Ab-Ab^3+a^3B+4ab^2B)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{2(12a^5A+22a^3Ab^2+11aAb^4-25a^4bB-14a^2b^3B-6b^5B+6(2a^4Ab+9a^2Ab^3-Ab^5+a^5B-9a^3b^2B-2ab^4B)\cos(c+dx)+b(2a^3Ab+13aAb^3+a^4B-10a^2b^2B-6b^4B)\cos(2(c+dx)))\sin(c+dx)}{(a+b\cos(c+dx))^3}}{24(a^2-b^2)^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-24*(-4*a^2*A*b - A*b^3 + a^3*B + 4*a*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*
x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(12*a^5*A + 22*a^3*A*b^2 +
11*a*A*b^4 - 25*a^4*b*B - 14*a^2*b^3*B - 6*b^5*B + 6*(2*a^4*A*b + 9*a^2*A*b
^3 - A*b^5 + a^5*B - 9*a^3*b^2*B - 2*a*b^4*B)*Cos[c + d*x] + b*(2*a^3*A*b +
 13*a*A*b^3 + a^4*B - 10*a^2*b^2*B - 6*b^4*B)*Cos[2*(c + d*x)])*Sin[c + d*x
])/(a + b*Cos[c + d*x])^3)/(24*(a^2 - b^2)^3*d)
```

Maple [A]
time = 0.63, size = 384, normalized size = 1.46

method	result

derivativedivides	$-\frac{2\left(-\frac{\left(2A\,a^{3}+2A\,a^{2}b+6Aa\,b^{2}+A\,b^{3}-a^{3}B-6a^{2}bB-2Ba\,b^{2}-2b^{3}B\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{3}+3a^{2}b+3b^{2}a+b^{3}\right)}-\frac{2\left(3A\,a^{3}+7Aa\,b^{2}-7a^{2}bB-3b^{3}B\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{3\left(a^{2}-2ab+b^{2}\right)\left(a^{2}+2ab+b^{2}\right)}}{\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^{3}}$
default	$-\frac{2\left(-\frac{\left(2A\,a^{3}+2A\,a^{2}b+6Aa\,b^{2}+A\,b^{3}-a^{3}B-6a^{2}bB-2Ba\,b^{2}-2b^{3}B\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{3}+3a^{2}b+3b^{2}a+b^{3}\right)}-\frac{2\left(3A\,a^{3}+7Aa\,b^{2}-7a^{2}bB-3b^{3}B\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{3\left(a^{2}-2ab+b^{2}\right)\left(a^{2}+2ab+b^{2}\right)}}{\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^{3}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(2*A*a^3+2*A*a^2*b+6*A*a*b^2+A*b^3-B*a^3-6*B*a^2*b-2*B*a*b^2-
2*B*b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(3*A*a^3+
7*A*a*b^2-7*B*a^2*b-3*B*b^3)/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/
2*c)^3-1/2*(2*A*a^3-2*A*a^2*b+6*A*a*b^2-A*b^3+B*a^3-6*B*a^2*b+2*B*a*b^2-2*B
*b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/
2*c)^2-b*tan(1/2*d*x+1/2*c)^2+a+b)^3-(4*A*a^2*b+A*b^3-B*a^3-4*B*a*b^2)/(a^6
-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-
b)/((a-b)*(a+b))^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(247) = 494$.
time = 0.45, size = 1232, normalized size = 4.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] [-1/12*(3*(B*a^6 - 4*A*a^5*b + 4*B*a^4*b^2 - A*a^3*b^3 + (B*a^3*b^3 - 4*A*a
^2*b^4 + 4*B*a*b^5 - A*b^6)*cos(d*x + c)^3 + 3*(B*a^4*b^2 - 4*A*a^3*b^3 + 4
*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + 3*(B*a^5*b - 4*A*a^4*b^2 + 4*B*a^3*b
^3 - A*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2
*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*
x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*
(6*A*a^7 - 13*B*a^6*b + 4*A*a^5*b^2 + 11*B*a^4*b^3 - 11*A*a^3*b^4 + 2*B*a^2
*b^5 + A*a*b^6 + (B*a^6*b + 2*A*a^5*b^2 - 11*B*a^4*b^3 + 11*A*a^3*b^4 + 4*B
*a^2*b^5 - 13*A*a*b^6 + 6*B*b^7)*cos(d*x + c)^2 + 3*(B*a^7 + 2*A*a^6*b - 10
*B*a^5*b^2 + 7*A*a^4*b^3 + 7*B*a^3*b^4 - 10*A*a^2*b^5 + 2*B*a*b^6 + A*b^7)*
cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 +
 b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 +
a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 +
a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b
^8)*d), 1/6*(3*(B*a^6 - 4*A*a^5*b + 4*B*a^4*b^2 - A*a^3*b^3 + (B*a^3*b^3 -
4*A*a^2*b^4 + 4*B*a*b^5 - A*b^6)*cos(d*x + c)^3 + 3*(B*a^4*b^2 - 4*A*a^3*b^
3 + 4*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + 3*(B*a^5*b - 4*A*a^4*b^2 + 4*B*
a^3*b^3 - A*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c)
+ b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (6*A*a^7 - 13*B*a^6*b + 4*A*a^5*b^2
+ 11*B*a^4*b^3 - 11*A*a^3*b^4 + 2*B*a^2*b^5 + A*a*b^6 + (B*a^6*b + 2*A*a^5*
b^2 - 11*B*a^4*b^3 + 11*A*a^3*b^4 + 4*B*a^2*b^5 - 13*A*a*b^6 + 6*B*b^7)*cos
(d*x + c)^2 + 3*(B*a^7 + 2*A*a^6*b - 10*B*a^5*b^2 + 7*A*a^4*b^3 + 7*B*a^3*b
^4 - 10*A*a^2*b^5 + 2*B*a*b^6 + A*b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^
3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2
 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b
 - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*
a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(247) = 494.
time = 0.55, size = 722, normalized size = 2.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(B*a^3 - 4*A*a^2*b + 4*B*a*b^2 - A*b^3)*(pi*floor(1/2*(d*x + c)/pi
+ 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b
^2)) - (6*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^5*tan(1/2*d*x + 1/2*c)^5 - 6
*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*A*
a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*A
*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*
A*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^5
*tan(1/2*d*x + 1/2*c)^5 - 6*B*b^5*tan(1/2*d*x + 1/2*c)^5 + 12*A*a^5*tan(1/2
*d*x + 1/2*c)^3 - 28*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 16*A*a^3*b^2*tan(1/2*
d*x + 1/2*c)^3 + 16*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 28*A*a*b^4*tan(1/2*d
*x + 1/2*c)^3 + 12*B*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^5*tan(1/2*d*x + 1/2
*c) + 3*B*a^5*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b*tan(1/2*d*x + 1/2*c) - 12*B*
a^4*b*tan(1/2*d*x + 1/2*c) + 12*A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*B*a^3*b
^2*tan(1/2*d*x + 1/2*c) + 27*A*a^2*b^3*tan(1/2*d*x + 1/2*c) - 12*B*a^2*b^3*
tan(1/2*d*x + 1/2*c) + 12*A*a*b^4*tan(1/2*d*x + 1/2*c) - 6*B*a*b^4*tan(1/2*
d*x + 1/2*c) - 3*A*b^5*tan(1/2*d*x + 1/2*c) - 6*B*b^5*tan(1/2*d*x + 1/2*c))
/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2
*d*x + 1/2*c)^2 + a + b)^3))/d
```

Mupad [B]
time = 4.03, size = 451, normalized size = 1.71

$$\frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\,A\,a^3-7\,B\,a^2\,b+7\,A\,a\,b^2-3\,B\,b^3\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A\,a^3-A\,b^3+B\,a^3-2\,B\,b^3+6\,A\,a\,b^2-2\,A\,a^2\,b+2\,B\,a\,b^2-6\,B\,a^2\,b\right)}{\left(a+b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,A\,a^3+A\,b^3-B\,a^3-2\,B\,b^3+6\,A\,a\,b^2+2\,A\,a^2\,b-2\,B\,a\,b^2-6\,B\,a^2\,b\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}}{d\,\left(3\,a\,b^2-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)+3\,a^2\,b+a^3+b^3+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)\right)}-\frac{\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}}\right)\,\left(-B\,a^3+4\,A\,a^2\,b-4\,B\,a\,b^2+A\,b^3\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^4,x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^3*(3*A*a^3 - 3*B*b^3 + 7*A*a*b^2 - 7*B*a^2*b))/(3*(a
 + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x)/2)*(2*A*a^3 - A*b^3 + B*a^3
 - 2*B*b^3 + 6*A*a*b^2 - 2*A*a^2*b + 2*B*a*b^2 - 6*B*a^2*b))/((a + b)*(3*a*
b^2 - 3*a^2*b + a^3 - b^3)) + (tan(c/2 + (d*x)/2)^5*(2*A*a^3 + A*b^3 - B*a^
3 - 2*B*b^3 + 6*A*a*b^2 + 2*A*a^2*b - 2*B*a*b^2 - 6*B*a^2*b))/((a + b)^3*(a
 - b)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b
^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b +
a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (atan(
(tan(c/2 + (d*x)/2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)
^(1/2)*(a - b)^(7/2)))*(A*b^3 - B*a^3 + 4*A*a^2*b - 4*B*a*b^2))/(d*(a + b)^
(7/2)*(a - b)^(7/2))
```

3.277 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=237

$$\frac{(2a^3A+3aAb^2-4a^2bB-b^3B)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}-\frac{(5aAb-}{6\left(a^2-\right.}$$

```
[Out] (2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(
a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/3*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/
(a+b*cos(d*x+c))^3-1/6*(5*A*a*b-2*B*a^2-3*B*b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(
a+b*cos(d*x+c))^2-1/6*(11*A*a^2*b+4*A*b^3-2*B*a^3-13*B*a*b^2)*sin(d*x+c)/(a
^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.28, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2833, 12, 2738, 211}

$$-\frac{(-2a^2B+5aAb-3b^2B)\sin(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{(2a^3A-4a^2bB+3aAb^2-b^3B)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{(-2a^3B+11a^2Ab-13ab^2B+4Ab^3)\sin(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - ((A*b - a*B)*Sin[c +
d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - ((5*a*A*b - 2*a^2*B - 3*b^
2*B)*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((11*a^2*A*
b + 4*A*b^3 - 2*a^3*B - 13*a*b^2*B)*Sin[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b
*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}-\frac{\int \frac{-3(aA-bB)+2(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}-\frac{\left(5aAb-2a^2B-3b^2B\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}+\cdots$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}-\frac{\left(5aAb-2a^2B-3b^2B\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}-\cdots$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}-\frac{\left(5aAb-2a^2B-3b^2B\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}-\cdots$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}-\frac{\left(5aAb-2a^2B-3b^2B\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}-\cdots$$

$$= \frac{\left(2a^3A+3aAb^2-4a^2bB-b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\cdots$$

Mathematica [A]
time = 2.31, size = 227, normalized size = 0.96

$$\frac{\frac{6\left(2a^3A+3aAb^2-4a^2bB-b^3B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{7/2}}+\frac{2(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^3}+\frac{\left(-5aAb+2a^2B+3b^2B\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2}+\frac{\left(-11a^2Ab-4Ab^3+2a^3B+13ab^2B\right)\sin(c+dx)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[((a - b)*Tan[(c + d*x
)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + (2*(-(A*b) + a*B)*Sin[c + d*x
```

```
])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3) + ((-5*a*A*b + 2*a^2*B + 3*b^2*
B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2) + ((-11*a^2*A
*b - 4*A*b^3 + 2*a^3*B + 13*a*b^2*B)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(a
+ b*Cos[c + d*x])))/(6*d)
```

Maple [A]
time = 0.55, size = 372, normalized size = 1.57 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b^3
)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(9*A*a^2*b+A*b^3
-3*B*a^3-7*B*a*b^2)/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/
2*(6*A*a^2*b-3*A*a*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a+b)/(a^
3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/
2*d*x+1/2*c)^2+a+b)^3+(2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*b^2+3*
a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. 2(222) = 444.
time = 0.46, size = 1228, normalized size = 5.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*A*a^6 - 4*B*a^5*b + 3*A*a^4*b^2 - B*a^3*b^3 + (2*A*a^3*b^3 - 4
*B*a^2*b^4 + 3*A*a*b^5 - B*b^6)*cos(d*x + c)^3 + 3*(2*A*a^4*b^2 - 4*B*a^3*b
^3 + 3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + 3*(2*A*a^5*b - 4*B*a^4*b^2 + 3
*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x +
```

```
c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b
)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^
2)) - 2*(6*B*a^7 - 18*A*a^6*b + 4*B*a^5*b^2 + 23*A*a^4*b^3 - 11*B*a^3*b^4 -
 7*A*a^2*b^5 + B*a*b^6 + 2*A*b^7 + (2*B*a^5*b^2 - 11*A*a^4*b^3 + 11*B*a^3*b
^4 + 7*A*a^2*b^5 - 13*B*a*b^6 + 4*A*b^7)*cos(d*x + c)^2 + 3*(2*B*a^6*b - 9*
A*a^5*b^2 + 7*B*a^4*b^3 + 8*A*a^3*b^4 - 10*B*a^2*b^5 + A*a*b^6 + B*b^7)*cos
(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^
11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b
^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2
*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)
*d), 1/6*(3*(2*A*a^6 - 4*B*a^5*b + 3*A*a^4*b^2 - B*a^3*b^3 + (2*A*a^3*b^3 -
 4*B*a^2*b^4 + 3*A*a*b^5 - B*b^6)*cos(d*x + c)^3 + 3*(2*A*a^4*b^2 - 4*B*a^3
*b^3 + 3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + 3*(2*A*a^5*b - 4*B*a^4*b^2 +
 3*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x
+ c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (6*B*a^7 - 18*A*a^6*b + 4*B*a^5
*b^2 + 23*A*a^4*b^3 - 11*B*a^3*b^4 - 7*A*a^2*b^5 + B*a*b^6 + 2*A*b^7 + (2*B
*a^5*b^2 - 11*A*a^4*b^3 + 11*B*a^3*b^4 + 7*A*a^2*b^5 - 13*B*a*b^6 + 4*A*b^7
)*cos(d*x + c)^2 + 3*(2*B*a^6*b - 9*A*a^5*b^2 + 7*B*a^4*b^3 + 8*A*a^3*b^4 -
 10*B*a^2*b^5 + A*a*b^6 + B*b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*
a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a
^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a
^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^
2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 691 vs. 2(222) = 444.
time = 0.46, size = 691, normalized size = 2.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 -
```

```
b^2)) - (6*B*a^5*tan(1/2*d*x + 1/2*c)^5 - 18*A*a^4*b*tan(1/2*d*x + 1/2*c)^
5 - 6*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5
+ 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5
- 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 +
12*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*A*b^5*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^
5*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^5*tan(1/2*d*x + 1/2*c)^3 - 36*A*a^4*b*tan
(1/2*d*x + 1/2*c)^3 + 16*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 32*A*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^3 - 28*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*A*b^5*tan(1/2*
d*x + 1/2*c)^3 + 6*B*a^5*tan(1/2*d*x + 1/2*c) - 18*A*a^4*b*tan(1/2*d*x + 1/
2*c) + 6*B*a^4*b*tan(1/2*d*x + 1/2*c) - 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c) +
 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + 27*
B*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a*b^4*tan(1/2*d*x + 1/2*c) + 12*B*a*b^
4*tan(1/2*d*x + 1/2*c) - 6*A*b^5*tan(1/2*d*x + 1/2*c) - 3*B*b^5*tan(1/2*d*x
 + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

Mupad [B]
time = 4.00, size = 440, normalized size = 1.86

$$\frac{\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}}\right)\,\left(2\,A\,a^3-4\,B\,a^2\,b+3\,A\,a\,b^2-B\,b^3\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}-\frac{\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(-3\,B\,a^3+9\,A\,a^2\,b-7\,B\,a\,b^2+A\,b^3\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,B\,a^3-2\,A\,b^3+B\,b^3-3\,A\,a\,b^2-6\,A\,a^2\,b+6\,B\,a\,b^2+2\,B\,a^2\,b\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A\,b^3-2\,B\,a^3+B\,b^3-3\,A\,a\,b^2+6\,A\,a^2\,b-6\,B\,a\,b^2+2\,B\,a^2\,b\right)}{\left(a+b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}}{d\,\left(3\,a\,b^2-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)+3\,a^2\,b+a^3+b^3+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^4,x)
```

```
[Out] (atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(
a + b)^(1/2)*(a - b)^(7/2)))*(2*A*a^3 - B*b^3 + 3*A*a*b^2 - 4*B*a^2*b))/(d*
(a + b)^(7/2)*(a - b)^(7/2)) - ((4*tan(c/2 + (d*x)/2)^3*(A*b^3 - 3*B*a^3 +
9*A*a^2*b - 7*B*a*b^2))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) - (tan(c/2 + (d*x
)/2)^5*(2*B*a^3 - 2*A*b^3 + B*b^3 - 3*A*a*b^2 - 6*A*a^2*b + 6*B*a*b^2 + 2*B
*a^2*b))/((a + b)^3*(a - b)) + (tan(c/2 + (d*x)/2)*(2*A*b^3 - 2*B*a^3 + B*b
^3 - 3*A*a*b^2 + 6*A*a^2*b - 6*B*a*b^2 + 2*B*a^2*b))/((a + b)*(3*a*b^2 - 3*
a^2*b + a^3 - b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b
- 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3)
 + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^
3)))
```

3.278 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=301

$$-\frac{(8a^6Ab-8a^4Ab^3+7a^2Ab^5-2Ab^7-2a^7B-3a^5b^2B)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d}+\frac{A\tanh^{-1}(\sin(c+d$$

```
[Out] -(8*A*a^6*b-8*A*a^4*b^3+7*A*a^2*b^5-2*A*b^7-2*B*a^7-3*B*a^5*b^2)*arctan((a-
b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(7/2)/(a+b)^(7/2)/d+A*ar
ctanh(sin(d*x+c))/a^4/d+1/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d
*x+c))^3+1/6*b*(8*A*a^2*b-3*A*b^3-5*B*a^3)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+
b*cos(d*x+c))^2+1/6*b*(26*A*a^4*b-17*A*a^2*b^3+6*A*b^5-11*B*a^5-4*B*a^3*b^2
)*sin(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.95, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3079, 3134, 3080, 3855, 2738, 211}

$$\frac{A\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{b(-5a^3B+8a^2Ab-3Ab^3)\sin(c+dx)}{6a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{b(-11a^5B+26a^4Ab-4a^3b^2B-17a^2Ab^3+6Ab^5)\sin(c+dx)}{6a^3d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}-\frac{(-2a^7B+8a^6Ab-3a^5b^2B-8a^4Ab^3+7a^2Ab^5-2Ab^7)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -(((8*a^6*A*b - 8*a^4*A*b^3 + 7*a^2*A*b^5 - 2*A*b^7 - 2*a^7*B - 3*a^5*b^2*B
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a
 + b)^(7/2)*d)) + (A*ArcTanh[Sin[c + d*x]])/(a^4*d) + (b*(A*b - a*B)*Sin[c
+ d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (b*(8*a^2*A*b - 3*A*b^
3 - 5*a^3*B)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) +
 (b*(26*a^4*A*b - 17*a^2*A*b^3 + 6*A*b^5 - 11*a^5*B - 4*a^3*b^2*B)*Sin[c +
d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{\int \frac{(3A(a^2-b^2)-3a(Ab-aB)\cos(c+dx)+}{(a+b\cos(c}}{3a\,(a^2}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{b(8a^2Ab-3Ab^3-5a^3B)\sin(c}{6a^2\,(a^2-b^2)^2\,d(a+b\cos(c+}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{b(8a^2Ab-3Ab^3-5a^3B)\sin(c}{6a^2\,(a^2-b^2)^2\,d(a+b\cos(c+}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{b(8a^2Ab-3Ab^3-5a^3B)\sin(c}{6a^2\,(a^2-b^2)^2\,d(a+b\cos(c+}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\cos(c+dx))^3} + \frac{b(8a}{6a^2}$$

$$= -\frac{(8a^6Ab-8a^4Ab^3+7a^2Ab^5-2Ab^7-2a^7B-3a^5b^2B)\tan^{-1}\left(\frac{\sqrt{}}{}\right.}{a^4(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A]
time = 1.74, size = 368, normalized size = 1.22

$$\frac{\cos(c+dx)(B+A\sec(c+dx))\left(\frac{24(-8a^6Ab+8a^4Ab^3-7a^2Ab^5+2Ab^7+2a^7B+3a^5b^2B)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} - 24A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 24A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) - \frac{2ab(-72a^6Ab+38a^4Ab^3-5a^2Ab^5-6Ab^7+36a^7B+a^5b^2B+8a^3b^4B+6ab(-20a^4Ab+15a^2Ab^3-5Ab^5+9a^5B+a^3b^2B)\cos(c+dx)+b^2(-26a^4Ab+17a^2Ab^3-6Ab^5+11a^5B+4a^3b^2B)\cos(2(c+dx)))\sin(c+dx)}{(a^2-b^2)^3(a+b\cos(c+dx))^3}\right)}{24a^4d(A+B\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*((24*(-8*a^6*A*b + 8*a^4*A*b^3 - 7*a^2*A
*b^5 + 2*A*b^7 + 2*a^7*B + 3*a^5*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/
Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - 24*A*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 24*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (2*a*b*(-72*a^
6*A*b + 38*a^4*A*b^3 - 5*a^2*A*b^5 - 6*A*b^7 + 36*a^7*B + a^5*b^2*B + 8*a^3
*b^4*B + 6*a*b*(-20*a^4*A*b + 15*a^2*A*b^3 - 5*A*b^5 + 9*a^5*B + a^3*b^2*B)
*Cos[c + d*x] + b^2*(-26*a^4*A*b + 17*a^2*A*b^3 - 6*A*b^5 + 11*a^5*B + 4*a^
3*b^2*B)*Cos[2*(c + d*x)])*Sin[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x]
)^3)))/(24*a^4*d*(A + B*Cos[c + d*x]))
```

Maple [A]
time = 1.40, size = 479, normalized size = 1.59

method	result

derivativedivides	$\frac{\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^4}-\frac{2\left(-\frac{\left(12Aa^4b+4Aa^3b^2-6Aa^2b^3-Aab^4+2Ab^5-6Ba^5-3Ba^4b-2Ba^3b^2\right)ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}-\frac{2\left(18Aa^4\right.}{\left(a\right.}\right.}{}}{}$
default	$\frac{\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^4}-\frac{2\left(-\frac{\left(12Aa^4b+4Aa^3b^2-6Aa^2b^3-Aab^4+2Ab^5-6Ba^5-3Ba^4b-2Ba^3b^2\right)ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}-\frac{2\left(18Aa^4\right.}{\left(a\right.}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A/a^4*ln(tan(1/2*d*x+1/2*c)+1)-2/a^4*((-1/2*(12*A*a^4*b+4*A*a^3*b^2-6*
A*a^2*b^3-A*a*b^4+2*A*b^5-6*B*a^5-3*B*a^4*b-2*B*a^3*b^2)*a*b/(a-b)/(a^3+3*a
^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(18*A*a^4*b-11*A*a^2*b^3+3*A*b^5
-9*B*a^5-B*a^3*b^2)*a*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3-1/2*(12*A*a^4*b-4*A*a^3*b^2-6*A*a^2*b^3+A*a*b^4+2*A*b^5-6*B*a^5+3*B*a^4*b
-2*B*a^3*b^2)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*ta
n(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2+a+b)^3+1/2*(8*A*a^6*b-8*A*a^4*b^3
+7*A*a^2*b^5-2*A*b^7-2*B*a^7-3*B*a^5*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a
-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))-A/a^
4*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1100 vs. 2(285) = 570.
time = 35.53, size = 2269, normalized size = 7.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(3*(2*B*a^10 - 8*A*a^9*b + 3*B*a^8*b^2 + 8*A*a^7*b^3 - 7*A*a^5*b^5 +
2*A*a^3*b^7 + (2*B*a^7*b^3 - 8*A*a^6*b^4 + 3*B*a^5*b^5 + 8*A*a^4*b^6 - 7*A*
a^2*b^8 + 2*A*b^10)*cos(d*x + c)^3 + 3*(2*B*a^8*b^2 - 8*A*a^7*b^3 + 3*B*a^6
*b^4 + 8*A*a^5*b^5 - 7*A*a^3*b^7 + 2*A*a*b^9)*cos(d*x + c)^2 + 3*(2*B*a^9*b
 - 8*A*a^8*b^2 + 3*B*a^7*b^3 + 8*A*a^6*b^4 - 7*A*a^4*b^6 + 2*A*a^2*b^8)*cos
(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 6*(A*a^11 - 4*A*a^9*b
^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8 + (A*a^8*b^3 - 4*A*a^6*b^5 + 6*A
*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*cos(d*x + c)^3 + 3*(A*a^9*b^2 - 4*A*a^7*b^
4 + 6*A*a^5*b^6 - 4*A*a^3*b^8 + A*a*b^10)*cos(d*x + c)^2 + 3*(A*a^10*b - 4*
A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^2*b^9)*cos(d*x + c))*log(sin(d*
x + c) + 1) - 6*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b
^8 + (A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*cos(d*x
 + c)^3 + 3*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6 - 4*A*a^3*b^8 + A*a*b^10
)*cos(d*x + c)^2 + 3*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 +
A*a^2*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(18*B*a^10*b - 36*A*a^9
*b^2 - 23*B*a^8*b^3 + 68*A*a^7*b^4 + 7*B*a^6*b^5 - 43*A*a^5*b^6 - 2*B*a^4*b
^7 + 11*A*a^3*b^8 + (11*B*a^8*b^3 - 26*A*a^7*b^4 - 7*B*a^6*b^5 + 43*A*a^5*b
^6 - 4*B*a^4*b^7 - 23*A*a^3*b^8 + 6*A*a*b^10)*cos(d*x + c)^2 + 3*(9*B*a^9*b
^2 - 20*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 - B*a^5*b^6 - 20*A*a^4*b^7 +
 5*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^12*b^3 - 4*a^10*b^5 + 6*a^8*b
^7 - 4*a^6*b^9 + a^4*b^11)*d*cos(d*x + c)^3 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*
a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c)^2 + 3*(a^14*b - 4*a^12*b^3 +
 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c) + (a^15 - 4*a^13*b^2 + 6*
a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d), 1/6*(3*(2*B*a^10 - 8*A*a^9*b + 3*B*a^8*
b^2 + 8*A*a^7*b^3 - 7*A*a^5*b^5 + 2*A*a^3*b^7 + (2*B*a^7*b^3 - 8*A*a^6*b^4
+ 3*B*a^5*b^5 + 8*A*a^4*b^6 - 7*A*a^2*b^8 + 2*A*b^10)*cos(d*x + c)^3 + 3*(2
*B*a^8*b^2 - 8*A*a^7*b^3 + 3*B*a^6*b^4 + 8*A*a^5*b^5 - 7*A*a^3*b^7 + 2*A*a*
b^9)*cos(d*x + c)^2 + 3*(2*B*a^9*b - 8*A*a^8*b^2 + 3*B*a^7*b^3 + 8*A*a^6*b^
4 - 7*A*a^4*b^6 + 2*A*a^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos
(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + 3*(A*a^11 - 4*A*a^9*b^2 +
6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8 + (A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^4*
b^7 - 4*A*a^2*b^9 + A*b^11)*cos(d*x + c)^3 + 3*(A*a^9*b^2 - 4*A*a^7*b^4 + 6
*A*a^5*b^6 - 4*A*a^3*b^8 + A*a*b^10)*cos(d*x + c)^2 + 3*(A*a^10*b - 4*A*a^8
*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^2*b^9)*cos(d*x + c))*log(sin(d*x + c
) + 1) - 3*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8 +
(A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*cos(d*x + c)
^3 + 3*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6 - 4*A*a^3*b^8 + A*a*b^10)*cos
(d*x + c)^2 + 3*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^2
```

```
*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - (18*B*a^10*b - 36*A*a^9*b^2 -
23*B*a^8*b^3 + 68*A*a^7*b^4 + 7*B*a^6*b^5 - 43*A*a^5*b^6 - 2*B*a^4*b^7 + 11
*A*a^3*b^8 + (11*B*a^8*b^3 - 26*A*a^7*b^4 - 7*B*a^6*b^5 + 43*A*a^5*b^6 - 4*
B*a^4*b^7 - 23*A*a^3*b^8 + 6*A*a*b^10)*cos(d*x + c)^2 + 3*(9*B*a^9*b^2 - 20
*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 - B*a^5*b^6 - 20*A*a^4*b^7 + 5*A*a^
2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*
a^6*b^9 + a^4*b^11)*d*cos(d*x + c)^3 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6
 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c)^2 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10
*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c) + (a^15 - 4*a^13*b^2 + 6*a^11*b^
4 - 4*a^9*b^6 + a^7*b^8)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 837 vs. 2(285) = 570.
time = 0.52, size = 837, normalized size = 2.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] 1/3*(3*(2*B*a^7 - 8*A*a^6*b + 3*B*a^5*b^2 + 8*A*a^4*b^3 - 7*A*a^2*b^5 + 2*A
*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^10 - 3*a^8*b^2
 + 3*a^6*b^4 - a^4*b^6)*sqrt(a^2 - b^2)) + 3*A*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/a^4 - 3*A*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - (18*B*a^7*b*tan(1
/2*d*x + 1/2*c)^5 - 36*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*B*a^6*b^2*tan(
1/2*d*x + 1/2*c)^5 + 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^5*b^3*tan(
1/2*d*x + 1/2*c)^5 + 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^4*b^4*tan(1
/2*d*x + 1/2*c)^5 - 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^3*b^5*tan(1
/2*d*x + 1/2*c)^5 + 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 15*A*a*b^7*tan(1/2
*d*x + 1/2*c)^5 - 6*A*b^8*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^7*b*tan(1/2*d*x +
 1/2*c)^3 - 72*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 32*B*a^5*b^3*tan(1/2*d*x
+ 1/2*c)^3 + 116*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^3*b^5*tan(1/2*d*x
 + 1/2*c)^3 - 56*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^8*tan(1/2*d*x +
```

```
1/2*c)^3 + 18*B*a^7*b*tan(1/2*d*x + 1/2*c) - 36*A*a^6*b^2*tan(1/2*d*x + 1/2
*c) + 27*B*a^6*b^2*tan(1/2*d*x + 1/2*c) - 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c)
 + 6*B*a^5*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c) + 3*
B*a^4*b^4*tan(1/2*d*x + 1/2*c) + 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c) + 6*B*a^
3*b^5*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c) - 15*A*a*b^7*
tan(1/2*d*x + 1/2*c) - 6*A*b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*
a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a
 + b)^3))/d
```

Mupad [B]
time = 12.81, size = 2500, normalized size = 8.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^4),x)
```

```
[Out] (A*atan(-((A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 -
 8*A^2*a*b^13 - 8*A^2*a^13*b - 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*
a^4*b^10 - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^
8*b^6 - 120*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b
^2 + 9*B^2*a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34
*A*B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^11*b^3))/(a^16*b + a^17 - a^6*b^11
 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b
^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) + (A*((8*(4*A*a^21 + 4*B*a^21 -
 4*A*a^8*b^13 + 2*A*a^9*b^12 + 26*A*a^10*b^11 - 14*A*a^11*b^10 - 70*A*a^12*
b^9 + 30*A*a^13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 110*A*a^16*b^5 + 20*
A*a^17*b^4 + 64*A*a^18*b^3 - 12*A*a^19*b^2 + 6*B*a^12*b^9 - 6*B*a^13*b^8 -
14*B*a^14*b^7 + 14*B*a^15*b^6 + 6*B*a^16*b^5 - 6*B*a^17*b^4 + 6*B*a^18*b^3
- 6*B*a^19*b^2 - 16*A*a^20*b - 4*B*a^20*b))/(a^19*b + a^20 - a^9*b^11 - a^1
0*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5
+ 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (8*A*tan(c/2 + (d*x)/2)*(8*a^21*
b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 +
 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 +
 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/(a^4*(a^16*b + a^17 - a^6*b^11 -
a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5
+ 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))))/a^4)*1i)/a^4 + (A*((8*tan(c/2 +
 (d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 - 8*A^2*a*b^13 - 8*A^2*a^13
*b - 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*a^4*b^10 - 120*A^2*a^5*b^9
 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b^6 - 120*A^2*a^9*b^5 -
92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b^2 + 9*B^2*a^10*b^4 + 12*B
^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 20*A*B*a^9*
b^5 - 16*A*B*a^11*b^3))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 +
5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*
b^3 - 5*a^15*b^2) - (A*((8*(4*A*a^21 + 4*B*a^21 - 4*A*a^8*b^13 + 2*A*a^9*b^
```

12 + 26*A*a^10*b^11 - 14*A*a^11*b^10 - 70*A*a^12*b^9 + 30*A*a^13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 110*A*a^16*b^5 + 20*A*a^17*b^4 + 64*A*a^18*b^3 - 12*A*a^19*b^2 + 6*B*a^12*b^9 - 6*B*a^13*b^8 - 14*B*a^14*b^7 + 14*B*a^15*b^6 + 6*B*a^16*b^5 - 6*B*a^17*b^4 + 6*B*a^18*b^3 - 6*B*a^19*b^2 - 16*A*a^20*b - 4*B*a^20*b))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (8*A*tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))))/a^4)*1i)/a^4)/((16*(4*A^3*b^13 + 4*A*B^2*a^13 - 4*A^2*B*a^13 - 2*A^3*a*b^12 + 16*A^3*a^12*b - 26*A^3*a^2*b^11 + 11*A^3*a^3*b^10 + 70*A^3*a^4*b^9 - 34*A^3*a^5*b^8 - 110*A^3*a^6*b^7 + 66*A^3*a^7*b^6 + 110*A^3*a^8*b^5 - 64*A^3*a^9*b^4 - 64*A^3*a^10*b^3 + 48*A^3*a^11*b^2 - 28*A^2*B*a^12*b + 9*A*B^2*a^9*b^4 + 12*A*B^2*a^11*b^2 + 6*A^2*B*a^4*b^9 + 6*A^2*B*a^5*b^8 - 20*A^2*B*a^6*b^7 - 14*A^2*B*a^7*b^6 + 14*A^2*B*a^8*b^5 + 6*A^2*B*a^9*b^4 - 22*A^2*B*a^10*b^3 + 6*A^2*B*a^11*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 - 8*A^2*a*b^13 - 8*A^2*a^13*b - 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*a^4*b^10 - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b^6 - 120*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b^2 + 9*B^2*a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^11*b^3))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) + (A*((8*(4*A*a^21 + 4*B*a^21 - 4*A*a^8*b^13 + 2*A*a^9*b^12 + 26*A*a^10*b^11 - 14*A*a^11*b^10 - 70*A*a^12*b^9 + 30*A*a^13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 110*A*a^16*b^5 + 20*A*a^17*b^4 + 64*A*a^18*b^3 - 12*A*a^19*b^2 + 6*B*a^12*b^9 - 6*B*a^13*b^8 - 14*B*a^14*b^7 + 14*B*a^15*b^6 + 6*B*a^16*b^5 - 6*B*a^17*b^4 + 6*B*a^18*b^3 - 6*B*a^19*b^2 - 16*A*a^20*b - 4*B*a^20*b))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (8*A*tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 ...

3.279 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=420

$$\frac{b\left(20a^6Ab-35a^4Ab^3+28a^2Ab^5-8Ab^7-8a^7B+8a^5b^2B-7a^3b^4B+2ab^6B\right)\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{7/2}(a+b)^{7/2}d}$$

```
[Out] b*(20*A*a^6*b-35*A*a^4*b^3+28*A*a^2*b^5-8*A*b^7-8*B*a^7+8*B*a^5*b^2-7*B*a^3
*b^4+2*B*a*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b
)^(7/2)/(a+b)^(7/2)/d-(4*A*b-B*a)*arctanh(sin(d*x+c))/a^5/d+1/6*(6*A*a^6-65
*A*a^4*b^2+68*A*a^2*b^4-24*A*b^6+26*B*a^5*b-17*B*a^3*b^3+6*B*a*b^5)*tan(d*x
+c)/a^4/(a^2-b^2)^3/d+1/3*b*(A*b-B*a)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x
+c))^3+1/6*b*(9*A*a^2*b-4*A*b^3-6*B*a^3+B*a*b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2
/d/(a+b*cos(d*x+c))^2+1/2*b*(12*A*a^4*b-11*A*a^2*b^3+4*A*b^5-6*B*a^5+2*B*a^
3*b^2-B*a*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 3.87, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3079, 3134, 3080, 3855, 2738, 211}

$$-\frac{(4Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^5d}+\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{b\left(-6a^3B+9a^2Ab+ab^2B-4Ab^3\right)\tan(c+dx)}{6a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{b\left(-6a^5B+12a^4Ab+2a^3b^2B-11a^2Ab^3-ab^4B+4Ab^5\right)\tan(c+dx)}{2a^3d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}+\frac{\left(6a^6A+26a^5bB-65a^4Ab^2-17a^3b^3B+68a^2Ab^4+6ab^5B-24Ab^6\right)\tan(c+dx)}{6a^4d\left(a^2-b^2\right)^3}+\frac{b\left(-8a^7B+20a^6Ab+8a^5b^2B-35a^4Ab^3-7a^3b^4B+28a^2Ab^5+2ab^6B-8Ab^7\right)\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (b*(20*a^6*A*b - 35*a^4*A*b^3 + 28*a^2*A*b^5 - 8*A*b^7 - 8*a^7*B + 8*a^5*b^
2*B - 7*a^3*b^4*B + 2*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d) - ((4*A*b - a*B)*ArcTanh[Sin[c
+ d*x]])/(a^5*d) + ((6*a^6*A - 65*a^4*A*b^2 + 68*a^2*A*b^4 - 24*A*b^6 + 26*
a^5*b*B - 17*a^3*b^3*B + 6*a*b^5*B)*Tan[c + d*x])/(6*a^4*(a^2 - b^2)^3*d) +
 (b*(A*b - a*B)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) +
(b*(9*a^2*A*b - 4*A*b^3 - 6*a^3*B + a*b^2*B)*Tan[c + d*x])/(6*a^2*(a^2 - b^
2)^2*d*(a + b*Cos[c + d*x])^2) + (b*(12*a^4*A*b - 11*a^2*A*b^3 + 4*A*b^5 -
6*a^5*B + 2*a^3*b^2*B - a*b^4*B)*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a +
b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{(3a^2A-4Ab^2+abB-3a(Ab-aB)\cos(}{(a+b}}{3a}$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b(9a^2Ab-4Ab^3-6a^3B+ab^2}{6a^2\left(a^2-b^2\right)^2d(a+b\cos}$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b(9a^2Ab-4Ab^3-6a^3B+ab^2}{6a^2\left(a^2-b^2\right)^2d(a+b\cos}$$

$$= \frac{(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5}{6a^4\left(a^2-b^2\right)^3d}$$

$$= \frac{(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5}{6a^4\left(a^2-b^2\right)^3d}$$

$$= -\frac{(4Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{(6a^6A-65a^4Ab^2+68a^2Ab^4}{}$$

$$= \frac{b(20a^6Ab-35a^4Ab^3+28a^2Ab^5-8Ab^7-8a^7B+8a^5b^2B-7a^3b^4}{a^5(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A]
time = 3.27, size = 549, normalized size = 1.31

$$\frac{-\frac{48b\left(-20a^6Ab+35a^4Ab^3-28a^2Ab^5+8Ab^7+8a^7B-8a^5b^2B+7a^3b^4B-2ab^6B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{7/2}}+48(4Ab-aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+48(-4Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2a\left(24a^9A-36a^7Ab^2-246a^5Ab^4+318a^3Ab^6-120aAb^8+120a^6b^3B-90a^4b^5B+30a^2b^7B+b\left(72a^8A-438a^6Ab^2+305a^4Ab^4+28a^2Ab^6-72Ab^8+144a^7bB-50a^5b^3B-7a^3b^5B+18ab^7B\right)\cos(c+dx)+6ab^2\left(6a^6A-53a^4Ab^2+57a^2Ab^4-20Ab^6+20a^5bB-15a^3b^3B+5ab^5B\right)\cos(2(c+dx))+6a^6Ab^3\cos(3(c+dx))-65a^4Ab^5\cos(3(c+dx))+68a^2Ab^7\cos(3(c+dx))-24Ab^9\cos(3(c+dx))+26a^5b^4B\cos(3(c+dx))-17a^3b^6B\cos(3(c+dx))+6ab^8B\cos(3(c+dx))\right)\tan(c+dx)}{\left(a^2-b^2\right)^3(a+b\cos(c+dx))^3}}{48a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-48*b*(-20*a^6*A*b + 35*a^4*A*b^3 - 28*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 8*
a^5*b^2*B + 7*a^3*b^4*B - 2*a*b^6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqr
t[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + 48*(4*A*b - a*B)*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]] + 48*(-4*A*b + a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + (2*a*(24*a^9*A - 36*a^7*A*b^2 - 246*a^5*A*b^4 + 318*a^3*A*b^6 - 120
*a*A*b^8 + 120*a^6*b^3*B - 90*a^4*b^5*B + 30*a^2*b^7*B + b*(72*a^8*A - 438*
a^6*A*b^2 + 305*a^4*A*b^4 + 28*a^2*A*b^6 - 72*A*b^8 + 144*a^7*b*B - 50*a^5*
b^3*B - 7*a^3*b^5*B + 18*a*b^7*B)*Cos[c + d*x] + 6*a*b^2*(6*a^6*A - 53*a^4*
```

```
A*b^2 + 57*a^2*A*b^4 - 20*A*b^6 + 20*a^5*b*B - 15*a^3*b^3*B + 5*a*b^5*B)*Co
s[2*(c + d*x)] + 6*a^6*A*b^3*Cos[3*(c + d*x)] - 65*a^4*A*b^5*Cos[3*(c + d*x
)] + 68*a^2*A*b^7*Cos[3*(c + d*x)] - 24*A*b^9*Cos[3*(c + d*x)] + 26*a^5*b^4
*B*Cos[3*(c + d*x)] - 17*a^3*b^6*B*Cos[3*(c + d*x)] + 6*a*b^8*B*Cos[3*(c +
d*x)])*Tan[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x])^3))/(48*a^5*d)
```

Maple [A]
time = 1.99, size = 587, normalized size = 1.40

method	result
derivativedivides	$\frac{-\frac{A}{a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-4Ab+aB)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^5}+\frac{2b\left(-\frac{\left(20Aa^4b+5Aa^3b^2-18Aa^2b^3-2Aab^4+6Ab^5-12Ba^5-4Ba^4b+\cdots\right.}{2(a-b)\left(a^3+3a^2b+3b^2a\cdots\right.}\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{A}{a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-4Ab+aB)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^5}+\frac{2b\left(-\frac{\left(20Aa^4b+5Aa^3b^2-18Aa^2b^3-2Aab^4+6Ab^5-12Ba^5-4Ba^4b+\cdots\right.}{2(a-b)\left(a^3+3a^2b+3b^2a\cdots\right.}\cdots\right.}{\cdots}}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-A/a^4/(tan(1/2*d*x+1/2*c)+1)+1/a^5*(-4*A*b+B*a)*ln(tan(1/2*d*x+1/2*c)
+1)+2*b/a^5*((-1/2*(20*A*a^4*b+5*A*a^3*b^2-18*A*a^2*b^3-2*A*a*b^4+6*A*b^5-1
2*B*a^5-4*B*a^4*b+6*B*a^3*b^2+B*a^2*b^3-2*B*a*b^4)*a*b/(a-b)/(a^3+3*a^2*b+3
*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(30*A*a^4*b-29*A*a^2*b^3+9*A*b^5-18*B*
a^5+11*B*a^3*b^2-3*B*a*b^4)*a*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3-1/2*(20*A*a^4*b-5*A*a^3*b^2-18*A*a^2*b^3+2*A*a*b^4+6*A*b^5-12*B*a
^5+4*B*a^4*b+6*B*a^3*b^2-B*a^2*b^3-2*B*a*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^
2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2+a
+b)^3+1/2*(20*A*a^6*b-35*A*a^4*b^3+28*A*a^2*b^5-8*A*b^7-8*B*a^7+8*B*a^5*b^2
-7*B*a^3*b^4+2*B*a*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*a
rctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))-A/a^4/(tan(1/2*d*x+1/2
*c)-1)+(4*A*b-B*a)/a^5*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1662 vs. $2(402) = 804$.
time = 72.79, size = 3393, normalized size = 8.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*((8*B*a^7*b^4 - 20*A*a^6*b^5 - 8*B*a^5*b^6 + 35*A*a^4*b^7 + 7*B*a^
3*b^8 - 28*A*a^2*b^9 - 2*B*a*b^10 + 8*A*b^11)*cos(d*x + c)^4 + 3*(8*B*a^8*b
^3 - 20*A*a^7*b^4 - 8*B*a^6*b^5 + 35*A*a^5*b^6 + 7*B*a^4*b^7 - 28*A*a^3*b^8
 - 2*B*a^2*b^9 + 8*A*a*b^10)*cos(d*x + c)^3 + 3*(8*B*a^9*b^2 - 20*A*a^8*b^3
 - 8*B*a^7*b^4 + 35*A*a^6*b^5 + 7*B*a^5*b^6 - 28*A*a^4*b^7 - 2*B*a^3*b^8 +
8*A*a^2*b^9)*cos(d*x + c)^2 + (8*B*a^10*b - 20*A*a^9*b^2 - 8*B*a^8*b^3 + 35
*A*a^7*b^4 + 7*B*a^6*b^5 - 28*A*a^5*b^6 - 2*B*a^4*b^7 + 8*A*a^3*b^8)*cos(d*
x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 6*((B*a^9*b^3 - 4*A*a^8*
b^4 - 4*B*a^7*b^5 + 16*A*a^6*b^6 + 6*B*a^5*b^7 - 24*A*a^4*b^8 - 4*B*a^3*b^9
 + 16*A*a^2*b^10 + B*a*b^11 - 4*A*b^12)*cos(d*x + c)^4 + 3*(B*a^10*b^2 - 4*
A*a^9*b^3 - 4*B*a^8*b^4 + 16*A*a^7*b^5 + 6*B*a^6*b^6 - 24*A*a^5*b^7 - 4*B*a
^4*b^8 + 16*A*a^3*b^9 + B*a^2*b^10 - 4*A*a*b^11)*cos(d*x + c)^3 + 3*(B*a^11
*b - 4*A*a^10*b^2 - 4*B*a^9*b^3 + 16*A*a^8*b^4 + 6*B*a^7*b^5 - 24*A*a^6*b^6
 - 4*B*a^5*b^7 + 16*A*a^4*b^8 + B*a^3*b^9 - 4*A*a^2*b^10)*cos(d*x + c)^2 +
(B*a^12 - 4*A*a^11*b - 4*B*a^10*b^2 + 16*A*a^9*b^3 + 6*B*a^8*b^4 - 24*A*a^7
*b^5 - 4*B*a^6*b^6 + 16*A*a^5*b^7 + B*a^4*b^8 - 4*A*a^3*b^9)*cos(d*x + c))*
log(sin(d*x + c) + 1) - 6*((B*a^9*b^3 - 4*A*a^8*b^4 - 4*B*a^7*b^5 + 16*A*a^
6*b^6 + 6*B*a^5*b^7 - 24*A*a^4*b^8 - 4*B*a^3*b^9 + 16*A*a^2*b^10 + B*a*b^11
 - 4*A*b^12)*cos(d*x + c)^4 + 3*(B*a^10*b^2 - 4*A*a^9*b^3 - 4*B*a^8*b^4 + 1
6*A*a^7*b^5 + 6*B*a^6*b^6 - 24*A*a^5*b^7 - 4*B*a^4*b^8 + 16*A*a^3*b^9 + B*a
^2*b^10 - 4*A*a*b^11)*cos(d*x + c)^3 + 3*(B*a^11*b - 4*A*a^10*b^2 - 4*B*a^9
*b^3 + 16*A*a^8*b^4 + 6*B*a^7*b^5 - 24*A*a^6*b^6 - 4*B*a^5*b^7 + 16*A*a^4*b
^8 + B*a^3*b^9 - 4*A*a^2*b^10)*cos(d*x + c)^2 + (B*a^12 - 4*A*a^11*b - 4*B*
a^10*b^2 + 16*A*a^9*b^3 + 6*B*a^8*b^4 - 24*A*a^7*b^5 - 4*B*a^6*b^6 + 16*A*a
```

^5*b^7 + B*a^4*b^8 - 4*A*a^3*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*
(6*A*a^12 - 24*A*a^10*b^2 + 36*A*a^8*b^4 - 24*A*a^6*b^6 + 6*A*a^4*b^8 + (6*
A*a^9*b^3 + 26*B*a^8*b^4 - 71*A*a^7*b^5 - 43*B*a^6*b^6 + 133*A*a^5*b^7 + 23
*B*a^4*b^8 - 92*A*a^3*b^9 - 6*B*a^2*b^10 + 24*A*a*b^11)*cos(d*x + c)^3 + 3*
(6*A*a^10*b^2 + 20*B*a^9*b^3 - 59*A*a^8*b^4 - 35*B*a^7*b^5 + 110*A*a^6*b^6
+ 20*B*a^5*b^7 - 77*A*a^4*b^8 - 5*B*a^3*b^9 + 20*A*a^2*b^10)*cos(d*x + c)^2
 + (18*A*a^11*b + 36*B*a^10*b^2 - 132*A*a^9*b^3 - 68*B*a^8*b^4 + 239*A*a^7*
b^5 + 43*B*a^6*b^6 - 169*A*a^5*b^7 - 11*B*a^4*b^8 + 44*A*a^3*b^9)*cos(d*x +
 c))*sin(d*x + c))/((a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^
11)*d*cos(d*x + c)^4 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a^8*b^8 +
a^6*b^10)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^
7 + a^7*b^9)*d*cos(d*x + c)^2 + (a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^
6 + a^8*b^8)*d*cos(d*x + c)), -1/6*(3*((8*B*a^7*b^4 - 20*A*a^6*b^5 - 8*B*a^
5*b^6 + 35*A*a^4*b^7 + 7*B*a^3*b^8 - 28*A*a^2*b^9 - 2*B*a*b^10 + 8*A*b^11)*
cos(d*x + c)^4 + 3*(8*B*a^8*b^3 - 20*A*a^7*b^4 - 8*B*a^6*b^5 + 35*A*a^5*b^6
 + 7*B*a^4*b^7 - 28*A*a^3*b^8 - 2*B*a^2*b^9 + 8*A*a*b^10)*cos(d*x + c)^3 +
3*(8*B*a^9*b^2 - 20*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 + 7*B*a^5*b^6 -
28*A*a^4*b^7 - 2*B*a^3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + (8*B*a^10*b - 20
*A*a^9*b^2 - 8*B*a^8*b^3 + 35*A*a^7*b^4 + 7*B*a^6*b^5 - 28*A*a^5*b^6 - 2*B*
a^4*b^7 + 8*A*a^3*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c
) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - 3*((B*a^9*b^3 - 4*A*a^8*b^4 - 4*B*
a^7*b^5 + 16*A*a^6*b^6 + 6*B*a^5*b^7 - 24*A*a^4*b^8 - 4*B*a^3*b^9 + 16*A*a^
2*b^10 + B*a*b^11 - 4*A*b^12)*cos(d*x + c)^4 + 3*(B*a^10*b^2 - 4*A*a^9*b^3
- 4*B*a^8*b^4 + 16*A*a^7*b^5 + 6*B*a^6*b^6 - 24*A*a^5*b^7 - 4*B*a^4*b^8 + 1
6*A*a^3*b^9 + B*a^2*b^10 - 4*A*a*b^11)*cos(d*x + c)^3 + 3*(B*a^11*b - 4*A*a
^10*b^2 - 4*B*a^9*b^3 + 16*A*a^8*b^4 + 6*B*a^7*b^5 - 24*A*a^6*b^6 - 4*B*a^5
*b^7 + 16*A*a^4*b^8 + B*a^3*b^9 - 4*A*a^2*b^10)*cos(d*x + c)^2 + (B*a^12 -
4*A*a^11*b - 4*B*a^10*b^2 + 16*A*a^9*b^3 + 6*B*a^8*b^4 - 24*A*a^7*b^5 - 4*B
*a^6*b^6 + 16*A*a^5*b^7 + B*a^4*b^8 - 4*A*a^3*b^9)*cos(d*x + c))*log(sin(d*
x + c) + 1) + 3*((B*a^9*b^3 - 4*A*a^8*b^4 - 4*B*a^7*b^5 + 16*A*a^6*b^6 + 6*
B*a^5*b^7 - 24*A*a^4*b^8 - 4*B*a^3*b^9 + 16*A*a^2*b^10 + B*a*b^11 - 4*A*b^1
2)*cos(d*x + c)^4 + 3*(B*a^10*b^2 - 4*A*a^9*b^3 - 4*B*a^8*b^4 + 16*A*a^7*b^
5 + 6*B*a^6*b^6 - 24*A*a^5*b^7 - 4*B*a^4*b^8 + 16*A*a^3*b^9 + B*a^2*b^10 -
4*A*a*b^11)*cos(d*x + c)^3 + 3*(B*a^11*b - 4*A*a^10*b^2 - 4*B*a^9*b^3 + 16*
A*a^8*b^4 + 6*B*a^7*b^5 - 24*A*a^6*b^6 - 4*B*a^5*b^7 + 16*A*a^4*b^8 + B*a^3
*b^9 - 4*A*a^2*b^10)*cos(d*x + c)^2 + (B*a^12 - 4*A*a^11*b - 4*B*a^10*b^2 +
 16*A*a^9*b^3 + 6*B*a^8*b^4 - 24*A*a^7*b^5 - 4*B*a^6*b^6 + 16*A*a^5*b^7 + B
*a^4*b^8 - 4*A*a^3*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - (6*A*a^12 -
24*A*a^10*b^2 + 36*A*a^8*b^4 - 24*A*a^6*b^6 + 6*A*a^4*b^8 + (6*A*a^9*b^3 +
26*B*a^8*b^4 - 71*A*a^7*b^5 - 43*B*a^6*b^6 + 133*A*a^5*b^7 + 23*B*a^4*b^8 -
 92*A*a^3*b^9 - 6*B*a^2*b^10 + 24*A*a*b^11)*cos...

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(402) = 804$.
time = 0.51, size = 996, normalized size = 2.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(8*B*a^7*b - 20*A*a^6*b^2 - 8*B*a^5*b^3 + 35*A*a^4*b^4 + 7*B*a^3*b^5
 - 28*A*a^2*b^6 - 2*B*a*b^7 + 8*A*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/s
qrt(a^2 - b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*sqrt(a^2 - b^2))
 + (36*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*A*a^6*b^3*tan(1/2*d*x + 1/2*c)
^5 - 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 105*A*a^5*b^4*tan(1/2*d*x + 1/2*
c)^5 - 6*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 24*A*a^4*b^5*tan(1/2*d*x + 1/2*
c)^5 + 45*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 - 117*A*a^3*b^6*tan(1/2*d*x + 1/
2*c)^5 - 6*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 24*A*a^2*b^7*tan(1/2*d*x + 1/
2*c)^5 - 15*B*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 + 42*A*a*b^8*tan(1/2*d*x + 1/2
*c)^5 + 6*B*a*b^8*tan(1/2*d*x + 1/2*c)^5 - 18*A*b^9*tan(1/2*d*x + 1/2*c)^5
+ 72*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^
3 - 116*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 + 236*A*a^4*b^5*tan(1/2*d*x + 1/2*
c)^3 + 56*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 - 152*A*a^2*b^7*tan(1/2*d*x + 1/
2*c)^3 - 12*B*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^9*tan(1/2*d*x + 1/2*c)^
3 + 36*B*a^7*b^2*tan(1/2*d*x + 1/2*c) - 60*A*a^6*b^3*tan(1/2*d*x + 1/2*c) +
 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c) - 105*A*a^5*b^4*tan(1/2*d*x + 1/2*c) - 6
*B*a^5*b^4*tan(1/2*d*x + 1/2*c) + 24*A*a^4*b^5*tan(1/2*d*x + 1/2*c) - 45*B*
a^4*b^5*tan(1/2*d*x + 1/2*c) + 117*A*a^3*b^6*tan(1/2*d*x + 1/2*c) - 6*B*a^3
*b^6*tan(1/2*d*x + 1/2*c) + 24*A*a^2*b^7*tan(1/2*d*x + 1/2*c) + 15*B*a^2*b^
7*tan(1/2*d*x + 1/2*c) - 42*A*a*b^8*tan(1/2*d*x + 1/2*c) + 6*B*a*b^8*tan(1/
2*d*x + 1/2*c) - 18*A*b^9*tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2 + 3*a^6*
b^4 - a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b
)^3) + 3*(B*a - 4*A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 3*(B*a - 4*
A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5 - 6*A*tan(1/2*d*x + 1/2*c)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)*a^4))/d
```

Mupad [B]
time = 18.11, size = 2500, normalized size = 5.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^4),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(18*A*a^8 + 72*A*b^8 - 236*A*a^2*b^6 + 47*A*a^3*b^5
+ 273*A*a^4*b^4 - 60*A*a^5*b^3 - 72*A*a^6*b^2 + 3*B*a^2*b^6 + 59*B*a^3*b^5
- 14*B*a^4*b^4 - 96*B*a^5*b^3 + 36*B*a^6*b^2 - 12*A*a*b^7 - 18*B*a*b^7))/(3
*a^4*(a + b)^2*(a - b)^3) - (tan(c/2 + (d*x)/2)^7*(24*A*a^2*b^5 - 8*A*b^7 -
 2*A*a^7 - 11*A*a^3*b^4 - 26*A*a^4*b^3 + 6*A*a^5*b^2 - B*a^2*b^5 - 6*B*a^3*
b^4 + 4*B*a^4*b^3 + 12*B*a^5*b^2 + 4*A*a*b^6 + 2*A*a^6*b + 2*B*a*b^6))/(a^4
*(a + b)^3*(a - b)) + (tan(c/2 + (d*x)/2)^5*(18*A*a^8 + 72*A*b^8 - 236*A*a^
2*b^6 - 47*A*a^3*b^5 + 273*A*a^4*b^4 + 60*A*a^5*b^3 - 72*A*a^6*b^2 - 3*B*a^
2*b^6 + 59*B*a^3*b^5 + 14*B*a^4*b^4 - 96*B*a^5*b^3 - 36*B*a^6*b^2 + 12*A*a*
b^7 - 18*B*a*b^7))/(3*a^4*(a + b)^3*(a - b)^2) + (tan(c/2 + (d*x)/2)*(2*A*a
^7 - 8*A*b^7 + 24*A*a^2*b^5 + 11*A*a^3*b^4 - 26*A*a^4*b^3 - 6*A*a^5*b^2 + B
*a^2*b^5 - 6*B*a^3*b^4 - 4*B*a^4*b^3 + 12*B*a^5*b^2 - 4*A*a*b^6 + 2*A*a^6*b
 + 2*B*a*b^6))/(a^4*(a + b)*(a - b)^3))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2 + (
d*x)/2)^4*(6*a^2*b - 6*b^3) - tan(c/2 + (d*x)/2)^2*(6*a*b^2 - 2*a^3 + 4*b^3
) - tan(c/2 + (d*x)/2)^6*(2*a^3 - 6*a*b^2 + 4*b^3) + a^3 + b^3 - tan(c/2 +
(d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (atan((((((4*A*b - B*a)*((8*
(4*B*a^24 + 16*A*a^10*b^14 - 8*A*a^11*b^13 - 104*A*a^12*b^12 + 50*A*a^13*b^
11 + 286*A*a^14*b^10 - 126*A*a^15*b^9 - 434*A*a^16*b^8 + 174*A*a^17*b^7 + 3
86*A*a^18*b^6 - 146*A*a^19*b^5 - 190*A*a^20*b^4 + 72*A*a^21*b^3 + 40*A*a^22
*b^2 - 4*B*a^11*b^13 + 2*B*a^12*b^12 + 26*B*a^13*b^11 - 14*B*a^14*b^10 - 70
*B*a^15*b^9 + 30*B*a^16*b^8 + 110*B*a^17*b^7 - 30*B*a^18*b^6 - 110*B*a^19*b
^5 + 20*B*a^20*b^4 + 64*B*a^21*b^3 - 12*B*a^22*b^2 - 16*A*a^23*b - 16*B*a^2
3*b))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10
*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b
^2) - (8*tan(c/2 + (d*x)/2)*(4*A*b - B*a)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*
b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^1
6*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*
b^3 - 8*a^22*b^2))/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 +
 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^1
6*b^3 - 5*a^17*b^2))))/a^5 - (8*tan(c/2 + (d*x)/2)*(128*A^2*b^16 + 4*B^2*a^
16 - 128*A^2*a*b^15 - 8*B^2*a^15*b - 768*A^2*a^2*b^14 + 768*A^2*a^3*b^13 +
1920*A^2*a^4*b^12 - 1920*A^2*a^5*b^11 - 2600*A^2*a^6*b^10 + 2560*A^2*a^7*b^
9 + 2025*A^2*a^8*b^8 - 1920*A^2*a^9*b^7 - 824*A^2*a^10*b^6 + 768*A^2*a^11*b
^5 + 80*A^2*a^12*b^4 - 128*A^2*a^13*b^3 + 64*A^2*a^14*b^2 + 8*B^2*a^2*b^14
- 8*B^2*a^3*b^13 - 48*B^2*a^4*b^12 + 48*B^2*a^5*b^11 + 117*B^2*a^6*b^10 - 1
20*B^2*a^7*b^9 - 164*B^2*a^8*b^8 + 160*B^2*a^9*b^7 + 156*B^2*a^10*b^6 - 120
*B^2*a^11*b^5 - 92*B^2*a^12*b^4 + 48*B^2*a^13*b^3 + 44*B^2*a^14*b^2 - 64*A*
```

B*a*b^15 - 32*A*B*a^15*b + 64*A*B*a^2*b^14 + 384*A*B*a^3*b^13 - 384*A*B*a^4
*b^12 - 948*A*B*a^5*b^11 + 960*A*B*a^6*b^10 + 1306*A*B*a^7*b^9 - 1280*A*B*a
^8*b^8 - 1128*A*B*a^9*b^7 + 960*A*B*a^10*b^6 + 592*A*B*a^11*b^5 - 384*A*B*a
^12*b^4 - 160*A*B*a^13*b^3 + 64*A*B*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 -
a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^
5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))*(4*A*b - B*a)*1i)/a^5 - ((((4*A
*b - B*a)*((8*(4*B*a^24 + 16*A*a^10*b^14 - 8*A*a^11*b^13 - 104*A*a^12*b^12
+ 50*A*a^13*b^11 + 286*A*a^14*b^10 - 126*A*a^15*b^9 - 434*A*a^16*b^8 + 174*
A*a^17*b^7 + 386*A*a^18*b^6 - 146*A*a^19*b^5 - 190*A*a^20*b^4 + 72*A*a^21*b
^3 + 40*A*a^22*b^2 - 4*B*a^11*b^13 + 2*B*a^12*b^12 + 26*B*a^13*b^11 - 14*B*
a^14*b^10 - 70*B*a^15*b^9 + 30*B*a^16*b^8 + 110*B*a^17*b^7 - 30*B*a^18*b^6
- 110*B*a^19*b^5 + 20*B*a^20*b^4 + 64*B*a^21*b^3 - 12*B*a^22*b^2 - 16*A*a^2
3*b - 16*B*a^23*b))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5
*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*
b^3 - 5*a^21*b^2) + (8*tan(c/2 + (d*x)/2)*(4*A*b - B*a)*(8*a^23*b - 8*a^10*
b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15
*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*
b^4 - 48*a^21*b^3 - 8*a^22*b^2))/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10
+ 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^
15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))))/a^5 + (8*tan(c/2 + (d*x)/2)*(128*A^2*b
^16 + 4*B^2*a^16 - 128*A^2*a*b^15 - 8*B^2*a^15*b - 768*A^2*a^2*b^14 + 768*A
^2*a^3*b^13 + 1920*A^2*a^4*b^12 - 1920*A^2*a^5*b^11 - 2600*A^2*a^6*b^10 + 2
560*A^2*a^7*b^9 + 2025*A^2*a^8*b^8 - 1920*A^2*a^9*b^7 - 824*A^2*a^10*b^6 +
768*A^2*a^11*b^5 + 80*A^2*a^12*b^4 - 128*A^2*a^13*b^3 + 64*A^2*a^14*b^2 + 8
*B^2*a^2*b^14 - 8*B^2*a^3*b^13 - 48*B^2*a^4*b^12 + 48*B^2*a^5*b^11 + 117*B^
2*a^6*b^10 - 120*B^2*a^7*b^9 - 164*B^2*a^8*b^8 + 160*B^2*a^9*b^7 + 156*B^2*
a^10*b^6 - 120*B^2*a^11*b^5 - 92*B^2*a^12*b^4 + 48*B^2*a^13*b^3 + 44*B^2*a^
14*b^2 - 64*A*B*a*b^15 - 32*A*B*a^15*b + 64*A*B*a^2*b^14 + 384*A*B*a^3*b^13
 - 384*A*B*a^4*b^12 - 948*A*B*a^5*b^11 + 960*A*B*a^6*b^10 + 1306*A*B*a^7*b^
9 - 1280*A*B*a^8*b^8 - 1128*A*B*a^9*b^7 + 960*A...

3.280 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=547

$$-\frac{b^2(40a^6Ab-84a^4Ab^3+69a^2Ab^5-20Ab^7-20a^7B+35a^5b^2B-28a^3b^4B+8ab^6B)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(}{\sqrt{a+}}\right.}{a^6(a-b)^{7/2}(a+b)^{7/2}d}$$

```
[Out] -b^2*(40*A*a^6*b-84*A*a^4*b^3+69*A*a^2*b^5-20*A*b^7-20*B*a^7+35*B*a^5*b^2-2
8*B*a^3*b^4+8*B*a*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a
^6/(a-b)^(7/2)/(a+b)^(7/2)/d+1/2*(A*a^2+20*A*b^2-8*B*a*b)*arctanh(sin(d*x+c
))/a^6/d-1/6*(24*A*a^6*b-146*A*a^4*b^3+167*A*a^2*b^5-60*A*b^7-6*B*a^7+65*B*
a^5*b^2-68*B*a^3*b^4+24*B*a*b^6)*tan(d*x+c)/a^5/(a^2-b^2)^3/d+1/2*(A*a^6-23
*A*a^4*b^2+27*A*a^2*b^4-10*A*b^6+12*B*a^5*b-11*B*a^3*b^3+4*B*a*b^5)*sec(d*x
+c)*tan(d*x+c)/a^4/(a^2-b^2)^3/d+1/3*b*(A*b-B*a)*sec(d*x+c)*tan(d*x+c)/a/(a
^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*b*(10*A*a^2*b-5*A*b^3-7*B*a^3+2*B*a*b^2)*s
ec(d*x+c)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*b*(48*A*a^4*b
-53*A*a^2*b^3+20*A*b^5-27*B*a^5+20*B*a^3*b^2-8*B*a*b^4)*sec(d*x+c)*tan(d*x+
c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 4.51, antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3079, 3134, 3080, 3855, 2738, 211}

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad(a^2-b^2)(a+b\cos(c+dx))^3}+\frac{(a^2A-8abB+20Ab^2)\tanh^{-1}(\sin(c+dx))}{2a^6d}+\frac{b(-7a^3B+10a^2Ab+2ab^2B-5Ab^3)\tan(c+dx)\sec(c+dx)}{6a^2d(a^2-b^2)^2(a+b\cos(c+dx))^2}+\frac{b(-27a^5B+48a^4Ab+20a^3b^2B-53a^2Ab^3-8ab^4B+20Ab^5)\tan(c+dx)\sec(c+dx)}{6a^3d(a^2-b^2)^3(a+b\cos(c+dx))}+\frac{(a^6A+12a^5bB-23a^4Ab^2-11a^3b^3B+27a^2Ab^4+4ab^5B-10Ab^6)\tan(c+dx)\sec(c+dx)}{2a^4d(a^2-b^2)^3}-\frac{b^2(-20a^7B+40a^6Ab+35a^5b^2B-84a^4Ab^3-28a^3b^4B+69a^2Ab^5+8ab^6B-20Ab^7)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^6d(a-b)^{7/2}(a+b)^{7/2}}-\frac{(-6a^7B+24a^6Ab+65a^5b^2B-146a^4Ab^3-68a^3b^4B+167a^2Ab^5+24ab^6B-60Ab^7)\tan(c+dx)}{6a^5d(a^2-b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((b^2*(40*a^6*A*b - 84*a^4*A*b^3 + 69*a^2*A*b^5 - 20*A*b^7 - 20*a^7*B + 35
*a^5*b^2*B - 28*a^3*b^4*B + 8*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a^6*(a - b)^(7/2)*(a + b)^(7/2)*d)) + ((a^2*A + 20*A*b^2 -
 8*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*a^6*d) - ((24*a^6*A*b - 146*a^4*A*b^3 +
 167*a^2*A*b^5 - 60*A*b^7 - 6*a^7*B + 65*a^5*b^2*B - 68*a^3*b^4*B + 24*a*b^
6*B)*Tan[c + d*x])/(6*a^5*(a^2 - b^2)^3*d) + ((a^6*A - 23*a^4*A*b^2 + 27*a^
2*A*b^4 - 10*A*b^6 + 12*a^5*b*B - 11*a^3*b^3*B + 4*a*b^5*B)*Sec[c + d*x]*Ta
n[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + (b*(A*b - a*B)*Sec[c + d*x]*Tan[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (b*(10*a^2*A*b - 5*A*b^3
- 7*a^3*B + 2*a*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a
 + b*Cos[c + d*x])^2) + (b*(48*a^4*A*b - 53*a^2*A*b^3 + 20*A*b^5 - 27*a^5*B
 + 20*a^3*b^2*B - 8*a*b^4*B)*Sec[c + d*x]*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^
3*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
```

```
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(3a^2A-5Ab^2+2abB-3a(Ab-a\ldots\right.}{}}{}$$

$$= \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(10a^2Ab-5Ab^3-7a^3B\ldots\right.}{6a^2\left(a^2-b^2\right)^2\ldots}$$

$$= \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(10a^2Ab-5Ab^3-7a^3B\ldots\right.}{6a^2\left(a^2-b^2\right)^2\ldots}$$

$$= \frac{\left(a^6A-23a^4Ab^2+27a^2Ab^4-10Ab^6+12a^5bB-11a^3b^3B+4ab\ldots\right.}{2a^4\left(a^2-b^2\right)^3d}$$

$$= -\frac{\left(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B+65a^5b^2B\ldots\right.}{6a^5\left(a^2-b^2\right)^3d}$$

$$= -\frac{\left(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B+65a^5b^2B\ldots\right.}{6a^5\left(a^2-b^2\right)^3d}$$

$$= \frac{\left(a^2A+20Ab^2-8abB\right)\tanh^{-1}(\sin(c+dx))}{2a^6d} - \frac{\left(24a^6Ab-146a^4\ldots\right.}{}$$

$$= -\frac{b^2\left(40a^6Ab-84a^4Ab^3+69a^2Ab^5-20Ab^7-20a^7B+35a^5b^2B\ldots\right.}{a^6(a-b)^{7/2}(a+b\ldots}$$

Mathematica [A]
time = 5.32, size = 781, normalized size = 1.43

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^4,x]
```

[Out] ((96*b^2*(-40*a^6*A*b + 84*a^4*A*b^3 - 69*a^2*A*b^5 + 20*A*b^7 + 20*a^7*B - 35*a^5*b^2*B + 28*a^3*b^4*B - 8*a*b^6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - 48*(a^2*A + 20*A*b^2 - 8*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 48*(a^2*A + 20*A*b^2 - 8*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a*(24*a^10*A - 324*a^8*A*b^2 + 1116*a^6*A*b^4 - 830*a^4*A*b^6 - 61*a^2*A*b^8 + 180*A*b^10 + 72*a^9*b*B - 438*a^7*b^3*B + 305*a^5*b^5*B + 28*a^3*b^7*B - 72*a*b^9*B + 6*a*(-20*a^8*A*b - 9*a^6*A*b^3 + 309*a^4*A*b^5 - 400*a^2*A*b^7 + 150*A*b^9 + 8*a^9*B - 6*a^7*b^2*B - 135*a^5*b^4*B + 163*a^3*b^6*B - 60*a*b^8*B)*Cos[c + d*x] + 12*b*(-21*a^8*A*b + 85*a^6*A*b^3 - 55*a^4*A*b^5 - 19*a^2*A*b^7 + 20*A*b^9 + 6*a^9*B - 36*a^7*b^2*B + 20*a^5*b^4*B + 8*a^3*b^6*B - 8*a*b^8*B)*Cos[2*(c + d*x)] - 138*a^7*A*b^3*Cos[3*(c + d*x)] + 738*a^5*A*b^5*Cos[3*(c + d*x)] - 840*a^3*A*b^7*Cos[3*(c + d*x)] + 300*a*A*b^9*Cos[3*(c + d*x)] + 36*a^8*b^2*B*Cos[3*(c + d*x)] - 318*a^6*b^4*B*Cos[3*(c + d*x)] + 342*a^4*b^6*B*Cos[3*(c + d*x)] - 120*a^2*b^8*B*Cos[3*(c + d*x)] - 24*a^6*A*b^4*Cos[4*(c + d*x)] + 146*a^4*A*b^6*Cos[4*(c + d*x)] - 167*a^2*A*b^8*Cos[4*(c + d*x)] + 60*A*b^10*Cos[4*(c + d*x)] + 6*a^7*b^3*B*Cos[4*(c + d*x)] - 65*a^5*b^5*B*Cos[4*(c + d*x)] + 68*a^3*b^7*B*Cos[4*(c + d*x)] - 24*a*b^9*B*Cos[4*(c + d*x)])*Sec[c + d*x]*Tan[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x])^3))/(96*a^6*d)

Maple [A]
time = 2.25, size = 672, normalized size = 1.23

method	result
derivativedivides	$-\frac{A}{2a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-aA-8Ab+2aB}{2a^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2A+20A\,b^2-8Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^6}-\frac{2b^2\left(-\frac{\left(30A\,a^4b+6A\,a^3b^2-34A\,a^2b\right.}{}\right.}{}$
default	$-\frac{A}{2a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-aA-8Ab+2aB}{2a^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2A+20A\,b^2-8Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^6}-\frac{2b^2\left(-\frac{\left(30A\,a^4b+6A\,a^3b^2-34A\,a^2b\right.}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2*A/a^4/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-A*a-8*A*b+2*B*a)/a^5/(tan(1/2*d*x+1/2*c)+1)+1/2*(A*a^2+20*A*b^2-8*B*a*b)/a^6*ln(tan(1/2*d*x+1/2*c)+1)-2*b^2/a^6*((-1/2*(30*A*a^4*b+6*A*a^3*b^2-34*A*a^2*b^3-3*A*a*b^4+12*A*b^5-20*

```
B*a^5-5*B*a^4*b+18*B*a^3*b^2+2*B*a^2*b^3-6*B*a*b^4)*a*b/(a-b)/(a^3+3*a^2*b+
3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(45*A*a^4*b-53*A*a^2*b^3+18*A*b^5-30*
B*a^5+29*B*a^3*b^2-9*B*a*b^4)*a*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3-1/2*(30*A*a^4*b-6*A*a^3*b^2-34*A*a^2*b^3+3*A*a*b^4+12*A*b^5-20*
B*a^5+5*B*a^4*b+18*B*a^3*b^2-2*B*a^2*b^3-6*B*a*b^4)*a*b/(a+b)/(a^3-3*a^2*b+
3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*
c)^2+a+b)^3+1/2*(40*A*a^6*b-84*A*a^4*b^3+69*A*a^2*b^5-20*A*b^7-20*B*a^7+35*
B*a^5*b^2-28*B*a^3*b^4+8*B*a*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b
))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))+1/2*A/a^4/(t
an(1/2*d*x+1/2*c)-1)^2-1/2*(-A*a-8*A*b+2*B*a)/a^5/(tan(1/2*d*x+1/2*c)-1)+1/
2/a^6*(-A*a^2-20*A*b^2+8*B*a*b)*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1875 vs. $2(525) = 1050$.
time = 97.31, size = 3819, normalized size = 6.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*((20*B*a^7*b^5 - 40*A*a^6*b^6 - 35*B*a^5*b^7 + 84*A*a^4*b^8 + 28*B
*a^3*b^9 - 69*A*a^2*b^10 - 8*B*a*b^11 + 20*A*b^12)*cos(d*x + c)^5 + 3*(20*B
*a^8*b^4 - 40*A*a^7*b^5 - 35*B*a^6*b^6 + 84*A*a^5*b^7 + 28*B*a^4*b^8 - 69*A
*a^3*b^9 - 8*B*a^2*b^10 + 20*A*a*b^11)*cos(d*x + c)^4 + 3*(20*B*a^9*b^3 - 4
0*A*a^8*b^4 - 35*B*a^7*b^5 + 84*A*a^6*b^6 + 28*B*a^5*b^7 - 69*A*a^4*b^8 - 8
*B*a^3*b^9 + 20*A*a^2*b^10)*cos(d*x + c)^3 + (20*B*a^10*b^2 - 40*A*a^9*b^3
- 35*B*a^8*b^4 + 84*A*a^7*b^5 + 28*B*a^6*b^6 - 69*A*a^5*b^7 - 8*B*a^4*b^8 +
 20*A*a^3*b^9)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (
2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d
*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3
```

*((A*a^10*b^3 - 8*B*a^9*b^4 + 16*A*a^8*b^5 + 32*B*a^7*b^6 - 74*A*a^6*b^7 - 48*B*a^5*b^8 + 116*A*a^4*b^9 + 32*B*a^3*b^10 - 79*A*a^2*b^11 - 8*B*a*b^12 + 20*A*b^13)*cos(d*x + c)^5 + 3*(A*a^11*b^2 - 8*B*a^10*b^3 + 16*A*a^9*b^4 + 32*B*a^8*b^5 - 74*A*a^7*b^6 - 48*B*a^6*b^7 + 116*A*a^5*b^8 + 32*B*a^4*b^9 - 79*A*a^3*b^10 - 8*B*a^2*b^11 + 20*A*a*b^12)*cos(d*x + c)^4 + 3*(A*a^12*b - 8*B*a^11*b^2 + 16*A*a^10*b^3 + 32*B*a^9*b^4 - 74*A*a^8*b^5 - 48*B*a^7*b^6 + 116*A*a^6*b^7 + 32*B*a^5*b^8 - 79*A*a^4*b^9 - 8*B*a^3*b^10 + 20*A*a^2*b^11)*cos(d*x + c)^3 + (A*a^13 - 8*B*a^12*b + 16*A*a^11*b^2 + 32*B*a^10*b^3 - 74*A*a^9*b^4 - 48*B*a^8*b^5 + 116*A*a^7*b^6 + 32*B*a^6*b^7 - 79*A*a^5*b^8 - 8*B*a^4*b^9 + 20*A*a^3*b^10)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((A*a^10*b^3 - 8*B*a^9*b^4 + 16*A*a^8*b^5 + 32*B*a^7*b^6 - 74*A*a^6*b^7 - 48*B*a^5*b^8 + 116*A*a^4*b^9 + 32*B*a^3*b^10 - 79*A*a^2*b^11 - 8*B*a*b^12 + 20*A*b^13)*cos(d*x + c)^5 + 3*(A*a^11*b^2 - 8*B*a^10*b^3 + 16*A*a^9*b^4 + 32*B*a^8*b^5 - 74*A*a^7*b^6 - 48*B*a^6*b^7 + 116*A*a^5*b^8 + 32*B*a^4*b^9 - 79*A*a^3*b^10 - 8*B*a^2*b^11 + 20*A*a*b^12)*cos(d*x + c)^4 + 3*(A*a^12*b - 8*B*a^11*b^2 + 16*A*a^10*b^3 + 32*B*a^9*b^4 - 74*A*a^8*b^5 - 48*B*a^7*b^6 + 116*A*a^6*b^7 + 32*B*a^5*b^8 - 79*A*a^4*b^9 - 8*B*a^3*b^10 + 20*A*a^2*b^11)*cos(d*x + c)^3 + (A*a^13 - 8*B*a^12*b + 16*A*a^11*b^2 + 32*B*a^10*b^3 - 74*A*a^9*b^4 - 48*B*a^8*b^5 + 116*A*a^7*b^6 + 32*B*a^6*b^7 - 79*A*a^5*b^8 - 8*B*a^4*b^9 + 20*A*a^3*b^10)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(3*A*a^13 - 12*A*a^11*b^2 + 18*A*a^9*b^4 - 12*A*a^7*b^6 + 3*A*a^5*b^8 + (6*B*a^10*b^3 - 24*A*a^9*b^4 - 71*B*a^8*b^5 + 170*A*a^7*b^6 + 133*B*a^6*b^7 - 313*A*a^5*b^8 - 92*B*a^4*b^9 + 227*A*a^3*b^10 + 24*B*a^2*b^11 - 60*A*a*b^12)*cos(d*x + c)^4 + 3*(6*B*a^11*b^2 - 23*A*a^10*b^3 - 59*B*a^9*b^4 + 146*A*a^8*b^5 + 110*B*a^7*b^6 - 263*A*a^6*b^7 - 77*B*a^5*b^8 + 190*A*a^4*b^9 + 20*B*a^3*b^10 - 50*A*a^2*b^11)*cos(d*x + c)^3 + (18*B*a^12*b - 63*A*a^11*b^2 - 132*B*a^10*b^3 + 342*A*a^9*b^4 + 239*B*a^8*b^5 - 590*A*a^7*b^6 - 169*B*a^6*b^7 + 421*A*a^5*b^8 + 44*B*a^4*b^9 - 110*A*a^3*b^10)*cos(d*x + c)^2 + 3*(2*B*a^13 - 5*A*a^12*b - 8*B*a^11*b^2 + 20*A*a^10*b^3 + 12*B*a^9*b^4 - 30*A*a^8*b^5 - 8*B*a^7*b^6 + 20*A*a^6*b^7 + 2*B*a^5*b^8 - 5*A*a^4*b^9)*cos(d*x + c))*sin(d*x + c))/((a^14*b^3 - 4*a^12*b^5 + 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d*cos(d*x + c)^5 + 3*(a^15*b^2 - 4*a^13*b^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + c)^4 + 3*(a^16*b - 4*a^14*b^3 + 6*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^3 + (a^17 - 4*a^15*b^2 + 6*a^13*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^2), 1/12*(6*((20*B*a^7*b^5 - 40*A*a^6*b^6 - 35*B*a^5*b^7 + 84*A*a^4*b^8 + 28*B*a^3*b^9 - 69*A*a^2*b^10 - 8*B*a*b^11 + 20*A*b^12)*cos(d*x + c)^5 + 3*(20*B*a^8*b^4 - 40*A*a^7*b^5 - 35*B*a^6*b^6 + 84*A*a^5*b^7 + 28*B*a^4*b^8 - 69*A*a^3*b^9 - 8*B*a^2*b^10 + 20*A*a*b^11)*cos(d*x + c)^4 + 3*(20*B*a^9*b^3 - 40*A*a^8*b^4 - 35*B*a^7*b^5 + 84*A*a^6*b^6 + 28*B*a^5*b^7 - 69*A*a^4*b^8 - 8*B*a^3*b^9 + 20*A*a^2*b^10)*cos(d*x + c)^3 + (20*B*a^10*b^2 - 40*A*a^9*b^3 - 35*B*a^8*b^4 + 84*A*a^7*b^5 + 28*B*a^6*b^6 - 69*A*a^5*b^7 - 8*B*a^4*b^8 + 20*A*a^3*b^9)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + 3*((A*a^10*b^3 - 8*B*a^9*b^4 + 16*A*a^8*b^5 + 32*B*a^7*b^6 - 74*A*a^6*b^7 - 48*B*a^5*b^8 + 116*A*a^4*b^9 + 32*B*a^3*b^10 - 79*A*a^2*b^11 - 8*B*a*b^12 + 20*A*b^13)*c

```
os(d*x + c)^5 + 3*(A*a^11*b^2 - 8*B*a^10*b^3 + 16*A*a^9*b^4 + 32*B*a^8*b^5
- 74*A*a^7*b^6 - 48*B*a^6*b^7 + 116*A*a^5*b^8 + 32*B*a^4*b^9 - 79*A*a^3*b^1
0 - 8*B*a^2*b^11 + 20*A*a*b^12)*cos(d*x + c)^4 + 3*(A*a^12*b - 8*B*a^11*b^2
 + 16*A*a^10*b^3 + 32*B*a^9*b^4 - 74*A*a^8*b^5 - 48*B*a^7*b^6 + 116*A*a^6*b
^7 + 32*B*a^5*b^8 - 79*A*a^4*b^9 - 8*B*a^3*b^10 + 20*A*a^2*b^11)*cos(d*x +
c)^3 + (A*a^13 - 8*B*a^12*b + 16*A*a^11*b^2 + 32*B*a^10*b^3 - 74*A*a^9*b^4
- 48*B*a^8*b^5 + 116*A*a^7*b^6 + 32*B*a^6*b^7 - 79*A*a^5*b^8 - 8*B*a^4*b^9
+ 20*A*a^3*b^10)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((A*a^10*b^3 - 8
*B*a^9*b^4 + 16*A*a^8*b^5 + 32*B*a^7*b^6 - 74*A*a^6*b^7 - 48*B*a^5*b^8 + 11
6*A*a^4*b^9 + 32*B*a^3*b^10 - 79*A*a^2*b^11 - 8*B*a*b^12 + 20*A*b^13)*cos(d
*x + c)^5 + 3*(A*a^11*b^2 - 8*B*a^10*b^3 + 16*A*a^9*b^4 + 32*B*a^8*b^5 - 74
*A*a^7*b^6 - 48*B*a^6*b^7 + 116*A*a^5*b^8 + 32*...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x))**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1090 vs. $2(525) = 1050$.
time = 0.54, size = 1090, normalized size = 1.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/6*(6*(20*B*a^7*b^2 - 40*A*a^6*b^3 - 35*B*a^5*b^4 + 84*A*a^4*b^5 + 28*B*a
^3*b^6 - 69*A*a^2*b^7 - 8*B*a*b^8 + 20*A*b^9)*(pi*floor(1/2*(d*x + c)/pi +
1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/
2*c))/sqrt(a^2 - b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*sqrt(a^2
 - b^2)) + 2*(60*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 - 90*A*a^6*b^4*tan(1/2*d*
x + 1/2*c)^5 - 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 + 162*A*a^5*b^5*tan(1/2
*d*x + 1/2*c)^5 - 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^4*b^6*tan(1/
2*d*x + 1/2*c)^5 + 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 213*A*a^3*b^7*tan
(1/2*d*x + 1/2*c)^5 - 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^2*b^8*ta
n(1/2*d*x + 1/2*c)^5 - 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 + 81*A*a*b^9*tan
(1/2*d*x + 1/2*c)^5 + 18*B*a*b^9*tan(1/2*d*x + 1/2*c)^5 - 36*A*b^10*tan(1/2
*d*x + 1/2*c)^5 + 120*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^3 - 180*A*a^6*b^4*tan(
```

```
1/2*d*x + 1/2*c)^3 - 236*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^3 + 392*A*a^4*b^6*t
an(1/2*d*x + 1/2*c)^3 + 152*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^3 - 284*A*a^2*b^
8*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*b^9*tan(1/2*d*x + 1/2*c)^3 + 72*A*b^10*ta
n(1/2*d*x + 1/2*c)^3 + 60*B*a^7*b^3*tan(1/2*d*x + 1/2*c) - 90*A*a^6*b^4*tan
(1/2*d*x + 1/2*c) + 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c) - 162*A*a^5*b^5*tan(
1/2*d*x + 1/2*c) - 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c) + 48*A*a^4*b^6*tan(1/2
*d*x + 1/2*c) - 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c) + 213*A*a^3*b^7*tan(1/2*
d*x + 1/2*c) - 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c) + 48*A*a^2*b^8*tan(1/2*d*x
 + 1/2*c) + 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c) - 81*A*a*b^9*tan(1/2*d*x + 1/
2*c) + 18*B*a*b^9*tan(1/2*d*x + 1/2*c) - 36*A*b^10*tan(1/2*d*x + 1/2*c))/((
a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1
/2*d*x + 1/2*c)^2 + a + b)^3) - 3*(A*a^2 - 8*B*a*b + 20*A*b^2)*log(abs(tan(
1/2*d*x + 1/2*c) + 1))/a^6 + 3*(A*a^2 - 8*B*a*b + 20*A*b^2)*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^6 - 6*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x
 + 1/2*c)^3 + 8*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + 2*B
*a*tan(1/2*d*x + 1/2*c) - 8*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c
)^2 - 1)^2*a^5))/d
```

Mupad [B]
time = 13.94, size = 2500, normalized size = 4.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^4),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(A*a^8 + 20*A*b^8 + 2*B*a^8 - 59*A*a^2*b^6 - 27*A*a^3*
b^5 + 57*A*a^4*b^4 + 21*A*a^5*b^3 - 11*A*a^6*b^2 - 4*B*a^2*b^6 + 24*B*a^3*b
^5 + 11*B*a^4*b^4 - 26*B*a^5*b^3 - 6*B*a^6*b^2 + 10*A*a*b^7 - 7*A*a^7*b - 8
*B*a*b^7 + 2*B*a^7*b))/(a^5*(a + b)*(a - b)^3) + (2*tan(c/2 + (d*x)/2)^5*(9
*A*a^10 + 180*A*b^10 - 611*A*a^2*b^8 + 740*A*a^4*b^6 - 324*A*a^6*b^4 + 36*A
*a^8*b^2 + 248*B*a^3*b^7 - 320*B*a^5*b^5 + 132*B*a^7*b^3 - 72*B*a*b^9 - 18*
B*a^9*b))/(3*a^5*(a + b)^3*(a - b)^3) + (tan(c/2 + (d*x)/2)^9*(A*a^8 + 20*A
*b^8 - 2*B*a^8 - 59*A*a^2*b^6 + 27*A*a^3*b^5 + 57*A*a^4*b^4 - 21*A*a^5*b^3
- 11*A*a^6*b^2 + 4*B*a^2*b^6 + 24*B*a^3*b^5 - 11*B*a^4*b^4 - 26*B*a^5*b^3 +
 6*B*a^6*b^2 - 10*A*a*b^7 + 7*A*a^7*b - 8*B*a*b^7 + 2*B*a^7*b))/(a^5*(a + b
)^3*(a - b)) + (2*tan(c/2 + (d*x)/2)^3*(6*A*a^9 - 120*A*b^9 + 6*B*a^9 + 364
*A*a^2*b^7 + 71*A*a^3*b^6 - 369*A*a^4*b^5 - 45*A*a^5*b^4 + 111*A*a^6*b^3 +
3*A*a^7*b^2 + 12*B*a^2*b^7 - 148*B*a^3*b^6 - 29*B*a^4*b^5 + 159*B*a^5*b^4 +
 18*B*a^6*b^3 - 30*B*a^7*b^2 - 30*A*a*b^8 - 21*A*a^8*b + 48*B*a*b^8 - 6*B*a
^8*b))/(3*a^5*(a + b)^2*(a - b)^3) + (2*tan(c/2 + (d*x)/2)^7*(6*A*a^9 + 120
*A*b^9 - 6*B*a^9 - 364*A*a^2*b^7 + 71*A*a^3*b^6 + 369*A*a^4*b^5 - 45*A*a^5*
b^4 - 111*A*a^6*b^3 + 3*A*a^7*b^2 + 12*B*a^2*b^7 + 148*B*a^3*b^6 - 29*B*a^4
*b^5 - 159*B*a^5*b^4 + 18*B*a^6*b^3 + 30*B*a^7*b^2 - 30*A*a*b^8 + 21*A*a^8*
b - 48*B*a*b^8 - 6*B*a^8*b))/(3*a^5*(a + b)^3*(a - b)^2))/(d*(tan(c/2 + (d*
```

x)/2)^4*(6*a*b^2 - 6*a^2*b - 2*a^3 + 10*b^3) - tan(c/2 + (d*x)/2)^2*(9*a*b^2 + 3*a^2*b - a^3 + 5*b^3) + tan(c/2 + (d*x)/2)^6*(6*a*b^2 + 6*a^2*b - 2*a^3 - 10*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^10*(3*a*b^2 - 3*a^2*b + a^3 - b^3) + tan(c/2 + (d*x)/2)^8*(3*a^2*b - 9*a*b^2 + a^3 + 5*b^3))) + (atan(((((8*tan(c/2 + (d*x)/2)*(800*A^2*a*b^17 - 800*A^2*b^18 - A^2*a^18 + 2*A^2*a^17*b + 4720*A^2*a^2*b^16 - 4720*A^2*a^3*b^15 - 11522*A^2*a^4*b^14 + 11522*A^2*a^5*b^13 + 14837*A^2*a^6*b^12 - 14812*A^2*a^7*b^11 - 10385*A^2*a^8*b^10 + 10430*A^2*a^9*b^9 + 3325*A^2*a^10*b^8 - 3640*A^2*a^11*b^7 + 45*A^2*a^12*b^6 + 350*A^2*a^13*b^5 - 209*A^2*a^14*b^4 + 68*A^2*a^15*b^3 - 35*A^2*a^16*b^2 - 128*B^2*a^2*b^16 + 128*B^2*a^3*b^15 + 768*B^2*a^4*b^14 - 768*B^2*a^5*b^13 - 1920*B^2*a^6*b^12 + 1920*B^2*a^7*b^11 + 2600*B^2*a^8*b^10 - 2560*B^2*a^9*b^9 - 2025*B^2*a^10*b^8 + 1920*B^2*a^11*b^7 + 824*B^2*a^12*b^6 - 768*B^2*a^13*b^5 - 80*B^2*a^14*b^4 + 128*B^2*a^15*b^3 - 64*B^2*a^16*b^2 + 640*A*B*a*b^17 + 16*A*B*a^17*b - 640*A*B*a^2*b^16 - 3808*A*B*a^3*b^15 + 3808*A*B*a^4*b^14 + 9408*A*B*a^5*b^13 - 9408*A*B*a^6*b^12 - 12430*A*B*a^7*b^11 + 12320*A*B*a^8*b^10 + 9200*A*B*a^9*b^9 - 8960*A*B*a^10*b^8 - 3360*A*B*a^11*b^7 + 3360*A*B*a^12*b^6 + 144*A*B*a^13*b^5 - 448*A*B*a^14*b^4 + 240*A*B*a^15*b^3 - 32*A*B*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2) + (((4*(4*A*a^27 - 80*A*a^12*b^15 + 40*A*a^13*b^14 + 516*A*a^14*b^13 - 248*A*a^15*b^12 - 1404*A*a^16*b^11 + 640*A*a^17*b^10 + 2076*A*a^18*b^9 - 896*A*a^19*b^8 - 1764*A*a^20*b^7 + 724*A*a^21*b^6 + 816*A*a^22*b^5 - 316*A*a^23*b^4 - 160*A*a^24*b^3 + 52*A*a^25*b^2 + 32*B*a^13*b^14 - 16*B*a^14*b^13 - 208*B*a^15*b^12 + 100*B*a^16*b^11 + 572*B*a^17*b^10 - 252*B*a^18*b^9 - 868*B*a^19*b^8 + 348*B*a^20*b^7 + 772*B*a^21*b^6 - 292*B*a^22*b^5 - 380*B*a^23*b^4 + 144*B*a^24*b^3 + 80*B*a^25*b^2 - 32*B*a^26*b))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) - (4*tan(c/2 + (d*x)/2)*(A*a^2 + 20*A*b^2 - 8*B*a*b)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/(a^6*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*(A*a^2 + 20*A*b^2 - 8*B*a*b))/(2*a^6))*(A*a^2 + 20*A*b^2 - 8*B*a*b)*1i)/(2*a^6) + (((8*tan(c/2 + (d*x)/2)*(800*A^2*a*b^17 - 800*A^2*b^18 - A^2*a^18 + 2*A^2*a^17*b + 4720*A^2*a^2*b^16 - 4720*A^2*a^3*b^15 - 11522*A^2*a^4*b^14 + 11522*A^2*a^5*b^13 + 14837*A^2*a^6*b^12 - 14812*A^2*a^7*b^11 - 10385*A^2*a^8*b^10 + 10430*A^2*a^9*b^9 + 3325*A^2*a^10*b^8 - 3640*A^2*a^11*b^7 + 45*A^2*a^12*b^6 + 350*A^2*a^13*b^5 - 209*A^2*a^14*b^4 + 68*A^2*a^15*b^3 - 35*A^2*a^16*b^2 - 128*B^2*a^2*b^16 + 128*B^2*a^3*b^15 + 768*B^2*a^4*b^14 - 768*B^2*a^5*b^13 - 1920*B^2*a^6*b^12 + 1920*B^2*a^7*b^11 + 2600*B^2*a^8*b^10 - 2560*B^2*a^9*b^9 - 2025*B^2*a^10*b^8 + 1920*B^2*a^11*b^7 + 824*B^2*a^12*b^6 - 768*B^2*a^13*b^5 - 80*B^2*a^14*b^4 + 128*B^2*a^15*b^3 - 64*B^2*a^16*b^2 + 640*A*B*a*b^17 + 16*A*B*a^17*b - 640*A*B*a^2*b^16 - 3808*A*B*a^3*b^15 + 3808*A*B*a^4*b^14 + 9408*A*B*a^5*

```
b^13 - 9408*A*B*a^6*b^12 - 12430*A*B*a^7*b^11 + 12320*A*B*a^8*b^10 + 9200*A
*B*a^9*b^9 - 8960*A*B*a^10*b^8 - 3360*A*B*a^11*b^7 + 3360*A*B*a^12*b^6 + 14
4*A*B*a^13*b^5 - 448*A*B*a^14*b^4 + 240*A*B*a^1...
```

3.281 $\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{B\sin(c+dx)}{d}-\frac{B\sin^3(c+dx)}{3d}$$

```
[Out] B*sin(d*x+c)/d-1/3*B*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {21, 2713}

$$\frac{B\sin(c+dx)}{d}-\frac{B\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*Sin[c + d*x])/d - (B*Sin[c + d*x]^3)/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx &= B\int \cos^3(c+dx)\,dx\\ &= -\frac{B\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\ &= \frac{B\sin(c+dx)}{d}-\frac{B\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.00

$$B\left(\frac{\sin(c+dx)}{d}-\frac{\sin^3(c+dx)}{3d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*(Sin[c + d*x]/d - Sin[c + d*x]^3/(3*d))
```

Maple [A]
time = 0.16, size = 23, normalized size = 0.82

method	result	size
derivativedivides	$\frac{B\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3d}$	23
default	$\frac{B\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3d}$	23
risch	$\frac{3B\sin(dx+c)}{4d}+\frac{B\sin(3dx+3c)}{12d}$	29
norman	$\frac{\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{10B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{10B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2B\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/3/d*B*(cos(d*x+c)^2+2)*sin(d*x+c)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.37, size = 25, normalized size = 0.89

$$\frac{\left(B\cos\left(dx+c\right)^2+2\,B\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/3*(B*cos(d*x + c)^2 + 2*B)*sin(d*x + c)/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(22) = 44$.
time = 0.46, size = 56, normalized size = 2.00

$$\begin{cases} \frac{2B\sin^3(c+dx)}{3d} + \frac{B\sin(c+dx)\cos^2(c+dx)}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\cos(c))\cos^3(c)}{a+b\cos(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((2*B*sin(c + d*x)**3/(3*d) + B*sin(c + d*x)*cos(c + d*x)**2/d, Ne
(d, 0)), (x*(B*a + B*b*cos(c))*cos(c)**3/(a + b*cos(c)), True))
```

Giac [A]
time = 0.43, size = 25, normalized size = 0.89

$$-\frac{B\sin(dx+c)^3 - 3B\sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] -1/3*(B*sin(d*x + c)^3 - 3*B*sin(d*x + c))/d
```

Mupad [B]
time = 0.48, size = 24, normalized size = 0.86

$$\frac{B(9\sin(c+dx) + \sin(3c+3dx))}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] (B*(9*sin(c + d*x) + sin(3*c + 3*d*x)))/(12*d)
```

3.282 $\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{Bx}{2}+\frac{B\cos(c+dx)\sin(c+dx)}{2d}$$

[Out] 1/2*B*x+1/2*B*cos(d*x+c)*sin(d*x+c)/d

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {21, 2715, 8}

$$\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]

[Out] (B*x)/2 + (B*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = B\int \cos^2(c+dx)\,dx$$
$$= \frac{B\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}B\int 1\,dx$$
$$= \frac{Bx}{2} + \frac{B\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A]
time = 0.03, size = 24, normalized size = 0.89

$$\frac{B(2(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
[Out] (B*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A]
time = 0.13, size = 28, normalized size = 1.04

method	result	size
risch	$\frac{Bx}{2}+\frac{B\sin(2dx+2c)}{4d}$	21
derivativedivides	$\frac{B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	28
default	$\frac{B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	28
norman	$\frac{\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{Bx}{2}-\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3Bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3Bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{Bx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURNVERB
OSE)
[Out] 1/d*B*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.43, size = 24, normalized size = 0.89

$$\frac{Bdx + B\cos(dx + c)\sin(dx + c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/2*(B*d*x + B*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(22) = 44$.
time = 0.39, size = 68, normalized size = 2.52

$$\begin{cases} \frac{Bx\sin^2(c+dx)}{2} + \frac{Bx\cos^2(c+dx)}{2} + \frac{B\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\cos(c))\cos^2(c)}{a+b\cos(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*x*sin(c + d*x)**2/2 + B*x*cos(c + d*x)**2/2 + B*sin(c + d*x)*c
os(c + d*x)/(2*d), Ne(d, 0)), (x*(B*a + B*b*cos(c))*cos(c)**2/(a + b*cos(c)
), True))
```

Giac [A]
time = 0.50, size = 33, normalized size = 1.22

$$\frac{(dx + c)B + \frac{B\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/2*((d*x + c)*B + B*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

Mupad [B]
time = 0.87, size = 50, normalized size = 1.85

$$\frac{Bx}{2}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] (B*x)/2 + (B*tan(c/2 + (d*x)/2) - B*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*
x)/2)^2 + 1)^2)
```

3.283 $\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=11

$$\frac{B\sin(c+dx)}{d}$$

```
[Out] B*sin(d*x+c)/d
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {21, 2717}

$$\frac{B\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*Sin[c + d*x])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx &= B\int \cos(c+dx)\,dx\\ &= \frac{B\sin(c+dx)}{d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.
time = 0.01, size = 23, normalized size = 2.09

$$B\left(\frac{\cos(dx)\sin(c)}{d}+\frac{\cos(c)\sin(dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*((Cos[d*x]*Sin[c])/d + (Cos[c]*Sin[d*x])/d)
```

Maple [A]
time = 0.10, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{B\sin(dx+c)}{d}$	12
default	$\frac{B\sin(dx+c)}{d}$	12
risch	$\frac{B\sin(dx+c)}{d}$	12
norman	$\frac{\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] B*sin(d*x+c)/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.39, size = 11, normalized size = 1.00

$$\frac{B\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] B*sin(d*x + c)/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(8) = 16$.
time = 0.32, size = 31, normalized size = 2.82

$$\begin{cases} \frac{B\sin{(c+dx)}}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\cos{(c)})\cos{(c)}}{a+b\cos{(c)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*sin(c + d*x)/d, Ne(d, 0)), (x*(B*a + B*b*cos(c))*cos(c)/(a + b
*cos(c)), True))
```

Giac [A]
time = 0.42, size = 11, normalized size = 1.00

$$\frac{B\sin{(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] B*sin(d*x + c)/d
```

Mupad [B]
time = 0.47, size = 11, normalized size = 1.00

$$\frac{B\,\sin{(c+d\,x)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] (B*sin(c + d*x))/d
```

3.284 $\int \frac{aB+bB\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=3

$$Bx$$

```
[Out] B*x
```

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {21, 8}

$$Bx$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{aB+bB\cos(c+dx)}{a+b\cos(c+dx)}\,dx &= B\int 1\,dx\\ &= Bx\end{aligned}$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*x
```

Maple [A]
time = 0.05, size = 4, normalized size = 1.33

method	result	size
default	Bx	4
risch	Bx	4
derivativedivides	$\frac{B(dx+c)}{d}$	11
norman	$\frac{Bx+Bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] B*x
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.37, size = 3, normalized size = 1.00

$$Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] B*x
```

Sympy [A]
time = 0.05, size = 2, normalized size = 0.67

$$Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] B*x
```

Giac [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.44, size = 10, normalized size = 3.33

$$\frac{(dx+c)B}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] (d*x + c)*B/d
```

Mupad [B]
time = 0.45, size = 3, normalized size = 1.00

$$B\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x)),x)
```

```
[Out] B*x
```

3.285 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=12

$$\frac{B\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] B*arctanh(sin(d*x+c))/d
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {21, 3855}

$$\frac{B\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx &= B\int \sec(c+dx)\,dx\\ &= \frac{B\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{B\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/d
```

Maple [A]
time = 0.12, size = 20, normalized size = 1.67

method	result	size
derivativedivides	$\frac{B\ln(\sec(dx+c)+\tan(dx+c))}{d}$	20
default	$\frac{B\ln(\sec(dx+c)+\tan(dx+c))}{d}$	20
norman	$\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	37
risch	$\frac{B\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{B\ln\left(e^{i(dx+c)}-i\right)}{d}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.
time = 0.39, size = 31, normalized size = 2.58

$$\frac{B\log\left(\sin\left(dx+c\right)+1\right)-B\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*(B*log(sin(d*x + c) + 1) - B*log(-sin(d*x + c) + 1))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(10) = 20$.
time = 2.26, size = 39, normalized size = 3.25

$$\begin{cases} \frac{B\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\cos\left(c\right))\sec\left(c\right)}{a+b\cos\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise(((B*log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(B*a + B*b*c
os(c))*sec(c)/(a + b*cos(c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(12) = 24$.
time = 0.47, size = 47, normalized size = 3.92

$$\frac{B\log\left(\left|\frac{1}{\sin\left(dx+c\right)}+\sin\left(dx+c\right)+2\right|\right)-B\log\left(\left|\frac{1}{\sin\left(dx+c\right)}+\sin\left(dx+c\right)-2\right|\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/4*(B*log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - B*log(abs(1/sin(d*x +
c) + sin(d*x + c) - 2)))/d
```

Mupad [B]
time = 0.49, size = 16, normalized size = 1.33

$$\frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))),x)
```

```
[Out] (2*B*atanh(tan(c/2 + (d*x)/2)))/d
```

3.286 $\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=11

$$\frac{B\tan(c+dx)}{d}$$

[Out] B*tan(d*x+c)/d

Rubi [A]

time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {21, 3852, 8}

$$\frac{B\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]

[Out] (B*Tan[c + d*x])/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx &= B\int \sec^2(c+dx)\,dx\\ &= -\frac{B\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}\\ &= \frac{B\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\frac{B\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
[Out] (B*Tan[c + d*x])/d
```

Maple [A]
time = 0.13, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{B\tan(dx+c)}{d}$	12
default	$\frac{B\tan(dx+c)}{d}$	12
risch	$\frac{2iB}{d\left(e^{2i(dx+c)}+1\right)}$	21
norman	$\frac{-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x,method=_RETURNVERB
OSE)
[Out] B*tan(d*x+c)/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="
maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.38, size = 19, normalized size = 1.73

$$\frac{B\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] B*sin(d*x + c)/(d*cos(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.
time = 1.82, size = 32, normalized size = 2.91

$$\begin{cases} \frac{B\tan(c+dx)}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\cos(c))\sec^2(c)}{a+b\cos(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*tan(c + d*x)/d, Ne(d, 0)), (x*(B*a + B*b*cos(c))*sec(c)**2/(a
+ b*cos(c)), True))
```

Giac [A]
time = 0.49, size = 11, normalized size = 1.00

$$\frac{B\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] B*tan(d*x + c)/d
```

Mupad [B]
time = 0.47, size = 30, normalized size = 2.73

$$-\frac{2B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))),x)
```

```
[Out] -(2*B*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.287 $\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{B\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*B*arctanh(sin(d*x+c))/d+1/2*B*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {21, 3853, 3855}

$$\frac{B\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(2*d) + (B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = B\int \sec^3(c+dx)\,dx$$
$$= \frac{B\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}B\int \sec(c+dx)\,dx$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))}{2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 1.00

$$B\left(\frac{\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\sec(c+dx)\tan(c+dx)}{2d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
[Out] B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))
```

Maple [A]
time = 0.18, size = 37, normalized size = 1.03

method	result	size
derivativedivides	$\frac{B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	37
default	$\frac{B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	37
risch	$-\frac{iB\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{B\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{B\ln\left(e^{i(dx+c)}-i\right)}{2d}$	81
norman	$\frac{\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x,method=_RETURNVERB
OSE)
[Out] 1/d*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.40, size = 64, normalized size = 1.78

$$\frac{B\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-B\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)+2\,B\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/4*(B*cos(d*x + c)^2*log(sin(d*x + c) + 1) - B*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*B*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int\sec^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

```
[Out] B*Integral(sec(c + d*x)**3, x)
```

Giac [A]
time = 0.48, size = 52, normalized size = 1.44

$$\frac{B\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-B\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,B\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/4*(B*log(abs(sin(d*x + c) + 1)) - B*log(abs(sin(d*x + c) - 1)) - 2*B*sin(
d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Mupad [B]
time = 0.86, size = 73, normalized size = 2.03

$$\frac{B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{B\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))),x)
```

```
[Out] (B*tan(c/2 + (d*x)/2) + B*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4 -
2*tan(c/2 + (d*x)/2)^2 + 1)) + (B*atanh(tan(c/2 + (d*x)/2)))/d
```

3.288 $\int \frac{(aB+bB\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{B\tan(c+dx)}{d}+\frac{B\tan^3(c+dx)}{3d}$$

```
[Out] B*tan(d*x+c)/d+1/3*B*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {21, 3852}

$$\frac{B\tan^3(c+dx)}{3d}+\frac{B\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*Tan[c + d*x])/d + (B*Tan[c + d*x]^3)/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(aB+bB\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx &= B\int \sec^4(c+dx)\,dx\\ &= -\frac{B\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{d}\\ &= \frac{B\tan(c+dx)}{d}+\frac{B\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 24, normalized size = 0.86

$$\frac{B\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A]
time = 0.20, size = 25, normalized size = 0.89

method	result	size
derivativedivides	$-\frac{B\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$	25
default	$-\frac{B\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$	25
risch	$\frac{4iB\left(3\,\mathrm{e}^{2i(dx+c)}+1\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	34
norman	$\frac{-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2B\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/d*B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.40, size = 32, normalized size = 1.14

$$\frac{\left(2 B \cos\left(dx+c\right)^{2}+B\right) \sin\left(dx+c\right)}{3 d \cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*B*cos(d*x + c)^2 + B)*sin(d*x + c)/(d*cos(d*x + c)^3)
```

Sympy [A]
time = 10.71, size = 42, normalized size = 1.50

$$\begin{cases} \frac{B\left(\frac{\tan^{3}\left(c+dx\right)}{3}+\tan\left(c+dx\right)\right)}{d} & \text{for}\: d \neq 0 \\ \frac{x(Ba+Bb\cos\left(c\right)) \sec^{4}\left(c\right)}{a+b\cos\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*(tan(c + d*x)**3/3 + tan(c + d*x))/d, Ne(d, 0)), (x*(B*a + B*b
*cos(c))*sec(c)**4/(a + b*cos(c)), True))
```

Giac [A]
time = 0.46, size = 25, normalized size = 0.89

$$\frac{B \tan\left(dx+c\right)^{3}+3 B \tan\left(dx+c\right)}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/3*(B*tan(d*x + c)^3 + 3*B*tan(d*x + c))/d
```

Mupad [B]
time = 0.52, size = 39, normalized size = 1.39

$$\frac{2 B \sin\left(c+d x\right) \cos\left(c+d x\right)^{2}+B \sin\left(c+d x\right)}{3 d \cos\left(c+d x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^4*(a + b*cos(c + d*x))),x)
```

```
[Out] (B*sin(c + d*x) + 2*B*cos(c + d*x)^2*sin(c + d*x))/(3*d*cos(c + d*x)^3)
```

3.289 $\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=114

$$\frac{(2a^2+b^2)\,Bx}{2b^3}-\frac{2a^3B\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d}-\frac{aB\sin(c+dx)}{b^2d}+\frac{B\cos(c+dx)\sin(c+dx)}{2bd}$$

```
[Out] 1/2*(2*a^2+b^2)*B*x/b^3-a*B*sin(d*x+c)/b^2/d+1/2*B*cos(d*x+c)*sin(d*x+c)/b/
d-2*a^3*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^3/d/(a-b)^(1
/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {21, 2872, 3102, 2814, 2738, 211}

$$-\frac{2a^3B\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\,\sqrt{a+b}}+\frac{Bx(2a^2+b^2)}{2b^3}-\frac{aB\sin(c+dx)}{b^2d}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*B*x)/(2*b^3) - (2*a^3*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*B*Sin[c + d*x])/(b^2*d
) + (B*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{B\cos(c+dx)\sin(c+dx)}{2bd} + \frac{B\int \frac{a+b\cos(c+dx)-2a\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2b}$$

$$= -\frac{aB\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd} + \frac{B\int \frac{ab+(2a^2+b^2}{a+b\cos}}{2b^2}$$

$$= \frac{(2a^2+b^2)\,Bx}{2b^3} - \frac{aB\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd} -$$

$$= \frac{(2a^2+b^2)\,Bx}{2b^3} - \frac{aB\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd} -$$

$$= \frac{(2a^2+b^2)\,Bx}{2b^3} - \frac{2a^3B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d} - \frac{aB\sin(}{b^2}$$

Mathematica [A]

time = 0.28, size = 98, normalized size = 0.86

$$\frac{B\left(2(2a^2+b^2)(c+dx) + \frac{8a^3\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - 4ab\sin(c+dx) + b^2\sin(2(c+dx))\right)}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] (B*(2*(2*a^2 + b^2)*(c + d*x) + (8*a^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/S
qrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 4*a*b*Sin[c + d*x] + b^2*Sin[2*(c + d*
x)]))/(4*b^3*d)
```

Maple [A]

time = 0.30, size = 139, normalized size = 1.22

method	result
derivativedivides	$\frac{2B\left(-\frac{a^3\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b^3\sqrt{(a-b)(a+b)}} + \frac{\frac{\left(-ab-\frac{1}{2}b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-ab+\frac{1}{2}b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} + \frac{\left(2a^2+b^2\right)\arctan\left(\tan\right.}{2}}{b^3}\right.}{d}$

default	$\frac{2B\left(-\frac{a^3\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b^3\sqrt{(a-b)(a+b)}}+\frac{\frac{\left(-ab-\frac{1}{2}b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-ab+\frac{1}{2}b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\left(2a^2+b^2\right)\arctan\left(\tan\left(\right.\right.}{2}}{b^3}\right.}{d}$
risch	$\frac{xB\,a^2}{b^3}+\frac{Bx}{2b}+\frac{i e^{i(dx+c)}aB}{2b^2d}-\frac{i e^{-i(dx+c)}aB}{2b^2d}-\frac{a^3\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+a\sqrt{-a^2+b^2}}{\sqrt{-a^2+b^2}\,b}\right)B}{\sqrt{-a^2+b^2}\,d\,b^3}+\frac{a^3\ln\left(e^{i(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/d*B*(-a^3/b^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*
(a+b))^(1/2))+1/b^3*(((-a*b-1/2*b^2)*tan(1/2*d*x+1/2*c)^3+(-a*b+1/2*b^2)*ta
n(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(2*a^2+b^2)*arctan(tan(1/2
*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.44, size = 350, normalized size = 3.07

$$\left[-\frac{\sqrt{-a^2+b^2}\,Ba^3\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-(2\,Ba^4-Ba^2b^2-Bb^4)dx+(2\,Ba^3b-2\,Bab^3-(Ba^2b^2-Bb^4)\cos(dx+c))\sin(dx+c)}{2\,(a^2b^3-b^5)d},\ -\frac{2\,\sqrt{a^2-b^2}\,Ba^3\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\,\sin(dx+c)}\right)-(2\,Ba^4-Ba^2b^2-Bb^4)dx+(2\,Ba^3b-2\,Bab^3-(Ba^2b^2-Bb^4)\cos(dx+c))\sin(dx+c)}{2\,(a^2b^3-b^5)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*a^3*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d
*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (2*B*a^4 - B*a^2*b^
2 - B*b^4)*d*x + (2*B*a^3*b - 2*B*a*b^3 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))
```

```
*sin(d*x + c))/((a^2*b^3 - b^5)*d), -1/2*(2*sqrt(a^2 - b^2)*B*a^3*arctan(-(
a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*B*a^4 - B*a^2*b^2
- B*b^4)*d*x + (2*B*a^3*b - 2*B*a*b^3 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))*s
in(d*x + c))/((a^2*b^3 - b^5)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 185, normalized size = 1.62

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Ba^3}{\sqrt{a^2-b^2}\,b^3}-\frac{(2Ba^2+Bb^2)(dx+c)}{b^3}+\frac{2\left(2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*a^3/(sqrt(a^2
- b^2)*b^3) - (2*B*a^2 + B*b^2)*(d*x + c)/b^3 + 2*(2*B*a*tan(1/2*d*x + 1/2*
c)^3 + B*b*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*tan(1/2*d*x + 1/2*c) - B*b*tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

Mupad [**B**]
time = 1.17, size = 173, normalized size = 1.52

$$\frac{B\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}+\frac{B\,\sin\left(2\,c+2\,d\,x\right)}{4\,b\,d}+\frac{2\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^3\,d}-\frac{B\,a\,\sin\left(c+d\,x\right)}{b^2\,d}-\frac{B\,a^3\,\mathrm{atan}\left(\frac{\left(a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)\,2{}\mathrm{i}}{b^3\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (B*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (B*sin(2*c + 2*d*x)
)/(4*b*d) + (2*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d) -
 (B*a*sin(c + d*x))/(b^2*d) - (B*a^3*atan(((a*sin(c/2 + (d*x)/2) - b*sin(c/
2 + (d*x)/2))*1i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)))*2i)/(b^3*d*(b^2 -
 a^2)^(1/2))
```

3.290 $\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{aBx}{b^2}+\frac{2a^2B\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d}+\frac{B\sin(c+dx)}{bd}$$

```
[Out] -a*B*x/b^2+B*sin(d*x+c)/b/d+2*a^2*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(
a+b)^(1/2))/b^2/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {21, 2825, 12, 2814, 2738, 211}

$$\frac{2a^2B\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}-\frac{aBx}{b^2}+\frac{B\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((a*B*x)/b^2) + (2*a^2*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (B*Sin[c + d*x])/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx\\
&= \frac{B\sin(c+dx)}{bd} - \frac{B\int \frac{a\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}\\
&= \frac{B\sin(c+dx)}{bd} - \frac{(aB)\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}\\
&= -\frac{aBx}{b^2} + \frac{B\sin(c+dx)}{bd} + \frac{(a^2B)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2}\\
&= -\frac{aBx}{b^2} + \frac{B\sin(c+dx)}{bd} + \frac{(2a^2B)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \text{ta}\right.}{b^2d}\\
&= -\frac{aBx}{b^2} + \frac{2a^2B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d} + \frac{B\sin(c+dx)}{bd}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 73, normalized size = 0.92

$$\frac{B\left(-a(c+dx) - \frac{2a^2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + b\sin(c+dx)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] (B*(-(a*(c + d*x)) - (2*a^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 +
b^2]])/Sqrt[-a^2 + b^2] + b*Sin[c + d*x]))/(b^2*d)
```

Maple [A]
time = 0.23, size = 98, normalized size = 1.24

method	result
derivativedivides	$\frac{2B\left(\frac{a^2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}-\frac{-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}\right)}{d}$
default	$\frac{2B\left(\frac{a^2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}-\frac{-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}\right)}{d}$
risch	$-\frac{aBx}{b^2}-\frac{i\mathrm{e}^{i(dx+c)}B}{2bd}+\frac{i\mathrm{e}^{-i(dx+c)}B}{2bd}-\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)B}{\sqrt{-a^2+b^2}\,d\,b^2}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+i}{\sqrt{}}\right.}{\sqrt{-a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/d*B*(1/b^2*a^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)
*(a+b))^(1/2))-1/b^2*(-b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+a*arct
an(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.40, size = 281, normalized size = 3.56

$$\left[-\frac{\sqrt{-a^{2}+b^{2}}\,Ba^{2}\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^{2}-b^{2})\cos(dx+c)^{2}+2\,\sqrt{-a^{2}+b^{2}}\,(a\cos(dx+c)+b)\sin(dx+c)-a^{2}+2\,b^{2}}{b^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+a^{2}}\right)+2\,(Ba^{3}-Bab^{2})dx-2\,(Ba^{2}b-Bb^{3})\sin(dx+c)}{2\,(a^{2}b^{2}-b^{4})d},\ \frac{\sqrt{a^{2}-b^{2}}\,Ba^{2}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^{2}-b^{2}}\sin(dx+c)}\right)-(Ba^{3}-Bab^{2})dx+(Ba^{2}b-Bb^{3})\sin(dx+c)}{(a^{2}b^{2}-b^{4})d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*a^2*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d
*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(B*a^3 - B*a*b^2)
*d*x - 2*(B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d), (sqrt(a^2 - b
^2)*B*a^2*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (B
*a^3 - B*a*b^2)*d*x + (B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 128, normalized size = 1.62

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)Ba^{2}}{\sqrt{a^{2}-b^{2}}\,b^{2}}-\frac{(dx+c)Ba}{b^{2}}+\frac{2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*a^2/(sqrt(a^2 - b^2
)*b^2) - (d*x + c)*B*a/b^2 + 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c
)^2 + 1)*b))/d
```

Mupad [B]
time = 0.89, size = 193, normalized size = 2.44

$$\frac{B\,\sin\left(c+d\,x\right)}{b\,d}-\frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d}-\frac{B\,a^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b-2\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^2+1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(b^2-a^2\right)}^{3/2}+a^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}-a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)\,2\mathrm{i}}{b^2\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (B*sin(c + d*x))/(b*d) - (2*B*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
)/(b^2*d) - (B*a^2*atan((b^3*sin(c/2 + (d*x)/2)*1i - a*b^2*sin(c/2 + (d*x)/
2)*2i + a^2*b*sin(c/2 + (d*x)/2)*1i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2)
+ a^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a*b*cos(c/2 + (d*x)/2)*(b^2 -
a^2)^(1/2)))*2i)/(b^2*d*(b^2 - a^2)^(1/2))
```

3.291 $\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=61

$$\frac{Bx}{b}-\frac{2aB\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}$$

```
[Out] B*x/b-2*a*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b/d/(a-b)^(1
/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {21, 2814, 2738, 211}

$$\frac{Bx}{b}-\frac{2aB\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*x)/b - (2*a*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[
a - b]*b*Sqrt[a + b]*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx\\ &= \frac{Bx}{b}-\frac{(aB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b}\\ &= \frac{Bx}{b}-\frac{(2aB)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\\ &= \frac{Bx}{b}-\frac{2aB\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 59, normalized size = 0.97

$$\frac{B\left(c+dx+\frac{2a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*(c + d*x + (2*a*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sq
rt[-a^2 + b^2]))/(b*d)
```

Maple [A]
time = 0.18, size = 66, normalized size = 1.08

method	result	size
derivativedivides	$\frac{2B\left(-\frac{a\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}+\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}\right)}{d}$	66

default	$\frac{2B\left(-\frac{a\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}+\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}\right)}{d}$	66
risch	$\frac{Bx}{b}-\frac{aB\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+a\sqrt{-a^2+b^2}}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,db}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)aB}{\sqrt{-a^2+b^2}\,db}$	153

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETURNVERB
OSE)
```

```
[Out] 2/d*B*(-1/b*a/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a
+b))^(1/2))+1/b*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.43, size = 231, normalized size = 3.79

$$\left[-\frac{\sqrt{-a^2+b^2}\,Ba\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-2\,(Ba^2-Bb^2)dx}{2\,(a^2b-b^3)d},\ -\frac{\sqrt{a^2-b^2}\,Ba\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\,\sin(dx+c)}\right)-(Ba^2-Bb^2)dx}{(a^2b-b^3)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*a*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(B*a^2 - B*b^2)*d*x
)/((a^2*b - b^3)*d), -(sqrt(a^2 - b^2)*B*a*arctan(-(a*cos(d*x + c) + b)/(sq
rt(a^2 - b^2)*sin(d*x + c))) - (B*a^2 - B*b^2)*d*x)/((a^2*b - b^3)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(52) = 104.
time = 0.46, size = 245, normalized size = 4.02

$$-\frac{\frac{\left(\sqrt{a^2-b^2}\,B(2a-b)|a-b|+\sqrt{a^2-b^2}\,B|a-b||b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{\frac{2a+\sqrt{-4(a+b)(a-b)+4a^2}}{a-b}}}\right)\right)}{(a^2-2ab+b^2)b^2+(a^3-2a^2b+ab^2)|b|}+\frac{(2Ba-Bb-B|b|)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{\frac{2a-\sqrt{-4(a+b)(a-b)+4a^2}}{a-b}}}\right)\right)}{b^2-a|b|}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -((sqrt(a^2 - b^2)*B*(2*a - b)*abs(a - b) + sqrt(a^2 - b^2)*B*abs(a - b)*ab
s(b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x +
1/2*c)/sqrt((2*a + sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/((a^2 - 2*a
*b + b^2)*b^2 + (a^3 - 2*a^2*b + a*b^2)*abs(b)) + (2*B*a - B*b - B*abs(b))*
(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)
/sqrt((2*a - sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/(b^2 - a*abs(b)))
/d
```

Mupad [B]
time = 0.80, size = 101, normalized size = 1.66

$$\frac{2\,B\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}+\frac{2\,B\,a\,\mathrm{atanh}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)}{b\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*B*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (2*B*a*atanh((a*s
in(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(
1/2))))/(b*d*(b^2 - a^2)^(1/2))
```

3.292 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=50

$$\frac{2B\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}\,d}$$

```
[Out] 2*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/d/(a-b)^(1/2)/(a+b)^
(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {21, 2738, 211}

$$\frac{2B\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{1}{a+b\cos(c+dx)}\,dx$$
$$= \frac{(2B)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$
$$= \frac{2B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}\,d}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.98

$$-\frac{2B\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*B*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2 + b^
2]*d)
```

Maple [A]
time = 0.12, size = 45, normalized size = 0.90

method	result	size
derivativedivides	$\frac{2B\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\sqrt{(a-b)(a+b)}}$	45
default	$\frac{2B\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\sqrt{(a-b)(a+b)}}$	45
risch	$-\frac{\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)B}{\sqrt{-a^2+b^2}\,d}+\frac{B\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d}$	141

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*B/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/
2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.39, size = 177, normalized size = 3.54

$$\left[-\frac{\sqrt{-a^2+b^2}\,B\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\,(a^2-b^2)d},\ \frac{B\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\,\sin(dx+c)}\right)}{\sqrt{a^2-b^2}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*B*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))/((a^2 - b^2)*d), B*arctan(
-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))/(sqrt(a^2 - b^2)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(42) = 84$.
time = 179.96, size = 190, normalized size = 3.80

$$\begin{cases} \frac{\tilde{\infty}Bx}{\cos(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} & \text{for } a=b \\ \frac{B}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } a=-b \\ \frac{x(Ba+Bb\cos(c))}{(a+b\cos(c))^2} & \text{for } d=0 \\ \frac{B\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{B\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*B*x/cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B*tan(c/2 + d*
x/2)/(b*d), Eq(a, b)), (B/(b*d*tan(c/2 + d*x/2)), Eq(a, -b)), (x*(B*a + B*b
*cos(c))/(a + b*cos(c))**2, Eq(d, 0)), (B*log(-sqrt(-a/(a - b) - b/(a - b))
 + tan(c/2 + d*x/2))/(a*d*sqrt(-a/(a - b) - b/(a - b)) - b*d*sqrt(-a/(a - b
) - b/(a - b))) - B*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a
*d*sqrt(-a/(a - b) - b/(a - b)) - b*d*sqrt(-a/(a - b) - b/(a - b))), True))
```

Giac [A]
time = 0.45, size = 78, normalized size = 1.56

$$\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)B}{\sqrt{a^2-b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
+ 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B/(sqrt(a^2 - b^2)*d)
```

Mupad [B]
time = 0.50, size = 44, normalized size = 0.88

$$\frac{2\,B\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a-b)}{\sqrt{a^2-b^2}}\right)}{d\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*B*atan((tan(c/2 + (d*x)/2)*(a - b))/(a^2 - b^2)^(1/2)))/(d*(a^2 - b^2)^(
1/2))
```

3.293 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$-\frac{2bB\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}\,d}+\frac{B\tanh^{-1}(\sin(c+dx))}{ad}$$

```
[Out] B*arctanh(sin(d*x+c))/a/d-2*b*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))/a/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {21, 2826, 3855, 2738, 211}

$$\frac{B\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2bB\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*
Sqrt[a + b]*d) + (B*ArcTanh[Sin[c + d*x]])/(a*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx\\
&= \frac{B\int \sec(c+dx)\,dx}{a}-\frac{(bB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a}\\
&= \frac{B\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(2bB)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}\right.\right.}{ad}\\
&= -\frac{2bB\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}\,d}+\frac{B\tanh^{-1}(\sin(c+dx))}{ad}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 103, normalized size = 1.47

$$\frac{B\left(\frac{2b\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*((2*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
b^2] - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]))/(a*d)
```

Maple [A]
time = 0.32, size = 86, normalized size = 1.23

method	result

derivativedivides	$\frac{2B\left(\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a}-\frac{b\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a}\right)}{d}$
default	$\frac{2B\left(\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a}-\frac{b\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a}\right)}{d}$
risch	$-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)B}{\sqrt{-a^2+b^2}\,da}+\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)B}{\sqrt{-a^2+b^2}\,da}+\frac{\ln(\mathrm{e}^{i(dx+c)}+i)}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x,method=_RETURNVERB
OSE)
```

```
[Out] 2/d*B*(1/2/a*ln(tan(1/2*d*x+1/2*c)+1)-b/a/((a-b)*(a+b))^(1/2)*arctan(tan(1/
2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/2/a*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.44, size = 292, normalized size = 4.17

$$\left[-\frac{\sqrt{-a^2+b^2}\,Bb\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-(Ba^2-Bb^2)\log(\sin(dx+c)+1)+(Ba^2-Bb^2)\log(-\sin(dx+c)+1)}{2\,(a^3-ab^2)d},\ -\frac{2\,\sqrt{a^2-b^2}\,Bb\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)-(Ba^2-Bb^2)\log(\sin(dx+c)+1)+(Ba^2-Bb^2)\log(-\sin(dx+c)+1)}{2\,(a^3-ab^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*b*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (B*a^2 - B*b^2)*log(s
in(d*x + c) + 1) + (B*a^2 - B*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d
), -1/2*(2*sqrt(a^2 - b^2)*B*b*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2
)*sin(d*x + c))) - (B*a^2 - B*b^2)*log(sin(d*x + c) + 1) + (B*a^2 - B*b^2)*
log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{\sec (c+dx)}{a+b\cos (c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] B*Integral(sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

Giac [A]
time = 0.52, size = 122, normalized size = 1.74

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Bb}{\sqrt{a^2-b^2}\,a}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}+\frac{B\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*b/(sqrt(a^2 - b^2)
*a) - B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + B*log(abs(tan(1/2*d*x + 1/2*
c) - 1))/a)/d
```

Mupad [B]
time = 0.76, size = 101, normalized size = 1.44

$$\frac{2\,B\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}+\frac{2\,B\,b\,\mathrm{atanh}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)}{a\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] (2*B*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) + (2*B*b*atanh((a*
sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^
(1/2))))/(a*d*(b^2 - a^2)^(1/2))
```

3.294 $\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=88

$$\frac{2b^2B\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}\,d}-\frac{bB\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{B\tan(c+dx)}{ad}$$

```
[Out] -b*B*arctanh(sin(d*x+c))/a^2/d+2*b^2*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c
)/(a+b)^(1/2))/a^2/d/(a-b)^(1/2)/(a+b)^(1/2)+B*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {21, 2881, 12, 2826, 3855, 2738, 211}

$$\frac{2b^2B\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{bB\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{B\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*b^2*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*Sqrt[a -
b]*Sqrt[a + b]*d) - (b*B*ArcTanh[Sin[c + d*x]])/(a^2*d) + (B*Tan[c + d*x])/
(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(aB + bB\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{B\tan(c+dx)}{ad} - \frac{B\int \frac{b\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$
$$= \frac{B\tan(c+dx)}{ad} - \frac{(bB)\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$
$$= \frac{B\tan(c+dx)}{ad} - \frac{(bB)\int \sec(c+dx)\,dx}{a^2} + \frac{(b^2B)\int \frac{1}{a+b\cos(c+dx)}\,d}{a^2}$$
$$= -\frac{bB\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{B\tan(c+dx)}{ad} + \frac{(2b^2B)\,\mathrm{Subst}\left(\int\right.}{}$$
$$= \frac{2b^2B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}\,d} - \frac{bB\tanh^{-1}(\sin(c+dx))}{a^2d}$$

Mathematica [A]
time = 0.41, size = 116, normalized size = 1.32

$$\frac{B\left(-\frac{2b^2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + b\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right) + a\tan(c+dx)\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] (B*((-2*b^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2
 + b^2] + b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]]) + a*Tan[c + d*x]))/(a^2*d)
```

Maple [A]
time = 0.35, size = 125, normalized size = 1.42

method	result
derivativedivides	$\frac{2B\left(-\frac{1}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} - \frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^2} + \frac{b^2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{a^2\sqrt{(a-b)(a+b)}} - \frac{1}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{b\ln\left(\tan\left(\right.\right.}{2}\right.}{d}$

default	$\frac{2B\left(-\frac{1}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^2}+\frac{b^2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{a^2\sqrt{(a-b)(a+b)}}-\frac{1}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{2a^2}\right.}{d}$
risch	$\frac{2iB}{da\left(e^{2i(dx+c)}+1\right)}-\frac{b^2\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)B}{\sqrt{-a^2+b^2}\,da^2}+\frac{b^2B\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,da^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/d*B*(-1/2/a/(tan(1/2*d*x+1/2*c)+1)-1/2/a^2*b*ln(tan(1/2*d*x+1/2*c)+1)+b^2
/a^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2
))-1/2/a/(tan(1/2*d*x+1/2*c)-1)+1/2/a^2*b*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(79) = 158.
time = 0.49, size = 398, normalized size = 4.52

$$\left[-\frac{\sqrt{-a^2+b^2}\,Bb^2\cos(dx+c)\log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2+2\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)+(Ba^2b-Bb^3)\cos(dx+c)\log(\sin(dx+c)+1)-(Ba^2b-Bb^3)\cos(dx+c)\log(-\sin(dx+c)+1)-2(Ba^3-Bab^2)\sin(dx+c)}{2(a^4-a^2b^2)d\cos(dx+c)},\ \frac{2\sqrt{a^2-b^2}\,Bb^2\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)\cos(dx+c)-(Ba^2b-Bb^3)\cos(dx+c)\log(\sin(dx+c)+1)+(Ba^2b-Bb^3)\cos(dx+c)\log(-\sin(dx+c)+1)+2(Ba^3-Bab^2)\sin(dx+c)}{2(a^4-a^2b^2)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*b^2*cos(d*x + c)*log((2*a*b*cos(d*x + c) + (2*a^2
 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x +
c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (B*a^2
*b - B*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) - (B*a^2*b - B*b^3)*cos(d*x
```

```
+ c)*log(-sin(d*x + c) + 1) - 2*(B*a^3 - B*a*b^2)*sin(d*x + c))/((a^4 - a^2
*b^2)*d*cos(d*x + c)), 1/2*(2*sqrt(a^2 - b^2)*B*b^2*arctan(-(a*cos(d*x + c)
 + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (B*a^2*b - B*b^3)*cos(
d*x + c)*log(sin(d*x + c) + 1) + (B*a^2*b - B*b^3)*cos(d*x + c)*log(-sin(d*
x + c) + 1) + 2*(B*a^3 - B*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*x
+ c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

```
[Out] B*Integral(sec(c + d*x)**2/(a + b*cos(c + d*x)), x)
```

Giac [A]
time = 0.49, size = 155, normalized size = 1.76

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Bb^2}{\sqrt{a^2-b^2}\,a^2}-\frac{Bb\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}+\frac{Bb\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*b^2/(sqrt(a^2 - b^2
)*a^2) - B*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + B*b*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^2 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 -
 1)*a))/d
```

Mupad [B]
time = 1.06, size = 326, normalized size = 3.70

$$\frac{2B\left(\frac{a^3\sin(c+dx)}{2}-\frac{ab^2\sin(c+dx)}{2}\right)}{a^2d\cos(c+dx)\,(a^2-b^2)}-\frac{2B\left(a^2b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)+b^2\,\mathrm{atanh}\left(\frac{a^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}+2b^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)(b^2-a^2)^{3/2}-2b^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}+3a^2b^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}-a^3b^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}-a^4b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)(ab^2-a^3)^2}\right)\sqrt{b^2-a^2}\right)}{a^2d\,(a^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^2),x)
```

```
[Out] (2*B*((a^3*sin(c + d*x))/2 - (a*b^2*sin(c + d*x))/2))/(a^2*d*cos(c + d*x)*(
a^2 - b^2)) - (2*B*(a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - b^
3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + b^2*atanh((a^5*sin(c/2 + (
```

```
d*x)/2)*(b^2 - a^2)^(1/2) + 2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 2*
b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*a^2*b^3*sin(c/2 + (d*x)/2)*(b^
2 - a^2)^(1/2) - a^3*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^4*b*sin(c
/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)^2))*(b^2
 - a^2)^(1/2)))/(a^2*d*(a^2 - b^2))
```

3.295 $\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=123

$$-\frac{2b^3B\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}\,d}+\frac{(a^2+2b^2)\,B\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{bB\tan(c+dx)}{a^2d}+\frac{B\sec(c+dx)\tan(c+dx)}{2ad}$$

```
[Out] 1/2*(a^2+2*b^2)*B*arctanh(sin(d*x+c))/a^3/d-2*b^3*B*arctan((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/d/(a-b)^(1/2)/(a+b)^(1/2)-b*B*tan(d*x+c)/a^
2/d+1/2*B*sec(d*x+c)*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.23, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {21, 2881, 3134, 3080, 3855, 2738, 211}

$$-\frac{2b^3B\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}-\frac{bB\tan(c+dx)}{a^2d}+\frac{B(a^2+2b^2)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{B\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^3*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a -
 b]*Sqrt[a + b]*d) + ((a^2 + 2*b^2)*B*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (b
*B*Tan[c + d*x])/(a^2*d) + (B*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{B\sec(c+dx)\tan(c+dx)}{2ad} + \frac{B\int \frac{(-2b+a\cos(c+dx)+b\cos^2(c+dx))\sec^2}{a+b\cos(c+dx)}}{2a}$$

$$= -\frac{bB\tan(c+dx)}{a^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2ad} + \frac{B\int \frac{(a^2+2b^2+a}{a}}{}$$

$$= -\frac{bB\tan(c+dx)}{a^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(b^3B)\int \frac{}{a+b\cos}}{a^3}$$

$$= \frac{(a^2+2b^2)\,B\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{bB\tan(c+dx)}{a^2d} + \frac{B\sec(c}{}$$

$$= -\frac{2b^3B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}\,d} + \frac{(a^2+2b^2)\,B\tanh^{-1}(\sin}{2a^3d}$$

Mathematica [A]

time = 1.13, size = 239, normalized size = 1.94

$$\frac{B\left(\frac{8b^3\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - 2a^2\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right) - 4b^2\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right) + 2a^2\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right) + 4b^2\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right) + \frac{a^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{a^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - 4ab\tan(c+dx)\right)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] (B*((8*b^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2
+ b^2] - 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*b^2*Log[Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] + 4*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 4*a
*b*Tan[c + d*x]))/(4*a^3*d)
```

Maple [A]

time = 0.44, size = 194, normalized size = 1.58

method	result
derivativedivides	$\frac{2B\left(-\frac{1}{4a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-a-2b}{4a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2+2b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4a^3}-\frac{b^3\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{a^3\sqrt{(a-b)(a+b)}}-\right.}{d}$

default	$\frac{2B\left(-\frac{1}{4a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{-a-2b}{4a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^{2}+2b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4a^{3}}-\frac{b^{3}\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{a^{3}\sqrt{(a-b)(a+b)}}+\frac{}{4}$
risch	$-\frac{iB\left(a\,e^{3i(dx+c)}+2b\,e^{2i(dx+c)}-a\,e^{i(dx+c)}+2b\right)}{d\,a^{2}\left(e^{2i(dx+c)}+1\right)^{2}}-\frac{b^{3}\ln\left(e^{i(dx+c)}-\frac{ia^{2}-ib^{2}-a\sqrt{-a^{2}+b^{2}}}{b\sqrt{-a^{2}+b^{2}}}\right)B}{\sqrt{-a^{2}+b^{2}}\,d\,a^{3}}+\frac{b^{3}\ln\left(e^{i(dx+c)}+\right.}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/d*B*(-1/4/a/(tan(1/2*d*x+1/2*c)+1)^2-1/4*(-a-2*b)/a^2/(tan(1/2*d*x+1/2*c)
+1)+1/4*(a^2+2*b^2)/a^3*ln(tan(1/2*d*x+1/2*c)+1)-b^3/a^3/((a-b)*(a+b))^(1/2
)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+1/4/a/(tan(1/2*d*x+1
/2*c)-1)^2-1/4*(-a-2*b)/a^2/(tan(1/2*d*x+1/2*c)-1)+1/4/a^3*(-a^2-2*b^2)*ln(
tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.52, size = 487, normalized size = 3.96

$$\left[-\frac{2\sqrt{-a^{2}+b^{2}}\,Bb^{3}\cos(dx+c)^{2}\log\left(\frac{2ab\cos(dx+c)+(2a^{2}-b^{2})\cos(dx+c)^{2}-2\sqrt{-a^{2}+b^{2}}(a\cos(dx+c)+b)\sin(dx+c)-a^{2}+2b^{2}}{b^{2}\cos(dx+c)^{2}+2ab\cos(dx+c)+a^{2}}\right)-(Ba^{4}+Ba^{2}b^{2}-2Bb^{4})\cos(dx+c)^{2}\log(\sin(dx+c)+1)+(Ba^{4}+Ba^{2}b^{2}-2Bb^{4})\cos(dx+c)^{2}\log(-\sin(dx+c)+1)-2(Ba^{4}-Ba^{2}b^{2}-2(Ba^{3}b-Bab^{3})\cos(dx+c))\sin(dx+c)}{4(a^{5}-a^{3}b^{2})d\cos(dx+c)^{2}},-\frac{4\sqrt{a^{2}-b^{2}}\,Bb^{3}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^{2}-b^{2}}\sin(dx+c)}\right)\cos(dx+c)^{2}-(Ba^{4}+Ba^{2}b^{2}-2Bb^{4})\cos(dx+c)^{2}\log(\sin(dx+c)+1)+(Ba^{4}+Ba^{2}b^{2}-2Bb^{4})\cos(dx+c)^{2}\log(-\sin(dx+c)+1)-2(Ba^{4}-Ba^{2}b^{2}-2(Ba^{3}b-Bab^{3})\cos(dx+c))\sin(dx+c)}{4(a^{5}-a^{3}b^{2})d\cos(dx+c)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] [-1/4*(2*sqrt(-a^2 + b^2)*B*b^3*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) + (2
*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*
x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (B
```

```
*a^4 + B*a^2*b^2 - 2*B*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (B*a^4 +
 B*a^2*b^2 - 2*B*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^4 - B*
a^2*b^2 - 2*(B*a^3*b - B*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2
)*d*cos(d*x + c)^2), -1/4*(4*sqrt(a^2 - b^2)*B*b^3*arctan(-(a*cos(d*x + c)
+ b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (B*a^4 + B*a^2*b^2 -
2*B*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (B*a^4 + B*a^2*b^2 - 2*B*b^
4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^4 - B*a^2*b^2 - 2*(B*a^3*
b - B*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)^2)
]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

```
[Out] B*Integral(sec(c + d*x)**3/(a + b*cos(c + d*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(110) = 220.
time = 0.51, size = 221, normalized size = 1.80

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Bb^3}{\sqrt{a^2-b^2}\,a^3}-\frac{(Ba^2+2Bb^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}+\frac{(Ba^2+2Bb^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}-\frac{2\left(Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*b^3/(sqrt(a^2
- b^2)*a^3) - (B*a^2 + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + (B
*a^2 + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 2*(B*a*tan(1/2*d*x
 + 1/2*c)^3 + 2*B*b*tan(1/2*d*x + 1/2*c)^3 + B*a*tan(1/2*d*x + 1/2*c) - 2*B
*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

Mupad [**B**]
time = 1.83, size = 1099, normalized size = 8.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^2),x)
```

```
[Out] ((B*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (B*b^2*sin(c + d*
x))/2 + (B*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x
))/2)/(a*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (a*((B*sin(c + d*x))/2
 + (B*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (B*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2))/(d*(a^2 - b^2)*(cos(2*c
+ 2*d*x)/2 + 1/2)) - (B*b*sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d
*x)/2 + 1/2)) - (B*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^3*d
*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B*b^3*sin(2*c + 2*d*x))/(2*a^2*
d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b^3*atan(((a^9*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*b^
9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2) + 3*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 3*a^5*b^4*s
in(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2) + 2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^8*b*sin(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(a^7 -
3*a^3*b^4 + 2*a^5*b^2)))*1i)/(a^3*d*(b^2 - a^2)^(1/2)*(cos(2*c + 2*d*x)/2 +
 1/2)) - (B*b^3*atan(((a^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin
(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*b^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1
/2) + 8*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*a^4*b^5*sin(c/2 +
(d*x)/2)*(b^2 - a^2)^(1/2) - 3*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)
 - 2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 2*a^7*b^2*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2) - a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))*1i)/(
cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(a^7 - 3*a^3*b^4 + 2*a^5*b^2)))*cos(2*c +
2*d*x)*1i)/(a^3*d*(b^2 - a^2)^(1/2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b^4*at
anh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/(a^3*d*(a^2 -
b^2)*(cos(2*c + 2*d*x)/2 + 1/2))
```

3.296 $\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=386

$$\frac{2(24a^3Ab+57aAb^3-16a^4B-24a^2b^2B+147b^4B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{315b^4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(a^2-b^2)\,(24a$$

```
[Out] -2/315*(36*A*a*b-24*B*a^2-49*B*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d
+2/21*(3*A*b-2*B*a)*cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/9*
B*cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/315*(24*A*a^2*b+75*A
*b^3-16*B*a^3-36*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/d+2/315*(24
*A*a^3*b+57*A*a*b^3-16*B*a^4-24*B*a^2*b^2+147*B*b^4)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1
/2))*(a+b*cos(d*x+c))^(1/2)/b^4/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/315*(a^2
-b^2)*(24*A*a^2*b+75*A*b^3-16*B*a^3-36*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*
((a+b*cos(d*x+c))/(a+b))^(1/2)/b^4/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3069, 3128, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(-24a^2B+36aAb-49b^2B)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^3d}+\frac{2(-16a^3B+24a^2Ab-36ab^2B+75Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315b^3d}-\frac{2(a^2-b^2)\left(-16a^3B+24a^2Ab-36ab^2B+75Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{315b^4d\sqrt{a+b\cos(c+dx)}}+\frac{2(-16a^4B+24a^3Ab-24a^2b^2B+57aAb^3+147b^4B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{315b^4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(3Ab-2aB)\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{21b^2d}+\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(24*a^3*A*b + 57*a*A*b^3 - 16*a^4*B - 24*a^2*b^2*B + 147*b^4*B)*Sqrt[a +
 b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^4*d*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(24*a^2*A*b + 75*A*b^3 - 16*a^
3*B - 36*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(315*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(24*a^2*A*b + 75
*A*b^3 - 16*a^3*B - 36*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315
*b^3*d) - (2*(36*a*A*b - 24*a^2*B - 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*Si
n[c + d*x])/(315*b^3*d) + (2*(3*A*b - 2*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(21*b^2*d) + (2*B*Cos[c + d*x]^2*(a + b*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(9*b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
```

```
n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{2B\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+a}{9bd}$$

$$= \frac{2(3Ab-2aB)\cos(c+dx)(a+b\cos(c+dx))^{3/}}{21b^2d}$$

$$= -\frac{2(36aAb-24a^2B-49b^2B)\,(a+b\cos(c+d}{315b^3d}$$

$$= \frac{2(24a^2Ab+75Ab^3-16a^3B-36ab^2B)\,\sqrt{a+}}{315b^3d}$$

$$= \frac{2(24a^2Ab+75Ab^3-16a^3B-36ab^2B)\,\sqrt{a+}}{315b^3d}$$

$$= \frac{2(24a^2Ab+75Ab^3-16a^3B-36ab^2B)\,\sqrt{a+}}{315b^3d}$$

$$= \frac{2(24a^3Ab+57aAb^3-16a^4B-24a^2b^2B+147}{315b^4d\sqrt{\frac{a+b}{}}}$$

Mathematica [**A**]
time = 1.70, size = 292, normalized size = 0.76

$$\frac{8\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\left(b^2(6a^2Ab+75Ab^3-4a^3B+111ab^2B)\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+(24a^3Ab+57aAb^3-16a^4B-24a^2b^2B+147b^4B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)-b(a+b\cos(c+dx))\,(-2(-48a^2Ab+345Ab^3+32a^3B+57ab^2B)\sin(c+dx)-b((36aAb-24a^2B+266b^2B)\sin(2(c+dx))+5b(2(9Ab+aB)\sin(3(c+dx))+7bB\sin(4(c+dx)))))}{1260b^4d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(6*a^2*A*b + 75*A*b^3 - 4*a^3*B
+ 111*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (24*a^3*A*b + 57*a*A
*b^3 - 16*a^4*B - 24*a^2*b^2*B + 147*b^4*B)*((a + b)*EllipticE[(c + d*x)/2,
 (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) - b*(a + b*Cos[
c + d*x])*(-2*(-48*a^2*A*b + 345*A*b^3 + 32*a^3*B + 57*a*b^2*B)*Sin[c + d*x
] - b*((36*a*A*b - 24*a^2*B + 266*b^2*B)*Sin[2*(c + d*x)] + 5*b*(2*(9*A*b +
 a*B)*Sin[3*(c + d*x)] + 7*b*B*Sin[4*(c + d*x)]))))/(1260*b^4*d*Sqrt[a + b*
Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1634 vs. 2(416) = 832.
time = 0.44, size = 1635, normalized size = 4.24

method	result	size
default	Expression too large to display	1635

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^5+(720*A*b^5+640*B*a*b^4+2240*B*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-432*A*a*b^4-1080*A*b^5+8*B*a^2*b^3-960*B*a*b^4-2072*B*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-12*A*a^2*b^3+432*A*a*b^4+840*A*b^5+8*B*a^3*b^2-8*B*a^2*b^3+728*B*a*b^4+952*B*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(24*A*a^3*b^2+6*A*a^2*b^3-258*A*a*b^4-240*A*b^5-16*B*a^4*b-4*B*a^3*b^2-24*B*a^2*b^3-204*B*a*b^4-168*B*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-24*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-51*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+75*A*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+24*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-24*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+57*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3-57*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4+16*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-36*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-16*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5+16*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-24*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+24*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*B*(sin

(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5)/b^4/(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/
2*d*x+1/2*c)^2*b+a+b)^(1/2)/d

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 639, normalized size = 1.66

$$\frac{\sqrt{2}\left(-32i\,Ba^5+48i\,Aa^4b-36i\,Ba^3b^2+96i\,Aa^2b^3-39i\,Bab^4-225i\,Ab^5\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right)+\sqrt{2}\left(32i\,Ba^5-48i\,Aa^4b+36i\,Ba^3b^2-96i\,Aa^2b^3+39i\,Bab^4+225i\,Ab^5\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)+2a}{3b}\right)-3\sqrt{2}\left(16i\,Ba^4b-24i\,Aa^3b^2+24i\,Ba^2b^3-57i\,Aab^4-147i\,Bb^5\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right)\right)-3\sqrt{2}\left(-16i\,Ba^4b+24i\,Aa^3b^2-24i\,Ba^2b^3+57i\,Aab^4+147i\,Bb^5\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)+2a}{3b}\right)\right)+6\left(35Bb^5\cos(dx+c)^3+8Ba^3b^2-12Aa^2b^3+13Bab^4+75Ab^5+5(Bab^4+9Ab^5)\cos(dx+c)^2-(6Ba^2b^3-9Aab^4-49Bb^5)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\,\sin(dx+c)}{945b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/945*(sqrt(2)*(-32*I*B*a^5 + 48*I*A*a^4*b - 36*I*B*a^3*b^2 + 96*I*A*a^2*b^
3 - 39*I*B*a*b^4 - 225*I*A*b^5)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*
b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*
x + c) + 2*a)/b) + sqrt(2)*(32*I*B*a^5 - 48*I*A*a^4*b + 36*I*B*a^3*b^2 - 96
*I*A*a^2*b^3 + 39*I*B*a*b^4 + 225*I*A*b^5)*sqrt(b)*weierstrassPInverse(4/3*
(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3
*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(16*I*B*a^4*b - 24*I*A*a^3*b^2 + 24
*I*B*a^2*b^3 - 57*I*A*a*b^4 - 147*I*B*b^5)*sqrt(b)*weierstrassZeta(4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^
2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*
sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(-16*I*B*a^4*b + 24*I*A*a^3*b^2 - 24*I*
B*a^2*b^3 + 57*I*A*a*b^4 + 147*I*B*b^5)*sqrt(b)*weierstrassZeta(4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 -
 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin
(d*x + c) + 2*a)/b)) + 6*(35*B*b^5*cos(d*x + c)^3 + 8*B*a^3*b^2 - 12*A*a^2*
b^3 + 13*B*a*b^4 + 75*A*b^5 + 5*(B*a*b^4 + 9*A*b^5)*cos(d*x + c)^2 - (6*B*a
^2*b^3 - 9*A*a*b^4 - 49*B*b^5)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d
*x + c))/(b^5*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^3\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

3.297 $\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=303

$$-\frac{2(14a^2Ab-63Ab^3-8a^3B-19ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(a^2-b^2)(14aAb-8a^2B-}{105}$$

```
[Out] 2/35*(7*A*b-4*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*B*cos(d*x+c)
*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d-2/105*(14*A*a*b-8*B*a^2-25*B*b^2)*si
n(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d-2/105*(14*A*a^2*b-63*A*b^3-8*B*a^3-19
*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/d/((a+b*cos
(d*x+c))/(a+b))^(1/2)+2/105*(a^2-b^2)*(14*A*a*b-8*B*a^2-25*B*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/
2)
```

Rubi [A]
time = 0.35, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3069, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(-8a^2B+14aAb-25b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d}+\frac{2(a^2-b^2)(-8a^2B+14aAb-25b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{a+b\cos(c+dx)}}-\frac{2(-8a^3B+14a^2Ab-19ab^2B-63Ab^3)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(7Ab-4aB)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{35b^2d}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*(14*a^2*A*b - 63*A*b^3 - 8*a^3*B - 19*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) + (2*(a^2 - b^2)*(14*a*A*b - 8*a^2*B - 25*b^2*B)*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt
[a + b*Cos[c + d*x]]) - (2*(14*a*A*b - 8*a^2*B - 25*b^2*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*b^2*d) + (2*(7*A*b - 4*a*B)*(a + b*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(35*b^2*d) + (2*B*Cos[c + d*x]*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(7*b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
```

```
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{2B\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx}{7bd}$$

$$= \frac{2(7Ab-4aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx}{35b^2d}$$

$$= -\frac{2(14aAb-8a^2B-25b^2B)\sqrt{a+b\cos(c+d}}{105b^2d}$$

$$= -\frac{2(14aAb-8a^2B-25b^2B)\sqrt{a+b\cos(c+d}}{105b^2d}$$

$$= -\frac{2(14aAb-8a^2B-25b^2B)\sqrt{a+b\cos(c+d}}{105b^2d}$$

$$= -\frac{2(14a^2Ab-63Ab^3-8a^3B-19ab^2B)\sqrt{a+}}{105b^3d\sqrt{\frac{a+b\cos(c}{a+b}}}$$

Mathematica [A]
time = 1.06, size = 232, normalized size = 0.77

$$\frac{4\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(49aAb+2a^2B+25b^2B)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+(-14a^2Ab+63Ab^3+8a^3B+19ab^2B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)+b(a+b\cos(c+dx))\left((28aAb-16a^2B+115b^2B)\sin(c+dx)+3b(2(7Ab+aB)\sin(2(c+dx))+5bB\sin(3(c+dx)))\right)}{210b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(49*a*A*b + 2*a^2*B + 25*b^2*B)*
EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B +
 19*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(
c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((28*a*A*b - 16*a^2*B
 + 115*b^2*B)*Sin[c + d*x] + 3*b*(2*(7*A*b + a*B)*Sin[2*(c + d*x)] + 5*b*B*
Sin[3*(c + d*x)])))/(210*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1304 vs. $2(337) = 674$.
time = 0.40, size = 1305, normalized size = 4.31

method	result	size
default	Expression too large to display	1305

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^4+(-168*A*b^4-144*B*a*b^3-360*B*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(112*A*a*b^3+168*A*b^4-4*B*a^2*b
^2+144*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A*a^
2*b^2-56*A*a*b^3-42*A*b^4+8*B*a^3*b+2*B*a^2*b^2-86*B*a*b^3-80*B*b^4)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+14*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^3*b-14*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b^3-14*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^3*b+14*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+
63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-63*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-8*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-17*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^4-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3
*b+19*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
```

```
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-19*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b^3/(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2
*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 561, normalized size = 1.85

$$\frac{\sqrt{2}\left(16iBa^4-28iAa^3b+32iBa^2b^2-21iAab^3-75iBb^4\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right)+\sqrt{2}\left(-16iBa^4+28iAa^3b-32iBa^2b^2+21iAab^3+75iBb^4\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)+2a}{3b}\right)-3\sqrt{2}\left(-8iBa^3b+14iAa^2b^2-19iBab^3-63iAb^4\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right)\right)-3\sqrt{2}\left(8iBa^3b-14iAa^2b^2+19iBab^3+63iAb^4\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)+2a}{3b}\right)\right)+6\left(15Bb^4\cos(dx+c)^2-4Ba^2b^2+7Aab^3+25Bb^4+3(Bab^3+7Ab^4)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\,\sin(dx+c)}{315\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/315*(sqrt(2)*(16*I*B*a^4 - 28*I*A*a^3*b + 32*I*B*a^2*b^2 - 21*I*A*a*b^3 -
 75*I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) +
sqrt(2)*(-16*I*B*a^4 + 28*I*A*a^3*b - 32*I*B*a^2*b^2 + 21*I*A*a*b^3 + 75*I*
B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt
(2)*(-8*I*B*a^3*b + 14*I*A*a^2*b^2 - 19*I*B*a*b^3 - 63*I*A*b^4)*sqrt(b)*wei
erstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstr
assPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*
cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(8*I*B*a^3*b - 14*
I*A*a^2*b^2 + 19*I*B*a*b^3 + 63*I*A*b^4)*sqrt(b)*weierstrassZeta(4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*si
n(d*x + c) + 2*a)/b)) + 6*(15*B*b^4*cos(d*x + c)^2 - 4*B*a^2*b^2 + 7*A*a*b^
3 + 25*B*b^4 + 3*(B*a*b^3 + 7*A*b^4)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)
*sin(d*x + c))/(b^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}\,\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*cos(c + d*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

3.298 $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=231

$$\frac{2(5aAb-2a^2B+9b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(a^2-b^2)(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{15b^2d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2/5*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/15*(5*A*b-2*B*a)*sin(d*x+c)*(
a+b*cos(d*x+c))^(1/2)/b/d+2/15*(5*A*a*b-2*B*a^2+9*B*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b)
)^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/15*(
a^2-b^2)*(5*A*b-2*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^
(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3047, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(a^2-b^2)(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2(-2a^2B+5aAb+9b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(5Ab-2aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{15bd}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(5*a*A*b - 2*a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a
^2 - b^2)*(5*A*b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*A*b -
2*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b*d) + (2*B*(a + b*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \int \sqrt{a+b\cos(c+dx)}\,\left(A\cos(c+dx)+B\cos^2(\right.$$

$$= \frac{2B(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{2\int\sqrt{a}}{}$$

$$= \frac{2(5Ab-2aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15bd}$$

$$= \frac{2(5Ab-2aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15bd}$$

$$= \frac{2(5Ab-2aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15bd}$$

$$= \frac{2(5aAb-2a^2B+9b^2B)\,\sqrt{a+b\cos(c+dx)}\,E}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [**A**]
time = 0.93, size = 179, normalized size = 0.77

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\left(b^2(5Ab+7aB)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+(5aAb-2a^2B+9b^2B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)+2b(a+b\cos(c+dx))(5Ab+aB+3bB\cos(c+dx))\sin(c+dx)}{15b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(5*A*b + 7*a*B)*EllipticF[(c + d
*x)/2, (2*b)/(a + b)] + (5*a*A*b - 2*a^2*B + 9*b^2*B)*((a + b)*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + 2*b
*(a + b*Cos[c + d*x])*(5*A*b + a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x])/(15*
b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(269) = 538.
time = 0.35, size = 993, normalized size = 4.30

method	result	size
default	Expression too large to display	993

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+(20*A*b^3+16*B*a*b^2+24*B*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3-2*B*a^2*b-8*B*a
*b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^2*b-5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a*b^2+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-
2*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+2*B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*b^3)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 492, normalized size = 2.13

$$\frac{\sqrt{2}\left(-4i\,Ba^{3}+10i\,Aa^{2}b-3i\,Bab^{2}-15i\,Ab^{3}\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cos\left(dx+c\right)+3i\,b\sin\left(dx+c\right)+2a}{3b}\right)+\sqrt{2}\left(4i\,Ba^{3}-10i\,Aa^{2}b+3i\,Bab^{2}+15i\,Ab^{3}\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cos\left(dx+c\right)-3i\,b\sin\left(dx+c\right)+2a}{3b}\right)-3\sqrt{2}\left(2i\,Ba^{2}b-5i\,Aab^{2}-9i\,Bb^{3}\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cos\left(dx+c\right)+3i\,b\sin\left(dx+c\right)+2a}{3b}\right)\right)-3\sqrt{2}\left(-2i\,Ba^{2}b+5i\,Aab^{2}+9i\,Bb^{3}\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cos\left(dx+c\right)-3i\,b\sin\left(dx+c\right)+2a}{3b}\right)\right)+6\left(3\,Bb^{3}\cos\left(dx+c\right)+Bab^{2}+5\,Ab^{3}\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{45\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/45*(sqrt(2)*(-4*I*B*a^3 + 10*I*A*a^2*b - 3*I*B*a*b^2 - 15*I*A*b^3)*sqrt(b
)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(4*I*B*a^3
- 10*I*A*a^2*b + 3*I*B*a*b^2 + 15*I*A*b^3)*sqrt(b)*weierstrassPInverse(4/3*
(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3
*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(2*I*B*a^2*b - 5*I*A*a*b^2 - 9*I*B*
b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^
2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2
)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(-
2*I*B*a^2*b + 5*I*A*a*b^2 + 9*I*B*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 -
 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 -
3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(
d*x + c) + 2*a)/b)) + 6*(3*B*b^3*cos(d*x + c) + B*a*b^2 + 5*A*b^3)*sqrt(b*c
os(d*x + c) + a)*sin(d*x + c))/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + B\cos\left(c + dx\right)\right)\sqrt{a + b\cos\left(c + dx\right)}\,\cos\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (c+d\,x)\ (A+B\ \cos (c+d\,x))\ \sqrt{a+b\ \cos (c+d\,x)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

3.299 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=171

$$\frac{2(3Ab+aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(a^2-b^2)\,B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2\ldots}{}$$

```
[Out] 2/3*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*(3*A*b+B*a)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b
))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(a^
2-b^2)*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b
*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2B(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2(aB+3Ab)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(3*A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(3*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*B*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d*S
qrt[a + b*Cos[c + d*x]]) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d
)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}(3aA+bB)+}{\sqrt{a-}}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d} - \frac{((a^2-b^2)\,B)\int \frac{}{\sqrt{a+}}}{3b}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{\left((3Ab+aB)\sqrt{a+b}\right.}{}$$

$$= \frac{2(3Ab+aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(a^2}{}$$

Mathematica [A]
time = 0.63, size = 146, normalized size = 0.85

$$\frac{2(a+b)(3Ab+aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2(a^2-b^2)\,B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2bB(a+b\cos(c+dx))\sin(c+dx)}{3bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(a + b)*(3*A*b + a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)] - 2*(a^2 - b^2)*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*B*(a + b*Cos[c + d*x])*Sin[c +
 d*x])/(3*b*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(213) = 426.
time = 0.32, size = 600, normalized size = 3.51

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4B\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2\left(\cos^2\left(\frac{d}{2}\right.\right.}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*cos(1
/2*d*x+1/2*c)^5*b^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)
^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b
-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+2*B*cos(1/2*d*x+1/
2*c)^3*a*b-6*B*cos(1/2*d*x+1/2*c)^3*b^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2+B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*B*cos(1/2*d*x+1/2*c)*a*b+2*B*cos(1/2*d*x+1
/2*c)*b^2)/b/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 435, normalized size = 2.54

$$\frac{6\sqrt{b\cos(dx+c)+a}\,Bb^2\sin(dx+c)+\sqrt{2}\,(2i\,Ba^2-3i\,Aab-3i\,Bb^2)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4a^2-3b^2)}{3b^2},-\frac{8\,(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2a}{3b}\right)+\sqrt{2}\,(-2i\,Ba^2+3i\,Aab+3i\,Bb^2)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4a^2-3b^2)}{3b^2},-\frac{8\,(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)+2a}{3b}\right)-3\sqrt{2}\,(-i\,Bab-3i\,Ab^2)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\,(4a^2-3b^2)}{3b^2},-\frac{8\,(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(4a^2-3b^2)}{3b^2},-\frac{8\,(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2a}{3b}\right)\right)-3\sqrt{2}\,(i\,Bab+3i\,Ab^2)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\,(4a^2-3b^2)}{3b^2},-\frac{8\,(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(4a^2-3b^2)}{3b^2},-\frac{8\,(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)+2a}{3b}\right)\right)}{9\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/9*(6*sqrt(b*cos(d*x + c) + a)*B*b^2*sin(d*x + c) + sqrt(2)*(2*I*B*a^2 - 3
*I*A*a*b - 3*I*B*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2
*a)/b) + sqrt(2)*(-2*I*B*a^2 + 3*I*A*a*b + 3*I*B*b^2)*sqrt(b)*weierstrassPI
nverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(-I*B*a*b - 3*I*A*b^2)*s
qrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3
, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(I*B*a*b
 + 3*I*A*b^2)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
```

```
 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/(b^2
*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos{(c + dx)})\sqrt{a + b\cos{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\,\cos(c + d\,x))\,\sqrt{a + b\,\cos(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
[Out] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

3.300 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=178

$$\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2aA\sqrt{\frac{a+b\cos(c+}{a+b}}}{d\sqrt{a+}}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(
a+b))^(1/2)+2*A*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)
/d/(a+b*cos(d*x+c))^(1/2)+2*a*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x
+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3081, 2734, 2732, 2882, 2742, 2740, 2886, 2884}

$$\frac{2Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2aA\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (2*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*
a*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2882

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
```

```
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx = A\int \sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\,dx + B$$

$$= (aA)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx + (Ab)\int$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} +$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} +$$

Mathematica [A]
time = 2.39, size = 107, normalized size = 0.60

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)BE\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+A\left(bF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*B*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + A*(b*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*EllipticPi[2,
(c + d*x)/2, (2*b)/(a + b)])))/(d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A]
time = 0.30, size = 247, normalized size = 1.39

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\left(Ab\,\text{Elliptic}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(A*b*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-a*A*EllipticPi(cos(1/2*d*x+1/2*c),
2,(-2*b/(a-b))^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-
B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c
)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x), x)
```

3.301 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=213

$$-\frac{A\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(aA+2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{(Ab+2aB)}{}$$

```
[Out] -A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a
+b))^(1/2)+(A*a+2*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^
(1/2)/d/(a+b*cos(d*x+c))^(1/2)+(A*b+2*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((
a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+A*(a+b*cos(d*x+c))^(1
/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.40, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3078, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(aA+2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{(2aB+Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}-\frac{A\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] -((A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])) + ((a*A + 2*b*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x
]]) + ((A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c
+ d*x]]*Tan[c + d*x])/d
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} + \int \frac{\left(\frac{1}{2}(Ab \ldots\right.}{\ldots}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} - \frac{1}{2}A\int \sqrt{\ldots}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} - \frac{1}{2}(-Ab \ldots$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \ldots$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \ldots$$

Mathematica [C] Result contains complex when optimal does not.

time = 18.38, size = 372, normalized size = 1.75

$$\frac{\frac{8bB\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab+4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{2iA\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)}{ab\sqrt{-\frac{1}{a+b}}}+4A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] ((8*b*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(A*b + 4*a*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*
x]] - ((2*I)*A*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c
+ d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] +
 b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d
*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*A*Sqrt[a + b*Cos[c
 + d*x]]*Tan[c + d*x])/(4*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(286) = 572$.
time = 0.48, size = 746, normalized size = 3.50

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2Bb\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))-2*(A*b+B*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2
))+2*a*A*(-cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
```

```
^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2
*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))\sqrt{a + b\cos(c + dx)}\sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^2, x)
```

3.302 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=292

$$-\frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(3Ab+4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\cdot$$

```
[Out] -1/4*(A*b+4*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/d/((a+
b*cos(d*x+c))/(a+b))^(1/2)+1/4*(3*A*b+4*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a
+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*A*a^2-A*b^2+4*B
*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*
x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b
*cos(d*x+c))^(1/2)+1/4*(A*b+4*B*a)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d+1/
2*A*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.61, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3078, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(4a^2A+4abB-Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{(4aB+Ab)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4ad}+\frac{(4aB+3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}-\frac{(4aB+Ab)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] -1/4*((A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*A*b + 4*a*B)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sq
rt[a + b*Cos[c + d*x]]) + ((4*a^2*A - A*b^2 + 4*a*b*B)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b
*Cos[c + d*x]]) + ((A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*
a*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+dx}{2d}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4ad}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4ad}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4ad}$$

$$= -\frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.38, size = 420, normalized size = 1.44

$$\frac{\frac{8Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(8a^2A-3Ab^2+4abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\sqrt{a+b\cos(c+dx)}}-\frac{2i(Ab+4aB)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)}{a^2b\sqrt{-\frac{1}{a+b}}}+\frac{4\sqrt{a+b\cos(c+dx)}(2aA+(Ab+4aB)\cos(c+dx))\sec(c+dx)\tan(c+dx)}{a}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] ((8*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A - 3*A*b^2 + 4*a*b*B)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*Sq
rt[a + b*Cos[c + d*x]]) - ((2*I)*(A*b + 4*a*B)*Sqrt[-((b*(-1 + Cos[c + d*x]
))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a -
b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Co
s[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a
 + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a^2*b*Sqrt[-(a
```

+ b)^(-1)]) + (4*Sqrt[a + b*Cos[c + d*x]]*(2*a*A + (A*b + 4*a*B)*Cos[c + d*
x])*Sec[c + d*x]*Tan[c + d*x])/a)/(16*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1289 vs. $2(353) = 706$.
time = 0.66, size = 1290, normalized size = 4.42

method	result	size
default	Expression too large to display	1290

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*B*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*a*A*(-1/2*cos(1/2*d*x+1/2*c)/a*(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*(A*b+B*a)*(-cos(1/2*d*x+1/2*c
)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)
^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
```

```
+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*b+a+b)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))\sqrt{a + b\cos(c + dx)}\sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sec(c + d*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^3,x)
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^3, x)
```

3.303 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=378

$$-\frac{(16a^2A-3Ab^2+6abB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(16a^2A-Ab^2+18abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{24ad\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -1/24*(16*A*a^2-3*A*b^2+6*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c)
)^(1/2)/a^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/24*(16*A*a^2-A*b^2+18*B*a*b)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x
+c))^(1/2)+1/8*(4*A*a^2*b+A*b^3+8*B*a^3-2*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(
1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)+1/24*(16*
A*a^2-3*A*b^2+6*B*a*b)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a^2/d+1/12*(A*b+6*
B*a)*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d+1/3*A*sec(d*x+c)^2*(a
+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]

time = 0.85, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3078, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(16a^2A+6abB-3Ab^2)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24a^2d}+\frac{(16a^2A+18abB-Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24ad\sqrt{a+b\cos(c+dx)}}-\frac{(16a^2A+6abB-3Ab^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(8a^3B+4a^2Ab-2ab^2B+Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{8a^2d\sqrt{a+b\cos(c+dx)}}+\frac{(6aB+Ab)\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{12ad}+\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] -1/24*((16*a^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((1
6*a^2*A - A*b^2 + 18*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2*A*b
+ A*b^3 + 8*a^3*B - 2*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticP
i[2, (c + d*x)/2, (2*b)/(a + b)])/(8*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((16
*a^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a^2*
d) + ((A*b + 6*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12
*a*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
```

```
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\,\sec^2(c+dx)\tan(c+dx}{3d}$$

$$= \frac{(Ab+6aB)\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\,\text{ta}}{12ad}$$

$$= \frac{(16a^2A-3Ab^2+6abB)\sqrt{a+b\cos(c+dx)}\,\text{t}}{24a^2d}$$

$$= \frac{(16a^2A-3Ab^2+6abB)\sqrt{a+b\cos(c+dx)}\,\text{t}}{24a^2d}$$

$$= \frac{(16a^2A-3Ab^2+6abB)\sqrt{a+b\cos(c+dx)}\,\text{t}}{24a^2d}$$

$$= -\frac{(16a^2A-3Ab^2+6abB)\sqrt{a+b\cos(c+dx)}}{24a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(16a^2A-3Ab^2+6abB)\sqrt{a+b\cos(c+dx)}}{24a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.56, size = 635, normalized size = 1.68

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((2*(4*a*A*b^2 + 24*a^2*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A*b + 9*A*
b^3 + 48*a^3*B - 18*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-16*a^2*
A*b + 3*A*b^3 - 6*a*b^2*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b
*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[
```

```
Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*El
lipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
 - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt
[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*
Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*C
os[c + d*x])^2)))/(96*a^2*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^2*(
A*b*Sin[c + d*x] + 6*a*B*Sin[c + d*x]))/(12*a) + (Sec[c + d*x]*(16*a^2*A*Si
n[c + d*x] - 3*A*b^2*Sin[c + d*x] + 6*a*b*B*Sin[c + d*x]))/(24*a^2) + (A*Se
c[c + d*x]^2*Tan[c + d*x])/3))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2212 vs. $2(435) = 870$.

time = 1.03, size = 2213, normalized size = 5.85

method	result	size
default	Expression too large to display	2213

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*A*(-1/3
*cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^
2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1
/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
```

```
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*b^
3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*(A*b+B*a)*(-1/2*cos(1/2*d*x+1
/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3
/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-
b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-
b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*B*b*(-cos(1/
2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/
a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d
*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^4, x)
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^4,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^4, x)
```

3.304 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=378

$$-\frac{2(18a^3Ab-246aAb^3-8a^4B-33a^2b^2B-147b^4B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{315b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(a^2-b^2)\,(18a$$

```
[Out] -2/315*(18*A*a*b-8*B*a^2-49*B*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+
2/63*(9*A*b-4*B*a)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+2/9*B*cos(d*x+c)
*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d-2/315*(18*A*a^2*b-75*A*b^3-8*B*a^3-3
9*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d-2/315*(18*A*a^3*b-246*A*
a*b^3-8*B*a^4-33*B*a^2*b^2-147*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d
*x+c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/315*(a^2-b^2)*(18*A*a^2
*b-75*A*b^3-8*B*a^3-39*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c)
)/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3069, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(-8a^2B+18aAb-49b^2B)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^2d}-\frac{2(-8a^3B+18a^2Ab-39ab^2B-75Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315b^2d}+\frac{2(a^2-b^2)\left(-8a^3B+18a^2Ab-39ab^2B-75Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{315b^3d\sqrt{a+b\cos(c+dx)}}-\frac{2(-8a^4B+18a^3Ab-33a^2b^2B-246aAb^3-147b^4B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{315b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(9Ab-4aB)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{63b^2d}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*(18*a^3*A*b - 246*a*A*b^3 - 8*a^4*B - 33*a^2*b^2*B - 147*b^4*B)*Sqrt[a
+ b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(18*a^2*A*b - 75*A*b^3 - 8*a^
3*B - 39*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(315*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(18*a^2*A*b - 75
*A*b^3 - 8*a^3*B - 39*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*
b^2*d) - (2*(18*a*A*b - 8*a^2*B - 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[
c + d*x])/(315*b^2*d) + (2*(9*A*b - 4*a*B)*(a + b*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(63*b^2*d) + (2*B*Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c +
d*x])/(9*b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
```

```
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2B\cos(c+dx)(a+b\cos(c+dx))^{5/2}\sin(c+ \ldots}{9bd}$$

$$= \frac{2(9Ab-4aB)(a+b\cos(c+dx))^{5/2}\sin(c+d \ldots}{63b^2d}$$

$$= -\frac{2(18aAb-8a^2B-49b^2B)\,(a+b\cos(c+d \ldots}{315b^2d}$$

$$= -\frac{2(18a^2Ab-75Ab^3-8a^3B-39ab^2B)\,\sqrt{a- \ldots}}{315b^2d}$$

$$= -\frac{2(18a^2Ab-75Ab^3-8a^3B-39ab^2B)\,\sqrt{a- \ldots}}{315b^2d}$$

$$= -\frac{2(18a^2Ab-75Ab^3-8a^3B-39ab^2B)\,\sqrt{a- \ldots}}{315b^2d}$$

$$= -\frac{2(18a^3Ab-246aAb^3-8a^4B-33a^2b^2B- \ldots}{315b^3d\sqrt{\frac{a- \ldots}{\ldots}}}$$

Mathematica [A]
time = 1.61, size = 291, normalized size = 0.77

$$\frac{8\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(153a^2Ab+75Ab^3+2a^3B+186ab^2B)\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+(-18a^3Ab+246aAb^3+8a^4B+33a^2b^2B+147b^4B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)+b(a+b\cos(c+dx))\left((72a^2Ab+690Ab^3-32a^3B+804ab^2B)\sin(c+dx)+b(2(144aAb+6a^2B+133b^2B)\sin(2(c+dx))+5b(2(9Ab+10aB)\sin(3(c+dx))+7bB\sin(4(c+dx))))\right)}{1260b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(153*a^2*A*b + 75*A*b^3 + 2*a^3*
B + 186*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-18*a^3*A*b + 246
*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*((a + b)*EllipticE[(c + d*x)
/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*C
os[c + d*x])*((72*a^2*A*b + 690*A*b^3 - 32*a^3*B + 804*a*b^2*B)*Sin[c + d*x
] + b*(2*(144*a*A*b + 6*a^2*B + 133*b^2*B)*Sin[2*(c + d*x)] + 5*b*(2*(9*A*b
 + 10*a*B)*Sin[3*(c + d*x)] + 7*b*B*Sin[4*(c + d*x)]))))/(1260*b^3*d*Sqrt[a
 + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1634 vs. $2(408) = 816$.
time = 0.44, size = 1635, normalized size = 4.33

method	result	size
default	Expression too large to display	1635

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^5+(720*A*b^5+1360*B*a*b^4+2240*
B*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-936*A*a*b^4-1080*A*b^5-424
*B*a^2*b^3-2040*B*a*b^4-2072*B*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)
+(324*A*a^2*b^3+936*A*a*b^4+840*A*b^5-4*B*a^3*b^2+424*B*a^2*b^3+1568*B*a*b^
4+952*B*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A*a^3*b^2-162*A*a
^2*b^3-384*A*a*b^4-240*A*b^5+8*B*a^4*b+2*B*a^3*b^2-282*B*a^2*b^3-444*B*a*b^
4-168*B*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+18*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-93*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+75*A*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-18*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^4*b+18*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^3*b^2+246*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*
b^3-246*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-8*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
```

```
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-31*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+39*a*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^5-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^4*b+33*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^3*b^2-33*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
^2*b^3+147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-1
47*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5)/b^3/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 639, normalized size = 1.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/945*(sqrt(2)*(16*I*B*a^5 - 36*I*A*a^4*b + 60*I*B*a^3*b^2 + 33*I*A*a^2*b^3
 - 264*I*B*a*b^4 - 225*I*A*b^5)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*
b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*
x + c) + 2*a)/b) + sqrt(2)*(-16*I*B*a^5 + 36*I*A*a^4*b - 60*I*B*a^3*b^2 - 3
3*I*A*a^2*b^3 + 264*I*B*a*b^4 + 225*I*A*b^5)*sqrt(b)*weierstrassPInverse(4/
3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) -
```

```
 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(-8*I*B*a^4*b + 18*I*A*a^3*b^2 -
33*I*B*a^2*b^3 - 246*I*A*a*b^4 - 147*I*B*b^5)*sqrt(b)*weierstrassZeta(4/3*(
4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I
*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(8*I*B*a^4*b - 18*I*A*a^3*b^2 + 33*I
*B*a^2*b^3 + 246*I*A*a*b^4 + 147*I*B*b^5)*sqrt(b)*weierstrassZeta(4/3*(4*a^
2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*s
in(d*x + c) + 2*a)/b)) + 6*(35*B*b^5*cos(d*x + c)^3 - 4*B*a^3*b^2 + 9*A*a^2
*b^3 + 88*B*a*b^4 + 75*A*b^5 + 5*(10*B*a*b^4 + 9*A*b^5)*cos(d*x + c)^2 + (3
*B*a^2*b^3 + 72*A*a*b^4 + 49*B*b^5)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*
sin(d*x + c))/(b^4*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

3.305 $\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=297

$$\frac{2(21a^2Ab+63Ab^3-6a^3B+82ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(a^2-b^2)(21aAb-6a^2B+2}{105b^2}$$

```
[Out] 2/35*(7*A*b-2*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*B*(a+b*cos(d*x
+c))^(5/2)*sin(d*x+c)/b/d+2/105*(21*A*a*b-6*B*a^2+25*B*b^2)*sin(d*x+c)*(a+b
*cos(d*x+c))^(1/2)/b/d+2/105*(21*A*a^2*b+63*A*b^3-6*B*a^3+82*B*a*b^2)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))
^(1/2)-2/105*(a^2-b^2)*(21*A*a*b-6*B*a^2+25*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2
))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3047, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{2(-6a^2B+21aAb+25b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105bd}-\frac{2(a^2-b^2)(-6a^2B+21aAb+25b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2(-6a^3B+21a^2Ab+82ab^2B+63Ab^3)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(7Ab-2aB)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{35bd}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(21*a^2*A*b + 63*A*b^3 - 6*a^3*B + 82*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]) - (2*(a^2 - b^2)*(21*a*A*b - 6*a^2*B + 25*b^2*B)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[
a + b*Cos[c + d*x]]) + (2*(21*a*A*b - 6*a^2*B + 25*b^2*B)*Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(105*b*d) + (2*(7*A*b - 2*a*B)*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(35*b*d) + (2*B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])
/(7*b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^{3/2}\left(A\cos(c+dx)+B\,\text{co}\right.$$

$$= \frac{2B(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{2\int(a-}{}$$

$$= \frac{2(7Ab-2aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx}{35bd}$$

$$= \frac{2(21aAb-6a^2B+25b^2B)\sqrt{a+b\cos(c+dx)}}{105bd}$$

$$= \frac{2(21aAb-6a^2B+25b^2B)\sqrt{a+b\cos(c+dx)}}{105bd}$$

$$= \frac{2(21aAb-6a^2B+25b^2B)\sqrt{a+b\cos(c+dx)}}{105bd}$$

$$= \frac{2(21a^2Ab+63Ab^3-6a^3B+82ab^2B)\sqrt{a+b}}{105b^2d\sqrt{\frac{a+b\cos(c+}{a+b}}}$$

Mathematica [A]
time = 1.13, size = 233, normalized size = 0.78

$$\frac{4\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(84aAb+51a^2B+25b^2B)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+(21a^2Ab+63Ab^3-6a^3B+82ab^2B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)+b(a+b\cos(c+dx))\left((168aAb+12a^2B+115b^2B)\sin(c+dx)+3b(2(7Ab+8aB)\sin(2(c+dx))+5bB\sin(3(c+dx)))\right)}{210b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(84*a*A*b + 51*a^2*B + 25*b^2*B)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (21*a^2*A*b + 63*A*b^3 - 6*a^3*B +
 82*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(
c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((168*a*A*b + 12*a^2*
```

```
B + 115*b^2*B)*Sin[c + d*x] + 3*b*(2*(7*A*b + 8*a*B)*Sin[2*(c + d*x)] + 5*b
*B*Sin[3*(c + d*x)])))/(210*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1304 vs. $2(331) = 662$.
time = 0.38, size = 1305, normalized size = 4.39

method	result	size
default	Expression too large to display	1305

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^4+(-168*A*b^4-312*B*a*b^3-360*B*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(252*A*a*b^3+168*A*b^4+108*B*a^2
*b^2+312*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-84*A*
a^2*b^2-126*A*a*b^3-42*A*b^4-6*B*a^3*b-54*B*a^2*b^2-128*B*a*b^3-80*B*b^4)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-21*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^3*b+21*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*b^3+21*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^3*b-21*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2
*b^2+63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-63*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+6*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-31*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))-6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^3*b+82*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2
-82*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b^2/(-2
```

```
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 562, normalized size = 1.89

$$\frac{\sqrt{2}\left(-12i\,Ba^{4}+42i\,Aa^{3}b+11i\,Ba^{2}b^{2}-126i\,Aab^{3}-75i\,Bb^{4}\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2a}{3b}\right)+\sqrt{2}\left(12i\,Ba^{4}-42i\,Aa^{3}b-11i\,Ba^{2}b^{2}+126i\,Aab^{3}+75i\,Bb^{4}\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)+2a}{3b}\right)-3\sqrt{2}\left(6i\,Ba^{3}b-21i\,Aa^{2}b^{2}-82i\,Bab^{3}-63i\,Ab^{4}\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2a}{3b}\right)\right)-3\sqrt{2}\left(-6i\,Ba^{3}b+21i\,Aa^{2}b^{2}+82i\,Bab^{3}+63i\,Ab^{4}\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)+2a}{3b}\right)\right)+6\left(15\,Bb^{4}\cos(dx+c)^{2}+3\,Ba^{2}b^{2}+42\,Aab^{3}+25\,Bb^{4}+3\left(8\,Bab^{3}+7\,Ab^{4}\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\,\sin(dx+c)}{315\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/315*(sqrt(2)*(-12*I*B*a^4 + 42*I*A*a^3*b + 11*I*B*a^2*b^2 - 126*I*A*a*b^3
 - 75*I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(
8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)
+ sqrt(2)*(12*I*B*a^4 - 42*I*A*a^3*b - 11*I*B*a^2*b^2 + 126*I*A*a*b^3 + 75*
I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sq
rt(2)*(6*I*B*a^3*b - 21*I*A*a^2*b^2 - 82*I*B*a*b^3 - 63*I*A*b^4)*sqrt(b)*we
ierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierst
rassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b
*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(-6*I*B*a^3*b + 2
1*I*A*a^2*b^2 + 82*I*B*a*b^3 + 63*I*A*b^4)*sqrt(b)*weierstrassZeta(4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^
2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*
sin(d*x + c) + 2*a)/b)) + 6*(15*B*b^4*cos(d*x + c)^2 + 3*B*a^2*b^2 + 42*A*a
*b^3 + 25*B*b^4 + 3*(8*B*a*b^3 + 7*A*b^4)*cos(d*x + c))*sqrt(b*cos(d*x + c)
 + a)*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

3.306 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=225

$$\frac{2(20aAb+3a^2B+9b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(a^2-b^2)(5Ab+3aB)\sqrt{\frac{a+b\cos(c+dx}{a+b}}}{15bd\sqrt{a+b\cos(c+dx}}$$

```
[Out] 2/5*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/15*(5*A*b+3*B*a)*sin(d*x+c)*(a+
b*cos(d*x+c))^(1/2)/d+2/15*(20*A*a*b+3*B*a^2+9*B*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(
1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/15*(a^2-b
^2)*(5*A*b+3*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)
/b/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(a^2-b^2)(3aB+5Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{a+b\cos(c+dx)}}+\frac{2(3a^2B+20aAb+9b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(3aB+5Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{15d}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^
2 - b^2)*(5*A*b + 3*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*A*b + 3*a
*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*B*(a + b*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2B(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{a+b\cos(c+$$

$$= \frac{2(5Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2B(a+b$$

$$= \frac{2(5Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2B(a+b$$

$$= \frac{2(5Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2B(a+b$$

$$= \frac{2(20aAb+3a^2B+9b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{15bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [**A**]
time = 0.83, size = 203, normalized size = 0.90

$$\frac{2\left(b(15a^2A+5Ab^2+12abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+(20aAb+3a^2B+9b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+b(a+b\cos(c+dx))(5Ab+6aB+3bB\cos(c+dx))\sin(c+dx)\right)}{15bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(b*(15*a^2*A + 5*A*b^2 + 12*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)] + (20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
 - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)]) + b*(a + b*Cos[c + d*x])*(5*A*b
 + 6*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x]))/(15*b*d*Sqrt[a + b*Cos[c + d*
x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(263) = 526$.
time = 0.36, size = 993, normalized size = 4.41

method	result	size
default	Expression too large to display	993

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+(20*A*b^3+36*B*a*b^2+24*B*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3-12*B*a^2*b-18*B
*a*b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^2*b-20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a*b^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^3+3*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+3*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-3*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3)/b/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 493, normalized size = 2.19

$$\frac{\sqrt{2}\left(6 i Ba^{3}-5 i Aa^{2}b-18 i Bab^{2}-15 i Ab^{3}\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cos(dx+c)+3i b\sin(dx+c)+2a}{3b}\right)+\sqrt{2}\left(-6 i Ba^{3}+5 i Aa^{2}b+18 i Bab^{2}+15 i Ab^{3}\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cos(dx+c)-3i b\sin(dx+c)+2a}{3b}\right)-3\sqrt{2}\left(-3 i Ba^{2}b-20 i Aab^{2}-9 i Bb^{3}\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cos(dx+c)+3i b\sin(dx+c)+2a}{3b}\right)\right)-3\sqrt{2}\left(3 i Ba^{2}b+20 i Aab^{2}+9 i Bb^{3}\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cos(dx+c)-3i b\sin(dx+c)+2a}{3b}\right)\right)+6\left(3Bb^{3}\cos(dx+c)+6Bab^{2}+5Ab^{3}\right)\sqrt{b\cos(dx+c)+a}\,\sin(dx+c)}{45\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/45*(sqrt(2)*(6*I*B*a^3 - 5*I*A*a^2*b - 18*I*B*a*b^2 - 15*I*A*b^3)*sqrt(b)
*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
```

```
1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(-6*I*B*a^3
+ 5*I*A*a^2*b + 18*I*B*a*b^2 + 15*I*A*b^3)*sqrt(b)*weierstrassPInverse(4/3*
(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3
*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(-3*I*B*a^2*b - 20*I*A*a*b^2 - 9*I*
B*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*
b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b
^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*
(3*I*B*a^2*b + 20*I*A*a*b^2 + 9*I*B*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*si
n(d*x + c) + 2*a)/b)) + 6*(3*B*b^3*cos(d*x + c) + 6*B*a*b^2 + 5*A*b^3)*sqrt
(b*cos(d*x + c) + a)*sin(d*x + c))/(b^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))(a + b\cos(c + dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\,\cos(c + d\,x))\,(a + b\,\cos(c + d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
[Out] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

3.307 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=236

$$\frac{2(3Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(3aAb-a^2B+b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{3d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2/3*b*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*(3*A*b+4*B*a)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/
(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(
3*A*a*b-B*a^2+B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1
/2)/d/(a+b*cos(d*x+c))^(1/2)+2*a^2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*co
s(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3069, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{2(a^2(-B)+3aAb+b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2a^2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2(4aB+3Ab)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*(3*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(3*a*A*b - a^2*B + b^
2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a^2*A*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]
]) + (2*b*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
```

```
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2}{3}\int$$

$$=\frac{2bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d}-\frac{2\int \frac{(-}{}}{}$$

$$=\frac{2bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d}+(a^2A)$$

$$=\frac{2(3Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$=\frac{2(3Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.68, size = 406, normalized size = 1.72

$$\frac{\frac{4(6aAb+3a^2B+b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(6a^2A+3Ab^2+4abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i(3Ab+4aB)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)}{ab\sqrt{-\frac{1}{a+b}}}+4bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] ((4*(6*a*A*b + 3*a^2*B + b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^2*A + 3*A
*b^2 + 4*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(3*A*b + 4*a*B)*Sqrt[-
((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Cs
c[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b
*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)
/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b
)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*b*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(6*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(303) = 606$.
time = 0.36, size = 738, normalized size = 3.13

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4B\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+3Aab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\right.}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*cos(1
/2*d*x+1/2*c)^5*b^2+3*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-3*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-3*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2,(-2*b/(a-b))^(1/2))+2*B*cos(1/2*d*x+1/2*c)^3*a*b-6*B*cos(1/2*d*x+1/2*c)^3
*b^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
```

```
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+B*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2-4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-
2*B*cos(1/2*d*x+1/2*c)*a*b+2*B*cos(1/2*d*x+1/2*c)*b^2)/(-2*sin(1/2*d*x+1/2*
c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\ (a+b\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x), x)
```

3.308 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=232

$$-\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(a^2A+2Ab^2+2abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -(A*a-2*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(
d*x+c))/(a+b))^(1/2)+(A*a^2+2*A*b^2+2*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a
+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+a*(3*A*b+2*B*a)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2
^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(
1/2)+a*A*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.44, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3068, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(a^2A+2abB+2Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a(2aB+3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] -(((a*A - 2*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + ((a^2*A + 2*A*b^2 + 2*a*b*B
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
/(d*Sqrt[a + b*Cos[c + d*x]]) + (a*(3*A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c +
 d*x]]) + (a*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
```

```
0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d}+\int \frac{\left(\frac{1}{2}\right.}{}$$

$$= \frac{aA\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d}-\frac{\int \frac{\left(-\frac{1}{2}\right.}{}}{}$$

$$= \frac{aA\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d}+\frac{1}{2}(a(3$$

$$= -\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.64, size = 398, normalized size = 1.72

$$\frac{\frac{8b(Ab+2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(5aAb+4a^2B+2b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i(-aA+2bB)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)}{ab\sqrt{-\frac{1}{a+b}}}+4aA\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] ((8*b*(A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(5*a*A*b + 4*a^2*B + 2*b^2
*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-(a*A) + 2*b*B)*Sqrt[-((b*(-1 + C
os[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*
(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*
x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sq
rt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*
Sqrt[-(a + b)^(-1)]) + 4*a*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1166 vs. $2(305) = 610$.
time = 0.41, size = 1167, normalized size = 5.03

method	result	size
default	Expression too large to display	1167

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b+(-2*A*a^2-2*A*a*b)*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2+2*A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-A*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+A*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a*b-3*A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
1/2))*a*b+2*B*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+2*B*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*B*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*B*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a
-b))^(1/2))*a^2)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^2+2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
```

```
-b))^(1/2))-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-3*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b+2*B*a*b*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a*b-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*b^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
*a^2)/(2*cos(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)
/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x
)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^2, x)
```

3.309 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=295

$$-\frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(7aAb+4a^2B+8b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{4d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -1/4*(5*A*b+4*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+
b*cos(d*x+c))/(a+b))^(1/2)+1/4*(7*A*a*b+4*B*a^2+8*B*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b)
)^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*A*a
^2+3*A*b^2+12*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))
^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*(5*A*b+4*B*a)*(a+b*cos(d*x+c))^(1/2)*ta
n(d*x+c)/d+1/2*a*A*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.68, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3068, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(4a^2B+7aAb+8b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{(4a^2A+12abB+3Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{(4aB+5Ab)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4d}-\frac{(4aB+5Ab)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] -1/4*((5*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((7*a*A*b + 4*a^2*B + 8
*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2*A + 3*A*b^2 + 12*a*b*B)*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/
(4*d*Sqrt[a + b*Cos[c + d*x]]) + ((5*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*
Tan[c + d*x])/(4*d) + (a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*
x])/(2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
```

```
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+d}{2d}$$

$$= \frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d}$$

$$= \frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d}$$

$$= \frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d}$$

$$= -\frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.02, size = 422, normalized size = 1.43

$$\frac{\frac{8b(aA+4bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(8a^2A+Ab^2+20abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{2i(5Ab+4aB)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)}{ab\sqrt{-\frac{1}{a+b}}}+4\sqrt{a+b\cos(c+dx)}(2aA+(5Ab+4aB)\cos(c+dx))\sec(c+dx)\tan(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] ((8*b*(a*A + 4*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A + A*b^2 + 20*a*b*
B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
 b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(5*A*b + 4*a*B)*Sqrt[-((b*(-1 + Cos
[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-
2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]
]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt
[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh
[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sq
```

```
rt[-(a + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*(2*a*A + (5*A*b + 4*a*B)*Co
s[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(16*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(356) = 712$.
time = 0.67, size = 1403, normalized size = 4.76

method	result	size
default	Expression too large to display	1403

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*b*(A*b+2*B*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))+2*a^2*A*(-1/2*cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*
d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2
*b/(a-b))^(1/2))*b^2)+2*a*(2*A*b+B*a)*(-cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*
x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1
/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+
1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Ellipti
```

```
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a
-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x
)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{3/2}}{\cos(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^3,x)
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^3, x)
```

3.310 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=375

$$-\frac{(16a^2A+3Ab^2+30abB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(16a^2A+17Ab^2+42abB)\sqrt{\frac{a+b\cos(c}{a+}}}{24d\sqrt{a+b\cos(c+}}$$

```
[Out] -1/24*(16*A*a^2+3*A*b^2+30*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c
))^(1/2)/a/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/24*(16*A*a^2+17*A*b^2+42*B*a*
b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x
+c))^(1/2)+1/8*(12*A*a^2*b-A*b^3+8*B*a^3+6*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^
(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+1/24*(16*A
*a^2+3*A*b^2+30*B*a*b)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d+1/12*(7*A*b+6*
B*a)*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*(a
+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.92, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3068, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(16a^2A+30abB+3Ab^2)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24ad}+\frac{(16a^2A+42abB+17Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{a+b\cos(c+dx)}}-\frac{(16a^2A+30abB+3Ab^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(8a^3B+12a^2Ab+6ab^2B-Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{8ad\sqrt{a+b\cos(c+dx)}}+\frac{(6aB+7Ab)\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{12d}+\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] -1/24*((16*a^2*A + 3*A*b^2 + 30*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16
*a^2*A + 17*A*b^2 + 42*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) + ((12*a^2*A*b
 - A*b^3 + 8*a^3*B + 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
Pi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((16*
a^2*A + 3*A*b^2 + 30*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a*d)
 + ((7*A*b + 6*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12
*d) + (a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
```

```
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA\sqrt{a+b\cos(c+dx)}\,\sec^2(c+dx)\tan(c+}{3d}$$

$$= \frac{(7Ab+6aB)\sqrt{a+b\cos(c+dx)}\,\sec(c+dx}{12d}$$

$$= \frac{(16a^2A+3Ab^2+30abB)\sqrt{a+b\cos(c+dx}}{24ad}$$

$$= \frac{(16a^2A+3Ab^2+30abB)\sqrt{a+b\cos(c+dx}}{24ad}$$

$$= \frac{(16a^2A+3Ab^2+30abB)\sqrt{a+b\cos(c+dx}}{24ad}$$

$$= -\frac{(16a^2A+3Ab^2+30abB)\sqrt{a+b\cos(c+}}{24ad\sqrt{\frac{a+b\cos(c+dx}{a+b}}}$$

$$= -\frac{(16a^2A+3Ab^2+30abB)\sqrt{a+b\cos(c+}}{24ad\sqrt{\frac{a+b\cos(c+dx}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.72, size = 634, normalized size = 1.69

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((2*(28*a*A*b^2 + 24*a^2*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(56*a^2*A*b - 9*
A*b^3 + 48*a^3*B + 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-16*a^2
*A*b - 3*A*b^3 - 30*a*b^2*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b +
 b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSin
```

```
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*
EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/
(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b
*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sq
rt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a +
b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b
*Cos[c + d*x])^2)))/(96*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^2*(
7*A*b*Sin[c + d*x] + 6*a*B*Sin[c + d*x]))/12 + (Sec[c + d*x]*(16*a^2*A*Sin[
c + d*x] + 3*A*b^2*Sin[c + d*x] + 30*a*b*B*Sin[c + d*x]))/(24*a) + (a*A*Sec
[c + d*x]^2*Tan[c + d*x])/3))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2326 vs. $2(432) = 864$.
time = 1.06, size = 2327, normalized size = 6.21

method	result	size
default	Expression too large to display	2327

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*B*b^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*a^2*A*(-1/3*cos(1/2*d*x+1/2*c)/a*(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)
/a^3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*
b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*
```

```
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/
a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*
b/(a-b))^(1/2)))+2*a*(2*A*b+B*a)*(-1/2*cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+
3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*
c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*b*(A*b+2*B*a)*(-cos(1/2*d*x+1/2*c)/
a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b
+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x
)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^4,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^4, x)
```

3.311 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=462

$$-\frac{2(110a^4Ab-3069a^2Ab^3-1617Ab^5-40a^5B-255a^3b^2B-3705ab^4B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\right.}{3465b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] -2/3465*(110*A*a^2*b-539*A*b^3-40*B*a^3-335*B*a*b^2)*(a+b*cos(d*x+c))^(3/2)
*sin(d*x+c)/b^2/d-2/693*(22*A*a*b-8*B*a^2-81*B*b^2)*(a+b*cos(d*x+c))^(5/2)*
sin(d*x+c)/b^2/d+2/99*(11*A*b-4*B*a)*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^2/
d+2/11*B*cos(d*x+c)*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d-2/3465*(110*A*a^3
*b-1254*A*a*b^3-40*B*a^4-285*B*a^2*b^2-675*B*b^4)*sin(d*x+c)*(a+b*cos(d*x+c
))^(1/2)/b^2/d-2/3465*(110*A*a^4*b-3069*A*a^2*b^3-1617*A*b^5-40*B*a^5-255*B
*a^3*b^2-3705*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^
3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3465*(a^2-b^2)*(110*A*a^3*b-1254*A*a*b
^3-40*B*a^4-285*B*a^2*b^2-675*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d
*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3069, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(-8a^2B+22aAb-81b^2B)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{693b^2d}-\frac{2(-40a^3B+110a^2Ab-335ab^2B-539Ab^3)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3465b^2d}-\frac{2(-40a^4B+110a^3Ab-285a^2b^2B-1254aAb^3-675b^4B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3465b^2d}+\frac{2(a^2-b^2)(-40a^4B+110a^3Ab-285a^2b^2B-1254aAb^3-675b^4B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3465b^3d\sqrt{a+b\cos(c+dx)}}-\frac{2(-40a^5B+110a^4Ab-255a^3b^2B-3069a^2Ab^3-3705ab^4B-1617Ab^5)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3465b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(11Ab-4aB)\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{99b^2d}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*(110*a^4*A*b - 3069*a^2*A*b^3 - 1617*A*b^5 - 40*a^5*B - 255*a^3*b^2*B -
 3705*a*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b
)])/(3465*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(110*a
^3*A*b - 1254*a*A*b^3 - 40*a^4*B - 285*a^2*b^2*B - 675*b^4*B)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3465*b^3*d*Sq
rt[a + b*Cos[c + d*x]]) - (2*(110*a^3*A*b - 1254*a*A*b^3 - 40*a^4*B - 285*a
^2*b^2*B - 675*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*b^2*d) -
 (2*(110*a^2*A*b - 539*A*b^3 - 40*a^3*B - 335*a*b^2*B)*(a + b*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(3465*b^2*d) - (2*(22*a*A*b - 8*a^2*B - 81*b^2*B)*(a +
 b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*b^2*d) + (2*(11*A*b - 4*a*B)*(a +
 b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^2*d) + (2*B*Cos[c + d*x]*(a + b*
Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
```

```
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{2B\cos(c+dx)(a+b\cos(c+dx))^{7/2}\sin(c+d}{11bd}$$

$$= \frac{2(11Ab-4aB)(a+b\cos(c+dx))^{7/2}\sin(c+}{99b^2d}$$

$$= -\frac{2(22aAb-8a^2B-81b^2B)\left(a+b\cos(c+dx\right.}{693b^2d}$$

$$= -\frac{2(110a^2Ab-539Ab^3-40a^3B-335ab^2B)\,(}{3465b^2d}$$

$$= -\frac{2(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2}{346}$$

$$= -\frac{2(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2}{346}$$

$$= -\frac{2(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2}{346}$$

$$= -\frac{2(110a^4Ab-3069a^2Ab^3-1617Ab^5-40a^5}{346}$$

Mathematica [A]
time = 2.22, size = 357, normalized size = 0.77

$$\frac{16\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(1705a^3Ab+2871aAb^3+10a^4B+3315a^2b^2B+675b^4B)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+(-110a^4Ab+3069a^2Ab^3+1617Ab^5+40a^5B+255a^3b^2B+3705ab^4B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)+b(a+b\cos(c+dx))((880a^3Ab+32868aAb^3-320a^4B+18660a^2b^2B+13050b^4B)\sin(c+dx)+b(4(1650a^2Ab+1463Ab^3+30a^3B+3095ab^2B)\sin(2(c+dx))+5b((836aAb+452a^2B+513b^2B)\sin(3(c+dx))+7b((22Ab+46aB)\sin(4(c+dx))+9bB\sin(5(c+dx))))))}{27720b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (16*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(1705*a^3*A*b + 2871*a*A*b^3 +
10*a^4*B + 3315*a^2*b^2*B + 675*b^4*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)
] + (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B
+ 3705*a*b^4*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((880*a^3*A*b + 32
868*a*A*b^3 - 320*a^4*B + 18660*a^2*b^2*B + 13050*b^4*B)*Sin[c + d*x] + b*(
4*(1650*a^2*A*b + 1463*A*b^3 + 30*a^3*B + 3095*a*b^2*B)*Sin[2*(c + d*x)] +
5*b*((836*a*A*b + 452*a^2*B + 513*b^2*B)*Sin[3*(c + d*x)] + 7*b*((22*A*b +
```

```
46*a*B)*Sin[4*(c + d*x)] + 9*b*B*Sin[5*(c + d*x)]))))))/(27720*b^3*d*Sqrt[a
+ b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1982 vs. $2(488) = 976$.
time = 0.48, size = 1983, normalized size = 4.29

method	result	size
default	Expression too large to display	1983

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*
B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12*b^6+40*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6-1617*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*b^6+675*b^6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))-40*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^6-255*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^3+
3069*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^3-3069*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4+1254*A*a*b
^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+255*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-390*a^2*b^4*B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1617*A*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^5-40*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^5*b+110*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^4*b^2-110*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a^5*b-245*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^4*b^2-3705*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
```

```
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b
^5+3705*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4+11
0*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b-1364*A*a^3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+(-12320*A*b^6-3
5840*B*a*b^5-50400*B*b^6)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(22880*A
*a*b^5+24640*A*b^6+21920*B*a^2*b^4+71680*B*a*b^5+56880*B*b^6)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-14960*A*a^2*b^4-34320*A*a*b^5-22792*A*b^6-4640
*B*a^3*b^3-32880*B*a^2*b^4-66160*B*a*b^5-34920*B*b^6)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(3520*A*a^3*b^3+14960*A*a^2*b^4+26488*A*a*b^5+10472*A*b^
6-20*B*a^4*b^2+4640*B*a^3*b^3+25120*B*a^2*b^4+30320*B*a*b^5+13860*B*b^6)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-110*A*a^4*b^2-1760*A*a^3*b^3-7326*A
*a^2*b^4-7524*A*a*b^5-1848*A*b^6+40*B*a^5*b+10*B*a^4*b^2-3210*B*a^3*b^3-708
0*B*a^2*b^4-6690*B*a*b^5-2790*B*b^6)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
))/b^3/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2
*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 726, normalized size = 1.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/10395*(sqrt(2)*(80*I*B*a^6 - 220*I*A*a^5*b + 480*I*B*a^4*b^2 + 1023*I*A*a
^3*b^3 - 2535*I*B*a^2*b^4 - 5379*I*A*a*b^5 - 2025*I*B*b^6)*sqrt(b)*weierstr
assPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*
cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(-80*I*B*a^6 + 220*I*
A*a^5*b - 480*I*B*a^4*b^2 - 1023*I*A*a^3*b^3 + 2535*I*B*a^2*b^4 + 5379*I*A*
a*b^5 + 2025*I*B*b^6)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
```

```
-8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2
*a)/b) - 3*sqrt(2)*(-40*I*B*a^5*b + 110*I*A*a^4*b^2 - 255*I*B*a^3*b^3 - 306
9*I*A*a^2*b^4 - 3705*I*B*a*b^5 - 1617*I*A*b^6)*sqrt(b)*weierstrassZeta(4/3*
(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(
4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*
I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(40*I*B*a^5*b - 110*I*A*a^4*b^2 + 2
55*I*B*a^3*b^3 + 3069*I*A*a^2*b^4 + 3705*I*B*a*b^5 + 1617*I*A*b^6)*sqrt(b)*
weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weier
strassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3
*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(315*B*b^6*cos(d*x + c)
^4 - 20*B*a^4*b^2 + 55*A*a^3*b^3 + 1025*B*a^2*b^4 + 1793*A*a*b^5 + 675*B*b^
6 + 35*(23*B*a*b^5 + 11*A*b^6)*cos(d*x + c)^3 + 5*(113*B*a^2*b^4 + 209*A*a*
b^5 + 81*B*b^6)*cos(d*x + c)^2 + (15*B*a^3*b^3 + 825*A*a^2*b^4 + 1145*B*a*b
^5 + 539*A*b^6)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^4*d
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

3.312 $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=372

$$\frac{2(45a^3Ab+435aAb^3-10a^4B+279a^2b^2B+147b^4B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{315b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(a^2-b^2)(45$$

```
[Out] 2/315*(45*A*a*b-10*B*a^2+49*B*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/
63*(9*A*b-2*B*a)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+2/9*B*(a+b*cos(d*x+c
))^(7/2)*sin(d*x+c)/b/d+2/315*(45*A*a^2*b+75*A*b^3-10*B*a^3+114*B*a*b^2)*si
n(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d+2/315*(45*A*a^3*b+435*A*a*b^3-10*B*a^4+
279*B*a^2*b^2+147*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/
b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/315*(a^2-b^2)*(45*A*a^2*b+75*A*b^3-1
0*B*a^3+114*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1
/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3047, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{2(-10a^2B+45aAb+49b^2B)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315bd}+\frac{2(-10a^3B+45a^2Ab+114ab^2B+75Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315bd}-\frac{2(a^2-b^2)(-10a^3B+45a^2Ab+114ab^2B+75Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{315b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2(-10a^4B+45a^3Ab+279a^2b^2B+435aAb^3+147b^4B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{315b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(9Ab-2aB)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{63bd}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(45*a^3*A*b + 435*a*A*b^3 - 10*a^4*B + 279*a^2*b^2*B + 147*b^4*B)*Sqrt[a
 + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(45*a^2*A*b + 75*A*b^3 - 10*
a^3*B + 114*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(45*a^2*A*b +
 75*A*b^3 - 10*a^3*B + 114*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(315*b*d) + (2*(45*a*A*b - 10*a^2*B + 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*
Sin[c + d*x])/(315*b*d) + (2*(9*A*b - 2*a*B)*(a + b*Cos[c + d*x])^(5/2)*Sin
[c + d*x])/(63*b*d) + (2*B*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^{5/2}\left(A\cos(c+dx)+B\cos\right.$$

$$= \frac{2B(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{2\int(a-}{}$$

$$= \frac{2(9Ab-2aB)(a+b\cos(c+dx))^{5/2}\sin(c+dx}{63bd}$$

$$= \frac{2(45aAb-10a^2B+49b^2B)\left(a+b\cos(c+dx)\right.}{315bd}$$

$$= \frac{2(45a^2Ab+75Ab^3-10a^3B+114ab^2B)\sqrt{a+}}{315bd}$$

$$= \frac{2(45a^2Ab+75Ab^3-10a^3B+114ab^2B)\sqrt{a+}}{315bd}$$

$$= \frac{2(45a^2Ab+75Ab^3-10a^3B+114ab^2B)\sqrt{a+}}{315bd}$$

$$= \frac{2(45a^3Ab+435aAb^3-10a^4B+279a^2b^2B+1}{315b^2d\sqrt{\frac{a+}{}}}$$

Mathematica [A]
time = 1.66, size = 291, normalized size = 0.78

$$\frac{8\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(405a^2Ab+75Ab^3+155a^3B+261ab^2B)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+(45a^3Ab+435aAb^3-10a^4B+279a^2b^2B+147b^4B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)+b(a+b\cos(c+dx))\left(2(540a^2Ab+345Ab^3+20a^3B+747ab^2B)\sin(c+dx)+b((540aAb+300a^2B+266b^2B)\sin(2(c+dx))+5b(2(9Ab+19aB)\sin(3(c+dx))+7bB\sin(4(c+dx))))\right)}{1260b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(405*a^2*A*b + 75*A*b^3 + 155*a^
3*B + 261*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (45*a^3*A*b + 43
5*a*A*b^3 - 10*a^4*B + 279*a^2*b^2*B + 147*b^4*B)*((a + b)*EllipticE[(c + d
*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a +
b*Cos[c + d*x])*(2*(540*a^2*A*b + 345*A*b^3 + 20*a^3*B + 747*a*b^2*B)*Sin[c
 + d*x] + b*((540*a*A*b + 300*a^2*B + 266*b^2*B)*Sin[2*(c + d*x)] + 5*b*(2*
(9*A*b + 19*a*B)*Sin[3*(c + d*x)] + 7*b*B*Sin[4*(c + d*x)])))))/(1260*b^2*d*
Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1634 vs. $2(402) = 804$.
time = 0.46, size = 1635, normalized size = 4.40

method	result	size
default	Expression too large to display	1635

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^5+(720*A*b^5+2080*B*a*b^4+2240*
B*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1440*A*a*b^4-1080*A*b^5-13
60*B*a^2*b^3-3120*B*a*b^4-2072*B*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1080*A*a^2*b^3+1440*A*a*b^4+840*A*b^5+320*B*a^3*b^2+1360*B*a^2*b^3+2408
*B*a*b^4+952*B*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-270*A*a^3*b^2
-540*A*a^2*b^3-510*A*a*b^4-240*A*b^5-10*B*a^4*b-160*B*a^3*b^2-666*B*a^2*b^3
-684*B*a*b^4-168*B*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-45*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-30*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+75*A*b^5*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^4*b-45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a^3*b^2+435*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a^2*b^3-435*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a*b^4+10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-124*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+114*a*B*(s
```

```
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-10*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5+10*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+279*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a^3*b^2-279*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^2*b^3+147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a*b^4-147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*b^5)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/si
n(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 639, normalized size = 1.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/945*(sqrt(2)*(-20*I*B*a^5 + 90*I*A*a^4*b + 93*I*B*a^3*b^2 - 345*I*A*a^2*b
^3 - 489*I*B*a*b^4 - 225*I*A*b^5)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 -
3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(
d*x + c) + 2*a)/b) + sqrt(2)*(20*I*B*a^5 - 90*I*A*a^4*b - 93*I*B*a^3*b^2 +
345*I*A*a^2*b^3 + 489*I*B*a*b^4 + 225*I*A*b^5)*sqrt(b)*weierstrassPInverse(
4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
 - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(10*I*B*a^4*b - 45*I*A*a^3*b^2
- 279*I*B*a^2*b^3 - 435*I*A*a*b^4 - 147*I*B*b^5)*sqrt(b)*weierstrassZeta(4/
3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3
*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) +
```

```
3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(-10*I*B*a^4*b + 45*I*A*a^3*b^2 +
 279*I*B*a^2*b^3 + 435*I*A*a*b^4 + 147*I*B*b^5)*sqrt(b)*weierstrassZeta(4/3
*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*
(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3
*I*b*sin(d*x + c) + 2*a)/b)) + 6*(35*B*b^5*cos(d*x + c)^3 + 5*B*a^3*b^2 + 1
35*A*a^2*b^3 + 163*B*a*b^4 + 75*A*b^5 + 5*(19*B*a*b^4 + 9*A*b^5)*cos(d*x +
c)^2 + (75*B*a^2*b^3 + 135*A*a*b^4 + 49*B*b^5)*cos(d*x + c))*sqrt(b*cos(d*x
 + c) + a)*sin(d*x + c))/(b^3*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)\,{\left(a+b\,\cos(c+d\,x)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

3.313 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=288

$$\frac{2(161a^2Ab+63Ab^3+15a^3B+145ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(a^2-b^2)(56aAb+15a^2B}{1}$$

```
[Out] 2/35*(7*A*b+5*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(a+b*cos(d*x+c
))^(5/2)*sin(d*x+c)/d+2/105*(56*A*a*b+15*B*a^2+25*B*b^2)*sin(d*x+c)*(a+b*co
s(d*x+c))^(1/2)/d+2/105*(161*A*a^2*b+63*A*b^3+15*B*a^3+145*B*a*b^2)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1/
2)-2/105*(a^2-b^2)*(56*A*a*b+15*B*a^2+25*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*
((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.31, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{2(15a^2B+56aAb+25b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2(a^2-b^2)(15a^2B+56aAb+25b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105bd\sqrt{a+b\cos(c+dx)}}+\frac{2(15a^3B+161a^2Ab+145ab^2B+63Ab^3)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(5aB+7Ab)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{35d}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) - (2*(a^2 - b^2)*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[
a + b*Cos[c + d*x]]) + (2*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A*b + 5*a*B)*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(35*d) + (2*B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7
*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{2B(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2}{7}\int (a+b\cos(c+$$

$$= \frac{2(7Ab+5aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2B(a+}{}$$

$$= \frac{2(56aAb+15a^2B+25b^2B)\sqrt{a+b\cos(c+dx)}\,\sin(c+d}{105d}$$

$$= \frac{2(56aAb+15a^2B+25b^2B)\sqrt{a+b\cos(c+dx)}\,\sin(c+d}{105d}$$

$$= \frac{2(56aAb+15a^2B+25b^2B)\sqrt{a+b\cos(c+dx)}\,\sin(c+d}{105d}$$

$$= \frac{2(161a^2Ab+63Ab^3+15a^3B+145ab^2B)\sqrt{a+b\cos(c+}}{105bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [**A**]
time = 1.14, size = 254, normalized size = 0.88

$$\frac{2b(105a^3A+119aAb^2+135a^2bB+25b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)+2(161a^2Ab+63Ab^3+15a^3B+145ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)\right)+b(a+b\cos(c+dx))(154aAb+90a^2B+65b^2B+6b(7Ab+15aB)\cos(c+dx)+15b^2B\cos(2(c+dx)))\sin(c+dx)}{105bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*b*(105*a^3*A + 119*a*A*b^2 + 135*a^2*b*B + 25*b^3*B)*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*(161*a^2*A*b + 63
*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b
)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a
 + b)]) + b*(a + b*Cos[c + d*x])*(154*a*A*b + 90*a^2*B + 65*b^2*B + 6*b*(7*
A*b + 15*a*B)*Cos[c + d*x] + 15*b^2*B*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*
b*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1304 vs. $2(322) = 644$.
time = 0.38, size = 1305, normalized size = 4.53

method	result	size
default	Expression too large to display	1305

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^4+(-168*A*b^4-480*B*a*b^3-360*B*b^4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(392*A*a*b^3+168*A*b^4+360*B*a^2*b^2+480*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-154*A*a^2*b^2-196*A*a*b^3-42*A*b^4-90*B*a^3*b-180*B*a^2*b^2-170*B*a*b^3-80*B*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+56*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+161*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b-161*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+145*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-145*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 562, normalized size = 1.95

$$\frac{\sqrt{2}\left(30i\,Ba^4+7i\,Aa^3b-115i\,Ba^2b^2-231i\,Aab^3-75i\,Bb^4\right)\sqrt{b}\,\text{weierstrassPInverse}\left(\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8a^3-9ab^2\right)}{27b^3},\frac{3b\cos\left(dx+c\right)+3i\,b\sin\left(dx+c\right)+2a}{3b}\right)+\sqrt{2}\left(-30i\,Ba^4-7i\,Aa^3b+115i\,Ba^2b^2+231i\,Aab^3+75i\,Bb^4\right)\sqrt{b}\,\text{weierstrassPInverse}\left(\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8a^3-9ab^2\right)}{27b^3},\frac{3b\cos\left(dx+c\right)-3i\,b\sin\left(dx+c\right)+2a}{3b}\right)-3\sqrt{2}\left(-15i\,Ba^3b-161i\,Aa^2b^2-145i\,Bab^3-63i\,Ab^4\right)\sqrt{b}\,\text{weierstrassZeta}\left(\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8a^3-9ab^2\right)}{27b^3},\text{weierstrassPInverse}\left(\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8a^3-9ab^2\right)}{27b^3},\frac{3b\cos\left(dx+c\right)+3i\,b\sin\left(dx+c\right)+2a}{3b}\right)\right)-3\sqrt{2}\left(15i\,Ba^3b+161i\,Aa^2b^2+145i\,Bab^3+63i\,Ab^4\right)\sqrt{b}\,\text{weierstrassZeta}\left(\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8a^3-9ab^2\right)}{27b^3},\text{weierstrassPInverse}\left(\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8a^3-9ab^2\right)}{27b^3},\frac{3b\cos\left(dx+c\right)-3i\,b\sin\left(dx+c\right)+2a}{3b}\right)\right)+6\left(15Bb^4\cos\left(dx+c\right)^2+45Ba^2b^2+77Aab^3+25Bb^4+3\left(15Bab^3+7Ab^4\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{315\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/315*(sqrt(2)*(30*I*B*a^4 + 7*I*A*a^3*b - 115*I*B*a^2*b^2 - 231*I*A*a*b^3
- 75*I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8
*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) +
 sqrt(2)*(-30*I*B*a^4 - 7*I*A*a^3*b + 115*I*B*a^2*b^2 + 231*I*A*a*b^3 + 75*
I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sq
rt(2)*(-15*I*B*a^3*b - 161*I*A*a^2*b^2 - 145*I*B*a*b^3 - 63*I*A*b^4)*sqrt(b
)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, wei
erstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*
(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(15*I*B*a^3*b
 + 161*I*A*a^2*b^2 + 145*I*B*a*b^3 + 63*I*A*b^4)*sqrt(b)*weierstrassZeta(4/
3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3
*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) -
3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(15*B*b^4*cos(d*x + c)^2 + 45*B*a^2*b^2 +
 77*A*a*b^3 + 25*B*b^4 + 3*(15*B*a*b^3 + 7*A*b^4)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)*sin(d*x + c))/(b^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+dx))(a+b\cos(c+dx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
[Out] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

3.314 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=292

$$\frac{2(35aAb+23a^2B+9b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(10a^2Ab+5Ab^3-8a^3B+8ab^2B)\sqrt{a+\dots}}{15d\sqrt{a+b\cos(\dots}}$$

```
[Out] 2/5*b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/15*b*(5*A*b+8*B*a)*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/d+2/15*(35*A*a*b+23*B*a^2+9*B*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+
b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/15*(10
*A*a^2*b+5*A*b^3-8*B*a^3+8*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*
x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+2*a^3*A*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^
(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3069, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{2a^3A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2(23a^2B+35aAb+9b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(-8a^3B+10a^2Ab+8ab^2B+5Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2b(8aB+5Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{15d}+\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*(35*a*A*b + 23*a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(10*
a^2*A*b + 5*A*b^3 - 8*a^3*B + 8*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[a + b*Cos[c + d*x]]) + (
2*a^3*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)
/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(5*A*b + 8*a*B)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(5*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
```

```
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2}{5}\int$$

$$= \frac{2b(5Ab+8aB)\sqrt{a+b\cos(c+dx)}\ \sin(c+d}{15d}$$

$$= \frac{2b(5Ab+8aB)\sqrt{a+b\cos(c+dx)}\ \sin(c+d}{15d}$$

$$= \frac{2b(5Ab+8aB)\sqrt{a+b\cos(c+dx)}\ \sin(c+d}{15d}$$

$$= \frac{2(35aAb+23a^2B+9b^2B)\sqrt{a+b\cos(c+dx}}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2(35aAb+23a^2B+9b^2B)\sqrt{a+b\cos(c+dx}}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.06, size = 453, normalized size = 1.55

$$\frac{\frac{4(45a^2Ab+5Ab^3+15a^3B+17ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(30a^3A+35aAb^2+23a^2bB+9b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i(35aAb+23a^2B+9b^2B)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{-\frac{b(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)}{ab\sqrt{-\frac{1}{a+b}}}+4b\sqrt{a+b\cos(c+dx)}(5Ab+11aB+3bB\cos(c+dx))\sin(c+dx)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] ((4*(45*a^2*A*b + 5*A*b^3 + 15*a^3*B + 17*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]]
+ (2*(30*a^3*A + 35*a*A*b^2 + 23*a^2*b*B + 9*b^3*B)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d
*x]] + ((2*I)*(35*a*A*b + 23*a^2*B + 9*b^2*B)*Sqrt[-((b*(-1 + Cos[c + d*x])
)/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a -
 b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(
```

```
a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a +
 b)^(-1)]) + 4*b*Sqrt[a + b*Cos[c + d*x]]*(5*A*b + 11*a*B + 3*b*B*Cos[c + d
*x])*Sin[c + d*x])/(30*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1066 vs. $2(355) = 710$.
time = 0.42, size = 1067, normalized size = 3.65

method	result	size
default	Expression too large to display	1067

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+(20*A*b^3+56*B*a*b^2+24*B*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3-22*B*a^2*b-28*B
*a*b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^2*b-35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a*b^2-15*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+8*a
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+23*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-23*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x), x)
```

3.315 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=296

$$-\frac{(3a^2A-6Ab^2-14abB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(3a^3A+12aAb^2+4a^2bB+2b^3B)\sqrt{\frac{a+}{}}}{3d\sqrt{a+b\cos(c}}$$

```
[Out] -1/3*b*(3*A*a-2*B*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d-1/3*(3*A*a^2-6*A*b
^2-14*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(
d*x+c))/(a+b))^(1/2)+1/3*(3*A*a^3+12*A*a*b^2+4*B*a^2*b+2*B*b^3)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(
b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+a^2
*(5*A*b+2*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(s
in(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)
/d/(a+b*cos(d*x+c))^(1/2)+a*A*(a+b*cos(d*x+c))^(3/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.66, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3068, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{(3a^2A-14abB-6Ab^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a^2(2aB+5Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{(3a^3A+4a^2bB+12aAb^2+2b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}-\frac{b(3aA-2bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] -1/3*((3*a^2*A - 6*A*b^2 - 14*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*a^3*
A + 12*a*A*b^2 + 4*a^2*b*B + 2*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (a^2*
(5*A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - (b*(3*a*A - 2*b*B)*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (a*A*(a + b*Cos[c + d*x])^(3/2)*Ta
n[c + d*x])/d
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
```

```
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^{3/2}\tan(c+dx)}{d} + \int \surd$$

$$= -\frac{b(3aA-2bB)\sqrt{a+b\cos(c+dx)}\,\sin(c+}{3d}$$

$$= -\frac{b(3aA-2bB)\sqrt{a+b\cos(c+dx)}\,\sin(c+}{3d}$$

$$= -\frac{b(3aA-2bB)\sqrt{a+b\cos(c+dx)}\,\sin(c+}{3d}$$

$$= -\frac{(3a^2A-6Ab^2-14abB)\sqrt{a+b\cos(c+d}}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(3a^2A-6Ab^2-14abB)\sqrt{a+b\cos(c+d}}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.03, size = 442, normalized size = 1.49

$$\frac{\frac{8b(9aAb+9a^2B+b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(27a^2Ab+6Ab^3+12a^3B+14ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i(-3a^2A+6Ab^2+14abB)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)}{ab\sqrt{-\frac{1}{a+b}}}+4\sqrt{a+b\cos(c+dx)}(3a^2A+2b^2B\cos(c+dx))\tan(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] ((8*b*(9*a*A*b + 9*a^2*B + b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipt
icF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(27*a^2*A*b
+ 6*A*b^3 + 12*a^3*B + 14*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-3
*a^2*A + 6*A*b^2 + 14*a*b*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[
(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*Arc
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-
2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
```

```
 + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*Sqrt
[a + b*Cos[c + d*x]]*(3*a^2*A + 2*b^2*B*Cos[c + d*x])*Tan[c + d*x])/(12*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1562 vs. $2(361) = 722$.
time = 0.49, size = 1563, normalized size = 5.28

method	result	size
default	Expression too large to display	1563

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+(12*A*a^2*b+8*B*a*b^2+16*B*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a^3-6*A*a^2*b-4*B*a*b^2-4*B*b^3
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(3*A*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+12*A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a*b^2-3*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+
3*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+6*A*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-6*A*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3-15*A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^
(1/2))*a^2*b+4*B*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+2*B
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+14*B*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-14*B*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*a*b^2-6*B*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1
/2))*a^3)*sin(1/2*d*x+1/2*c)^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^3+12*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+3*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+6*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-6*A*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b+4*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+2*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
```

```
a-b))^(1/2))+14*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
2*b-14*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-6*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3)/(-2*sin(1/2*d
*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/
sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x
)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^2, x)
```

3.316 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=315

$$-\frac{(9aAb+4a^2B-8b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(11a^2Ab+8Ab^3+4a^3B+16ab^2B)\sqrt{\frac{a}{}}}{4d\sqrt{a+b\cos(c}}$$

```
[Out] -1/4*(9*A*a*b+4*B*a^2-8*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^
(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/4*(11*A*a^2*b+8*A*b^3+4*B*a^3+16*B
*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*co
s(d*x+c))^(1/2)+1/4*a*(4*A*a^2+15*A*b^2+20*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1
/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/2*a*A*(a+b*c
os(d*x+c))^(3/2)*sec(d*x+c)*tan(d*x+c)/d+1/4*a*(7*A*b+4*B*a)*(a+b*cos(d*x+c
))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.68, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3068, 3126, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{(4a^2B+9aAb-8b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a(4a^2A+20abB+15Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{(4a^3B+11a^2Ab+16ab^2B+8Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{a(4aB+7Ab)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4d}+\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] -1/4*((9*a*A*b + 4*a^2*B - 8*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((11*a^2*
A*b + 8*A*b^3 + 4*a^3*B + 16*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(4
*a^2*A + 15*A*b^2 + 20*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(7*A*b
 + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a*A*(a + b*Cos[c
+ d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
```

```
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx &= \frac{aA(a+b\cos(c+dx))^{3/2}\sec(c+dx)\tan(c+
}{2d} \\
&= \frac{a(7Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx}{4d} \\
&= \frac{a(7Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx}{4d} \\
&= \frac{a(7Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx}{4d} \\
&= -\frac{(9aAb+4a^2B-8b^2B)\sqrt{a+b\cos(c+dx)}}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&= -\frac{(9aAb+4a^2B-8b^2B)\sqrt{a+b\cos(c+dx)}}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.01, size = 451, normalized size = 1.43

$$
\frac{\frac{8b(a^2A+4Ab^2+12abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(8a^3A+21aAb^2+36a^2bB+8b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i(-9aAb-4a^2B+8b^2B)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)}{ab\sqrt{-\frac{1}{a+b}}}+4a\sqrt{a+b\cos(c+dx)}(2aA+(9Ab+4aB)\cos(c+dx))\sec(c+dx)\tan(c+dx)}{16d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] ((8*b*(a^2*A + 4*A*b^2 + 12*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^3*A +
21*a*A*b^2 + 36*a^2*b*B + 8*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-9
*a*A*b - 4*a^2*B + 8*b^2*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(
b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2
*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*a*Sqr
```

```
t[a + b*Cos[c + d*x]]*(2*a*A + (9*A*b + 4*a*B)*Cos[c + d*x])*Sec[c + d*x]*T
an[c + d*x])/(16*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1741 vs. $2(376) = 752$.

time = 0.74, size = 1742, normalized size = 5.53

method	result	size
default	Expression too large to display	1742

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b^2*B*(a
-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2)))+2*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+6*B*a*b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))-2*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-6*a*b*(A
*b+B*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A*a^3*(-1/2*cos(1/2*d*x+1/2*
c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*
b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))
^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*a^2*(3*A*b+B*a)
```

```
*(-cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-
1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*
sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^3, x)
```

3.317 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=376

$$-\frac{(16a^2A+33Ab^2+54abB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(16a^3A+59aAb^2+66a^2bB+48b^3B)\ \cdots}{24d\sqrt{a+b\,\mathrm{c}\cdots}}$$

```
[Out] -1/24*(16*A*a^2+33*A*b^2+54*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+
c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/24*(16*A*a^3+59*A*a*b^2+66*B*a
^2*b+48*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/
(a+b*cos(d*x+c))^(1/2)+1/8*(20*A*a^2*b+5*A*b^3+8*B*a^3+30*B*a*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(
1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/
2)+1/3*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2*tan(d*x+c)/d+1/24*(16*A*a^2+
33*A*b^2+54*B*a*b)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/4*a*(3*A*b+2*B*a)*
sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.92, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3068, 3126, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(16a^2A+54abB+33Ab^2)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}-\frac{(16a^2A+54abB+33Ab^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(16a^3A+66a^2bB+59aAb^2+48b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{a+b\cos(c+dx)}}+\frac{(8a^3B+20a^2Ab+30ab^2B+5Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{8d\sqrt{a+b\cos(c+dx)}}+\frac{a(2aB+3Ab)\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{4d}+\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] -1/24*((16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*
a^3*A + 59*a*A*b^2 + 66*a^2*b*B + 48*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]])
+ ((20*a^2*A*b + 5*A*b^3 + 8*a^3*B + 30*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c +
 d*x]]) + ((16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c
+ d*x])/(24*d) + (a*(3*A*b + 2*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*T
an[c + d*x])/(4*d) + (a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c +
 d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
```

```
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\tan(c\cdots}{3d}$$

$$= \frac{a(3Ab+2aB)\sqrt{a+b\cos(c+dx)}\ \sec(c+d\cdots}{4d}$$

$$= \frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+d\cdots}}{24d}$$

$$= \frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+d\cdots}}{24d}$$

$$= \frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+d\cdots}}{24d}$$

$$= -\frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+\cdots}}{24d\sqrt{\frac{a+b\cos(c+dx\cdots}{a+b}}}$$

$$= -\frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+\cdots}}{24d\sqrt{\frac{a+b\cos(c+dx\cdots}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.22, size = 486, normalized size = 1.29

$$\frac{\frac{8b(13aAb+6a^2B+24b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(104a^2Ab-3Ab^3+48a^3B+126ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{2i(16a^2A+33Ab^2+54abB)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)}{ab\sqrt{-\frac{1}{a+b}}}+4\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\left(2a(13Ab+6aB)\sin(c+dx)+\left(8a^2A+\frac{33Ab^2}{2}+27abB\right)\sin(2(c+dx))+8a^2A\tan(c+dx)\right)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((8*b*(13*a*A*b + 6*a^2*B + 24*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(104*a^2
*A*b - 3*A*b^3 + 48*a^3*B + 126*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*
I)*(16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))
]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*Elliptic
E[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]
 + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]
], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]
*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) +
 4*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*(2*a*(13*A*b + 6*a*B)*Sin[c + d*
x] + (8*a^2*A + (33*A*b^2)/2 + 27*a*b*B)*Sin[2*(c + d*x)] + 8*a^2*A*Tan[c +
 d*x]))/(96*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2437 vs. $2(433) = 866$.
time = 1.10, size = 2438, normalized size = 6.48

method	result	size
default	Expression too large to display	2438

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*b^2*(A*b+3*B*a)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-
b))^(1/2))+2*A*a^3*(-1/3*cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x
+1/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*
c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2
```

```
*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+
5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b)
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2
*b/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*a^2*(
3*A*b+B*a)*(-1/2*cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-
1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-
b))^(1/2))*b^2)+6*a*b*(A*b+B*a)*(-cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*
c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x
)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^4,x)
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^4, x)
```

3.318 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=465

$$-\frac{(284a^2Ab+15Ab^3+128a^3B+264ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{192ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(356a^2Ab+133Ab^3+12$$

```
[Out] -1/192*(284*A*a^2*b+15*A*b^3+128*B*a^3+264*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/
2))*(a+b*cos(d*x+c))^(1/2)/a/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/192*(356*A*
a^2*b+133*A*b^3+128*B*a^3+472*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos
(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/64*(48*A*a^4+120*A*a^2*b^2
-5*A*b^4+160*B*a^3*b+40*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*
x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+1/4*a*A*(a+b*cos(d*x+c))^(3/2
)*sec(d*x+c)^3*tan(d*x+c)/d+1/192*(284*A*a^2*b+15*A*b^3+128*B*a^3+264*B*a*b
^2)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d+1/96*(36*A*a^2+59*A*b^2+104*B*a*b
)*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/24*a*(11*A*b+8*B*a)*sec(
d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 1.20, antiderivative size = 465, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3068, 3126, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(36a^2A+104abB+59Ab^2)\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{96d}+\frac{(128a^3B+284a^2Ab+264ab^2B+15Ab^3)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{192ad}+\frac{(128a^3B+356a^2Ab+472ab^2B+133Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{192d\sqrt{a+b\cos(c+dx)}}-\frac{(128a^3B+284a^2Ab+264ab^2B+15Ab^3)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{192ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(48a^4A+160a^3bB+120a^2Ab^2+40ab^3B-5Ab^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{64ad\sqrt{a+b\cos(c+dx)}}+\frac{a(8aB+11Ab)\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] -1/192*((284*a^2*A*b + 15*A*b^3 + 128*a^3*B + 264*a*b^2*B)*Sqrt[a + b*Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]) + ((356*a^2*A*b + 133*A*b^3 + 128*a^3*B + 472*a*b^2*B)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(192*d*
Sqrt[a + b*Cos[c + d*x]]) + ((48*a^4*A + 120*a^2*A*b^2 - 5*A*b^4 + 160*a^3*
b*B + 40*a*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x
)/2, (2*b)/(a + b)])/(64*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((284*a^2*A*b + 15
*A*b^3 + 128*a^3*B + 264*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(1
92*a*d) + ((36*a^2*A + 59*A*b^2 + 104*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c
 + d*x]*Tan[c + d*x])/(96*d) + (a*(11*A*b + 8*a*B)*Sqrt[a + b*Cos[c + d*x]]
```

```
*Sec[c + d*x]^2*Tan[c + d*x])/(24*d) + (a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[
c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
```

```
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\tan(c+}{4d}$$

$$= \frac{a(11Ab+8aB)\sqrt{a+b\cos(c+dx)}\,\sec^2(c+}{24d}$$

$$= \frac{(36a^2A+59Ab^2+104abB)\sqrt{a+b\cos(c+d}}{96d}$$

$$= \frac{(284a^2Ab+15Ab^3+128a^3B+264ab^2B)\sqrt{a}}{192ad}$$

$$= \frac{(284a^2Ab+15Ab^3+128a^3B+264ab^2B)\sqrt{a}}{192ad}$$

$$= \frac{(284a^2Ab+15Ab^3+128a^3B+264ab^2B)\sqrt{a}}{192ad}$$

$$= -\frac{(284a^2Ab+15Ab^3+128a^3B+264ab^2B)\sqrt{}}{192ad\sqrt{\frac{a+b\cos}{a-}}}$$

$$= -\frac{(284a^2Ab+15Ab^3+128a^3B+264ab^2B)\sqrt{}}{192ad\sqrt{\frac{a+b\cos}{a-}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.77, size = 729, normalized size = 1.57

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] ((2*(144*a^3*A*b + 236*a*A*b^3 + 416*a^2*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (
2*(288*a^4*A + 436*a^2*A*b^2 - 45*A*b^4 + 832*a^3*b*B - 24*a*b^3*B)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt
```

```
[a + b*Cos[c + d*x]] - ((2*I)*(-284*a^2*A*b^2 - 15*A*b^4 - 128*a^3*b*B - 26
4*a*b^3*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(
a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSin
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*Ellip
ticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (
a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x
]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)
/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2))
)/(768*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^3*(17*a*A*b*Sin[c +
d*x] + 8*a^2*B*Sin[c + d*x]))/24 + (Sec[c + d*x]^2*(36*a^2*A*Sin[c + d*x] +
 59*A*b^2*Sin[c + d*x] + 104*a*b*B*Sin[c + d*x]))/96 + (Sec[c + d*x]*(284*a
^2*A*b*Sin[c + d*x] + 15*A*b^3*Sin[c + d*x] + 128*a^3*B*Sin[c + d*x] + 264*
a*b^2*B*Sin[c + d*x]))/(192*a) + (a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/4))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3547 vs. $2(518) = 1036$.
time = 1.65, size = 3548, normalized size = 7.63

method	result	size
default	Expression too large to display	3548

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^3*(-1
/4*cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^4+7/24/a^2*b*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2-1)^3-1/96*(36*a^2+35*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+5/192*b
*(20*a^2+21*b^2)/a^4*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-7/96*b/a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-35/384*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/
2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+25/96
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-25/96*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))+35/128/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
```

2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-35/128*b^4/a^4
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
-3/16/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2-35/128/a^4*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2,(-2*b/(a-b))^(1/2))*b^4)-2*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1
/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2
a^2(3*A*b+B*a)*(-1/3*cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)
^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/
16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b
/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+6*a*b*(A*
b+B*a)*(-1/2*cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)

```
^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^5, x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^5, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^5,x)
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^5, x)
```

3.319 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=320

$$\frac{2(56a^2Ab+63Ab^3-48a^3B-44ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{105b^4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(56a^3Ab+49aAb^3-48a^4$$

[Out] -2/105*(28*A*a*b-24*B*a^2-25*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/d
+2/35*(7*A*b-6*B*a)*cos(d*x+c)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2/7*
B*cos(d*x+c)^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d+2/105*(56*A*a^2*b+63*A
*b^3-48*B*a^3-44*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)
/b^4/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/105*(56*A*a^3*b+49*A*a*b^3-48*B*a^4
-32*B*a^2*b^2-25*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))
^(1/2)/b^4/d/(a+b*cos(d*x+c))^(1/2)

Rubi [A]
time = 0.39, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3069, 3128, 3102, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(-24a^2B+28aAb-25b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^3d}+\frac{2(-48a^3B+56a^2Ab-44ab^2B+63Ab^3)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{105b^4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(-48a^4B+56a^3Ab-32a^2b^2B+49aAb^3-25b^4B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{105b^4d\sqrt{a+b\cos(c+dx)}}+\frac{2(7Ab-6aB)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{35b^2d}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*(56*a^2*A*b + 63*A*b^3 - 48*a^3*B - 44*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^4*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) - (2*(56*a^3*A*b + 49*a*A*b^3 - 48*a^4*B - 32*a^2*b^2*B - 25*b^
4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(105*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(28*a*A*b - 24*a^2*B - 25*b^2
*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b^3*d) + (2*(7*A*b - 6*a*B)
*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*b^2*d) + (2*B*Cos[
c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*b*d)

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2B\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7bd} + \frac{2\int \frac{\cos(c+dx)(2a}{}}{}$$

$$= \frac{2(7Ab-6aB)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35b^2d} + \frac{2B}{}$$

$$= -\frac{2(28aAb-24a^2B-25b^2B)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105b^3d} +$$

$$= -\frac{2(28aAb-24a^2B-25b^2B)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105b^3d} +$$

$$= -\frac{2(28aAb-24a^2B-25b^2B)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105b^3d} +$$

$$= \frac{2(56a^2Ab+63Ab^3-48a^3B-44ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\tfrac{1}{2}(\right.}{105b^4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A]

time = 1.12, size = 230, normalized size = 0.72

$$\frac{4\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(14aAb-12a^2B+25b^2B)\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-(-56a^2Ab-63Ab^3+48a^3B+44ab^2B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)+2b(a+b\cos(c+dx))\left(-56aAb+48a^2B+65b^2B+6b(7Ab-6aB)\cos(c+dx)+15b^2B\cos(2(c+dx))\right)\sin(c+dx)}{210b^4d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(14*a*A*b - 12*a^2*B + 25*b^2*B)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (-56*a^2*A*b - 63*A*b^3 + 48*a^3*B
 + 44*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])) + 2*b*(a + b*Cos[c + d*x])*(-56*a*A*b + 48*a
^2*B + 65*b^2*B + 6*b*(7*A*b - 6*a*B)*Cos[c + d*x] + 15*b^2*B*Cos[2*(c + d*
x)])*Sin[c + d*x])/(210*b^4*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1304 vs. 2(354) = 708.
time = 0.41, size = 1305, normalized size = 4.08

method	result	size
default	Expression too large to display	1305

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^4+(-168*A*b^4+24*B*a*b^3-360*B*b^4
)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-28*A*a*b^3+168*A*b^4+24*B*a^2*b
^2-24*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(56*A*a^2*
b^2+14*A*a*b^3-42*A*b^4-48*B*a^3*b-12*B*a^2*b^2-44*B*a*b^3-80*B*b^4)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^3*b-49*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b^3+56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^3*b-56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+
63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-63*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+48*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+32*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*
```

```
x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-48*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^4+48*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^3*b-44*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+4
4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b^4/(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 562, normalized size = 1.76

$$\frac{\sqrt{2}\left(-96i\,Ba^4+112i\,Aa^3b-52i\,Ba^2b^2+84i\,Aab^3-75i\,Bb^4\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2a}{3b}\right)+\sqrt{2}\left(96i\,Ba^4-112i\,Aa^3b+52i\,Ba^2b^2-84i\,Aab^3+75i\,Bb^4\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)+2a}{3b}\right)-3\sqrt{2}\left(48i\,Ba^3b-56i\,Aa^2b^2+44i\,Bab^3-63i\,Ab^4\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2a}{3b}\right)\right)-3\sqrt{2}\left(-48i\,Ba^3b+56i\,Aa^2b^2-44i\,Bab^3+63i\,Ab^4\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)+2a}{3b}\right)\right)+6\left(15\,Bb^4\cos(dx+c)^2+24\,Ba^2b^2-28\,Aab^3+25\,Bb^4-3\left(6\,Bab^3-7\,Ab^4\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\,\sin(dx+c)}{315\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/315*(sqrt(2)*(-96*I*B*a^4 + 112*I*A*a^3*b - 52*I*B*a^2*b^2 + 84*I*A*a*b^3
 - 75*I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(
8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)
+ sqrt(2)*(96*I*B*a^4 - 112*I*A*a^3*b + 52*I*B*a^2*b^2 - 84*I*A*a*b^3 + 75*
I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sq
rt(2)*(48*I*B*a^3*b - 56*I*A*a^2*b^2 + 44*I*B*a*b^3 - 63*I*A*b^4)*sqrt(b)*w
eierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weiers
trassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*
b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(-48*I*B*a^3*b +
 56*I*A*a^2*b^2 - 44*I*B*a*b^3 + 63*I*A*b^4)*sqrt(b)*weierstrassZeta(4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*
```

```
b*sin(d*x + c) + 2*a)/b)) + 6*(15*B*b^4*cos(d*x + c)^2 + 24*B*a^2*b^2 - 28*
A*a*b^3 + 25*B*b^4 - 3*(6*B*a*b^3 - 7*A*b^4)*cos(d*x + c))*sqrt(b*cos(d*x +
 c) + a)*sin(d*x + c))/(b^5*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2), x)
```

3.320 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=246

$$-\frac{2(10aAb-8a^2B-9b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(10a^2Ab+5Ab^3-8a^3B-7ab^2B)\sqrt{}}{15b^3d\sqrt{a+b\,c}}$$

```
[Out] 2/15*(5*A*b-4*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2/5*B*cos(d*x+c)
*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d-2/15*(10*A*a*b-8*B*a^2-9*B*b^2)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b)
)^(1/2)+2/15*(10*A*a^2*b+5*A*b^3-8*B*a^3-7*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/
2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3069, 3102, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(-8a^2B+10aAb-9b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(-8a^3B+10a^2Ab-7ab^2B+5Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2(5Ab-4aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{15b^2d}+\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(10*a*A*b - 8*a^2*B - 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*
(10*a^2*A*b + 5*A*b^3 - 8*a^3*B - 7*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[a + b*Cos[c + d*
x]]) + (2*(5*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^2*d)
 + (2*B*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2B\cos(c+dx)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5bd} + \frac{2\int \frac{aB+\frac{3}{2}bB\cos(c}{\sqrt{a}}}{}$$

$$= \frac{2(5Ab-4aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15b^2d} + \frac{2B\cos(c+dx)}{}$$

$$= \frac{2(5Ab-4aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15b^2d} + \frac{2B\cos(c+dx)}{}$$

$$= \frac{2(5Ab-4aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15b^2d} + \frac{2B\cos(c+dx)}{}$$

$$= -\frac{2(10aAb-8a^2B-9b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A]
time = 0.95, size = 180, normalized size = 0.73

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(5Ab+2aB)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+(-10aAb+8a^2B+9b^2B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)+2b(a+b\cos(c+dx))(5Ab-4aB+3bB\cos(c+dx))\sin(c+dx)}{15b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(5*A*b + 2*a*B)*EllipticF[(c + d
*x)/2, (2*b)/(a + b)] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*((a + b)*EllipticE[
(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + 2
*b*(a + b*Cos[c + d*x])*(5*A*b - 4*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x])/
(15*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(284) = 568$.
time = 0.36, size = 993, normalized size = 4.04

method	result	size
default	Expression too large to display	993

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+(20*A*b^3-4*B*a*b^2+24*B*b^3)*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3+8*B*a^2*b+2*B*a*
b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^2*b+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a*b^2-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
3-7*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+8*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-8*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*b^3)/b^3/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 493, normalized size = 2.00

$$\frac{\sqrt{2}\,(16i\,Ba^3-20i\,Aa^2b+12i\,Bab^2-15i\,Ab^3)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4a^2-3b^2)}{3b^2},-\frac{8\,(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2a}{3b}\right)+\sqrt{2}\,(-16i\,Ba^3+20i\,Aa^2b-12i\,Bab^2+15i\,Ab^3)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4a^2-3b^2)}{3b^2},-\frac{8\,(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)+2a}{3b}\right)-3\sqrt{2}\,(-8i\,Ba^2b+10i\,Aab^2-9i\,Bb^3)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\,(4a^2-3b^2)}{3b^2},-\frac{8\,(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(4a^2-3b^2)}{3b^2},-\frac{8\,(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2a}{3b}\right)\right)-3\sqrt{2}\,(8i\,Ba^2b-10i\,Aab^2+9i\,Bb^3)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\,(4a^2-3b^2)}{3b^2},-\frac{8\,(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(4a^2-3b^2)}{3b^2},-\frac{8\,(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)+2a}{3b}\right)\right)+6\,(3\,Bb^3\cos(dx+c)-4\,Bab^2+5\,Ab^3)\sqrt{b\cos(dx+c)+a}\,\sin(dx+c)}{45\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/45*(sqrt(2)*(16*I*B*a^3 - 20*I*A*a^2*b + 12*I*B*a*b^2 - 15*I*A*b^3)*sqrt(
b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3
, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(-16*I*B*a
^3 + 20*I*A*a^2*b - 12*I*B*a*b^2 + 15*I*A*b^3)*sqrt(b)*weierstrassPInverse(
4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
 - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(-8*I*B*a^2*b + 10*I*A*a*b^2 -
9*I*B*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9
*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt
(2)*(8*I*B*a^2*b - 10*I*A*a*b^2 + 9*I*B*b^3)*sqrt(b)*weierstrassZeta(4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*
b*sin(d*x + c) + 2*a)/b)) + 6*(3*B*b^3*cos(d*x + c) - 4*B*a*b^2 + 5*A*b^3)*
sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)})\cos^2{(c + dx)}}{\sqrt{a + b\cos{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)**2/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\sqrt{a+b\,\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2), x)
```

3.321 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=183

$$\frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(3aAb-2a^2B-b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2/3*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d+2/3*(3*A*b-2*B*a)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/
(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2
/3*(3*A*a*b-2*B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+
b))^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3047, 3102, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(-2a^2B+3aAb-b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(3*A*b - 2*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(3*a*A*b - 2*a^2*
B - b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*B*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(3*b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{bB}{2}+\frac{1}{2}(3Ab-2aB)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{3b}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{(3Ab-2aB)\int \sqrt{a+b\cos(}}{3b^2}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{\left((3Ab-2aB)\sqrt{a+b\cos(c}\right.}{3b^2\sqrt{}}$$

$$= \frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(3aAb-}{}$$

Mathematica [A]
time = 0.73, size = 154, normalized size = 0.84

$$\frac{-2(a+b)(-3Ab+2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)+2(-3aAb+2a^2B+b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)+2bB(a+b\cos(c+dx))\sin(c+dx)}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(a + b)*(-3*A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)] + 2*(-3*a*A*b + 2*a^2*B + b^2*B)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*B*(a + b*Co
s[c + d*x])*Sin[c + d*x])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(225) = 450$.
time = 0.33, size = 671, normalized size = 3.67

method	result

default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-4B\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+3Aab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\left(\right.\right.}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1
/2*d*x+1/2*c)^5*b^2+3*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+3*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*B*cos(1/2*d*x+1/2*c)^3*a*b+6*B*cos(
1/2*d*x+1/2*c)^3*b^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
2-B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a*b+2*B*cos(1/2*d*x+1/2*c)*a*b-2*B*cos(1/2*d*x+1/2*c)*b^2)/b^
2/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 435, normalized size = 2.38

$$\frac{6\sqrt{b\cos(dx+c)+a}\,Bb^2\sin(dx+c)+\sqrt{2}\left(-4i\,Ba^2+6i\,Aab-3i\,Bb^2\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2a}{3b}\right)+\sqrt{2}\left(4i\,Ba^2-6i\,Aab+3i\,Bb^2\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)+2a}{3b}\right)-3\sqrt{2}\left(2i\,Bab-3i\,Ab^2\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2a}{3b}\right)\right)-3\sqrt{2}\left(-2i\,Bab+3i\,Ab^2\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)+2a}{3b}\right)\right)}{9b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/9*(6*sqrt(b*cos(d*x + c) + a)*B*b^2*sin(d*x + c) + sqrt(2)*(-4*I*B*a^2 +
6*I*A*a*b - 3*I*B*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
 -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) +
2*a)/b) + sqrt(2)*(4*I*B*a^2 - 6*I*A*a*b + 3*I*B*b^2)*sqrt(b)*weierstrassPI
nverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(2*I*B*a*b - 3*I*A*b^2)*
sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^
3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3
, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(-2*I*B
*a*b + 3*I*A*b^2)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8
*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/
(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\cos(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [B]
time = 0.80, size = 199, normalized size = 1.09

$$\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}+\frac{2A\left(\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|\frac{2b}{a+b}\right)(a+b)-a\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|\frac{2b}{a+b}\right)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|\frac{2b}{a+b}\right)(2a^2+b^2)-2a\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|\frac{2b}{a+b}\right)(a+b)\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*B*sin(c + d*x)*(a + b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*(ellipticE(c/2
 + (d*x)/2, (2*b)/(a + b))*(a + b) - a*ellipticF(c/2 + (d*x)/2, (2*b)/(a +
b)))*((a + b*cos(c + d*x))/(a + b))^(1/2))/(b*d*(a + b*cos(c + d*x))^(1/2))
 + (2*B*((a + b*cos(c + d*x))/(a + b))^(1/2)*(ellipticF(c/2 + (d*x)/2, (2*b
)/(a + b))*(2*a^2 + b^2) - 2*a*ellipticE(c/2 + (d*x)/2, (2*b)/(a + b))*(a +
 b)))/(3*b^2*d*(a + b*cos(c + d*x))^(1/2))
```

3.322 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=130

$$\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))
/(a+b))^(1/2)+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b
))^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2831, 2742, 2740, 2734, 2732}

$$\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{B\int \sqrt{a+b\cos(c+dx)}\,dx}{b} + \frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b}$$

$$= \frac{\left(B\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left((Ab-aB)\sqrt{\frac{a+}{}}\right.}{}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F}{bd\sqrt{a+b\cos(c+d}}$$

Mathematica [A]
time = 3.43, size = 93, normalized size = 0.72

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)BE\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)+(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*B*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + (A*b - a*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(b*d*Sqrt
[a + b*Cos[c + d*x]])
```

Maple [A]

time = 0.31, size = 249, normalized size = 1.92

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\,\left(Ab\,\text{Ellip}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\,(}}$
risch	$-\frac{iB\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)\sqrt{2}\,e^{-i(dx+c)}}{bd\sqrt{\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)e^{-i(dx+c)}}}-\frac{i\left(\frac{2A\left(a+\sqrt{a^2-b^2}\right)\sqrt{\frac{\left(e^{i(dx+c)}+\frac{a+\sqrt{a^2-b^2}}{b}\right)b}{a+\sqrt{a^2-b^2}}}\,\sqrt{\cdots}}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(A*b*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-B*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-B*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4
*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 371, normalized size = 2.85

$$\frac{3i\sqrt{2}\,Bb^{\frac{3}{2}}\text{weierstrassZeta}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right)\right)-3i\sqrt{2}\,Bb^{\frac{3}{2}}\text{weierstrassZeta}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)-3i\,b\sin(dx+c)+2\,a}{3\,b}\right)\right)+\sqrt{2}\,(2i\,Ba-3i\,Ab)\sqrt{b}\,\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right)+\sqrt{2}\,(-2i\,Ba+3i\,Ab)\sqrt{b}\,\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)-3i\,b\sin(dx+c)+2\,a}{3\,b}\right)}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*I*sqrt(2)*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(
8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8
*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b))
- 3*I*sqrt(2)*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^
3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + s
qrt(2)*(2*I*B*a - 3*I*A*b)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/
b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c
) + 2*a)/b) + sqrt(2)*(-2*I*B*a + 3*I*A*b)*sqrt(b)*weierstrassPInverse(4/3*
(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3
*I*b*sin(d*x + c) + 2*a)/b))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{\left(c + dx \right)}}{\sqrt{a + b\cos{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [B]

time = 0.89, size = 135, normalized size = 1.04

$$\frac{2\,A\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\sqrt{\frac{a+b\,\cos\left(c+d\,x\right)}{a+b}}}{d\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}+\frac{2\,B\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\left(a+b\right)-a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\right)\,\sqrt{\frac{a+b\,\cos\left(c+d\,x\right)}{a+b}}}{b\,d\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*A*ellipticF(c/2 + (d*x)/2, (2*b)/(a + b))*((a + b*cos(c + d*x))/(a + b))
^(1/2))/(d*(a + b*cos(c + d*x))^(1/2)) + (2*B*(ellipticE(c/2 + (d*x)/2, (2*
b)/(a + b))*(a + b) - a*ellipticF(c/2 + (d*x)/2, (2*b)/(a + b)))*((a + b*co
s(c + d*x))/(a + b))^(1/2))/(b*d*(a + b*cos(c + d*x))^(1/2))
```

3.323 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*
x+c))^(1/2)+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2
)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3081, 2742, 2740, 2886, 2884}

$$\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = A\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx + B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{\left(A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}} + \frac{\Big(B\sqrt{\ldots}}{\ldots}$$

$$= \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \frac{2A\sqrt{\frac{a+b\cos(c+d\ldots}{a+b}}\ldots}{d\sqrt{a+b\ldots}}$$

Mathematica [A]
time = 0.21, size = 81, normalized size = 0.69

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(BF\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)+A\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(B*EllipticF[(c + d*x)/2, (2*b)/(a +
b)] + A*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]))/(d*Sqrt[a + b*Cos[c + d
*x]])
```

Maple [A]
time = 0.28, size = 194, normalized size = 1.64

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\,\left(A\,\mathrm{EllipticPi}\left(\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(A*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-B*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos(c + d\,x)}{\cos(c + d\,x)\,\sqrt{a + b\,\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2)), x)
```

3.324 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=216

$$-\frac{A\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{(Ab-2aB)\sqrt{\frac{a+b\,\mathrm{c}}{a}}}{ad\sqrt{a}}$$

```
[Out] -A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/d/((a+b*cos(d*x+c))/
(a+b))^(1/2)+A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/
(a+b*cos(d*x+c))^(1/2)-(A*b-2*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(
d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+A*(a+b*cos(d*x+c))^(1/2)*ta
n(d*x+c)/a/d
```

Rubi [A]
time = 0.39, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3079, 3139, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}+\frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{A\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -((A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)])) + (A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - ((A*
b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2
*b)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c + d*x]]*
Tan[c + d*x])/(a*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
```

```
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3139

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{ad} + \frac{\int \frac{\left(\frac{1}{2}(-Ab+2aB)-\frac{1}{2}Ab\cos^2(c+dx)\right)\text{s}}{\sqrt{a+b\cos(c+dx)}}}{a}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{ad} - \frac{A\int \sqrt{a+b\cos(c+dx)}\,dx}{2a}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{ad} + \frac{1}{2}A\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,d$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{a+b\cos(c+dx}}{ad}$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{\frac{a+b\cos(c+dx}{a+b}}}{d\sqrt{a+b\cos}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.78, size = 320, normalized size = 1.48

$$\frac{\frac{2(-3Ab+4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{2iA\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)}{ab\sqrt{-\frac{1}{a+b}}}+4A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((2*(-3*A*b + 4*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*A*Sqrt[-((b*(-1 +
 Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x
]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*Arc
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*
b*Sqrt[-(a + b)^(-1)]) + 4*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(289) = 578$.
time = 0.46, size = 639, normalized size = 2.96

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A*(-cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1
/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
```

```
)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/sqrt(a + b*cos(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^2\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2)),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2)), x)
```

3.325 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=299

$$\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\ldots (4$$

```
[Out] 1/4*(3*A*b-4*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^2/d/(
(a+b*cos(d*x+c))/(a+b))^(1/2)-1/4*(A*b-4*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((
a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*A*a^2+3*A*b^
2-4*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2
/d/(a+b*cos(d*x+c))^(1/2)-1/4*(3*A*b-4*B*a)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+
c)/a^2/d+1/2*A*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.61, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$,
Rules used = {3079, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(4a^2A-4abB+3Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(4*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((A*b - 4*a*B)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*S
qrt[a + b*Cos[c + d*x]]) + ((4*a^2*A + 3*A*b^2 - 4*a*b*B)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[
a + b*Cos[c + d*x]]) - ((3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*
x])/(4*a^2*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*a
*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(\frac{1}{2}(-3Ab+4aB)+\right.}{}}{}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos(}{}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos(}{}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos(}{}$$

$$= \frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{(3Ab-4a}{}$$

$$= \frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{(Ab-4a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.98, size = 420, normalized size = 1.40

$$\frac{\frac{8aAb\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2(8a^2A+9Ab^2-12abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i(3Ab-4aB)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)}{ab\sqrt{-\frac{1}{a+b}}} + 4\sqrt{a+b\cos(c+dx)}\,(2aA+(-3Ab+4aB)\cos(c+dx))\sec(c+dx)\tan(c+dx)}{16a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((8*a*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A + 9*A*b^2 - 12*a*b*B)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sq
rt[a + b*Cos[c + d*x]] + ((2*I)*(3*A*b - 4*a*B)*Sqrt[-((b*(-1 + Cos[c + d*x
]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a -
 b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(
```

```
a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a +
 b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*(2*a*A + (-3*A*b + 4*a*B)*Cos[c + d
*x])*Sec[c + d*x]*Tan[c + d*x])/(16*a^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1181 vs. $2(360) = 720$.
time = 0.65, size = 1182, normalized size = 3.95

method	result	size
default	Expression too large to display	1182

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(-1/2*c
os(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1
)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*
B*(-cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/
(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2
*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)})\sec^3{(c + dx)}}{\sqrt{a + b\cos{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,\cos{(c + d\,x)}}{{\cos{(c + d\,x)}}^3\,\sqrt{a + b\,\cos{(c + d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2)),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2)), x)
```

3.326 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=387

$$-\frac{2(40a^3Ab-25aAb^3-48a^4B+24a^2b^2B+9b^4B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^4(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(40a^2Ab+5A$$

```
[Out] 2*a*(A*b-B*a)*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+
2/15*(20*A*a^2*b-5*A*b^3-24*B*a^3+9*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1
/2)/b^3/(a^2-b^2)/d-2/5*(5*A*a*b-6*B*a^2+B*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*
cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d-2/15*(40*A*a^3*b-25*A*a*b^3-48*B*a^4+24*B
*a^2*b^2+9*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^4/(a^
2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/15*(40*A*a^2*b+5*A*b^3-48*B*a^3-1
2*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^4/d/
(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3068, 3128, 3102, 2831, 2742, 2740, 2734, 2732}

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2(-6a^2B+5aAb+b^2B)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d(a^2-b^2)}+\frac{2(-24a^3B+20a^2Ab+9ab^2B-5Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{15b^3d(a^2-b^2)}+\frac{2(-48a^3B+40a^2Ab-12ab^2B+5Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^4d\sqrt{a+b\cos(c+dx)}}-\frac{2(-48a^4B+40a^3Ab+24a^2b^2B-25aAb^3+9b^4B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^4d(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(40*a^3*A*b - 25*a*A*b^3 - 48*a^4*B + 24*a^2*b^2*B + 9*b^4*B)*Sqrt[a +
b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^4*(a^2 - b^2)*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(40*a^2*A*b + 5*A*b^3 - 48*a^3*B
 - 12*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/(15*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Cos[c
+ d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(20*
a^2*A*b - 5*A*b^3 - 24*a^3*B + 9*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(15*b^3*(a^2 - b^2)*d) - (2*(5*a*A*b - 6*a^2*B + b^2*B)*Cos[c + d*x]*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^2*(a^2 - b^2)*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\cos(c+dx)(-2a(Ab-aB)+\ldots}{\ldots}}{\ldots}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2(5aAb-6a^2B+b^2B)\ldots}{\ldots}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2(20a^2Ab-5Ab^3-24a\ldots}{\ldots}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2(20a^2Ab-5Ab^3-24a\ldots}{\ldots}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2(20a^2Ab-5Ab^3-24a\ldots}{\ldots}$$

$$= -\frac{2(40a^3Ab-25aAb^3-48a^4B+24a^2b^2B+9b^4B)\sqrt{a+b\cos(c\ldots}}{15b^4\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A]
time = 1.92, size = 304, normalized size = 0.79

$$\frac{\frac{2b^2(-10a^2Ab-5Ab^3+12a^3B+3ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{(a-b)(a+b)}+\frac{2(-40a^3Ab+25aAb^3+48a^4B-24a^2b^2B-9b^4B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)\right)}{(a-b)(a+b)}+\frac{30a^3b(-Ab+aB)\sin(c+dx)}{-a^2+b^2}+2b(5Ab-9aB)(a+b\cos(c+dx))\sin(c+dx)+3b^2B(a+b\cos(c+dx))\sin(2(c+dx))}{15b^4d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] ((2*b^2*(-10*a^2*A*b - 5*A*b^3 + 12*a^3*B + 3*a*b^2*B)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/((a - b)*(a + b)) + (
2*(-40*a^3*A*b + 25*a*A*b^3 + 48*a^4*B - 24*a^2*b^2*B - 9*b^4*B)*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a
*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/((a - b)*(a + b)) + (30*a^3*b*(-(A
*b) + a*B)*Sin[c + d*x])/(-a^2 + b^2) + 2*b*(5*A*b - 9*a*B)*(a + b*Cos[c +
d*x])*Sin[c + d*x] + 3*b^2*B*(a + b*Cos[c + d*x])*Sin[2*(c + d*x)])/(15*b^4
*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1311 vs. $2(423) = 846$.
time = 0.74, size = 1312, normalized size = 3.39

method	result	size
default	Expression too large to display	1312

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16/b*B*(-1/
10/b*cos(1/2*d*x+1/2*c)^3*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+8/b
^2*(A*b-B*a-3*B*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/6/b*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))-1/12/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
```

```
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2/b^4*(A*a*b+2*A*b^2-B*a^2-2*B*a*b-
3*B*b^2)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2)))+2*(A*a^2*b+A*a*b^2+A*b^3-B*a^3-B*a^2*b-B*a*b^2-B*b
^3)/b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*a^3*(A*b-B*a)/b^4/sin(1/2*d*x+1
/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)+(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b))/sin(1/2*d*x+1/2*c)/(-2*sin(1/
2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 910, normalized size = 2.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/45*(6*(24*B*a^4*b^2 - 20*A*a^3*b^3 - 9*B*a^2*b^4 + 5*A*a*b^5 - 3*(B*a^2*
b^4 - B*b^6)*cos(d*x + c)^2 + (6*B*a^3*b^3 - 5*A*a^2*b^4 - 6*B*a*b^5 + 5*A*
b^6)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c) + (sqrt(2)*(-96*I*
B*a^5*b + 80*I*A*a^4*b^2 + 84*I*B*a^3*b^3 - 80*I*A*a^2*b^4 + 27*I*B*a*b^5 -
 15*I*A*b^6)*cos(d*x + c) + sqrt(2)*(-96*I*B*a^6 + 80*I*A*a^5*b + 84*I*B*a^
4*b^2 - 80*I*A*a^3*b^3 + 27*I*B*a^2*b^4 - 15*I*A*a*b^5))*sqrt(b)*weierstras
sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*co
```

```
s(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + (sqrt(2)*(96*I*B*a^5*b - 80*I*A
*a^4*b^2 - 84*I*B*a^3*b^3 + 80*I*A*a^2*b^4 - 27*I*B*a*b^5 + 15*I*A*b^6)*cos
(d*x + c) + sqrt(2)*(96*I*B*a^6 - 80*I*A*a^5*b - 84*I*B*a^4*b^2 + 80*I*A*a^
3*b^3 - 27*I*B*a^2*b^4 + 15*I*A*a*b^5))*sqrt(b)*weierstrassPInverse(4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*
b*sin(d*x + c) + 2*a)/b) - 3*(sqrt(2)*(48*I*B*a^4*b^2 - 40*I*A*a^3*b^3 - 24
*I*B*a^2*b^4 + 25*I*A*a*b^5 - 9*I*B*b^6)*cos(d*x + c) + sqrt(2)*(48*I*B*a^5
*b - 40*I*A*a^4*b^2 - 24*I*B*a^3*b^3 + 25*I*A*a^2*b^4 - 9*I*B*a*b^5))*sqrt(
b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, we
ierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3
*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(sqrt(2)*(-48*I*B*a^
4*b^2 + 40*I*A*a^3*b^3 + 24*I*B*a^2*b^4 - 25*I*A*a*b^5 + 9*I*B*b^6)*cos(d*x
 + c) + sqrt(2)*(-48*I*B*a^5*b + 40*I*A*a^4*b^2 + 24*I*B*a^3*b^3 - 25*I*A*a
^2*b^4 + 9*I*B*a*b^5))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/
b)))/((a^2*b^6 - b^8)*d*cos(d*x + c) + (a^3*b^5 - a*b^7)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2), x)
```

3.327 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=262

$$\frac{2(6a^2Ab-3Ab^3-8a^3B+5ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(6aAb-8a^2B-b^2B)\sqrt{\frac{a+b}{}}}{3b^3d\sqrt{a+b}}$$

```
[Out] -2*a^2*(A*b-B*a)*sin(d*x+c)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2/3*B*si
n(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2/3*(6*A*a^2*b-3*A*b^3-8*B*a^3+5*B*a*
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)/d/((a+
b*cos(d*x+c))/(a+b))^(1/2)-2/3*(6*A*a*b-8*B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^
(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3067, 3102, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2(-8a^2B+6aAb-b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2(-8a^3B+6a^2Ab+5ab^2B-3Ab^3)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(6*a^2*A*b - 3*A*b^3 - 8*a^3*B + 5*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) - (2*(6*a*A*b - 8*a^2*B - b^2*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*d*Sqrt[a + b*Cos[c
 + d*x]]) - (2*a^2*(A*b - a*B)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*
Cos[c + d*x]]) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}ab(Ab-aB)+\frac{1}{2}(2a^2-b^2)(Ab-}{\sqrt{a+}}}{b^2}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin}{3b^2d}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin}{3b^2d}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin}{3b^2d}$$

$$= \frac{2(6a^2Ab-3Ab^3-8a^3B+5ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{3b^3(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A]
time = 1.56, size = 189, normalized size = 0.72

$$\frac{2\left(\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((6a^2Ab-3Ab^3-8a^3B+5ab^2B)E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)+(a-b)(-6aAb+8a^2B+b^2B)F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)\right)}{a-b}+b\left(\frac{a(3aAb-4a^2B+b^2B)}{-a^2+b^2}+bB\cos(c+dx)\right)\sin(c+dx)\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (2*((Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((6*a^2*A*b - 3*A*b^3 - 8*a^3*B + 5
*a*b^2*B)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + (a - b)*(-6*a*A*b + 8*a^2
*B + b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b) + b*((a*(3*a*A*
b - 4*a^2*B + b^2*B))/(-a^2 + b^2) + b*B*Cos[c + d*x])*Sin[c + d*x]))/(3*b^
3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1335 vs. $2(302) = 604$.
time = 0.63, size = 1336, normalized size = 5.10

method	result	size
default	Expression too large to display	1336

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/b^3/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2*b^2-4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^4+6*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b^2-6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+6*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b-6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-8*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3*b-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b^2+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b^3+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^4+8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-7*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 789, normalized size = 3.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/9*(6*(4*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + (B*a^2*b^3 - B*b^5)*cos(d*x +
 c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c) - (sqrt(2)*(16*I*B*a^4*b - 12*I*
A*a^3*b^2 - 16*I*B*a^2*b^3 + 15*I*A*a*b^4 - 3*I*B*b^5)*cos(d*x + c) + sqrt(
2)*(16*I*B*a^5 - 12*I*A*a^4*b - 16*I*B*a^3*b^2 + 15*I*A*a^2*b^3 - 3*I*B*a*b
^4))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*
a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) - (sqrt(2)
*(-16*I*B*a^4*b + 12*I*A*a^3*b^2 + 16*I*B*a^2*b^3 - 15*I*A*a*b^4 + 3*I*B*b^
5)*cos(d*x + c) + sqrt(2)*(-16*I*B*a^5 + 12*I*A*a^4*b + 16*I*B*a^3*b^2 - 15
*I*A*a^2*b^3 + 3*I*B*a*b^4))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2
)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x +
 c) + 2*a)/b) + 3*(sqrt(2)*(-8*I*B*a^3*b^2 + 6*I*A*a^2*b^3 + 5*I*B*a*b^4 -
3*I*A*b^5)*cos(d*x + c) + sqrt(2)*(-8*I*B*a^4*b + 6*I*A*a^3*b^2 + 5*I*B*a^2
*b^3 - 3*I*A*a*b^4))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)
) + 3*(sqrt(2)*(8*I*B*a^3*b^2 - 6*I*A*a^2*b^3 - 5*I*B*a*b^4 + 3*I*A*b^5)*co
s(d*x + c) + sqrt(2)*(8*I*B*a^4*b - 6*I*A*a^3*b^2 - 5*I*B*a^2*b^3 + 3*I*A*a
*b^4))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*
b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b
^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^2*b^5 -
 b^7)*d*cos(d*x + c) + (a^3*b^4 - a*b^6)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2), x)
```

3.328 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{2(aAb-2a^2B+b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{b^2d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*(A*a*b-2*B*
a^2+B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/
d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2*(A*b-2*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(
(a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3047, 3100, 2831, 2742, 2740, 2734, 2732}

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2(-2a^2B+aAb+b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a*A*b - 2*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*b)/(a + b)])/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
 (2*(A*b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{1}{2}b(Ab-aB)+\frac{1}{2}(aAb-2a^2B+b^2B)\,c}{\sqrt{a+b\cos(c+dx)}}}{b(a^2-b^2)}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{(Ab-2aB)\int \frac{1}{\sqrt{a+b\cos(c+}}}{b^2}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{\left((aAb-2a^2B+b^2B)\sqrt{a+b\right.}{b^2(a^2-}$$

$$= -\frac{2(aAb-2a^2B+b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} +$$

Mathematica [A]
time = 0.88, size = 170, normalized size = 0.83

$$-\frac{2\left(-\left((a+b)(-aAb+2a^2B-b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+(a^2-b^2)(-Ab+2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+ab(-Ab+aB)\sin(c+dx)\right)}{(a-b)b^2(a+b)d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(-((a + b)*(-(a*A*b) + 2*a^2*B - b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]) + (a^2 - b^2)*(-(A*b) + 2*a*B)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a
*b*(-(A*b) + a*B)*Sin[c + d*x]))/((a - b)*b^2*(a + b)*d*Sqrt[a + b*Cos[c +
d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(252) = 504.
time = 0.46, size = 519, normalized size = 2.54

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2/(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(A*b*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-B
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)+2*a*(A*b-B*a)/b^2/sin(
1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*sin(1/2*d*x+1
/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)+(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b))/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 683, normalized size = 3.35

$$-\frac{6\left(Ba^2b^2-Aab^3\right)\sqrt{b\cos(dx+c)+a}\sin(dx+c)+\left(\sqrt{2}\left(-4i\,Ba^3b+2i\,Aa^2b^2+5i\,Bab^3-3i\,Ab^4\right)\cos(dx+c)+\sqrt{2}\left(-4i\,Ba^4+2i\,Aa^3b+5i\,Ba^2b^2-3i\,Aab^3\right)\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2a}{3b}\right)+\left(\sqrt{2}\left(4i\,Ba^3b-2i\,Aa^2b^2-5i\,Bab^3+3i\,Ab^4\right)\cos(dx+c)+\sqrt{2}\left(4i\,Ba^4-2i\,Aa^3b-5i\,Ba^2b^2+3i\,Aab^3\right)\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)+2a}{3b}\right)-3\left(\sqrt{2}\left(2i\,Ba^2b^2-i\,Aab^3-i\,Bb^4\right)\cos(dx+c)+\sqrt{2}\left(2i\,Ba^3b-i\,Aa^2b^2-i\,Bab^3\right)\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2a}{3b}\right)\right)-3\left(\sqrt{2}\left(-2i\,Ba^2b^2+i\,Aab^3+i\,Bb^4\right)\cos(dx+c)+\sqrt{2}\left(-2i\,Ba^3b+i\,Aa^2b^2+i\,Bab^3\right)\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)+2a}{3b}\right)\right)}{3\left((a^2b^4-b^6)d\cos(dx+c)+(a^3b^3-ab^5)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/3*(6*(B*a^2*b^2 - A*a*b^3)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c) + (sqrt
(2)*(-4*I*B*a^3*b + 2*I*A*a^2*b^2 + 5*I*B*a*b^3 - 3*I*A*b^4)*cos(d*x + c) +
 sqrt(2)*(-4*I*B*a^4 + 2*I*A*a^3*b + 5*I*B*a^2*b^2 - 3*I*A*a*b^3))*sqrt(b)*
weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1
/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + (sqrt(2)*(4*I*B*a^3*b
 - 2*I*A*a^2*b^2 - 5*I*B*a*b^3 + 3*I*A*b^4)*cos(d*x + c) + sqrt(2)*(4*I*B*a
^4 - 2*I*A*a^3*b - 5*I*B*a^2*b^2 + 3*I*A*a*b^3))*sqrt(b)*weierstrassPInvers
e(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x +
c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(sqrt(2)*(2*I*B*a^2*b^2 - I*A*a*b^3 -
 I*B*b^4)*cos(d*x + c) + sqrt(2)*(2*I*B*a^3*b - I*A*a^2*b^2 - I*B*a*b^3))*s
qrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3
, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(sqrt(2)*(-2*I*B
*a^2*b^2 + I*A*a*b^3 + I*B*b^4)*cos(d*x + c) + sqrt(2)*(-2*I*B*a^3*b + I*A*
a^2*b^2 + I*B*a*b^3))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b
)))/((a^2*b^4 - b^6)*d*cos(d*x + c) + (a^3*b^3 - a*b^5)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)}{{\left(a+b\,\cos(c+d\,x)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2), x)
```

3.329 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=185

$$\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-}{(a^2-b^2)\,d}$$

```
[Out] -2*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*(A*b-B*a)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/(a^2-b^2)/d/((a+b*cos(d*x+c
))/(a+b))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/
2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b
)])/(b*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*B*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a
+ b*Cos[c + d*x]]) - (2*(A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b
*Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-aA+bB)-\frac{1}{2}(Ab-aB)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} + \frac{(Ab-a}{}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{\left((Ab-aB)\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{}}{b\,(a^2-b^2)\sqrt{\frac{a+b\cos(}{a+}}}$$

$$= \frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{b\,(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{bd\sqrt{a+b\cos}}$$

Mathematica [A]
time = 0.60, size = 151, normalized size = 0.82

$$\frac{2\left(-\left((a+b)(-Ab+aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)\right)+(a^2-b^2)\,B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)+b(-Ab+aB)\sin(c+dx)\right)}{(a-b)b(a+b)d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(-((a + b)*(-(A*b) + a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)]) + (a^2 - b^2)*B*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(-(A*b) + a*B)*Sin[c + d*x]))
/((a - b)*b*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A]
time = 0.45, size = 432, normalized size = 2.34

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\,\mathrm{El}}{b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))-2*(A*b-B*a)/b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*
d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*b*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*b))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+
b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 610, normalized size = 3.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(6*(B*a*b^2 - A*b^3)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c) - (sqrt(2)*(
2*I*B*a^2*b + I*A*a*b^2 - 3*I*B*b^3)*cos(d*x + c) + sqrt(2)*(2*I*B*a^3 + I*
A*a^2*b - 3*I*B*a*b^2))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2
```

```
, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) +
 2*a)/b) - (sqrt(2)*(-2*I*B*a^2*b - I*A*a*b^2 + 3*I*B*b^3)*cos(d*x + c) + s
qrt(2)*(-2*I*B*a^3 - I*A*a^2*b + 3*I*B*a*b^2))*sqrt(b)*weierstrassPInverse(
4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
 - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*(sqrt(2)*(-I*B*a*b^2 + I*A*b^3)*cos(d*x
 + c) + sqrt(2)*(-I*B*a^2*b + I*A*a*b^2))*sqrt(b)*weierstrassZeta(4/3*(4*a^
2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*s
in(d*x + c) + 2*a)/b)) + 3*(sqrt(2)*(I*B*a*b^2 - I*A*b^3)*cos(d*x + c) + sq
rt(2)*(I*B*a^2*b - I*A*a*b^2))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/
b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b
^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c)
 + 2*a)/b)))/((a^2*b^3 - b^5)*d*cos(d*x + c) + (a^3*b^2 - a*b^4)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(3/2),x)
[Out] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(3/2), x)
```

3.330 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}+\frac{2b(A}{a\left(a^2-b\right.}$$

```
[Out] 2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*(A*b-B*a)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/(a^2-b^2)/d/((a+b*cos(d*
x+c))/(a+b))^(1/2)+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b
))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3079, 3138, 2734, 2732, 12, 2886, 2884}

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(a*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*A*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sq
rt[a + b*Cos[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*S
qrt[a + b*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\left(\frac{1}{2}A(a^2-b^2)-\frac{1}{2}a(Ab-aB)\cos(c+dx\right.}{\sqrt{a+b\cos}}}{a(a^2}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{2\int -\frac{Ab(a^2-b^2)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}\,dx}{ab(a^2-b^2)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{A\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a} -$$

$$= -\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{a(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b(Ab-}{a(a^2-b^2)\,c}$$

$$= -\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{a(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2A\sqrt{\frac{a+b\,c}{}}}{a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.09, size = 460, normalized size = 2.42

$$\frac{\cos(c+dx)(B+A\sec(c+dx))\left(-\frac{\frac{4a(-Ab+aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(2a^2A-3Ab^2+abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{2i(Ab-aB)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)\right)\right)}{ab\sqrt{-\frac{1}{a+b}}}}{(-a+b)(a+b)}+\frac{4b(Ab-aB)\sin(c+dx)}{(a^2-b^2)\sqrt{a+b\cos(c+dx)}}\right)}{2ad(A+B\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*(-(((4*a*(-(A*b) + a*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
+ d*x]] + (2*(2*a^2*A - 3*A*b^2 + a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*
I)*(A*b - a*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c
+ d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
```

```
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] +
 b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d
*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]))/((-a + b)*(a + b))) +
 (4*b*(A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]])))/(2
*a*d*(A + B*Cos[c + d*x]))
```

Maple [A]
time = 0.45, size = 433, normalized size = 2.28

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2(-Ab+aB)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(-A*b+B*
a)/a/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)+(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a-(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)-2*A/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)})\sec{(c + dx)}}{(a + b\cos{(c + dx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos{(c + d\,x)}}{\cos{(c + d\,x)}\,\left(a + b\,\cos{(c + d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)), x)
```

3.331 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=303

$$-\frac{(a^2A-3Ab^2+2abB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}-$$

```
[Out] b*(A*a^2-3*A*b^2+2*B*a*b)*sin(d*x+c)/a^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
-(A*a^2-3*A*b^2+2*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/
a^2/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+A*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))
*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)-(3*A*b-2*B*a)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c)
,2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d
*x+c))^(1/2)+A*tan(d*x+c)/a/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3079, 3135, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{b(a^2A+2abB-3Ab^2)\sin(c+dx)}{a^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{(a^2A+2abB-3Ab^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(3Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}+\frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] -(((a^2*A - 3*A*b^2 + 2*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]))
 + (A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) - ((3*A*b - 2*a*B)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b
*Cos[c + d*x]]) + (b*(a^2*A - 3*A*b^2 + 2*a*b*B)*Sin[c + d*x])/(a^2*(a^2 -
b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*Sqrt[a + b*Cos[c +
 d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{2}(-3Ab+2aB)+\frac{1}{2}Ab\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{a}$$

$$= \frac{b(a^2A-3Ab^2+2abB)\sin(c+dx)}{a^2(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{2 \ldots}{\ldots}$$

$$= \frac{b(a^2A-3Ab^2+2abB)\sin(c+dx)}{a^2(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} - \frac{2 \ldots}{\ldots}$$

$$= \frac{b(a^2A-3Ab^2+2abB)\sin(c+dx)}{a^2(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{A \ldots}{\ldots}$$

$$= -\frac{(a^2A-3Ab^2+2abB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{a^2(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \ldots$$

$$= -\frac{(a^2A-3Ab^2+2abB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{a^2(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \ldots$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.84, size = 482, normalized size = 1.59

$$\frac{\frac{-\frac{8ab(-Ab+aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(-7a^2Ab+9Ab^3+4a^3B-6ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i(a^2A-3Ab^2+2abB)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{-\frac{b(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\left(2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)-b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)\right)\right)}{ab\sqrt{-\frac{1}{a+b}}}}{(a-b)(a+b)}+\frac{4(aA(a^2-b^2)+b(a^2A-3Ab^2+2abB)\cos(c+dx))\tan(c+dx)}{(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (((-8*a*b*(-(A*b) + a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(-7*a^2*A*b + 9*A*b^3
 + 4*a^3*B - 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(a^2*A - 3*A*b
^2 + 2*a*b*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c
 + d*x]))/(a - b))]*Csc[c + d*x]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I
```

```
*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] -
b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*
x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]))/((a - b)*(a + b)) + (4
*(a*A*(a^2 - b^2) + b*(a^2*A - 3*A*b^2 + 2*a*b*B)*Cos[c + d*x])*Tan[c + d*x
])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))/(4*a^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(374) = 748$.
time = 0.76, size = 912, normalized size = 3.01

method	result	size
default	Expression too large to display	912

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(A*b-B*a
)*b/a^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a-(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)-2*(-A*b+B
*a)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2/a*A*(-cos(1/2*d*x+1/2*c)/a*(
-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+
b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**(3/2),
x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,\cos(c + d\,x)}{\cos(c + d\,x)^2\,(a + b\,\cos(c + d\,x))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2)),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2)), x)
```

3.332 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=398

$$\frac{(7a^2Ab-15Ab^3-4a^3B+12ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4a^3(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(5Ab-4aB)\sqrt{\frac{a+b\cos(c+dx}{a+b}}}{4a^2d\sqrt{a+b\cos(c}}$$

```
[Out] -1/4*b*(7*A*a^2*b-15*A*b^3-4*B*a^3+12*B*a*b^2)*sin(d*x+c)/a^3/(a^2-b^2)/d/(
a+b*cos(d*x+c))^(1/2)+1/4*(7*A*a^2*b-15*A*b^3-4*B*a^3+12*B*a*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/((a+b*cos(d*x+c))/
(a+b))^(1/2)-1/4*(5*A*b-4*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))
/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*A*a^2+15*A*b^2-12*B*a*b)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*
c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*cos
(d*x+c))^(1/2)-1/4*(5*A*b-4*B*a)*tan(d*x+c)/a^2/d/(a+b*cos(d*x+c))^(1/2)+1/
2*A*sec(d*x+c)*tan(d*x+c)/a/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3079, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{(5Ab-4aB)\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{(5Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}+\frac{(4a^2A-12abB+15Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4a^3d\sqrt{a+b\cos(c+dx)}}-\frac{b(-4a^3B+7a^2Ab+12ab^2B-15Ab^3)\sin(c+dx)}{4a^3d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{(-4a^3B+7a^2Ab+12ab^2B-15Ab^3)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4a^3d(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) - ((5*A*b - 4*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((
4*a^2*A + 15*A*b^2 - 12*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticP
i[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^3*d*Sqrt[a + b*Cos[c + d*x]]) - (b*(
7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Sin[c + d*x])/(4*a^3*(a^2 - b^
2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b - 4*a*B)*Tan[c + d*x])/(4*a^2*d*Sq
rt[a + b*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sqrt[a + b*C
os[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
```

```
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{2}(-5Ab+4aB)+aA\cos(c+dx)+\frac{3}{2}Ab\cos^2(c+\right.}{(a+b\cos(c+dx))^{3/2}}}{2a}$$

$$= -\frac{(5Ab-4aB)\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{4}(\right.}{}}{}$$

$$= -\frac{b(7a^2Ab-15Ab^3-4a^3B+12ab^2B)\sin(c+dx)}{4a^3(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{(5Ab-4aB}{4a^2d\sqrt{a+}}$$

$$= -\frac{b(7a^2Ab-15Ab^3-4a^3B+12ab^2B)\sin(c+dx)}{4a^3(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{(5Ab-4aB}{4a^2d\sqrt{a+}}$$

$$= -\frac{b(7a^2Ab-15Ab^3-4a^3B+12ab^2B)\sin(c+dx)}{4a^3(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{(5Ab-4aB}{4a^2d\sqrt{a+}}$$

$$= \frac{(7a^2Ab-15Ab^3-4a^3B+12ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{4a^3(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{(7a^2Ab-15Ab^3-4a^3B+12ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{4a^3(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.86, size = 678, normalized size = 1.70

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] -1/16*((2*(4*a^3*A*b - 20*a*A*b^3 + 16*a^2*b^2*B)*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] +
 (2*(8*a^4*A + 29*a^2*A*b^2 - 45*A*b^4 - 28*a^3*b*B + 36*a*b^3*B)*Sqrt[(a +
 b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a
```

```
 + b*Cos[c + d*x]] - ((2*I)*(7*a^2*A*b^2 - 15*A*b^4 - 4*a^3*b*B + 12*a*b^3*
B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]
*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt
[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-
(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a
 + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(
a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqr
t[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(
2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(a^3*(
-a + b)*(a + b)*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]*(-7*A*b*Sin[c
 + d*x] + 4*a*B*Sin[c + d*x]))/(4*a^3) - (2*(-(A*b^4*Sin[c + d*x]) + a*b^3*
B*Sin[c + d*x]))/(a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (A*Sec[c + d*x]*T
an[c + d*x])/(2*a^2)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1567 vs. $2(455) = 910$.

time = 1.01, size = 1568, normalized size = 3.94

method	result	size
default	Expression too large to display	1568

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)
*b^2/a^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)+(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a-(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)-2*(A*b-B
*a)/a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A/a*(-1/2*cos(1/2*d*x+1/2*
c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*
b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
```

```
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))
^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*(-A*b+B*a)/a^2*
(-cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1
/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*s
in(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)})\sec^{3}{(c + dx)}}{(a + b\cos{(c + dx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x))**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2)), x)
```

3.333 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=550

$$-\frac{2\left(80a^5Ab-140a^3Ab^3+40aAb^5-128a^6B+212a^4b^2B-55a^2b^4B-9b^6B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\tfrac{1}{2}(c+\right.}{15b^5\left(a^2-b^2\right)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 2/3*a*(A*b-B*a)*cos(d*x+c)^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2
)+2/3*a*(5*A*a^2*b-9*A*b^3-8*B*a^3+12*B*a*b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/
(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/15*(40*A*a^4*b-65*A*a^2*b^3+5*A*b^5-
64*B*a^5+98*B*a^3*b^2-14*B*a*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^4/(a^
2-b^2)^2/d-2/15*(30*A*a^3*b-50*A*a*b^3-48*B*a^4+71*B*a^2*b^2-3*B*b^4)*cos(d
*x+c)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d-2/15*(80*A*a^5*b-
140*A*a^3*b^3+40*A*a*b^5-128*B*a^6+212*B*a^4*b^2-55*B*a^2*b^4-9*B*b^6)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^5/(a^2-b^2)^2/d/((a+b*cos(d
*x+c))/(a+b))^(1/2)+2/15*(80*A*a^4*b-80*A*a^2*b^3-5*A*b^5-128*B*a^5+116*B*a
^3*b^2+17*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2
)/b^5/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.78, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3068, 3126, 3128, 3102, 2831, 2742, 2740, 2734, 2732}

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(-8a^3B+5a^2Ab+12ab^2B-9Ab^3\right)\sin(c+dx)\cos^2(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-48a^4B+30a^3Ab+71a^2b^2B-50aAb^3-3b^4B\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{15b^3d\left(a^2-b^2\right)^2}+\frac{2\left(-64a^5B+40a^4Ab+98a^3b^2B-65a^2Ab^3-14ab^4B+5Ab^5\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{15b^4d\left(a^2-b^2\right)^2}+\frac{2\left(-128a^5B+80a^4Ab+116a^3b^2B-80a^2Ab^3+17ab^4B-5Ab^5\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^5d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-128a^6B+80a^5Ab+212a^4b^2B-140a^3Ab^3-55a^2b^4B+40aAb^5-9b^6B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^5d\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(80*a^5*A*b - 140*a^3*A*b^3 + 40*a*A*b^5 - 128*a^6*B + 212*a^4*b^2*B -
55*a^2*b^4*B - 9*b^6*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)])/(15*b^5*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
(2*(80*a^4*A*b - 80*a^2*A*b^3 - 5*A*b^5 - 128*a^5*B + 116*a^3*b^2*B + 17*a*
b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])/(15*b^5*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Co
s[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) +
 (2*a*(5*a^2*A*b - 9*A*b^3 - 8*a^3*B + 12*a*b^2*B)*Cos[c + d*x]^2*Sin[c + d
*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(40*a^4*A*b - 65
*a^2*A*b^3 + 5*A*b^5 - 64*a^5*B + 98*a^3*b^2*B - 14*a*b^4*B)*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(15*b^4*(a^2 - b^2)^2*d) - (2*(30*a^3*A*b - 50*a*A
```

```
*b^3 - 48*a^4*B + 71*a^2*b^2*B - 3*b^4*B)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(15*b^3*(a^2 - b^2)^2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos^2(c+dx)\left(-3a(Ab-aB)+\frac{3}{2}\right.}{}}{}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a(5a^2Ab-9Ab^3-8a^3B}{3b^2\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a(5a^2Ab-9Ab^3-8a^3B}{3b^2\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a(5a^2Ab-9Ab^3-8a^3B}{3b^2\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a(5a^2Ab-9Ab^3-8a^3B}{3b^2\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a(5a^2Ab-9Ab^3-8a^3B}{3b^2\left(a^2-b^2\right)}$$

$$= -\frac{2(80a^5Ab-140a^3Ab^3+40aAb^5-128a^6B+212a^4b^2B-55a^2b^4}{15b^5\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos}{a}}}$$

Mathematica [A]
time = 4.02, size = 372, normalized size = 0.68

$$-\frac{\frac{2\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(b^2\left(20a^4Ab-35a^2Ab^3-5Ab^5-32a^5B+44a^3b^2B+8ab^4B\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-\left(-80a^5Ab+140a^3Ab^3-40aAb^5+128a^6B-212a^4b^2B+55a^2b^4B+9b^6B\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)}{(a-b)^2(a+b)}+b\left(\frac{10a^4(-Ab+aB)\sin(c+dx)}{a^2-b^2}-\frac{10a^3\left(-8a^2Ab+12Ab^3+11a^3B-15ab^2B\right)(a+b\cos(c+dx))\sin(c+dx)}{\left(a^2-b^2\right)^2}+2(5Ab-14aB)(a+b\cos(c+dx))^2\sin(c+dx)+3bB(a+b\cos(c+dx))^2\sin(2(c+dx))\right)}{15b^5d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] ((-2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(20*a^4*A*b - 35*a^2*A*b^3 -
 5*A*b^5 - 32*a^5*B + 44*a^3*b^2*B + 8*a*b^4*B)*EllipticF[(c + d*x)/2, (2*b
)/(a + b)] - (-80*a^5*A*b + 140*a^3*A*b^3 - 40*a*A*b^5 + 128*a^6*B - 212*a^
4*b^2*B + 55*a^2*b^4*B + 9*b^6*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b)) + b*
((10*a^4*(-(A*b) + a*B)*Sin[c + d*x])/(a^2 - b^2) - (10*a^3*(-8*a^2*A*b + 1
```

```
2*A*b^3 + 11*a^3*B - 15*a*b^2*B)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(a^2 -
b^2)^2 + 2*(5*A*b - 14*a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x] + 3*b*B*(a
+ b*Cos[c + d*x])^2*Sin[2*(c + d*x)]))/(15*b^5*d*(a + b*Cos[c + d*x])^(3/2)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1749 vs. $2(580) = 1160$.
time = 1.74, size = 1750, normalized size = 3.18

method	result	size
default	Expression too large to display	1750

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16/b^2*B*(-
1/10/b*cos(1/2*d*x+1/2*c)^3*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+8
/b^3*(A*b-2*B*a-3*B*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*
b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/6/b*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))-1/12/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2/b^5*(2*A*a*b+2*A*b^2-3*B*a^2-
4*B*a*b-3*B*b^2)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+2*(3*A*a^2*b+2*A*a*b^2+A*b^3-4*B*a^3-3*B*a
^2*b-2*B*a*b^2-B*b^3)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*
c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*a^4*(A*b-B*a)
/b^5*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*sin(1
/2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/
2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*
```

```
b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2*a^3
/b^5*(4*A*b-5*B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2
-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a-(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b
))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.68, size = 1538, normalized size = 2.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/45*(6*(64*B*a^7*b^2 - 40*A*a^6*b^3 - 98*B*a^5*b^4 + 65*A*a^4*b^5 + 14*B*
a^3*b^6 - 5*A*a^2*b^7 - 3*(B*a^4*b^5 - 2*B*a^2*b^7 + B*b^9)*cos(d*x + c)^3
+ (8*B*a^5*b^4 - 5*A*a^4*b^5 - 16*B*a^3*b^6 + 10*A*a^2*b^7 + 8*B*a*b^8 - 5*
A*b^9)*cos(d*x + c)^2 + 5*(16*B*a^6*b^3 - 10*A*a^5*b^4 - 25*B*a^4*b^5 + 16*
A*a^3*b^6 + 5*B*a^2*b^7 - 2*A*a*b^8)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)
*sin(d*x + c) - (sqrt(2)*(256*I*B*a^7*b^2 - 160*I*A*a^6*b^3 - 520*I*B*a^5*b
^4 + 340*I*A*a^4*b^5 + 242*I*B*a^3*b^6 - 185*I*A*a^2*b^7 + 42*I*B*a*b^8 - 1
5*I*A*b^9)*cos(d*x + c)^2 - 2*sqrt(2)*(-256*I*B*a^8*b + 160*I*A*a^7*b^2 + 5
20*I*B*a^6*b^3 - 340*I*A*a^5*b^4 - 242*I*B*a^4*b^5 + 185*I*A*a^3*b^6 - 42*I
*B*a^2*b^7 + 15*I*A*a*b^8)*cos(d*x + c) + sqrt(2)*(256*I*B*a^9 - 160*I*A*a^
8*b - 520*I*B*a^7*b^2 + 340*I*A*a^6*b^3 + 242*I*B*a^5*b^4 - 185*I*A*a^4*b^5
```

```
 + 42*I*B*a^3*b^6 - 15*I*A*a^2*b^7))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*s
in(d*x + c) + 2*a)/b) - (sqrt(2)*(-256*I*B*a^7*b^2 + 160*I*A*a^6*b^3 + 520*
I*B*a^5*b^4 - 340*I*A*a^4*b^5 - 242*I*B*a^3*b^6 + 185*I*A*a^2*b^7 - 42*I*B*
a*b^8 + 15*I*A*b^9)*cos(d*x + c)^2 - 2*sqrt(2)*(256*I*B*a^8*b - 160*I*A*a^7
*b^2 - 520*I*B*a^6*b^3 + 340*I*A*a^5*b^4 + 242*I*B*a^4*b^5 - 185*I*A*a^3*b^
6 + 42*I*B*a^2*b^7 - 15*I*A*a*b^8)*cos(d*x + c) + sqrt(2)*(-256*I*B*a^9 + 1
60*I*A*a^8*b + 520*I*B*a^7*b^2 - 340*I*A*a^6*b^3 - 242*I*B*a^5*b^4 + 185*I*
A*a^4*b^5 - 42*I*B*a^3*b^6 + 15*I*A*a^2*b^7))*sqrt(b)*weierstrassPInverse(4
/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
 - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*(sqrt(2)*(-128*I*B*a^6*b^3 + 80*I*A*a^5*
b^4 + 212*I*B*a^4*b^5 - 140*I*A*a^3*b^6 - 55*I*B*a^2*b^7 + 40*I*A*a*b^8 - 9
*I*B*b^9)*cos(d*x + c)^2 + 2*sqrt(2)*(-128*I*B*a^7*b^2 + 80*I*A*a^6*b^3 + 2
12*I*B*a^5*b^4 - 140*I*A*a^4*b^5 - 55*I*B*a^3*b^6 + 40*I*A*a^2*b^7 - 9*I*B*
a*b^8)*cos(d*x + c) + sqrt(2)*(-128*I*B*a^8*b + 80*I*A*a^7*b^2 + 212*I*B*a^
6*b^3 - 140*I*A*a^5*b^4 - 55*I*B*a^4*b^5 + 40*I*A*a^3*b^6 - 9*I*B*a^2*b^7))
*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b
^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^
3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*(sqrt(2)*(128*
I*B*a^6*b^3 - 80*I*A*a^5*b^4 - 212*I*B*a^4*b^5 + 140*I*A*a^3*b^6 + 55*I*B*a
^2*b^7 - 40*I*A*a*b^8 + 9*I*B*b^9)*cos(d*x + c)^2 + 2*sqrt(2)*(128*I*B*a^7*
b^2 - 80*I*A*a^6*b^3 - 212*I*B*a^5*b^4 + 140*I*A*a^4*b^5 + 55*I*B*a^3*b^6 -
 40*I*A*a^2*b^7 + 9*I*B*a*b^8)*cos(d*x + c) + sqrt(2)*(128*I*B*a^8*b - 80*I
*A*a^7*b^2 - 212*I*B*a^6*b^3 + 140*I*A*a^5*b^4 + 55*I*B*a^4*b^5 - 40*I*A*a^
3*b^6 + 9*I*B*a^2*b^7))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)
/b)))/((a^4*b^8 - 2*a^2*b^10 + b^12)*d*cos(d*x + c)^2 + 2*(a^5*b^7 - 2*a^3*
b^9 + a*b^11)*d*cos(d*x + c) + (a^6*b^6 - 2*a^4*b^8 + a^2*b^10)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,(A+B\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x)
```

3.334 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=413

$$\frac{2\left(8a^4Ab-15a^2Ab^3+3Ab^5-16a^5B+28a^3b^2B-8ab^4B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^4\left(a^2-b^2\right)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(8a^3Ab}{}$$

```
[Out] 2/3*a*(A*b-B*a)*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2
)-2/3*a^2*(3*A*a^2*b-7*A*b^3-6*B*a^3+10*B*a*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2
/d/(a+b*cos(d*x+c))^(1/2)-2/3*(A*a*b-2*B*a^2+B*b^2)*sin(d*x+c)*(a+b*cos(d*x
+c))^(1/2)/b^3/(a^2-b^2)/d+2/3*(8*A*a^4*b-15*A*a^2*b^3+3*A*b^5-16*B*a^5+28*
B*a^3*b^2-8*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^4/
(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(8*A*a^3*b-9*A*a*b^3-16*B*
a^4+16*B*a^2*b^2+B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))
^(1/2)/b^4/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3068, 3110, 3102, 2831, 2742, 2740, 2734, 2732}

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-2a^2B+aAb+b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3b^3d\left(a^2-b^2\right)}-\frac{2a^2\left(-6a^3B+3a^2Ab+10ab^2B-7Ab^3\right)\sin(c+dx)}{3b^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-16a^4B+8a^3Ab+16a^2b^2B-9aAb^3+b^4B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^4d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-16a^5B+8a^4Ab+28a^3b^2B-15a^2Ab^3-8ab^4B+3Ab^5\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^4d\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(8*a^4*A*b - 15*a^2*A*b^3 + 3*A*b^5 - 16*a^5*B + 28*a^3*b^2*B - 8*a*b^4*
B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(
a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(8*a^3*A*b - 9*a*A*
b^3 - 16*a^4*B + 16*a^2*b^2*B + b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]]) + (2*a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*
d*(a + b*Cos[c + d*x])^(3/2)) - (2*a^2*(3*a^2*A*b - 7*A*b^3 - 6*a^3*B + 10*
a*b^2*B)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (
2*(a*A*b - 2*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(
a^2 - b^2)*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos(c+dx)(-2a(Ab-aB)+\cdots}{\cdots}}{\cdots}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2(3a^2Ab-7Ab^3-6a\cdots}{3b^3\left(a^2-b^2\right)^2d\sqrt{\cdots}}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2(3a^2Ab-7Ab^3-6a\cdots}{3b^3\left(a^2-b^2\right)^2d\sqrt{\cdots}}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2(3a^2Ab-7Ab^3-6a\cdots}{3b^3\left(a^2-b^2\right)^2d\sqrt{\cdots}}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2(3a^2Ab-7Ab^3-6a\cdots}{3b^3\left(a^2-b^2\right)^2d\sqrt{\cdots}}$$

$$= \frac{2(8a^4Ab-15a^2Ab^3+3Ab^5-16a^5B+28a^3b^2B-8ab^4B)\sqrt{a+\cdots}}{3b^4\left(a^2-b^2\right)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\cdots}$$

Mathematica [A]
time = 3.04, size = 334, normalized size = 0.81

$$\frac{2\left(\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(b^2(2a^3Ab-6aAb^3-4a^4B+7a^2b^2B+b^4B)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-(-8a^4Ab+15a^2Ab^3-3Ab^5+16a^5B-28a^3b^2B+8ab^4B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)}{(a-b)^2(a+b)}+\frac{b\left(-8a^5Ab+16a^3Ab^3+16a^6B-25a^4b^2B+b^6B+2ab(-5a^3Ab+9aAb^3+10a^4B-16a^2b^2B+2b^4B)\cos(c+dx)+(-a^2b+b^3)^2B\cos(2(c+dx))\right)\sin(c+dx)}{2(a^2-b^2)^2}\right)}{3b^4d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(2*a^3*A*b - 6*a*A*b^3 - 4*a
^4*B + 7*a^2*b^2*B + b^4*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (-8*a^4
*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*((a +
b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])))/((a - b)^2*(a + b)) + (b*(-8*a^5*A*b + 16*a^3*A*b^3 + 16*a^6*B -
 25*a^4*b^2*B + b^6*B + 2*a*b*(-5*a^3*A*b + 9*a*A*b^3 + 10*a^4*B - 16*a^2*b
^2*B + 2*b^4*B)*Cos[c + d*x] + (-(a^2*b) + b^3)^2*B*Cos[2*(c + d*x)])*Sin[c
 + d*x])/(2*(a^2 - b^2)^2)))/(3*b^4*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1411 vs. $2(447) = 894$.
time = 1.42, size = 1412, normalized size = 3.42

method	result	size
default	Expression too large to display	1412

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^4/(-
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^2+9*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a*b+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+2*
B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b+2*B*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2*b^2-17*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))-B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+8*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-8*B*(sin(1/2*d*x+1
```

```
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b)-2*a^3*(A*b-B*a)/b^4*(1/6/b/(a
-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*sin(1/2*d*x+1/2*c)^
2*b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-2*a^2/b^4*(3*A*b-4*
B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)+(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a-(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b))/sin(1/2*d*x
+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 1348, normalized size = 3.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/9*(6*(8*B*a^6*b^2 - 4*A*a^5*b^3 - 13*B*a^4*b^4 + 8*A*a^3*b^5 + B*a^2*b^6
+ (B*a^4*b^4 - 2*B*a^2*b^6 + B*b^8)*cos(d*x + c)^2 + (10*B*a^5*b^3 - 5*A*a^
4*b^4 - 16*B*a^3*b^5 + 9*A*a^2*b^6 + 2*B*a*b^7)*cos(d*x + c))*sqrt(b*cos(d*
x + c) + a)*sin(d*x + c) + (sqrt(2)*(-32*I*B*a^6*b^2 + 16*I*A*a^5*b^3 + 68*
I*B*a^4*b^4 - 36*I*A*a^3*b^5 - 37*I*B*a^2*b^6 + 24*I*A*a*b^7 - 3*I*B*b^8)*c
```

```
os(d*x + c)^2 - 2*sqrt(2)*(32*I*B*a^7*b - 16*I*A*a^6*b^2 - 68*I*B*a^5*b^3 +
 36*I*A*a^4*b^4 + 37*I*B*a^3*b^5 - 24*I*A*a^2*b^6 + 3*I*B*a*b^7)*cos(d*x +
c) + sqrt(2)*(-32*I*B*a^8 + 16*I*A*a^7*b + 68*I*B*a^6*b^2 - 36*I*A*a^5*b^3
- 37*I*B*a^4*b^4 + 24*I*A*a^3*b^5 - 3*I*B*a^2*b^6))*sqrt(b)*weierstrassPInv
erse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x
 + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + (sqrt(2)*(32*I*B*a^6*b^2 - 16*I*A*a^
5*b^3 - 68*I*B*a^4*b^4 + 36*I*A*a^3*b^5 + 37*I*B*a^2*b^6 - 24*I*A*a*b^7 + 3
*I*B*b^8)*cos(d*x + c)^2 - 2*sqrt(2)*(-32*I*B*a^7*b + 16*I*A*a^6*b^2 + 68*I
*B*a^5*b^3 - 36*I*A*a^4*b^4 - 37*I*B*a^3*b^5 + 24*I*A*a^2*b^6 - 3*I*B*a*b^7
)*cos(d*x + c) + sqrt(2)*(32*I*B*a^8 - 16*I*A*a^7*b - 68*I*B*a^6*b^2 + 36*I
*A*a^5*b^3 + 37*I*B*a^4*b^4 - 24*I*A*a^3*b^5 + 3*I*B*a^2*b^6))*sqrt(b)*weie
rstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(
3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(sqrt(2)*(16*I*B*a^5*b^
3 - 8*I*A*a^4*b^4 - 28*I*B*a^3*b^5 + 15*I*A*a^2*b^6 + 8*I*B*a*b^7 - 3*I*A*b
^8)*cos(d*x + c)^2 + 2*sqrt(2)*(16*I*B*a^6*b^2 - 8*I*A*a^5*b^3 - 28*I*B*a^4
*b^4 + 15*I*A*a^3*b^5 + 8*I*B*a^2*b^6 - 3*I*A*a*b^7)*cos(d*x + c) + sqrt(2)
*(16*I*B*a^7*b - 8*I*A*a^6*b^2 - 28*I*B*a^5*b^3 + 15*I*A*a^4*b^4 + 8*I*B*a^
3*b^5 - 3*I*A*a^2*b^6))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)
/b)) - 3*(sqrt(2)*(-16*I*B*a^5*b^3 + 8*I*A*a^4*b^4 + 28*I*B*a^3*b^5 - 15*I*
A*a^2*b^6 - 8*I*B*a*b^7 + 3*I*A*b^8)*cos(d*x + c)^2 + 2*sqrt(2)*(-16*I*B*a^
6*b^2 + 8*I*A*a^5*b^3 + 28*I*B*a^4*b^4 - 15*I*A*a^3*b^5 - 8*I*B*a^2*b^6 + 3
*I*A*a*b^7)*cos(d*x + c) + sqrt(2)*(-16*I*B*a^7*b + 8*I*A*a^6*b^2 + 28*I*B*
a^5*b^3 - 15*I*A*a^4*b^4 - 8*I*B*a^3*b^5 + 3*I*A*a^2*b^6))*sqrt(b)*weierstr
assZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPI
nverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^4*b^7 - 2*a^2*b^9 + b^11)*d*co
s(d*x + c)^2 + 2*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^6*b^5 -
 2*a^4*b^7 + a^2*b^9)*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x)
```

3.335 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{2(2a^3Ab-6aAb^3-8a^4B+15a^2b^2B-3b^4B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3(a^2-b^2)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(2a^2Ab-3Ab^3-8}{3}$$

```
[Out] -2/3*a^2*(A*b-B*a)*sin(d*x+c)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*a*
(2*A*a^2*b-6*A*b^3-5*B*a^3+9*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos
(d*x+c))^(1/2)-2/3*(2*A*a^3*b-6*A*a*b^3-8*B*a^4+15*B*a^2*b^2-3*B*b^4)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d/((a+b*cos(d*
x+c))/(a+b))^(1/2)+2/3*(2*A*a^2*b-3*A*b^3-8*B*a^3+9*B*a*b^2)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(
a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/(a^2-b^2)/d/(a+b*cos(d*x+c)
)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3067, 3100, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}+\frac{2a(-5a^3B+2a^2Ab+9ab^2B-6Ab^3)\sin(c+dx)}{3b^2d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}}+\frac{2(-8a^3B+2a^2Ab+9ab^2B-3Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2(-8a^4B+2a^3Ab+15a^2b^2B-6aAb^3-3b^4B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d(a^2-b^2)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(2*a^3*A*b - 6*a*A*b^3 - 8*a^4*B + 15*a^2*b^2*B - 3*b^4*B)*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)^2*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2*A*b - 3*A*b^3 - 8*a^3*B + 9
*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*(A*b - a*B
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*a*(2*
a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2
*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
```

```
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3}{2}ab(Ab-aB)+\frac{1}{2}(2a^2-3b^2)(A}{(a}}{3}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{2a(2a^2Ab-6Ab^3-5a^3B}{3b^2(a^2-b^2)^2\,d\sqrt{a}}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{2a(2a^2Ab-6Ab^3-5a^3B}{3b^2(a^2-b^2)^2\,d\sqrt{a}}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{2a(2a^2Ab-6Ab^3-5a^3B}{3b^2(a^2-b^2)^2\,d\sqrt{a}}$$

$$= -\frac{2(2a^3Ab-6aAb^3-8a^4B+15a^2b^2B-3b^4B)\sqrt{a+b\cos(c+dx}}{3b^3(a^2-b^2)^2\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A]
time = 2.43, size = 274, normalized size = 0.83

$$\frac{2\left(\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(b^2(a^2Ab+3Ab^3+2a^3B-6ab^2B)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+(-2a^3Ab+6aAb^3+8a^4B-15a^2b^2B+3b^4B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)}{(a-b)^2(a+b)}-\frac{ab(a(-a^2Ab+5Ab^3+4a^3B-8ab^2B)+b(-2a^2Ab+6Ab^3+5a^3B-9ab^2B)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2}\right)}{3b^3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(a^2*A*b + 3*A*b^3 + 2*a^3*B
 - 6*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-2*a^3*A*b + 6*a*A*b
^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b
)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b))
 - (a*b*(a*(-(a^2*A*b) + 5*A*b^3 + 4*a^3*B - 8*a*b^2*B) + b*(-2*a^2*A*b + 6
*A*b^3 + 5*a^3*B - 9*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(
3*b^3*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(369) = 738$.

time = 1.23, size = 954, normalized size = 2.88

method	result	size
default	Expression too large to display	954

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*b*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3*B*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-B
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)+2*a^2*(A*b-B*a)/b^3*(1
/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*sin(1/2*d*x+
1/2*c)^2*b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*
b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2*a/b^3*(2*A
*b-3*B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a-(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b))/sin(1/
2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 1193, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/9*(6*(4*B*a^5*b^2 - A*a^4*b^3 - 8*B*a^3*b^4 + 5*A*a^2*b^5 + (5*B*a^4*b^3
 - 2*A*a^3*b^4 - 9*B*a^2*b^5 + 6*A*a*b^6)*cos(d*x + c))*sqrt(b*cos(d*x + c)
 + a)*sin(d*x + c) - (sqrt(2)*(16*I*B*a^5*b^2 - 4*I*A*a^4*b^3 - 36*I*B*a^3*
b^4 + 9*I*A*a^2*b^5 + 24*I*B*a*b^6 - 9*I*A*b^7)*cos(d*x + c)^2 - 2*sqrt(2)*
(-16*I*B*a^6*b + 4*I*A*a^5*b^2 + 36*I*B*a^4*b^3 - 9*I*A*a^3*b^4 - 24*I*B*a^
2*b^5 + 9*I*A*a*b^6)*cos(d*x + c) + sqrt(2)*(16*I*B*a^7 - 4*I*A*a^6*b - 36*
I*B*a^5*b^2 + 9*I*A*a^4*b^3 + 24*I*B*a^3*b^4 - 9*I*A*a^2*b^5))*sqrt(b)*weie
rstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(
3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) - (sqrt(2)*(-16*I*B*a^5*b^2
 + 4*I*A*a^4*b^3 + 36*I*B*a^3*b^4 - 9*I*A*a^2*b^5 - 24*I*B*a*b^6 + 9*I*A*b^
7)*cos(d*x + c)^2 - 2*sqrt(2)*(16*I*B*a^6*b - 4*I*A*a^5*b^2 - 36*I*B*a^4*b^
3 + 9*I*A*a^3*b^4 + 24*I*B*a^2*b^5 - 9*I*A*a*b^6)*cos(d*x + c) + sqrt(2)*(-
16*I*B*a^7 + 4*I*A*a^6*b + 36*I*B*a^5*b^2 - 9*I*A*a^4*b^3 - 24*I*B*a^3*b^4
+ 9*I*A*a^2*b^5))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/
b) + 3*(sqrt(2)*(-8*I*B*a^4*b^3 + 2*I*A*a^3*b^4 + 15*I*B*a^2*b^5 - 6*I*A*a*
b^6 - 3*I*B*b^7)*cos(d*x + c)^2 + 2*sqrt(2)*(-8*I*B*a^5*b^2 + 2*I*A*a^4*b^3
 + 15*I*B*a^3*b^4 - 6*I*A*a^2*b^5 - 3*I*B*a*b^6)*cos(d*x + c) + sqrt(2)*(-8
*I*B*a^6*b + 2*I*A*a^5*b^2 + 15*I*B*a^4*b^3 - 6*I*A*a^3*b^4 - 3*I*B*a^2*b^5
))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)
/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/
b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*(sqrt(2)*(8*
I*B*a^4*b^3 - 2*I*A*a^3*b^4 - 15*I*B*a^2*b^5 + 6*I*A*a*b^6 + 3*I*B*b^7)*cos
(d*x + c)^2 + 2*sqrt(2)*(8*I*B*a^5*b^2 - 2*I*A*a^4*b^3 - 15*I*B*a^3*b^4 + 6
*I*A*a^2*b^5 + 3*I*B*a*b^6)*cos(d*x + c) + sqrt(2)*(8*I*B*a^6*b - 2*I*A*a^5
*b^2 - 15*I*B*a^4*b^3 + 6*I*A*a^3*b^4 + 3*I*B*a^2*b^5))*sqrt(b)*weierstrass
Zeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInve
rse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x
+ c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^4*b^6 - 2*a^2*b^8 + b^10)*d*cos(d
*x + c)^2 + 2*(a^5*b^5 - 2*a^3*b^7 + a*b^9)*d*cos(d*x + c) + (a^6*b^4 - 2*a
^4*b^6 + a^2*b^8)*d)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x)
```

3.336 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=307

$$-\frac{2(a^2Ab+3Ab^3+2a^3B-6ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2(a^2-b^2)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(aAb+2a^2B-3b^2B)\sqrt{\frac{a+b\cos}{}}}{3b^2(a^2-b^2)d\sqrt{a}}$$

```
[Out] 2/3*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*(A*a^2*
b+3*A*b^3+2*B*a^3-6*B*a*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1
/2)-2/3*(A*a^2*b+3*A*b^3+2*B*a^3-6*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b
*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(A*
a*b+2*B*a^2-3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1
/2)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3047, 3100, 2833, 2831, 2742, 2740, 2734, 2732}

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}}+\frac{2(2a^2B+aAb-3b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{2(2a^3B+a^2Ab-6ab^2B+3Ab^3)\sin(c+dx)}{3bd(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(2a^3B+a^2Ab-6ab^2B+3Ab^3)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d(a^2-b^2)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]) + (2*(a*A*b + 2*a^2*B - 3*b^2*B)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqr
t[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)) + (2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*
Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
```

```
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3}{2}b(Ab-aB)-\frac{1}{2}\left(aAb+2a^2B-3b^2B\right.}{(a+b\cos(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2(a^2Ab+3Ab^3+2a^3B-6ab}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{co}}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2(a^2Ab+3Ab^3+2a^3B-6ab}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{co}}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2(a^2Ab+3Ab^3+2a^3B-6ab}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{co}}}$$

$$= -\frac{2(a^2Ab+3Ab^3+2a^3B-6ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A]
time = 2.11, size = 224, normalized size = 0.73

$$\frac{2\left(-\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left((a^2Ab+3Ab^3+2a^3B-6ab^2B)E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-(a-b)(aAb+2a^2B-3b^2B)F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)\right)}{(a-b)^2}+\frac{b\left(a(2a^2Ab+2Ab^3+a^3B-5ab^2B)+b(a^2Ab+3Ab^3+2a^3B-6ab^2B)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2}\right)}{3b^2d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((a^2*A*b + 3*A*b^3 + 2*a^3*B -
 6*a*b^2*B)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - (a - b)*(a*A*b + 2*a^2*
B - 3*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2) + (b*(a*(2*
a^2*A*b + 2*A*b^3 + a^3*B - 5*a*b^2*B) + b*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6
*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^2*d*(a + b*Cos[c
 + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 863 vs. $2(345) = 690$.

time = 1.08, size = 864, normalized size = 2.81

method	result	size
default	Expression too large to display	864

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*a*(A*b-B*a)/b^2*(1/6/b/(a-b)/(a+b)*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*sin(1/2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)
^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-2/b^2*(A*b-2*B*a)/sin(1/2*d*x+1/2*c)
^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 1076, normalized size = 3.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/9*(6*(B*a^4*b^2 + 2*A*a^3*b^3 - 5*B*a^2*b^4 + 2*A*a*b^5 + (2*B*a^3*b^3 +
A*a^2*b^4 - 6*B*a*b^5 + 3*A*b^6)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin
(d*x + c) + (sqrt(2)*(-4*I*B*a^4*b^2 - 2*I*A*a^3*b^3 + 9*I*B*a^2*b^4 + 6*I*
A*a*b^5 - 9*I*B*b^6)*cos(d*x + c)^2 - 2*sqrt(2)*(4*I*B*a^5*b + 2*I*A*a^4*b^
2 - 9*I*B*a^3*b^3 - 6*I*A*a^2*b^4 + 9*I*B*a*b^5)*cos(d*x + c) + sqrt(2)*(-4
*I*B*a^6 - 2*I*A*a^5*b + 9*I*B*a^4*b^2 + 6*I*A*a^3*b^3 - 9*I*B*a^2*b^4))*sq
rt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/
b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + (sqrt(2)*(4*I*B
*a^4*b^2 + 2*I*A*a^3*b^3 - 9*I*B*a^2*b^4 - 6*I*A*a*b^5 + 9*I*B*b^6)*cos(d*x
 + c)^2 - 2*sqrt(2)*(-4*I*B*a^5*b - 2*I*A*a^4*b^2 + 9*I*B*a^3*b^3 + 6*I*A*a
^2*b^4 - 9*I*B*a*b^5)*cos(d*x + c) + sqrt(2)*(4*I*B*a^6 + 2*I*A*a^5*b - 9*I
*B*a^4*b^2 - 6*I*A*a^3*b^3 + 9*I*B*a^2*b^4))*sqrt(b)*weierstrassPInverse(4/
3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) -
 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(sqrt(2)*(2*I*B*a^3*b^3 + I*A*a^2*b^4 - 6
*I*B*a*b^5 + 3*I*A*b^6)*cos(d*x + c)^2 + 2*sqrt(2)*(2*I*B*a^4*b^2 + I*A*a^3
*b^3 - 6*I*B*a^2*b^4 + 3*I*A*a*b^5)*cos(d*x + c) + sqrt(2)*(2*I*B*a^5*b + I
*A*a^4*b^2 - 6*I*B*a^3*b^3 + 3*I*A*a^2*b^4))*sqrt(b)*weierstrassZeta(4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*
b*sin(d*x + c) + 2*a)/b)) - 3*(sqrt(2)*(-2*I*B*a^3*b^3 - I*A*a^2*b^4 + 6*I*
B*a*b^5 - 3*I*A*b^6)*cos(d*x + c)^2 + 2*sqrt(2)*(-2*I*B*a^4*b^2 - I*A*a^3*b
^3 + 6*I*B*a^2*b^4 - 3*I*A*a*b^5)*cos(d*x + c) + sqrt(2)*(-2*I*B*a^5*b - I*
A*a^4*b^2 + 6*I*B*a^3*b^3 - 3*I*A*a^2*b^4))*sqrt(b)*weierstrassZeta(4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b
*sin(d*x + c) + 2*a)/b)))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2 + 2
*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^6*b^3 - 2*a^4*b^5 + a^2*
b^7)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x)
```

3.337 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=275

$$\frac{2(4aAb-a^2B-3b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b(a^2-b^2)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx\right.}{3b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -2/3*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-2/3*(4*A*a*b-B
*a^2-3*B*b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(4*A*a*b-
B*a^2-3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/(a^2-b^2
)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2
))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(a^2(-B)+4aAb-3b^2B)\sin(c+dx)}{3d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}-\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{2(a^2(-B)+4aAb-3b^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd(a^2-b^2)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(4*a*A*b - a^2*B - 3*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(3*b*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
) - (2*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b -
a*B)*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*(4*a*A
*b - a^2*B - 3*b^2*B)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d
*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(aA-bB)+\frac{1}{2}(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \cdots$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \cdots$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \cdots$$

$$= \frac{2\left(4aAb-a^2B-3b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(Ab-aB)\cdots}{3b(\cdots}$$

Mathematica [A]
time = 1.74, size = 193, normalized size = 0.70

$$\frac{2\left(-\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(-4aAb+a^2B+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-(a-b)(-Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)\right)}{(a-b)^2b}+\frac{\left(-5a^2Ab+Ab^3+2a^3B+2ab^2B+b\left(-4aAb+a^2B+3b^2B\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2}\right)}{3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((-4*a*A*b + a^2*B + 3*b^2*B)*E
llipticE[(c + d*x)/2, (2*b)/(a + b)] - (a - b)*(-(A*b) + a*B)*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)]))/((a - b)^2*b)) + ((-5*a^2*A*b + A*b^3 + 2*a^3*B
+ 2*a*b^2*B + b*(-4*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(a
^2 - b^2)^2))/(3*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 753 vs. 2(313) = 626.
time = 0.99, size = 754, normalized size = 2.74

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2(Ab-aB)\left(\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b \cdots}}{6b(a-b)(a+b)\left(\cos^2\left(\frac{dx}{2} \cdots\right)\right)} \cdots\right)}{\cdots} \cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)
/b*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*sin(1/2
*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*
c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^
3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))-2*B/b/s
in(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*sin(1/2*d*
x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b))/sin(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 956, normalized size = 3.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(6*(2*B*a^3*b^2 - 5*A*a^2*b^3 + 2*B*a*b^4 + A*b^5 + (B*a^2*b^3 - 4*A*a*
b^4 + 3*B*b^5)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c) + (sqrt(
2)*(-2*I*B*a^3*b^2 - I*A*a^2*b^3 + 6*I*B*a*b^4 - 3*I*A*b^5)*cos(d*x + c)^2
- 2*sqrt(2)*(2*I*B*a^4*b + I*A*a^3*b^2 - 6*I*B*a^2*b^3 + 3*I*A*a*b^4)*cos(d
*x + c) + sqrt(2)*(-2*I*B*a^5 - I*A*a^4*b + 6*I*B*a^3*b^2 - 3*I*A*a^2*b^3))
*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^
2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + (sqrt(2)*(2*
I*B*a^3*b^2 + I*A*a^2*b^3 - 6*I*B*a*b^4 + 3*I*A*b^5)*cos(d*x + c)^2 - 2*sqr
t(2)*(-2*I*B*a^4*b - I*A*a^3*b^2 + 6*I*B*a^2*b^3 - 3*I*A*a*b^4)*cos(d*x + c
) + sqrt(2)*(2*I*B*a^5 + I*A*a^4*b - 6*I*B*a^3*b^2 + 3*I*A*a^2*b^3))*sqrt(b
)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(sqrt(2)*(I*B*a^2
*b^3 - 4*I*A*a*b^4 + 3*I*B*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(I*B*a^3*b^2 - 4
*I*A*a^2*b^3 + 3*I*B*a*b^4)*cos(d*x + c) + sqrt(2)*(I*B*a^4*b - 4*I*A*a^3*b
^2 + 3*I*B*a^2*b^3))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)
) - 3*(sqrt(2)*(-I*B*a^2*b^3 + 4*I*A*a*b^4 - 3*I*B*b^5)*cos(d*x + c)^2 + 2*
sqrt(2)*(-I*B*a^3*b^2 + 4*I*A*a^2*b^3 - 3*I*B*a*b^4)*cos(d*x + c) + sqrt(2)
*(-I*B*a^4*b + 4*I*A*a^3*b^2 - 3*I*B*a^2*b^3))*sqrt(b)*weierstrassZeta(4/3*
(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(
4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*
I*b*sin(d*x + c) + 2*a)/b)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c)^2
+ 2*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 2*a^4*b^4 + a
^2*b^6)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(5/2), x)
```

3.338 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=349

$$-\frac{2(7a^2Ab-3Ab^3-4a^3B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2(a^2-b^2)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{3a(a^2-b^2)d\sqrt{a+b\cos(c+dx}}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*b*(7*A*
a^2*b-3*A*b^3-4*B*a^3)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-
2/3*(7*A*a^2*b-3*A*b^3-4*B*a^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))
^(1/2)/a^2/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(A*b-B*a)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/(a^2-b^2)/d/(a+b*cos
(d*x+c))^(1/2)+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
Pi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(
1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.69, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3079, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}}+\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\sqrt{a+b\cos(c+dx)}}+\frac{2b(-4a^3B+7a^2Ab-3Ab^3)\sin(c+dx)}{3a^2d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(-4a^3B+7a^2Ab-3Ab^3)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2d(a^2-b^2)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(7*a^2*A*b - 3*A*b^3 - 4*a^3*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]) + (2*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*
A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(7*a^2*A*b - 3*A*b^3 - 4*
a^3*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\left(\frac{3}{2}A\left(a^2-b^2\right)-\frac{3}{2}a(Ab-aB)\cos(c\right.}{(a+b}}{3a}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b(7a^2Ab-3Ab^3-4a^3B)\,\mathrm{s}}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+b\cos}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b(7a^2Ab-3Ab^3-4a^3B)\,\mathrm{s}}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+b\cos}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b(7a^2Ab-3Ab^3-4a^3B)\,\mathrm{s}}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+b\cos}}$$

$$= -\frac{2(7a^2Ab-3Ab^3-4a^3B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2\left(a^2-b^2\right)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{2(7a^2Ab-3Ab^3-4a^3B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2\left(a^2-b^2\right)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.85, size = 743, normalized size = 2.13

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*((2*(-12*a^3*A*b + 4*a*A*b^3 + 6*a^4*B +
 2*a^2*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^4*A - 19*a^2*A*b^2 + 9*A*b^
4 + 4*a^3*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2
, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-7*a^2*A*b^2 + 3*A*b^4
 + 4*a^3*b*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x]
)/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)
^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*Arc
```

```
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*El
lipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]]
, (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c +
d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])
^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^
2))))/(6*a^2*(a - b)^2*(a + b)^2*d*(A + B*Cos[c + d*x])) + (Cos[c + d*x]*Sq
rt[a + b*Cos[c + d*x]]*(B + A*Sec[c + d*x])*((-2*(-(A*b^2*Sin[c + d*x]) + a
*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(-7*a^2*A
*b^2*Sin[c + d*x] + 3*A*b^4*Sin[c + d*x] + 4*a^3*b*B*Sin[c + d*x]))/(3*a^2*
(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/(d*(A + B*Cos[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(412) = 824$.
time = 1.26, size = 858, normalized size = 2.46

method	result	size
default	Expression too large to display	858

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b+B*a
)/a*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*sin(1/
2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2
*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b
^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2*A*b/
a^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a-(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)-2*A/a^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
2*b+a+b)^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)), x)
```

3.339 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}} \, dx$

Optimal. Leaf size=437

$$-\frac{(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^3(a^2-b^2)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{(3a^2A-5Ab^2+2a}{3a^2\,(a}$$

```
[Out] 1/3*b*(3*A*a^2-5*A*b^2+2*B*a*b)*sin(d*x+c)/a^2/(a^2-b^2)/d/(a+b*cos(d*x+c))
^(3/2)+1/3*b*(3*A*a^4-26*A*a^2*b^2+15*A*b^4+14*B*a^3*b-6*B*a*b^3)*sin(d*x+c
)/a^3/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-1/3*(3*A*a^4-26*A*a^2*b^2+15*A*b
^4+14*B*a^3*b-6*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/
a^3/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/3*(3*A*a^2-5*A*b^2+2*B*a
*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/(a^2-b^2)
/d/(a+b*cos(d*x+c))^(1/2)-(5*A*b-2*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b
*cos(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*cos(d*x+c))^(1/2)+A*tan(d*x+c)/a/d/(a+
b*cos(d*x+c))^(3/2)
```

Rubi [A]
time = 0.95, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3079, 3135, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^3d\sqrt{a+b\cos(c+dx)}}+\frac{b(3a^2A+2abB-5Ab^2)\sin(c+dx)}{3a^2d\,(a^2-b^2)\,(a+b\cos(c+dx))^{3/2}}+\frac{(3a^2A+2abB-5Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2d\,(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{b(3a^4A+14a^3bB-26a^2Ab^2-6ab^3B+15Ab^4)\sin(c+dx)}{3a^3d\,(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}}-\frac{(3a^4A+14a^3bB-26a^2Ab^2-6ab^3B+15Ab^4)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^3d\,(a^2-b^2)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] -1/3*((3*a^4*A - 26*a^2*A*b^2 + 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Sqrt[a +
 b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a^3*(a^2 - b^2)^2*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*a^2*A - 5*A*b^2 + 2*a*b*B)*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*a^
2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b - 2*a*B)*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^3*d*Sqrt
[a + b*Cos[c + d*x]]) + (b*(3*a^2*A - 5*A*b^2 + 2*a*b*B)*Sin[c + d*x])/(3*a
^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (b*(3*a^4*A - 26*a^2*A*b^2 +
 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sq
rt[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*(a + b*Cos[c + d*x])^(3/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
```

```
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a,
```

```
0]))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{1}{2}(-5Ab+2aB)+\frac{3}{2}Ab\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,d}{a}$$

$$= \frac{b(3a^2A-5Ab^2+2abB)\sin(c+dx)}{3a^2(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}$$

$$= \frac{b(3a^2A-5Ab^2+2abB)\sin(c+dx)}{3a^2(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{b(3a^4A-26a^2Ab^2+15A}{3a^3(a^2-b^2)^2}$$

$$= \frac{b(3a^2A-5Ab^2+2abB)\sin(c+dx)}{3a^2(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{b(3a^4A-26a^2Ab^2+15A}{3a^3(a^2-b^2)^2}$$

$$= \frac{b(3a^2A-5Ab^2+2abB)\sin(c+dx)}{3a^2(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{b(3a^4A-26a^2Ab^2+15A}{3a^3(a^2-b^2)^2}$$

$$= -\frac{(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B)\sqrt{a+b\cos(c+}}{3a^3(a^2-b^2)^2\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B)\sqrt{a+b\cos(c+}}{3a^3(a^2-b^2)^2\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.31, size = 750, normalized size = 1.72

$$\frac{\frac{2(36a^3Ab^2-20aAb^4-24a^4bB+8a^2b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(-33a^4Ab+86a^2Ab^3-45Ab^5+12a^5B-38a^3b^2B+18ab^4B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{2i(-3a^4Ab+26a^2Ab^3-15Ab^5-14a^3b^2B+6ab^4B)\sqrt{\frac{b-b\cos(c+dx)}{a+b}}\sqrt{-\frac{b+b\cos(c+dx)}{a-b}}\cos(2(c+dx))\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)-b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sin(c+dx)}{a\sqrt{-\frac{1}{a+b}}\sqrt{1-\cos^2(c+dx)}\sqrt{-\frac{a^2-b^2-2a(a+b\cos(c+dx))+(a+b\cos(c+dx))^2}{b^2}}\,(2a^2-b^2-4a(a+b\cos(c+dx))+2(a+b\cos(c+dx))^2)}}{12a^3(-a+b)^2(a+b)^2d}+\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{2(-Ab^3\sin(c+dx)+ab^2B\sin(c+dx))}{3a^2(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{2(-10a^2Ab^3\sin(c+dx)+6Ab^5\sin(c+dx)+7a^3b^2B\sin(c+dx)-3ab^4B\sin(c+dx))}{3a^3(a^2-b^2)^2(a+b\cos(c+dx))}+\frac{A\tan(c+dx)}{a^3}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] ((2*(36*a^3*A*b^2 - 20*a*A*b^4 - 24*a^4*b*B + 8*a^2*b^3*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
+ d*x]] + (2*(-33*a^4*A*b + 86*a^2*A*b^3 - 45*A*b^5 + 12*a^5*B - 38*a^3*b^2
*B + 18*a*b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-3*a^4*A*b + 26*a^2*
A*b^3 - 15*A*b^5 - 14*a^3*b^2*B + 6*a*b^4*B)*Sqrt[(b - b*Cos[c + d*x])/(a +
 b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*El
lipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
 - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-
(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c
 + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c +
d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(12*a^3*(-a + b)^2*(a + b)^2*d) + (Sqrt
[a + b*Cos[c + d*x]]*((2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x]))/(3
*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-10*a^2*A*b^3*Sin[c + d*x] +
 6*A*b^5*Sin[c + d*x] + 7*a^3*b^2*B*Sin[c + d*x] - 3*a*b^4*B*Sin[c + d*x]))
/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (A*Tan[c + d*x])/a^3))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(498) = 996$.
time = 1.78, size = 1345, normalized size = 3.08

method	result	size
default	Expression too large to display	1345

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)
*b/a^2*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*sin
(1/2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+
1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-
3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
```

```
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-2*b
*(2*A*b-B*a)/a^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b
^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a-(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)-
2*(-2*A*b+B*a)/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A/a^2*(-cos(1/2*d
*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^2\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2)), x)
```

3.340 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=532

$$\frac{(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{12a^4(a^2-b^2)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(2$$

```
[Out] -1/12*b*(27*A*a^2*b-35*A*b^3-12*B*a^3+20*B*a*b^2)*sin(d*x+c)/a^3/(a^2-b^2)/
d/(a+b*cos(d*x+c))^(3/2)-1/12*b*(33*A*a^4*b-170*A*a^2*b^3+105*A*b^5-12*B*a^
5+104*B*a^3*b^2-60*B*a*b^4)*sin(d*x+c)/a^4/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(
1/2)+1/12*(33*A*a^4*b-170*A*a^2*b^3+105*A*b^5-12*B*a^5+104*B*a^3*b^2-60*B*a
*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/(
(a+b*cos(d*x+c))/(a+b))^(1/2)-1/12*(27*A*a^2*b-35*A*b^3-12*B*a^3+20*B*a*b^2
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^3/(a^2-b^2)/d
/(a+b*cos(d*x+c))^(1/2)+1/4*(4*A*a^2+35*A*b^2-20*B*a*b)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+
b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^4/d/(a+b*cos(d*x+c))^(1/2)-1/4*
(7*A*b-4*B*a)*tan(d*x+c)/a^2/d/(a+b*cos(d*x+c))^(3/2)+1/2*A*sec(d*x+c)*tan(
d*x+c)/a/d/(a+b*cos(d*x+c))^(3/2)
```

Rubi [A]
time = 1.25, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3079, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{(7Ab-4aB)\tan(c+dx)}{4a^2d(a+b\cos(c+dx))^{3/2}}+\frac{(4a^2A-20abB+35Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^4d\sqrt{a+b\cos(c+dx)}}-\frac{b(-12a^3B+27a^2Ab+20ab^2B-35Ab^3)\sin(c+dx)}{12a^3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}-\frac{(-12a^3B+27a^2Ab+20ab^2B-35Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{12a^3d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{b(-12a^5B+33a^4Ab+104a^3b^2B-170a^2Ab^3-60ab^4B+105Ab^5)\sin(c+dx)}{12a^4d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}}+\frac{(-12a^5B+33a^4Ab+104a^3b^2B-170a^2Ab^3-60ab^4B+105Ab^5)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{12a^4d(a^2-b^2)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((33*a^4*A*b - 170*a^2*A*b^3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*
b^4*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(12*
a^4*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((27*a^2*A*b - 35
*A*b^3 - 12*a^3*B + 20*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/(12*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x
]]) + ((4*a^2*A + 35*A*b^2 - 20*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^4*d*Sqrt[a + b*Cos[c + d*x]]
) - (b*(27*a^2*A*b - 35*A*b^3 - 12*a^3*B + 20*a*b^2*B)*Sin[c + d*x])/(12*a^
3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (b*(33*a^4*A*b - 170*a^2*A*b^
3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*b^4*B)*Sin[c + d*x])/(12*a^
```

```
4*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((7*A*b - 4*a*B)*Tan[c + d*x]
)/(4*a^2*d*(a + b*Cos[c + d*x])^(3/2)) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a
*d*(a + b*Cos[c + d*x])^(3/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{1}{2}(-7Ab+4aB)+aA\cos(c+dx)+\frac{5}{2}Ab\cos^2(c+d}{(a+b\cos(c+dx))^{5/2}}}{2a}$$

$$= -\frac{(7Ab-4aB)\tan(c+dx)}{4a^2d(a+b\cos(c+dx))^{3/2}} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{1}{4}(4}{}}{}$$

$$= -\frac{b(27a^2Ab-35Ab^3-12a^3B+20ab^2B)\sin(c+dx)}{12a^3(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} - \frac{(7Ab-4a}{4a^2d(a+b}$$

$$= -\frac{b(27a^2Ab-35Ab^3-12a^3B+20ab^2B)\sin(c+dx)}{12a^3(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} - \frac{b(33a^4Ab}{}$$

$$= -\frac{b(27a^2Ab-35Ab^3-12a^3B+20ab^2B)\sin(c+dx)}{12a^3(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} - \frac{b(33a^4Ab}{}$$

$$= -\frac{b(27a^2Ab-35Ab^3-12a^3B+20ab^2B)\sin(c+dx)}{12a^3(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} - \frac{b(33a^4Ab}{}$$

$$= \frac{(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B)\sqrt{}}{12a^4(a^2-b^2)^2\,d\sqrt{\frac{a+b\cos(c+}{a+b}}}$$

$$= \frac{(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B)\sqrt{}}{12a^4(a^2-b^2)^2\,d\sqrt{\frac{a+b\cos(c+}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.42, size = 820, normalized size = 1.54

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] ((2*(12*a^5*A*b - 216*a^3*A*b^3 + 140*a*A*b^5 + 144*a^4*b^2*B - 80*a^2*b^4*
B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
)/Sqrt[a + b*Cos[c + d*x]] + (2*(24*a^6*A + 195*a^4*A*b^2 - 566*a^2*A*b^4 +
 315*A*b^6 - 132*a^5*b*B + 344*a^3*b^3*B - 180*a*b^5*B)*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
 + d*x]] - ((2*I)*(33*a^4*A*b^2 - 170*a^2*A*b^4 + 105*A*b^6 - 12*a^5*b*B +
104*a^3*b^3*B - 60*a*b^5*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b +
b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh
[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*E
llipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(
a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*
Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqr
t[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b
*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*
Cos[c + d*x])^2)))/(48*a^4*(a - b)^2*(a + b)^2*d) + (Sqrt[a + b*Cos[c + d*x
]]*((Sec[c + d*x]*(-11*A*b*Sin[c + d*x] + 4*a*B*Sin[c + d*x]))/(4*a^4) - (2
*(-(A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(a + b*
Cos[c + d*x])^2) - (2*(-13*a^2*A*b^4*Sin[c + d*x] + 9*A*b^6*Sin[c + d*x] +
10*a^3*b^3*B*Sin[c + d*x] - 6*a*b^5*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(
a + b*Cos[c + d*x])) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a^3)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2003 vs. $2(585) = 1170$.
time = 2.66, size = 2004, normalized size = 3.77

method	result	size
default	Expression too large to display	2004

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(A*b-B*a
)*b^2/a^3*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*
sin(1/2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d
*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b
^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+
```

```
2*b^2*(3*A*b-2*B*a)/a^4/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)
/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
b*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a-(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*b)-2*b*(3*A*b-2*B*a)/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*(-2*A*b
+B*a)/a^3*(-cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*A/a^2*(-1/2*c
os(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1
)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2))/s
in(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(5/2)), x)
```

3.341 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*
x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {21, 2742, 2740}

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx &= B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx\\
&= \frac{\left(B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}}\\
&= \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.06, size = 58, normalized size = 1.00

$$
\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 76, normalized size = 1.31

method	result	size
default	$\frac{2B\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a+b}}\;\mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle|\frac{\sqrt{2}\,\sqrt{b}}{\sqrt{a+b}}\right)}{d\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*B/d/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/
(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)/(a+b)^(1/2)*b^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 148, normalized size = 2.55

$$\frac{-i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right)+i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)-3i\,b\sin(dx+c)+2\,a}{3\,b}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] (-I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8
*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) +
 I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b))/(
b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int\frac{1}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] B*Integral(1/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(3/2),x)
[Out] int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(3/2), x)
```

3.342 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=59

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos
(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {21, 2886, 2884}

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
 + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{\left(B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A]
time = 0.09, size = 59, normalized size = 1.00

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
 + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A]
time = 0.26, size = 167, normalized size = 2.83

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\,\text{EllipticPi}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2,(-2*b/(a-b))^(1/2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b
)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2),
 x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] B*Integral(sec(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)), x)
```

3.343 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=108

$$\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2bB\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -2*b*B*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b)
)^(1/2))*(a+b*cos(d*x+c))^(1/2)/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {21, 2743, 2734, 2732}

$$\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\,(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2bB\sin(c+dx)}{d\,(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/((a^2
- b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*b*B*Sin[c + d*x])/((a^2 -
 b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx &= B\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx\\
&= -\frac{2bB\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{(2B)\int \frac{-\frac{a}{2}-\frac{1}{2}b\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}\\
&= -\frac{2bB\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{B\int \sqrt{a+b\cos(c+dx)}\,dx}{a^2-b^2}\\
&= -\frac{2bB\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{\left(B\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b}{a+b}\cos(c+dx)}\,dx}{(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\
&= \frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2bB\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 84, normalized size = 0.78

$$\frac{B\left(2(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-2b\sin(c+dx)\right)}{(a-b)(a+b)d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (B*(2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] - 2*b*Sin[c + d*x]))/((a - b)*(a + b)*d*Sqrt[a + b*Cos[c + d*x]
])
```

Maple [A]
time = 0.32, size = 218, normalized size = 2.02

method	result
default	$-\frac{2B\left(2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right.\right.}{(a-b)(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*B*(2*b*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b)/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+
b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 491, normalized size = 4.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/3*(6*sqrt(b*cos(d*x + c) + a)*B*b^2*sin(d*x + c) + (I*sqrt(2)*B*a*b*cos(
d*x + c) + I*sqrt(2)*B*a^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)
/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x +
c) + 2*a)/b) + (-I*sqrt(2)*B*a*b*cos(d*x + c) - I*sqrt(2)*B*a^2)*sqrt(b)*we
ierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3
```

```
*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(I*sqrt(2)*B*b^2*cos(
d*x + c) + I*sqrt(2)*B*a*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2
, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
 -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) +
2*a)/b)) - 3*(-I*sqrt(2)*B*b^2*cos(d*x + c) - I*sqrt(2)*B*a*b)*sqrt(b)*weie
rstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstra
ssPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^2*b^2 - b^4)*d*cos(d*x + c
) + (a^3*b - a*b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int\frac{1}{a\sqrt{a+b\cos{(c+dx)}}+b\sqrt{a+b\cos{(c+dx)}}\cos{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] B*Integral(1/(a*sqrt(a + b*cos(c + d*x)) + b*sqrt(a + b*cos(c + d*x))*cos(c
 + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(5/2), x)
```

3.344 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=179

$$-\frac{2bB\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*b^2*B*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*b*B*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/
(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))
^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d
/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {21, 2881, 3138, 2734, 2732, 12, 2886, 2884}

$$\frac{2b^2B\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2bB\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*
(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*B*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b
*Cos[c + d*x]]) + (2*b^2*B*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
+ d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = B\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{(2B)\int \frac{\left(\frac{1}{2}(a^2-b^2)-\frac{1}{2}ab\cos(c+dx)-\cdots}{\sqrt{a+b\cos\cdots}}}{a(a^2-\cdots}$$

$$= \frac{2b^2B\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{(2B)\int -\frac{b(a^2-b^2)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx}\cdots}}{ab(a^2-b^2)}$$

$$= \frac{2b^2B\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{B\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a}$$

$$= -\frac{2bB\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{a(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b^2B\sin(\cdots}{a(a^2-b^2)\,d\sqrt{a\cdots}}$$

$$= -\frac{2bB\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{a(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2B\sqrt{\frac{a+b\cos(c\cdots}{a+b}}}{ad\sqrt{a\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.56, size = 403, normalized size = 2.25

$$\frac{B\left(-\frac{-\frac{4ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(2a^2-3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{2i\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)\right)\right)}{a\sqrt{-\frac{1}{a+b}}}}{(-a+b)(a+b)}+\frac{4b^2\sin(c+dx)}{(a^2-b^2)\sqrt{a+b\cos(c+dx)}}\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (B*(-(((-4*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(2*a^2 - 3*b^2)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*C
os[c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 +
 Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Ell
ipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
- b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Co
s[c + d*x]]], (a + b)/(a - b)])))/(a*Sqrt[-(a + b)^(-1)]))/((-a + b)*(a + b
))) + (4*b^2*Sin[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]])))/(2*a*d)
```

Maple [A]
time = 0.38, size = 377, normalized size = 2.11

method	result
default	$\frac{2B\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{2}+\sqrt{-\frac{2b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 2*B*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2+(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*b^2+(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)
)*a^2-(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)/a/(a-b
)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2),
 x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)), x)
```

3.345 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=170

$$\frac{2(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{10(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{10(Ab+aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}+$$

```
[Out] 2/15*(9*A*a+7*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+10/21*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*(9*A*a+7*
B*b)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*(A*b+B*a)*cos(d*x+c)^(5/2)*sin(d*x+c
)/d+2/9*b*B*cos(d*x+c)^(7/2)*sin(d*x+c)/d+10/21*(A*b+B*a)*sin(d*x+c)*cos(d*
x+c)^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3047, 3102, 2827, 2715, 2719, 2720}

$$\frac{10(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(aB+Ab)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2(9aA+7bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{10(aB+Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(9*a*A + 7*b*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*(A*b + a*B)*Elli
pticF[(c + d*x)/2, 2])/(21*d) + (10*(A*b + a*B)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) + (2*(9*a*A + 7*b*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) +
 (2*(A*b + a*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*b*B*Cos[c + d*x
]^(7/2)*Sin[c + d*x])/(9*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx &= \int \cos^{\frac{5}{2}}(c+dx)\left(aA+(Ab+aB)\cos(c+dx)+\right.\\
&= \frac{2bB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}+\frac{2}{9}\int \cos^{\frac{5}{2}}(c+d\\
&= \frac{2bB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}+(Ab+aB)\int \mathrm{co}\\
&= \frac{2(9aA+7bB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}+\frac{2(Ab}{}\\
&= \frac{2(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d}+\frac{10(Ab+aB)}{}\\
&= \frac{2(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d}+\frac{10(Ab+aB)}{2}\end{aligned}$$

Mathematica [**A**]

time = 1.35, size = 125, normalized size = 0.74

$$\frac{84(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+300(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(7(36aA+43bB)\cos(c+dx)+5(78Ab+78aB+18(Ab+aB)\cos(2(c+dx))+7bB\cos(3(c+dx))))\sin(c+dx)}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (84*(9*a*A + 7*b*B)*EllipticE[(c + d*x)/2, 2] + 300*(A*b + a*B)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*a*A + 43*b*B)*Cos[c + d*x] + 5*(
78*A*b + 78*a*B + 18*(A*b + a*B)*Cos[2*(c + d*x)] + 7*b*B*Cos[3*(c + d*x)])
)*Sin[c + d*x])/(630*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(202) = 404$.
time = 0.31, size = 451, normalized size = 2.65

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(720Ab+720aB+2240Bb)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b+(720*A*b+720*B*a+2240*B*b)*sin(1/2*
d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*a-1080*A*b-1080*B*a-2072*B*b)*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(504*A*a+840*A*b+840*B*a+952*B*b)*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*A*a-240*A*b-240*B*a-168*B*b)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a+75*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 211, normalized size = 1.24

$$\frac{2\left(35\,Bb\cos\left(dx+c\right)^{3}+45\left(Ba+Ab\right)\cos\left(dx+c\right)^{2}+75\,Ba+75\,Ab+7\left(9\,Aa+7\,Bb\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-75\,\sqrt{2}\left(i\,Ba+i\,Ab\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)-75\,\sqrt{2}\left(-i\,Ba-i\,Ab\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-21\,\sqrt{2}\left(-9i\,Aa-7i\,Bb\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-21\,\sqrt{2}\left(9i\,Aa+7i\,Bb\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/315*(2*(35*B*b*cos(d*x + c)^3 + 45*(B*a + A*b)*cos(d*x + c)^2 + 75*B*a +
75*A*b + 7*(9*A*a + 7*B*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) -
75*sqrt(2)*(I*B*a + I*A*b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 75*sqrt(2)*(-I*B*a - I*A*b)*weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c)) - 21*sqrt(2)*(-9*I*A*a - 7*I*B*b)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2
)*(9*I*A*a + 7*I*B*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [B]
time = 1.35, size = 177, normalized size = 1.04

$$-\frac{2\,A\,a\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,A\,b\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,B\,a\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,B\,b\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

```
[Out] - (2*A*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
+ d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*b*cos(c + d*x)^(9/2)*sin(c +
 d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1
/2)) - (2*B*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, c
os(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b*cos(c + d*x)^(11/2)*s
in(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*
x)^2)^(1/2))
```

3.346 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{6(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(7aA+5bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(7aA+5bB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2(}{}$$

```
[Out] 6/5*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(7*A*a+5*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*(A*b+B*a)*co
s(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*b*B*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/21*(7*A
*a+5*B*b)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3047, 3102, 2827, 2715, 2720, 2719}

$$\frac{2(7aA+5bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2(7aA+5bB)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a*A + 5*b*B)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*(7*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(21*d) + (2*(A*b + a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*
b*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^{\frac{3}{2}}(c+dx)\left(aA+(Ab+aB)\cos(c+dx)\right.$$

$$= \frac{2bB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \cos^{\frac{3}{2}}(c+$$

$$= \frac{2bB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + (Ab+aB)\int$$

$$= \frac{2(7aA+5bB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2(}{}$$

$$= \frac{6(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(7aA+5bB)}{2}$$

Mathematica [A]

time = 0.90, size = 103, normalized size = 0.74

$$\frac{126(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7aA+5bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(70aA+65bB+42(Ab+aB)\cos(c+dx)+15bB\cos(2(c+dx)))\sin(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (126*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + 10*(7*a*A + 5*b*B)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*a*A + 65*b*B + 42*(A*b + a*B)*Cos[c
 + d*x] + 15*b*B*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(176) = 352$.
time = 0.29, size = 413, normalized size = 2.95

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(-168Ab-168aB-360Bb)\left(\sin^6\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b+(-168*A*b-168*B*a-360*B*b)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*a+168*A*b+168*B*a+280*B*b)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*a-42*A*b-42*B*a-80*B*b)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+35*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b+25*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 192, normalized size = 1.37

$$\frac{2\left(15\,Bb\cos\left(dx+c\right)^2+35\,Aa+25\,Bb+21\,\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-5\,\sqrt{2}\,\left(7i\,Aa+5i\,Bb\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)-5\,\sqrt{2}\,\left(-7i\,Aa-5i\,Bb\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-63\,\sqrt{2}\,\left(-i\,Ba-i\,Ab\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-63\,\sqrt{2}\,\left(i\,Ba+i\,Ab\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/105*(2*(15*B*b*cos(d*x + c)^2 + 35*A*a + 25*B*b + 21*(B*a + A*b)*cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sqrt(2)*(7*I*A*a + 5*I*B*b)*weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-7*I*A*a
- 5*I*B*b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*s
qrt(2)*(-I*B*a - I*A*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) - 63*sqrt(2)*(I*B*a + I*A*b)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [**B**]
time = 1.16, size = 166, normalized size = 1.19

$$\frac{2\,A\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}-\frac{2\,A\,b\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,a\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,b\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

```
[Out] (2*A*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) - (2*A*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos
(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a*cos(c + d*x)^(7/2)*sin(
c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)
^(1/2)) - (2*B*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4
, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.347 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=108

$$\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2bB$$

```
[Out] 2/5*(5*A*a+3*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b*B*cos(d*x+c
)^(3/2)*sin(d*x+c)/d+2/3*(A*b+B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3047, 3102, 2827, 2719, 2715, 2720}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(5*a*A + 3*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(3*d) + (2*b*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx &= \int \sqrt{\cos(c+dx)}\,(aA+(Ab+aB)\cos(c+\\
&= \frac{2bB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{\cos(\\
&= \frac{2bB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+(Ab+aB)\int\\
&= \frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(Ab+aB}{}\\
&= \frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(Ab+aB}{}\end{aligned}$$

Mathematica [A]
time = 0.44, size = 86, normalized size = 0.80

$$\frac{2\left(3(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(5Ab+5aB+3bB\cos(c+dx))\sin(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(3*(5*a*A + 3*b*B)*EllipticE[(c + d*x)/2, 2] + 5*(A*b + a*B)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A*b + 5*a*B + 3*b*B*Cos[c + d*x])*Si
n[c + d*x]))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(148) = 296$.
time = 0.29, size = 371, normalized size = 3.44

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(20Ab+20aB+24Bb)\left(\sin^4\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b+(20*A*b+20*B*a+24*B*b)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b-10*B*a-6*B*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+5*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+
5*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 175, normalized size = 1.62

$$\frac{2\,(3\,Bb\cos(dx+c)+5\,Ba+5\,Ab)\sqrt{\cos(dx+c)}\,\sin(dx+c)-5\,\sqrt{2}\,(i\,Ba+i\,Ab)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-5\,\sqrt{2}\,(-i\,Ba-i\,Ab)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3\,\sqrt{2}\,(-5i\,Aa-3i\,Bb)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3\,\sqrt{2}\,(5i\,Aa+3i\,Bb)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/15*(2*(3*B*b*cos(d*x + c) + 5*B*a + 5*A*b)*sqrt(cos(d*x + c))*sin(d*x + c
) - 5*sqrt(2)*(I*B*a + I*A*b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) - 5*sqrt(2)*(-I*B*a - I*A*b)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-5*I*A*a - 3*I*B*b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2
)*(5*I*A*a + 3*I*B*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Mupad [**B**]
time = 1.01, size = 128, normalized size = 1.19

$$\frac{2\,A\,b\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,a\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}-\frac{2\,B\,b\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

```
[Out] (2*A*b*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) + (2*B*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2))
)/(3*d) + (2*A*a*ellipticE(c/2 + (d*x)/2, 2))/d - (2*B*b*cos(c + d*x)^(7/2)
*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*
x)^2)^(1/2))
```

3.348 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2bB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/d+2/3*(3*A*a+B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*B*sin(d*x+c)*co
s(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3047, 3102, 2827, 2720, 2719}

$$\frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a*A + b*B)*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*b*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2bB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}(3aA+bB)+\frac{3}{2}(A}{\sqrt{\cos(c}}$$

$$= \frac{2bB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + (Ab+aB)\int \sqrt{\cos(c+}$$

$$= \frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\right|}{3d}$$

Mathematica [A]
time = 0.24, size = 67, normalized size = 0.89

$$\frac{2\left(3(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+bB\sqrt{\cos(c+dx)}\,\sin(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(3*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + (3*a*A + b*B)*EllipticF[(c +
d*x)/2, 2] + b*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(121) = 242$.
time = 0.28, size = 326, normalized size = 4.35

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+3aA\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\mathrm{s}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b+3*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*b-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+B*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 156, normalized size = 2.08

$$\frac{2\,Bb\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\sqrt{2}\,(-3i\,Aa-i\,Bb)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\,(3i\,Aa+i\,Bb)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-3\,\sqrt{2}\,(-i\,Ba-i\,Ab)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-3\,\sqrt{2}\,(i\,Ba+i\,Ab)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*B*b*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-3*I*A*a - I*B*b)*wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*A*a +
 I*B*b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(
2)*(-I*B*a - I*A*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
```

```
*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*B*a + I*A*b)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)})\,(a + b\cos{(c + dx)})}{\sqrt{\cos{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 0.99, size = 85, normalized size = 1.13

$$\frac{2\,B\,b\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*B*b*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) + (2*A*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*b*ellipticE(c/2 + (d*x)/2
, 2))/d + (2*B*a*ellipticE(c/2 + (d*x)/2, 2))/d
```

3.349 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=71

$$-\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*(A*a-B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/d+2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*A*sin(d*x+c)/d/cos(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3047, 3100, 2827, 2720, 2719}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a*A - b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(A*b + a*B)*EllipticF[(c
+ d*x)/2, 2])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}(Ab+aB)-\frac{1}{2}(aA-bB)\cos(c+$$
$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + (Ab+aB)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + (-a$$
$$= -\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\right.}{d}$$

Mathematica [A]
time = 0.37, size = 64, normalized size = 0.90

$$\frac{2\left((-aA+bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{aA\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*((-(a*A) + b*B)*EllipticE[(c + d*x)/2, 2] + (A*b + a*B)*EllipticF[(c + d
*x)/2, 2] + (a*A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(121) = 242.
time = 0.33, size = 246, normalized size = 3.46

method	result
default	$\frac{4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2Ab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2A\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a-A*b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*a-a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 185, normalized size = 2.61

$$\frac{2\,Aa\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\sqrt{2}\left(-i\,Ba-i\,Ab\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(i\,Ba+i\,Ab\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(-i\,Aa+i\,Bb\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+\sqrt{2}\left(i\,Aa-i\,Bb\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] (2*A*a*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-I*B*a - I*A*b)*cos(d*x +
 c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*
B*a + I*A*b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) + sqrt(2)*(-I*A*a + I*B*b)*cos(d*x + c)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*A*a -
I*B*b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c))))/(d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Mupad [**B**]
time = 1.44, size = 96, normalized size = 1.35

$$\frac{2\,A\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*A*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*B*b*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*a*sin(c + d*x)*hypergeom(
[-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(
1/2))
```

3.350 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=103

$$-\frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(Ab+aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*a+3*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*A*sin(d*x+c)/d
/cos(d*x+c)^(3/2)+2*(A*b+B*a)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3047, 3100, 2827, 2716, 2719, 2720}

$$\frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aB+Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a*A + 3*b*B)*EllipticF[(
c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*
(A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \int \frac{aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\frac{3}{2}(Ab+aB)+\frac{1}{2}(aA+3bB)\cos(c
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+(Ab+aB)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\frac{1}{3}(aA \\
&= \frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(Ab+}{d\sqrt{}} \\
&= -\frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx\right.}{3d}
\end{aligned}
$$

Mathematica [A]
time = 0.50, size = 107, normalized size = 1.04

$$\frac{2\left(-3(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+(aA+3bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+3Ab\sin(c+dx)+3aB\sin(c+dx)+aA\tan(c+dx)\right)}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]

[Out] (2*(-3*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x] + a*A*Tan[c + d*x]))/(3*d*Sqrt[Cos[c + d*x]])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(147) = 294$.
time = 0.54, size = 401, normalized size = 3.89

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2Bb\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{El}}{\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b+B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 213, normalized size = 2.07

$$\frac{\sqrt{2}\left(-iAa-3iBb\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\sqrt{2}\left(iAa+3iBb\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-3\sqrt{2}\left(iBa+iAb\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-3\sqrt{2}\left(-iBa-iAb\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+2\left(Aa+3\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A*a - 3*I*B*b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a + 3*I*B*b)*cos(d*x + c)^2*we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*B*a
+ I*A*b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*B*a - I*A*b)*cos(d*x + c)^2*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c))) + 2*(A*a + 3*(B*a + A*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3881 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Mupad [**B**]
time = 1.97, size = 150, normalized size = 1.46

$$\frac{2\,B\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,A\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,B\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*B*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a*sin(c + d*x)*hypergeom([-3/4
, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2
)) + (2*A*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a*sin(c + d*x)*hypergeom([-
1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/
2))
```

3.351 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$-\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+$$

```
[Out] -2/5*(3*A*a+5*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*A*sin(d*x+
c)/d/cos(d*x+c)^(5/2)+2/3*(A*b+B*a)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*(3*A*
a+5*B*b)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3047, 3100, 2827, 2716, 2720, 2719}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(3aA+5bB)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a*A + 5*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(3*a*A + 5*b*
B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \int \frac{aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx \\
&= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{\frac{5}{2}(Ab+aB)+\frac{1}{2}(3aA+5bB)\cos(c}{\cos^{\frac{5}{2}}(c+dx)} \\
&= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+(Ab+aB)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx+\frac{1}{5}(3aA \\
&= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(3aA+5bE}{5d\sqrt{\cos}} \\
&= -\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(Ab+aB)F\left(\frac{1}{2}(c+d\right.}{3d}
\end{aligned}
$$

Mathematica [A]
time = 0.86, size = 134, normalized size = 0.96

$$
\frac{-6(3aA+5bB)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(Ab+aB)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+10Ab\sin(c+dx)+10aB\sin(c+dx)+9aA\sin(2(c+dx))+15bB\sin(2(c+dx))+6aA\tan(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*(3*a*A + 5*b*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*(A*b
+ a*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*A*b*Sin[c + d*x] +
 10*a*B*Sin[c + d*x] + 9*a*A*Sin[2*(c + d*x)] + 15*b*B*Sin[2*(c + d*x)] + 6
*a*A*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(176) = 352$.
time = 0.72, size = 636, normalized size = 4.54

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2(Ab+aB)\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2}}{6\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b+B*a)*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*B*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/
2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*a*A/(8
*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin
(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 235, normalized size = 1.68

$$-\frac{5\sqrt{2}\,(i\,Ba+i\,Ab)\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\,(-i\,Ba-i\,Ab)\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+3\sqrt{2}\,(3i\,Aa+5i\,Bb)\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+3\sqrt{2}\,(-3i\,Aa-5i\,Bb)\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-2\,(3\,(3\,Aa+5\,Bb)\cos(dx+c)^2+3\,Aa+5\,(Ba+Ab)\cos(dx+c))\sqrt{\cos(dx+c)}\,\sin(dx+c)}{15\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(I*B*a + I*A*b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a - I*A*b)*cos(d*x + c)^3*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(3*I*
A*a + 5*I*B*b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-3*I*A*a - 5*I*B*b)*cos(d
*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(3*(3*A*a + 5*B*b)*cos(d*x + c)^2 + 3*A*a + 5*(B*a + A
*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Mupad [B]
time = 2.39, size = 177, normalized size = 1.26

$$\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{5\,d\,{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,A\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,B\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,B\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*A*a*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(
c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*b*sin(c + d*x)*hypergeom([-3/
4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/
2)) + (2*B*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d
*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*b*sin(c + d*x)*hypergeom
([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^
(1/2))
```

3.352 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=264

$$\frac{2(9a^2A+7Ab^2+14abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{10(9b^2B+11a(2Ab+aB))\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{10(9b^2B+11a(\cdots$$

```
[Out] 2/15*(9*A*a^2+7*A*b^2+14*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+10/231*(9*b^2*B+11*a*(2*A*b+B*
a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2/45*(9*A*a^2+7*A*b^2+14*B*a*b)*cos(d*x+c)^(3/2)*sin(d*x+c
)/d+2/77*(9*b^2*B+11*a*(2*A*b+B*a))*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/99*b*(1
1*A*b+13*B*a)*cos(d*x+c)^(7/2)*sin(d*x+c)/d+2/11*b*B*cos(d*x+c)^(7/2)*(a+b*
cos(d*x+c))*sin(d*x+c)/d+10/231*(9*b^2*B+11*a*(2*A*b+B*a))*sin(d*x+c)*cos(d
*x+c)^(1/2)/d
```

Rubi [A]

time = 0.24, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3069, 3102, 2827, 2715, 2719, 2720}

$$\frac{2(9a^2A+14abB+7Ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(9a^2A+14abB+7Ab^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{10(11a(aB+2Ab)+9b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2(11a(aB+2Ab)+9b^2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{77d}+\frac{10(11a(aB+2Ab)+9b^2B)\sin(c+dx)\sqrt{\cos(c+dx)}}{231d}+\frac{2b(13aB+11Ab)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{99d}+\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(9*a^2*A + 7*A*b^2 + 14*a*b*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*(
9*b^2*B + 11*a*(2*A*b + a*B))*EllipticF[(c + d*x)/2, 2])/(231*d) + (10*(9*b
^2*B + 11*a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(9
*a^2*A + 7*A*b^2 + 14*a*b*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(
9*b^2*B + 11*a*(2*A*b + a*B))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*
b*(11*A*b + 13*a*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*b*B*Cos[c
+ d*x]^(7/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(11*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+d}{11d}$$
$$= \frac{2b(11Ab+13aB)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d} +$$
$$= \frac{2b(11Ab+13aB)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d} +$$
$$= \frac{2(9a^2A+7Ab^2+14abB)\cos^{\frac{3}{2}}(c+dx)\sin(c+}{45d}$$
$$= \frac{2(9a^2A+7Ab^2+14abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d} +$$
$$= \frac{2(9a^2A+7Ab^2+14abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d} +$$

Mathematica [A]
time = 1.84, size = 196, normalized size = 0.74

$$\frac{3696(9a^2A+7Ab^2+14abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+1200(22aAb+11a^2B+9b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+2\sqrt{\cos(c+dx)}\,(154(36a^2A+43Ab^2+86abB)\cos(c+dx)+180(22aAb+11a^2B+16b^2B)\cos(2(c+dx))+770b(Ab+2aB)\cos(3(c+dx))+15(1144aAb+572a^2B+531b^2B+21b^2B\cos(4(c+dx))))\sin(c+dx)}{27720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3696*(9*a^2*A + 7*A*b^2 + 14*a*b*B)*EllipticE[(c + d*x)/2, 2] + 1200*(22*a
*A*b + 11*a^2*B + 9*b^2*B)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]
*(154*(36*a^2*A + 43*A*b^2 + 86*a*b*B)*Cos[c + d*x] + 180*(22*a*A*b + 11*a^
2*B + 16*b^2*B)*Cos[2*(c + d*x)] + 770*b*(A*b + 2*a*B)*Cos[3*(c + d*x)] + 1
5*(1144*a*A*b + 572*a^2*B + 531*b^2*B + 21*b^2*B*Cos[4*(c + d*x)]))*Sin[c +
 d*x])/(27720*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(292) = 584$.
time = 0.33, size = 666, normalized size = 2.52

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(20160B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+\left(-12320A\,b^2-24640Bab\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12*b^2+(-12320*A*b^2-24640*B*a*b-50400*
B*b^2)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(15840*A*a*b+24640*A*b^2+79
20*B*a^2+49280*B*a*b+56880*B*b^2)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(
-5544*A*a^2-23760*A*a*b-22792*A*b^2-11880*B*a^2-45584*B*a*b-34920*B*b^2)*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(5544*A*a^2+18480*A*a*b+10472*A*b^2+9
240*B*a^2+20944*B*a*b+13860*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-1386*A*a^2-5280*A*a*b-1848*A*b^2-2640*B*a^2-3696*B*a*b-2790*B*b^2)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1650*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2-1617*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+825*B*a^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+675*B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3234*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 299, normalized size = 1.13

$$\frac{2\left(315\,Bb^2\cos\left(dx+c\right)^4+385\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^3+825\,Ba^2+1650\,Aab+675\,Bb^2+45\left(11\,Ba^2+22\,Aab+9\,Bb^2\right)\cos\left(dx+c\right)^2+77\left(9\,Aa^2+14\,Bab+7\,Ab^2\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-75\sqrt{2}\left(11i\,Ba^2+22i\,Aab+9i\,Bb^2\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-75\sqrt{2}\left(-11i\,Ba^2-22i\,Aab-9i\,Bb^2\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-231\sqrt{2}\left(-9i\,Aa^2-14i\,Bab-7i\,Ab^2\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-231\sqrt{2}\left(9i\,Aa^2+14i\,Bab+7i\,Ab^2\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/3465*(2*(315*B*b^2*cos(d*x + c)^4 + 385*(2*B*a*b + A*b^2)*cos(d*x + c)^3
+ 825*B*a^2 + 1650*A*a*b + 675*B*b^2 + 45*(11*B*a^2 + 22*A*a*b + 9*B*b^2)*c
```

```
os(d*x + c)^2 + 77*(9*A*a^2 + 14*B*a*b + 7*A*b^2)*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c) - 75*sqrt(2)*(11*I*B*a^2 + 22*I*A*a*b + 9*I*B*b^2)*wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 75*sqrt(2)*(-11*I*
B*a^2 - 22*I*A*a*b - 9*I*B*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c)) - 231*sqrt(2)*(-9*I*A*a^2 - 14*I*B*a*b - 7*I*A*b^2)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
 231*sqrt(2)*(9*I*A*a^2 + 14*I*B*a*b + 7*I*A*b^2)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x
)
```

Mupad [**B**]
time = 1.53, size = 275, normalized size = 1.04

$$-\frac{2\,A\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,a^2\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,A\,b^2\,{\cos\left(c+d\,x\right)}^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,b^2\,{\cos\left(c+d\,x\right)}^{13/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{13}{4};\ \frac{17}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{13\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{4\,A\,a\,b\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{4\,B\,a\,b\,{\cos\left(c+d\,x\right)}^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

```
[Out] - (2*A*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(
c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^2*cos(c + d*x)^(9/2)*sin
(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2
)^(1/2)) - (2*A*b^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4],
 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b^2*cos(c + d*
x)^(13/2)*sin(c + d*x)*hypergeom([1/2, 13/4], 17/4, cos(c + d*x)^2))/(13*d*
(sin(c + d*x)^2)^(1/2)) - (4*A*a*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeo
m([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (4*B*a*
b*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x
)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

3.353 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=223

$$\frac{2(7b^2B+9a(2Ab+aB))\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d}+\frac{2(7a^2A+5Ab^2+10abB)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d}+\frac{2(7a^2A+5Ab^2+10}{}$$

```
[Out] 2/15*(7*b^2*B+9*a*(2*A*b+B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(7*A*a^2+5*A*b^2+10*B*a*b)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2))/d+2/45*(7*b^2*B+9*a*(2*A*b+B*a))*cos(d*x+c)^(3/2)*sin(d*x+c)/d+
2/63*b*(9*A*b+11*B*a)*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*b*B*cos(d*x+c)^(5/2
)*(a+b*cos(d*x+c))*sin(d*x+c)/d+2/21*(7*A*a^2+5*A*b^2+10*B*a*b)*sin(d*x+c)*
cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.22, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3069, 3102, 2827, 2715, 2720, 2719}

$$\frac{2(7a^2A+10abB+5Ab^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d}+\frac{2(7a^2A+10abB+5Ab^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2(9a(aB+2Ab)+7b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d}+\frac{2(9a(aB+2Ab)+7b^2B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2b(11aB+9Ab)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}+\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(7*b^2*B + 9*a*(2*A*b + a*B))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(7*
a^2*A + 5*A*b^2 + 10*a*b*B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(7*a^2*A
 + 5*A*b^2 + 10*a*b*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(7*b^2*
B + 9*a*(2*A*b + a*B))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(9*A*
b + 11*a*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b*B*Cos[c + d*x]^(
5/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx}{9d}$$
$$= \frac{2b(9Ab+11aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2b}{}$$
$$= \frac{2b(9Ab+11aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2b}{}$$
$$= \frac{2(7a^2A+5Ab^2+10abB)\sqrt{\cos(c+dx)}\sin(c}{21d}$$
$$= \frac{2(7b^2B+9a(2Ab+aB))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2}{}$$

Mathematica [A]
time = 1.50, size = 167, normalized size = 0.75

$$\frac{84(18aAb+9a^2B+7b^2B)E\left(\frac{1}{2}(c+dx)\middle|2\right)+60(7a^2A+5Ab^2+10abB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}(7(72aAb+36a^2B+43b^2B)\cos(c+dx)+5(84a^2A+78Ab^2+156abB+18b(Ab+2aB)\cos(2(c+dx))+7b^2B\cos(3(c+dx))))\sin(c+dx)}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (84*(18*a*A*b + 9*a^2*B + 7*b^2*B)*EllipticE[(c + d*x)/2, 2] + 60*(7*a^2*A
+ 5*A*b^2 + 10*a*b*B)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(72
*a*A*b + 36*a^2*B + 43*b^2*B)*Cos[c + d*x] + 5*(84*a^2*A + 78*A*b^2 + 156*a
*b*B + 18*b*(A*b + 2*a*B)*Cos[2*(c + d*x)] + 7*b^2*B*Cos[3*(c + d*x)]))*Sin
[c + d*x])/(630*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(255) = 510$.
time = 0.34, size = 610, normalized size = 2.74

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+(720A\,b^2+1440Bab+2240B\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^2+(720*A*b^2+1440*B*a*b+2240*B*b^2)
```

```
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1008*A*a*b-1080*A*b^2-504*B*a^2-
2160*B*a*b-2072*B*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a^2+1
008*A*a*b+840*A*b^2+504*B*a^2+1680*B*a*b+952*B*b^2)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-210*A*a^2-252*A*a*b-240*A*b^2-126*B*a^2-480*B*a*b-168*B*
b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*a^2*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+75*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-378*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+1
50*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-147*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 271, normalized size = 1.22

$$\frac{2\left(35\,Bb^{2}\cos\left(dx+c\right)^{3}+105\,Aa^{2}+150\,Bab+75\,Ab^{2}+45\left(2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)^{2}+7\left(9\,Ba^{2}+18\,Aab+7\,Bb^{2}\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-15\sqrt{2}\left(7i\,Aa^{2}+10i\,Bab+5i\,Ab^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-15\sqrt{2}\left(-7i\,Aa^{2}-10i\,Bab-5i\,Ab^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-21\sqrt{2}\left(-9i\,Ba^{2}-18i\,Aab-7i\,Bb^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-21\sqrt{2}\left(9i\,Ba^{2}+18i\,Aab+7i\,Bb^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/315*(2*(35*B*b^2*cos(d*x + c)^3 + 105*A*a^2 + 150*B*a*b + 75*A*b^2 + 45*(
2*B*a*b + A*b^2)*cos(d*x + c)^2 + 7*(9*B*a^2 + 18*A*a*b + 7*B*b^2)*cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*(7*I*A*a^2 + 10*I*B*a*b
+ 5*I*A*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 15
*sqrt(2)*(-7*I*A*a^2 - 10*I*B*a*b - 5*I*A*b^2)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 21*sqrt(2)*(-9*I*B*a^2 - 18*I*A*a*b - 7*I*B
*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) - 21*sqrt(2)*(9*I*B*a^2 + 18*I*A*a*b + 7*I*B*b^2)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

Mupad [B]
time = 1.35, size = 264, normalized size = 1.18

$$\frac{2\,A\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}-\frac{2\,B\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,A\,b^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,b^2\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{4\,A\,a\,b\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{4\,B\,a\,b\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(
3*d) - (2*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4,
 cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*b^2*cos(c + d*x)^(9/2
)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d
*x)^2)^(1/2)) - (2*B*b^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 1
1/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (4*A*a*b*cos(c
 + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*
d*(sin(c + d*x)^2)^(1/2)) - (4*B*a*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hyperg
eom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.354 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=182

$$\frac{2(5a^2A+3Ab^2+6abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(5b^2B+7a(2Ab+aB))\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(5b^2B+7a(2Ab+$$

```
[Out] 2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(5*b^2*B+7*a*(2*A*b+B*a))*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))/d+2/35*b*(7*A*b+9*B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*b*B*cos(
d*x+c)^(3/2)*(a+b*cos(d*x+c))*sin(d*x+c)/d+2/21*(5*b^2*B+7*a*(2*A*b+B*a))*s
in(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3069, 3102, 2827, 2719, 2715, 2720}

$$\frac{2(5a^2A+6abB+3Ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(7a(aB+2Ab)+5b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(7a(aB+2Ab)+5b^2B)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2b(9aB+7Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d}+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*b
^2*B + 7*a*(2*A*b + a*B))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*b^2*B +
 7*a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(7*A*b +
 9*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) + (2*b*B*Cos[c + d*x]^(3/2)
*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+}{7d}$$
$$= \frac{2b(7Ab+9aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} +$$
$$= \frac{2b(7Ab+9aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} +$$
$$= \frac{2(5a^2A+3Ab^2+6abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} +$$
$$= \frac{2(5a^2A+3Ab^2+6abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} +$$

Mathematica [A]
time = 1.19, size = 139, normalized size = 0.76

$$\frac{42(5a^2A+3Ab^2+6abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+10(14aAb+7a^2B+5b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\sqrt{\cos(c+dx)}\,(42b(Ab+2aB)\cos(c+dx)+5(28aAb+14a^2B+13b^2B+3b^2B\cos(2(c+dx))))\sin(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (42*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2] + 10*(14*a*A*b
+ 7*a^2*B + 5*b^2*B)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(42*b*(
A*b + 2*a*B)*Cos[c + d*x] + 5*(28*a*A*b + 14*a^2*B + 13*b^2*B + 3*b^2*B*Cos
[2*(c + d*x)]))*Sin[c + d*x])/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(218) = 436$.
time = 0.32, size = 548, normalized size = 3.01

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+\left(-168A\,b^2-336Bab-360B\,b\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^2+(-168*A*b^2-336*B*a*b-360*B*b^2)*sin
```

```
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a*b+168*A*b^2+140*B*a^2+336*B*a
*b+280*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-140*A*a*b-42*A*b^2-
70*B*a^2-84*B*a*b-80*B*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*A*a*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-63*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*b^2+35*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*b^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-126*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 243, normalized size = 1.34

$$\frac{2\left(15\,Bb^2\cos\left(dx+c\right)^2+35\,Ba^2+70\,Aab+25\,Bb^2+21\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-5\,\sqrt{2}\left(7i\,Ba^2+14i\,Aab+5i\,Bb^2\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5\,\sqrt{2}\left(-7i\,Ba^2-14i\,Aab-5i\,Bb^2\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-21\,\sqrt{2}\left(-5i\,Aa^2-6i\,Bab-3i\,Ab^2\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-21\,\sqrt{2}\left(5i\,Aa^2+6i\,Bab+3i\,Ab^2\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/105*(2*(15*B*b^2*cos(d*x + c)^2 + 35*B*a^2 + 70*A*a*b + 25*B*b^2 + 21*(2*
B*a*b + A*b^2)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sqrt(2)*(7
*I*B*a^2 + 14*I*A*a*b + 5*I*B*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) - 5*sqrt(2)*(-7*I*B*a^2 - 14*I*A*a*b - 5*I*B*b^2)*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*sqrt(2)*(-5*I*A*a^2
 - 6*I*B*a*b - 3*I*A*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(5*I*A*a^2 + 6*I*B*a*b + 3*I*
A*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

Mupad [**B**]
time = 1.34, size = 229, normalized size = 1.26

$$\frac{2\,B\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,b\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}-\frac{2\,A\,b^2\,\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,B\,b^2\,\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{4\,B\,a\,b\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*B*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(
3*d) + (2*A*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*a*b*((2*cos(c + d*x)^
(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d - (2*A*b^2*co
s(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/
(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hyp
ergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (4
*B*a*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.355 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{2(3b^2B+5a(2Ab+aB))\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{2(3a^2A+Ab^2+2abB)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d}+\frac{2b(5Ab+7aB)\sqrt{\cos(\cdots}}{15\cdots}$$

```
[Out] 2/5*(3*b^2*B+5*a*(2*A*b+B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(3*A*a^2+A*b^2+2*B*a*b)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))/d+2/15*b*(5*A*b+7*B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+2/5*b*B*(a+b*co
s(d*x+c))*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.18, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3069, 3102, 2827, 2720, 2719}

$$\frac{2(3a^2A+2abB+Ab^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d}+\frac{2(5a(aB+2Ab)+3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{2b(7aB+5Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(3*b^2*B + 5*a*(2*A*b + a*B))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(3*a
^2*A + A*b^2 + 2*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*(5*A*b + 7*
a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*B*Sqrt[Cos[c + d*x]]*(a
 + b*Cos[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2bB\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int$$

$$= \frac{2b(5Ab+7aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2bB\sqrt{\cos(c}{}$$

$$= \frac{2b(5Ab+7aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2bB\sqrt{\cos(c}{}$$

$$= \frac{2(3b^2B+5a(2Ab+aB))\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{2(3a^2A+A}{}$$

Mathematica [A]
time = 0.63, size = 106, normalized size = 0.76

$$\frac{2\left(3(10aAb+5a^2B+3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5(3a^2A+Ab^2+2abB)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+b\sqrt{\cos(c+dx)}\,(5Ab+10aB+3bB\cos(c+dx))\sin(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (2*(3*(10*a*A*b + 5*a^2*B + 3*b^2*B)*EllipticE[(c + d*x)/2, 2] + 5*(3*a^2*A
 + A*b^2 + 2*a*b*B)*EllipticF[(c + d*x)/2, 2] + b*Sqrt[Cos[c + d*x]]*(5*A*b
 + 10*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(180) = 360$.
time = 0.31, size = 487, normalized size = 3.48

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+\left(20A\,b^2+40Bab+24B\,b^2\right)\left(\sin\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^2+(20*A*b^2+40*B*a*b+24*B*b^2)*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b^2-20*B*a*b-6*B*b^2)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+15*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*A*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+10*B*a*b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b
^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 216, normalized size = 1.54

$$\frac{2\,(3\,Bb^2\cos(dx+c)+10\,Bab+5\,Ab^2)\sqrt{\cos(dx+c)}\,\sin(dx+c)-5\,\sqrt{2}\,(3i\,Aa^2+2i\,Bab+i\,Ab^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-5\,\sqrt{2}\,(-3i\,Aa^2-2i\,Bab-i\,Ab^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3\,\sqrt{2}\,(-5i\,Ba^2-10i\,Aab-3i\,Bb^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3\,\sqrt{2}\,(5i\,Ba^2+10i\,Aab+3i\,Bb^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/15*(2*(3*B*b^2*cos(d*x + c) + 10*B*a*b + 5*A*b^2)*sqrt(cos(d*x + c))*sin(
d*x + c) - 5*sqrt(2)*(3*I*A*a^2 + 2*I*B*a*b + I*A*b^2)*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-3*I*A*a^2 - 2*I*B*a*b -
 I*A*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqr
t(2)*(-5*I*B*a^2 - 10*I*A*a*b - 3*I*B*b^2)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(5*I*B*a^2 +
10*I*A*a*b + 3*I*B*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

Mupad [B]
time = 1.34, size = 177, normalized size = 1.26

$$\frac{A\,b^2\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,A\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,b\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{4\,A\,a\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}-\frac{2\,B\,b^2\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^(1/2),x)
```

```
[Out] (A*b^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d + (2*A*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a^2*ellipticE(c
/2 + (d*x)/2, 2))/d + (2*B*a*b*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*
ellipticF(c/2 + (d*x)/2, 2))/3))/d + (4*A*a*b*ellipticE(c/2 + (d*x)/2, 2))/
d - (2*B*b^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, co
s(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.356 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$-\frac{2(a^2A-Ab^2-2abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(6aAb+3a^2B+b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2}{}$$

```
[Out] -2*(A*a^2-A*b^2-2*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(6*A*a*b+3*B*a^2+B*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
)/d+2*a^2*A*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*b^2*B*sin(d*x+c)*cos(d*x+c)^(
1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3067, 3102, 2827, 2720, 2719}

$$\frac{2(3a^2B+6aAb+b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(a^2A-2abB-Ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*A*b +
3*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*A*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-2\int \frac{-\frac{1}{2}a(2Ab+aB)+\frac{1}{2}(a^2A-Ab^2-\\ &= \frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}-\frac{4}{3}\int\\ &= \frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}-(a^2A\\ &= -\frac{2(a^2A-Ab^2-2abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d}+\frac{2(6aAb+3a^2B\end{aligned}$$

Mathematica [A]
time = 0.67, size = 102, normalized size = 0.84

$$\frac{2\left((-3a^2A+3Ab^2+6abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+(6aAb+3a^2B+b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\frac{(3a^2A+b^2B\cos(c+dx))\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

[Out] (2*((-3*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2] + (6*a*A*b + 3 *a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2] + ((3*a^2*A + b^2*B*Cos[c + d*x]) *Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(3*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(165) = 330$.
time = 0.35, size = 405, normalized size = 3.35

method	result
default	$-\frac{-\frac{8B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{3}+4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2-4Aab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/3*(-4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^2+6*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2-6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2-3*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 240, normalized size = 1.98

$$\frac{\sqrt{2}\left(-3i\,Ba^2-6i\,Aab-i\,Bb^2\right)\cos\left(dx+c\right)\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(3i\,Ba^2+6i\,Aab+i\,Bb^2\right)\cos\left(dx+c\right)\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\sqrt{2}\left(i\,Aa^2-2i\,Bab-i\,Ab^2\right)\cos\left(dx+c\right)\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\sqrt{2}\left(-i\,Aa^2+2i\,Bab+i\,Ab^2\right)\cos\left(dx+c\right)\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\left(Bb^2\cos\left(dx+c\right)+3\,Aa^2\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*B*a^2 - 6*I*A*a*b - I*B*b^2)*cos(d*x + c)*weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*B*a^2 + 6*I*A*a*
b + I*B*b^2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 3*sqrt(2)*(I*A*a^2 - 2*I*B*a*b - I*A*b^2)*cos(d*x + c)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
3*sqrt(2)*(-I*A*a^2 + 2*I*B*a*b + I*A*b^2)*cos(d*x + c)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(B*b^2*c
os(d*x + c) + 3*A*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

Mupad [B]
time = 1.57, size = 158, normalized size = 1.31

$$\frac{B\,b^2\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,A\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,A\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,B\,a\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^(3/2),x)
```

```
[Out] (B*b^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d + (2*A*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*a^2*ellipticF(c
/2 + (d*x)/2, 2))/d + (4*A*a*b*ellipticF(c/2 + (d*x)/2, 2))/d + (4*B*a*b*el
lipticE(c/2 + (d*x)/2, 2))/d + (2*A*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2],
 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.357 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=126

$$-\frac{2(2aAb+a^2B-b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(a^2A+3Ab^2+6abB)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(\ldots}{}$$

```
[Out] -2*(2*A*a*b+B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*a^2+3*A*b^2+6*B*a*b)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
)/d+2/3*a^2*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*a*(2*A*b+B*a)*sin(d*x+c)/d/co
s(d*x+c)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3067, 3100, 2827, 2720, 2719}

$$\frac{2(a^2A+6abB+3Ab^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(a^2B+2aAb-b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(aB+2Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2*A + 3*
A*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*A*Sin[c + d*x])/
(3*d*Cos[c + d*x]^(3/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c +
 d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2}{3}\int \frac{-\frac{3}{2}a(2Ab+aB)-\frac{1}{2}(a^2A+3Ab^2\\ &= \frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(2Ab+aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{4}{3}\int \frac{\frac{1}{4}(-a\\ &= \frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(2Ab+aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{1}{3}\big(-a^2A\\ &= -\frac{2(2aAb+a^2B-b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d}+\frac{2(a^2A+3Ab^2\end{aligned}$$

Mathematica [A]
time = 1.23, size = 105, normalized size = 0.83

$$\frac{2\left(-3(2aAb+a^2B-b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+(a^2A+3Ab^2+6abB)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\frac{a(aA+3(2Ab+aB)\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2), x]

[Out] (2*(-3*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2] + (a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2] + (a*(a*A + 3*(2*A*b + a*B)*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(170) = 340$.
time = 0.56, size = 650, normalized size = 5.16

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2Bb^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a^2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a*(2*A*b+B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 255, normalized size = 2.02

$$\frac{\sqrt{2}\left(-i\,Aa^2-6i\,Bab-3i\,Ab^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(i\,Aa^2+6i\,Bab+3i\,Ab^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\,\sqrt{2}\left(i\,Ba^2+2i\,Aab-i\,Bb^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\,\sqrt{2}\left(-i\,Ba^2-2i\,Aab+i\,Bb^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\left(Aa^2+3\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A*a^2 - 6*I*B*a*b - 3*I*A*b^2)*cos(d*x + c)^2*weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a^2 + 6*I*B*a*
b + 3*I*A*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c)) - 3*sqrt(2)*(I*B*a^2 + 2*I*A*a*b - I*B*b^2)*cos(d*x + c)^2*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
))) - 3*sqrt(2)*(-I*B*a^2 - 2*I*A*a*b + I*B*b^2)*cos(d*x + c)^2*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*
(A*a^2 + 3*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))
/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5990 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x
)
```

Mupad [B]
time = 2.29, size = 194, normalized size = 1.54

$$\frac{2\,A\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,B\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{4\,A\,a\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*A*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*b^2*ellipticE(c/2 + (d*x)/2,
 2))/d + (4*B*a*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^2*sin(c + d*x)*hy
pergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c +
 d*x)^2)^(1/2)) + (2*B*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c +
 d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (4*A*a*b*sin(c +
d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(si
n(c + d*x)^2)^(1/2))
```

3.358 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$-\frac{2(3a^2A+5Ab^2+10abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(2aAb+a^2B+3b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+2$$

```
[Out] -2/5*(3*A*a^2+5*A*b^2+10*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(2*A*a*b+B*a^2+3*B*b^2)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))/d+2/5*a^2*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/3*a*(2*A*b+B*a)*sin(d*
x+c)/d/cos(d*x+c)^(3/2)+2/5*(3*A*a^2+5*A*b^2+10*B*a*b)*sin(d*x+c)/d/cos(d*x
+c)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3067, 3100, 2827, 2716, 2719, 2720}

$$\frac{2(a^2B+2aAb+3b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(3a^2A+10abB+5Ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(3a^2A+10abB+5Ab^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a(aB+2Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(2
*a*A*b + a^2*B + 3*b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*A*Sin[c
 + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(3*d*C
os[c + d*x]^(3/2)) + (2*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Sin[c + d*x])/(5*d*S
qrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(2Ab+aB)-\frac{1}{2}(3a^2A+5Ab^2}{\text{co}}$$

$$= \frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}(}{}$$

$$= \frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{5}(-3a^2A$$

$$= \frac{2(2aAb+a^2B+3b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d} + \frac{2a^2A\sin(c+dx}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{2(3a^2A+5Ab^2+10abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{2(2aAb+a}{}$$

Mathematica [A]
time = 1.16, size = 175, normalized size = 1.02

$$\frac{-6(3a^2A+5Ab^2+10abB)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+10(2aAb+a^2B+3b^2B)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+20aAb\sin(c+dx)+10a^2B\sin(c+dx)+9a^2A\sin(2(c+dx))+15Ab^2\sin(2(c+dx))+30abB\sin(2(c+dx))+6a^2A\tan(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] (-6*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2
, 2] + 10*(2*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x
)/2, 2] + 20*a*A*b*Sin[c + d*x] + 10*a^2*B*Sin[c + d*x] + 9*a^2*A*Sin[2*(c
+ d*x)] + 15*A*b^2*Sin[2*(c + d*x)] + 30*a*b*B*Sin[2*(c + d*x)] + 6*a^2*A*T
an[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(208) = 416$.
time = 0.83, size = 723, normalized size = 4.20

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2Bb^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a
*(2*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(A*b+2*B*a)/sin(
1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))+2/5*a^2*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6
*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 286, normalized size = 1.66

$$-\frac{5\sqrt{2}\left(iBa^2+2iAab+3iBb^2\right)\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-iBa^2-2iAab-3iBb^2\right)\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+3\sqrt{2}\left(3iAa^2+10iBab+5iAb^2\right)\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(-3iAa^2-10iBab-5iAb^2\right)\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-2\left(3Aa^2+3\left(3Aa^2+10Bab+5Ab^2\right)\cos\left(dx+c\right)^2+5\left(Ba^2+2Aab\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(I*B*a^2 + 2*I*A*a*b + 3*I*B*b^2)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a^2 - 2*
```

```
I*A*a*b - 3*I*B*b^2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 3*sqrt(2)*(3*I*A*a^2 + 10*I*B*a*b + 5*I*A*b^2)*cos(d*x
 + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) + 3*sqrt(2)*(-3*I*A*a^2 - 10*I*B*a*b - 5*I*A*b^2)*cos(d*x +
c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c))) - 2*(3*A*a^2 + 3*(3*A*a^2 + 10*B*a*b + 5*A*b^2)*cos(d*x + c)^2
+ 5*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos
(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x
)
```

Mupad [**B**]
time = 2.62, size = 227, normalized size = 1.32

$$\frac{6\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+30\,A\,b^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)+20\,A\,a\,b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{15\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{1-\cos\left(c+d\,x\right)^2}}+\frac{2\,B\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{4\,B\,a\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^(7/2),x)
```

```
[Out] (6*A*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*A*b
^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2)
+ 20*A*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*
x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (2*B*b^2*elli
pticF(c/2 + (d*x)/2, 2))/d + (2*B*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1
/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (4*B
*a*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d
*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.359 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=305

$$\frac{2(27a^2Ab+7Ab^3+9a^3B+21ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(77a^3A+165aAb^2+165a^2bB+45b^3B)\,F\left(\frac{1}{2}(c+\right.}{231d}$$

```
[Out] 2/15*(27*A*a^2*b+7*A*b^3+9*B*a^3+21*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/231*(77*A*a^3+1
65*A*a*b^2+165*B*a^2*b+45*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*(27*A*a^2*b+7*A*b^3+9*B*
a^3+21*B*a*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/77*b*(33*A*a*b+26*B*a^2+9*B
*b^2)*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/99*b^2*(11*A*b+15*B*a)*cos(d*x+c)^(7/
2)*sin(d*x+c)/d+2/11*b*B*cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+2
/231*(77*A*a^3+165*A*a*b^2+165*B*a^2*b+45*B*b^3)*sin(d*x+c)*cos(d*x+c)^(1/2
)/d
```

Rubi [A]
time = 0.35, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3069, 3112, 3102, 2827, 2715, 2720, 2719}

$$\frac{2b(26a^2B+33aAb+9b^2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{77d}+\frac{2(77a^3A+165a^2bB+165aAb^2+45b^3B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2(9a^3B+27a^2Ab+21ab^2B+7Ab^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(9a^3B+27a^2Ab+21ab^2B+7Ab^3)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2(77a^3A+165a^2bB+165aAb^2+45b^3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{231d}+\frac{2b^2(15aB+11Ab)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{99d}+\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(27*a^2*A*b + 7*A*b^3 + 9*a^3*B + 21*a*b^2*B)*EllipticE[(c + d*x)/2, 2])
/(15*d) + (2*(77*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 45*b^3*B)*EllipticF[(c
 + d*x)/2, 2])/(231*d) + (2*(77*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 45*b^3*
B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(27*a^2*A*b + 7*A*b^3 + 9*
a^3*B + 21*a*b^2*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(33*a*A*
b + 26*a^2*B + 9*b^2*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*b^2*(1
1*A*b + 15*a*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*b*B*Cos[c + d*
x]^(5/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(11*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
```

```
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\sin(c+}{11d}$$

$$= \frac{2b^2(11Ab+15aB)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d} +$$

$$= \frac{2b(33aAb+26a^2B+9b^2B)\cos^{\frac{5}{2}}(c+dx)\sin(}{77d}$$

$$= \frac{2b(33aAb+26a^2B+9b^2B)\cos^{\frac{5}{2}}(c+dx)\sin(}{77d}$$

$$= \frac{2(77a^3A+165aAb^2+165a^2bB+45b^3B)\sqrt{c}}{231d}$$

$$= \frac{2(27a^2Ab+7Ab^3+9a^3B+21ab^2B)E\left(\frac{1}{2}(c+\right.}{15d}$$

Mathematica [A]
time = 2.04, size = 235, normalized size = 0.77

$$\frac{3696(27a^2Ab+7Ab^3+9a^3B+21ab^2B)E\left(\frac{1}{2}(c+dx)\middle|2\right)+240(77a^3A+165aAb^2+165a^2bB+45b^3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}\,(154(108a^2Ab+43Ab^3+36a^3B+129ab^2B)\cos(c+dx)+180b(33aAb+33a^2B+16b^2B)\cos(2(c+dx))+770b^2(Ab+3aB)\cos(3(c+dx))+15(616a^3A+1716aAb^2+1716a^2bB+531b^3B+21b^3B\cos(4(c+dx))))\sin(c+dx)}{27720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3696*(27*a^2*A*b + 7*A*b^3 + 9*a^3*B + 21*a*b^2*B)*EllipticE[(c + d*x)/2,
2] + 240*(77*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 45*b^3*B)*EllipticF[(c + d
*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(154*(108*a^2*A*b + 43*A*b^3 + 36*a^3*B +
129*a*b^2*B)*Cos[c + d*x] + 180*b*(33*a*A*b + 33*a^2*B + 16*b^2*B)*Cos[2*(c
 + d*x)] + 770*b^2*(A*b + 3*a*B)*Cos[3*(c + d*x)] + 15*(616*a^3*A + 1716*a*
A*b^2 + 1716*a^2*b*B + 531*b^3*B + 21*b^3*B*Cos[4*(c + d*x)]))*Sin[c + d*x]
)/(27720*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 824 vs. 2(333) = 666.
time = 0.37, size = 825, normalized size = 2.70

method	result	size
default	Expression too large to display	825

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12*b^3+(-12320*A*b^3-36960*B*a*b^2-5040
0*B*b^3)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(23760*A*a*b^2+24640*A*b^
3+23760*B*a^2*b+73920*B*a*b^2+56880*B*b^3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x
+1/2*c)+(-16632*A*a^2*b-35640*A*a*b^2-22792*A*b^3-5544*B*a^3-35640*B*a^2*b-
68376*B*a*b^2-34920*B*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(4620*A*
a^3+16632*A*a^2*b+27720*A*a*b^2+10472*A*b^3+5544*B*a^3+27720*B*a^2*b+31416*
B*a*b^2+13860*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2310*A*a^3-4
158*A*a^2*b-7920*A*a*b^2-1848*A*b^3-1386*B*a^3-7920*B*a^2*b-5544*B*a*b^2-27
90*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1155*A*a^3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+2475*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6237*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*a^2*b-1617*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+2475*a^2*b*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+675*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a^3-4851*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 358, normalized size = 1.17

$$\frac{2\,\left(315\,Bb^3\cos\left(dx+c\right)^4+1155\,Aa^3+2475\,Ba^2b+2475\,Aab^2+675\,Bb^3+385\,\left(3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)^3+135\,\left(11\,Ba^2b+11\,Aab^2+3\,Bb^3\right)\cos\left(dx+c\right)^2+77\,\left(9\,Ba^3+27\,Aa^2b+21\,Bab^2+7\,Ab^3\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-15\,\sqrt{2}\,\left(77i\,Aa^3+165i\,Ba^2b+165i\,Aab^2+45i\,Bb^3\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)-15\,\sqrt{2}\,\left(-77i\,Aa^3-165i\,Ba^2b-165i\,Aab^2-45i\,Bb^3\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-231\,\sqrt{2}\,\left(-9i\,Ba^3-27i\,Aa^2b-21i\,Bab^2-7i\,Ab^3\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-231\,\sqrt{2}\,\left(9i\,Ba^3+27i\,Aa^2b+21i\,Bab^2+7i\,Ab^3\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/3465*(2*(315*B*b^3*cos(d*x + c)^4 + 1155*A*a^3 + 2475*B*a^2*b + 2475*A*a*
b^2 + 675*B*b^3 + 385*(3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + 135*(11*B*a^2*b
+ 11*A*a*b^2 + 3*B*b^3)*cos(d*x + c)^2 + 77*(9*B*a^3 + 27*A*a^2*b + 21*B*a*
b^2 + 7*A*b^3)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*(
77*I*A*a^3 + 165*I*B*a^2*b + 165*I*A*a*b^2 + 45*I*B*b^3)*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 15*sqrt(2)*(-77*I*A*a^3 - 165*I*B
*a^2*b - 165*I*A*a*b^2 - 45*I*B*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c)) - 231*sqrt(2)*(-9*I*B*a^3 - 27*I*A*a^2*b - 21*I*B*a*b^2
 - 7*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c))) - 231*sqrt(2)*(9*I*B*a^3 + 27*I*A*a^2*b + 21*I*B*a*b^2
 + 7*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7319 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x
)
```

Mupad [B]
time = 1.74, size = 364, normalized size = 1.19

$$\frac{A\,a^3\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}-\frac{2\,B\,a^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,A\,b^3\,{\cos\left(c+d\,x\right)}^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,b^3\,{\cos\left(c+d\,x\right)}^{13/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{13}{4};\ \frac{17}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{13\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{6\,A\,a^2\,b\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,A\,a\,b^2\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,a^2\,b\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{6\,B\,a\,b^2\,{\cos\left(c+d\,x\right)}^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

```
[Out] (A*a^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d - (2*B*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4],
 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*b^3*cos(c + d*x
)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(
sin(c + d*x)^2)^(1/2)) - (2*B*b^3*cos(c + d*x)^(13/2)*sin(c + d*x)*hypergeo
m([1/2, 13/4], 17/4, cos(c + d*x)^2))/(13*d*(sin(c + d*x)^2)^(1/2)) - (6*A*
a^2*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d
*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a*b^2*cos(c + d*x)^(9/2)*sin(c
+ d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(
1/2)) - (2*B*a^2*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13
/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2)) - (6*B*a*b^2*cos(c + d*x)
^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(s
in(c + d*x)^2)^(1/2))
```

3.360 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=255

$$\frac{2(15a^3A+27aAb^2+27a^2bB+7b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(21a^2Ab+5Ab^3+7a^3B+15ab^2B)\,F\left(\frac{1}{2}(c+dx)\right.$$

```
[Out] 2/15*(15*A*a^3+27*A*a*b^2+27*B*a^2*b+7*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(21*A*a^2*b
+5*A*b^3+7*B*a^3+15*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*b*(27*A*a*b+22*B*a^2+7*B*b^2
)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/63*b^2*(9*A*b+13*B*a)*cos(d*x+c)^(5/2)*si
n(d*x+c)/d+2/9*b*B*cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+2/21*(2
1*A*a^2*b+5*A*b^3+7*B*a^3+15*B*a*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.32, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3069, 3112, 3102, 2827, 2719, 2715, 2720}

$$\frac{2b(22a^2B+27aAb+7b^2B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2(7a^3B+21a^2Ab+15ab^2B+5Ab^3)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(15a^3A+27a^2bB+27aAb^2+7b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(7a^3B+21a^2Ab+15ab^2B+5Ab^3)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2b^2(13aB+9Ab)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*EllipticE[(c + d*x)/2, 2]
)/(15*d) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*EllipticF[(c +
d*x)/2, 2])/(21*d) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(27*a*A*b + 22*a^2*B + 7*b^2*B)*C
os[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b^2*(9*A*b + 13*a*B)*Cos[c + d*
x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b*B*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^2*Sin[c + d*x])/(9*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\sin(c+}{9d}$$
$$= \frac{2b^2(9Ab+13aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d}$$
$$= \frac{2b(27aAb+22a^2B+7b^2B)\cos^{\frac{3}{2}}(c+dx)\sin}{45d}$$
$$= \frac{2b(27aAb+22a^2B+7b^2B)\cos^{\frac{3}{2}}(c+dx)\sin}{45d}$$
$$= \frac{2(15a^3A+27aAb^2+27a^2bB+7b^3B)\,E\left(\frac{1}{2}\right.}{15d}$$
$$= \frac{2(15a^3A+27aAb^2+27a^2bB+7b^3B)\,E\left(\frac{1}{2}\right.}{15d}$$

Mathematica [A]
time = 1.27, size = 197, normalized size = 0.77

$$\frac{84(15a^3A+27aAb^2+27a^2bB+7b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+60(21a^2Ab+5Ab^3+7a^3B+15ab^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(7b(108aAb+108a^2B+43b^2B)\cos(c+dx)+5(252a^2Ab+78Ab^3+84a^3B+234ab^2B+18b^2(Ab+3aB)\cos(2(c+dx))+7b^3B\cos(3(c+dx))))\sin(c+dx)}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (84*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*EllipticE[(c + d*x)/2, 2
] + 60*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*EllipticF[(c + d*x)/2,
 2] + Sqrt[Cos[c + d*x]]*(7*b*(108*a*A*b + 108*a^2*B + 43*b^2*B)*Cos[c + d*
x] + 5*(252*a^2*A*b + 78*A*b^3 + 84*a^3*B + 234*a*b^2*B + 18*b^2*(A*b + 3*a
*B)*Cos[2*(c + d*x)] + 7*b^3*B*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 744 vs. 2(287) = 574.
time = 0.35, size = 745, normalized size = 2.92

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+(720A\,b^3+2160Ba\,b^2+22\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^3+(720*A*b^3+2160*B*a*b^2+2240*B*b^
3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*A*a*b^2-1080*A*b^3-1512*B
*a^2*b-3240*B*a*b^2-2072*B*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(12
60*A*a^2*b+1512*A*a*b^2+840*A*b^3+420*B*a^3+1512*B*a^2*b+2520*B*a*b^2+952*B
*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-630*A*a^2*b-378*A*a*b^2-240
*A*b^3-210*B*a^3-378*B*a^2*b-720*B*a*b^2-168*B*b^3)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+315*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+75*A*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b^2+105*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*B*a*b^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-567*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-147*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 321, normalized size = 1.26

$$\frac{2\left(35\,Bb^3\cos(dx+c)^3+105\,Ba^3+315\,Aa^2b+225\,Bab^2+75\,Ab^3+45\left(3\,Bab^2+Ab^3\right)\cos(dx+c)^2+7\left(27\,Ba^2b+27\,Aab^2+7\,Bb^3\right)\cos(dx+c)\right)\sqrt{\cos(dx+c)}\sin(dx+c)-15\sqrt{2}\left(7i\,Ba^3+21i\,Aa^2b+15i\,Bab^2+5i\,Ab^3\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-15\sqrt{2}\left(-7i\,Ba^3-21i\,Aa^2b-15i\,Bab^2-5i\,Ab^3\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21\sqrt{2}\left(-15i\,Aa^3-27i\,Ba^2b-27i\,Aab^2-7i\,Bb^3\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21\sqrt{2}\left(15i\,Aa^3+27i\,Ba^2b+27i\,Aab^2+7i\,Bb^3\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/315*(2*(35*B*b^3*cos(d*x + c)^3 + 105*B*a^3 + 315*A*a^2*b + 225*B*a*b^2 +
 75*A*b^3 + 45*(3*B*a*b^2 + A*b^3)*cos(d*x + c)^2 + 7*(27*B*a^2*b + 27*A*a*
b^2 + 7*B*b^3)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*(
```

```
7*I*B*a^3 + 21*I*A*a^2*b + 15*I*B*a*b^2 + 5*I*A*b^3)*weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c)) - 15*sqrt(2)*(-7*I*B*a^3 - 21*I*A*a^2*b
 - 15*I*B*a*b^2 - 5*I*A*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 21*sqrt(2)*(-15*I*A*a^3 - 27*I*B*a^2*b - 27*I*A*a*b^2 - 7*I*B
*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) - 21*sqrt(2)*(15*I*A*a^3 + 27*I*B*a^2*b + 27*I*A*a*b^2 + 7*I*B
*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x
)
```

Mupad [B]
time = 1.54, size = 328, normalized size = 1.29

$$\frac{2\left(A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+A\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+A\,a^2\,b\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}+\frac{B\,a^3\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}-\frac{2\,A\,b^3\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,b^3\,{\cos\left(c+d\,x\right)}^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{6\,A\,a\,b^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{6\,B\,a^2\,b\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,a\,b^2\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

```
[Out] (2*(A*a^3*ellipticE(c/2 + (d*x)/2, 2) + A*a^2*b*ellipticF(c/2 + (d*x)/2, 2)
 + A*a^2*b*cos(c + d*x)^(1/2)*sin(c + d*x)))/d + (B*a^3*((2*cos(c + d*x)^(1
/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d - (2*A*b^3*cos(
c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9
*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hype
rgeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (
6*A*a*b^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
 + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (6*B*a^2*b*cos(c + d*x)^(7/2)*si
n(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^
2)^(1/2)) - (2*B*a*b^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4]
, 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2))
```

3.361 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=205

$$\frac{2(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(21a^3A+21aAb^2+21a^2bB+5b^3B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

```
[Out] 2/5*(15*A*a^2*b+3*A*b^3+5*B*a^3+9*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(21*A*a^3+21*A
*a*b^2+21*B*a^2*b+5*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/35*b^2*(7*A*b+11*B*a)*cos(d*x+c)^
(3/2)*sin(d*x+c)/d+2/21*b*(21*A*a*b+18*B*a^2+5*B*b^2)*sin(d*x+c)*cos(d*x+c)
^(1/2)/d+2/7*b*B*(a+b*cos(d*x+c))^2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.31, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3069, 3112, 3102, 2827, 2720, 2719}

$$\frac{2b(18a^2B+21aAb+5b^2B)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2(21a^3A+21a^2bB+21aAb^2+5b^3B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(5a^3B+15a^2Ab+9ab^2B+3Ab^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b^2(11aB+7Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d}+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/
(5*d) + (2*(21*a^3*A + 21*a*A*b^2 + 21*a^2*b*B + 5*b^3*B)*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*b*(21*a*A*b + 18*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(21*d) + (2*b^2*(7*A*b + 11*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(35*d) + (2*b*B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x]
)/(7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2bB\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2}{7}\int$$

$$= \frac{2b^2(7Ab+11aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2bB\sqrt{\cos(c+}}{}$$

$$= \frac{2b(21aAb+18a^2B+5b^2B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2}{}$$

$$= \frac{2b(21aAb+18a^2B+5b^2B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2}{}$$

$$= \frac{2(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(2}{}$$

Mathematica [A]
time = 1.39, size = 158, normalized size = 0.77

$$\frac{42(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(21a^3A+21aAb^2+21a^2bB+5b^3B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sqrt{\cos(c+dx)}\,(42b(Ab+3aB)\cos(c+dx)+5(42aAb+42a^2B+13b^2B+3b^2B\cos(2(c+dx))))\sin(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (42*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2]
+ 10*(21*a^3*A + 21*a*A*b^2 + 21*a^2*b*B + 5*b^3*B)*EllipticF[(c + d*x)/2,
2] + b*Sqrt[Cos[c + d*x]]*(42*b*(A*b + 3*a*B)*Cos[c + d*x] + 5*(42*a*A*b +
42*a^2*B + 13*b^2*B + 3*b^2*B*Cos[2*(c + d*x)]))*Sin[c + d*x])/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(241) = 482.
time = 0.36, size = 664, normalized size = 3.24

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+(-168A\,b^3-504Ba\,b^2-360b^3B\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^3+(-168*A*b^3-504*B*a*b^2-360*B*b^3)*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a*b^2+168*A*b^3+420*B*a^2*b+5
04*B*a*b^2+280*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*A*a*b^2
-42*A*b^3-210*B*a^2*b-126*B*a*b^2-80*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)+105*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*A*a*b^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^
3+105*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10
5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 284, normalized size = 1.39

$$\frac{2\left(15\,Bb^3\cos\left(dx+c\right)^2+105\,Ba^2b+105\,Aab^2+25\,Bb^3+21\left(3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-5\sqrt{2}\left(21i\,Aa^3+21i\,Ba^2b+21i\,Aab^2+5i\,Bb^3\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5\sqrt{2}\left(-21i\,Aa^3-21i\,Ba^2b-21i\,Aab^2-5i\,Bb^3\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-21\sqrt{2}\left(-5i\,Ba^3-15i\,Aa^2b-9i\,Bab^2-3i\,Ab^3\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-21\sqrt{2}\left(5i\,Ba^3+15i\,Aa^2b+9i\,Bab^2+3i\,Ab^3\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/105*(2*(15*B*b^3*cos(d*x + c)^2 + 105*B*a^2*b + 105*A*a*b^2 + 25*B*b^3 +
21*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sq
rt(2)*(21*I*A*a^3 + 21*I*B*a^2*b + 21*I*A*a*b^2 + 5*I*B*b^3)*weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-21*I*A*a^3 - 21*I
*B*a^2*b - 21*I*A*a*b^2 - 5*I*B*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c
```

```
) - I*sin(d*x + c)) - 21*sqrt(2)*(-5*I*B*a^3 - 15*I*A*a^2*b - 9*I*B*a*b^2 -
 3*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) - 21*sqrt(2)*(5*I*B*a^3 + 15*I*A*a^2*b + 9*I*B*a*b^2 + 3
*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x
)
```

Mupad [B]
time = 1.43, size = 275, normalized size = 1.34

$$\frac{2\left(B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+B\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+B\,a^2\,b\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}+\frac{2\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a^2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{3\,A\,a\,b^2\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}-\frac{2\,A\,b^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,b^3\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{6\,B\,a\,b^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*(B*a^3*ellipticE(c/2 + (d*x)/2, 2) + B*a^2*b*ellipticF(c/2 + (d*x)/2, 2)
 + B*a^2*b*cos(c + d*x)^(1/2)*sin(c + d*x)))/d + (2*A*a^3*ellipticF(c/2 + (
d*x)/2, 2))/d + (6*A*a^2*b*ellipticE(c/2 + (d*x)/2, 2))/d + (3*A*a*b^2*((2*
cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d
- (2*A*b^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(
c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b^3*cos(c + d*x)^(9/2)*sin
(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2
)^(1/2)) - (6*B*a*b^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4],
 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.362 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=202

$$-\frac{2(5a^3A-15aAb^2-15a^2bB-3b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(9a^2Ab+Ab^3+3a^3B+3ab^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] -2/5*(5*A*a^3-15*A*a*b^2-15*B*a^2*b-3*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(9*A*a^2*b+A*
b^3+3*B*a^3+3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d-2/5*b^2*(5*A*a-B*b)*cos(d*x+c)^(3/2)*si
n(d*x+c)/d+2*a*A*(a+b*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)-2/3*b*(6*
A*a^2-A*b^2-3*B*a*b)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.30, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3068, 3112, 3102, 2827, 2720, 2719}

$$-\frac{2b(6a^2A-3abB-Ab^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2(3a^3B+9a^2Ab+3ab^2B+Ab^3)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(5a^3A-15a^2bB-15aAb^2-3b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{2b^2(5aA-bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*EllipticE[(c + d*x)/2, 2]
)/(5*d) + (2*(9*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B)*EllipticF[(c + d*x)/
2, 2])/(3*d) - (2*b*(6*a^2*A - A*b^2 - 3*a*b*B)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(3*d) - (2*b^2*(5*a*A - b*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) +
 (2*a*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+b\cos(c+$$

$$= -\frac{2b^2(5aA-bB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2aA(a+b\cos}{d\sqrt{}}$$

$$= -\frac{2b(6a^2A-Ab^2-3abB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2}{}$$

$$= -\frac{2b(6a^2A-Ab^2-3abB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2}{}$$

$$= -\frac{2(5a^3A-15aAb^2-15a^2bB-3b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} +$$

Mathematica [A]
time = 1.20, size = 150, normalized size = 0.74

$$\frac{(-30a^3A+90aAb^2+90a^2bB+18b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(9a^2Ab+Ab^3+3a^3B+3ab^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{(10b^2(Ab+3aB)\cos(c+dx)+3(10a^3A+b^3B+b^3B\cos(2(c+dx))))\sin(c+dx)}{\sqrt{\cos(c+dx)}}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] ((-30*a^3*A + 90*a*A*b^2 + 90*a^2*b*B + 18*b^3*B)*EllipticE[(c + d*x)/2, 2]
 + 10*(9*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B)*EllipticF[(c + d*x)/2, 2] +
 ((10*b^2*(A*b + 3*a*B)*Cos[c + d*x] + 3*(10*a^3*A + b^3*B + b^3*B*Cos[2*(c
 + d*x)]))*Sin[c + d*x])/Sqrt[Cos[c + d*x]])/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(240) = 480$.
time = 0.41, size = 641, normalized size = 3.17

method	result
default	$-\frac{2\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+20A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+60B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2+24B\cos\left(\frac{dx}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15*(-24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+20*A*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^4*b^3+60*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4
*a*b^2+24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^3-30*A*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2*a^3-10*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*
b^3+45*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15
*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*a^3-45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-30*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b^2-6*B*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2*b^3+15*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*a*b^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-45*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-9*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*b^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x
)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 302, normalized size = 1.50

$$-\frac{5\sqrt{2}\left(3i\,Ba^3+9i\,Aa^2b+3i\,Bab^2+i\,Ab^3\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-3i\,Ba^3-9i\,Aa^2b-3i\,Bab^2-i\,Ab^3\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+3\sqrt{2}\left(5i\,Aa^3-15i\,Ba^2b-15i\,Aab^2-3i\,Bb^3\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(-5i\,Aa^3+15i\,Ba^2b+15i\,Aab^2+3i\,Bb^3\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-2\left(3\,Bb^3\cos\left(dx+c\right)^2+15\,Aa^3+5\left(3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(3*I*B*a^3 + 9*I*A*a^2*b + 3*I*B*a*b^2 + I*A*b^3)*cos(d*x
+ c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*
(-3*I*B*a^3 - 9*I*A*a^2*b - 3*I*B*a*b^2 - I*A*b^3)*cos(d*x + c)*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(5*I*A*a^3 - 15*
I*B*a^2*b - 15*I*A*a*b^2 - 3*I*B*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-5*I
```

```
*A*a^3 + 15*I*B*a^2*b + 15*I*A*a*b^2 + 3*I*B*b^3)*cos(d*x + c)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(
3*B*b^3*cos(d*x + c)^2 + 15*A*a^3 + 5*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4850 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x
)
```

Mupad [B]
time = 1.46, size = 248, normalized size = 1.23

$$\frac{A\,b^3\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,B\,a^2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{3\,B\,a\,b^2\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,b^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^(3/2),x)
```

```
[Out] (A*b^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d + (2*B*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*A*a*b^2*ellipticE
(c/2 + (d*x)/2, 2))/d + (6*A*a^2*b*ellipticF(c/2 + (d*x)/2, 2))/d + (6*B*a^
2*b*ellipticE(c/2 + (d*x)/2, 2))/d + (3*B*a*b^2*((2*cos(c + d*x)^(1/2)*sin(
c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d + (2*A*a^3*sin(c + d*x)
*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c
+ d*x)^2)^(1/2)) - (2*B*b^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2,
 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.363 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=192

$$-\frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(a^3A+9aAb^2+9a^2bB+b^3B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(\cdots}{}$$

```
[Out] -2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*a^3+9*A*a*b^2+9*B*
a^2*b+B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*A*(a+b*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c
)^(3/2)+2/3*a^2*(7*A*b+3*B*a)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-2/3*b^2*(A*a-B*
b)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.29, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3068, 3110, 3102, 2827, 2720, 2719}

$$\frac{2a^2(3aB+7Ab)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2(a^3A+9a^2bB+9aAb^2+b^3B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(a^3B+3a^2Ab-3ab^2B-Ab^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b^2(aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/d +
(2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*B)*EllipticF[(c + d*x)/2, 2])/(3*d)
 + (2*a^2*(7*A*b + 3*a*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(
a*A - b*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + b*Cos[c + d
*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+b\cos(c+
$$

$$= \frac{2a^2(7Ab+3aB)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2aA(a+b\cos(c+dx))^2\sin(}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(7Ab+3aB)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} - \frac{2b^2(aA-bB)\sqrt{\cos(c+dx)}}{3d}$$

$$= \frac{2a^2(7Ab+3aB)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} - \frac{2b^2(aA-bB)\sqrt{\cos(c+dx)}}{3d}$$

$$= -\frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{2(a^3}{}$$

Mathematica [A]
time = 1.17, size = 165, normalized size = 0.86

$$\frac{-6(3a^2Ab-Ab^3+a^3B-3ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+2(a^3A+9aAb^2+9a^2bB+b^3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+18a^2Ab\sin(c+dx)+6a^3B\sin(c+dx)+b^3B\sin(2(c+dx))+2a^3A\tan(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] (-6*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2] + 2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*B)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + 18*a^2*A*b*Sin[c + d*x] + 6*a^3*B*Sin[c + d*x
] + b^3*B*Sin[2*(c + d*x)] + 2*a^3*A*Tan[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1209 vs. $2(230) = 460$.
time = 0.68, size = 1210, normalized size = 6.30

method	result	size
default	Expression too large to display	1210

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(-8*B*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+36*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
```

```
)^4*a^2*b-2*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*a^3-18*A*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*a*b^2-18*A*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*sin(1/2*d*x+1/2*c)^2*a^2*b+6*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)
^2*b^3+12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^3+8*B*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4*b^3-18*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*
c)^2*a^2*b-2*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*b^3-6*B*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*a^3+18*B*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*s
in(1/2*d*x+1/2*c)^2*a*b^2-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3-1
8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b+A*a^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+9*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b
-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2*a^3-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+9*a^2*b*B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a^3-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.14, size = 306, normalized size = 1.59

$$\frac{\sqrt{2}\left(-i\,Aa^3-9i\,Ba^2b-9i\,Aab^2-i\,Bb^3\right)\cos\left(dx+c\right)^2\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(i\,Aa^3+9i\,Ba^2b+9i\,Aab^2+i\,Bb^3\right)\cos\left(dx+c\right)^2\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\sqrt{2}\left(i\,Ba^3+3i\,Aa^2b-3i\,Bab^2-i\,Ab^3\right)\cos\left(dx+c\right)^2\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\sqrt{2}\left(-i\,Ba^3-3i\,Aa^2b+3i\,Bab^2+i\,Ab^3\right)\cos\left(dx+c\right)^2\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\left(Bb^3\cos\left(dx+c\right)^2+Aa^3+3\left(Ba^3+3\,Aa^2b\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A*a^3 - 9*I*B*a^2*b - 9*I*A*a*b^2 - I*B*b^3)*cos(d*x + c)^
2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*
a^3 + 9*I*B*a^2*b + 9*I*A*a*b^2 + I*B*b^3)*cos(d*x + c)^2*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*B*a^3 + 3*I*A*a^2*b
 - 3*I*B*a*b^2 - I*A*b^3)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*B*a^3 - 3*I
*A*a^2*b + 3*I*B*a*b^2 + I*A*b^3)*cos(d*x + c)^2*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(B*b^3*cos(d*x
+ c)^2 + A*a^3 + 3*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x
)
```

Mupad [**B**]
time = 2.34, size = 255, normalized size = 1.33

$$\frac{2\left(A\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\,b^3+3\,A\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\,b^2\right)}{d}+\frac{B\,b^3\left(\frac{2\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{6\,B\,a\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,B\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\sqrt{\cos\left(c+d\,x\right)}\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{6\,A\,a^2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\sqrt{\cos\left(c+d\,x\right)}\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^(5/2),x)

[Out] (2*(A*b^3*ellipticE(c/2 + (d*x)/2, 2) + 3*A*a*b^2*ellipticF(c/2 + (d*x)/2, 2)))/d + (B*b^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d + (6*B*a*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (6*B*a^2*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (6*A*a^2*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))

3.364 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=204

$$-\frac{2(3a^3A+15aAb^2+15a^2bB-5b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(3a^2Ab+3Ab^3+a^3B+9ab^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-$$

```
[Out] -2/5*(3*A*a^3+15*A*a*b^2+15*B*a^2*b-5*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(3*A*a^2*b+3*
A*b^3+B*a^3+9*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/15*a^2*(9*A*b+5*B*a)*sin(d*x+c)/d/cos
(d*x+c)^(3/2)+2/5*a*A*(a+b*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/5*
a*(3*A*a^2+14*A*b^2+15*B*a*b)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3068, 3110, 3100, 2827, 2720, 2719}

$$\frac{2a(3a^2A+15abB+14Ab^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a^2(5aB+9Ab)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a^3B+3a^2Ab+9ab^2B+3Ab^3)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(3a^3A+15a^2bB+15aAb^2-5b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*EllipticE[(c + d*x)/2, 2]
)/(5*d) + (2*(3*a^2*A*b + 3*A*b^3 + a^3*B + 9*a*b^2*B)*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*a^2*(9*A*b + 5*a*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2
)) + (2*a*(3*a^2*A + 14*A*b^2 + 15*a*b*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*a*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)
)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\cos(c+d}{}$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+b\cos(c+dx))^2\sin(}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(3a^2A+14Ab^2+15abB)}{5d\sqrt{\cos(c+dx)}}$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(3a^2A+14Ab^2+15abB)}{5d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2(3a^3A+15aAb^2+15a^2bB-5b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} +$$

Mathematica [A]
time = 2.30, size = 176, normalized size = 0.86

$$\frac{-6(3a^3A+15aAb^2+15a^2bB-5b^3B)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(3a^2Ab+3Ab^3+a^3B+9ab^2B)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+10a^2(3Ab+aB)\sin(c+dx)+9a(a^2A+5Ab^2+5abB)\sin(2(c+dx))+6a^3A\tan(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] (-6*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Cos[c + d*x]^(3/2)*Ellipt
icE[(c + d*x)/2, 2] + 10*(3*a^2*A*b + 3*A*b^3 + a^3*B + 9*a*b^2*B)*Cos[c +
d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a^2*(3*A*b + a*B)*Sin[c + d*x] +
9*a*(a^2*A + 5*A*b^2 + 5*a*b*B)*Sin[2*(c + d*x)] + 6*a^3*A*Tan[c + d*x])/(1
5*d*Cos[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 969 vs. 2(240) = 480.
time = 0.83, size = 970, normalized size = 4.75

method	result	size
default	Expression too large to display	970

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
```

```
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*a*b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^3*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
2*a^2*(3*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*a*b*(A*b+B*a)
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))+2/5*A*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c
)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 326, normalized size = 1.60

$$-\frac{5\sqrt{2}\left(i\,Ba^3+3i\,Aa^2b+9i\,Bab^2+3i\,Ab^3\right)\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-i\,Ba^3-3i\,Aa^2b-9i\,Bab^2-3i\,Ab^3\right)\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+3\sqrt{2}\left(3i\,Aa^3+15i\,Ba^2b+15i\,Aab^2-5i\,Bb^3\right)\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(-3i\,Aa^3-15i\,Ba^2b-15i\,Aab^2+5i\,Bb^3\right)\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-2\left(3\,Aa^3+9\left(Aa^3+5\,Ba^2b+5\,Aab^2\right)\cos\left(dx+c\right)^2+5\left(Ba^3+3\,Aa^2b\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(I*B*a^3 + 3*I*A*a^2*b + 9*I*B*a*b^2 + 3*I*A*b^3)*cos(d*x
+ c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2
)*(-I*B*a^3 - 3*I*A*a^2*b - 9*I*B*a*b^2 - 3*I*A*b^3)*cos(d*x + c)^3*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(3*I*A*a^3 +
 15*I*B*a^2*b + 15*I*A*a*b^2 - 5*I*B*b^3)*cos(d*x + c)^3*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)
*(-3*I*A*a^3 - 15*I*B*a^2*b - 15*I*A*a*b^2 + 5*I*B*b^3)*cos(d*x + c)^3*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)) - 2*(3*A*a^3 + 9*(A*a^3 + 5*B*a^2*b + 5*A*a*b^2)*cos(d*x + c)^2 + 5*(B*a
^3 + 3*A*a^2*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x
)
```

Mupad [B]
time = 3.59, size = 291, normalized size = 1.43

$$\frac{2\,\left(B\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\,b^3+3\,B\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\,b^2\right)}{d}+\frac{2\,A\,b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{5\,d\,{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,B\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{6\,A\,a\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,A\,a^2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{6\,B\,a^2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*(B*b^3*ellipticE(c/2 + (d*x)/2, 2) + 3*B*a*b^2*ellipticF(c/2 + (d*x)/2,
2)))/d + (2*A*b^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^3*sin(c + d*x)*hy
pergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c
+ d*x)^2)^(1/2)) + (2*B*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c
```

```
+ d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (6*A*a*b^2*sin
(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2
)*(sin(c + d*x)^2)^(1/2)) + (2*A*a^2*b*sin(c + d*x)*hypergeom([-3/4, 1/2],
1/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (6*B*
a^2*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c +
d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.365 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=182

$$-\frac{2(5aAb-5a^2B-3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}+\frac{2(3a^2+b^2)\,(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d}-\frac{2a^3(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\right.}{b^4(a+b)d}$$

```
[Out] -2/5*(5*A*a*b-5*B*a^2-3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d+2/3*(3*a^2+b^2)*(A*b-B*a)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))/b^4/d-2*a^3*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b^4/(a+b)/d+2/5*B*cos
(d*x+c)^(3/2)*sin(d*x+c)/b/d+2/3*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/
d
```

Rubi [A]
time = 0.50, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3069, 3128, 3138, 2719, 3081, 2720, 2884}

$$-\frac{2a^3(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^4d(a+b)}+\frac{2(3a^2+b^2)\,(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d}-\frac{2(-5a^2B+5aAb-3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}+\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*EllipticE[(c + d*x)/2, 2])/(5*b^3*d) + (2
*(3*a^2 + b^2)*(A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(3*b^4*d) - (2*a^3*(A
*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^4*(a + b)*d) + (2*(
A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*B*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5bd} + \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3aB}{2}+\frac{3}{2}bB\cos(c+dx\right.}{a+b\cos(c+dx)}}{5b}$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5bd}$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5bd}$$

$$= -\frac{2(5aAb-5a^2B-3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d} + \frac{2(Ab-aB)\sqrt{\cos(c}}{3b^2}$$

$$= -\frac{2(5aAb-5a^2B-3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d} + \frac{2(3a^2+b^2)(Ab-a}{3b}$$

Mathematica [A]
time = 12.53, size = 260, normalized size = 1.43

$$\frac{\frac{2b^2(-5aAb+5a^2B+9b^2B)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + 2b^2(5Ab+4aB)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right) - \frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right) + 4b^2\sqrt{\cos(c+dx)}\,(5Ab-5aB+3bB\cos(c+dx))\sin(c+dx) + \frac{6(-5aAb+5a^2B+3b^2B)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(-2a^2+b^2)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a\sqrt{\sin^2(c+dx)}}}{30b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*b^2*(-5*a*A*b + 5*a^2*B + 9*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2])/(a + b) + 2*b^2*(5*A*b + 4*a*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*
EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + 4*b^2*Sqrt[Cos[c + d*
x]]*(5*A*b - 5*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x] + (6*(-5*a*A*b + 5*a^
2*B + 3*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a +
 b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-
(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^
2]))/(30*b^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(248) = 496$.
time = 0.36, size = 1074, normalized size = 5.90

method	result	size

default	Expression too large to display	1074

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*B*a*b^3
+24*B*b^4)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*a*b^3-20*A*b^4-20*
B*a^2*b^2+44*B*a*b^3-24*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10
*A*a*b^3+10*A*b^4+10*B*a^2*b^2-16*B*a*b^3+6*B*b^4)*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-15*A*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*a^2*b^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-5*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*b^4+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a*b^3-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^3*b-15*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^4+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-5*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*a^2*b^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-15*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a^3*b+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a*b^3+9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))*a^4)/b^4/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(b*cos(d*
x + c) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{5/2}\,\left(A+B\,\cos(c+d\,x)\right)}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

3.366 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=137

$$\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2(3aAb-3a^2B-b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}+\frac{2a^2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3(a+b)d}$$

```
[Out] 2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/b^2/d-2/3*(3*A*a*b-3*B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d+2
*a^2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(s
in(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b^3/(a+b)/d+2/3*B*sin(d*x+c)*cos(d*x+c
)^(1/2)/b/d
```

Rubi [A]
time = 0.32, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3069, 3138, 2719, 3081, 2720, 2884}

$$\frac{2a^2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2(-3a^2B+3aAb-b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}+\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b^2*d) - (2*(3*a*A*b - 3*a^2*B -
 b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*b^3*d) + (2*a^2*(A*b - a*B)*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*B*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3*b*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{aB}{2}+\frac{1}{2}bB\cos(c+dx)+\frac{3}{2}(Ab-aB)\cos^2}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+d}}{3b}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} - \frac{2\int \frac{-\frac{1}{2}abB+\frac{1}{2}(3aAb-3a^2B-b^2B)\cos(c+}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx)}}{3b^2}$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} +$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} - \frac{2(3aAb-3a^2B-b^2B)\,F\left(\frac{1}{2}(c+\right.}{3b^3d}$$

Mathematica [**A**]
time = 11.47, size = 207, normalized size = 1.51

$$\frac{\frac{(3Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + B\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right) - \frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right) + 2B\sqrt{\cos(c+dx)}\,\sin(c+dx) + \frac{3(Ab-aB)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(-2a^2+b^2)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab^2\sqrt{\sin^2(c+dx)}}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (((3*A*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + B*(2*E
llipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
(a + b)) + 2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x] + (3*(A*b - a*B)*(-2*a*b*Ell
ipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c +
 d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/(3*b*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 821 vs. $2(209) = 418$.
time = 0.47, size = 822, normalized size = 6.00

method	result	size
default	Expression too large to display	822

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2+4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^4*b^3+3*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
```

```
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*a*b^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-3*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2*b+2*B*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2*a*b^2-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3-3*a^3*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+3*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*a*b^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a^2*b+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b
),2^(1/2))*a^3)/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3066 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

3.367 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=89

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a+b)d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/b/d+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d-2*a*(A*b-B*a)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),
2^(1/2))/b^2/(a+b)/d
```

Rubi [A]

time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3081, 2719, 2882, 2720, 2884}

$$\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(b*d) + (2*(A*b - a*B)*EllipticF[(c + d*x)/
2, 2])/(b^2*d) - (2*a*(A*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(b^2*(a + b)*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2882

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{B\int \sqrt{\cos(c+dx)}\,dx}{b} - \frac{(-Ab+aB)\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx}{b}$$

$$= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} + \frac{(Ab-aB)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2} - \frac{(a(A$$

$$= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} + \frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} - \frac{2a(Ab-$$

Mathematica [A]
time = 10.94, size = 128, normalized size = 1.44

$$\frac{Ab\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)-\frac{2B\left(bE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+a\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{\sqrt{\sin^2(c+dx)}}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (A*b*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(a + b)) - (2*B*(b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a
+ b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + a*EllipticPi[-(b/a), ArcSi
n[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/Sqrt[Sin[c + d*x]^2])/(b^2*d)
```

Maple [A]
time = 0.28, size = 295, normalized size = 3.31

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{EllipticF}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-A*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a*b-B*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*a*b+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+B*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2)/b^2/(a-b)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

3.368 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=61

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/b/d+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b/(a+b)/d
```

Rubi [A]
time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3081, 2720, 2884}

$$\frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(b*d) + (2*(A*b - a*B)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(b*(a + b)*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx = \frac{B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b} - \frac{(-Ab+aB)\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c\cdots}}{b}$$
$$= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{bd} + \frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)}{b(a+b)d}$$

Mathematica [A]
time = 0.22, size = 58, normalized size = 0.95

$$\frac{2\left((a+b)BF\left(\frac{1}{2}(c+dx)\middle|\,2\right)+(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)\right)}{b(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*((a + b)*B*EllipticF[(c + d*x)/2, 2] + (A*b - a*B)*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]))/(b*(a + b)*d)
```

Maple [A]
time = 0.29, size = 217, normalized size = 3.56

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{Ellipti}\cdots\right.}{b(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\mathrm{si}\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))*b+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))*a)/b/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))), x)
```

3.369 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=86

$$-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d}+\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] -2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a/d-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(a+b)/d+2*A*sin(d*x
+c)/a/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}=0.152$, Rules used = {3079, 3138, 2719, 12, 2884}

$$-\frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (-2*A*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2])/(a*(a + b)*d) + (2*A*Sin[c + d*x])/(a*d*Sqrt[Cos[c
+ d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}(-Ab+aB)-\frac{1}{2}aA\cos(c+dx)-\frac{1}{2}Ab\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{a}$$

$$= \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{A\int\sqrt{\cos(c+dx)}\,dx}{a} - \frac{2\int \frac{b(Ab-aB)}{2\sqrt{\cos(c+dx)}\,(a\cdots}}{ab}$$

$$= -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(Ab-aB)\int\frac{}{\sqrt{\cos(c\cdots}}}{a\cdots}$$

$$= -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d} + \frac{2A\sin\cdots}{ad\sqrt{\cos\cdots}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 206 vs. 2(86) = 172.

time = 12.59, size = 206, normalized size = 2.40

$$\frac{\frac{2(-3Ab+2aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}-\frac{2aA\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{b}+\frac{4A\sin(c+dx)}{\sqrt{\cos(c+dx)}}-\frac{2A\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(-2a^2+b^2)\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] ((2*(-3*A*b + 2*a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (
2*a*A*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*
x)/2, 2])/(a + b)))/b + (4*A*Sin[c + d*x])/Sqrt[Cos[c + d*x]] - (2*A*(-2*a*
b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[
Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Co
s[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(2*a*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(136) = 272$.
time = 0.39, size = 300, normalized size = 3.49

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4(-Ab+aB)b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{a\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(-A*b+B*a)/a
/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x
+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))), x)
```

3.370 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=150

$$\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2b(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a+b)d}+\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-$$

```
[Out] 2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/a^2/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+2*b*(A*b-B*a)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b
),2^(1/2))/a^2/(a+b)/d+2/3*A*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-2*(A*b-B*a)*si
n(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3079, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*A*EllipticF[(c + d*x
)/2, 2])/(3*a*d) + (2*b*(A*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a^2*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*(A
*b - a*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
```

```
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{-\frac{3}{2}(Ab-aB)+\frac{1}{2}aA\cos(c+dx)+\frac{1}{2}Ab\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{4\int \frac{\frac{1}{4}(a^2A+3Ab^2-3abB \ldots}{\ldots}}{\ldots}$$

$$= \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{4\int \frac{-\frac{1}{4}b(a^2A+3Ab^2-3a \ldots}{\sqrt{\cos(c+d \ldots}}}{3 \ldots}$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin \ldots}{a^2d\sqrt{\cos(c \ldots}}$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{2b(Ab-aB \ldots}{\ldots}$$

Mathematica [A]
time = 12.29, size = 260, normalized size = 1.73

$$\frac{\frac{2a(2a^2A+9Ab^2-9abB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{a(8aAb-6a^2B)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{b}+\frac{4a^2A\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}+\frac{12a(-Ab+aB)\sin(c+dx)}{\sqrt{\cos(c+dx)}}+\frac{6(Ab-aB)\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(-2a^2+b^2)\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{b\sqrt{\sin^2(c+dx)}}}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] ((2*a*(2*a^2*A + 9*A*b^2 - 9*a*b*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a + b) + (a*(8*a*A*b - 6*a^2*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (4*a^2*A*Sin[c + d*x
])/Cos[c + d*x]^(3/2) + (12*a*(-(A*b) + a*B)*Sin[c + d*x])/Sqrt[Cos[c + d*x
]] + (6*(A*b - a*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a
*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*Ellipti
cPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c +
d*x]^2]))/(6*a^3*d)
```

Maple [A]
time = 0.70, size = 441, normalized size = 2.94

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4(Ab-aB)b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{a^2\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(A*b-B*a)*b^
2/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2
+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2)))+2*(-A*b+B*a)/a^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3883 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))), x)
```

3.371 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=303

$$\frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3(a^2-b^2)\,d}-\frac{(9a^3Ab-12aAb^3-15a^4B+16a^2b^2B+2b^4B)\,F\left(\frac{1}{2}(c+\right.}{3b^4(a^2-b^2)\,d}$$

```
[Out] (3*A*a^2*b-2*A*b^3-5*B*a^3+4*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)/d-1/3*(9*A*a
^3*b-12*A*a*b^3-15*B*a^4+16*B*a^2*b^2+2*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^4/(a^2-b^2)/d+a
^2*(3*A*a^2*b-5*A*b^3-5*B*a^3+7*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)/b^4/(a
+b)^2/d+a*(A*b-B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+
c))-1/3*(3*A*a*b-5*B*a^2+2*B*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)
/d
```

Rubi [A]
time = 0.60, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3068, 3128, 3138, 2719, 3081, 2720, 2884}

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{(-5a^2B+3aAb+2b^2B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d(a^2-b^2)}+\frac{(-5a^3B+3a^2Ab+4ab^2B-2Ab^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a^2-b^2)}+\frac{a^2(-5a^3B+3a^2Ab+7ab^2B-5Ab^3)\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^4d(a-b)(a+b)^2}-\frac{(-15a^4B+9a^3Ab+16a^2b^2B-12aAb^3+2b^4B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b^
3*(a^2 - b^2)*d) - ((9*a^3*A*b - 12*a*A*b^3 - 15*a^4*B + 16*a^2*b^2*B + 2*b
^4*B)*EllipticF[(c + d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d) + (a^2*(3*a^2*A*b -
5*A*b^3 - 5*a^3*B + 7*a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(
(a - b)*b^4*(a + b)^2*d) - ((3*a*A*b - 5*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \sqrt{\cos(c+dx)}\,\left(-\frac{3}{2}a(Ab-a\right.}{}$$

$$= -\frac{\left(3aAb-5a^2B+2b^2B\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB}{b\left(a^2-\right.}$$

$$= -\frac{\left(3aAb-5a^2B+2b^2B\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB}{b\left(a^2-\right.}$$

$$= \frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} - \frac{(3aAb-5a^2}{}$$

$$= \frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} - \frac{(9a^3Ab-12}{}$$

Mathematica [A]
time = 13.27, size = 318, normalized size = 1.05

$$\frac{4\sqrt{\cos(c+dx)}\left(2B+\frac{3a^2(-Ab+aB)}{(a^2-b^2)(a+b\cos(c+dx))}\right)\sin(c+dx)-\frac{\frac{2(-3a^2Ab+6Ab^3+5a^3B-8ab^2B)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8(-3aAb+2a^2B+b^2B)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{6(-3a^2Ab+2Ab^3+5a^3B-4ab^2B)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(-2a^2+b^2)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab^2\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]

[Out] (4*Sqrt[Cos[c + d*x]]*(2*B + (3*a^2*(-(A*b) + a*B))/((a^2 - b^2)*(a + b*Cos
[c + d*x])))*Sin[c + d*x] - ((2*(-3*a^2*A*b + 6*A*b^3 + 5*a^3*B - 8*a*b^2*B
)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(-3*a*A*b + 2*a^2
*B + b^2*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]))/(a + b) + (6*(-3*a^2*A*b + 2*A*b^3 + 5*a^3*B - 4*a*b^2*
B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Elliptic
F[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSi
n[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a
 - b)*(a + b)))/(12*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. $2(373) = 746$.
time = 0.82, size = 1066, normalized size = 3.52

method	result	size
default	Expression too large to display	1066

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^4/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^4*b^2+6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^2+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2-9*B*a^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b)-4*a^2/b^3*(3*A*b-4*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b-
B*a)/b^4*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*
b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x)
```

3.372 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=224

$$-\frac{(aAb-3a^2B+2b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d}+\frac{(a^2Ab-2Ab^3-3a^3B+4ab^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d}-\frac{a(a^2Ab-3A$$

```
[Out] -(A*a*b-3*B*a^2+2*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d+(A*a^2*b-2*A*b^3-3*B*a^
3+4*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)/d-a*(A*a^2*b-3*A*b^3-3*B*a^3+5*B*a*b^
2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1
/2*c),2*b/(a+b),2^(1/2))/(a-b)/b^3/(a+b)^2/d+a*(A*b-B*a)*sin(d*x+c)*cos(d*x
+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.40, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3068, 3138, 2719, 3081, 2720, 2884}

$$-\frac{(-3a^2B+aAb+2b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{(-3a^3B+a^2Ab+4ab^2B-2Ab^3)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}-\frac{a(-3a^3B+a^2Ab+5ab^2B-3Ab^3)\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -(((a*A*b - 3*a^2*B + 2*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*
d)) + ((a^2*A*b - 2*A*b^3 - 3*a^3*B + 4*a*b^2*B)*EllipticF[(c + d*x)/2, 2])
/(b^3*(a^2 - b^2)*d) - (a*(a^2*A*b - 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*Ellipti
cPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) + (a*(A*b - a
*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}a(Ab-aB)+b(Ab-aB)\,c}{\sqrt{\cos(c}}}{}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}ab(Ab-aB)+\frac{1}{2}\left(a^2Ab-2A\right.}{\sqrt{\cos(c+dx}}}{b^2\left(a\right.}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\sqrt{\cos(c}}{b\left(a^2-b^2\right)d(a+}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} + \frac{\left(a^2Ab-2Ab^3-3a^3\right.}{b^3}$$

Mathematica [**A**]
time = 12.77, size = 280, normalized size = 1.25

$$\frac{-\frac{4a(-Ab+aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{2\left(aAb+a^2B-2b^2B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8(-Ab+aB)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{2\left(-aAb+3a^2B-2b^2B\right)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(-2a^2+b^2\right)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab^2\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] ((-4*a*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*
Cos[c + d*x])) + ((2*(a*A*b + a^2*B - 2*b^2*B)*EllipticPi[(2*b)/(a + b), (c
 + d*x)/2, 2])/(a + b) + (8*(-(A*b) + a*B)*((a + b)*EllipticF[(c + d*x)/2,
2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (2*(-(a*A*b) +
 3*a^2*B - 2*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a
*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*Ellipti
cPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[
c + d*x]^2]))/((a - b)*(a + b)))/(4*b*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(300) = 600$.
time = 0.68, size = 849, normalized size = 3.79

method	result	size
default	Expression too large to display	849

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2
*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*b*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))+4*a/b^2*(2*A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(A*b-
B*a)/b^3*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*
b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4851 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x)
```

3.373 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=198

$$\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d}+\frac{\left(aAb+a^2B-2b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d}-\frac{\left(a^2Ab+Ab^3+a^3B-3ab^2B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\right.}{(a-b)b^2(a+b)^2d}$$

```
[Out] (A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)/d+(A*a*b+B*a^2-2*B*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^
2-b^2)/d-(A*a^2*b+A*b^3+B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)/b^2/(a
+b)^2/d-(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.35, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3078, 3138, 2719, 3081, 2720, 2884}

$$\frac{\left(a^2B+aAb-2b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((a*A*b + a^2*B
 - 2*b^2*B)*EllipticF[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - ((a^2*A*b + A*
b^3 + a^3*B - 3*a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b
)*b^2*(a + b)^2*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 -
b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}(Ab-aB)-(aA-bB)\cos(\ldots}{\sqrt{\cos(c+d\ldots}}}{-\ldots}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{1}{2}b(Ab-aB)+\frac{1}{2}(aAb+a^2\ldots}{\sqrt{\cos(c+dx)}\,(\ldots}}{b\,(a^2-\ldots}$$

$$= \frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b\,(a^2-b^2)\,d} - \frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+\ldots}{(a^2-b^2)\,d(a+b\cos(c+dx))}$$

$$= \frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b\,(a^2-b^2)\,d} + \frac{(aAb+a^2B-2b^2B)\,F\left(\frac{1}{2}(c+dx\ldots\right.}{b^2\,(a^2-b^2)\,d}$$

Mathematica [A]
time = 12.40, size = 260, normalized size = 1.31

$$\frac{\frac{4(-Ab+aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))} - \frac{\frac{2(-Ab+aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{(4aA-4bB)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{b} + \frac{2(Ab-aB)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(-2a^2+b^2)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab^2\sqrt{\sin^2(c+dx)}}}{(-a+b)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] ((4*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos
[c + d*x])) - ((2*(-(A*b) + a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/(a + b) + ((4*a*A - 4*b*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(A*b - a*B)*(-2*a*b*Ellipt
icE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos
[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*
x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)))/
(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(274) = 548$.
time = 0.63, size = 808, normalized size = 4.08

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{E}}{b^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a*(A*b-B*a)/b^2*(-b^2/a/(a^2-
b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*
b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b
^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x
)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x)
```

3.374 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d}-\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d}+\frac{\left(3a^2Ab-Ab^3-a^3B-ab^2B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\right.}{a(a-b)b(a+b)^2d}$$

```
[Out] -(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)/d-(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)/d+(3*A
*a^2*b-A*b^3-B*a^3-B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(a-b)/b/(a+b)^2/d+b*(A*
b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.39, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3079, 3138, 2719, 3081, 2720, 2884}

$$-\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] -(((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d)) - ((A*b - a*B)
*EllipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((3*a^2*A*b - A*b^3 - a^3*B
 - a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a*(a - b)*b*(a + b)
^2*d) + (b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}(2a^2A-Ab^2-abB)-a(A}{\sqrt{\cos(c}}}{}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}b(2a^2A-Ab^2-abB)+}{\sqrt{\cos(c+dx)}}}{ab\,(a^2}$$

$$= -\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\,(a^2-b^2)\,d} + \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(}{a\,(a^2-b^2)\,d(a+b\cos(c+}$$

$$= -\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\,(a^2-b^2)\,d} - \frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\,(a^2-b^2)\,d} + $$

Mathematica [**A**]
time = 12.71, size = 274, normalized size = 1.37

$$\frac{\frac{4b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{2(4a^2A-3Ab^2-abB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{4a(-Ab+aB)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{b}+\frac{2(-Ab+aB)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(-2a^2+b^2)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),
x]
```

```
[Out] ((4*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[
c + d*x])) + ((2*(4*a^2*A - 3*A*b^2 - a*b*B)*EllipticPi[(2*b)/(a + b), (c +
 d*x)/2, 2])/(a + b) + (4*a*(-(A*b) + a*B)*(2*EllipticF[(c + d*x)/2, 2] - (
2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(-(A*b) + a
*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Ellipti
cF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcS
in[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a
- b)*(a + b)))/(4*a*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(276) = 552$.
time = 0.78, size = 721, normalized size = 3.60

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{E}}{\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\mathrm{s}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/(-2*a*b+2*
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+2*(A*b-B*a)/b*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b
+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-
b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2), x)
```

3.375 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=256

$$-\frac{(2a^2A-3Ab^2+abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2\,(a^2-b^2)\,d}+\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a\,(a^2-b^2)\,d}-\frac{(5a^2Ab-3Ab^3-3a^3B+ab^2B)\,\Pi\left(\right.}{a^2(a-b)(a+b)^2d}$$

```
[Out] -(2*A*a^2-3*A*b^2+B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d+(A*b-B*a)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/
(a^2-b^2)/d-(5*A*a^2*b-3*A*b^3-3*B*a^3+B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^2/(
a-b)/(a+b)^2/d+(2*A*a^2-3*A*b^2+B*a*b)*sin(d*x+c)/a^2/(a^2-b^2)/d/cos(d*x+c
)^(1/2)+b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))/cos(d*x+c)^(1
/2)
```

Rubi [A]
time = 0.60, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3079, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{ad\,(a^2-b^2)}-\frac{(2a^2A+abB-3Ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2d\,(a^2-b^2)}+\frac{(2a^2A+abB-3Ab^2)\sin(c+dx)}{a^2d\,(a^2-b^2)\sqrt{\cos(c+dx)}}+\frac{b(Ab-aB)\sin(c+dx)}{ad\,(a^2-b^2)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}-\frac{(-3a^3B+5a^2Ab+ab^2B-3Ab^3)\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] -(((2*a^2*A - 3*A*b^2 + a*b*B)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*
d)) + ((A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) - ((5*a^2*A
*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(a^2*(a - b)*(a + b)^2*d) + ((2*a^2*A - 3*A*b^2 + a*b*B)*Sin[c + d*x])/(a
^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))} + \frac{\int \frac{1}{2}\left(2a^2A-3Ab^2+abB\right.}{}$$

$$= \frac{\left(2a^2A-3Ab^2+abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b(Ab-aB)\sin(c-}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a}$$

$$= \frac{\left(2a^2A-3Ab^2+abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b(Ab-aB)\sin(c-}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a}$$

$$= -\frac{\left(2a^2A-3Ab^2+abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{\left(2a^2A-3Ab^2+abB\right)s}{a^2\left(a^2-b^2\right)d\sqrt{\cos(}}$$

$$= -\frac{\left(2a^2A-3Ab^2+abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{(Ab-aB)F\left(\frac{1}{2}(c+dx\right.}{a\left(a^2-b^2\right)d}$$

Mathematica [A]
time = 14.28, size = 316, normalized size = 1.23

$$-\frac{\frac{\frac{2\left(-10a^2Ab+9Ab^3+4a^3B-3ab^2B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}-\frac{8a\left(a^2A-2Ab^2+abB\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{b(a+b)}-\frac{2\left(2a^2A-3Ab^2+abB\right)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(-2a^2+b^2\right)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{(-a+b)(a+b)}+4\sqrt{\cos(c+dx)}\left(\frac{b^2(Ab-aB)\sin(c+dx)}{\left(-a^2+b^2\right)(a+b\cos(c+dx))}+2A\tan(c+dx)\right)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),
x]
```

```
[Out] (-(((2*(-10*a^2*A*b + 9*A*b^3 + 4*a^3*B - 3*a*b^2*B)*EllipticPi[(2*b)/(a +
b), (c + d*x)/2, 2])/(a + b) - (8*a*(a^2*A - 2*A*b^2 + a*b*B)*((a + b)*Elli
pticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a
 + b)) - (2*(2*a^2*A - 3*A*b^2 + a*b*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c
 + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-
```

```
2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*
x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b))) + 4*Sqrt[Cos[c + d*x]]*
((b^2*(A*b - a*B)*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x])) + 2*A*T
an[c + d*x]))/(4*a^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(330) = 660$.
time = 0.74, size = 856, normalized size = 3.34

method	result	size
default	Expression too large to display	856

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^2/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b+B*a)/a*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3
*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*
b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2)))+2*A/a^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3067 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2), x)
```

3.376 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=345

$$\frac{(4a^2Ab-5Ab^3-2a^3B+3ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^3\,(a^2-b^2)\,d}+\frac{(2a^2A-5Ab^2+3abB)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3a^2\,(a^2-b^2)\,d}+\frac{b(7a^2Ab-5A}{}$$

```
[Out] (4*A*a^2*b-5*A*b^3-2*B*a^3+3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)/d+1/3*(2*A*a
^2-5*A*b^2+3*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d+b*(7*A*a^2*b-5*A*b^3-5*B*a^3
+3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(
1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^3/(a-b)/(a+b)^2/d+1/3*(2*A*a^2-5*A*b^2+
3*B*a*b)*sin(d*x+c)/a^2/(a^2-b^2)/d/cos(d*x+c)^(3/2)+b*(A*b-B*a)*sin(d*x+c)
/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))-(4*A*a^2*b-5*A*b^3-2*B*a^3
+3*B*a*b^2)*sin(d*x+c)/a^3/(a^2-b^2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.83, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3079, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{(2a^2A+3abB-5Ab^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3a^2d\,(a^2-b^2)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\,(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}+\frac{(2a^2A+3abB-5Ab^2)\sin(c+dx)}{3a^2d\,(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)}+\frac{(-2a^3B+4a^2Ab+3ab^2B-5Ab^3)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^3d\,(a^2-b^2)}+\frac{b(-5a^3B+7a^2Ab+3ab^2B-5Ab^3)\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)}{a^3d(a-b)(a+b)^2}-\frac{(-2a^3B+4a^2Ab+3ab^2B-5Ab^3)\sin(c+dx)}{a^3d\,(a^2-b^2)\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(a^
3*(a^2 - b^2)*d) + ((2*a^2*A - 5*A*b^2 + 3*a*b*B)*EllipticF[(c + d*x)/2, 2]
)/(3*a^2*(a^2 - b^2)*d) + (b*(7*a^2*A*b - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) + ((2*a^
2*A - 5*A*b^2 + 3*a*b*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3
/2)) - ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Sin[c + d*x])/(a^3*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)
*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} + \frac{\int \frac{1}{2}\left(2a^2A-5Ab^2+3abB\right.\cdots}{\cdots}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{b(Ab-aB)\sin(\cdots}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(\cdots}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(4a^2Ab-5Ab^3-2a^3B+3\cdots\right.}{a^3\left(a^2-b^2\right)d\sqrt{\cos\cdots}}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(4a^2Ab-5Ab^3-2a^3B+3\cdots\right.}{a^3\left(a^2-b^2\right)d\sqrt{\cos\cdots}}$$

$$= \frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A-5A\cdots\right.}{3a^2\left(a^2-\cdots\right.}$$

$$= \frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A-5A\cdots\right.}{\cdots}$$

Mathematica [A]
time = 16.96, size = 427, normalized size = 1.24

$$\frac{\frac{2\left(4a^4A+44a^2Ab^2-45Ab^4-30a^3bB+27ab^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{\left(28a^3Ab-40aAb^3-12a^4B+24a^2b^2B\right)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{b}+\frac{2\left(12a^2Ab^2-15Ab^4-6a^3bB+9ab^3B\right)\cos(2(c+dx))\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(-2a^2+b^2\right)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab^2\sqrt{1-\cos^2(c+dx)}\left(-1+2\cos^2(c+dx)\right)}}{12a^3(a-b)(a+b)d}+\frac{\sqrt{\cos(c+dx)}\left(\frac{2\sec(c+dx)(-2Ab\sin(c+dx)+aB\sin(c+dx))}{a^3}+\frac{Ab^4\sin(c+dx)-ab^3B\sin(c+dx)}{a^3\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{2A\sec(c+dx)\tan(c+dx)}{3a^2}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),
x]
```

```
[Out] ((2*(4*a^4*A + 44*a^2*A*b^2 - 45*A*b^4 - 30*a^3*b*B + 27*a*b^3*B)*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + ((28*a^3*A*b - 40*a*A*b^3 - 12*
```

```
a^4*B + 24*a^2*b^2*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/
(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(12*a^2*A*b^2 - 15*A*b^4 - 6*a^3
*b*B + 9*a*b^3*B)*Cos[2*(c + d*x)]*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*
x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2
 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(
a*b^2*Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)))/(12*a^3*(a - b)*(a
 + b)*d) + (Sqrt[Cos[c + d*x]]*((2*Sec[c + d*x]*(-2*A*b*Sin[c + d*x] + a*B*
Sin[c + d*x]))/a^3 + (A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x])/(a^3*(a^2
- b^2)*(a + b*Cos[c + d*x])) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(3*a^2)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. $2(413) = 826$.
time = 1.39, size = 1004, normalized size = 2.91

method	result	size
default	Expression too large to display	1004

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*(2*A*b-B
*a)/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^
2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))+2*(A*b-B*a)*b/a^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-
b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2
)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),
2^(1/2)))+2*(-2*A*b+B*a)/a^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
```

```
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)),
 x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5992 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2), x)
```

3.377 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=367

$$-\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d}+\frac{(3a^4Ab-5a^2Ab^3+8Ab^5-15a^5B+33a}{4b^4\left(a^2-b^2\right.}$$

```
[Out] -1/4*(3*A*a^3*b-9*A*a*b^3-15*B*a^4+29*B*a^2*b^2-8*B*b^4)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^
2-b^2)^2/d+1/4*(3*A*a^4*b-5*A*a^2*b^3+8*A*b^5-15*B*a^5+33*B*a^3*b^2-24*B*a*
b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))/b^4/(a^2-b^2)^2/d-1/4*a*(3*A*a^4*b-6*A*a^2*b^3+15*A*b^5-15*
B*a^5+38*B*a^3*b^2-35*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)^2/b^4/(a+b)^3/d+
1/2*a*(A*b-B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^
2+1/4*a*(A*a^2*b-7*A*b^3-5*B*a^3+11*B*a*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^
2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.66, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3068, 3126, 3138, 2719, 3081, 2720, 2884}

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{a(-5a^3B+a^2Ab+11ab^2B-7Ab^3)\sin(c+dx)\sqrt{\cos(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}+\frac{(-15a^5B+3a^4Ab+33a^3b^2B-5a^2Ab^3-24ab^4B+8Ab^5)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{a(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5)\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -1/4*((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*EllipticE
[(c + d*x)/2, 2])/(b^3*(a^2 - b^2)^2*d) + ((3*a^4*A*b - 5*a^2*A*b^3 + 8*A*b
^5 - 15*a^5*B + 33*a^3*b^2*B - 24*a*b^4*B)*EllipticF[(c + d*x)/2, 2])/(4*b^
4*(a^2 - b^2)^2*d) - (a*(3*a^4*A*b - 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38
*a^3*b^2*B - 35*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a -
 b)^2*b^4*(a + b)^3*d) + (a*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2
*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (a*(a^2*A*b - 7*A*b^3 - 5*a^3*B
+ 11*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a +
b*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(Ab-\right.}{}$$

$$= \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a(a^2Ab-7Ab^3-5a^3B-}{4b^2\left(a^2-b\right.}$$

$$= \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a(a^2Ab-7Ab^3-5a^3B-}{4b^2\left(a^2-b\right.}$$

$$= -\frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2d}$$

Mathematica [A]
time = 15.11, size = 390, normalized size = 1.06

$$\frac{-\frac{2a\sqrt{\cos(c+dx)}\left(a\left(-a^2Ab+7Ab^3+5a^3B-11ab^2B\right)+b\left(-3a^2Ab+9Ab^3+7a^3B-13ab^2B\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{\frac{\left(-a^3Ab-5aAb^3+5a^4B-7a^2b^2B+8b^4B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{\left(-3a^3Ab+9aAb^3+15a^4B-29a^2b^2B+8b^4B\right)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(-2a^2+b^2\right)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab^2\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{8b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] ((-2*a*Sqrt[Cos[c + d*x]]*(a*(-(a^2*A*b) + 7*A*b^3 + 5*a^3*B - 11*a*b^2*B)
+ b*(-3*a^2*A*b + 9*A*b^3 + 7*a^3*B - 13*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x
])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (((-(a^3*A*b) - 5*a*A*b^3 + 5*a
```

```
^4*B - 7*a^2*b^2*B + 8*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a
 + b) + (8*(a^2*A*b + 2*A*b^3 + a^3*B - 4*a*b^2*B)*((a + b)*EllipticF[(c +
d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + ((-3*a
^3*A*b + 9*a*A*b^3 + 15*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*(-2*a*b*EllipticE[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]],
 -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8
*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1976 vs. $2(431) = 862$.
time = 1.39, size = 1977, normalized size = 5.39

method	result	size
default	Expression too large to display	1977

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^4/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3
*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*b*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))+12/b^3*a*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2/b^4*(3
*A*b-4*B*a)*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/
(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2*a^3*(A*b
-B*a)/b^4*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^
2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin
```

```
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^
2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*
b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3, x)
```

3.378 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=344

$$-\frac{(a^2Ab+5Ab^3+3a^3B-9ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2d}+\frac{(a^3Ab-7aAb^3+3a^4B-5a^2b^2B+8b^4B)\,F\left(\frac{1}{2}(c+dx\right.$$

```
[Out] -1/4*(A*a^2*b+5*A*b^3+3*B*a^3-9*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d+1/4*(
A*a^3*b-7*A*a*b^3+3*B*a^4-5*B*a^2*b^2+8*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)^2/d
-1/4*(A*a^4*b-10*A*a^2*b^3-3*A*b^5+3*B*a^5-6*B*a^3*b^2+15*B*a*b^4)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(
a+b),2^(1/2))/(a-b)^2/b^3/(a+b)^3/d+1/2*a*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(
1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/4*(A*a^2*b+5*A*b^3+3*B*a^3-9*B*a*b^
2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A]

time = 0.64, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3068, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{(3a^3B+a^2Ab-9ab^2B+5Ab^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{(3a^3B+a^2Ab-9ab^2B+5Ab^3)\sin(c+dx)\sqrt{\cos(c+dx)}}{4bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{(3a^4B+a^3Ab-5a^2b^2B-7aAb^3+8b^4B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}-\frac{(3a^5B+a^4Ab-6a^3b^2B-10a^2Ab^3+15ab^4B-3Ab^5)\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -1/4*((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/
(b^2*(a^2 - b^2)^2*d) + ((a^3*A*b - 7*a*A*b^3 + 3*a^4*B - 5*a^2*b^2*B + 8*b
^4*B)*EllipticF[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) - ((a^4*A*b - 10*a
^2*A*b^3 - 3*A*b^5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*EllipticPi[(2*b)/(
a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) + (a*(A*b - a*B)*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) +
((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(4*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}a(Ab-aB)+2b(Ab-aB)}{\sqrt{\cos(c}}}{}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{(a^2Ab+5Ab^3+3a^3B}{4b\left(a^2-b\right.}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{(a^2Ab+5Ab^3+3a^3B}{4b\left(a^2-b\right.}$$

$$= -\frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2d} + \frac{a(Ab-aB}{2b\left(a^2-\right.}$$

$$= -\frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2d} + \frac{(a^3Ab-7a}{}$$

Mathematica [A]
time = 13.79, size = 360, normalized size = 1.05

$$\frac{\frac{2\sqrt{\cos(c+dx)}\left(a\left(3a^2Ab+3Ab^3+a^3B-7ab^2B\right)+b\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{\frac{\left(-5a^2Ab-Ab^3+a^3B+5ab^2B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}-\frac{8\left(-3aAb+a^2B+2b^2B\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(-2a^2+b^2\right)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab^2\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] ((2*Sqrt[Cos[c + d*x]]*(a*(3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B) + b*(a^
2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2 -
b^2)^2*(a + b*Cos[c + d*x])^2) - (((-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)
*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*(-3*a*A*b + a^2*B
+ 2*b^2*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b),
```

```
 (c + d*x)/2, 2]))/(a + b) + ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*(-2
*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt
[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^
2*(a + b)^2))/(8*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1936 vs. $2(408) = 816$.

time = 1.28, size = 1937, normalized size = 5.63

method	result	size
default	Expression too large to display	1937

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
^2*(A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a/b^3*(2*A*b-3*B*a)*(-b^2/a
/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/
(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*a^2*(A*b-B*a)/b^3*(-1/2*b^2
/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2
)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
```

```
(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/
(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)
*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3, x)
```

3.379 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=337

$$\frac{(5a^2Ab+Ab^3-a^3B-5ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\,(a^2-b^2)^2\,d}+\frac{(3a^2Ab+3Ab^3+a^3B-7ab^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\,(a^2-b^2)^2\,d}-\frac{(3a^4A$$

```
[Out] 1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/b/(a^2-b^2)^2/d+1/4*(3*A
*a^2*b+3*A*b^3+B*a^3-7*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d-1/4*(3*A*a^4*b
+10*A*a^2*b^3-A*b^5+B*a^5-10*B*a^3*b^2-3*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(
a-b)^2/b^2/(a+b)^3/d-1/2*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/
(a+b*cos(d*x+c))^2-1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*sin(d*x+c)*cos(d*x
+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A]

time = 0.61, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3078, 3134, 3138, 2719, 3081, 2720, 2884}

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\,(a^2-b^2)\,(a+b\cos(c+dx))^2}+\frac{(a^3B+3a^2Ab-7ab^2B+3Ab^3)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\,(a^2-b^2)^2}+\frac{(a^3(-B)+5a^2Ab-5ab^2B+Ab^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\,(a^2-b^2)^2}-\frac{(a^3(-B)+5a^2Ab-5ab^2B+Ab^3)\sin(c+dx)\sqrt{\cos(c+dx)}}{4ad\,(a^2-b^2)^2\,(a+b\cos(c+dx))}-\frac{(a^5B+3a^4Ab-10a^3b^2B+10a^2Ab^3-3ab^4B-Ab^5)\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4ab^2d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*a*b*
(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)*EllipticF[(c
+ d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) - ((3*a^4*A*b + 10*a^2*A*b^3 - A*b^5
+ a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(4*a*(a - b)^2*b^2*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((5*a^2*A*b + A*b^3 - a
^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(a
+ b*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\frac{1}{2}(Ab-aB)-2(aA-bB)\,\cdots}{\sqrt{\cos(c+\cdots}}}{2\cdots}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(5a^2Ab+Ab^3-a^3B\cdots\right.}{4a\left(a^2-\cdots\right.}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(5a^2Ab+Ab^3-a^3B\cdots\right.}{4a\left(a^2-\cdots\right.}$$

$$= \frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\left(a^2-b^2\right)^2d} - \frac{(Ab-aB)\cdots}{2\left(a^2-b\cdots\right.}$$

$$= \frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\left(a^2-b^2\right)^2d} + \frac{\left(3a^2Ab+3\cdots\right.}{\cdots}$$

Mathematica [A]
time = 14.52, size = 365, normalized size = 1.08

$$\frac{\frac{4\sqrt{\cos(c+dx)}\left(a\left(-7a^2Ab+Ab^3+3a^3B+3ab^2B\right)+b\left(-5a^2Ab-Ab^3+a^3B+5ab^2B\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{\frac{2\left(-9a^2Ab+3Ab^3+5a^3B+ab^2B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{16a\left(2a^2A+Ab^2-3abB\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{b(a+b)}-\frac{2\left(-5a^2Ab-Ab^3+a^3B+5ab^2B\right)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(-2a^2+b^2\right)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab^2\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(a*(-7*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B) + b*(-
5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2 -
b^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^
2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*a*(2*a^2*A +
A*b^2 - 3*a*b*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]))/(b*(a + b)) - (2*(-5*a^2*A*b - A*b^3 + a^3*B + 5*a
```

```
*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Ell
ipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a),
ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2])
)/((a - b)^2*(a + b)^2))/(16*a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1849 vs. $2(401) = 802$.
time = 1.22, size = 1850, normalized size = 5.49

method	result	size
default	Expression too large to display	1850

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/b/(-2*a*b+
2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))+2*(A*b-2*B*a)/b^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a
/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2
*b/(a-b),2^(1/2)))-2*a*(A*b-B*a)/b^2*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-
b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/
a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
```

```
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^
2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b
/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/
(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3066 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3, x)
```

3.380 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=345

$$-\frac{\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2d}-\frac{\left(7a^2Ab-Ab^3-3a^3B-3ab^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\left(a^2-b^2\right)^2d}+\frac{(15$$

```
[Out] -1/4*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)^2/d-1/4*(
7*A*a^2*b-A*b^3-3*B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/b/(a^2-b^2)^2/d+1/4*(15*A*a
^4*b-6*A*a^2*b^3+3*A*b^5-3*B*a^5-10*B*a^3*b^2+B*a*b^4)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2)
)/a^2/(a-b)^2/b/(a+b)^3/d+1/2*b*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^
2-b^2)/d/(a+b*cos(d*x+c))^2+1/4*b*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*sin(d
*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A]

time = 0.69, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3079, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}-\frac{\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{b\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{\left(-3a^5B+15a^4Ab-10a^3b^2B-6a^2Ab^3+ab^4B+3Ab^5\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2bd(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] -1/4*((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*EllipticE[(c + d*x)/2, 2])/
(a^2*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*Elliptic
F[(c + d*x)/2, 2])/(4*a*b*(a^2 - b^2)^2*d) + ((15*a^4*A*b - 6*a^2*A*b^3 + 3
*A*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d
*x)/2, 2])/(4*a^2*(a - b)^2*b*(a + b)^3*d) + (b*(A*b - a*B)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b*(9*a^2*A*
b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a
^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}(4a^2A-3Ab^2-abB)-2a}{\sqrt{\cos(}}}{}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\cos(c+dx))^2} + \frac{b(9a^2Ab-3Ab^3-5a}{4a^2\,(a^2-}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\cos(c+dx))^2} + \frac{b(9a^2Ab-3Ab^3-5a}{4a^2\,(a^2-}$$

$$= -\frac{(9a^2Ab-3Ab^3-5a^3B-ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4a^2\,(a^2-b^2)^2\,d} + \frac{b(Ab-a}{2a\,(a^2}$$

$$= -\frac{(9a^2Ab-3Ab^3-5a^3B-ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4a^2\,(a^2-b^2)^2\,d} - \frac{(7a^2Ab-}{}$$

Mathematica [A]
time = 14.90, size = 383, normalized size = 1.11

$$\frac{-\frac{2b\sqrt{\cos(c+dx)}\,\left(a\left(-11a^2Ab+5Ab^3+7a^3B-ab^2B\right)+b\left(-9a^2Ab+3Ab^3+5a^3B+ab^2B\right)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2}+\frac{\frac{\left(16a^4A-19a^2Ab^2+9Ab^4-9a^3bB+3ab^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8a\left(-4a^2Ab+Ab^3+2a^3B+ab^2B\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{b(a+b)}+\frac{\left(-9a^2Ab+3Ab^3+5a^3B+ab^2B\right)\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(-2a^2+b^2\right)\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),
x]
```

```
[Out] ((-2*b*Sqrt[Cos[c + d*x]]*(a*(-11*a^2*A*b + 5*A*b^3 + 7*a^3*B - a*b^2*B) +
b*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((
a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (((16*a^4*A - 19*a^2*A*b^2 + 9*A*b^4
 - 9*a^3*b*B + 3*a*b^3*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b
) + (8*a*(-4*a^2*A*b + A*b^3 + 2*a^3*B + a*b^2*B)*((a + b)*EllipticF[(c + d
```

```
*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) + ((-
9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[
c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (
-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d
*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8*a^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1743 vs. $2(409) = 818$.
time = 1.24, size = 1744, normalized size = 5.06

method	result	size
default	Expression too large to display	1744

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b*(-b^2/a/(
a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a
^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*(A*b-B*a)/b*(-1/2*b^2/a/(a^2-
b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b
+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2
```

```
)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2
)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3067 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3), x)
```

3.381 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=420

$$-\frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2d}+\frac{\left(11a^2Ab-5Ab^3-7a^3B+ab^2B\right)F\left(\frac{1}{2}(c\right.}{4a^2\left(a^2-b^2\right)^2d}$$

```
[Out] -1/4*(8*A*a^4-29*A*a^2*b^2+15*A*b^4+9*B*a^3*b-3*B*a*b^3)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^
2-b^2)^2/d+1/4*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2
)^2/d-1/4*(35*A*a^4*b-38*A*a^2*b^3+15*A*b^5-15*B*a^5+6*B*a^3*b^2-3*B*a*b^4)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2
*c),2*b/(a+b),2^(1/2))/a^3/(a-b)^2/(a+b)^3/d+1/4*(8*A*a^4-29*A*a^2*b^2+15*A
*b^4+9*B*a^3*b-3*B*a*b^3)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)+1/2
*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2)+1
/4*b*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b
*cos(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.96, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3079, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}+\frac{\left(-7a^3B+11a^2Ab+ab^2B-5Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{b\left(-7a^3B+11a^2Ab+ab^2B-5Ab^3\right)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}-\frac{\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}+\frac{\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)}{4a^3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}-\frac{\left(-15a^5B+35a^4Ab+6a^3b^2B-38a^2Ab^3-3ab^4B+15Ab^5\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] -1/4*((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*EllipticE
[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)^2*d) + ((11*a^2*A*b - 5*A*b^3 - 7*a^3*B
+ a*b^2*B)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) - ((35*a^4*A*
b - 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*a^3*b^2*B - 3*a*b^4*B)*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^3*(a - b)^2*(a + b)^3*d) + ((8*a^4*A
 - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sin[c + d*x])/(4*a^3*(a
^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^2
- b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2) + (b*(11*a^2*A*b - 5*A*
b^3 - 7*a^3*B + a*b^2*B)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c +
d*x]]*(a + b*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
```

```
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2} + \frac{\int \frac{1}{2}\left(4a^2A-5Ab^2 \cdots\right.}{\cdots}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2} + \frac{b(11a^2Ab- \cdots}{4a^2\left(a^2-b^2\right)^2 \cdots}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{\cdots}{2a\,(\cdots}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{\cdots}{2a\,(\cdots}$$

$$= -\frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} \cdots$$

$$= -\frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} \cdots$$

Mathematica [A]
time = 15.56, size = 458, normalized size = 1.09

$$\frac{-\frac{\frac{\left(56a^4Ab-95a^2Ab^3+45Ab^5-16a^5B+19a^3b^2B-9ab^4B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8a\left(2a^4A-10a^2Ab^2+5Ab^4+4a^3bB-ab^3B\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{b(a+b)}+\frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\left(-2abE\left(\operatorname{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\operatorname{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(-2a^2+b^2\right)\Pi\left(-\frac{b}{a};\operatorname{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}+\frac{\sqrt{\cos(c+dx)}\left(2ab\left(16a^4A-47a^2Ab^2+25Ab^4+11a^3bB-5ab^3B\right)\sin(c+dx)+b^2\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sin(2(c+dx))+16A\left(a^3-ab^2\right)^2\tan(c+dx)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),
x]
```

```
[Out] (-((((56*a^4*A*b - 95*a^2*A*b^3 + 45*A*b^5 - 16*a^5*B + 19*a^3*b^2*B - 9*a*
b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*a*(2*a^4*A -
 10*a^2*A*b^2 + 5*A*b^4 + 4*a^3*b*B - a*b^3*B)*((a + b)*EllipticF[(c + d*x)
/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) + ((8*a^
4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*(-2*a*b*EllipticE[Ar
cSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d
*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]],
-1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) + (Sq
rt[Cos[c + d*x]]*(2*a*b*(16*a^4*A - 47*a^2*A*b^2 + 25*A*b^4 + 11*a^3*b*B -
5*a*b^3*B)*Sin[c + d*x] + b^2*(8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*
B - 3*a*b^3*B)*Sin[2*(c + d*x)] + 16*A*(a^3 - a*b^2)^2*Tan[c + d*x]))/((a^2
 - b^2)^2*(a + b*Cos[c + d*x])^2))/(8*a^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1974 vs. $2(480) = 960$.
time = 1.55, size = 1975, normalized size = 4.70

method	result	size
default	Expression too large to display	1975

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^3/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))-2*A*b/a^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^
2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a
^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/
```

```
(a-b),2^(1/2)))+2*A/a^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(-A*b+B*a)/a*(-1/2*b^2/a/(a^2
-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^
2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^
2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4852 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3), x)
```

3.382 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=523

$$\frac{\left(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2d}+\frac{\left(8a^4A-61a^2Ab^2+35Ab^4+\right.}{12a^3}$$

```
[Out] 1/4*(24*A*a^4*b-65*A*a^2*b^3+35*A*b^5-8*B*a^5+29*B*a^3*b^2-15*B*a*b^4)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2))/a^4/(a^2-b^2)^2/d+1/12*(8*A*a^4-61*A*a^2*b^2+35*A*b^4+33*B*a^3*b-15*
B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)^2/d+1/4*b*(63*A*a^4*b-86*A*a^2*b^3+35*A*b
^5-35*B*a^5+38*B*a^3*b^2-15*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^4/(a-b)^2/(a+b
)^3/d+1/12*(8*A*a^4-61*A*a^2*b^2+35*A*b^4+33*B*a^3*b-15*B*a*b^3)*sin(d*x+c)
/a^3/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)+1/2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/
d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2+1/4*b*(13*A*a^2*b-7*A*b^3-9*B*a^3+3*B
*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))-1/4*
(24*A*a^4*b-65*A*a^2*b^3+35*A*b^5-8*B*a^5+29*B*a^3*b^2-15*B*a*b^4)*sin(d*x+
c)/a^4/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 1.26, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3079, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}+\frac{b\left(-9a^3B+13a^2Ab+3ab^2B-7Ab^3\right)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}+\frac{\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^3d\left(a^2-b^2\right)^2}+\frac{\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{12a^3d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)}+\frac{\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15ab^4B+35Ab^5\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{b\left(-35a^5B+63a^4Ab+38a^3b^2B-86a^2Ab^3-15ab^4B+35Ab^5\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d(a-b)^2(a+b)^3}-\frac{\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15ab^4B+35Ab^5\right)\sin(c+dx)}{4a^4d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] ((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*
B)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^4*A - 61*a^2*
A*b^2 + 35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B)*EllipticF[(c + d*x)/2, 2])/(12*
a^3*(a^2 - b^2)^2*d) + (b*(63*a^4*A*b - 86*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B
+ 38*a^3*b^2*B - 15*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*
a^4*(a - b)^2*(a + b)^3*d) + ((8*a^4*A - 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3*b
*B - 15*a*b^3*B)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2))
- ((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^
4*B)*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a
*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]
)^2) + (b*(13*a^2*A*b - 7*A*b^3 - 9*a^3*B + 3*a*b^2*B)*Sin[c + d*x])/(4*a^2
*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
```

```
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2A-7Ab^2+3\right.}{}}{}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} + \frac{b\left(13a^2Ab-7A\right.}{4a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} + \frac{}{2a}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} - \frac{(2}{}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} - \frac{(2}{}$$

$$= \frac{\left(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B\right)E\left(\frac{1}{2}(\right.}{4a^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B\right)E\left(\frac{1}{2}(\right.}{4a^4\left(a^2-b^2\right)^2 d}$$

Mathematica [A]
time = 17.33, size = 570, normalized size = 1.09

$$\frac{\frac{2(16a^6A+328a^4Ab^2-641a^2Ab^4+315Ab^6-168a^5bB+285a^3b^3B-135ab^5B)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{(160a^5Ab-512a^3Ab^3+280aAb^5-48a^6B+240a^4b^2B-120a^2b^4B)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{b}+\frac{2(72a^4Ab^2-195a^2Ab^4+105Ab^6-24a^5bB+87a^3b^3B-45ab^5B)\cos(2(c+dx))\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(-2a^2+b^2)\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab^2\sqrt{1-\cos^2(c+dx)}\,(-1+2\cos^2(c+dx))}}{48a^4(a-b)^2(a+b)^2d}+\frac{\sqrt{\cos(c+dx)}\left(\frac{2\sec(c+dx)(-3Ab\sin(c+dx)+aB\sin(c+dx))}{a^4}+\frac{Ab^4\sin(c+dx)-ab^3B\sin(c+dx)}{2a^3(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{17a^2Ab^4\sin(c+dx)-11Ab^6\sin(c+dx)-13a^3b^3B\sin(c+dx)+7ab^5B\sin(c+dx)}{4a^4(a^2-b^2)^2(a+b\cos(c+dx))}+\frac{2A\sec(c+dx)\tan(c+dx)}{3a^3}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),
x]
```

```
[Out] ((2*(16*a^6*A + 328*a^4*A*b^2 - 641*a^2*A*b^4 + 315*A*b^6 - 168*a^5*b*B + 2
85*a^3*b^3*B - 135*a*b^5*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a +
 b) + ((160*a^5*A*b - 512*a^3*A*b^3 + 280*a*A*b^5 - 48*a^6*B + 240*a^4*b^2*
B - 120*a^2*b^4*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(a + b)))/b + (2*(72*a^4*A*b^2 - 195*a^2*A*b^4 + 105
*A*b^6 - 24*a^5*b*B + 87*a^3*b^3*B - 45*a*b^5*B)*Cos[2*(c + d*x)]*(-2*a*b*E
llipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqr
t[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c
 + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c
 + d*x]^2)))/(48*a^4*(a - b)^2*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*((2*Sec[c
 + d*x]*(-3*A*b*Sin[c + d*x] + a*B*Sin[c + d*x]))/a^4 + (A*b^4*Sin[c + d*x]
 - a*b^3*B*Sin[c + d*x])/(2*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (17*a
^2*A*b^4*Sin[c + d*x] - 11*A*b^6*Sin[c + d*x] - 13*a^3*b^3*B*Sin[c + d*x] +
 7*a*b^5*B*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*
Sec[c + d*x]*Tan[c + d*x])/(3*a^3)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2130 vs. 2(579) = 1158.
time = 2.87, size = 2131, normalized size = 4.07

method	result	size
default	Expression too large to display	2131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*(3*A*b-B
*a)/a^4/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/a^3*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^
2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))+2*b*(2*A*b-B*a)/a^3*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+
```

a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b
^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b
),2^(1/2)))+2*(-3*A*b+B*a)/a^4/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2
-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b-B*a)*b/a^2*(-1/2*
b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-
b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b
^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a
^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a
^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b
^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^3),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^3), x)
```

3.383 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

```
[Out] 6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/5*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {21, 2715, 2719}

$$\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = B\int \cos^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}(3B)\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

Mathematica [A]
time = 0.06, size = 41, normalized size = 0.93

$$\frac{B\left(6E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,\sin(2(c+dx))\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (B*(6*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[2*(c + d*x)]))/(5*
d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(64) = 128.
time = 0.24, size = 203, normalized size = 4.61

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(-8*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 77, normalized size = 1.75

$$\frac{2 B \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 3i\sqrt{2} B\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) - 3i\sqrt{2} B\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)))}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] 1/5*(2*B*cos(d*x + c)^(3/2)*sin(d*x + c) + 3*I*sqrt(2)*B*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(
2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a),
 x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
[Out] int((cos(c + d*x)^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

3.384 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*B*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {21, 2715, 2720}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = B\int \cos^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

Mathematica [A]
time = 0.05, size = 37, normalized size = 0.84

$$\frac{2B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,\sin(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(64) = 128.
time = 0.26, size = 180, normalized size = 4.09

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 71, normalized size = 1.61

$$\frac{2\,B\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right) - i\,\sqrt{2}\,B\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)) + i\,\sqrt{2}\,B\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] 1/3*(2*B*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*B*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3066 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\cos\left(c+d\,x\right)}^{3/2}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{a+b\,\cos\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
[Out] int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

3.385 $\int \frac{\sqrt{\cos(c+dx)}\,(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=17

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {21, 2719}

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}\,(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx &= B\int \sqrt{\cos(c+dx)}\,dx\\ &= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(43) = 86.
time = 0.24, size = 134, normalized size = 7.88

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticE}\left(\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ d}$
risch	$-\frac{iB\sqrt{2}\,\sqrt{\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{d}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}+1\right)}{\sqrt{\left(e^{2i(dx+c)}+1\right)e^{i(dx+c)}}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\right.}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 59, normalized size = 3.47

$$\frac{i\sqrt{2}\,B\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - i\sqrt{2}\,B\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] (I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c))) - I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

3.386 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=17

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/d
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {21, 2720}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx &= B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])
),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 19, normalized size = 1.12

method	result	size
default	$\frac{2B\,\mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle|\sqrt{2}\right)}{d}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] 2*B/d*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 53, normalized size = 3.12

$$\frac{-i\sqrt{2}\,B\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,B\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] (-I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I
*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))),x)
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))), x)
```

3.387 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=40

$$-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

[Out] -2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*B*sin(d*x+c)/d/cos(d*x+c)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {21, 2716, 2719}

$$\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]

[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx &= B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - B\int \sqrt{\cos(c+dx)}\,dx \\
&= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 1.00

$$
B\left(-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
),x]
```

```
[Out] B*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]
]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(64) = 128.
time = 0.25, size = 183, normalized size = 4.58

method	result
default	$-\frac{2B\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\right.\right.}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -2*B*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 96, normalized size = 2.40

$$\frac{-i\sqrt{2}\,B\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+i\sqrt{2}\,B\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\,B\sqrt{\cos(dx+c)}\,\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] (-I*sqrt(2)*B*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*B*cos(d*x + c)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*B*s
qrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))),x)
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))), x)
```

3.388 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=44

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*B*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {21, 2716, 2720}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*B*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.08, size = 37, normalized size = 0.84

$$\frac{2B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])
),x]
```

```
[Out] (2*B*(EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(64) = 128$.
time = 0.22, size = 214, normalized size = 4.86

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*B*((2*cos(1/2*d*x+1/2*c)^2-
1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 95, normalized size = 2.16

$$\frac{-i\sqrt{2}\,B\cos\left(dx+c\right)^{2}\operatorname{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+i\sqrt{2}\,B\cos\left(dx+c\right)^{2}\operatorname{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+2\,B\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*B*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + I*sqrt(2)*B*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 2*B*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3883 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))),x)
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))), x)
```

3.389 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2} dx$

Optimal. Leaf size=116

$$-\frac{2aBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2(3a^2+b^2)\,BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a^3B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3(a+b)d}+\frac{2B\sqrt{\cos(c+dx)}\,\sin(c}{3bd}$$

```
[Out] -2*a*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/b^2/d+2/3*(3*a^2+b^2)*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d-2*a^3*B*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(
a+b),2^(1/2))/b^3/(a+b)/d+2/3*B*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d
```

Rubi [A]
time = 0.25, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {21, 2872, 3138, 2719, 3081, 2720, 2884}

$$-\frac{2a^3B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}+\frac{2B(3a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2aBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*a*B*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*(3*a^2 + b^2)*B*EllipticF[(
c + d*x)/2, 2])/(3*b^3*d) - (2*a^3*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2])/(b^3*(a + b)*d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{(2B)\int \frac{\frac{a}{2}+\frac{1}{2}b\cos(c+dx)-\frac{3}{2}a\cos^2(c+}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+}}{3b}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} - \frac{(2B)\int \frac{-\frac{ab}{2}-\frac{1}{2}(3a^2+b^2)\cos(c+dx}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+}}{3b^2}$$

$$= -\frac{2aBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} - \frac{(a^3B}{}$$

$$= -\frac{2aBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2(3a^2+b^2)\,BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d} - \frac{2a^3E}{}$$

Mathematica [A]
time = 1.82, size = 159, normalized size = 1.37

$$\frac{B\left(4F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{6a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+4\sqrt{\cos(c+dx)}\,\sin(c+dx)-\frac{6\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(-2a^2+b^2)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{b^2\sqrt{\sin^2(c+dx)}}\right)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^2,x]
```

```
[Out] (B*(4*EllipticF[(c + d*x)/2, 2] - (6*a*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2])/(a + b) + 4*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (6*(-2*a*b*EllipticE[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]],
 -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2])))/(6*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(188) = 376$.
time = 0.33, size = 553, normalized size = 4.77

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(4*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2-4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^4*b^3-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b^2+2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2*b^3+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^2*b+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^3
+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*a^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^
2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^
2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x
)
```

3.390 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=78

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2aBF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2a^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a+b)d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/b/d-2*a*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d+2*a^2*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b^2
/(a+b)/d
```

Rubi [A]
time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {21, 2883, 2719, 2882, 2720, 2884}

$$\frac{2a^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2aBF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(b*d) - (2*a*B*EllipticF[(c + d*x)/2, 2])/(
b^2*d) + (2*a^2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^2*(a + b)*d
)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2882

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2883

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*Sin[e + f*x]], x], x]
 - Dist[(b*c - a*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx\\
&= \frac{B\int \sqrt{\cos(c+dx)}\,dx}{b} - \frac{(aB)\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx}{b}\\
&= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{(aB)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2} + \frac{(a^2B)\int \frac{}{\sqrt{c}}}{}\\
&= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2aBF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2a^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c\right.}{b^2(a+b)}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 82, normalized size = 1.05

$$-\frac{2B\left(bE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+a\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^2,x]
```

```
[Out] (-2*B*(b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + a*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]
]], -1])*Sin[c + d*x])/(b^2*d*Sqrt[Sin[c + d*x]^2])
```

Maple [A]
time = 0.26, size = 228, normalized size = 2.92

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\text{EllipticF}\right.}{b^2(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETU
RNVERBOSE)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a^2-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-a^2*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^2/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4851 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x
)
```

3.391 $\int \frac{\sqrt{\cos(c+dx)}\,(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2aB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/b/d-2*a*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b/(a+b)/d
```

Rubi [A]

time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 2882, 2720, 2884}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2aB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a*B*EllipticPi[(2*b)/(a + b), (c
 + d*x)/2, 2])/(b*(a + b)*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2882

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}\,(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx\\ &= \frac{B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b} - \frac{(aB)\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,d}{b}\\ &= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2aB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.89

$$\frac{B\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^2,x]
```

```
[Out] (B*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2])/(a + b)))/(b*d)
```

Maple [A]
time = 0.29, size = 189, normalized size = 3.44

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\text{EllipticF}\right.}{b(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x,method=_RETU
RNVERBOSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-a*EllipticPi(cos(1/2
*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^
2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^
2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x
)
```

3.392 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=30

$$\frac{2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2*b/(a+b),2^(1/2))/(a+b)/d
```

Rubi [A]

time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{21, 2884\}$

$$\frac{2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] (2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}\,dx &= B\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx\\ &= \frac{2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 30, normalized size = 1.00

$$\frac{2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])
^2),x]
```

```
[Out] (2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(56) = 112.
time = 0.21, size = 151, normalized size = 5.03

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticPi}(}{(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x,method=_RETU
RNVERBOSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c
))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c
))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2), x
)
```

3.393 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=80

$$-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2bB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d}+\frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] -2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a/d-2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(a+b)/d+2*B*sin(d*x+c)/a/
d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$,
Rules used = {21, 2881, 3138, 2719, 12, 2884}

$$-\frac{2bB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*b*B*EllipticPi[(2*b)/(a + b), (
c + d*x)/2, 2])/(a*(a + b)*d) + (2*B*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$

$$= \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{(2B)\int \frac{-\frac{b}{2}-\frac{1}{2}a\cos(c+dx)-\frac{1}{2}b\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{a}$$

$$= \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{B\int \sqrt{\cos(c+dx)}\,dx}{a} - \frac{(2B)\int \frac{}{2\sqrt{\cos(c+d}}}{ab}$$

$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(bB)\int \frac{1}{\sqrt{\cos(c+dx)}}}{a}$$

$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2bB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d} + \frac{2B\sin(c+d}{ad\sqrt{\cos(c+}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 196 vs. 2(80) = 160.
time = 2.75, size = 196, normalized size = 2.45

$$-\frac{B\left(\frac{6b\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{2a\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{b}-\frac{4\sin(c+dx)}{\sqrt{\cos(c+dx)}}+\frac{2\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(-2a^2+b^2)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
^2),x]
```

```
[Out] -1/2*(B*((6*b*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (2*a*(2*
EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/(a + b)))/b - (4*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*(-2*a*b*EllipticE[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]],
 -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2])))/(a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(130) = 260.
time = 0.31, size = 355, normalized size = 4.44

method	result

default	$-\frac{2B\left(-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\mathrm{si}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x,method=_RETU
RNVERBOSE)
```

```
[Out] -2*B*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/a/(a-b)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/
2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3067 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2), x
)
```

3.394 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2b^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a+b)d}+\frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^2/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+2*b^2*B*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a
^2/(a+b)/d+2/3*B*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-2*b*B*sin(d*x+c)/a^2/d/cos
(d*x+c)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {21, 2881, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{2b^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] (2*b*B*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*B*EllipticF[(c + d*x)/2, 2])
/(3*a*d) + (2*b^2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a + b)
*d) + (2*B*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*b*B*Sin[c + d*x])/
(a^2*d*Sqrt[Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$

$$= \frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{(2B)\int \frac{-\frac{3b}{2}+\frac{1}{2}a\cos(c+dx)+\frac{1}{2}b\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx}{3a}$$

$$= \frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{(4B)\int \frac{\frac{1}{4}(a^2+3b^2)+ab\cos(c\cdots}{\sqrt{\cos(c+dx}\cdots}}{3a^2\cdots}$$

$$= \frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(4B)\int \frac{-\frac{1}{4}b(a^2+3b^2)-\frac{1}{4}a\cdots}{\sqrt{\cos(c+dx)}\,(\cdots}}{3a^2b}$$

$$= \frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \cdots$$

$$= \frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{2b^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+\cdots\right.}{a^2(a+b)d}$$

Mathematica [A]

time = 4.12, size = 211, normalized size = 1.59

$$\frac{B\left(\frac{2(2a^2+9b^2)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)|2\right)}{a+b} + 8a\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right) - \frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)|2\right)}{a+b}\right) + \frac{4(a-3b\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} + \frac{6\left(-2abE\left(\operatorname{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\operatorname{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(-2a^2+b^2)\Pi\left(-\frac{b}{a};\operatorname{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a\sqrt{\sin^2(c+dx)}}\right)}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])
^2),x]
```

```
[Out] (B*((2*(2*a^2 + 9*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) +
 8*a*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(a + b)) + (4*(a - 3*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3
/2) + (6*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*El
```

```
lipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a),
 ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2])))/
(6*a^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(203) = 406$.
time = 0.57, size = 425, normalized size = 3.20

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\; B\left(-\frac{2b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{a^2\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x,method=_RETU
RNVERBOSE)
```

```
[Out] -2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(-2*b^3/a^2/
(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
)-1/a^2*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/
2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5992 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2),x)
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2), x
)
```

3.395 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=560

$$-\frac{(a-b)\sqrt{a+b}\,(6aAb-3a^2B+16b^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{24ab^2d}$$

```
[Out] 1/3*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d+1/24*(6*A*a*b-
3*B*a^2+16*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)+
1/4*(2*A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/b/d-1/24
*(a-b)*(6*A*a*b-3*B*a^2+16*B*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d+1/24*(a+2*b)*(
6*A*b-3*B*a+8*B*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+1/8*(2*A*a^2*b-8*A*b^3-B*a^3-4*
B*a*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d
```

Rubi [A]
time = 0.98, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3069, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$-\frac{(a-b)\sqrt{a+b}\,(-3a^2B+6aAb+16b^2B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{24ab^2d}+\frac{(-3a^2B+6aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24b^2d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\,(a^3(-B)+2a^2Ab-4ab^2B-8Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{8b^3d}+\frac{\sqrt{a+b}\,(a+2b)(-3aB+6Ab+8bB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{24b^2d}+\frac{(2Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{4bd}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] -1/24*((a - b)*Sqrt[a + b]*(6*a*A*b - 3*a^2*B + 16*b^2*B)*Cot[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(a*b^2*d) + (Sqrt[a + b]*(a + 2*b)*(6*A*b - 3*a*B + 8*b*B
)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^2*d) + (Sqrt[a + b]*(2*a^2*A*b -
 8*A*b^3 - a^3*B - 4*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(8*b^3*d) + ((6*a*A*b - 3*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(24*b^2*d*Sqrt[Cos[c + d*x]]) + ((2*A*b - a*B)*Sqrt[Cos[c + d*x]]
```

```
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d) + (B*Sqrt[Cos[c + d*x]]*(a
+ b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{B\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}\sin(c\cdots}{3bd}$$

$$= \frac{(2Ab-aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+d\cdots}}{4bd}$$

$$= \frac{(6aAb-3a^2B+16b^2B)\,\sqrt{a+b\cos(c+dx)}\cdots}{24b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{(6aAb-3a^2B+16b^2B)\,\sqrt{a+b\cos(c+dx)}\cdots}{24b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{a+b}\,(2a^2Ab-8Ab^3-a^3B-4ab^2B)\cot(\cdots}{\cdots}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(6aAb-3a^2B+16b^2B)\cot\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.34, size = 1224, normalized size = 2.19

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] -1/48*((-4*a*(-18*a*A*b + a^2*B - 16*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4
*a*(-24*A*b^2 - 28*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4
)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Co
```

```
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])) + 2*(-6*a*A*b + 3*a^2*B - 16*b^2*B)*((I*Cos[(c + d*x)/2]*Sq
rt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*
x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x
]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(b*d) + (Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]]*(((6*A*b + a*B)*Sin[c + d*x])/(12*b) + (B*Sin[2
*(c + d*x)])/6))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2948 vs. $2(512) = 1024$.
time = 1.95, size = 2949, normalized size = 5.27

method	result	size
default	Expression too large to display	2949

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/24/d/(a+b*cos(d*x+c))^(1/2)*(16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*B*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+2*B*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-28*B*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^2+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)*b^3-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
```

(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3+8*B*cos(d*x+c)^5*b^3+8*B*co
s(d*x+c)^3*b^3-3*B*cos(d*x+c)^2*a^3-16*B*cos(d*x+c)^2*b^3+3*B*cos(d*x+c)*a^
3+12*A*cos(d*x+c)^4*b^3-12*A*cos(d*x+c)^2*b^3+6*A*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+6*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)*a*b^2-12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+12*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*B*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b+16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b^2+24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+2*B*sin(d*x+c)*cos(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
-28*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^2+48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-3*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+16*B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+6*B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-3
B(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)*a^3+16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+6*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+6*A*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*

```
a*b^2-12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^2*b+12*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+3*B*cos(d*x+c)^2*a^2*b+6*B*cos(d*
x+c)^2*a*b^2-2*B*cos(d*x+c)*a^2*b-16*B*cos(d*x+c)*a*b^2+10*B*cos(d*x+c)^4*a
*b^2-B*cos(d*x+c)^3*a^2*b+18*A*cos(d*x+c)^3*a*b...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(
cos(d*x + c)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^{3/2}\,(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

3.396 $\int \sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=473

$$-\frac{(a-b)\sqrt{a+b}\,(4Ab+aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4abd}$$

```
[Out] 1/4*(4*A*b+B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)+1/2*
B*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/4*(a-b)*(4*A*b+B*a
)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(
d*x+c))/(a-b))^(1/2)/a/b/d+1/4*(4*A*b+(a+2*b)*B)*cot(d*x+c)*EllipticF((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^
(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d-1/4
*(4*A*a*b-B*a^2+4*B*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A]
time = 0.67, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3082, 3140, 3132, 2888, 3077, 2895, 3073}

$$-\frac{\sqrt{a+b}\,(a^2(-B)+4aAb+4b^2B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}+\frac{\sqrt{a+b}\,(B(a+2b)+4Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}-\frac{(a-b)\sqrt{a+b}\,(aB+4Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4abd}+\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4bd\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] -1/4*((a - b)*Sqrt[a + b]*(4*A*b + a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(a*b*d) + (Sqrt[a + b]*(4*A*b + (a + 2*b)*B)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(4*b*d) - (Sqrt[a + b]*(4*a*A*b - a^2*B + 4*b^2*B)*Cot[c + d*x]*Ellipt
icPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(4*b^2*d) + ((4*A*b + a*B)*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3082

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a +
 b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)
*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(
a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
```

```
[n^2, 1/4]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{B\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,\sin(c+}{2d}$$

$$= \frac{(4Ab+aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}}$$

$$= \frac{(4Ab+aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\,(4aAb-a^2B+4b^2B)\cot(c+dx)}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(4Ab+aB)\cot(c+dx)E\Big(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.69, size = 1175, normalized size = 2.48

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] (B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-4*a
*(4*A*b + 3*a*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*a*A + 4*b*B)*((Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*A*b + a*B)*
((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]
) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Co
s[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8
*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2051 vs. $2(431) = 862$.
time = 0.32, size = 2052, normalized size = 4.34

method	result	size

default	Expression too large to display	2052

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*cos(d*x+c)^4*b^2-2*B*cos(d*x+c)^2*b^2+B*cos(d*x+c)^2*a^2-B*cos(d*x+c)*a^2+4*A*cos(d*x+c)^3*b^2-4*A*cos(d*x+c)^2*b^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+B*sin(d*x+c)*cos(d*x

```
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*B
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*b^2-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+3*B*cos(d*x+c
)^3*a*b-B*cos(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*b+4*A*cos(d*x+c)^2*a*b-4*A*cos(
d*x+c)*a*b)/sin(d*x+c)/b/cos(d*x+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))\sqrt{a + b\cos(c + dx)}\sqrt{\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

3.397 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=385

$$-\frac{(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+}{}}}{ad}$$

```
[Out] B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(a-b)*B*cot(d*x+c)*E
llipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/a/d+(2*A+B)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-(2*A*b+B*a)*cot(d*x+c)*EllipticPi((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/b/d
```

Rubi [A]

time = 0.46, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3082, 3132, 2888, 3077, 2895, 3073}

$$\frac{\sqrt{a+b}\,(2A+B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}-\frac{\sqrt{a+b}\,(aB+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}-\frac{B(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)) + (Sqrt[
a + b]*(2*A + B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (Sqrt[a + b]*(2*A*b
 + a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (B*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
```

```
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3082

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a +
 b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)
*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(
a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\int \frac{-aB+2aA\cos}{\cos^{\frac{3}{2}}(}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\int \frac{-aB+2a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= -\frac{\sqrt{a+b}\,(2Ab+aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+b}}\right.\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\,\sqrt{\cos(}}\right.\right.}{}$$

Mathematica [A]
time = 10.96, size = 408, normalized size = 1.06

$$\frac{\sqrt{\cos(c+dx)}\left(2(a+b)B\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)-4(Ab+a(-A+B))\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)+8Ab\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)+4aB\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)+bB\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{3}{2}(c+dx)\right)+2aB\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\tan\left(\frac{1}{2}(c+dx)\right)-bB\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{2d\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(a + b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(A*
b + a*(-A + B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 8*A*b*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)] + 4*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b
*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))
/2] + 2*a*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*B*Sq
```

```
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(2*d*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1692 vs. $2(357) = 714$.
time = 0.53, size = 1693, normalized size = 4.40

method	result	size
default	Expression too large to display	1693

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(4*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*b+2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*A*sin(d*x+c)*cos(d*x+c)^2
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+8*A*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+
b))^(1/2))*b+4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a-4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+4*A*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b+2*A*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*A*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*B*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*a+B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a+B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*B*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+2*B*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))
```

```
/(a+b))^(1/2)*a+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*B*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+B*cos(d*x+c)
^4*b+B*cos(d*x+c)^3*a-B*cos(d*x+c)^3*b-B*cos(d*x+c)^2*a)/(a+b*cos(d*x+c))^(
1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)),
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)),
x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)})\sqrt{a + b\cos{(c + dx)}}}{\sqrt{\cos{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/sqrt(cos(c + d*x)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}}{\sqrt{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2), x
)
```

3.398 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=351

$$\frac{2A(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+}{}}}{ad}$$

```
[Out] 2*A*(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+2*(A*b-a*(A-B))*cot(d*x+c)*EllipticF((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^
(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-2*B
*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A]
time = 0.33, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3070, 2888, 3077, 2895, 3073}

$$\frac{2\sqrt{a+b}\,(Ab-a(A-B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+\frac{2A(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2B\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*Sqrt
[a + b]*(A*b - a*(A - B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqr
t[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
```

```
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3070

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)]])/((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[B*(d
/b^2), Int[Sqrt[b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Int[(A*c
 + (B*c + A*d)*Sin[e + f*x])/((b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = (bB)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx + \int \frac{aA+(Ab+aB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+}}$$

$$= -\frac{2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\,\sqrt{\cos(c}}\right.\right.}{}$$

$$= \frac{2A(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c-}}{\sqrt{a+b}\,\sqrt{\cos(c}}\right.\right.}{}$$

Mathematica [A]
time = 12.77, size = 273, normalized size = 0.78

$$\frac{-2A(a+b)\sqrt{1+\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+2(b(A-B)+a(A+B))\sqrt{1+\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+4bB\sqrt{1+\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\Pi\left(-1;\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+\frac{2A(a+b\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] (-2*A*(a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2
*(b*(A - B) + a*(A + B))*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] + 4*b*B*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)] + (2*A*(a + b*Cos[c + d*x])*Tan[(c + d*x)/2])/Sqrt[Cos[c + d*x]])/(d*Sq
rt[a + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1686 vs. $2(327) = 654$.
time = 0.87, size = 1687, normalized size = 4.81

method	result	size
default	Expression too large to display	1687

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/d*(B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
```

```
(d*x+c)*cos(d*x+c)^2*a-B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b+2*B*(cos(d*x
+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)*a-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*cos(d*x+c)*b+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a+A*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*
b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)^2*a-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b+B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a-B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b+2*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b+A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a+A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*cos(d*x+c)*a-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b+A*cos(d*x+c)^3*b+A*cos(d*x+c)^2*a-A*
cos(d*x+c)^2*b-A*cos(d*x+c)*a)/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2)/sin(
d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{a + b\cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(3/2),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\,\cos(c + d\,x))\,\sqrt{a + b\,\cos(c + d\,x)}}{{\cos(c + d\,x)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2), x
)
```

3.399 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=284

$$\frac{2(a-b)\sqrt{a+b}\,(Ab+3aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3a^2d}$$

```
[Out] 2/3*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/3*(a-b)*(A*b+3
*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/a^2/d+2/3*(a-b)*(A-3*B)*cot(d*x+c)*EllipticF((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^
(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A]
time = 0.33, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3078, 3077, 2895, 3073}

$$\frac{2(a-b)\sqrt{a+b}\,(3aB+Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}+\frac{2(a-b)\sqrt{a+b}\,(A-3B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad}+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b + 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*d) + (2*(a - b)*Sqrt[a + b]*(A - 3*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(3*a*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(
3/2))
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}(Ab+3aB)+}{\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}((a-b)(A-3B))$$

$$= \frac{2(a-b)\sqrt{a+b}\,(Ab+3aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+b}}\right.\right.}{}$$

Mathematica [A]

time = 13.52, size = 407, normalized size = 1.43

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)^{5/2}\left(\frac{\cos(c+dx)}{1+\cos(c+dx)}\right)^{3/2}\sqrt{1+\cos(c+dx)}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(-2(a+b)(Ab+3aB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+2a(a+b)(A+3B)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)-(Ab+3aB)\cos(c+dx)(a+b\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)(Ab\sin(c+dx)+3aB\sin(c+dx))}{3a}+\frac{2}{3}A\sec(c+dx)\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] (4*(Cos[(c + d*x)/2]^2)^(5/2)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[
1 + Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(a + b)*(A*b +
3*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)] + 2*a*(a + b)*(A + 3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)] - (A*b + 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a*d*Cos[c + d*x]^(5/2)*Sqrt[a +
b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c +
 d*x]*(A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(3*a) + (2*A*Sec[c + d*x]*Ta
n[c + d*x])/3))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1728 vs. $2(258) = 516$.
time = 0.31, size = 1729, normalized size = 6.09

method	result	size
default	Expression too large to display	1729

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3/d*(a^2*A+A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2+3*B*cos(d*x+c)^2*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-3*B
*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-A*(cos(d*x+c)/(1+cos(
```

```
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos
(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+3*B*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*cos(d*x+c)^2*a^2-3*B*cos(d*x+c)^2*a^2+3
*B*cos(d*x+c)*a^2-A*cos(d*x+c)^3*b^2+A*cos(d*x+c)^2*b^2+A*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-
A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b+A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x
+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*b^2+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-A*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^3*
a*b+3*B*cos(d*x+c)^2*a*b-A*cos(d*x+c)^2*a*b+2*A*cos(d*x+c)*a*b)/(a+b*cos(d*
x+c))^(1/2)/a/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos{(c+dx)})\sqrt{a+b\cos{(c+dx)}}}{\cos^{\frac{5}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(5/2),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2), x
)
```

3.400 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=350

$$\frac{2(a-b)\sqrt{a+b}\,(9a^2A-2Ab^2+5abB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\text{se}}{a}}}{15a^3d}$$

```
[Out] 2/5*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/15*(A*b+5*B*a)
*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9*A*a^2
-2*A*b^2+5*B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d-2/15*(a-b)*(9*A*a+2*A*b-5*B*a)*c
ot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/a^2/d
```

Rubi [A]

time = 0.54, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3078, 3134, 3077, 2895, 3073}

$$-\frac{2(a-b)\sqrt{a+b}\,(9aA-5aB+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^2d}+\frac{2(a-b)\sqrt{a+b}\,(9a^2A+5abB-2Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}+\frac{2(5aB+Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{15ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(9*a*A + 2*A*b - 5*a*B)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(A*b + 5*a*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2))
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

```
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
```

```
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

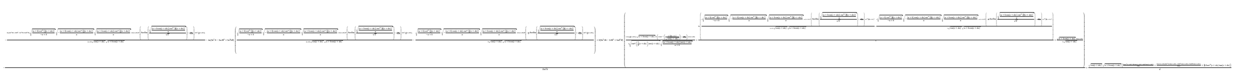
$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}(Ab+5aB)+}{\mathrm{c}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+}}{15ad\,\mathrm{c}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+}}{15ad\,\mathrm{c}}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(9a^2A-2Ab^2+5abB)\cot(c+dx)E\Big(\sin}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.40, size = 1315, normalized size = 3.76



Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] -1/15*((-4*a*(2*a^2*A*b - 2*A*b^3 - 5*a^3*B + 5*a*b^2*B)*Sqrt[((a + b)*Cot[
(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/
a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
F[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)
/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]]) - 4*a*(9*a^3*A - 2*a*A*b^2 + 5*a^2*b*B)*((Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
```

```
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b - 2*A*b^3 + 5*a*b^2*
B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c
 + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a +
b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b
), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))
/(a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*
(A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/(15*a) + (2*Sec[c + d*x]*(9*a^2*A*
Sin[c + d*x] - 2*A*b^2*Sin[c + d*x] + 5*a*b*B*Sin[c + d*x]))/(15*a^2) + (2*
A*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2480 vs. $2(318) = 636$.
time = 0.37, size = 2481, normalized size = 7.09

method	result	size
default	Expression too large to display	2481

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/15/d*(3*A*a^3-5*B*cos(d*x+c)^3*a^3-9*A*cos(d*x+c)^3*a^3-2*A*cos(d*x+c)^3*
b^3+6*A*cos(d*x+c)^2*a^3-7*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)^2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+5*B*
sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*a^2*b+5*B*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2-5*B*sin(d*x+c)*cos(d*x+c)^2*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+9*A*sin(d*
x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
```

```
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2-7*A*sin(d*x+c)*cos(d*x+c)^3*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+2*A*sin(d*x+c)*c
os(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*a*b^2+5*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b+5*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-5*B*sin(d*x+c)*cos(d*x
+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b
-5*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*a^3+9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-2*A*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-9*A*sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^3-5*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*a^3+9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-2*A*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-9*
A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3+5*B*cos(d*x+c)*a^3+2*A*cos(d*x+c)^4*b^3+9*A*sin(d*x+c)*c
os(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*a*b^2-5*B*cos(d*x+c)^4*a^2*b+5*B*cos(d*x+c)^3*a*b^2
+5*A*cos(d*x+c)^3*a^2*b-9*A*cos(d*x+c)^4*a^2*b-A*cos(d*x+c)^4*a*b^2+10*B*co
s(d*x+c)^2*a^2*b-5*B*cos(d*x+c)^4*a*b^2-5*B*cos(d*x+c)^3*a^2*b+2*A*cos(d*x+
c)^3*a*b^2-A*cos(d*x+c)^2*a*b^2+4*A*cos(d*x+c)*a^2*b)/(a+b*cos(d*x+c))^(1/2
)/a^2/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2),
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2),
 x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\cos(c+dx)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2), x
)
```

3.401 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=433

$$\frac{2(a-b)\sqrt{a+b}\,(19a^2Ab+8Ab^3+63a^3B-14ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a-}{a-}\right.}{105a^4d}$$

```
[Out] 2/7*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/35*(A*b+7*B*a)
*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+2/105*(25*A*a^2-4*A
*b^2+7*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(3/2)+2/10
5*(a-b)*(19*A*a^2*b+8*A*b^3+63*B*a^3-14*B*a*b^2)*cot(d*x+c)*EllipticE((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^
(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2
/105*(a-b)*(8*A*b^2+a^2*(25*A-63*B)+2*a*b*(3*A-7*B))*cot(d*x+c)*EllipticF((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3
/d
```

Rubi [A]
time = 0.77, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3078, 3134, 3077, 2895, 3073}

$$\frac{2(25a^2A+7abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,(a^2(25A-63B)+2ab(3A-7B)+8Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{105a^3d}+\frac{2(a-b)\sqrt{a+b}\,(63a^3B+19a^2Ab-14ab^2B+8Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{105a^4d}+\frac{2(7aB+Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{35ad\cos^{\frac{5}{2}}(c+dx)}+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]*(8*A*b^2
 + a^2*(25*A - 63*B) + 2*a*b*(3*A - 7*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(105*a^3*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*
x]^(7/2)) + (2*(A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*a*d
*Cos[c + d*x]^(5/2)) + (2*(25*a^2*A - 4*A*b^2 + 7*a*b*B)*Sqrt[a + b*Cos[c +
 d*x]]*Sin[c + d*x])/(105*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
```

```
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
```

```
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}(Ab+7aB)}{}$$

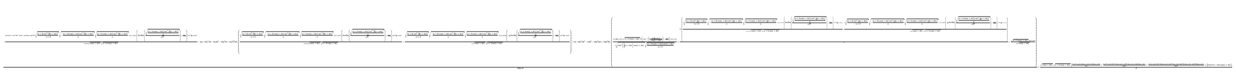
$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+}}{35ad}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+}}{35ad}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+}}{35ad}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(19a^2Ab+8Ab^3+63a^3B-14ab^2B)\cot(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.48, size = 1408, normalized size = 3.25



Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2
),x]
```

```
[Out] ((-4*a*(25*a^4*A - 17*a^2*A*b^2 - 8*A*b^4 - 14*a^3*b*B + 14*a*b^3*B)*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - 4*a*(-19*a^3*A*b - 8*a*A*b^3 - 63*a^4*B + 14*a^2*b^
```

```
2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(
-19*a^2*A*b^2 - 8*A*b^4 - 63*a^3*b*B + 14*a*b^3*B)*((I*Cos[(c + d*x)/2]*Sqr
t[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x
]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a^3*d) + (Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(A*b*Sin[c + d*x] + 7*a
*B*Sin[c + d*x]))/(35*a) + (2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x] - 4*A*b
^2*Sin[c + d*x] + 7*a*b*B*Sin[c + d*x]))/(105*a^2) + (2*Sec[c + d*x]*(19*a^
2*A*b*Sin[c + d*x] + 8*A*b^3*Sin[c + d*x] + 63*a^3*B*Sin[c + d*x] - 14*a*b^
2*B*Sin[c + d*x]))/(105*a^3) + (2*A*Sec[c + d*x]^3*Tan[c + d*x])/7))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3426 vs. $2(395) = 790$.
time = 0.49, size = 3427, normalized size = 7.91

method	result	size
default	Expression too large to display	3427

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*(25*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^4-8*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+63*B*sin(d*x+c)*cos(d*
```

x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-
63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^4+25*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^4-8*A*sin(d*x+c)*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+63*B*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^4-63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-15*A*a^4+25*A*cos(d*x+c)^4*a^4-10*A*cos(d
*x+c)^2*a^4+63*B*cos(d*x+c)^4*a^4-42*B*cos(d*x+c)^3*a^4-21*B*cos(d*x+c)*a^4
+8*A*cos(d*x+c)^5*b^4-8*A*cos(d*x+c)^4*b^4+19*A*sin(d*x+c)*cos(d*x+c)^4*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-4*A*cos(
d*x+c)^3*a*b^3+A*cos(d*x+c)^2*a^2*b^2-18*A*cos(d*x+c)*a^3*b+25*A*cos(d*x+c)
^5*a^3*b+19*A*cos(d*x+c)^5*a^2*b^2-4*A*cos(d*x+c)^5*a*b^3+19*A*cos(d*x+c)^4
*a^3*b-20*A*cos(d*x+c)^4*a^2*b^2+8*A*cos(d*x+c)^4*a*b^3-26*A*cos(d*x+c)^3*a
^3*b-28*B*cos(d*x+c)^2*a^3*b+63*B*cos(d*x+c)^5*a^3*b+7*B*cos(d*x+c)^5*a^2*b
^2-14*B*cos(d*x+c)^5*a*b^3-35*B*cos(d*x+c)^4*a^3*b-14*B*cos(d*x+c)^4*a^2*b^
2+14*B*cos(d*x+c)^4*a*b^3+7*B*cos(d*x+c)^3*a^2*b^2+2*A*sin(d*x+c)*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2
+8*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^3-19*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-19*A*sin(d*x+c)*cos(d*x+c)^
4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8
A(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)^4*a*b^3+49*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-14*B*sin(d*x+c)*cos(d*x+c)^4*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-63*
B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3*b+14*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+14*B*sin(d*x+c)*cos(d*x+c)^4

```
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+19*A
*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^3*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b^2+8*A*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^3-19*A*si
n(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^3*b-19*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)^3*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+49*B*sin(
d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2),
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(9/2), x
)
```

3.402 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=670

$$-\frac{(a-b)\sqrt{a+b}\,(24a^2Ab+128Ab^3-9a^3B+156ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\right.}{192ab^2d}$$

```
[Out] 1/24*(8*A*b-3*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d+1
/4*B*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d+1/192*(24*A*a^2
*b+128*A*b^3-9*B*a^3+156*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d/c
os(d*x+c)^(1/2)+1/32*(8*A*a*b-3*B*a^2+12*B*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)
*(a+b*cos(d*x+c))^(1/2)/b/d-1/192*(a-b)*(24*A*a^2*b+128*A*b^3-9*B*a^3+156*B
*a*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d-1/192*(9*a^3*B-6*a^2*b*(4*A+B)-8*b^3*(16
*A+9*B)-4*a*b^2*(28*A+39*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a
+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+1/64*(8*A*a^3*b-96*A*
a*b^3-3*B*a^4-24*B*a^2*b^2-48*B*b^4)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/
2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d
```

Rubi [A]
time = 1.36, antiderivative size = 670, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3069, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] -1/192*((a - b)*Sqrt[a + b]*(24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B
)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*d) - (Sqrt[a + b]*(9*a^3*B - 6*
a^2*b*(4*A + B) - 8*b^3*(16*A + 9*B) - 4*a*b^2*(28*A + 39*B))*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(192*b^2*d) + (Sqrt[a + b]*(8*a^3*A*b - 96*a*A*b^3 -
3*a^4*B - 24*a^2*b^2*B - 48*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
```

```
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(64*b^3*d) + ((24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(192*b^2*d*Sqrt[Cos[c + d*x]]) + ((8*a*A*
b - 3*a^2*B + 12*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(32*b*d) + ((8*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^
(3/2)*Sin[c + d*x])/(24*b*d) + (B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(
5/2)*Sin[c + d*x])/(4*b*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
```

```
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{B\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}\sin(c}{4bd}$$

$$= \frac{(8Ab-3aB)\sqrt{\cos(c+dx)}\,(a+b\cos(c+d}{24bd}$$

$$= \frac{(8aAb-3a^2B+12b^2B)\sqrt{\cos(c+dx)}\,\sqrt{a}{32bd}$$

$$= \frac{(24a^2Ab+128Ab^3-9a^3B+156ab^2B)\sqrt{a}{192b^2d\sqrt{\cos(c+d}}$$

$$= \frac{(24a^2Ab+128Ab^3-9a^3B+156ab^2B)\sqrt{a}{192b^2d\sqrt{\cos(c+d}}$$

$$= \frac{\sqrt{a+b}\,(8a^3Ab-96aAb^3-3a^4B-24a^2b^2}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(24a^2Ab+128Ab^3-9a^3B}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.45, size = 1284, normalized size = 1.92

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] -1/384*((-4*a*(-136*a^2*A*b - 128*A*b^3 + 3*a^3*B - 228*a*b^2*B)*Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Cos[c + d*x]]) - 4*a*(-416*a*A*b^2 - 228*a^2*b*B - 144*b^3*B)*((Sqrt[((a
```

```
 + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-24*a^2*A*b - 1
28*A*b^3 + 9*a^3*B - 156*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a -
 b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Co
s[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(b*Sqrt[Cos[c + d*x]])))/(b*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]*(((56*a*A*b + 3*a^2*B + 42*b^2*B)*Sin[c + d*x])/(96*b) + ((8*A*b
+ 9*a*B)*Sin[2*(c + d*x)])/48 + (b*B*Sin[3*(c + d*x)])/16))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4047 vs. $2(616) = 1232$.
time = 0.64, size = 4048, normalized size = 6.04

method	result	size
default	Expression too large to display	4048

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/192/d/(a+b*cos(d*x+c))^(1/2)*(64*A*cos(d*x+c)^3*b^4-128*A*cos(d*x+c)^2*b
^4-3*B*cos(d*x+c)^3*a^3*b+108*B*cos(d*x+c)^3*a*b^3+24*A*cos(d*x+c)^2*a^3*b-
48*A*cos(d*x+c)^2*a*b^3-112*A*cos(d*x+c)*a^2*b^2-128*A*cos(d*x+c)*a*b^3+136
*A*cos(d*x+c)^3*a^2*b^2+78*B*cos(d*x+c)^2*a^2*b^2-156*B*cos(d*x+c)^2*a*b^3-
6*B*cos(d*x+c)*a^3*b-156*B*cos(d*x+c)*a^2*b^2-72*B*cos(d*x+c)*a*b^3+24*B*co
s(d*x+c)^4*b^4-72*B*cos(d*x+c)^2*b^4-9*B*cos(d*x+c)^2*a^4-228*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+72*
```

B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^3+144*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a
-b)/(a+b))^(1/2))*a^2*b^2+24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2+128*A*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^3+11
2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*a^2*b^2-416*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^3-48*A*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b+576
*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*a*b^3-9*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b+156*B*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2+156*B
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)*a*b^3+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b-228*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2+72*B*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*a*b^3+144*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2+24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b+128*A*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-9*B
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^4-144*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*b^4+18*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-

```
1,(-(a-b)/(a+b))^(1/2))*a^4+288*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+48*B*cos(d*x+c)^6*b^4+9*B*cos(d*x
+c)*a^4+64*A*cos(d*x+c)^5*b^4-24*A*cos(d*x+c)^2*a^2*b^2-24*A*cos(d*x+c)*a^3
*b+176*A*cos(d*x+c)^4*a*b^3+9*B*cos(d*x+c)^2*a^3*b+120*B*cos(d*x+c)^5*a*b^3
+78*B*cos(d*x+c)^4*a^2*b^2+128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^4-9*B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^4-144*B*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)*b^4+18*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^3 + A*a*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)^
2)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

3.403 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=566

$$-\frac{(a-b)\sqrt{a+b}\,(30aAb+3a^2B+16b^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{24abd}$$

```
[Out] 1/3*b*B*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+1/24*(30*A*a*b
+3*B*a^2+16*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)+1
/12*(6*A*b+7*B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/24
*(a-b)*(30*A*a*b+3*B*a^2+16*B*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+1/24*(30*A*a*b+
12*A*b^2+3*B*a^2+14*B*a*b+16*B*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d-1/8*(6*A*a^2*b+8
*A*b^3-B*a^3+12*B*a*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A]
time = 1.09, antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3069, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$\frac{\sqrt{a+b}\,(3a^2B+30aAb+14abB+12Ab^2+16b^2B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{24bd}-\frac{(a-b)\sqrt{a+b}\,(3a^2B+30aAb+16b^2B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{24abd}+\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\,(a^3(-B)+6a^2Ab+12ab^2B+8Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{8b^2d}+\frac{(7aB+6Ab)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{12d}+\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] -1/24*((a - b)*Sqrt[a + b]*(30*a*A*b + 3*a^2*B + 16*b^2*B)*Cot[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(a*b*d) + (Sqrt[a + b]*(30*a*A*b + 12*A*b^2 + 3*a^2*B +
14*a*b*B + 16*b^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d) - (Sqrt[a
+ b]*(6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a
+ b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(8*b^2*d) + ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x]]) + ((6*A*b + 7*a*B)
```

```
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d) + (b*B*Co
s[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{bB\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,\sin(c}{3d}$$

$$= \frac{(6Ab+7aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c}{12d}$$

$$= \frac{(30aAb+3a^2B+16b^2B)\,\sqrt{a+b\cos(c+}{24bd\sqrt{\cos(c+dx)}}$$

$$= \frac{(30aAb+3a^2B+16b^2B)\,\sqrt{a+b\cos(c+}{24bd\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\,(6a^2Ab+8Ab^3-a^3B+12ab^2B}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(30aAb+3a^2B+16b^2B}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.33, size = 1227, normalized size = 2.17

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] ((-4*a*(42*a*A*b + 17*a^2*B + 16*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(
48*a^2*A + 24*A*b^2 + 52*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b
)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
```

```
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])) + 2*(30*a*A*b + 3*a^2*B + 16*b^2*B)*((I*Cos[(c + d*x)
/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[
c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(48*d) + (Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(((6*A*b + 7*a*B)*Sin[c + d*x])/12 + (b*
B*Sin[2*(c + d*x)])/6))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3138 vs. $2(518) = 1036$.
time = 0.44, size = 3139, normalized size = 5.55

method	result	size
default	Expression too large to display	3139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/24/d/(a+b*cos(d*x+c))^(1/2)*(16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+72*B*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+14*B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-52*B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b^2+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)*b^3-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3+8*B*cos(d*x+c)^5*b^3+8*B*c
```

os(d*x+c)^3*b^3+3*B*cos(d*x+c)^2*a^3-16*B*cos(d*x+c)^2*b^3-3*B*cos(d*x+c)*a
^3+12*A*cos(d*x+c)^4*b^3-12*A*cos(d*x+c)^2*b^3+30*A*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+30*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)*a*b^2+36*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+12*A*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+3*B*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b+16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2+72*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+14*B*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b-52*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^2-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+48*A*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*A*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+3*B*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
a^3+16*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*b^3-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*a^3-48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+16*B*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-
6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*a^3+30*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x

```
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+30*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+36*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+12*A*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos (c+d\,x)}\ (A+B\ \cos (c+d\,x))\ (a+b\ \cos (c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

3.404 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=472

$$-\frac{(a-b)\sqrt{a+b}\,(4Ab+5aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4ad}$$

```
[Out] 1/4*(4*A*b+5*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*
b*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/4*(a-b)*(4*A*b+5
*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/a/d+1/4*(8*A*a+4*A*b+5*B*a+2*B*b)*cot(d*x+c)*Ellip
ticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/d-1/4*(12*A*a*b+3*B*a^2+4*B*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^
(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2
)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A]
time = 0.76, antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3069, 3140, 3132, 2888, 3077, 2895, 3073}

$$-\frac{\sqrt{a+b}\,(3a^2B+12aAb+4b^2B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}+\frac{\sqrt{a+b}\,(8aA+5aB+4Ab+2bB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4d}-\frac{(a-b)\sqrt{a+b}\,(5aB+4Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4ad}+\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4d\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -1/4*((a - b)*Sqrt[a + b]*(4*A*b + 5*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*d) + (Sqrt[a + b]*(8*a*A + 4*A*b + 5*a*B + 2*b*B)*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(4*d) - (Sqrt[a + b]*(12*a*A*b + 3*a^2*B + 4*b^2*B)*Cot[c +
d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + ((4*A*b + 5*a*B)*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{bB\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{2}\int$$

$$= \frac{(4Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{\cos}}{}$$

$$= \frac{(4Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{\cos}}{}$$

$$= -\frac{\sqrt{a+b}\,(12aAb+3a^2B+4b^2B)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(4Ab+5aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.38, size = 1198, normalized size = 2.54

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (b*B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-4
*a*(8*a^2*A + 4*A*b^2 + 7*a*b*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(16*a*A*
b + 8*a^2*B + 4*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])) + 2*(4*A*b^2 + 5*a*b*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
+ d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-
a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b
*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2429 vs. $2(430) = 860$.
time = 0.37, size = 2430, normalized size = 5.15

method	result	size

default	Expression too large to display	2430

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+5*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+5*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*cos(d*x+c)^4*b^2-2*B*cos(d*x+c)^2*b^2+5*B*cos(d*x+c)^2*a^2-5*B*cos(d*x+c)*a^2+4*A*cos(d*x+c)^3*b^2-4*A*cos(d*x+c)^2*b^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+5*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))

```
*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^2+5*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+7*B*cos(d*x+c)^3*a*b-5*B*cos(d*x+c
)^2*a*b-2*B*cos(d*x+c)*a*b+4*A*cos(d*x+c)^2*a*b-4*A*cos(d*x+c)*a*b+8*A*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-
8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))(a + b\cos(c + dx))^{\frac{3}{2}}}{\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(3/2)/sqrt(cos(c + d*x)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\cos(c + d\,x))\,(a + b\cos(c + d\,x))^{3/2}}{\sqrt{\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2), x
)
```

3.405 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=449

$$\frac{(a-b)\sqrt{a+b}\,(2aA-bB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{ad}$$

```
[Out] 2*a*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(2*A*a-B*b)*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+(a-b)*(2*A*a-B*b)*cot(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a
-b))^(1/2)/a/d-(2*a*(A-B)-b*(4*A+B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^
(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-
sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-(2*A*b+3*B*a)*cot
(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b
)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A]
time = 0.76, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3068, 3140, 3132, 2888, 3077, 2895, 3073}

$$-\frac{\sqrt{a+b}\,(2a(A-B)-b(4A+B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}+\frac{(a-b)\sqrt{a+b}\,(2aA-bB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{\sqrt{a+b}\,(3aB+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}-\frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(2*a*A - b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
 - (Sqrt[a + b]*(2*a*(A - B) - b*(4*A + B))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/d - (Sqrt[a + b]*(2*A*b + 3*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/d + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]) - ((2*a*A - b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
```

```
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}a(2Ab+a}{}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(2aA-bB)\sqrt{a}}{d\sqrt{}}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(2aA-bB)\sqrt{a}}{d\sqrt{}}$$

$$= -\frac{\sqrt{a+b}\,(2Ab+3aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,(2aA-bB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.37, size = 1196, normalized size = 2.66

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((4*
a*(-2*a*A*b - 2*a^2*B - b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 4*a*(2*a^2*A -
 2*A*b^2 - 4*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
+ b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-
(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]])) - 2*(2*a*A*b - b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b
)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[
c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(b*Sqrt[Cos[c + d*x]])))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2184 vs. $2(417) = 834$.
time = 0.31, size = 2185, normalized size = 4.87

method	result	size

default	Expression too large to display	2185

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(-2*a^2*A+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2+4*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*co
s(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-2*A*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-
2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b-B*cos(d*x+c)^2*b^2-2*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b+2*A*cos(d*x+c)*a^2+B*cos(d*x+c)^3*b^2+6*B*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b-2*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

```
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-2*A*sin(d*x+c)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-2*A*sin(d*x+c)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2+B*sin(d*x+c)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+B*cos(d*x+c)^2
*a*b-B*cos(d*x+c)*a*b+2*A*cos(d*x+c)^2*a*b-2*A*cos(d*x+c)*a*b+2*A*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+2*B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)})\,(a + b\cos{(c + dx)})^{\frac{3}{2}}}{\cos^{\frac{3}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(3/2)/cos(c + d*x)**(3/
2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2), x
)
```

3.406 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=419

$$\frac{2(a-b)\sqrt{a+b}\,(4Ab+3aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3ad}$$

```
[Out] 2/3*a*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/3*(a-b)*(4*A
*b+3*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+2/3*(3*A*b^2+a^2*(A-3*B)-a*(4*A*b-6*B*b))*
cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((
-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*
x+c))/(a-b))^(1/2)/a/d-2*b*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(
a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A]
time = 0.55, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3068, 3132, 2888, 3077, 2895, 3073}

$$\frac{2\sqrt{a+b}\,(a^2(A-3B)-a(4Ab-6bB)+3Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad}+\frac{2(a-b)\sqrt{a+b}\,(3aB+4Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2bB\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(4*A*b + 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
3*a*d) + (2*Sqrt[a + b]*(3*A*b^2 + a^2*(A - 3*B) - a*(4*A*b - 6*b*B))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) - (2*b*Sqrt[a + b]*B*Cot[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
```

```
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}a(4Ab+3a \cdots}{\cdots}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}a(4Ab+3a \cdots}{\cos \cdots}$$

$$= -\frac{2b\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c \cdots}}{\sqrt{a+b}\,\sqrt{\cos(c \cdots}}\right.\right.}{\cdots}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(4Ab+3aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a \cdots}}{\sqrt{a+ \cdots}}\right.\right.}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.40, size = 1236, normalized size = 2.95

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] ((-4*a*(a^2*A - A*b^2 + 3*a*b*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-4*a*A*
```

```
b - 3*a^2*B + 3*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])) + 2*(-4*A*b^2 - 3*a*b*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
 + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(
-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a +
b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
+ d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*((2*Sec[c + d*x]*(4*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/3
 + (2*a*A*Sec[c + d*x]*Tan[c + d*x])/3))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2320 vs. $2(385) = 770$.
time = 0.32, size = 2321, normalized size = 5.54

method	result	size
default	Expression too large to display	2321

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3/d*(a^2*A+4*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2+3*B*cos(d*x+c)^2*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-3
*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
```

c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-3*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^
2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)^2*b^2-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2-4*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x
+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+3*B*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b-A*cos(d*x+c)^2*a^2-3*A*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-3*B*cos
(d*x+c)^2*a^2+3*B*cos(d*x+c)*a^2-4*A*cos(d*x+c)^3*b^2+4*A*cos(d*x+c)^2*b^2+
4*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b-4*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b-6*B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*
x+c)*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+3*B*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+3*B*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^2-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-A*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^3*a*b+3*
B*cos(d*x+c)^2*a*b-4*A*cos(d*x+c)^2*a*b+5*A*cos(d*x+c)*a*b)/(a+b*cos(d*x+c)

```
)^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\,(a + b\cos(c + dx))^{\frac{3}{2}}}{\cos^{\frac{5}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(3/2)/cos(c + d*x)**(5/
2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\ (a+b\cos(c+dx))^{3/2}}{\cos(c+dx)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2), x
)
```

3.407 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=353

$$\frac{2(a-b)\sqrt{a+b}\,(9a^2A+3Ab^2+20abB)\cot(c+dx)E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{15a^2d}$$

```
[Out] 2/5*a*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/15*(6*A*b+5*
B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9*A*a
^2+3*A*b^2+20*B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2
)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-2/15*(a-b)*(9*A*a-3*A*b-5*B*a
+15*B*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)
^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A]
time = 0.57, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3068, 3134, 3077, 2895, 3073}

$$\frac{2(a-b)\sqrt{a+b}\,(9a^2A+20abB+3Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^2d}-\frac{2(a-b)\sqrt{a+b}\,(9aA-5aB-3Ab+15bB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15ad}+\frac{2(5aB+6Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^2*d) - (2*(a - b)*Sqrt[a + b]*(9*a*A - 3*A*b - 5*a*B +
15*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d) + (2*a*A*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(6*A*b + 5*a*B)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

```
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
```

```
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}a(6Ab+5 \cdots}{\cdots}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(6Ab+5aB)\sqrt{\cdots}}{15\cdots}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(6Ab+5aB)\sqrt{\cdots}}{15\cdots}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(9a^2A+3Ab^2+20abB)\cot(c+dx)E\Big(\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.47, size = 1314, normalized size = 3.72

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] -1/15*((-4*a*(-3*a^2*A*b + 3*A*b^3 - 5*a^3*B + 5*a*b^2*B)*Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - 4*a*(9*a^3*A + 3*a*A*b^2 + 20*a^2*b*B)*((Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[A
rcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-
a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
```

```
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b + 3*A*b^3 + 20*a*b
^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin
[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a
 + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b
)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(
a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]
)))/(a*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^2
*(6*A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/15 + (2*Sec[c + d*x]*(9*a^2*A*S
in[c + d*x] + 3*A*b^2*Sin[c + d*x] + 20*a*b*B*Sin[c + d*x]))/(15*a) + (2*a*
A*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2665 vs. $2(321) = 642$.
time = 0.37, size = 2666, normalized size = 7.55

method	result	size
default	Expression too large to display	2666

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*(15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^3*a*b^2+15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2-3*A*a^3+5*B*cos(d*
x+c)^3*a^3+9*A*cos(d*x+c)^3*a^3-3*A*cos(d*x+c)^3*b^3-6*A*cos(d*x+c)^2*a^3+1
2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-20*B*sin(d*x+c)*cos(d*x+c)^2*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
```

os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b-20*B*
sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*a*b^2+20*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b-9*A*sin(d*x+c)*cos(d*x+c)^3*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b-3*A*sin(d
*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*a*b^2+12*A*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+3*A*sin(d*x+c)*cos(d*x+c)^3*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2-20*B*sin(d*x+c
)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b-20*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+20*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+5*B*sin(d*x+c)*co
s(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
a^3-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3-3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9*A*sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+5*B*sin(
d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*a^3-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9*A*sin(d*x+c)*cos(d
x+c)^3(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
-5*B*cos(d*x+c)*a^3+3*A*cos(d*x+c)^4*b^3-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-3*A*sin(d*x
+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^

```
(1/2)*a*b^2+5*B*cos(d*x+c)^4*a^2*b-20*B*cos(d*x+c)^3*a*b^2+9*A*cos(d*x+c)^4
*a^2*b+6*A*cos(d*x+c)^4*a*b^2-25*B*cos(d*x+c)^2*a^2*b+20*B*cos(d*x+c)^4*a*b
^2+20*B*cos(d*x+c)^3*a^2*b+3*A*cos(d*x+c)^3*a*b^2-9*A*cos(d*x+c)^2*a*b^2-9*
A*cos(d*x+c)*a^2*b)/(a+b*cos(d*x+c))^(1/2)/a/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{3/2}}{\cos(c+dx)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(7/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(7/2), x
)
```

3.408 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=433

$$\frac{2(a-b)\sqrt{a+b}\,(82a^2Ab-6Ab^3+63a^3B+21ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{105a^3d}$$

```
[Out] 2/7*a*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/35*(8*A*b+7*
B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/105*(25*A*a^2+3
*A*b^2+42*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)+2/1
05*(a-b)*(82*A*a^2*b-6*A*b^3+63*B*a^3+21*B*a*b^2)*cot(d*x+c)*EllipticE((a+b
*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d-
2/105*(a-b)*(6*A*b^2-a^2*(25*A-63*B)+3*a*b*(19*A-7*B))*cot(d*x+c)*EllipticF
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a
^2/d
```

Rubi [A]
time = 0.82, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3068, 3134, 3077, 2895, 3073}

$$-\frac{2(a-b)\sqrt{a+b}\,(-(a^2(25A-63B))+3ab(19A-7B)+6Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{105a^2d}+\frac{2(25a^2A+42abB+3Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,(63a^3B+82a^2Ab+21ab^2B-6Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{105a^3d}+\frac{2(7aB+8Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d) - (2*(a - b)*Sqrt[a + b]*(6*A*b^2
 - a^2*(25*A - 63*B) + 3*a*b*(19*A - 7*B))*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(105*a^2*d) + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c +
 d*x]^(7/2)) + (2*(8*A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3
5*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(105*a*d*Cos[c + d*x]^(3/2))
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
```

```
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
```

```
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}a(8Ab+7a}{}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a}}{35a}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a}}{35a}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a}}{35a}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(82a^2Ab-6Ab^3+63a^3B+21ab^2B)\cot(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.54, size = 1407, normalized size = 3.25

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9
/2),x]
```

```
[Out] ((-4*a*(25*a^4*A - 31*a^2*A*b^2 + 6*A*b^4 + 21*a^3*b*B - 21*a*b^3*B)*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
```

```
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - 4*a*(-82*a^3*A*b + 6*a*A*b^3 - 63*a^4*B - 21*a^2*b^
2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(
-82*a^2*A*b^2 + 6*A*b^4 - 63*a^3*b*B - 21*a*b^3*B)*((I*Cos[(c + d*x)/2]*Sqr
t[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x
]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a^2*d) + (Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(8*A*b*Sin[c + d*x] + 7
*a*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x] + 3*A*b^2
*Sin[c + d*x] + 42*a*b*B*Sin[c + d*x]))/(105*a) + (2*Sec[c + d*x]*(82*a^2*A
*b*Sin[c + d*x] - 6*A*b^3*Sin[c + d*x] + 63*a^3*B*Sin[c + d*x] + 21*a*b^2*B
*Sin[c + d*x]))/(105*a^2) + (2*a*A*Sec[c + d*x]^3*Tan[c + d*x])/7))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3412 vs. $2(395) = 790$.
time = 0.48, size = 3413, normalized size = 7.88

method	result	size
default	Expression too large to display	3413

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*(25*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^4+6*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+
```

cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+63*B*sin(d*x+c)*cos(d*
x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-
63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^4+25*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^4+6*A*sin(d*x+c)*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+63*B*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^4-63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-15*A*a^4+25*A*cos(d*x+c)^4*a^4-10*A*cos(d
*x+c)^2*a^4+63*B*cos(d*x+c)^4*a^4-42*B*cos(d*x+c)^3*a^4-21*B*cos(d*x+c)*a^4
-6*A*cos(d*x+c)^5*b^4+6*A*cos(d*x+c)^4*b^4+82*A*sin(d*x+c)*cos(d*x+c)^4*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+3*A*cos(
d*x+c)^3*a*b^3-27*A*cos(d*x+c)^2*a^2*b^2-39*A*cos(d*x+c)*a^3*b+25*A*cos(d*x
+c)^5*a^3*b+82*A*cos(d*x+c)^5*a^2*b^2+3*A*cos(d*x+c)^5*a*b^3+82*A*cos(d*x+c
)^4*a^3*b-55*A*cos(d*x+c)^4*a^2*b^2-6*A*cos(d*x+c)^4*a*b^3-68*A*cos(d*x+c)^
3*a^3*b-63*B*cos(d*x+c)^2*a^3*b+63*B*cos(d*x+c)^5*a^3*b+42*B*cos(d*x+c)^5*a
^2*b^2+21*B*cos(d*x+c)^5*a*b^3+21*B*cos(d*x+c)^4*a^2*b^2-21*B*cos(d*x+c)^4*
a*b^3-63*B*cos(d*x+c)^3*a^2*b^2+51*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-6*A*sin(d*x+c)*co
s(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^3-82*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^3*b-82*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*
b^3+84*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3*b+21*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-63*B*sin(d*x+c)*cos(d
x+c)^4(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
*b-21*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos

```
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b^2-21*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+82*A*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*
b+51*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)^3*a^2*b^2-6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^3-82*A*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-
82*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^2+6*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+84*B*sin(d*x+c)*cos(d*x+c)
^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(9/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2), x
)
```

3.409 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=522

$$\frac{2(a-b)\sqrt{a+b}\,(147a^4A+33a^2Ab^2+8Ab^4+246a^3bB-18ab^3B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\,\sqrt{\cos(c}}\right.\right.}{315a^4d}$$

```
[Out] 2/9*a*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+2/63*(10*A*b+9
*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/315*(49*A*a^2+
3*A*b^2+72*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+2/
315*(88*A*a^2*b-4*A*b^3+75*B*a^3+9*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/
2)/a^2/d/cos(d*x+c)^(3/2)+2/315*(a-b)*(147*A*a^4+33*A*a^2*b^2+8*A*b^4+246*B
*a^3*b-18*B*a*b^3)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/315*(a-b)*(8*A*b^3-a^3*(147*A
-75*B)+3*a^2*b*(13*A-57*B)+6*a*b^2*(A-3*B))*cot(d*x+c)*EllipticF((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)
*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d
```

Rubi [A]
time = 1.17, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3068, 3134, 3077, 2895, 3073}

$$\frac{2(49a^2A+72abB+3Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{5}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,(-(a^3(147A-75B))+3a^2b(13A-57B)+6ab^2(A-3B)+8Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{315a^3d}+\frac{2(75a^3B+88a^2Ab+9ab^2B-4Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,(147a^4A+246a^3bB+33a^2Ab^2-18ab^3B+8Ab^4)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{315a^4d}+\frac{2(9aB+10Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{63d\cos^{\frac{7}{2}}(c+dx)}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x
]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B -
18*a*b^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*d) + (2*(a - b)*Sqr
t[a + b]*(8*A*b^3 - a^3*(147*A - 75*B) + 3*a^2*b*(13*A - 57*B) + 6*a*b^2*(A
 - 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) + (2*a*A*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (2*(10*A*b + 9*a*B)
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*(49*
a^2*A + 3*A*b^2 + 72*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d
*Cos[c + d*x]^(5/2)) + (2*(88*a^2*A*b - 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqr
t[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\frac{1}{2}a(10Ab+$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(10Ab+9aB)}{6}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(10Ab+9aB)}{6}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(10Ab+9aB)}{6}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(10Ab+9aB)}{6}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(147a^4A+33a^2Ab^2+8Ab^4+246a^3bB}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.66, size = 1515, normalized size = 2.90

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
1/2),x]
```

```
[Out] -1/315*((-4*a*(-39*a^4*A*b + 31*a^2*A*b^3 + 8*A*b^5 - 75*a^5*B + 93*a^3*b^2
*B - 18*a*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(147*a^5*A + 33*a^3*A*b^
2 + 8*a*A*b^4 + 246*a^4*b*B - 18*a^2*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
 (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*A*b + 33*a^2*A*b^3 + 8*A*b^5
 + 246*a^3*b^2*B - 18*a*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]
]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)
]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c
 + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (
a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(b*Sqrt[Cos[c + d*x]])))/(a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]]*((2*Sec[c + d*x]^4*(10*A*b*Sin[c + d*x] + 9*a*B*Sin[c + d*x]))/63
+ (2*Sec[c + d*x]^3*(49*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 72*a*b*
B*Sin[c + d*x]))/(315*a) + (2*Sec[c + d*x]^2*(88*a^2*A*b*Sin[c + d*x] - 4*A
*b^3*Sin[c + d*x] + 75*a^3*B*Sin[c + d*x] + 9*a*b^2*B*Sin[c + d*x]))/(315*a
^2) + (2*Sec[c + d*x]*(147*a^4*A*Sin[c + d*x] + 33*a^2*A*b^2*Sin[c + d*x] +
 8*A*b^4*Sin[c + d*x] + 246*a^3*b*B*Sin[c + d*x] - 18*a*b^3*B*Sin[c + d*x])
)/(315*a^3) + (2*a*A*Sec[c + d*x]^4*Tan[c + d*x])/9))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4391 vs. $2(478) = 956$.

time = 0.65, size = 4392, normalized size = 8.41

method	result	size
default	Expression too large to display	4392

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)

[Out] 2/315/d*(-8*A*cos(d*x+c)^6*b^5-147*A*cos(d*x+c)^5*a^5+8*A*cos(d*x+c)^5*b^5+98*A*cos(d*x+c)^4*a^5+14*A*cos(d*x+c)^2*a^5-75*B*cos(d*x+c)^5*a^5+30*B*cos(d*x+c)^3*a^5+45*B*cos(d*x+c)*a^5-75*B*cos(d*x+c)^6*a^4*b-246*B*cos(d*x+c)^6*a^3*b^2-9*B*cos(d*x+c)^6*a^2*b^3+18*B*cos(d*x+c)^6*a*b^4-246*B*cos(d*x+c)^5*a^4*b+165*B*cos(d*x+c)^5*a^3*b^2+18*B*cos(d*x+c)^5*a^2*b^3-18*B*cos(d*x+c)^5*a*b^4+10*A*cos(d*x+c)^5*a^4*b-33*A*cos(d*x+c)^5*a^3*b^2+34*A*cos(d*x+c)^5*a^2*b^3-8*A*cos(d*x+c)^5*a*b^4+68*A*cos(d*x+c)^4*a^3*b^2+4*A*cos(d*x+c)^4*a*b^4+52*A*cos(d*x+c)^3*a^4*b-A*cos(d*x+c)^3*a^2*b^3+53*A*cos(d*x+c)^2*a^3*b^2+246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-153*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-75*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-75*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E

```
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+35*A*a^5+33
*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^3*b^2+33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+8*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^5*sin
(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-18
6*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^4*b-33*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-2*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^5*sin(
d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-8
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^4+246*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+246*B*cos(d*x+c)^5*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-1
8*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b^3-18*B*cos(d*x+c)^5*sin(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/
2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(11/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/
2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(11/2),x
)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(11/2),
x)
```

3.410 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=779

$$-\frac{(a-b)\sqrt{a+b}\,(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2B+1024b^4B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{1920ab^2d}$$

[Out] 1/240*(50*A*a*b-15*B*a^2+64*B*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*
x+c)^(1/2)/b/d+1/40*(10*A*b-3*B*a)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)*cos(d*
x+c)^(1/2)/b/d+1/5*B*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d
+1/1920*(150*A*a^3*b+2840*A*a*b^3-45*B*a^4+1692*B*a^2*b^2+1024*B*b^4)*sin(d
x+c)(a+b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)+1/320*(50*A*a^2*b+120*A
*b^3-15*B*a^3+172*B*a*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/
2)/b/d-1/1920*(a-b)*(150*A*a^3*b+2840*A*a*b^3-45*B*a^4+1692*B*a^2*b^2+1024*
B*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(
1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1
+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d-1/1920*(45*a^4*B-30*a^3*b*(5*A+B)-16*b^4*
(45*A+64*B)-8*a*b^3*(355*A+193*B)-4*a^2*b^2*(295*A+423*B))*cot(d*x+c)*Ellip
ticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/b^2/d+1/128*(10*A*a^4*b-240*A*a^2*b^3-96*A*b^5-3*B*a^5-40*B*a^3*b^2-240*
B*a*b^4)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d

Rubi [A]
time = 2.06, antiderivative size = 779, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$,
Rules used = {3069, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]

[Out] -1/1920*((a - b)*Sqrt[a + b]*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*
a^2*b^2*B + 1024*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*d) - (Sqr
t[a + b]*(45*a^4*B - 30*a^3*b*(5*A + B) - 16*b^4*(45*A + 64*B) - 8*a*b^3*(3
55*A + 193*B) - 4*a^2*b^2*(295*A + 423*B))*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)

```
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(1920*b^2*d) + (Sqrt[a + b]*(10*a^4*A*b - 240*a^2*A*b^3 - 96*A*b^5 - 3*a
^5*B - 40*a^3*b^2*B - 240*a*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(128*b^3*d) + ((150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2
*B + 1024*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(1920*b^2*d*Sqrt[Co
s[c + d*x]]) + ((50*a^2*A*b + 120*A*b^3 - 15*a^3*B + 172*a*b^2*B)*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(320*b*d) + ((50*a*A*b - 1
5*a^2*B + 64*b^2*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d
*x])/(240*b*d) + ((10*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(40*b*d) + (B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(
7/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
```

```
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{B\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{7/2}\sin(c}{5bd}$$

$$= \frac{(10Ab-3aB)\sqrt{\cos(c+dx)}\,(a+b\cos(c+}{40bd}$$

$$= \frac{(50aAb-15a^2B+64b^2B)\sqrt{\cos(c+dx)}\,(a}{240bd}$$

$$= \frac{(50a^2Ab+120Ab^3-15a^3B+172ab^2B)\sqrt{}}{32}$$

$$= \frac{(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2}{1920b^2d\sqrt{}}$$

$$= \frac{(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2}{1920b^2d\sqrt{}}$$

$$= \frac{\sqrt{a+b}\,(10a^4Ab-240a^2Ab^3-96Ab^5-3a^5}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(150a^3Ab+2840aAb^3-4}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.54, size = 1353, normalized size = 1.74

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] -1/3840*((-4*a*(-1330*a^3*A*b - 3560*a*A*b^3 + 15*a^4*B - 3236*a^2*b^2*B -
1024*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos
[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-6440*a^2*A*b^2 - 1440*A*b^4
 - 2292*a^3*b*B - 4624*a*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b
)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])) + 2*(-150*a^3*A*b - 2840*a*A*b^3 + 45*a^4*B - 1692*a^
2*b^2*B - 1024*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ellipti
cE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c +
 d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*
Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sq
rt[Cos[c + d*x]])))/(b*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((
(590*a^2*A*b + 420*A*b^3 + 15*a^3*B + 898*a*b^2*B)*Sin[c + d*x])/(960*b) +
((170*a*A*b + 93*a^2*B + 88*b^2*B)*Sin[2*(c + d*x)])/480 + (b*(10*A*b + 21*
a*B)*Sin[3*(c + d*x)])/160 + (b^2*B*Sin[4*(c + d*x)])/40))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5163 vs. $2(719) = 1438$.
time = 0.95, size = 5164, normalized size = 6.63

method	result	size
default	Expression too large to display	5164

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2
), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (c+d\,x)^{3/2}\,(A+B\,\cos (c+d\,x))\,\left(a+b\,\cos (c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

3.411 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=664

$$-\frac{(a-b)\sqrt{a+b}\,(264a^2Ab+128Ab^3+15a^3B+284ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\right.}{192abd}$$

```
[Out] 1/4*b*B*cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+1/24*(8*A*b+11
*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+1/192*(264*A*a^2
*b+128*A*b^3+15*B*a^3+284*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/co
s(d*x+c)^(1/2)+1/32*(24*A*a*b+5*B*a^2+12*B*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)
*(a+b*cos(d*x+c))^(1/2)/d-1/192*(a-b)*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B
*a*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+1/192*(15*a^3*B+8*b^3*(16*A+9*B)+2*a^2*b*(
132*A+59*B)+4*a*b^2*(52*A+71*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d-1/64*(40*A*a^3*b+16
0*A*a*b^3-5*B*a^4+120*B*a^2*b^2+48*B*b^4)*cot(d*x+c)*EllipticPi((a+b*cos(d*
x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b
)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A]
time = 1.43, antiderivative size = 664, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3069, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$\frac{(5a^2B+24aAb+12b^2B)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{32d}+\frac{\sqrt{a+b}\,(15a^3B+2a^2b(132A+59B)+4ab^2(52A+71B)+8b^3(16A+9B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{192bd}-\frac{(a-b)\sqrt{a+b}\,(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{192abd}+\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{192bd\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\,(-5a^4B+40a^3Ab+120a^2b^2B+160aAb^3+48b^4B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{64b^2d}+\frac{(11aB+8Ab)\sin(c+dx)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}{24d}+\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] -1/192*((a - b)*Sqrt[a + b]*(264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2
*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d) + (Sqrt[a + b]*(15*a^3*B + 8
*b^3*(16*A + 9*B) + 2*a^2*b*(132*A + 59*B) + 4*a*b^2*(52*A + 71*B))*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(192*b*d) - (Sqrt[a + b]*(40*a^3*A*b + 160*a*A*
b^3 - 5*a^4*B + 120*a^2*b^2*B + 48*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b
, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
```

```
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(64*b^2*d) + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2
*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(192*b*d*Sqrt[Cos[c + d*x]]) + (
(24*a*A*b + 5*a^2*B + 12*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(32*d) + ((8*A*b + 11*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(24*d) + (b*B*Cos[c + d*x]^(3/2)*(a + b*Cos[c +
d*x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
```

```
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{bB\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c}{4d}$$

$$= \frac{(8Ab+11aB)\sqrt{\cos(c+dx)}\,(a+b\cos(c+}{24d}$$

$$= \frac{(24aAb+5a^2B+12b^2B)\,\sqrt{\cos(c+dx)}\,\sqrt{}}{32d}$$

$$= \frac{(264a^2Ab+128Ab^3+15a^3B+284ab^2B)\,\sqrt{}}{192bd\sqrt{\cos(c+}}$$

$$= \frac{(264a^2Ab+128Ab^3+15a^3B+284ab^2B)\,\sqrt{}}{192bd\sqrt{\cos(c+}}$$

$$= -\frac{\sqrt{a+b}\,(40a^3Ab+160aAb^3-5a^4B+12}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(264a^2Ab+128Ab^3+15a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.44, size = 1287, normalized size = 1.94

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] ((-4*a*(472*a^2*A*b + 128*A*b^3 + 133*a^3*B + 356*a*b^2*B)*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]) - 4*a*(384*a^3*A + 608*a*A*b^2 + 644*a^2*b*B + 144*b^3*B)*((Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
```

```
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(264*a^2*A*
b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos
[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)
/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a
+ b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(384*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*(((104*a*A*b + 59*a^2*B + 42*b^2*B)*Sin[c + d*x])/96 + (b
*(8*A*b + 17*a*B)*Sin[2*(c + d*x)])/48 + (b^2*B*Sin[3*(c + d*x)])/16))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4237 vs. $2(610) = 1220$.
time = 0.61, size = 4238, normalized size = 6.38

method	result	size
default	Expression too large to display	4238

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/192/d/(a+b*cos(d*x+c))^(1/2)*(64*A*cos(d*x+c)^3*b^4-128*A*cos(d*x+c)^2*b
^4+133*B*cos(d*x+c)^3*a^3*b+172*B*cos(d*x+c)^3*a*b^3+264*A*cos(d*x+c)^2*a^3
*b-144*A*cos(d*x+c)^2*a*b^3-208*A*cos(d*x+c)*a^2*b^2-128*A*cos(d*x+c)*a*b^3
+472*A*cos(d*x+c)^3*a^2*b^2+30*B*cos(d*x+c)^2*a^2*b^2-284*B*cos(d*x+c)^2*a*
b^3-118*B*cos(d*x+c)*a^3*b-284*B*cos(d*x+c)*a^2*b^2-72*B*cos(d*x+c)*a*b^3+2
4*B*cos(d*x+c)^4*b^4-72*B*cos(d*x+c)^2*b^4+15*B*cos(d*x+c)^2*a^4-644*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b^2+72*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
```

cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b^3+720*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+264*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2+128*A*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
a*b^3+208*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)*a^2*b^2-608*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^3+240*A*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
a^3*b+960*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)*a*b^3+15*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b+284*B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2
*b^2+284*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*a*b^3+118*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b-644*B*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2
+72*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)*a*b^3+720*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2+264*A*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b-
384*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^3*b+128*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+15*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-144*B*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-30*B*sin(d*x+c)*(cos(d*

```
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4+288*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
b^4+48*B*cos(d*x+c)^6*b^4-15*B*cos(d*x+c)*a^4+64*A*cos(d*x+c)^5*b^4-264*A*c
os(d*x+c)^2*a^2*b^2-264*A*cos(d*x+c)*a^3*b+272*A*cos(d*x+c)^4*a*b^3-15*B*co
s(d*x+c)^2*a^3*b+184*B*cos(d*x+c)^5*a*b^3+254*B*cos(d*x+c)^4*a^2*b^2+128*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)*b^4+15*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^4-144*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)
), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

3.412 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=564

$$-\frac{(a-b)\sqrt{a+b}\,(54aAb+33a^2B+16b^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{a(}}{24ad}$$

[Out] 1/3*b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+1/24*(54*A*a*b+33*B*a^2+16*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/4*b*(2*A*b+3*B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/24*(a-b)*(54*A*a*b+33*B*a^2+16*B*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+1/24*(4*b^2*(3*A+4*B)+a^2*(48*A+33*B)+a*(54*A*b+26*B*b))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/8*(30*A*a^2*b+8*A*b^3+5*B*a^3+20*B*a*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d

Rubi [A]

time = 1.08, antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3069, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

Antiderivative was successfully verified.

[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]

[Out] -1/24*((a - b)*Sqrt[a + b]*(54*a*A*b + 33*a^2*B + 16*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (Sqrt[a + b]*(4*b^2*(3*A + 4*B) + a^2*(48*A + 33*B) + a*(54*A*b + 26*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d) - (Sqrt[a + b]*(30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b*d) + ((54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + (b*(2*A*b +

```
3*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (
b*B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{bB\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\int$$

$$= \frac{b(2Ab+3aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,\sin(c+}{4d}$$

$$= \frac{(54aAb+33a^2B+16b^2B)\,\sqrt{a+b\cos(c+dx)}\,\sin(c+dx}{24d\sqrt{\cos(c+dx)}}$$

$$= \frac{(54aAb+33a^2B+16b^2B)\,\sqrt{a+b\cos(c+dx)}\,\sin(c+dx}{24d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\,(30a^2Ab+8Ab^3+5a^3B+20ab^2B)\cot(c+dx)}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(54aAb+33a^2B+16b^2B)\cot(c+dx)E}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.53, size = 1251, normalized size = 2.22

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] ((-4*a*(48*a^3*A + 66*a*A*b^2 + 59*a^2*b*B + 16*b^3*B)*Sqrt[((a + b)*Cot[(c
 + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]]) - 4*a*(144*a^2*A*b + 24*A*b^3 + 48*a^3*B + 76*a*b^2*B)*((Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
```

```
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(54*a*A*b^2 + 33*a
^2*b*B + 16*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[
I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*
x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec
[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[
Cos[c + d*x]])))/(48*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((b*
(6*A*b + 13*a*B)*Sin[c + d*x])/12 + (b^2*B*Sin[2*(c + d*x)])/6))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3511 vs. $2(516) = 1032$.
time = 0.48, size = 3512, normalized size = 6.23

method	result	size
default	Expression too large to display	3512

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/24/d/(a+b*cos(d*x+c))^(1/2)*(16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+120*B*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+26*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-76*B*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^2+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)*b^3-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3+48*A*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
```

pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+8*B*cos(d*x+c)^5*b^3+8*B*cos(d*x+c)^3*b^3+33*B*cos(d*x+c)^2*a^3-16*B*cos(d*x+c)^2*b^3-33*B*cos(d*x+c)*a^3+12*A*cos(d*x+c)^4*b^3-12*A*cos(d*x+c)^2*b^3+54*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+54*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+180*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+12*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+33*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+120*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+26*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-76*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-144*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+33*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+16*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-144*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)

```
,(-(a-b)/(a+b))^(1/2))*a^2*b+33*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+16*B*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+30*B*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)
), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2), x
)
```

3.413 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=547

$$\frac{(a-b)\sqrt{a+b}\,(8a^2A-4Ab^2-9abB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\text{se}}{a}}}{4ad}$$

```
[Out] 2*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-1/4*(8*A*a^2-4*A
*b^2-9*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/2*b*(4
*A*a-B*b)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d+1/4*(a-b)*(8
*A*a^2-4*A*b^2-9*B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-1/4*(8*a^2*(A-B)-2*b^2*(2*A+
B)-3*a*b*(8*A+3*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b)
)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(20*A*a*b+15*B*a^2+4*B*b^2)*co
t(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+
b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1
+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A]
time = 1.05, antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3068, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$-\frac{\sqrt{a+b}\,(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4d}+\frac{(a-b)\sqrt{a+b}\,(8a^2A-9abB-4Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4ad}-\frac{\sqrt{a+b}\,(15a^2B+20aAb+4b^2B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4d}-\frac{(8a^2A-9abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4d\sqrt{\cos(c+dx)}}-\frac{b(4aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(8*a^2*A - 4*A*b^2 - 9*a*b*B)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(4*a*d) - (Sqrt[a + b]*(8*a^2*(A - B) - 2*b^2*(2*A + B) - 3*a*b*
(8*A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (Sqrt[a + b]*(20*a*
A*b + 15*a^2*B + 4*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*d) - ((8*a^2*A - 4*A*b^2 - 9*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(4*d*Sqrt[Cos[c + d*x]]) - (b*(4*a*A - b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
```

```
Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\sqrt{a+b\cos(}{}$$

$$= -\frac{b(4aA-bB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,\sin(c+}{2d}$$

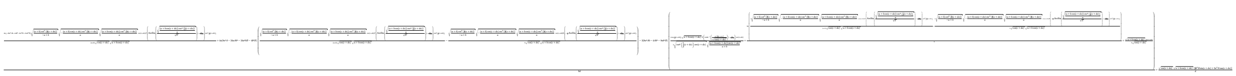
$$= -\frac{(8a^2A-4Ab^2-9abB)\,\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}$$

$$= -\frac{(8a^2A-4Ab^2-9abB)\,\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\,(20aAb+15a^2B+4b^2B)\cot(c+dx)\Pi\left(\frac{a+b}{b};\text{si}\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,(8a^2A-4Ab^2-9abB)\cot(c+dx)E\left(\sin^{\cdot}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.54, size = 1241, normalized size = 2.27



Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] ((4*a*(-16*a^2*A*b - 4*A*b^3 - 8*a^3*B - 11*a*b^2*B)*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) + 4*a*(8*a^3*A - 24*a*A*b^2 - 24*a^2*b*B - 4*b^3*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
```

```
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(8*a^2*A*b - 4*A*b^3 -
9*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSin
h[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x
])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((
a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[
(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/
a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
Pi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c +
d*x]])))/(8*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((b^2*B*Sin[c
 + d*x])/2 + 2*a^2*A*Tan[c + d*x]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3269 vs. $2(501) = 1002$.
time = 0.39, size = 3270, normalized size = 5.98

method	result	size
default	Expression too large to display	3270

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*(-8*A*a^3+4*A*cos(d*x+c)^3*b^3+9*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-24*B*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+8*A*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^3+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+c
```

os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-4*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x
+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+
b))^(1/2))*b^3+40*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*a*b^2+30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+2*B*cos(d*x+c)^4*b^3+8*A*cos(d
*x+c)*a^3-2*B*cos(d*x+c)^2*b^3-4*A*cos(d*x+c)^2*b^3-8*A*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+11
*B*cos(d*x+c)^3*a*b^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-24*A*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+9*B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b+9*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2-24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+
40*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*a*b^2+30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+8*B*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^3+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*b^3-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+24*A*(cos(d*x+c)/(1+cos(

```
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+24
*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2*b-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^2*b+4*A*sin(d*x+c)*(cos(d*x+c)/(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2
), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}}{\cos(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2), x
)
```

3.414 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=536

$$\frac{(a-b)\sqrt{a+b}\,(14aAb+6a^2B-3b^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-s}{}}}{3ad}$$

```
[Out] 2/3*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*a*(2*A*b+B*a
)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*(14*A*a*b+6*B*a^
2-3*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/3*(a-b)*(
14*A*a*b+6*B*a^2-3*B*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))
/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-1/3*(2*a*b*(7*A-9*B)-2*a^2
*(A-3*B)-3*b^2*(6*A+B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-b*(2*A*b+5*B*a)*cot(d*x+c)*Ell
ipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)
/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/d
```

Rubi [A]
time = 1.05, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3068, 3126, 3140, 3132, 2888, 3077, 2895, 3073}

$$-\frac{\sqrt{a+b}\,(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6A+B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3d}+\frac{(a-b)\sqrt{a+b}\,(6a^2B+14aAb-3b^2B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad}-\frac{(6a^2B+14aAb-3b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}-\frac{b\sqrt{a+b}\,(5aB+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}+\frac{2a(aB+2Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(14*a*A*b + 6*a^2*B - 3*b^2*B)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3*a*d) - (Sqrt[a + b]*(2*a*b*(7*A - 9*B) - 2*a^2*(A - 3*B) - 3
*b^2*(6*A + B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d) - (b*Sqrt[a + b]*(
2*A*b + 5*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*(2*A
*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (
(14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*
```

```
Sqrt[Cos[c + d*x]]) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*
Cos[c + d*x]^(3/2))
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\sqrt{a+b\cos(}}{}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aA(a-}{}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(14aAb}{}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(14aAb}{}$$

$$= -\frac{b\sqrt{a+b}\,(2Ab+5aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,(14aAb+6a^2B-3b^2B)\cot(c+dx)E\Big(\sin}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.52, size = 1269, normalized size = 2.37

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] ((-4*a*(2*a^3*A + 4*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - 4*a*(-14*a^2*A*b + 6*A*b^3 - 6*a^3*B + 18*a*b^2*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
```

```
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-14*a*A*b^2 - 6*a^2*b*
B + 3*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcS
inh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d
*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c
+ d*x]])))/(6*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c +
 d*x]*(7*a*A*b*Sin[c + d*x] + 3*a^2*B*Sin[c + d*x]))/3 + (2*a^2*A*Sec[c + d
*x]*Tan[c + d*x])/3))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3203 vs. $2(492) = 984$.
time = 0.35, size = 3204, normalized size = 5.98

method	result	size
default	Expression too large to display	3204

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*(18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*cos(d*x+c)^2*a*b^2+2*A*a^3-2*A*cos(d*x+c)^3*a^2*b-2*A*cos(d*x+c
)^2*a^3-12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*sin(d*x+c)*cos(d*x+c)*b^3+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3-30*B*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*
b^2-14*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b-18*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
```

```
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+6*B*sin(d*x+c)*cos(d*x+
c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b-
3*B*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*a*b^2-18*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)^2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-6*B*sin
(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*a^3-3*B*cos(d*x+c)^4*b^3+3*B*cos(d*x+c)^3*b^3-6*B*cos(d*x+c)^2*a^
3+6*B*cos(d*x+c)*a^3+14*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+14*A*sin(d*x+c)*cos(d*x+c)^2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+14*A*sin
(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*a*b^2+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3+6*B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*
x+c)*a^3-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)^2*sin(d*x+c)*b^3-3*B*cos(d*x+c)^3*a*b^2+14*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-
18*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^2+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-30*B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a*b^2-18*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+18*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
```

```
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-2*A*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3-14*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+6*B...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2
), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2
), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2), x
)
```

3.415 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=493

$$\frac{2(a-b)\sqrt{a+b}\,(9a^2A+23Ab^2+35abB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{15ad}$$

```
[Out] 2/5*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/15*a*(8*A*b+
5*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9*A
*a^2+23*A*b^2+35*B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+2/15*(15*A*b^3-a*b^2*(23*A-4
5*B)+a^2*b*(17*A-35*B)-a^3*(9*A-5*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-2*b^2*B*cot(d*
x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b
,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A]
time = 0.78, antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3068, 3126, 3132, 2888, 3077, 2895, 3073}

$$\frac{2(a-b)\sqrt{a+b}\,(9a^2A+35abB+23Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15ad}+\frac{2\sqrt{a+b}\,(-(a^3(9A-5B))+a^2b(17A-35B)-ab^2(23A-45B)+15Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15ad}+\frac{2a(5aB+8Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}-\frac{2b^2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(15*a*d) + (2*Sqrt[a + b]*(15*A*b^3 - a*b^2*(23*A - 45*B) +
a^2*b*(17*A - 35*B) - a^3*(9*A - 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
15*a*d) - (2*b^2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/d + (2*a*(8*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Co
s[c + d*x]^(3/2)) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Co
s[c + d*x]^(5/2))
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\sqrt{a+b\cos(}}{}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a}{}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a}{}$$

$$= -\frac{2b^2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\,\sqrt{\cos(}}\right.\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(9a^2A+23Ab^2+35abB)\cot(c+dx)E\Big(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.57, size = 1319, normalized size = 2.68

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] ((4*a*(-8*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) + 4*a*(9*a^3*A + 23*a*A*b^2 + 35*a^2*b*B - 15*b^3*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(9*a^2*A*b + 23*A*b^3 +
 35*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcS
```

```
inh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d
*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c
+ d*x]])))/(15*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c
+ d*x]^2*(11*a*A*b*Sin[c + d*x] + 5*a^2*B*Sin[c + d*x]))/15 + (2*Sec[c + d*
x]*(9*a^2*A*Sin[c + d*x] + 23*A*b^2*Sin[c + d*x] + 35*a*b*B*Sin[c + d*x]))/
15 + (2*a^2*A*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3273 vs. $2(453) = 906$.
time = 0.38, size = 3274, normalized size = 6.64

method	result	size
default	Expression too large to display	3274

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*(45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^3*a*b^2+45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2-3*A*a^3+5*A*cos(d*
x+c)^3*a^2*b+5*B*cos(d*x+c)^3*a^3+9*A*cos(d*x+c)^3*a^3-23*A*cos(d*x+c)^3*b^
3-6*A*cos(d*x+c)^2*a^3+17*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+23*A*sin(d*x+c)*cos(d*x+c)^2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-35*B
*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*a^2*b-35*B*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+35*B*sin(d*x+c)*cos(d*x+c)^2*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos
```

(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b-9*A*sin
(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*a^2*b-23*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+17*A*sin(d*x+c)*cos(d*x+c)^3*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+23*A*sin(d*
x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*a*b^2-35*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-35*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+35*B*sin(d*x+c
)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*a^2*b+5*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*a^3-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-23*A*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9
*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^3+5*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-23*A*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*b^3+9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3-5*B*cos(d*x+c)*a^3+23*A*cos(d*x+c)^4*b^3-9*A*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b-23*A*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+15*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3+5*B*cos(d*
x+c)^4*a^2*b-35*B*cos(d*x+c)^3*a*b^2+9*A*cos(d*x+c)^4*a^2*b+11*A*cos(d*x+c)
^4*a*b^2-40*B*cos(d*x+c)^2*a^2*b+35*B*cos(d*x+c)^4*a*b^2+35*B*cos(d*x+c)^3*

```
a^2*b+23*A*cos(d*x+c)^3*a*b^2-34*A*cos(d*x+c)^2*a*b^2-14*A*cos(d*x+c)*a^2*b
-15*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*b^3+30*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2
), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2), x
)
```

3.416 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=434

$$\frac{2(a-b)\sqrt{a+b}\,(145a^2Ab+15Ab^3+63a^3B+161ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\right|}{105a^2d}$$

```
[Out] 2/7*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/35*a*(10*A*b
+7*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/105*(25*A*a^
2+45*A*b^2+77*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2
/105*(a-b)*(145*A*a^2*b+15*A*b^3+63*B*a^3+161*B*a*b^2)*cot(d*x+c)*EllipticE
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a
^2/d+2/105*(a-b)*(a^2*(25*A-63*B)+15*b^2*(A-7*B)-8*a*b*(15*A-7*B))*cot(d*x+
c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a
-b))^(1/2)/a/d
```

Rubi [A]
time = 0.83, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3068, 3126, 3134, 3077, 2895, 3073}

$$\frac{2(a-b)\sqrt{a+b}\,(a^2(25A-63B)-8ab(15A-7B)+15b^2(A-7B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{105ad}+\frac{2(25a^2A+77abB+45Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,(63a^3B+145a^2Ab+161ab^2B+15Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{105a^2d}+\frac{2a(7aB+10Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d) + (2*(a - b)*Sqrt[a + b]*(a^2*
(25*A - 63*B) + 15*b^2*(A - 7*B) - 8*a*b*(15*A - 7*B))*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(105*a*d) + (2*a*(10*A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2*A + 45*A*b^2 + 77*a*b*B
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*a*
A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
```

```
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
```

```
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\sqrt{a+b\,\mathrm{co}}}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA}{}$$

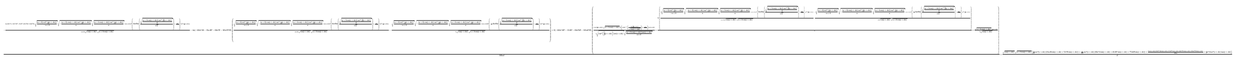
$$= \frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(25}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(25}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(145a^2Ab+15Ab^3+63a^3B+161ab^2B}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 6.64, size = 1409, normalized size = 3.25



Warning: Unable to verify antiderivative.

[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9
/2),x]

[Out] ((-4*a*(25*a^4*A - 10*a^2*A*b^2 - 15*A*b^4 + 56*a^3*b*B - 56*a*b^3*B)*Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - 4*a*(-145*a^3*A*b - 15*a*A*b^3 - 63*a^4*B - 161*a^
2*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) +
 2*(-145*a^2*A*b^2 - 15*A*b^4 - 63*a^3*b*B - 161*a*b^3*B)*((I*Cos[(c + d*x)
/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[
c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a*d) + (Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(15*a*A*b*Sin[c +
d*x] + 7*a^2*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x]
 + 45*A*b^2*Sin[c + d*x] + 77*a*b*B*Sin[c + d*x]))/105 + (2*Sec[c + d*x]*(1
45*a^2*A*b*Sin[c + d*x] + 15*A*b^3*Sin[c + d*x] + 63*a^3*B*Sin[c + d*x] + 1
61*a*b^2*B*Sin[c + d*x]))/(105*a) + (2*a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/7
))/d

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3627 vs.

2(396) = 792.
time = 0.45, size = 3628, normalized size = 8.36

method	result	size
default	Expression too large to display	3628

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/105/d*(-25*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^4+15*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-63*B*sin(d*x+c)*cos(d
*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4
+63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^4-25*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^4+15*A*sin(d*x+c)*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-63*B*sin
(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^4+63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+15*A*a^4-25*A*cos(d*x+c)^4*a^4+10*A*cos
(d*x+c)^2*a^4-63*B*cos(d*x+c)^4*a^4+42*B*cos(d*x+c)^3*a^4+21*B*cos(d*x+c)*a
^4-15*A*cos(d*x+c)^5*b^4+15*A*cos(d*x+c)^4*b^4-145*A*sin(d*x+c)*cos(d*x+c)^
4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+60*
A*cos(d*x+c)^3*a*b^3+90*A*cos(d*x+c)^2*a^2*b^2+60*A*cos(d*x+c)*a^3*b-25*A*c
os(d*x+c)^5*a^3*b-145*A*cos(d*x+c)^5*a^2*b^2-45*A*cos(d*x+c)^5*a*b^3-145*A*
cos(d*x+c)^4*a^3*b+55*A*cos(d*x+c)^4*a^2*b^2-15*A*cos(d*x+c)^4*a*b^3+110*A*
cos(d*x+c)^3*a^3*b+98*B*cos(d*x+c)^2*a^3*b-63*B*cos(d*x+c)^5*a^3*b-77*B*cos
(d*x+c)^5*a^2*b^2-161*B*cos(d*x+c)^5*a*b^3-35*B*cos(d*x+c)^4*a^3*b-161*B*co
s(d*x+c)^4*a^2*b^2+161*B*cos(d*x+c)^4*a*b^3+238*B*cos(d*x+c)^3*a^2*b^2-105*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^4*a*b^3-105*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-135*A*sin(d*x+c)*cos(d*x+c)^4
```

```
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-15
*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^3+145*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+145*A*sin(d*x+c)*cos(d*x+c)^
4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+1
5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)^4*a*b^3-119*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-161*B*sin(d*x+c)*cos(d*x+c)
^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+
63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3*b+161*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+161*B*sin(d*x+c)*cos(d*x
+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3
-145*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3*b-135*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b^2-15*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^
3+145*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3*b+145*A*sin(d*x+c)*cos...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2
), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(9/2), x
)
```

3.417 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=522

$$\frac{2(a-b)\sqrt{a+b}\,(147a^4A+279a^2Ab^2-10Ab^4+435a^3bB+45ab^3B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\,\sqrt{\cos(c}}\right.\right.}{315a^3d}$$

```
[Out] 2/9*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)+2/21*a*(4*A*b+
3*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/315*(49*A*a^2
+75*A*b^2+135*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2
/315*(163*A*a^2*b+5*A*b^3+75*B*a^3+135*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))
^(1/2)/a/d/cos(d*x+c)^(3/2)+2/315*(a-b)*(147*A*a^4+279*A*a^2*b^2-10*A*b^4+4
35*B*a^3*b+45*B*a*b^3)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a
+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d-2/315*(a-b)*(10*A*b^3-6*a^2
*b*(19*A-60*B)+3*a^3*(49*A-25*B)+15*a*b^2*(11*A-3*B))*cot(d*x+c)*EllipticF(
(a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(
a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^
2/d
```

Rubi [A]
time = 1.17, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3068, 3126, 3134, 3077, 2895, 3073}

$$\frac{2(49a^2A+135abB+75Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315d\cos^{\frac{5}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\,(3a^3(49A-25B)-6a^2b(19A-60B)+15ab^2(11A-3B)+10Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{315a^2d}+\frac{2(75a^3B+163a^2Ab+135ab^2B+5Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,(147a^4A+435a^3bB+279a^2Ab^2+45ab^3B-10Ab^4)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{315a^3d}+\frac{2a(3aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d\cos^{\frac{7}{2}}(c+dx)}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x
]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B
+ 45*a*b^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) - (2*(a - b)*S
qrt[a + b]*(10*A*b^3 - 6*a^2*b*(19*A - 60*B) + 3*a^3*(49*A - 25*B) + 15*a*b
^2*(11*A - 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d) + (2*a*(4*A
*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)
) + (2*(49*a^2*A + 75*A*b^2 + 135*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(315*d*Cos[c + d*x]^(5/2)) + (2*(163*a^2*A*b + 5*A*b^3 + 75*a^3*B + 13
```

```
5*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(3/
2)) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/
2))
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\sqrt{a+b\cos}}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aA(}{}$$

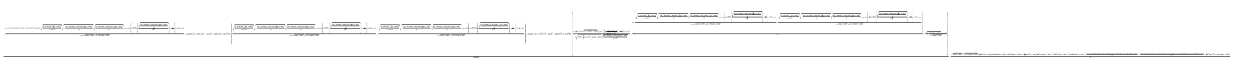
$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(49a}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(49a}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(49a}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(147a^4A+279a^2Ab^2-10Ab^4+435a^3b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.76, size = 1517, normalized size = 2.91



Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
1/2),x]
```

```
[Out] -1/315*((-4*a*(-114*a^4*A*b + 124*a^2*A*b^3 - 10*A*b^5 - 75*a^5*B + 30*a^3*
b^2*B + 45*a*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(147*a^5*A + 279*a^3*
A*b^2 - 10*a*A*b^4 + 435*a^4*b*B + 45*a^2*b^3*B)*((Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
```

/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*A*b + 279*a^2*A*b^3 - 1
0*A*b^5 + 435*a^3*b^2*B + 45*a*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
 + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(
-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a +
b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
+ d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*((2*Sec[c + d*x]^4*(19*a*A*b*Sin[c + d*x] + 9*a^2*B*Sin[c +
 d*x]))/63 + (2*Sec[c + d*x]^3*(49*a^2*A*Sin[c + d*x] + 75*A*b^2*Sin[c + d*
x] + 135*a*b*B*Sin[c + d*x]))/315 + (2*Sec[c + d*x]^2*(163*a^2*A*b*Sin[c +
d*x] + 5*A*b^3*Sin[c + d*x] + 75*a^3*B*Sin[c + d*x] + 135*a*b^2*B*Sin[c + d
*x]))/(315*a) + (2*Sec[c + d*x]*(147*a^4*A*Sin[c + d*x] + 279*a^2*A*b^2*Sin
[c + d*x] - 10*A*b^4*Sin[c + d*x] + 435*a^3*b*B*Sin[c + d*x] + 45*a*b^3*B*S
in[c + d*x]))/(315*a^2) + (2*a^2*A*Sec[c + d*x]^4*Tan[c + d*x])/9))/d

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4391 vs. $2(478) = 956$.
time = 0.64, size = 4392, normalized size = 8.41

method	result	size
default	Expression too large to display	4392

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)

[Out] -2/315/d*(-10*A*cos(d*x+c)^6*b^5+147*A*cos(d*x+c)^5*a^5+10*A*cos(d*x+c)^5*b
^5-98*A*cos(d*x+c)^4*a^5-14*A*cos(d*x+c)^2*a^5+75*B*cos(d*x+c)^5*a^5-30*B*c
os(d*x+c)^3*a^5-45*B*cos(d*x+c)*a^5+75*B*cos(d*x+c)^6*a^4*b+435*B*cos(d*x+c
)^6*a^3*b^2+135*B*cos(d*x+c)^6*a^2*b^3+45*B*cos(d*x+c)^6*a*b^4+435*B*cos(d*
x+c)^5*a^4*b-165*B*cos(d*x+c)^5*a^3*b^2+45*B*cos(d*x+c)^5*a^2*b^3-45*B*cos(
d*x+c)^5*a*b^4+65*A*cos(d*x+c)^5*a^4*b+279*A*cos(d*x+c)^5*a^3*b^2-199*A*cos
(d*x+c)^5*a^2*b^3-10*A*cos(d*x+c)^5*a*b^4-272*A*cos(d*x+c)^4*a^3*b^2+5*A*co
s(d*x+c)^4*a*b^4-82*A*cos(d*x+c)^3*a^4*b-80*A*cos(d*x+c)^3*a^2*b^3-170*A*co
s(d*x+c)^2*a^3*b^2-435*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)

```
)/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-45*B*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*
x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-45*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b^4+435*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+405*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d
*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+45
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b^3-147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+10*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^5*sin(d
*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+147*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^5+75*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+10*A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos
(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*b^5+147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^5+75*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*A*cos(d*x+c)^5*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
4*b-35*A*a^5-279*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+10*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^4+261*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+279*A*cos(d*x+c)^5*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
```

```
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+155*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*b^3-10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-435*B*cos(d*x+c)^5*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-435*B*
cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^3*b^2-45*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/
2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(11/
2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/
2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}}{\cos(c+d\,x)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2),x
)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2),
x)
```

3.418 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=622

$$\frac{2(a-b)\sqrt{a+b}\,(3705a^4Ab+255a^2Ab^3+40Ab^5+1617a^5B+3069a^3b^2B-110ab^4B)\cot(c+dx)E\Big(\text{ArcSin}}{3465a^4d}$$

```
[Out] 2/11*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(11/2)+2/99*a*(14*A
*b+11*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+2/693*(81*A
*a^2+113*A*b^2+209*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7
/2)+2/3465*(1145*A*a^2*b+15*A*b^3+539*B*a^3+825*B*a*b^2)*sin(d*x+c)*(a+b*co
s(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+2/3465*(675*A*a^4+1025*A*a^2*b^2-20*A*
b^4+1793*B*a^3*b+55*B*a*b^3)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*
x+c)^(3/2)+2/3465*(a-b)*(3705*A*a^4*b+255*A*a^2*b^3+40*A*b^5+1617*B*a^5+306
9*B*a^3*b^2-110*B*a*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/3465*(a-b)*(40*A*b^4+3*
a^4*(225*A-539*B)-6*a^3*b*(505*A-209*B)+15*a^2*b^2*(19*A-121*B)+10*a*b^3*(3
*A-11*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d
```

Rubi [A]

time = 1.66, antiderivative size = 622, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3068, 3126, 3134, 3077, 2895, 3073}

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2),x
]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*
B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3465*
a^4*d) + (2*(a - b)*Sqrt[a + b]*(40*A*b^4 + 3*a^4*(225*A - 539*B) - 6*a^3*b
*(505*A - 209*B) + 15*a^2*b^2*(19*A - 121*B) + 10*a*b^3*(3*A - 11*B))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(3465*a^3*d) + (2*a*(14*A*b + 11*a*B)*Sqrt[a
```

```
+ b*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*(81*a^2*A +
113*A*b^2 + 209*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*d*Cos[c
+ d*x]^(7/2)) + (2*(1145*a^2*A*b + 15*A*b^3 + 539*a^3*B + 825*a*b^2*B)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*a*d*Cos[c + d*x]^(5/2)) + (2*(675*
a^4*A + 1025*a^2*A*b^2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(3465*a^2*d*Cos[c + d*x]^(3/2)) + (2*a*A*(a + b*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
```

```
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2}{11}\int \frac{\sqrt{a+b\,\mathrm{c}}}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2a}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(3705a^4Ab+255a^2Ab^3+40Ab^5+1617}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.87, size = 1640, normalized size = 2.64

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
3/2),x]
```

```
[Out] ((-4*a*(675*a^6*A - 390*a^4*A*b^2 - 245*a^2*A*b^4 - 40*A*b^6 + 1254*a^5*b*B
 - 1364*a^3*b^3*B + 110*a*b^5*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-3705*a
^5*A*b - 255*a^3*A*b^3 - 40*a*A*b^5 - 1617*a^6*B - 3069*a^4*b^2*B + 110*a^2
*b^4*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
```

```
^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) +
2*(-3705*a^4*A*b^2 - 255*a^2*A*b^4 - 40*A*b^6 - 1617*a^5*b*B - 3069*a^3*b^3
*B + 110*a*b^5*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I
*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x
])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[
c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[C
os[c + d*x]])))/(3465*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
*((2*Sec[c + d*x]^5*(23*a*A*b*Sin[c + d*x] + 11*a^2*B*Sin[c + d*x]))/99 + (
2*Sec[c + d*x]^4*(81*a^2*A*Sin[c + d*x] + 113*A*b^2*Sin[c + d*x] + 209*a*b*
B*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^3*(1145*a^2*A*b*Sin[c + d*x] + 15*A*
b^3*Sin[c + d*x] + 539*a^3*B*Sin[c + d*x] + 825*a*b^2*B*Sin[c + d*x]))/(346
5*a) + (2*Sec[c + d*x]^2*(675*a^4*A*Sin[c + d*x] + 1025*a^2*A*b^2*Sin[c + d
*x] - 20*A*b^4*Sin[c + d*x] + 1793*a^3*b*B*Sin[c + d*x] + 55*a*b^3*B*Sin[c
+ d*x]))/(3465*a^2) + (2*Sec[c + d*x]*(3705*a^4*A*b*Sin[c + d*x] + 255*a^2*
A*b^3*Sin[c + d*x] + 40*A*b^5*Sin[c + d*x] + 1617*a^5*B*Sin[c + d*x] + 3069
*a^3*b^2*B*Sin[c + d*x] - 110*a*b^4*B*Sin[c + d*x]))/(3465*a^3) + (2*a^2*A*
Sec[c + d*x]^5*Tan[c + d*x])/11))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5372 vs. $2(572) = 1144$.
time = 0.94, size = 5373, normalized size = 8.64

method	result	size
default	Expression too large to display	5373

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x,method=_RET
URNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/
2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(13/
2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/
2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{5/2}}{\cos(c+dx)^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(13/2),x
)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(13/2),
x)
```

3.419 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=418

$$\frac{2(a-b)\sqrt{a+b}\,(a^2+3b^2)\,B\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{ad}$$

```
[Out] b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*(a-b)*(a^2+3*b^2
)*B*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a/d-(a-3*b)*(2*a^2-a*b+3*b^2)*B*cot(d*x+c)*EllipticF
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a
/d-b*(5*a+3*b^2/a)*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/
2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x
+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A]
time = 0.60, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {3068, 3070, 2888, 3077, 2895, 3073}

$$-\frac{B(a-3b)\sqrt{a+b}\,(2a^2-ab+3b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+\frac{2B(a-b)\sqrt{a+b}\,(a^2+3b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{bB\sqrt{a+b}\left(\frac{3b^2}{a}+5a\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}+\frac{bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*((3*b*B)/(2*a) + B*Cos[c + d*x]))/Cos[c + d
*x]^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(a^2 + 3*b^2)*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
a*d) - ((a - 3*b)*Sqrt[a + b]*(2*a^2 - a*b + 3*b^2)*B*Cot[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(a*d) - (b*Sqrt[a + b]*(5*a + (3*b^2)/a)*B*Cot[c + d*x]*Ellip
ticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/d + (b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*
x])/(d*Cos[c + d*x]^(3/2))
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
```

```
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3070

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)]])/((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[B*(d
/b^2), Int[Sqrt[b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Int[(A*c
 + (B*c + A*d)*Sin[e + f*x])/((b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\sqrt{a+b\,c\cdots}}{\cdots}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\frac{3}{2}a(a^2+3\cdots}{\cdots}$$

$$= -\frac{b\sqrt{a+b}\left(5a+\frac{3b^2}{a}\right)B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(a^2+3b^2)\,B\cot(c+dx)E\left(\sin^{-1}\left(\cdots\right.\right.}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 19.45, size = 1236, normalized size = 2.96

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*((3*b*B)/(2*a) + B*Cos[c + d*x]))/Cos
[c + d*x]^(5/2),x]
```

```
[Out] -1/2*(B*((-4*a*(-5*a^3*b - 3*a*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
 b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d
*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(2*a^
4 + a^2*b^2 - 3*b^4)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
```

```
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
 + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[
-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]])) + 2*(2*a^3*b + 6*a*b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c +
d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a
- b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*C
os[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)
/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
 - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d
*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(b*Sqrt[Cos[c + d*x]]))))/(a*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]*(Sec[c + d*x]*(2*a^2*B*Sin[c + d*x] + 7*b^2*B*Sin[c + d*x]) + a
*b*B*Sec[c + d*x]*Tan[c + d*x]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2345 vs. $2(390) = 780$.
time = 0.38, size = 2346, normalized size = 5.61

method	result	size
default	Expression too large to display	2346

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,meth
od=_RETURNVERBOSE)
```

```
[Out] -B/a/d*(-a^3*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2-6*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3+7
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
```

```
*cos(d*x+c)*a^3*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3+2*cos(d*x+c)^2*a^4+
cos(d*x+c)^4*a*b^3-cos(d*x+c)^2*a^3*b-7*cos(d*x+c)^2*a*b^3-8*cos(d*x+c)*a^2
*b^2-2*a^4*cos(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4-2*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-6*sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b^2-6*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b^3+7*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+9*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b^3+2*cos(d*x+c)^3*a^3*b+6*cos(d*x+c)^3*a*b^3+6*cos(d*x+c)^2*a
^2*b^2+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^4+10*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+2*cos(d*x+c)^3*a^2*b^2+
10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)*a^2*b^2+6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+2*sin(d*x+c)*cos(d*x+c)^2*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)^2*b^4-2*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^4+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^4-3*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4)/(a+b*
cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),
x, algorithm="maxima")
```

```
[Out] 1/2*integrate((2*B*cos(d*x + c) + 3*B*b/a)*(b*cos(d*x + c) + a)^(5/2)/cos(d
*x + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),
x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*B*a*b^2*cos(d*x + c)^3 + 3*B*a^2*b + (4*B*a^2*b + 3*B*b^3)*
cos(d*x + c)^2 + 2*(B*a^3 + 3*B*a*b^2)*cos(d*x + c))*sqrt(b*cos(d*x + c) +
a)/(a*cos(d*x + c)^(5/2)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)**(5/2
),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),
x, algorithm="giac")
```

```
[Out] integrate(1/2*(2*B*cos(d*x + c) + 3*B*b/a)*(b*cos(d*x + c) + a)^(5/2)/cos(d
*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,\cos\left(c+d\,x\right)+\frac{3\,B\,b}{2\,a}\right)\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((B*cos(c + d*x) + (3*B*b)/(2*a))*(a + b*cos(c + d*x))^(5/2))/cos(c + d
*x)^(5/2),x)
```

```
[Out] int(((B*cos(c + d*x) + (3*B*b)/(2*a))*(a + b*cos(c + d*x))^(5/2))/cos(c + d
*x)^(5/2), x)
```

3.420 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=479

$$-\frac{(a-b)\sqrt{a+b}\,(4Ab-3aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx)}{a+b}}}{4ab^2d}$$

```
[Out] 1/4*(4*A*b-3*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)+
1/2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/b/d-1/4*(a-b)*(4*A
*b-3*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d+1/4*(4*A*b-3*B*a+2*B*b)*cot(d*x+c)*Ell
ipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/b^2/d+1/4*(4*A*a*b-3*B*a^2-4*B*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x
+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d
```

Rubi [A]

time = 0.68, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3069, 3140, 3132, 2888, 3077, 2895, 3073}

$$\frac{\sqrt{a+b}\,(-3a^2B+4aAb-4b^2B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^3d}+\frac{\sqrt{a+b}\,(-3aB+4Ab+2bB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}-\frac{(a-b)\sqrt{a+b}\,(4Ab-3aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4ab^2d}+\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4b^2d\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -1/4*((a - b)*Sqrt[a + b]*(4*A*b - 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*b^2*d) + (Sqrt[a + b]*(4*A*b - 3*a*B + 2*b*B)*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(4*b^2*d) + (Sqrt[a + b]*(4*a*A*b - 3*a^2*B - 4*b^2*B)*Cot[c + d
*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d) + ((4*A*b - 3*a*B)*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{B\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2bd} + \frac{\int \frac{\frac{aB}{2}+bB\cos(c+\ldots}{\sqrt{\cos(c+d\ldots}}}{\ldots}$$

$$= \frac{(4Ab-3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\ldots}{\ldots}$$

$$= \frac{(4Ab-3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\ldots}{\ldots}$$

$$= \frac{\sqrt{a+b}\,(4aAb-3a^2B-4b^2B)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+\ldots}}{\sqrt{a+b}\ldots}\right.\right.}{\ldots}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(4Ab-3aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos\ldots}}{\sqrt{a+b}\,\sqrt{\mathrm{co}\ldots}}\right.\right.}{4a\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.12, size = 1175, normalized size = 2.45

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] (B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d) + ((-4
*a*(4*A*b - a*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 16*a*b*B*((Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*A*b - 3*a*B)*((I*Cos[(
c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/S
qrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*
((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1870 vs. $2(437) = 874$.
time = 0.35, size = 1871, normalized size = 3.91

method	result	size
default	Expression too large to display	1871

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*cos(d*x+c)^4*b^2-2*B*cos(d*x+c)^2*b^2-3*B*cos(d*x+c)^2*a^2+3*B*cos(d*x+c)*a^2+4*A*cos(d*x+c)^3*b^2-4*A*cos(d*x+c)^2*b^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-

```
(a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+c)^3*a*b+3*B*cos(d
*x+c)^2*a*b-2*B*cos(d*x+c)*a*b+4*A*cos(d*x+c)^2*a*b-4*A*cos(d*x+c)*a*b)/sin
(d*x+c)/b^2/cos(d*x+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c))/sqrt(b*cos(
d*x + c) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2), x
)
```

3.421 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=427

$$-\frac{(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+ …}{…}}}{abd}$$

```
[Out] a*B*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos
(d*x+c)^(1/2)/d/(a+b*cos(d*x+c))^(1/2)-(a-b)*B*cot(d*x+c)*EllipticE((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1
/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+B*c
ot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/b/d-(2*A*b-B*a)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A]
time = 0.69, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3082, 3130, 2888, 3072, 3077, 2895, 3073}

$$-\frac{\sqrt{a+b}\,(2Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{B\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}-\frac{B(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{aB\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) - (Sqrt[a + b]*(2*A*b -
 a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (a*B*Sin[c +
 d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + (B*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
```

```
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3082

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a +
 b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)
*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(
a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3130

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}}+\frac{1}{2}\int \frac{aB+2aA\cos(c+dx)+(}{\sqrt{\cos(c+dx)}\,(a+}$$

$$= \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}}+\frac{\int \frac{abB+(2aAb-a(2Ab-aB))\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/}}}{2b}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\,\sqrt{\cos}}\right.\right.}{}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\,\sqrt{\cos}}\right.\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx}}\right.\right.}{abd}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.28, size = 4017, normalized size = 9.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] ((1 + Cos[c + d*x])^(3/2)*((A*Sqrt[Cos[c + d*x]])/Sqrt[a + b*Cos[c + d*x]]
+ (B*Cos[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*x]])*Sec[(c + d*x)/2]^2*((2*I
)*(a - b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] +
 (4*I)*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a -
 b))] - (8*I)*A*b*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]
], -((a + b)/(a - b))] + (4*I)*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*
Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)
/2] - b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(
c + d*x)/2]))/(4*b*Sqrt[(a - b)/(a + b)]*d*Sqrt[a + b*Cos[c + d*x]]*(((1 +
Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*Sin[c + d*x]*((2*I)*(a - b)*B*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[
(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*(A*b - a*B)
*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSin
h[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] - (8*I)*A*b*
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/
(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a -
b))] + (4*I)*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]]
, -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a +
b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a -
b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(8*S
qrt[(a - b)/(a + b)]*(a + b*Cos[c + d*x])^(3/2)) - (3*Sqrt[1 + Cos[c + d*x]
]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*((2*I)*(a - b)*B*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*T
an[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*(A*b - a*B)*Sqrt[(a + b*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a
+ b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] - (8*I)*A*b*Sqrt[(a + b*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh
[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*a*B*S
qrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(
a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b
))] + b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(
c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(a + b)]*B*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(8*b*Sqrt[(a - b)/(a +
```

```
b)]*Sqrt[a + b*Cos[c + d*x]]) + ((1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]*((2*I)*(a - b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2
]], -((a + b)/(a - b))] + (4*I)*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c +
 d*x)/2]], -((a + b)/(a - b))] - (8*I)*A*b*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/
(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*a*B*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcS
inh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(
a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Si
n[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(4*b*Sqrt[(a - b)/(a + b)]*Sqrt[a +
b*Cos[c + d*x]]) + ((1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*((3*b*Sqrt[
(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Cos[(3*(c + d*x))/
2]*Sec[(c + d*x)/2])/2 + a*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sec[(c + d*x)/2]^2 - (b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^2)/2 + (I*(a - b)*B*EllipticE[I*A
rcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*(-((b*S
in[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c +
d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))] + ((2*I)*(A*b - a*B)*EllipticF[I*ArcSinh[Sqrt[(a - b)/(
a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*(-((b*Sin[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*S...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. $2(395) = 790$.
time = 0.45, size = 1005, normalized size = 2.35

method	result	size
default	Expression too large to display	1005

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d/(a+b*cos(d*x+c))^(1/2)*(2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b-4*A*sin(d*x+c)*cos(d*x+c)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b+2*B*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*a-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
```

```
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*A*sin(d*x+c)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b-4*A*sin(d*x+c)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b+2*B*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a-B*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-B*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
-B*cos(d*x+c)^3*b-B*cos(d*x+c)^2*a+b*B*cos(d*x+c)^2+B*cos(d*x+c)*a)/sin(d*x
+c)/b/cos(d*x+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)})\sqrt{\cos{(c + dx)}}}{\sqrt{a + b\cos{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/sqrt(a + b*cos(c + d*x)),
x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\sqrt{a+b\,\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2), x
)
```

3.422 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=228

$$\frac{2A\sqrt{a+b}\,\cot(c+dx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\sec(c+\cdots}{a-b}}}{ad}$$

```
[Out] 2*A*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a/d-2*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)
/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A]
time = 0.17, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3085, 2888, 2895}

$$\frac{2A\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2B\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]*
B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
```

```
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3085

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[B/d, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Dist
[(B*c - A*d)/d, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = A\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx + B\int \frac{\sqrt{\cos(c}}{\sqrt{a+b\cos}}$$

$$= \frac{2A\sqrt{a+b}\,\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{ad}$$

Mathematica [A]
time = 1.63, size = 144, normalized size = 0.63

$$\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left((A-B)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+2B\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*((A - B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] +
 2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2])
```

Maple [A]
time = 0.30, size = 197, normalized size = 0.86

method	result

default	$\frac{2\left(A\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)-B\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)+2B\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\sqrt{-\frac{a-b}{a+b}}\right.\right.}{d\sqrt{a+b\cos\left(dx+c\right)}\,(-1+\cos(dx+c))\sqrt{\cos\left(dx+c\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/d*(A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-B*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))+2*B*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))*sin(d*x+c)^2*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^2 + a*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+B\cos\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}\,\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x)))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2)), x
)
```

3.423 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=230

$$\frac{2A(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\text{se}}{a}}}{a^2d}$$

```
[Out] 2*A*(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-2*(A-B)*cot(d*x+c)*EllipticF((a+b*cos(d*
x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A]
time = 0.20, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3077, 2895, 3073}

$$\frac{2A(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}\,(A-B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sq
rt[a + b]*(A - B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
```

```
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = A\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx + (-A+B)\int \frac{}{\sqrt{\cos(}}$$

$$= \frac{2A(a-b)\sqrt{a+b}\ \cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\ \sqrt{\cos(c+dx)}}\right.\right.}{a^2d}$$

Mathematica [A]
time = 13.12, size = 299, normalized size = 1.30

$$\frac{2\left(A(a+b\cos(c+dx))\sin(c+dx)-\frac{2\sqrt{2}\cos^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\left(2A(a+b)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)-2a(A+B)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)+A\cos(c+dx)(a+b\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(1+\cos(c+dx))^{3/2}}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]
),x]
```

```
[Out] (2*(A*(a + b*Cos[c + d*x])*Sin[c + d*x] - (2*Sqrt[2]*(Cos[(c + d*x)/2]^2)^(
3/2)*(2*A*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(A + B)*Cos[(c + d*x)/2]^2*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + A*Cos[c
 + d*x]*(a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(1 + Cos[c + d*x])^(3/2)))/
(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(214) = 428$.

time = 0.30, size = 935, normalized size = 4.07

method	result	size
default	Expression too large to display	935

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/d*(B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)^2*a+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a-A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a-A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)^2*b+B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*a+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a-A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b+A*cos(d*x+
c)^3*b+A*cos(d*x+c)^2*a-A*cos(d*x+c)^2*b-A*cos(d*x+c)*a)/(a+b*cos(d*x+c))^(
1/2)/a/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^3 + a*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + b\cos(c + dx)}\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3/2)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,\cos(c + d\,x)}{{\cos(c + d\,x)}^{3/2}\,\sqrt{a + b\,\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2)), x
)
```

3.424 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=290

$$-\frac{2(a-b)\sqrt{a+b}\,(2Ab-3aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{3a^3d}$$

```
[Out] 2/3*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)-2/3*(a-b)*(2*A
*b-3*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d+2/3*(2*A*b+a*(A-3*B))*cot(d*x+c)*Ellipti
cF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2)
)*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)
/a^2/d
```

Rubi [A]
time = 0.32, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3079, 3077, 2895, 3073}

$$-\frac{2(a-b)\sqrt{a+b}\,(2Ab-3aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d}+\frac{2\sqrt{a+b}\,(a(A-3B)+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b - 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(3*a^3*d) + (2*Sqrt[a + b]*(2*A*b + a*(A - 3*B))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(3*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos
[c + d*x]^(3/2))
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{\frac{1}{2}(-2Ab+3aB)+\frac{1}{2}aA\cos(c+}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+}}}{3a}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{(2Ab+a(A-3B))\int \frac{}{\sqrt{\cos}}}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(2Ab-3aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\,\sqrt{c}}\right.\right.}{3}$$

Mathematica [A]
time = 16.02, size = 416, normalized size = 1.43

$$\frac{8\cos^2\left(\frac{1}{2}(c+dx)\right)^{7/2}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(-2(a+b)(-2Ab+3aB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)+2a(-2Ab+a(A+3B))\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)+(2Ab-3aB)\cos(c+dx)(a+b\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))^{3/2}\sqrt{a+b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)(-2Ab\sin(c+dx)+3aB\sin(c+dx))}{3a^2}+\frac{2A\sec(c+dx)\tan(c+dx)}{3a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]
),x]
```

```
[Out] (8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Co
s[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(a + b)*(-2*A*b + 3*a*B)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(-2*A*b + a
*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] + (2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])^(3/
2)*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
*((2*Sec[c + d*x]*(-2*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(3*a^2) + (2*
A*Sec[c + d*x]*Tan[c + d*x])/(3*a)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1535 vs. $2(264) = 528$.
time = 0.31, size = 1536, normalized size = 5.30

method	result	size
default	Expression too large to display	1536

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(-a^2*A+2*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2-3*B*cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
+3*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-2*A*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+
c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b+A*cos(d*x+c)^2*a^2+3*B*cos(d*x+c)^2*a^2-3*B*cos(d*x+c)
*a^2-2*A*cos(d*x+c)^3*b^2+2*A*cos(d*x+c)^2*b^2+2*A*cos(d*x+c)^2*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*A*cos
(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*c
os(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-3*
B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2+A*cos(d*x+c)^3*a*b+3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a
*b-2*A*cos(d*x+c)^2*a*b+A*cos(d*x+c)*a*b)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*
x+c)/cos(d*x+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^4 + a*cos(d*x + c)^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{(c + dx)}}{\sqrt{a + b\cos{(c + dx)}}\cos^{\frac{5}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(5/2)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,\cos\left(c + d\,x\right)}{{\cos\left(c + d\,x\right)}^{5/2}\,\sqrt{a + b\,\cos\left(c + d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(1/2)), x
)
```

3.425 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=363

$$\frac{2(a-b)\sqrt{a+b}\,(9a^2A+8Ab^2-10abB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{15a^4d}$$

```
[Out] 2/5*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)-2/15*(4*A*b-5*
B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9
*A*a^2+8*A*b^2-10*B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d-2/15*(8*A*b^2+a^2*(9*A-5*
B)-2*a*b*(A+5*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d
```

Rubi [A]
time = 0.54, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3079, 3134, 3077, 2895, 3073}

$$-\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{15a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,(9a^2A-10abB+8Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^4d}-\frac{2\sqrt{a+b}\,(a^2(9A-5B)-2ab(A+5B)+8Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^4*d) - (2*Sqrt[a + b]*(8*A*b^2 + a^2*(9*A - 5*B) - 2*a*
b*(A + 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d) + (2*A*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) - (2*(4*A*b - 5*a*
B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

```
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
```

```
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{\frac{1}{2}(-4Ab+5aB)+\frac{3}{2}aA\cos(c+d\ldots}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos\ldots}}}{5a}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\cos(\ldots}}{15a^2d\cos^{\frac{3}{2}}(c\ldots}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\cos(\ldots}}{15a^2d\cos^{\frac{3}{2}}(c\ldots}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(9a^2A+8Ab^2-10abB)\cot(c+dx)E\left(\sin^{-1}\left(\ldots\right.\right.}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.44, size = 1319, normalized size = 3.63

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Cos[c + d*x]]
),x]
```

```
[Out] -1/15*((-4*a*(7*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - 4*a*(9*a^3*A + 8*a*A*b^2 - 10*a^2*b*B)*((Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[A
rcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-
```

```
a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b + 8*A*b^3 - 10*a*b
^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin
[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a
 + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b
)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(
a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]
)))/(a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]
^2*(-4*A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/(15*a^2) + (2*Sec[c + d*x]*(
9*a^2*A*Sin[c + d*x] + 8*A*b^2*Sin[c + d*x] - 10*a*b*B*Sin[c + d*x]))/(15*a
^3) + (2*A*Sec[c + d*x]^2*Tan[c + d*x])/(5*a)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2479 vs. $2(331) = 662$.
time = 0.37, size = 2480, normalized size = 6.83

method	result	size
default	Expression too large to display	2480

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*(-3*A*a^3-10*A*cos(d*x+c)^3*a^2*b+5*B*cos(d*x+c)^3*a^3+9*A*cos(d*x+
c)^3*a^3-8*A*cos(d*x+c)^3*b^3-6*A*cos(d*x+c)^2*a^3+2*A*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8
*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^2+10*B*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+10*B*sin(d*x+c)*cos(d*x+c)^2*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
```

os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2-10*B*
sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*a^2*b-9*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b-8*A*sin(d*x+c)*cos(d*x+c)^3*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+2*A*sin(d*
x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*a^2*b+8*A*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+10*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+10*B*sin(d*x+c)
*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b^2-10*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+5*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3-9*A*sin(d*x+c)*cos(d*
x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-
8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^3+9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+5*B*sin(d*x+c)*cos(d*x+c)^2*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3-9*A*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3-8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*b^3+9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-5*B*cos(d*x+c)*a^3+8*A*c
os(d*x+c)^4*b^3-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-8*A*sin(d*x+c)*cos(d*x+c)^2*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+5*B*cos(d*x+c
)^4*a^2*b+10*B*cos(d*x+c)^3*a*b^2+9*A*cos(d*x+c)^4*a^2*b-4*A*cos(d*x+c)^4*a
*b^2+5*B*cos(d*x+c)^2*a^2*b-10*B*cos(d*x+c)^4*a*b^2-10*B*cos(d*x+c)^3*a^2*b

```
+8*A*cos(d*x+c)^3*a*b^2-4*A*cos(d*x+c)^2*a*b^2+A*cos(d*x+c)*a^2*b)/(a+b*cos
(d*x+c))^(1/2)/a^3/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/2)
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^5 + a*cos(d*x + c)^4), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b*cos(c + d*x))^(1/2)), x
)
```

3.426 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=500

$$\frac{(2aAb-3a^2B+b^2B)\cot(c+dx)E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1}{}}}{ab^2\sqrt{a+b}\,d}$$

```
[Out] 2*a*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1
/2)-(2*A*a*b-3*B*a^2+B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)
/d/cos(d*x+c)^(1/2)+(2*A*a*b-3*B*a^2+B*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d
*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d/(a+b)^(1/2)-(2*A
*b-(3*a+b)*B)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d
*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(
d*x+c))/(a-b))^(1/2)/b^2/d/(a+b)^(1/2)-(2*A*b-3*B*a)*cot(d*x+c)*EllipticPi(
(a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^
(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^
(1/2)/b^3/d
```

Rubi [A]
time = 0.83, antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3068, 3140, 3132, 2888, 3077, 2895, 3073}

$$\frac{(-3a^2B+2aAb+b^2B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ab^2d\sqrt{a+b}}-\frac{(-3a^2B+2aAb+b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{b^2d(a^2-b^2)\sqrt{\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\sqrt{a+b}(2Ab-3aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^3d}-\frac{(2Ab-B(3a+b))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((2*a*A*b - 3*a^2*B + b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*Sqrt
[a + b]*d) - ((2*A*b - (3*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2
*Sqrt[a + b]*d) - (Sqrt[a + b]*(2*A*b - 3*a*B)*Cot[c + d*x]*EllipticPi[(a +
 b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(b^3*d) + (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*
B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d
*x]])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}a(Ab-aB)+\frac{1}{2}b(Ab-}{\sqrt{\cos(c}}}{}$$

$$= \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{(2aAb-3a^2B+b^2B)}{b^2\,(a^2-}$$

$$= \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{(2aAb-3a^2B+b^2B)}{b^2\,(a^2-}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-3aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\,\sqrt{\cos}}\right.\right.}{b}$$

$$= \frac{(2aAb-3a^2B+b^2B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\,\sqrt{\cos(c+d}}\right.\right.}{ab^2\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.43, size = 1234, normalized size = 2.47

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(-(a*A*b*Sin[c + d*x]) + a^2*B*Sin[c + d*x]))/(b*(-a^
2 + b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*B - b^2*B)*Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - 4*a*(-2*A*b^2 + 2*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
```

```
[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-2*a*A*b + 3*a^2*B - b^2*B)*((I*Cos[(c
 + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sq
rt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*(
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
 b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(2*(a - b)*b
*(a + b)*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2880 vs. $2(468) = 936$.
time = 0.36, size = 2881, normalized size = 5.76

method	result	size
default	Expression too large to display	2881

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2-6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*a*b^2+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+4*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3-2*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)*b^3+B*cos(d*x+c)^3*b^3-3*B*cos(d*x+c)^2*a^3-B*cos(d*x+c)^2*b^3+3*
B*cos(d*x+c)*a^3+2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
```

in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-4*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^2-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+B*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-6*B*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*a*b^2+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+4*A*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-2
*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*b^3-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*a^3+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*sin(d*x+c)*b^3+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+B*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+6*B*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(
a+b))^(1/2))*a^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*a^2*b+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-2*A*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*B*sin(d*x+c)

```
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+3*B*
cos(d*x+c)^2*a^2*b+B*cos(d*x+c)^2*a*b^2-2*B*cos(d*x+c)*a^2*b-B*cos(d*x+c)*a
*b^2-B*cos(d*x+c)^3*a^2*b+2*A*cos(d*x+c)^2*a^2*b-2*A*cos(d*x+c)^2*a*b^2-2*A
*cos(d*x+c)*a^2*b+2*A*cos(d*x+c)*a*b^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/b
^2/(a^2-b^2)/cos(d*x+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2), x
)
```

3.427 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=416

$$-\frac{2(Ab-aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\sec(c}{a-b}}}{ab\sqrt{a+b}\,d}$$

[Out] 2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)-2*(A*b-B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/(a+b)^(1/2)+2*(A*b-B*a)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/(a+b)^(1/2)-2*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d

Rubi [A]
time = 0.39, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3071, 2888, 2873, 2874, 2895, 3073}

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd\sqrt{a+b}}-\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd\sqrt{a+b}}-\frac{2B\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]

[Out] (-2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) + (2*(A*b - a*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (2*a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])

Rule 2873

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*a*d*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b
```

```
*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2874

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3071

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> D
ist[B/b, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Di
st[(A*b - a*B)/b, Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
```

```
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} + \frac{(Ab-aB)\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx}{b}$$

$$= -\frac{2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{b^2d}$$

$$= -\frac{2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{b^2d}$$

$$= -\frac{2(Ab-aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\right.}{ab\sqrt{a+b}\,a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.11, size = 1012, normalized size = 2.43

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(-(A*b*Sin[c + d*x]) + a*B*Sin[c + d*x]))/((a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]]) - (-4*a*(a*A - b*B)*((Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
```

```
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(A*b - a*B)*((I*Cos[(c + d*x)/2
]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c
+ d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d
*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/((-a + b)*(a + b)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2012 vs. $2(388) = 776$.
time = 0.32, size = 2013, normalized size = 4.84

method	result	size
default	Expression too large to display	2013

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/d/(a+b*cos(d*x+c))^(1/2)*(-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-2*B*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+2*B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
```

```
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+B*cos(d*x+c
)^2*a^2-B*cos(d*x+c)*a^2+A*cos(d*x+c)^2*b^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x
+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*b^2+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-2*B*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*a^2+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*b^2-A*cos(d*x+c)*b^2+A*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-B*cos(d*x+c)^2*a*b+B
*cos(d*x+c)*a*b-A*cos(d*x+c)^2*a*b+A*cos(d*x+c)*a*b)/sin(d*x+c)/b/(a^2-b^2)
/cos(d*x+c)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(3/2
), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2), x
)
```

3.428 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=284

$$\frac{2(Ab-aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\sec(c+}{a-b}}}{a^2\sqrt{a+b}\,d}$$

```
[Out] -2*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)
+2*(A*b-B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)+2*(A+B)*cot(d*x+c)*EllipticF((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3072, 3077, 2895, 3073}

$$\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{a+b}}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}+\frac{2(A+B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d) + (2*
(A + B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) - (2*(A*b - a*
B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

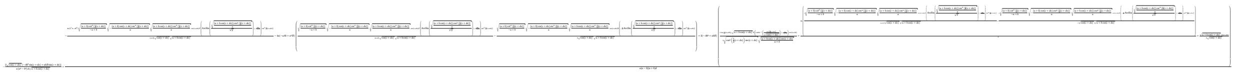
$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{Ab-a}{\cos^{\frac{3}{2}}(c+dx}}{}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}} + \frac{(A+B)\int}{}$$

$$= \frac{2(Ab-aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|\right.}{a^2\sqrt{a+b}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 6.39, size = 1223, normalized size = 4.31



Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c + d*x]))/(a*(a^
2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*A - A*b^2)*Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - 4*a*(-(a*A*b) + a^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[
(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-(A*b^2) + a*b*B)*((I*Cos[(c + d*x)/2]*S
qrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d
*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a*(a - b)*(a + b)*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1632 vs. 2(264) = 528.
time = 0.31, size = 1633, normalized size = 5.75

method	result	size
default	Expression too large to display	1633

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/d*(A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*
cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-B
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
*sin(d*x+c)*a^2-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+B*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*b^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*cos(d*x+c)^2*a*b+A*cos(d*x+c)^2*b
^2+B*cos(d*x+c)^2*a^2-B*cos(d*x+c)^2*a*b+A*cos(d*x+c)*a*b-A*cos(d*x+c)*b^2-
B*cos(d*x+c)*a^2+B*cos(d*x+c)*a*b)/(a+b*cos(d*x+c))^(1/2)/(a^2-b^2)/a/sin(d
*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^3 + 2*a*b*cos(d*x + c)^2 + a^2*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{\frac{3}{2}}\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x
))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)), x
)
```

3.429 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=305

$$\frac{2(a^2A-2Ab^2+abB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1}{}}}{a^3\sqrt{a+b}\,d}$$

```
[Out] 2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1
/2)+2*(A*a^2-2*A*b^2+B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)-2*(2*A*b+a*(A-B))*cot(
d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a^2/d/(a+b)^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3079, 3077, 2895, 3073}

$$-\frac{2(a(A-B)+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{a+b}}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2(a^2A+abB-2Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*(a^2*A - 2*A*b^2 + a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3*Sqrt[a
 + b]*d) - (2*(2*A*b + a*(A - B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*S
qrt[a + b]*d) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}(a^2A-2Ab^2}{\cos^{\frac{3}{2}}(c+dx}}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}} - \frac{((a-b)(2Ab}{}$$

$$= \frac{2(a^2A-2Ab^2+abB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\,\sqrt{\cos(c+d}}\right.\right.}{a^3\sqrt{a+b}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.52, size = 1281, normalized size = 4.20

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] ((-4*a*(2*a^2*A*b - 2*A*b^3 - a^3*B + a*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- 4*a*(a^3*A - 2*a*A*b^2 + a^2*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]])) + 2*(a^2*A*b - 2*A*b^3 + a*b^2*B)*((I*Cos[(c + d*
x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Co
s[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec
[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
```

```
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a^2*(-a + b)*(a
+ b)*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(A*b^3*Sin[c +
 d*x]) + a*b^2*B*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2
*A*Tan[c + d*x])/a^2))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2281 vs. $2(285) = 570$.
time = 0.33, size = 2282, normalized size = 7.48

method	result	size
default	Expression too large to display	2282

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/d/(a+b*cos(d*x+c))^(1/2)*(A*a*b^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*s
in(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-A*
a^3-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^3+A*cos(d*x+c)*a^3+2*A*cos(d*x+c)*b^3-2*A*cos
(d*x+c)^2*b^3-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+2*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-2*A*sin
```

```
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^2-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+B*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+A*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^3+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-A*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+2*A*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-
2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*b-B*cos(d*x+c)^2*a^2*b+B*cos(d*x+c)^2*a*b^2+B*cos(d*x+c)*a^
2*b-B*cos(d*x+c)*a*b^2+A*cos(d*x+c)^2*a^2*b+A*cos(d*x+c)^2*a*b^2-A*cos(d*x+
c)*a^2*b-2*A*cos(d*x+c)*a*b^2)/a^2/(a^2-b^2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{(c + dx)}}{(a + b\cos{(c + dx)})^{\frac{3}{2}}\cos^{\frac{3}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))**(3/2)*cos(c + d*x)**(3
/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,\cos\left(c + d\,x\right)}{{\cos\left(c + d\,x\right)}^{3/2}\,{\left(a + b\,\cos\left(c + d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)), x
)
```

3.430 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=393

$$-\frac{2(5a^2Ab-8Ab^3-3a^3B+6ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c}{a+b}}}{3a^4\sqrt{a+b}\,d}$$

```
[Out] 2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1
/2)+2/3*(A*a^2-4*A*b^2+3*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-
b^2)/d/cos(d*x+c)^(3/2)-2/3*(5*A*a^2*b-8*A*b^3-3*B*a^3+6*B*a*b^2)*cot(d*x+c
)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-
b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^
4/d/(a+b)^(1/2)+2/3*(a+2*b)*(4*A*b+a*(A-3*B))*cot(d*x+c)*EllipticF((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)
```

Rubi [A]

time = 0.61, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3079, 3134, 3077, 2895, 3073}

$$\frac{2(a+2b)(a(A-3B)+4Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d\sqrt{a+b}}+\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}-\frac{2(-3a^3B+5a^2Ab+6ab^2B-8Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^4d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (-2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(3*a^4*Sqrt[a + b]*d) + (2*(a + 2*b)*(4*A*b + a*(A - 3*B))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d) + (2*b*(A*b - a*B)*Sin[c +
 d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(
a^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a
^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

```
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
```

```
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}(a^2A-4Ab^2+3}{c}}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2(a^2A-4Ab^2+}{3a}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2(a^2A-4Ab^2+}{3a}$$

$$= -\frac{2(5a^2Ab-8Ab^3-3a^3B+6ab^2B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{3a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.71, size = 1357, normalized size = 3.45

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] ((-4*a*(a^4*A + 7*a^2*A*b^2 - 8*A*b^4 - 6*a^3*b*B + 6*a*b^3*B)*Sqrt[((a + b
)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Cos[c + d*x]]) - 4*a*(5*a^3*A*b - 8*a*A*b^3 - 3*a^4*B + 6*a^2*b^2*B)*((Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
```

```
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(5*a^2*A*b^
2 - 8*A*b^4 - 3*a^3*b*B + 6*a*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
+ d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-
a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b
*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a^3*(a - b)*(a + b)*d) + (Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]*(-5*A*b*Sin[c + d*x] + 3*
a*B*Sin[c + d*x]))/(3*a^3) - (2*(-(A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*
x]))/(a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*A*Sec[c + d*x]*Tan[c + d*x
])/(3*a^2)))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3333 vs. $2(363) = 726$.

time = 0.36, size = 3334, normalized size = 8.48

method	result	size
default	Expression too large to display	3334

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(A*a^2*b^2+8*A*cos(d*x+c)^3*b^4-8*A*cos(d*x+c)^2*b^4+3*B*cos(d*x+c)^
3*a^3*b-6*B*cos(d*x+c)^3*a*b^3-5*A*cos(d*x+c)^2*a^3*b+8*A*cos(d*x+c)^2*a*b^
3-4*A*cos(d*x+c)*a*b^3-5*A*cos(d*x+c)^3*a^2*b^2-6*B*cos(d*x+c)^2*a^2*b^2+6*
B*cos(d*x+c)^2*a*b^3+3*B*cos(d*x+c)*a^2*b^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^4-8*A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)^2*b^4+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
```

+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)^2*a^4-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^4+A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4+3*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos
(d*x+c)*a^4+3*B*cos(d*x+c)^2*a^4-5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b+2*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^
2*b^2+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)^2*a*b^3+5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b+5*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^
2-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)^2*a*b^3-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b-6*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2-3*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*a^3*b+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^3+5*A*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)*a^2*b^2-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^3+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2+8*A*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
*a*b^3-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+

```
b))^(1/2))*cos(d*x+c)*a^3*b+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2+6*B*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^3-3*B
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)*a^3*b-6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2-A*a^4+5*A*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b-5*A*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^5 + 2*a*b*cos(d*x + c)^4 + a^2*cos(d*x + c)^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(3/2)), x
)
```

3.431 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=674

$$\frac{(6a^3Ab-14aAb^3-15a^4B+26a^2b^2B-3b^4B)\cot(c+dx)E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{3a(a-b)b^3(a+b)^{3/2}d}$$

```
[Out] 2/3*a*(A*b-B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^
(3/2)+2/3*a*(2*A*a^2*b-6*A*b^3-5*B*a^3+9*B*a*b^2)*sin(d*x+c)*cos(d*x+c)^(1/
2)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-1/3*(6*A*a^3*b-14*A*a*b^3-15*B*
a^4+26*B*a^2*b^2-3*B*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)^2
/d/cos(d*x+c)^(1/2)+1/3*(6*A*a^3*b-14*A*a*b^3-15*B*a^4+26*B*a^2*b^2-3*B*b^4
)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b
))^(1/2)/a/(a-b)/b^3/(a+b)^(3/2)/d-1/3*(6*A*a^2*b+2*A*a*b^2-12*A*b^3-15*B*a
^3-5*B*a^2*b+21*B*a*b^2+3*B*b^3)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^3/(a+b)^(3/2)/d-(2*A*b-5*B*
a)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d
```

Rubi [A]
time = 1.42, antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3068, 3126, 3140, 3132, 2888, 3077, 2895, 3073}

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d) - ((6*a^2*A*b + 2*a
*A*b^2 - 12*A*b^3 - 15*a^3*B - 5*a^2*b*B + 21*a*b^2*B + 3*b^3*B)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(3*(a - b)*b^3*(a + b)^(3/2)*d) - (Sqrt[a + b]*(2*
A*b - 5*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
```

c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^4*d) + (2*a*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]])

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
```

```
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
```

```
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \sqrt{\cos(c+dx)}\,\left(-\tfrac{3}{2}a(A\right.}{}$$

$$= \frac{2a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{2a(2a^2Ab-6Ab^3-5a^3B}{3b^2(a^2-b^2}$$

$$= \frac{2a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{2a(2a^2Ab-6Ab^3-5a^3B}{3b^2(a^2-b^2}$$

$$= \frac{2a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{2a(2a^2Ab-6Ab^3-5a^3B}{3b^2(a^2-b^2}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-5aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\,\sqrt{\cos(c}}\right.\right.}{b^4d}$$

$$= \frac{(6a^3Ab-14aAb^3-15a^4B+26a^2b^2B-3b^4B)\cot(c+dx)E\left(\sin^{-1}\right.}{3a(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.71, size = 1396, normalized size = 2.07

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(-(a^2*A*b*Sin[c + d*x])
+ a^3*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) - (2*(-3
*a^3*A*b*Sin[c + d*x] + 7*a*A*b^3*Sin[c + d*x] + 6*a^4*B*Sin[c + d*x] - 10*
a^2*b^2*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d +
((-4*a*(-2*a^3*A*b + 2*a*A*b^3 + 5*a^4*B - 8*a^2*b^2*B + 3*b^4*B)*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
```

```
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]) - 4*a*(2*a^2*A*b^2 + 6*A*b^4 + 4*a^3*b*B - 12*a*b^3*B)*(
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-6*a^3
*A*b + 14*a*A*b^3 + 15*a^4*B - 26*a^2*b^2*B + 3*b^4*B)*((I*Cos[(c + d*x)/2]
*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c +
 d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(6*(a - b)^2*b^2*(a +
b)^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8610 vs. $2(628) = 1256$.
time = 0.57, size = 8611, normalized size = 12.78

method	result	size
default	Expression too large to display	8611

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x
)
```

3.432 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=545

$$\frac{2(4Ab^3+3a^3B-7ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{3a(a-b)b^2(a+b)^{3/2}d}$$

```
[Out] 2/3*a*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^
(3/2)-2/3*a*(4*A*b^3+3*B*a^3-7*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/cos(d*
x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2/3*(4*A*b^3+3*B*a^3-7*B*a*b^2)*cot(d*x+c
)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-
b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/
(a-b)/b^2/(a+b)^(3/2)/d+2/3*(A*a*b^2-3*A*b^3-3*B*a^3-B*a^2*b+6*B*a*b^2)*cot
(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/a/(a-b)/b^2/(a+b)^(3/2)/d-2*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d
```

Rubi [A]
time = 0.89, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3068, 3130, 2888, 3072, 3077, 2895, 3073}

$$\frac{2(3a^3B-7ab^2B+4Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ab^2d(a-b)(a+b)^{3/2}}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2(-3a^3B-a^2bB+aAb^2+6ab^2B-3Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ab^2d(a-b)(a+b)^{3/2}}-\frac{2a(3a^3B-7ab^2B+4Ab^3)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*
(a - b)*b^2*(a + b)^(3/2)*d) + (2*(a*A*b^2 - 3*A*b^3 - 3*a^3*B - a^2*b*B +
6*a*b^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2
)*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3
*d) + (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)) - (2*a*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Sin[c +
d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
```

```
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3130

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{1}{2}a(Ab-aB)+\frac{3}{2}b(Ab-}{\sqrt{\cos(c-}}}{3}$$

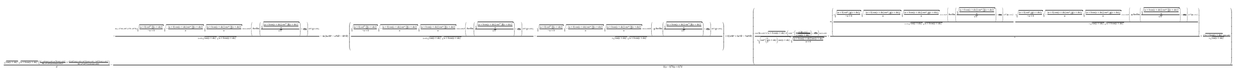
$$= \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{1}{2}ab(Ab-aB)+\left(\frac{3}{2}a(a^2\right.}{\sqrt{\cos(c+}}}{3b^2}$$

$$= -\frac{2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{b^3d}$$

$$= -\frac{2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{b^3d}$$

$$= \frac{2(4Ab^3+3a^3B-7ab^2B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\,\sqrt{\cos(c}}\right.\right.}{3a(a-b)b^2(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.52, size = 1342, normalized size = 2.46



Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(a*A*b*Sin[c + d*x]) + a
^2*B*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(4*A*b^3
*Sin[c + d*x] + 3*a^3*B*Sin[c + d*x] - 7*a*b^2*B*Sin[c + d*x]))/(3*b*(-a^2
+ b^2)^2*(a + b*Cos[c + d*x]))))/d - ((-4*a*(-(a^2*A*b) + A*b^3 + a^3*B - a
*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(4*a*A*b^2 - a^2*b*B - 3*b^3*B)*(
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*A*b^
3 + 3*a^3*B - 7*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Elli
pticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[
c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x
])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b
*Sqrt[Cos[c + d*x]])))/(3*(a - b)^2*b*(a + b)^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5748 vs. $2(505) = 1010$.
time = 0.43, size = 5749, normalized size = 10.55

method	result	size
default	Expression too large to display	5749

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*
x + c) + a^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x
)
```

3.433 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=391

$$-\frac{2(3a^2A+Ab^2-4abB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{}}{3a^2(a-b)(a+b)^{3/2}d}$$

```
[Out] -2/3*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/
2)+2/3*(3*A*a^2+A*b^2-4*B*a*b)*sin(d*x+c)/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a
+b*cos(d*x+c))^(1/2)-2/3*(3*A*a^2+A*b^2-4*B*a*b)*cot(d*x+c)*EllipticE((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-
sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/
2)/d+2/3*(3*A*a-A*b+B*a-3*B*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d
```

Rubi [A]
time = 0.56, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3078, 3072, 3077, 2895, 3073}

$$-\frac{2(3a^2A-4abB+Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d(a-b)(a+b)^{3/2}}+\frac{2(3a^2A-4abB+Ab^2)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2(3aA+aB-Ab-3bB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad(a-b)(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(3*a^2*A + A*b^2 - 4*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*(
a - b)*(a + b)^(3/2)*d) + (2*(3*a*A - A*b + a*B - 3*b*B)*Cot[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d) - (2*(A*b - a*B)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(3*
a^2*A + A*b^2 - 4*a*b*B)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
```

```
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3\,(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(Ab-aB)-\frac{3}{2}(aA}{\sqrt{\cos(c+dx)}}}{3\,(a^2-}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3\,(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{2(3a^2A+A}{3\,(a^2-b^2)^2\,d\sqrt{\cos}}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3\,(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{2(3a^2A+A}{3\,(a^2-b^2)^2\,d\sqrt{\cos}}$$

$$= -\frac{2(3a^2A+Ab^2-4abB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\,\sqrt{\cos(}}\right.\right.}{3a^2(a-b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.46, size = 1335, normalized size = 3.41

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(A*b*Sin[c + d*x]) + a*B
*Sin[c + d*x]))/(3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(3*a^2*A*b*Sin[
c + d*x] + A*b^3*Sin[c + d*x] - 4*a*b^2*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2
*(a + b*Cos[c + d*x]))))/d + ((-4*a*(-(a^2*A*b) + A*b^3 + a^3*B - a*b^2*B)*
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^3*A + a*A*b^2 - 4*a^2*b*B)*((Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^2*A*b + A*
```

```
b^3 - 4*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*
ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x]
)/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c
 + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b
)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Co
s[c + d*x]])))/(3*a*(a - b)^2*(a + b)^2*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4240 vs. $2(359) = 718$.
time = 0.42, size = 4241, normalized size = 10.85

method	result	size
default	Expression too large to display	4241

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3/d/(a+b*cos(d*x+c))^(3/2)*(A*cos(d*x+c)^3*b^4-A*cos(d*x+c)^2*b^4-4*B*cos
(d*x+c)^3*a*b^3+6*A*cos(d*x+c)^2*a^3*b+2*A*cos(d*x+c)^2*a*b^3+A*cos(d*x+c)*
a^2*b^2+3*A*cos(d*x+c)^3*a^2*b^2-8*B*cos(d*x+c)^2*a^2*b^2+4*B*cos(d*x+c)^2*
a*b^3-4*B*cos(d*x+c)*a^3*b+3*B*cos(d*x+c)*a^2*b^2-A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^4+3*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)*a^4-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)*a^4+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b+4*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b
^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)^2*a*b^3-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
```

b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b-3*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2-A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)^2*a*b^3-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2+4*B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)^2*a^2*b^2+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^3-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-4*A*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2-2*A
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)*a*b^3+5*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^3+4*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
a^3*b+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)*a^2*b^2+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^3-5*B*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b-7*B*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)*a^2*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^3-6*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b+7*A*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3*b-3*A*cos(d*x+c)^2*a^4-B*cos(d*x+c)^3*a^4+B*cos(d*x+c)*a^4+3*A*sin(d*x

```
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*A
*cos(d*x+c)*a^4-2*A*cos(d*x+c)^3*a*b^3-4*A*cos(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{\cos(c + dx)}}{(a + b\cos(c + dx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(5/2
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x
)
```

3.434 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=429

$$\frac{2(6a^2Ab-2Ab^3-3a^3B-ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{3a^3(a-b)(a+b)^{3/2}d}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^
(3/2)-2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/co
s(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^
2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-
b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d-2/3*(2*A*b^2-3*a^2*(A+B)+a*b*(3*A+B))*cot
(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/a^2/(a^2-b^2)/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3079, 3072, 3077, 2895, 3073}

$$-\frac{2(-3a^2(A+B)+ab(3A+B)+2Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{a+b}\,(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\,(a^2-b^2)\,(a+b\cos(c+dx))^{3/2}}+\frac{2(-3a^3B+6a^2Ab-ab^2B-2Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d(a-b)(a+b)^{3/2}}-\frac{2(-3a^3B+6a^2Ab-ab^2B-2Ab^3)\sin(c+dx)}{3ad\,(a^2-b^2)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d) - (2*(2*A*b^2 - 3*a^2*(A + B) + a*b*(3
*A + B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*
d) + (2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(
a + b*Cos[c + d*x])^(3/2)) - (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*S
in[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
```

```
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
```

)

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}(3a^2A-2Ab^2-abB)}{\sqrt{\cos(c+dx)}}}{3a\,(a}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} - \frac{2(6a^2Ab-2Ab^3}{3a\,(a^2-b^2)^2\,d\sqrt{\cos}}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} - \frac{2(6a^2Ab-2Ab^3}{3a\,(a^2-b^2)^2\,d\sqrt{\cos}}$$

$$= \frac{2(6a^2Ab-2Ab^3-3a^3B-ab^2B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a-}}{\sqrt{a+}}\right.\right.}{3a^3(a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.59, size = 1384, normalized size = 3.23

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(-(A*b^2*Sin[c + d*x]) +
a*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(-6*a^2*
A*b^2*Sin[c + d*x] + 2*A*b^4*Sin[c + d*x] + 3*a^3*b*B*Sin[c + d*x] + a*b^3*
B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(3
*a^4*A - 5*a^2*A*b^2 + 2*A*b^4 - a^3*b*B + a*b^3*B)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - 4*a*(-6*a^3*A*b + 2*a*A*b^3 + 3*a^4*B + a^2*b^2*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
```

```
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-6*a^2*A*b^2 + 2*A*b^4
 + 3*a^3*b*B + a*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ellip
ticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c
 + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x]
)*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*
Sqrt[Cos[c + d*x]])))/(3*a^2*(a - b)^2*(a + b)^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5202 vs. $2(397) = 794$.
time = 0.45, size = 5203, normalized size = 12.13

method	result	size
default	Expression too large to display	5203

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^4 + 3*a*b^2*cos(d*x + c)^3 + 3*a^2*b*cos(d*x + c)^2 + a^3*
cos(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2)), x
)
```

3.435 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=456

$$\frac{2(3a^4A-15a^2Ab^2+8Ab^4+6a^3bB-2ab^3B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a}{}}}{3a^4(a-b)(a+b)^{3/2}d}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^
(1/2)+2/3*b*(8*A*a^2*b-4*A*b^3-5*B*a^3+B*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/
d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2/3*(3*A*a^4-15*A*a^2*b^2+8*A*b^4
+6*B*a^3*b-2*B*a*b^3)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/
2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d+2/3*(8*A*b^3-3*a^3*(A-
B)+2*a*b^2*(3*A-B)-3*a^2*b*(3*A+B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a^2-b^2)/d/(a+b)^(1/2)
```

Rubi [A]

time = 0.74, antiderivative size = 456, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3079, 3134, 3077, 2895, 3073}

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+\frac{2(-3a^3(A-B)-3a^2b(3A+B)+2ab^2(3A-B)+8Ab^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d\sqrt{a+b}(a^2-b^2)}+\frac{2b(-5a^3B+8a^2Ab+ab^2B-4Ab^3)\sin(c+dx)}{3a^2d(a^2-b^2)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2(3a^4A+6a^3bB-15a^2Ab^2-2ab^3B+8Ab^4)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^4d(a-b)(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d) + (2*(8*A*b^3 - 3*a^3
*(A - B) + 2*a*b^2*(3*A - B) - 3*a^2*b*(3*A + B))*Cot[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(3*a^3*Sqrt[a + b]*(a^2 - b^2)*d) + (2*b*(A*b - a*B)*Sin[c + d*x]
)/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) + (2*b*
(8*a^2*A*b - 4*A*b^3 - 5*a^3*B + a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^
2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
```

```
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
```

```
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}(3a^2A \cdots}{\cdots}}{\cdots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}} + \frac{2b(8a^2A \cdots}{3a^2\left(a^2-\cdots\right.}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}} + \frac{2b(8a^2A \cdots}{3a^2\left(a^2-\cdots\right.}$$

$$= \frac{2(3a^4A-15a^2Ab^2+8Ab^4+6a^3bB-2ab^3B)\cot(c+dx)E\left(\sin \cdots\right.}{3 \cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.72, size = 1431, normalized size = 3.14

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] -1/3*((-4*a*(9*a^4*A*b - 17*a^2*A*b^3 + 8*A*b^5 - 3*a^5*B + 5*a^3*b^2*B - 2
*a*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^5*A - 15*a^3*A*b^2 + 8*a*A
*b^4 + 6*a^4*b*B - 2*a^2*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b
```

```
)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])) + 2*(3*a^4*A*b - 15*a^2*A*b^3 + 8*A*b^5 + 6*a^3*b^2*B
 - 2*a*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*Arc
Sinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(
b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c +
d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c
 + d*x]])))/(a^3*(a - b)^2*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]*((2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x]))/(3*a^2*(a^2
 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-9*a^2*A*b^3*Sin[c + d*x] + 5*A*b^5*S
in[c + d*x] + 6*a^3*b^2*B*Sin[c + d*x] - 2*a*b^4*B*Sin[c + d*x]))/(3*a^3*(a
^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/a^3))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6499 vs. $2(424) = 848$.
time = 0.51, size = 6500, normalized size = 14.25

method	result	size
default	Expression too large to display	6500

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^5 + 3*a*b^2*cos(d*x + c)^4 + 3*a^2*b*cos(d*x + c)^3 + a^3*
cos(d*x + c)^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(5/2)),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(5/2)), x
)
```

3.436 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=567

$$-\frac{2(8a^4Ab-28a^2Ab^3+16Ab^5-3a^5B+15a^3b^2B-8ab^4B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{3a^5(a-b)(a+b)^{3/2}d}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^
(3/2)+2/3*b*(10*A*a^2*b-6*A*b^3-7*B*a^3+3*B*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)
^2/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2)+2/3*(A*a^4-13*A*a^2*b^2+8*A*b^
4+8*B*a^3*b-4*B*a*b^3)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d/
cos(d*x+c)^(3/2)-2/3*(8*A*a^4*b-28*A*a^2*b^3+16*A*b^5-3*B*a^5+15*B*a^3*b^2-
8*B*a*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/a^5/(a-b)/(a+b)^(3/2)/d-2/3*(16*A*b^4-a^4*(A-3*B)+4*a*b^3
*(3*A-2*B)-9*a^3*b*(A-B)-2*a^2*b^2*(8*A+3*B))*cot(d*x+c)*EllipticF((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a^2-b^2)/d/(a+b)^
(1/2)
```

Rubi [A]
time = 1.20, antiderivative size = 567, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3079, 3134, 3077, 2895, 3073}

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\,(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}+\frac{2b(-7a^3B+10a^2Ab+3ab^2B-6Ab^3)\sin(c+dx)}{3a^2d\,(a^2-b^2)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}-\frac{2(-(a^4(A-3B))-9a^3b(A-B)-2a^2b^2(8A+3B)+4ab^3(3A-2B)+16Ab^4)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^4d\sqrt{a+b}\,(a^2-b^2)}+\frac{2(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d\,(a^2-b^2)^2\cos^{\frac{3}{2}}(c+dx)}-\frac{2(-3a^5B+8a^4Ab+15a^3b^2B-28a^2Ab^3-8ab^4B+16Ab^5)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^5d(a-b)(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (-2*(8*a^4*A*b - 28*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4
*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5*(a - b)*(a + b)^(3/2)*d) - (2
*(16*A*b^4 - a^4*(A - 3*B) + 4*a*b^3*(3*A - 2*B) - 9*a^3*b*(A - B) - 2*a^2*
b^2*(8*A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2
 - b^2)*d) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]
^(3/2)*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B +
 3*a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a
+ b*Cos[c + d*x]]) + (2*(a^4*A - 13*a^2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*b
```

```
^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Cos[c +
 d*x]^(3/2))
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3}{2}(a^2A-2A\cdots}{\cdots}}{\cdots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2b(10a^2Ab\cdots}{3a^2\left(a^2-b^2\cdots\right.}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2b(10a^2Ab\cdots}{3a^2\left(a^2-b^2\cdots\right.}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2b(10a^2Ab\cdots}{3a^2\left(a^2-b^2\cdots\right.}$$

$$= -\frac{2(8a^4Ab-28a^2Ab^3+16Ab^5-3a^5B+15a^3b^2B-8ab^4B)\cot\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.96, size = 1499, normalized size = 2.64

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] ((-4*a*(a^6*A + 15*a^4*A*b^2 - 32*a^2*A*b^4 + 16*A*b^6 - 9*a^5*b*B + 17*a^3
*b^3*B - 8*a*b^5*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*a^5*A*b - 28*a^3*A
*b^3 + 16*a*A*b^5 - 3*a^6*B + 15*a^4*b^2*B - 8*a^2*b^4*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(8*a^4*A*b^2 - 28*a^2*A
*b^4 + 16*A*b^6 - 3*a^5*b*B + 15*a^3*b^3*B - 8*a*b^5*B)*((I*Cos[(c + d*x)/2
]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c
+ d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d
*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a^4*(a - b)^2*(a +
 b)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]*(-
8*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(3*a^4) - (2*(-(A*b^4*Sin[c + d*x
]) + a*b^3*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2
*(-12*a^2*A*b^4*Sin[c + d*x] + 8*A*b^6*Sin[c + d*x] + 9*a^3*b^3*B*Sin[c + d
*x] - 5*a*b^5*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) +
 (2*A*Sec[c + d*x]*Tan[c + d*x])/(3*a^3)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8092 vs. $2(529) = 1058$.
time = 0.53, size = 8093, normalized size = 14.27

method	result	size

default	Expression too large to display	8093

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/
2)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^6 + 3*a*b^2*cos(d*x + c)^5 + 3*a^2*b*cos(d*x + c)^4 + a^3*
cos(d*x + c)^3), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(5/2)), x
)
```

3.437 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=419

$$-\frac{(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+}{}}}{abd}$$

```
[Out] a*B*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos
(d*x+c)^(1/2)/d/(a+b*cos(d*x+c))^(1/2)-(a-b)*B*cot(d*x+c)*EllipticE((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1
/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+B*c
ot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/b/d+a*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b
)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A]
time = 0.51, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {21, 2899, 2888, 3082, 3072, 12, 2880, 2895, 3073}

$$\frac{aB\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}-\frac{B(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{aB\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (a*Sqrt[a + b]*B*Cot[
c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (a*B*Sin[c + d*x])/(b*
d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2880

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 2899

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> Dist[(-a)*(d/(2*b)), Int[Sqrt[d*Sin[e + f*x]]/Sqrt
[a + b*Sin[e + f*x]], x], x] + Dist[d/(2*b), Int[Sqrt[d*Sin[e + f*x]]*((a +
 2*b*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
```

```
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3082

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a +
 b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)
*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(
a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{B\int \frac{\sqrt{\cos(c+dx)}\,(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx}{2b} - \frac{(aB)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+d}}}{2b}$$

$$= \frac{a\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\right.}{b^2d}$$

$$= \frac{a\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\right.}{b^2d}$$

$$= \frac{a\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\right.}{b^2d}$$

$$= \frac{a\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\right.}{b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{abd}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.57, size = 480, normalized size = 1.15

$$\frac{B\sqrt{\cos(c+dx)}\left(2i(a-b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(i\sinh^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\frac{a+b}{a-b}\right)-4ia\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(i\sinh^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\frac{a+b}{a-b}\right)+4ia\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\Pi\left(\frac{a+b}{a-b};i\sinh^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\frac{a+b}{a-b}\right)+b\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{3}{2}(c+dx)\right)+2a\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\tan\left(\frac{1}{2}(c+dx)\right)-b\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{2b\sqrt{\frac{a-b}{a+b}}\,d\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^(3/2),x]
```

```
[Out] (B*Sqrt[Cos[c + d*x]]*((2*I)*(a - b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2
]], -((a + b)/(a - b))] - (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]],
 -((a + b)/(a - b))] + (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan
[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[c +
```

d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(2*b*Sqrt[(a - b)/(a + b)]*d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])

Maple [A]
time = 0.34, size = 623, normalized size = 1.49

method	result
default	$-\frac{B\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\mathrm{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos(dx+c)\sin(dx+c)a+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -B/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*b-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a+(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+cos(d*x+c)^3
*b+cos(d*x+c)^2*a-cos(d*x+c)^2*b-a*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/sin(d
*x+c)/cos(d*x+c)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int\frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] B*Integral(cos(c + d*x)**(3/2)/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\cos(c+d\,x)^{3/2}\,(B\,a+B\,b\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2
),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2
), x)
```

3.438 $\int \frac{\sqrt{\cos(c+dx)}\,(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+s}{}}}{bd}$$

```
[Out] -2*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A]

time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {21, 2888}

$$-\frac{2B\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (-2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx}}\right.\right.}{bd}$$

Mathematica [A]
time = 0.15, size = 131, normalized size = 1.12

$$-\frac{2B\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left(F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)-2\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\right)}{d\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^(3/2),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*EllipticPi
[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A]
time = 0.30, size = 160, normalized size = 1.37

method	result
default	$-\frac{2B\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)-2\,\text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\sqrt{-\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+}{(1+\cos(dx+c))(}}}{d\sqrt{a+b\cos\left(dx+c\right)}\,(-1+\cos(dx+c))\sqrt{\cos\left(dx+c\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -2*B/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-2*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)/(a+b*cos(d*x
+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^
(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int\frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] B*Integral(sqrt(cos(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^
(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos (c+d\,x)}\ (B\,a+B\,b\,\cos (c+d\,x))}{(a+b\,\cos (c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2
),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2
), x)
```

3.439 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=110

$$\frac{2\sqrt{a+b}\,B\cot(c+dx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

```
[Out] 2*B*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {21, 2895}

$$\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)
),x]
```

```
[Out] (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{a+b}\,B\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\right.}{ad}$$

Mathematica [A]
time = 0.98, size = 171, normalized size = 1.55

$$-\frac{4(a+b)B\cos^{\frac{3}{2}}(c+dx)\sqrt{-\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{a-b}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+dx)F\left(\text{ArcSin}\left(\sqrt{-\frac{a+b\cos(c+dx)}{a(-1+\cos(c+dx))}}\right)|\frac{2a}{a-b}\right)}{ad\sqrt{a+b\cos(c+dx)}\left(-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])
^(3/2)),x]
```

```
[Out] (-4*(a + b)*B*Cos[c + d*x]^(3/2)*Sqrt[-(((a + b)*Cot[(c + d*x)/2]^2)/(a - b
))]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
F[ArcSin[Sqrt[-((a + b*Cos[c + d*x])/(a*(-1 + Cos[c + d*x])))]], (2*a)/(a -
 b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]*(-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a))^(3/2))
```

Maple [A]
time = 0.28, size = 124, normalized size = 1.13

method	result	size
default	$-\frac{2B\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\left(\sin^4(dx+c)\right)}{d\sqrt{a+b\cos(dx+c)}\,\cos(dx+c)^{\frac{3}{2}}(-1+\cos(dx+c))^2}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -2*B/d*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(a+b*cos(d*x+c))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*sin(d*x+c)^4/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x
 + c))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))/(b*cos(d*x + c)^2 +
a*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int\frac{1}{\sqrt{a+b\cos{(c+dx)}}\sqrt{\cos{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] B*Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x
 + c))), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,a+B\,b\,\cos \left (c+d\,x\right)}{\sqrt {\cos \left (c+d\,x\right)}\,{\left (a+b\,\cos \left (c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)
),x)

[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)
), x)
```

3.440 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=226

$$\frac{2(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+}{}}}{a^2d}$$

```
[Out] 2*(a-b)*B*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-2*B*cot(d*x+c)*EllipticF((a+b*cos(d*x+c)
)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A]
time = 0.17, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {21, 2880, 2895, 3073}

$$\frac{2B(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2B\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)
),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sq
rt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2880

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
```

```
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= -\left(B\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx\right)+B\int \frac{}{\cos^{\frac{3}{2}}(c}$$
$$= \frac{2(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{a^2d}$$

Mathematica [A]
time = 2.21, size = 212, normalized size = 0.94

$$\frac{2B\left(-\left((a+b)\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\tfrac{1}{2}(c+dx)\right)\right)|\tfrac{-a+b}{a+b}\right)\right)+a\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\tfrac{1}{2}(c+dx)\right)\right)|\tfrac{-a+b}{a+b}\right)+(a+b\cos(c+dx))\tan\left(\tfrac{1}{2}(c+dx)\right)\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
^(3/2)),x]
```

[Out] (2*B*(-((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]) + a*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(210) = 420$.
time = 0.29, size = 613, normalized size = 2.71

method	result
default	$-\frac{2B\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos(dx+c)\sin(dx+c)a-\sqrt{\frac{\cos(dx+c}{1+\cos(dx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2*B/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+cos(d*x+c)^2*b+a*cos(d*x+c)-b*cos(d*x+c)-a)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)/a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))/(b*cos(d*x + c)^3 +
a*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int\frac{1}{\sqrt{a+b\cos{(c+dx)}}\,\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] B*Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)
^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)
), x)
```

3.441 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=72

$$-\frac{\cot(c+dx)E\left(\operatorname{ArcSin}\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(c+dx)}\sqrt{1-\sec(c+dx)}}{d}$$

```
[Out] -cot(d*x+c)*EllipticE(1/5*(2+3*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/2),5
^(1/2))*(-1-sec(d*x+c))^(1/2)*(1-sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {3073}

$$-\frac{\cot(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\,E\left(\operatorname{ArcSin}\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

```
[Out] -((Cot[c + d*x]*EllipticE[ArcSin[Sqrt[2 + 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos
[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/d)
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx = -\frac{\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(}}{d}$$

Mathematica [F]
time = 45.89, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[2 + 3*Cos[c + d*x]]),
x]
[Out] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[2 + 3*Cos[c + d*x]]),
 x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 657 vs. 2(64) = 128.
time = 2.44, size = 658, normalized size = 9.14

method	result
default	$-\frac{2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)\sqrt{2}\,\sin(dx+c)\left(\cos^{2}(dx+c)\right)+4\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x,method=_RETURN
VERBOSE)
[Out] -1/10/d/(2+3*cos(d*x+c))^(1/2)*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*10^(1/2
)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),1/5*5^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+4*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),1/5*5^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)+2*2^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*10^(1/2)*((2+3*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))-5*2^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))*sin(d*x+c)*cos
(d*x+c)^2+2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2
))*sin(d*x+c)*cos(d*x+c)^2-5*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(
1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),1/5*5^(1/2))*sin(d*x+c)*cos(d*x+c)+2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),1/5*5^(1/2))*sin(d*x+c)*cos(d*x+c)+30*cos(d*x+c)^3-10
*cos(d*x+c)^2-20*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2)),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(3*cos(d*x + c) + 2)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(3*
cos(d*x + c)^3 + 2*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)+1}{\sqrt{3\cos(c+dx)+2}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(3*cos(c + d*x) + 2)*cos(c + d*x)**(3/2)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)+1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{3\,\cos\left(c+d\,x\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3*cos(c + d*x) + 2)^(1/2)),x)
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3*cos(c + d*x) + 2)^(1/2)), x)
```

3.442 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{5}\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{d}$$

```
[Out] -cot(d*x+c)*EllipticE((-2+3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),1/5*5^(1/2))
*5^(1/2)*(-1+sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {3073}

$$-\frac{\sqrt{5}\cot(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\,E\left(\text{ArcSin}\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[5]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[-2 + 3*Cos[c + d*x]]/Sqrt[Cos
[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d)
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx = -\frac{\sqrt{5}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-}}{d}$$

Mathematica [F]
time = 49.13, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-2 + 3*Cos[c + d*x]])
,x]
[Out] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-2 + 3*Cos[c + d*x]])
, x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(61) = 122.
time = 1.80, size = 600, normalized size = 8.57

method	result
default	$-\frac{2\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)\left(\cos^2(dx+c)\right)\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}+4\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x,method=_RETUR
NVERBOSE)
[Out] -1/d/(-2+3*cos(d*x+c))^(1/2)*(2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2
))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((-2+3*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)+4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((-2+3*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)+2*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2
+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5
^(1/2))*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*((-2+3*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2
))+cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)+2*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)+cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)-3*cos(d*x+c)^3
+5*cos(d*x+c)^2-2*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2)),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(sqrt(3*cos(d*x + c) - 2)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(3*
cos(d*x + c)^3 - 2*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)+1}{\sqrt{3\cos(c+dx)-2}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(3*cos(c + d*x) - 2)*cos(c + d*x)**(3/2)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)+1}{{\cos(c+d\,x)}^{3/2}\,\sqrt{3\,\cos(c+d\,x)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3*cos(c + d*x) - 2)^(1/2)),x)
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3*cos(c + d*x) - 2)^(1/2)), x)
```

3.443 $\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{\sqrt{5}\,\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\operatorname{ArcSin}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}}{d}$$

```
[Out] csc(d*x+c)*EllipticE((2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),1/5*5^(1/2)
)*5^(1/2)*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+sec(d*x+c))^(1/2)*(1+sec
(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.14, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {3074, 3073}

$$\frac{\sqrt{5}\,\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\sec(c+dx)-1}\,\sqrt{\sec(c+dx)+1}\,E\left(\operatorname{ArcSin}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Sqrt[2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] (Sqrt[5]*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcS
in[Sqrt[2 - 3*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*
x]]*Sqrt[1 + Sec[c + d*x]])/d
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3074

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-Sqrt
[(-b)*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]], Int[(A + B*Sin[e + f*x])/(((-b)*S
in[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\sqrt{-\cos(c+dx)}\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}(-\cos(c+dx))^{3/2}}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\Big(\sin^{-1}\Big(\frac{\sqrt{2-}}{\sqrt{-}}}{d}$$

Mathematica [**F**]
time = 44.42, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Sqrt[2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),
x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Sqrt[2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),
 x]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 610 vs. 2(80) = 160.
time = 0.89, size = 611, normalized size = 6.57

method	result
default	$\frac{\sqrt{2-3\cos(dx+c)}\left(2\,\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)(\cos^2(dx+c))\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}+4\,\text{El}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2-3*cos(d*x+c))^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/d*(2-3*cos(d*x+c))^(1/2)*(2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))
*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((-2+3*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)+4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((-2+3*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)+2*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(
1/2))*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*((-2+3*cos(
```

```
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))
+cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)+2*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)+cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)-3*cos(d*x+c)^3+5
*cos(d*x+c)^2-2*cos(d*x+c))/(-2+3*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2-3*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2))
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2-3*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(cos(d*x + c) + 1)*sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(
3*cos(d*x + c)^3 - 2*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)+1}{\sqrt{2-3\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(2 - 3*cos(c + d*x))*cos(c + d*x)**(3/2)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2-3*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2))
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\,\sqrt{2-3\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2 - 3*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2 - 3*cos(c + d*x))^(1/2)), x)
```

3.444 $\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(c+dx)}}{d}$$

```
[Out] csc(d*x+c)*EllipticE(1/5*(-2-3*cos(d*x+c))^(1/2)*5^(1/2)/(-cos(d*x+c))^(1/2
),5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1-sec(d*x+c))^(1/2)*(1-se
c(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {3074, 3073}

$$\frac{\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}E\left(\text{ArcSin}\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Sqrt[-2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] (Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[
-2 - 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]])/d
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3074

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-Sqrt
[(-b)*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]], Int[(A + B*Sin[e + f*x])/(((-b)*S
in[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\sqrt{-\cos(c+dx)}\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}(-\cos(c+dx))^{3/2}}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{-2-}}{\sqrt{5}\sqrt{-}}\right.\right.}{d}$$

Mathematica [F]
time = 38.58, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Sqrt[-2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
,x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Sqrt[-2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
, x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 704 vs. 2(83) = 166.
time = 0.90, size = 705, normalized size = 7.42

method	result
default	$\frac{\sqrt{-2-3\cos(dx+c)}\left(2\sqrt{10}\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\text{EllipticF}\left(\frac{\sqrt{5}(-1+\cos(dx+c))}{5\sin(dx+c)},\sqrt{5}\right)\sqrt{5}\,(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2-3*cos(d*x+c))^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/10/d*(-2-3*cos(d*x+c))^(1/2)*(2*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF(1/5*5^(1/2)*(-1
+cos(d*x+c))/sin(d*x+c),5^(1/2))*5^(1/2)*cos(d*x+c)^2*sin(d*x+c)+4*10^(1/2)
*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*5^(1/2)*c
os(d*x+c)*sin(d*x+c)+2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*5^(1/2)*10
^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(1/5*5^(1/2)*(-1+co
```

```
s(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)-2*10^(1/2)*((2+3*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(1/5*5^
(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*5^(1/2)*cos(d*x+c)^2*sin(d*x+c)-1
0^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*EllipticE(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*5^
(1/2)*cos(d*x+c)^2*sin(d*x+c)-2*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(1/5*5^(1/2)*(-1+c
os(d*x+c))/sin(d*x+c),5^(1/2))*5^(1/2)*cos(d*x+c)*sin(d*x+c)-10^(1/2)*((2+3
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*EllipticE(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*5^(1/2)*cos(d*x
+c)*sin(d*x+c)+30*cos(d*x+c)^3-10*cos(d*x+c)^2-20*cos(d*x+c))/(2+3*cos(d*x+
c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2-3*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2))
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2-3*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(-(cos(d*x + c) + 1)*sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(
3*cos(d*x + c)^3 + 2*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)+1}{\sqrt{-3\cos(c+dx)-2}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(-3*cos(c + d*x) - 2)*cos(c + d*x)**(3/2))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2-3*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2))
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)+1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{-3\,\cos\left(c+d\,x\right)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(- 3*cos(c + d*x) - 2)^(1/2)),x)
```

```
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(- 3*cos(c + d*x) - 2)^(1/2)), x
)
```

3.445 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{2\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{3d}$$

```
[Out] 2/3*cot(d*x+c)*EllipticE(1/5*(3+2*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/2
),I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {3073}

$$\frac{2\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,E\left(\text{ArcSin}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos
[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx = \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(}}{3d}$$

Mathematica [F]
time = 48.75, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[3 + 2*Cos[c + d*x]]),
x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[3 + 2*Cos[c + d*x]]),
 x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 664 vs. 2(65) = 130.
time = 2.00, size = 665, normalized size = 9.24

method	result
default	$-\frac{3\sqrt{2}\,\sin(dx+c)(\cos^2(dx+c))\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)+6\sqrt{2}\,\sin(dx+c)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/15/d/(3+2*cos(d*x+c))^(1/2)*(3*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+
c)/(1+cos(d*x+c)))^(3/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))+6*2^(1/2)*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))-5*2^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))*sin(d*x+c)*c
os(d*x+c)^2+3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^
(1/2))*sin(d*x+c)*cos(d*x+c)^2+3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*
10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),1/5*I*5^(1/2))*sin(d*x+c)-5*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),1/5*I*5^(1/2))*sin(d*x+c)*cos(d*x+c)+3*2^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))*sin(d*x+c)*cos(d*x+c)+20*c
os(d*x+c)^3+10*cos(d*x+c)^2-30*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2)),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(2*cos(d*x + c) + 3)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(2*
cos(d*x + c)^3 + 3*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)+1}{\sqrt{2\cos(c+dx)+3}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(2*cos(c + d*x) + 3)*cos(c + d*x)**(3/2)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)+1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{2\,\cos\left(c+d\,x\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2*cos(c + d*x) + 3)^(1/2)),x)
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2*cos(c + d*x) + 3)^(1/2)), x)
```

3.446 $\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{2\sqrt{5}\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{3d}$$

```
[Out] 2/3*cot(d*x+c)*EllipticE((3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),1/5*I*5^(1
/2))*5^(1/2)*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {3073}

$$\frac{2\sqrt{5}\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,E\left(\text{ArcSin}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Sqrt[3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] (2*Sqrt[5]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[
c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{5}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-}}{3d}$$

Mathematica [F]
time = 48.61, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Sqrt[3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),
x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Sqrt[3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),
 x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(64) = 128.
time = 2.43, size = 663, normalized size = 8.96

method	result
default	$\frac{\sqrt{3-2\cos(dx+c)}\left(3\sqrt{2}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\,\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, i\sqrt{5}\right)(\cos^2(dx+c))\sin\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3-2*cos(d*x+c))^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/3/d*(3-2*cos(d*x+c))^(1/2)*(3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(
-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),I*5^(1/2))*cos(d*x+c)^2*sin(d*x+c)+6*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),I*5^(1/2))*cos(d*x+c)*sin(d*x+c)+3*2^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),I*5^(1/2))*sin(d*x+c)-cos(d*x+c)^2*sin(d*x+c)*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))+3*cos(d*x+c)^2*sin(d*x
+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))-cos(d*x+c)*s
in(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))+3*cos(
d*x+c)*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2)
)-4*cos(d*x+c)^3+10*cos(d*x+c)^2-6*cos(d*x+c))/(-3+2*cos(d*x+c))/cos(d*x+c)
^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3-2*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2))
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3-2*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(cos(d*x + c) + 1)*sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(
2*cos(d*x + c)^3 - 3*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)+1}{\sqrt{3-2\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(3 - 2*cos(c + d*x))*cos(c + d*x)**(3/2)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3-2*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2))
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\,\sqrt{3-2\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3 - 2*cos(c + d*x))^(1/2)),x)
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3 - 2*cos(c + d*x))^(1/2)), x)
```

3.447 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\sqrt{5}\,\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-\sec(c+}{3d}$$

```
[Out] -2/3*csc(d*x+c)*EllipticE((-3+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),1/5*I
*5^(1/2))*5^(1/2)*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(1-sec(d*x+c))^(1/2)
*(1+sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {3074, 3073}

$$-\frac{2\sqrt{5}\,\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,E\left(\text{ArcSin}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[5]*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3074

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-Sqrt
[(-b)*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]], Int[(A + B*Sin[e + f*x])/(((-b)*S
in[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx = -\frac{\sqrt{-\cos(c+dx)}\int \frac{1+\cos(c+dx)}{(-\cos(c+dx))^{3/2}\sqrt{-3+2\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{2\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\Big(\sin^{-1}\Big(\frac{\sqrt{}}{}$$

Mathematica [F]
time = 51.97, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]])
,x]
[Out] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]])
, x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 713 vs. 2(84) = 168.
time = 0.66, size = 714, normalized size = 7.29

method	result
default	$\frac{3i\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\sqrt{2}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)(\cos^2(dx+c))\sin(dx+c)\sqrt{5}+6i\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x,method=_RETUR
NVERBOSE)
[Out] 1/15/d/(-3+2*cos(d*x+c))^(1/2)*(3*I*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF(I*(-1+cos(d*x+c))*
5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))*cos(d*x+c)^2*sin(d*x+c)*5^(1/2)+6*I*(-2*(
-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)*EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))*cos(d*x
+c)*sin(d*x+c)*5^(1/2)+3*I*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*
x+c)*5^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+co
```

```
s(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))-3*I*cos(d*x+c)^2*sin(d*x+c)*(-2
*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))*5^(1/
2)+5*I*cos(d*x+c)^2*sin(d*x+c)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))*5^(1/
2)/sin(d*x+c),1/5*I*5^(1/2))*5^(1/2)-3*I*cos(d*x+c)*sin(d*x+c)*(-2*(-3+2*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*E
llipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))*5^(1/2)+5*I*co
s(d*x+c)*sin(d*x+c)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+
c),1/5*I*5^(1/2))*5^(1/2)+20*cos(d*x+c)^3-50*cos(d*x+c)^2+30*cos(d*x+c))/co
s(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2)),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(sqrt(2*cos(d*x + c) - 3)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(2*
cos(d*x + c)^3 - 3*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c + dx) + 1}{\sqrt{2\cos(c + dx) - 3}\,\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(2*cos(c + d*x) - 3)*cos(c + d*x)**(3/2)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)+1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{2\,\cos\left(c+d\,x\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2*cos(c + d*x) - 3)^(1/2)),x)
```

```
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2*cos(c + d*x) - 3)^(1/2)), x)
```

3.448 $\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(c+d$$

$$3d$$

```
[Out] -2/3*csc(d*x+c)*EllipticE(1/5*(-3-2*cos(d*x+c))^(1/2)*5^(1/2)/(-cos(d*x+c))
^(1/2),I*5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(1-sec(d*x+c))^(1/2)
*(1+sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {3074, 3073}

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,E\left(\text{ArcSin}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Sqrt[-3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sq
rt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3074

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-Sqrt
[(-b)*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]], Int[(A + B*Sin[e + f*x])/(((-b)*S
in[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\sqrt{-\cos(c+dx)}\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\,(-\cos(c+dx))^{3/2}}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{-3}}{\sqrt{5}}\right.\right.}{}$$

Mathematica [F]
time = 38.61, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Sqrt[-3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
,x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Sqrt[-3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
, x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 739 vs. 2(85) = 170.
time = 0.56, size = 740, normalized size = 7.71

method	result
default	$-\frac{\sqrt{-3-2\cos(dx+c)}\left(3i\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{5\sin(dx+c)}, i\sqrt{5}\right)(\cos^2(dx+c))\sqrt{10}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}\left(\frac{\cos}{1+\cos}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3-2*cos(d*x+c))^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/15/d*(-3-2*cos(d*x+c))^(1/2)*(3*I*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2
)/sin(d*x+c),I*5^(1/2))*cos(d*x+c)^2*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*5^(1/2)+6*I
*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*cos(d*x+c)*1
0^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(3/2)*sin(d*x+c)*5^(1/2)-3*I*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(1/5*I*(-1+c
```

```
os(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*cos(d*x+c)^2*sin(d*x+c)*5^(1/2)+3*
I*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*5^(1/2)*10^(1/2)*((3
+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2
)/sin(d*x+c),I*5^(1/2))+I*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(1/5*I*(-1+cos(d*x+c))*5
^(1/2)/sin(d*x+c),I*5^(1/2))*cos(d*x+c)^2*sin(d*x+c)*5^(1/2)-3*I*10^(1/2)*(
(3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*cos(d*x
+c)*sin(d*x+c)*5^(1/2)+I*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(1/5*I*(-1+cos(d*x+c))*5^
(1/2)/sin(d*x+c),I*5^(1/2))*cos(d*x+c)*sin(d*x+c)*5^(1/2)-20*cos(d*x+c)^3-1
0*cos(d*x+c)^2+30*cos(d*x+c))/(3+2*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3-2*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2))
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3-2*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(-(cos(d*x + c) + 1)*sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(
2*cos(d*x + c)^3 + 3*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)+1}{\sqrt{-2\cos(c+dx)-3}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(-2*cos(c + d*x) - 3)*cos(c + d*x)**(3/2))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3-2*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2))
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)+1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{-2\,\cos\left(c+d\,x\right)-3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(- 2*cos(c + d*x) - 3)^(1/2)),x)
```

```
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(- 2*cos(c + d*x) - 3)^(1/2)), x
)
```

3.449 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=36

$$\text{Int}((c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx)),x)$$

```
[Out] Unintegrable((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]),x]
[Out] Defer[Int][(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]), x]
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx = \int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A$$

Mathematica [A]
time = 7.25, size = 0, normalized size = 0.00

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]),x]
[Out] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]), x
]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(a+b\cos\left(fx+e\right)\right)^{n}\left(A+B\cos\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*cos(f*x + e))^m, x
)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*cos(f*x + e))^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c\cos(e+fx))^m (A+B\cos(e+fx)) (a+b\cos(e+fx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**n*(A+B*cos(f*x+e)),x)
```

```
[Out] Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))**n,
x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*cos(f*x + e))^m, x
)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int (c\,\cos\left(e+f\,x\right))^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^n,x)
```

```
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^n, x)
```

3.450 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^4(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=595

$$\frac{b\left(Ab^3(15+8m+m^2)+4ab^2B(15+8m+m^2)+2a^3B(28+10m+m^2)+a^2Ab(110+47m+5m^2)\right)(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)(4+m)(5+m)}$$

```
[Out] b*(A*b^3*(m^2+8*m+15)+4*a*b^2*B*(m^2+8*m+15)+2*a^3*B*(m^2+10*m+28)+a^2*A*b*
(5*m^2+47*m+110))*(c*cos(f*x+e))^(1+m)*sin(f*x+e)/c/f/(5+m)/(m^2+6*m+8)+b^2
*(b^2*B*(4+m)^2+2*a*A*b*(5+m)^2+a^2*B*(m^2+11*m+36))*cos(f*x+e)*(c*cos(f*x+
e))^(1+m)*sin(f*x+e)/c/f/(3+m)/(4+m)/(5+m)+b*(A*b*(5+m)+a*B*(8+m))*(c*cos(f
*x+e))^(1+m)*(a+b*cos(f*x+e))^2*sin(f*x+e)/c/f/(4+m)/(5+m)+b*B*(c*cos(f*x+e
))^(1+m)*(a+b*cos(f*x+e))^3*sin(f*x+e)/c/f/(5+m)-(A*b^4*(m^2+4*m+3)+4*a*b^3
*B*(m^2+4*m+3)+6*a^2*A*b^2*(m^2+5*m+4)+4*a^3*b*B*(m^2+5*m+4)+a^4*A*(m^2+6*m
+8))*(c*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)
^2)*sin(f*x+e)/c/f/(4+m)/(m^2+3*m+2)/(sin(f*x+e)^2)^(1/2)-(b^4*B*(m^2+6*m+8
)+4*a*A*b^3*(m^2+7*m+10)+6*a^2*b^2*B*(m^2+7*m+10)+4*a^3*A*b*(m^2+8*m+15)+a^
4*B*(m^2+8*m+15))*(c*cos(f*x+e))^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c
os(f*x+e)^2)*sin(f*x+e)/c^2/f/(2+m)/(3+m)/(5+m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 1.31, antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3069, 3128, 3112, 3102, 2827, 2722}

$$\frac{b^2\sin(e+fx)\cos(e+fx)\left(a^2B(m^2+11m+36)+2aAb(m+5)^2+b^2B(m+4)^2\right)(c\cos(e+fx))^{m+1}}{cf(m+3)(m+4)(m+5)}+\frac{b\sin(e+fx)\left(2a^3B(m^2+10m+28)+a^2Ab(5m^2+47m+110)+4ab^2B(m^2+8m+15)+Ab^3(m^2+8m+15)\right)(c\cos(e+fx))^{m+1}}{cf(m+2)(m+4)(m+5)}-\frac{\sin(e+fx)\left(a^4B(m^2+8m+15)+4a^3Ab(m^2+8m+15)+6a^2b^2B(m^2+7m+10)+4aAb^3(m^2+7m+10)+b^4B(m^2+6m+8)\right)(c\cos(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(e+fx)\right)}{c^2f(m+2)(m+3)(m+5)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+fx)\left(a^4A(m^2+6m+8)+4a^3bB(m^2+5m+4)+6a^2Ab^2(m^2+5m+4)+4ab^3B(m^2+4m+3)+Ab^4(m^2+4m+3)\right)(c\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{cf(m+1)(m+2)(m+4)\sqrt{\sin^2(e+fx)}}+\frac{b\sin(e+fx)(aB(m+8)+Ab(m+5))(a+b\cos(e+fx))^2(c\cos(e+fx))^{m+1}}{cf(m+4)(m+5)}+\frac{bB\sin(e+fx)(a+b\cos(e+fx))^3(c\cos(e+fx))^{m+1}}{cf(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x]),x]
```

```
[Out] (b*(A*b^3*(15 + 8*m + m^2) + 4*a*b^2*B*(15 + 8*m + m^2) + 2*a^3*B*(28 + 10*
m + m^2) + a^2*A*b*(110 + 47*m + 5*m^2))*(c*Cos[e + f*x])^(1 + m)*Sin[e + f
*x])/(c*f*(2 + m)*(4 + m)*(5 + m)) + (b^2*(b^2*B*(4 + m)^2 + 2*a*A*b*(5 + m
)^2 + a^2*B*(36 + 11*m + m^2))*Cos[e + f*x]*(c*Cos[e + f*x])^(1 + m)*Sin[e
 + f*x])/(c*f*(3 + m)*(4 + m)*(5 + m)) + (b*(A*b*(5 + m) + a*B*(8 + m))*(c*C
os[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^2*Sin[e + f*x])/(c*f*(4 + m)*(5 +
 m)) + (b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^3*Sin[e + f*x])/(
c*f*(5 + m)) - ((A*b^4*(3 + 4*m + m^2) + 4*a*b^3*B*(3 + 4*m + m^2) + 6*a^2*
A*b^2*(4 + 5*m + m^2) + 4*a^3*b*B*(4 + 5*m + m^2) + a^4*A*(8 + 6*m + m^2))*
(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e
 + f*x]^2]*Sin[e + f*x])/(c*f*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[e + f*x]^2])
 - ((b^4*B*(8 + 6*m + m^2) + 4*a*A*b^3*(10 + 7*m + m^2) + 6*a^2*b^2*B*(10 +
 7*m + m^2) + 4*a^3*A*b*(15 + 8*m + m^2) + a^4*B*(15 + 8*m + m^2))*(c*Cos[e
```

```
 + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^
2]*Sin[e + f*x])/(c^2*f*(2 + m)*(3 + m)*(5 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
```

```
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^4(A+B\cos(e+fx))\,dx = \frac{bB(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))^3 s}{cf(5+m)}$$
$$= \frac{b(Ab(5+m)+aB(8+m))(c\cos(e+fx))}{cf(4+m)(5}$$
$$= \frac{b^2(b^2B(4+m)^2+2aAb(5+m)^2+a^2B(36}{cf(3}$$
$$= \frac{b(Ab^3(15+8m+m^2)+4ab^2B(15+8m+}{}$$
$$= \frac{b(Ab^3(15+8m+m^2)+4ab^2B(15+8m+}{}$$
$$= \frac{b(Ab^3(15+8m+m^2)+4ab^2B(15+8m+}{}$$

Mathematica [A]
time = 6.22, size = 487, normalized size = 0.82

$$-\frac{a^4A\cos(e+fx)(c\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin(e+fx)}{f(1+m)\sqrt{\sin^2(e+fx)}}-\frac{a^3(4Ab+aB)\cos^2(e+fx)(c\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\cos^2(e+fx)\right)\sin(e+fx)}{f(2+m)\sqrt{\sin^2(e+fx)}}-\frac{2a^2b(3Ab+2aB)\cos^3(e+fx)(c\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};\cos^2(e+fx)\right)\sin(e+fx)}{f(3+m)\sqrt{\sin^2(e+fx)}}-\frac{2ab^2(2Ab+3aB)\cos^4(e+fx)(c\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{4+m}{2};\frac{6+m}{2};\cos^2(e+fx)\right)\sin(e+fx)}{f(4+m)\sqrt{\sin^2(e+fx)}}-\frac{b^3(Ab+4aB)\cos^5(e+fx)(c\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{5+m}{2};\frac{7+m}{2};\cos^2(e+fx)\right)\sin(e+fx)}{f(5+m)\sqrt{\sin^2(e+fx)}}-\frac{b^4B\cos^6(e+fx)(c\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{6+m}{2};\frac{8+m}{2};\cos^2(e+fx)\right)\sin(e+fx)}{f(6+m)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x]),x]
[Out] -((a^4*A*Cos[e + f*x]*(c*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 + m)*Sqrt[Sin[e + f*x]^2]))
```

```
- (a^3*(4*A*b + a*B)*Cos[e + f*x]^2*(c*Cos[e + f*x])^m*Hypergeometric2F1[1/
2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(2 + m)*Sqrt[Sin[
e + f*x]^2]) - (2*a^2*b*(3*A*b + 2*a*B)*Cos[e + f*x]^3*(c*Cos[e + f*x])^m*H
ypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(
f*(3 + m)*Sqrt[Sin[e + f*x]^2]) - (2*a*b^2*(2*A*b + 3*a*B)*Cos[e + f*x]^4*(
c*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[e + f*x]
^2]*Sin[e + f*x])/(f*(4 + m)*Sqrt[Sin[e + f*x]^2]) - (b^3*(A*b + 4*a*B)*Cos
[e + f*x]^5*(c*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2,
 Cos[e + f*x]^2]*Sin[e + f*x])/(f*(5 + m)*Sqrt[Sin[e + f*x]^2]) - (b^4*B*Co
s[e + f*x]^6*(c*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (6 + m)/2, (8 + m)/2
, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(6 + m)*Sqrt[Sin[e + f*x]^2])
```

Maple [**F**]
time = 0.58, size = 0, normalized size = 0.00

$$\int \left(c \cos \left(f x +e \right)\right)^{m} \left(a +b \cos \left(f x +e \right)\right)^{4} \left(A +B \cos \left(f x +e \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*cos(f*x + e))^m, x
)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^4*cos(f*x + e)^5 + A*a^4 + (4*B*a*b^3 + A*b^4)*cos(f*x + e)^4
 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*cos(f*x + e)^3 + 2*(2*B*a^3*b + 3*A*a^2*b^2)
*cos(f*x + e)^2 + (B*a^4 + 4*A*a^3*b)*cos(f*x + e))*(c*cos(f*x + e))^m, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**4*(A+B*cos(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*cos(f*x + e))^m, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,\cos(e+f\,x))^m\,(A+B\,\cos(e+f\,x))\,\left(a+b\,\cos(e+f\,x)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^4,x)
```

```
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^4, x)
```

3.451 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=406

$$\frac{b(b^2B(3+m)+3aAb(4+m)+2a^2B(5+m))\,(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)(4+m)}+\frac{b^2(Ab(4+m)+aB(6+m}{c}$$

```
[Out] b*(b^2*B*(3+m)+3*a*A*b*(4+m)+2*a^2*B*(5+m))*(c*cos(f*x+e))^(1+m)*sin(f*x+e)
/c/f/(2+m)/(4+m)+b^2*(A*b*(4+m)+a*B*(6+m))*cos(f*x+e)*(c*cos(f*x+e))^(1+m)*
sin(f*x+e)/c/f/(3+m)/(4+m)+b*B*(c*cos(f*x+e))^(1+m)*(a+b*cos(f*x+e))^2*sin(
f*x+e)/c/f/(4+m)-(a^2*(2+m)*(b*B*(1+m)+a*A*(4+m))+b*(1+m)*(b^2*B*(3+m)+3*a*
A*b*(4+m)+2*a^2*B*(5+m)))*(c*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[
3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c/f/(1+m)/(2+m)/(4+m)/(sin(f*x+e)^2)^(1
/2)-(A*b^3*(2+m)+3*a*b^2*B*(2+m)+3*a^2*A*b*(3+m)+a^3*B*(3+m))*(c*cos(f*x+e)
)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c^2/f/(
2+m)/(3+m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3069, 3112, 3102, 2827, 2722}

$$-\frac{\sin(e+fx)\,(b(m+1)\,(2a^2B(m+5)+3aAb(m+4)+b^2B(m+3))+a^2(m+2)(aA(m+4)+bB(m+1)))\,(c\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{cf(m+1)(m+2)(m+4)\sqrt{\sin^2(e+fx)}}+\frac{b\sin(e+fx)\,(2a^2B(m+5)+3aAb(m+4)+b^2B(m+3))\,(c\cos(e+fx))^{m+1}}{cf(m+2)(m+4)}-\frac{\sin(e+fx)\,(a^3B(m+3)+3a^2Ab(m+3)+3ab^2B(m+2)+Ab^3(m+2))\,(c\cos(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(e+fx)\right)}{c^2f(m+2)(m+3)\sqrt{\sin^2(e+fx)}}+\frac{b^2\sin(e+fx)\cos(e+fx)(aB(m+6)+Ab(m+4))(c\cos(e+fx))^{m+1}}{cf(m+3)(m+4)}+\frac{bB\sin(e+fx)(a+b\cos(e+fx))^2(c\cos(e+fx))^{m+1}}{cf(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x]),x]
```

```
[Out] (b*(b^2*B*(3 + m) + 3*a*A*b*(4 + m) + 2*a^2*B*(5 + m))*(c*Cos[e + f*x])^(1
+ m)*Sin[e + f*x])/(c*f*(2 + m)*(4 + m)) + (b^2*(A*b*(4 + m) + a*B*(6 + m))
*Cos[e + f*x]*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(c*f*(3 + m)*(4 + m))
+ (b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^2*Sin[e + f*x])/(c*f*(
4 + m)) - ((a^2*(2 + m)*(b*B*(1 + m) + a*A*(4 + m)) + b*(1 + m)*(b^2*B*(3 +
 m) + 3*a*A*b*(4 + m) + 2*a^2*B*(5 + m)))*(c*Cos[e + f*x])^(1 + m)*Hypergeo
metric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c*f*(1
+ m)*(2 + m)*(4 + m)*Sqrt[Sin[e + f*x]^2]) - ((A*b^3*(2 + m) + 3*a*b^2*B*(2
 + m) + 3*a^2*A*b*(3 + m) + a^3*B*(3 + m))*(c*Cos[e + f*x])^(2 + m)*Hyperge
ometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c^2*f*
(2 + m)*(3 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx = \frac{bB(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))^2}{cf(4+m)}$$
$$= \frac{b^2(Ab(4+m)+aB(6+m))\cos(e+fx)(}{cf(3+m)(4+}$$
$$= \frac{b(b^2B(3+m)+3aAb(4+m)+2a^2B(5}{cf(2+m)(}$$
$$= \frac{b(b^2B(3+m)+3aAb(4+m)+2a^2B(5}{cf(2+m)(}$$
$$= \frac{b(b^2B(3+m)+3aAb(4+m)+2a^2B(5}{cf(2+m)(}$$

Mathematica [A]
time = 2.87, size = 269, normalized size = 0.66

$$\frac{\cos(e+fx)(c\cos(e+fx))^m\left(-\frac{a^3A\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)}{1+m}+\cos(e+fx)\left(-\frac{a^2(3Ab+aB)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\cos^2(e+fx)\right)}{2+m}+b\cos(e+fx)\left(-\frac{3a(Ab+aB)\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};\cos^2(e+fx)\right)}{3+m}+b\cos(e+fx)\left(-\frac{(Ab+3aB)\,{}_2F_1\left(\frac{1}{2},\frac{4+m}{2};\frac{6+m}{2};\cos^2(e+fx)\right)}{4+m}-\frac{bB\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{5+m}{2};\frac{7+m}{2};\cos^2(e+fx)\right)}{5+m}\right)\right)\right)\right)\sin(e+fx)}{f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x]),x]
[Out] (Cos[e + f*x]*(c*Cos[e + f*x])^m*(-((a^3*A*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Cos[e + f*x]^2])/(1 + m)) + Cos[e + f*x]*(-((a^2*(3*A*b + a*B)
*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2])/(2 + m)) + b
*Cos[e + f*x]*((-3*a*(A*b + a*B)*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/
2, Cos[e + f*x]^2])/(3 + m) + b*Cos[e + f*x]*(-(((A*b + 3*a*B)*Hypergeometr
ic2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[e + f*x]^2])/(4 + m)) - (b*B*Cos[e + f
*x]*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, Cos[e + f*x]^2])/(5 + m)))
))*Sin[e + f*x])/(f*Sqrt[Sin[e + f*x]^2])
```

Maple [F]
time = 0.49, size = 0, normalized size = 0.00

$$\int \left(c\cos\left(fx+e\right)\right)^m\left(a+b\cos\left(fx+e\right)\right)^3\left(A+B\cos\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x)
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*cos(f*x + e))^m, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(f*x + e)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(f*x + e)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(f*x + e)^2 + (B*a^3 + 3*A*a^2*b)*cos(f*x + e))
*(c*cos(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**3*(A+B*cos(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*cos(f*x + e))^m, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,\cos(e+f\,x))^m\,(A+B\,\cos(e+f\,x))\,(a+b\,\cos(e+f\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^3,x)
```

```
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^3, x)
```

3.452 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=287

$$\frac{b(Ab(3+m)+aB(4+m))(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)(3+m)}+\frac{bB(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))\sin(e$$

```
[Out] b*(A*b*(3+m)+a*B*(4+m))*(c*cos(f*x+e))^(1+m)*sin(f*x+e)/c/f/(2+m)/(3+m)+b*B
*(c*cos(f*x+e))^(1+m)*(a+b*cos(f*x+e))*sin(f*x+e)/c/f/(3+m)-(A*b^2*(1+m)+2*
a*b*B*(1+m)+a^2*A*(2+m))*(c*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3
/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c/f/(1+m)/(2+m)/(sin(f*x+e)^2)^(1/2)-(b^
2*B*(2+m)+a*(2*A*b+B*a)*(3+m))*(c*cos(f*x+e))^(2+m)*hypergeom([1/2, 1+1/2*m
],[2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c^2/f/(2+m)/(3+m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3069, 3102, 2827, 2722}

$$-\frac{\sin(e+fx)\left(a^2A(m+2)+2abB(m+1)+Ab^2(m+1)\right)(c\cos(e+fx))^{m+1}{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{cf(m+1)(m+2)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+fx)\left(a(m+3)(aB+2Ab)+b^2B(m+2)\right)(c\cos(e+fx))^{m+2}{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(e+fx)\right)}{c^2f(m+2)(m+3)\sqrt{\sin^2(e+fx)}}+\frac{b\sin(e+fx)(aB(m+4)+Ab(m+3))(c\cos(e+fx))^{m+1}}{cf(m+2)(m+3)}+\frac{bB\sin(e+fx)(a+b\cos(e+fx))(c\cos(e+fx))^{m+1}}{cf(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x]),x]
```

```
[Out] (b*(A*b*(3 + m) + a*B*(4 + m))*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(c*f*
(2 + m)*(3 + m)) + (b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])*Sin[e
 + f*x])/(c*f*(3 + m)) - ((A*b^2*(1 + m) + 2*a*b*B*(1 + m) + a^2*A*(2 + m))
*(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[
e + f*x]^2]*Sin[e + f*x])/(c*f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*x]^2]) - ((b^
2*B*(2 + m) + a*(2*A*b + a*B)*(3 + m))*(c*Cos[e + f*x])^(2 + m)*Hypergeomet
ric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c^2*f*(2 +
 m)*(3 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx = \frac{bB(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))\,\mathrm{si}}{cf(3+m)}$$
$$= \frac{b(Ab(3+m)+aB(4+m))(c\cos(e+fx))}{cf(2+m)(3+m)}$$
$$= \frac{b(Ab(3+m)+aB(4+m))(c\cos(e+fx))}{cf(2+m)(3+m)}$$
$$= \frac{b(Ab(3+m)+aB(4+m))(c\cos(e+fx))}{cf(2+m)(3+m)}$$

Mathematica [A]
time = 1.77, size = 217, normalized size = 0.76

$$\frac{\cos(e+fx)(c\cos(e+fx))^m\left(-\frac{a^2A\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)}{1+m}+\cos(e+fx)\left(-\frac{a(2Ab+aB)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\cos^2(e+fx)\right)}{2+m}+b\cos(e+fx)\left(-\frac{(Ab+2aB)\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};\cos^2(e+fx)\right)}{3+m}-\frac{bB\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{4+m}{2};\frac{6+m}{2};\cos^2(e+fx)\right)}{4+m}\right)\right)\right)\sin(e+fx)}{f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*(c*Cos[e + f*x])^m*(-((a^2*A*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Cos[e + f*x]^2])/(1 + m)) + Cos[e + f*x]*(-((a*(2*A*b + a*B)*H
ypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2])/(2 + m)) + b*C
os[e + f*x]*(-(((A*b + 2*a*B)*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2,
Cos[e + f*x]^2])/(3 + m)) - (b*B*Cos[e + f*x]*Hypergeometric2F1[1/2, (4 + m
)/2, (6 + m)/2, Cos[e + f*x]^2])/(4 + m))))*Sin[e + f*x])/(f*Sqrt[Sin[e + f
*x]^2])
```

Maple [F]
time = 0.67, size = 0, normalized size = 0.00

$$\int (c\cos(fx+e))^m (a+b\cos(fx+e))^2 (A+B\cos(fx+e))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*cos(f*x + e))^m, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(f*x + e)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(f*x + e)^2 +
 (B*a^2 + 2*A*a*b)*cos(f*x + e))*(c*cos(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**2*(A+B*cos(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*cos(f*x + e))^m, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,\cos\left(e+f\,x\right))^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^2,x)
```

```
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^2, x)
```

3.453 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=196

$$\frac{bB(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)}-\frac{(bB(1+m)+aA(2+m))(c\cos(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+\right.}{cf(1+m)(2+m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] b*B*(c*cos(f*x+e))^(1+m)*sin(f*x+e)/c/f/(2+m)-(b*B*(1+m)+a*A*(2+m))*(c*cos(
f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+
e)/c/f/(1+m)/(2+m)/(sin(f*x+e)^2)^(1/2)-(A*b+B*a)*(c*cos(f*x+e))^(2+m)*hype
rgeom([1/2, 1+1/2*m],[2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c^2/f/(2+m)/(sin(f*
x+e)^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3047, 3102, 2827, 2722}

$$-\frac{(aB+Ab)\sin(e+fx)(c\cos(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(e+fx)\right)}{c^2f(m+2)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+fx)(aA(m+2)+bB(m+1))(c\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{cf(m+1)(m+2)\sqrt{\sin^2(e+fx)}}+\frac{bB\sin(e+fx)(c\cos(e+fx))^{m+1}}{cf(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x]),x]
```

```
[Out] (b*B*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(c*f*(2 + m)) - ((b*B*(1 + m) +
 a*A*(2 + m))*(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3
 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c*f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*
x]^2]) - ((A*b + a*B)*(c*Cos[e + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 +
m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c^2*f*(2 + m)*Sqrt[Sin[e +
f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx = \int (c\cos(e+fx))^m\left(aA+(Ab+aB)\cos(e\right.$$

$$= \frac{bB(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)} + \frac{\int (c\cos}{}$$

$$= \frac{bB(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)} + \frac{(Ab+}{}$$

$$= \frac{bB(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)} - \frac{\left(aA+\right.}{}$$

Mathematica [A]
time = 0.35, size = 151, normalized size = 0.77

$$-\frac{\cos(e+fx)(c\cos(e+fx))^m\sin(e+fx)\left((bB(1+m)+aA(2+m))\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)+(1+m)\left((Ab+aB)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\cos^2(e+fx)\right)-bB\sqrt{\sin^2(e+fx)}\right)\right)}{f(1+m)(2+m)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(c*Cos[e + f*x])^m*Sin[e + f*x]*((b*B*(1 + m) + a*A*(2 + m)
)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2] + (1 + m)*((
A*b + a*B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e
+ f*x]^2] - b*B*Sqrt[Sin[e + f*x]^2])))/(f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*x
]^2]))
```

Maple [F]
time = 0.59, size = 0, normalized size = 0.00

$$\int (c\cos(fx+e))^m (a+b\cos(fx+e)) (A+B\cos(fx+e)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x)
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x, algorithm="
maxima")
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x, algorithm="
fricas")
[Out] integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*(c*cos(f*x +
 e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c\cos(e+fx))^m (A+B\cos(e+fx)) (a+b\cos(e+fx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x)
[Out] Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c \cos(e + f x))^m (A + B \cos(e + f x)) (a + b \cos(e + f x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)),x)
```

```
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)), x)
```

3.454 $\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx$

Optimal. Leaf size=286

$$\frac{a(Ab-aB)cF_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)(c\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}\sin(e+fx)}{b\left(a^2-b^2\right)f}-$$

```
[Out] a*(A*b-B*a)*c*AppellF1(1/2,1/2-1/2*m,1,3/2,sin(f*x+e)^2,-b^2*sin(f*x+e)^2/(
a^2-b^2))*(c*cos(f*x+e))^(-1+m)*(cos(f*x+e)^2)^(1/2-1/2*m)*sin(f*x+e)/b/(a^
2-b^2)/f-(A*b-B*a)*AppellF1(1/2,-1/2*m,1,3/2,sin(f*x+e)^2,-b^2*sin(f*x+e)^2
/(a^2-b^2))*(c*cos(f*x+e))^m*sin(f*x+e)/(a^2-b^2)/f/((cos(f*x+e)^2)^(1/2*m)
)-B*(c*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^
2)*sin(f*x+e)/b/c/f/(1+m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3081, 2722, 2902, 3268, 440}

$$\frac{ac(Ab-aB)\sin(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)}{bf\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(e+fx)\cos^2(e+fx)^{-m/2}(c\cos(e+fx))^mF_1\left(\frac{1}{2};-\frac{m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{B\sin(e+fx)(c\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{bcf(m+1)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x]),x]
```

```
[Out] (a*(A*b - a*B)*c*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Si
n[e + f*x]^2)/(a^2 - b^2))]*(c*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1
- m)/2)*Sin[e + f*x])/(b*(a^2 - b^2)*f) - ((A*b - a*B)*AppellF1[1/2, -1/2*m
, 1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*x]^2)/(a^2 - b^2))]*(c*Cos[e + f
*x])^m*Sin[e + f*x])/((a^2 - b^2)*f*(Cos[e + f*x]^2)^(m/2)) - (B*(c*Cos[e +
 f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]
*Sin[e + f*x])/(b*c*f*(1 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
 &&  !IntegerQ[2*n]
```

Rule 2902

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx &= \frac{B\int (c\cos(e+fx))^m\,dx}{b}-\frac{(-Ab+aB)\int \frac{(c\cos(e+fx))^m}{a+b\cos(e+fx)}\,dx}{b}\\ &= -\frac{B(c\cos(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin(e+}{bcf(1+m)\sqrt{\sin^2(e+fx)}}\\ &= -\frac{B(c\cos(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin(e+}{bcf(1+m)\sqrt{\sin^2(e+fx)}}\\ &= \frac{a(Ab-aB)cF_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)(c\cos}{b\left(a^2-b^2\right)f}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 10482 vs. $2(286) = 572$.

time = 27.02, size = 10482, normalized size = 36.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(c\cos(fx+e))^{m} (A+B\cos(fx+e))}{a+b\cos(fx+e)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[0,1,0,0]%%%} / %%%{1,[0,0,1,0]%%%}+%%%{-1,[0,0,0,1]%%%} E
rror:
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\cos(e+fx))^m (A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x)),x)
```

```
[Out] int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x)), x)
```

3.455 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=181

$$\frac{2bB(c\cos(e+fx))^{1+m}\sqrt{a+b\cos(e+fx)}\,\sin(e+fx)}{cf(5+2m)}+\frac{2\mathrm{Int}\left(\frac{(c\cos(e+fx))^m\left(\frac{1}{2}ac(2bB(1+m)+2aA\left(\frac{5}{2}+m\right))+\frac{1}{2}c(b^2B(3\right.}{\sqrt{}}\right.}{}$$

```
[Out] 2*b*B*(c*cos(f*x+e))^(1+m)*sin(f*x+e)*(a+b*cos(f*x+e))^(1/2)/c/f/(5+2*m)+2*
Unintegrable((c*cos(f*x+e))^m*(1/2*a*c*(2*b*B*(1+m)+2*a*A*(5/2+m))+1/2*c*(b
^2*B*(3+2*m)+a*(2*A*b+B*a)*(5+2*m))*cos(f*x+e)+1/2*b*c*(2*a*B*(3+m)+A*b*(5+
2*m))*cos(f*x+e)^2)/(a+b*cos(f*x+e))^(1/2),x)/c/(5+2*m)
```

Rubi [A]
time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x]),x]
```

```
[Out] (2*b*B*(c*Cos[e + f*x])^(1 + m)*Sqrt[a + b*Cos[e + f*x]]*Sin[e + f*x])/(c*f
*(5 + 2*m)) + (2*Defer[Int][((c*Cos[e + f*x])^m*((a*c*(2*b*B*(1 + m) + 2*a*
A*(5/2 + m)))/2 + (c*(b^2*B*(3 + 2*m) + a*(2*A*b + a*B)*(5 + 2*m))*Cos[e +
f*x])/2 + (b*c*(2*a*B*(3 + m) + A*b*(5 + 2*m))*Cos[e + f*x]^2)/2))/Sqrt[a +
 b*Cos[e + f*x]], x])/(c*(5 + 2*m))
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx=\frac{2bB(c\cos(e+fx))^{1+m}\sqrt{a+b\cos(e+}}{cf(5+2m)}$$

Mathematica [A]
time = 75.98, size = 0, normalized size = 0.00

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x]
),x]
```

```
[Out] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x]
), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(c \cos \left(f x+e\right)\right)^{m} \left(a+b \cos \left(f x+e\right)\right)^{\frac{3}{2}} \left(A+B \cos \left(f x+e\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*cos(f*x + e))^
m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*sqrt(b*cos(f
*x + e) + a)*(c*cos(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**(3/2)*(A+B*cos(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*cos(f*x + e))^
m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,\cos\left(e+f\,x\right))^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(3/2),x)
```

```
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(3/2), x)
```

3.456 $\int (c\cos(e+fx))^m \sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=38

$$\text{Int}\left((c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx)),x\right)$$

```
[Out] Unintegrable((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x]),x]
```

```
[Out] Defer[Int][(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])
, x]
```

Rubi steps

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))\,dx = \int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}\,(A$$

Mathematica [A]
time = 8.89, size = 0, normalized size = 0.00

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x]),
x]
```

```
[Out] Integrate[(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x]),
 x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int (c\cos(fx+e))^m (A+B\cos(fx+e))\sqrt{a+b\cos(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x)
```

```
[Out] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m,
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m,
x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c\cos(e+fx))^m (A+B\cos(e+fx))\sqrt{a+b\cos(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))*sqrt(a + b*cos(e + f*x)),
 x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m,
 x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int (c\,\cos\left(e+f\,x\right))^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\sqrt{a+b\,\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(1/2),x)
```

```
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(1/2), x)
```

3.457 $\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}},x\right)$$

```
[Out] Unintegrable((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x]],x]
```

```
[Out] Defer[Int][((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x
]], x]
```

Rubi steps

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx = \int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Mathematica [A]
time = 9.95, size = 0, normalized size = 0.00

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x]
],x]
```

```
[Out] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x]
], x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(c\cos(fx+e))^m (A+B\cos(fx+e))}{\sqrt{a+b\cos(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x)
```

```
[Out] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\cos(e+fx))^m (A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))/sqrt(a + b*cos(e + f*x)),
 x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c\,\cos(e+f\,x))^m\,(A+B\,\cos(e+f\,x))}{\sqrt{a+b\,\cos(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(1/2),x)
```

```
[Out] int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(1/2), x
)
```

3.458 $\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=191

$$\frac{2b(Ab-aB)(c\cos(e+fx))^{1+m}\sin(e+fx)}{a\left(a^2-b^2\right)cf\sqrt{a+b\cos(e+fx)}}+\frac{2\text{Int}\left(\frac{(c\cos(e+fx))^m\left(\frac{1}{2}c\left(a(aA-bB)+2b(Ab-aB)\left(\frac{1}{2}+m\right)\right)-\frac{1}{2}a(Ab-aB)c\cos(e+f}{\sqrt{a+b\cos(e+fx)}}\right.}{a\left(a^2-b^2\right)c}$$

```
[Out] 2*b*(A*b-B*a)*(c*cos(f*x+e))^(1+m)*sin(f*x+e)/a/(a^2-b^2)/c/f/(a+b*cos(f*x+
e))^(1/2)+2*Unintegrable((c*cos(f*x+e))^m*(1/2*c*(a*(A*a-B*b)+2*b*(A*b-B*a)
*(1/2+m))-1/2*a*(A*b-B*a)*c*cos(f*x+e)-1/2*b*(A*b-B*a)*c*(3+2*m)*cos(f*x+e)
^2)/(a+b*cos(f*x+e))^(1/2),x)/a/(a^2-b^2)/c
```

Rubi [A]
time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x])^(3/2),x]
```

```
[Out] (2*b*(A*b - a*B)*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*(a^2 - b^2)*c*f*
Sqrt[a + b*Cos[e + f*x]]) + (2*Defer[Int][((c*Cos[e + f*x])^m*((c*(a*(a*A -
 b*B) + 2*b*(A*b - a*B)*(1/2 + m)))/2 - (a*(A*b - a*B)*c*Cos[e + f*x])/2 -
(b*(A*b - a*B)*c*(3 + 2*m)*Cos[e + f*x]^2)/2))/Sqrt[a + b*Cos[e + f*x]], x]
)/(a*(a^2 - b^2)*c)
```

Rubi steps

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx=\frac{2b(Ab-aB)(c\cos(e+fx))^{1+m}\sin(e+fx)}{a\left(a^2-b^2\right)cf\sqrt{a+b\cos(e+fx)}}+\frac{2\int \frac{(c\cos(e+fx))^m}{}}{}$$

Mathematica [A]
time = 11.54, size = 0, normalized size = 0.00

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x])^(3
/2),x]
```

```
[Out] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x])^(3
/2), x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\left(c\cos\left(fx+e\right)\right)^{m}\left(A+B\cos\left(fx+e\right)\right)}{\left(a+b\cos\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x)
```

```
[Out] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a)^(3/2
), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m/(
b^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\cos\left(e+fx\right)\right)^{m}\left(A+B\cos\left(e+fx\right)\right)}{\left(a+b\cos\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))**(3/2),x)
```

```
[Out] Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))/(a + b*cos(e + f*x))**(3/
2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a)^(3/2
), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,\cos\left(e+f\,x\right))^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)}{(a+b\,\cos\left(e+f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(3/2),x)
```

```
[Out] int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(3/2), x
)
```

3.459 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=172

$$-\frac{2a(3A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] 2/3*a*(A+B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*A*sec(d*x+c)^(5/2)*sin(d*x+
c)/d+2/5*a*(3*A+5*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*a*(3*A+5*B)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3039, 4082, 3872, 3853, 3856, 2719, 2720}

$$\frac{2a(A+B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(3A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-2*a*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*d) + (2*a*(3*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*d) + (2*a*(A + B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*A*Sec[
c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx &= \int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))(B+A\sec(\\
&= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sec^{\frac{3}{2}}(c+ \\
&= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + (a(A+B))\int \\
&= \frac{2a(3A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a(}{} \\
&= \frac{2a(3A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a(}{} \\
&= -\frac{2a(3A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.08, size = 292, normalized size = 1.70

$$
\frac{ae^{-ic}(-1+e^{2ic})\,(1+\cos(c+dx))\csc(c)\left(5A+5B-3Ae^{i(c+dx)}-15Be^{i(c+dx)}-24Ae^{3i(c+dx)}-30Be^{3i(c+dx)}-5Ae^{4i(c+dx)}-5Be^{4i(c+dx)}-9Ae^{5i(c+dx)}-15Be^{5i(c+dx)}-5i(A+B)\left(1+e^{2i(c+dx)}\right)^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(3A+5B)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}}{30d\left(1+e^{2i(c+dx)}\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*(-1 + E^((2*I)*c))*(1 + Cos[c + d*x])*Csc[c]*(5*A + 5*B - 3*A*E^(I*(c +
d*x)) - 15*B*E^(I*(c + d*x)) - 24*A*E^((3*I)*(c + d*x)) - 30*B*E^((3*I)*(c
+ d*x)) - 5*A*E^((4*I)*(c + d*x)) - 5*B*E^((4*I)*(c + d*x)) - 9*A*E^((5*I)*
(c + d*x)) - 15*B*E^((5*I)*(c + d*x)) - (5*I)*(A + B)*(1 + E^((2*I)*(c + d*
x)))^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (3*A + 5*B)*E^(I*(c +
 d*x))*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))])*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]])/(30*d*E^(I*c)*(1
 + E^((2*I)*(c + d*x)))^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(200) = 400$.
time = 0.88, size = 634, normalized size = 3.69

method	result

default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(\left(\frac{A}{2}+\frac{B}{2}\right)\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2}}{6\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)

[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*((1/2*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/10*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/2*B/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 219, normalized size = 1.27

$$\frac{-5i\sqrt{2}(A+B)a\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(A+B)a\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}(3A+5B)a\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}(3A+5B)a\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3(3A+5B)a\cos(dx+c)^2+5(A+B)a\cos(dx+c)+3Aa\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
fricas")

[Out] 1/15*(-5*I*sqrt(2)*(A + B)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(A + B)*a*cos(d*x + c)^2*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(3*A + 5*B)*
a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(3*A + 5*B)*a*cos(d*x + c)^2*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) +
 2*(3*(3*A + 5*B)*a*cos(d*x + c)^2 + 5*(A + B)*a*cos(d*x + c) + 3*A*a)*sin(
d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
giac")

[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x)),x)

[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x)), x)

3.460 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=135

$$-\frac{2a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a(A+3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{3d}$$

```
[Out] 2/3*a*A*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*(A+B)*sin(d*x+c)*sec(d*x+c)^(1/2)
/d-2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(A+3*B)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3039, 4082, 3872, 3856, 2720, 3853, 2719}

$$\frac{2a(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a*(A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a
*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))(B+A\sec($$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2}{3}\int \sqrt{\sec(c+}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + (a(A+B))\int \mathrm{s}$$

$$= \frac{2a(A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2aA\sec}{}$$

$$= \frac{2a(A+3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{3d}$$

$$= -\frac{2a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{s}}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.33, size = 225, normalized size = 1.67

$$\frac{a(1+\cos(c+dx))\left((A+3B)\left(1+e^{2i(c+dx)}\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+i\left(A-3Ae^{i(c+dx)}-3Be^{i(c+dx)}-Ae^{2i(c+dx)}-3Ae^{3i(c+dx)}-3Be^{3i(c+dx)}+(A+B)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}}{3d\left(1+e^{2i(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*((A + 3*B)*(1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] + I*(A - 3*A*E^(I*(c + d*x)) - 3*B*E^(I*(c + d
*x)) - A*E^((2*I)*(c + d*x)) - 3*A*E^((3*I)*(c + d*x)) - 3*B*E^((3*I)*(c +
d*x)) + (A + B)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(3/2)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))*Sec[(c + d*x)/2]^2*Sqrt[Sec[c
 + d*x]])/(3*d*(1 + E^((2*I)*(c + d*x))))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(171) = 342$.
time = 0.63, size = 399, normalized size = 2.96

method	result

default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(\frac{B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1
/2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2*A+1/2*B)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 188, normalized size = 1.39

$$\frac{-i\sqrt{2}\,(A+3\,B)a\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+i\sqrt{2}\,(A+3\,B)a\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3i\sqrt{2}\,(A+B)a\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3i\sqrt{2}\,(A+B)a\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+\frac{2\,(3\,(A+B)a\cos(dx+c)+Aa)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(A + 3*B)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*B)*a*cos(d*x + c)*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(A + B)*a*cos(d*x
 + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) + 3*I*sqrt(2)*(A + B)*a*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*(A + B)*a*c
os(d*x + c) + A*a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x)),x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x)), x)
```

3.461 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=106

$$-\frac{2a(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{d}$$

```
[Out] 2*a*A*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3039, 4082, 3872, 3856, 2719, 2720}

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=\int \frac{(a+a\sec(c+dx))(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$=\frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+2\int \frac{-\frac{1}{2}a(A-$$

$$=\frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}-(a(A-B))\int$$

$$=\frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}-\Big(a(A-B)\sqrt{}$$

$$=-\frac{2a(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.15, size = 157, normalized size = 1.48

$$\frac{2ae^{-idx}\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(-3iA\cos(c+dx)+3iB\cos(c+dx)+3(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+i(A-B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+3A\sin(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((-3*I)*A*Cos[c + d*x] + (3
*I)*B*Cos[c + d*x] + 3*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
 + I*(A - B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F
1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 3*A*Sin[c + d*x]))/(3*d*E^(I*d*x))
```

Maple [A]
time = 0.40, size = 242, normalized size = 2.28

method	result
default	$\frac{2a\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-A\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2*a*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 141, normalized size = 1.33

$$\frac{-i\sqrt{2}\,(A+B)a\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,(A+B)a\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-i\sqrt{2}\,(A-B)a\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\,(A-B)a\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2Aa\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] (-I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) - I*sqrt(2)*(A - B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(A - B)*a*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*A*a
*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x)),x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x)), x)
```

3.462 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\, d$

Optimal. Leaf size=110

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\, E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a(3A+B)\sqrt{\cos(c+dx)}\, F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\text{se}}}{3d}$$

```
[Out] 2/3*a*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+2/3*a*(3*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3039, 4081, 3872, 3856, 2719, 2720}

$$\frac{2a(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\, F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\, E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx &= \int \frac{(a+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx \\ &= \frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int \frac{-\frac{3}{2}a(A+B)-\frac{1}{2}a(3 \\ &= \frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+(a(A+B))\int \frac{1}{\sqrt{\sec(c+ \\ &= \frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\left(a(A+B)\sqrt{\cos(c+dx)} \\ &= \frac{2a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.37, size = 148, normalized size = 1.35

$$\frac{2ae^{-idx}\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left((3A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-i(A+B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(3i(A+B)+B\sin(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

[Out] (2*a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - I*(A + B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((3*I)*(A + B) + B*Sin[c + d*x])))/(3*d*E^(I*d*x))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(148) = 296$.
time = 0.36, size = 321, normalized size = 2.92

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\,(}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 142, normalized size = 1.29

$$\frac{2\,Ba\sqrt{\cos(dx+c)}\,\sin(dx+c) - i\sqrt{2}\,(3A+B)a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}\,(3A+B)a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)) + 3i\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - 3i\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*B*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*(3*A + B)*a*weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(3*A + B)*a*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(A +
 B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) - 3*I*sqrt(2)*(A + B)*a*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sqrt{\sec\left(c+dx\right)}\,dx+\int A\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}\,dx+\int B\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}\,dx+\int B\cos^{2}\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A*sqrt(sec(c + d*x)), x) + Integral(A*cos(c + d*x)*sqrt(sec(c +
 d*x)), x) + Integral(B*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(B*co
s(c + d*x)**2*sqrt(sec(c + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(a+a\,\cos\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x)),x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x)), x)
```

3.463 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2a(5A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}$$

```
[Out] 2/5*a*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*a*(A+B)*sin(d*x+c)/d/sec(d*x+c)^(
1/2)+2/5*a*(5*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(
A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.13, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3039, 4081, 3872, 3854, 3856, 2720, 2719}

$$\frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*d) + (2*a*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx &= \int \frac{(a+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(A+B)-\frac{1}{2}a(5A+3B)\sec(c
}{\sec^{\frac{3}{2}}(c+dx)} \\
&= \frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{5}(a(5 \\
&= \frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(a(A+B)) \\
&= \frac{2a(5A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{5d} \\
&= \frac{2a(5A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{5d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.70, size = 148, normalized size = 1.05

$$
\frac{a\sqrt{\sec(c+dx)}\left(10(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-2i(5A+3B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(6i(5A+3B)+10(A+B)\sin(c+dx)+3B\sin(2(c+dx)))\right)}{15d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(10*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2] - (2*I)*(5*A + 3*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyper
geometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((6*I)*(5*
A + 3*B) + 10*(A + B)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)])))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(173) = 346.
time = 0.38, size = 355, normalized size = 2.52

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+44B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+44*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-10*A-16*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 169, normalized size = 1.20

$$\frac{-5i\sqrt{2}(A+B)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(A+B)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}(5A+3B)a\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}(5A+3B)a\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3Ba\cos(dx+c)^2+5(A+B)a\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 3*I*sqrt(2)*(5*A + 3*B)*a*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(5*A +
3*B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) + 2*(3*B*a*cos(d*x + c)^2 + 5*(A + B)*a*cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{A}{\sqrt{\sec(c+dx)}}\,dx+\int\frac{A\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx+\int\frac{B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx+\int\frac{B\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(A*cos(c + d*x)/sqrt(sec(c +
 d*x)), x) + Integral(B*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(B*co
s(c + d*x)**2/sqrt(sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/(1/cos(c + d*x))^(1/2), x)
```

3.464 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{6a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(7A+5B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{21d}$$

```
[Out] 2/7*a*B*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*a*(A+B)*sin(d*x+c)/d/sec(d*x+c)^(
3/2)+2/21*a*(7*A+5*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*a*(A+B)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*a*(7*A+5*B)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3039, 4081, 3872, 3854, 3856, 2719, 2720}

$$\frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(7A+5B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (6*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(5*d) + (2*a*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(21*d) + (2*a*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) +
(2*a*(A + B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(7*A + 5*B)*Sin[
c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(A+B) - \frac{1}{2}a(7A+5B)\sec(c+}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \frac{1}{7}(a(7A$$

$$= \frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(7A+5B)\,\mathrm{s}}{21d\sqrt{\sec(c}}$$

$$= \frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(7A+5B)\,\mathrm{s}}{21d\sqrt{\sec(c}}$$

$$= \frac{6a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.36, size = 182, normalized size = 1.06

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(20(7A+5B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-84i(A+B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(252i(A+B)+5(28A+23B)\sin(c+dx)+42(A+B)\sin(2(c+dx))+15B\sin(3(c+dx)))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(20*(7*A + 5*B)*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2] - (84*I)*(A + B)*E^(I*(c + d*x))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
+ Cos[c + d*x]*((252*I)*(A + B) + 5*(28*A + 23*B)*Sin[c + d*x] + 42*(A + B)
*Sin[2*(c + d*x)] + 15*B*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x))
```

Maple [A]
time = 0.36, size = 383, normalized size = 2.23

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-528B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-528*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(308*A+448*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-11
2*A-122*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 187, normalized size = 1.09

$$\frac{-5i\sqrt{2}\,(7\,A+5\,B)a\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5i\sqrt{2}\,(7\,A+5\,B)a\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}-i\,\sin{(dx+c)})+63i\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)}))-63i\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}-i\,\sin{(dx+c)}))+\frac{2\left(15\,Ba\cos(dx+c)^3+21\,(A+B)a\cos(dx+c)^2+5\,(7\,A+5\,B)a\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos{(dx+c)}}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(7*A + 5*B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 5*I*sqrt(2)*(7*A + 5*B)*a*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*(A + B)*a*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*
(A + B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(15*B*a*cos(d*x + c)^3 + 21*(A + B)*a*cos(d*x + c)^2 +
 5*(7*A + 5*B)*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{A}{\sec^{\frac{3}{2}}{\left(c+dx\right)}}\,dx+\int \frac{A\cos{\left(c+dx\right)}}{\sec^{\frac{3}{2}}{\left(c+dx\right)}}\,dx+\int \frac{B\cos{\left(c+dx\right)}}{\sec^{\frac{3}{2}}{\left(c+dx\right)}}\,dx+\int \frac{B\cos^{2}{\left(c+dx\right)}}{\sec^{\frac{3}{2}}{\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(A*cos(c + d*x)/sec(c + d*x
)**(3/2), x) + Integral(B*cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(B
*cos(c + d*x)**2/sec(c + d*x)**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/(1/cos(c + d*x))^(3/2), x)
```

3.465 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=199

$$-\frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\right.}{3d}$$

```
[Out] 2/15*a^2*(7*A+5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*A*sec(d*x+c)^(3/2)*(a^
2+a^2*sec(d*x+c))*sin(d*x+c)/d+4/5*a^2*(4*A+5*B)*sin(d*x+c)*sec(d*x+c)^(1/2
)/d-4/5*a^2*(4*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a^
2*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.22, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4103, 4082, 3872, 3856, 2720, 3853, 2719}

$$\frac{2a^2(7A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{4a^2(4A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-4*a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^2*(4*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(5*d) + (2*a^2*(7*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d
) + (2*A*Sec[c + d*x]^(3/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a + a\cos(c+dx))^2 (A + B\cos(c+dx)) \sec^{\frac{7}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(a + a\sec(c+dx))^2 (B + A$$

$$= \frac{2A\sec^{\frac{3}{2}}(c+dx)\,(a^2 + a^2\sec(c+dx))\sin(c+}{5d}$$

$$= \frac{2a^2(7A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2A}{}$$

$$= \frac{2a^2(7A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2A}{}$$

$$= \frac{4a^2(4A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2}{}$$

$$= \frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d}$$

$$= -\frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\right.}{5d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.14, size = 299, normalized size = 1.50

$$\frac{a^2e^{-ic}(-1+e^{2ic})\,(1+\cos(c+dx))^2\csc(c)\left(10A+5B-18Ae^{i(c+dx)}-30Be^{i(c+dx)}-54Ae^{3i(c+dx)}-60Be^{3i(c+dx)}-10Ae^{4i(c+dx)}-5Be^{4i(c+dx)}-24Ae^{5i(c+dx)}-30Be^{5i(c+dx)}-10i(A+2B)\left(1+e^{2i(c+dx)}\right)^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+2(4A+5B)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}}{60d\left(1+e^{2i(c+dx)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*(-1 + E^((2*I)*c))*(1 + Cos[c + d*x])^2*Csc[c]*(10*A + 5*B - 18*A*E^(I
*(c + d*x)) - 30*B*E^(I*(c + d*x)) - 54*A*E^((3*I)*(c + d*x)) - 60*B*E^((3*
I)*(c + d*x)) - 10*A*E^((4*I)*(c + d*x)) - 5*B*E^((4*I)*(c + d*x)) - 24*A*E
^((5*I)*(c + d*x)) - 30*B*E^((5*I)*(c + d*x)) - (10*I)*(A + 2*B)*(1 + E^((2
*I)*(c + d*x)))^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(4*A + 5
*B)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]])/(60
*d*E^(I*c)*(1 + E^((2*I)*(c + d*x)))^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 713 vs. 2(227) = 454.
time = 0.84, size = 714, normalized size = 3.59

method	result

default	$-\frac{8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2\left(\frac{B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{4\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{d}{}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+1/20*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c
)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1
/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/2*A+1/4*B)*(-1/6*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*
c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))+(1/4*A+1/2*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2
-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 239, normalized size = 1.20

$$-\frac{2\left(5i\sqrt{2}(A+2B)a^2\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(A+2B)a^2\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}(4A+5B)a^2\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}(4A+5B)a^2\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(6(4A+5B)a^2\cos(dx+c)^2+5(2A+B)a^2\cos(dx+c)+3Aa^2\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{15d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*(A + 2*B)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(A + 2*B)*a^2*cos(d*x + c)^2*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(4*A
 + 5*B)*a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(4*A + 5*B)*a^2*cos(d*x + c
)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) - (6*(4*A + 5*B)*a^2*cos(d*x + c)^2 + 5*(2*A + B)*a^2*cos(d*x +
c) + 3*A*a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}\,(a+a\,\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2, x)
```

3.466 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=160

$$-\frac{4a^2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2(2A+3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}$$

```
[Out] 2/3*a^2*(5*A+3*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/3*A*(a^2+a^2*sec(d*x+c))*
sin(d*x+c)*sec(d*x+c)^(1/2)/d-4*a^2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+4/3*a^2*(2*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4103, 4082, 3872, 3856, 2719, 2720}

$$\frac{2a^2(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{4a^2(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2\sec(c+dx)+a^2\right)}{3d}-\frac{4a^2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
d + (4*a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a^2*(5*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*A*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n
, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{\sec(c+dx)}\,(a^2+a^2\sec(c+dx))\sin(c+}{3d}$$

$$= \frac{2a^2(5A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2A}{}$$

$$= \frac{2a^2(5A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2A}{}$$

$$= \frac{2a^2(5A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2A}{}$$

$$= -\frac{4a^2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.46, size = 279, normalized size = 1.74

$$\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4i\sqrt{2}\,e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\left(3Ae^{ic}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{idx}\left((2A+3B)(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)+Ae^{i(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\right)}{-1+e^{2ic}}+\sqrt{\sec(c+dx)}\,(3(4A+B-B\cos(2c))\cos(dx)\csc(c)+6B\cos(c)\sin(dx)+2A\tan(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((-4*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(3*A*E^(I*
c)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E^(I*d*x)*((2*
A + 3*B)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c +
 d*x))] + A*E^(I*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))])))/(E^(I*d*x)*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(3*(4*A + B
- B*Cos[2*c])*Cos[d*x]*Csc[c] + 6*B*Cos[c]*Sin[d*x] + 2*A*Tan[c + d*x])))/(
12*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(194) = 388.
time = 0.47, size = 513, normalized size = 3.21

method	result

default	$-\frac{4\left(6\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(2A+B)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/3*(6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A+B)*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(7*A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2)))*sin(1/2*d*x+1/2*c)^2+2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))*a^2/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*
x+1/2*c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 202, normalized size = 1.26

$$-\frac{2\left(i\sqrt{2}\,(2A+3B)a^2\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}\,(2A+3B)a^2\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}\,Aa^2\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,Aa^2\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{(3(2A+B)a^2\cos(dx+c)+Aa^2)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] -2/3*(I*sqrt(2)*(2*A + 3*B)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) - I*sqrt(2)*(2*A + 3*B)*a^2*cos(d*x + c)*weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*A*a^2*cos
(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c))) - 3*I*sqrt(2)*A*a^2*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*(2*A + B)*a^2
*cos(d*x + c) + A*a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2, x)
```

3.467 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{4a^2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2(3A+2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{3d}$$

```
[Out] 2/3*B*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/3*a^2*(3*A-B)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d+4*a^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/d+4/3*a^2*(3*A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.21, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4102, 4082, 3872, 3856, 2719, 2720}

$$\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{4a^2(3A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (4*a^2*(3*A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a^2*(3*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d)
 + (2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2B(a^2+a^2\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a$$

$$= \frac{2a^2(3A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2B$$

$$= \frac{2a^2(3A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2B$$

$$= \frac{2a^2(3A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2B$$

$$= \frac{4a^2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.98, size = 302, normalized size = 1.89

$$\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{4i\sqrt{2}e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(3Be^{ic}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{idx}\left(-\left((3A+2B)(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)+Be^{i(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\right)}{-1+e^{2ic}}+\sqrt{\sec(c+dx)}\left(-3(-A+2B+(A+2B)\cos(2c))\cos(dx)\csc(c)+B\cos(2dx)\sin(2c)+6(A+2B)\cos(c)\sin(dx)+B\cos(2c)\sin(2dx)\right)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*Sqrt[E^(I*(c +
 d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(3*B*E^(I*c
)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E^(I*d*x)*(-((3
*A + 2*B)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c
+ d*x))]) + B*E^(I*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))])))/(E^(I*d*x)*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(-3*(-A +
2*B + (A + 2*B)*Cos[2*c])*Cos[d*x]*Csc[c] + B*Cos[2*d*x]*Sin[2*c] + 6*(A +
2*B)*Cos[c]*Sin[d*x] + B*Cos[2*c]*Sin[2*d*x])))/(12*d)
```

Maple [A]
time = 0.43, size = 244, normalized size = 1.52

method	result

default	$\frac{4a^2\left(-2B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 4/3*a^2*(-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 166, normalized size = 1.04

$$-\frac{2\left(i\sqrt{2}\,(3A+2B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}\,(3A+2B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}\,Ba^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\,Ba^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{(Ba^2\cos(dx+c)+3\,Aa^2)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] -2/3*(I*sqrt(2)*(3*A + 2*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - I*sqrt(2)*(3*A + 2*B)*a^2*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*a^2*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*a^2*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) - (B*a^2*cos(d*x + c) + 3*A*a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="giac")
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2,x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2, x)
```

3.468 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,d$

Optimal. Leaf size=166

$$\frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(2A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{3d}$$

```
[Out] 2/5*B*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/15*a^2*(5*A+7*B)
*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4/5*a^2*(5*A+4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+4/3*a^2*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d
```

Rubi [A]
time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4102, 4081, 3872, 3856, 2719, 2720}

$$\frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a^2*(5*A + 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*(5*A + 7*B)*Sin[c + d*x])/(15*d*Sqrt
[Sec[c + d*x]]) + (2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c +
d*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx &= \int \frac{(a+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,d \\
&= \frac{2B(a^2+a^2\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2}{5}\int \frac{(}{} \\
&= \frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2B(a^2+a^2\sec(}{5d\sec} \\
&= \frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2B(a^2+a^2\sec(}{5d\sec} \\
&= \frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2B(a^2+a^2\sec(}{5d\sec} \\
&= \frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.77, size = 153, normalized size = 0.92

$$
\frac{a^2\sqrt{\sec(c+dx)}\left(20(2A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-4i(5A+4B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(60iA+48iB+10(A+2B)\sin(c+dx)+3B\sin(2(c+dx)))\right)}{15d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(20*(2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2] - (4*I)*(5*A + 4*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((60*I
)*A + (48*I)*B + 10*(A + 2*B)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)])))/(15*d)
```

Maple [A]
time = 0.37, size = 357, normalized size = 2.15

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-12B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(10A+32B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\mathrm{co}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(10*A+32*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-5*A-13*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-12*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 187, normalized size = 1.13

$$-\frac{2\left(5i\sqrt{2}\,(2A+B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,(2A+B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}\,(5A+4B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\,(5A+4B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(3Ba^2\cos(dx+c)^2+5(A+2B)a^2\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*(2*A + B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) - 5*I*sqrt(2)*(2*A + B)*a^2*weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(5*A + 4*B)*a^2*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(
2)*(5*A + 4*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) - (3*B*a^2*cos(d*x + c)^2 + 5*(A + 2*B)*a^2*cos(d
*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sqrt{\sec(c+dx)}\,dx+\int 2A\cos(c+dx)\sqrt{\sec(c+dx)}\,dx+\int A\cos^2(c+dx)\sqrt{\sec(c+dx)}\,dx+\int B\cos(c+dx)\sqrt{\sec(c+dx)}\,dx+\int 2B\cos^2(c+dx)\sqrt{\sec(c+dx)}\,dx+\int B\cos^3(c+dx)\sqrt{\sec(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(A*sqrt(sec(c + d*x)), x) + Integral(2*A*cos(c + d*x)*sqrt(se
c(c + d*x)), x) + Integral(A*cos(c + d*x)**2*sqrt(sec(c + d*x)), x) + Integ
ral(B*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(2*B*cos(c + d*x)**2*sq
rt(sec(c + d*x)), x) + Integral(B*cos(c + d*x)**3*sqrt(sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2, x)
```

3.469 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=201

$$\frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(7A+6B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{21d}$$

```
[Out] 2/35*a^2*(7*A+9*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*B*(a^2+a^2*sec(d*x+c))
*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/21*a^2*(7*A+6*B)*sin(d*x+c)/d/sec(d*x+c)^(
1/2)+4/5*a^2*(4*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*
a^2*(7*A+6*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.24, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4102, 4081, 3872, 3854, 3856, 2720, 2719}

$$\frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(7A+6B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(7A+6B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a^2*(4*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(7*A + 6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(7*A + 9*B)*Sin[c + d*x])/(35*d*S
ec[c + d*x]^(3/2)) + (4*a^2*(7*A + 6*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*
x]]) + (2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2B(a^2+a^2\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\sec(c+$$

$$= \frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2B(a^2+a^2\sec(c+dx))\sin(}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2B(a^2+a^2\sec(c+dx))\sin(}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2(7A+6B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} +$$

$$= \frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}{5d}$$

$$= \frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}{5d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.47, size = 193, normalized size = 0.96

$$\frac{a^2e^{-idx}\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(40(7A+6B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-56i(4A+3B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(672iA+504iB+5(56A+51B)\sin(c+dx)+42(A+2B)\sin(2(c+dx))+15B\sin(3(c+dx)))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(7*A + 6*B)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(4*A + 3*B)*E^(I*(c + d*x))*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))] + Cos[c + d*x]*((672*I)*A + (504*I)*B + 5*(56*A + 51*B)*Sin[c + d*x]
+ 42*(A + 2*B)*Sin[2*(c + d*x)] + 15*B*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x)
)
```

Maple [A]
time = 0.38, size = 385, normalized size = 1.92

method	result

default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2\left(120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-84A-348B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-348*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(224*A+378*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-9
1*A-117*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-84*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+30*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 211, normalized size = 1.05

$$-\frac{2\left(5i\sqrt{2}\,(7A+6B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,(7A+6B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21i\sqrt{2}\,(4A+3B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}\,(4A+3B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(15Ba^2\cos(dx+c)^3+21(A+2B)a^2\cos(dx+c)^2+10(7A+6B)a^2\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/105*(5*I*sqrt(2)*(7*A + 6*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 5*I*sqrt(2)*(7*A + 6*B)*a^2*weierstrassPInverse(-4, 0,
```

```
 cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(4*A + 3*B)*a^2*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*
I*sqrt(2)*(4*A + 3*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - (15*B*a^2*cos(d*x + c)^3 + 21*(A + 2*B)*
a^2*cos(d*x + c)^2 + 10*(7*A + 6*B)*a^2*cos(d*x + c))*sin(d*x + c)/sqrt(cos
(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{A}{\sqrt{\sec \left(c+dx\right)}}\, dx + \int \frac{2A\cos \left(c+dx\right)}{\sqrt{\sec \left(c+dx\right)}}\, dx + \int \frac{A\cos^{2}\left(c+dx\right)}{\sqrt{\sec \left(c+dx\right)}}\, dx + \int \frac{B\cos \left(c+dx\right)}{\sqrt{\sec \left(c+dx\right)}}\, dx + \int \frac{2B\cos^{2}\left(c+dx\right)}{\sqrt{\sec \left(c+dx\right)}}\, dx + \int \frac{B\cos^{3}\left(c+dx\right)}{\sqrt{\sec \left(c+dx\right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(2*A*cos(c + d*x)/sqrt(se
c(c + d*x)), x) + Integral(A*cos(c + d*x)**2/sqrt(sec(c + d*x)), x) + Integ
ral(B*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(2*B*cos(c + d*x)**2/sq
rt(sec(c + d*x)), x) + Integral(B*cos(c + d*x)**3/sqrt(sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2), x
)
```

3.470 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=244

$$-\frac{4a^3(7A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3(13A+21B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{21d}$$

```
[Out] 4/105*a^3*(41*A+42*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a*A*sec(d*x+c)^(3/2
)*(a+a*sec(d*x+c))^2*sin(d*x+c)/d+2/35*(11*A+7*B)*sec(d*x+c)^(3/2)*(a^3+a^3
*sec(d*x+c))*sin(d*x+c)/d+4/5*a^3*(7*A+9*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4
/5*a^3*(7*A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*a^3*(1
3*A+21*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.32, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4103, 4082, 3872, 3856, 2720, 3853, 2719}

$$\frac{4a^3(41A+42B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(11A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d}+\frac{4a^3(7A+9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3(13A+21B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(7A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (-4*a^3*(7*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(13*A + 21*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(7*A + 9*B)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(5*d) + (4*a^3*(41*A + 42*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(105*d) + (2*a*A*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/
(7*d) + (2*(11*A + 7*B)*Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c +
 d*x])/(35*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3(B+A\,\mathrm{se}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+d}{7d}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+d}{7d}$$

$$= \frac{4a^3(41A+42B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2a}{}$$

$$= \frac{4a^3(41A+42B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2a}{}$$

$$= \frac{4a^3(7A+9B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{4a}{}$$

$$= \frac{4a^3(13A+21B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right.}{21d}$$

$$= -\frac{4a^3(7A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right.}{5d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 4.49, size = 435, normalized size = 1.78

$$\frac{a^3e^{-idx}(1+\cos(c+dx))^3\csc(c)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(7\sqrt{2}(7A+9B)e^{2idx}(-1+e^{2ic})\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-\frac{e^{-i(c-dx)}(-1+e^{2ic})\left(21B(-5+16e^{i(c+dx)}-5e^{2i(c+dx)}+54e^{3i(c+dx)}+5e^{4i(c+dx)}+56e^{5i(c+dx)}+5e^{6i(c+dx)}+18e^{7i(c+dx)})+2A(-65+84e^{i(c+dx)}-95e^{2i(c+dx)}+441e^{3i(c+dx)}+95e^{4i(c+dx)}+504e^{5i(c+dx)}+65e^{6i(c+dx)}+147e^{7i(c+dx)})+10i(13A+21B)(1+e^{2i(c+dx)})^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)\sqrt{\sec(c+dx)}}{2(1+e^{2i(c+dx)})^3}\right)}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Csc[c]*Sec[(c + d*x)/2]^6*(7*Sqrt[2]*(7*A + 9*B)*
E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*
x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))] - ((-1 + E^((2*I)*c))*(21*B*(-5 + 16*E^(I*(c + d*x)) - 5*E^
((2*I)*(c + d*x)) + 54*E^((3*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 56*E^(
(5*I)*(c + d*x)) + 5*E^((6*I)*(c + d*x)) + 18*E^((7*I)*(c + d*x))) + 2*A*(-
65 + 84*E^(I*(c + d*x)) - 95*E^((2*I)*(c + d*x)) + 441*E^((3*I)*(c + d*x))
+ 95*E^((4*I)*(c + d*x)) + 504*E^((5*I)*(c + d*x)) + 65*E^((6*I)*(c + d*x))
 + 147*E^((7*I)*(c + d*x))) + (10*I)*(13*A + 21*B)*(1 + E^((2*I)*(c + d*x))
)^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[Sec[c + d*x]])/(2*E^
(I*(c - d*x))*(1 + E^((2*I)*(c + d*x)))^3)))/(420*d*E^(I*d*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 901 vs. 2(268) = 536.

time = 1.04, size = 902, normalized size = 3.70

method	result	size
default	Expression too large to display	902

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1/5*(3/8*A+1/8*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+
8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(3/8*A+3/8*B)*(-1/6*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(
1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/
2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/8*A+3/8*B)/sin(1/2*d*x+1/2*c)^2/(2*s
in(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 263, normalized size = 1.08

$$-\frac{2\left(5i\sqrt{2}\,(13A+21B)a^3\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,(13A+21B)a^3\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21i\sqrt{2}\,(7A+9B)a^3\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}\,(7A+9B)a^3\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(42(7A+9B)a^3\cos(dx+c)^3+5(26A+21B)a^3\cos(dx+c)^2+21(3A+B)a^3\cos(dx+c)+15Aa^3\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{105\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] -2/105*(5*I*sqrt(2)*(13*A + 21*B)*a^3*cos(d*x + c)^3*weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(13*A + 21*B)*a^3*cos(d*x
 + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sq
rt(2)*(7*A + 9*B)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(7*A + 9*B)*a^3*
cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c))) - (42*(7*A + 9*B)*a^3*cos(d*x + c)^3 + 5*(26*A + 21*B
)*a^3*cos(d*x + c)^2 + 21*(3*A + B)*a^3*cos(d*x + c) + 15*A*a^3)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}\,(a+a\,\cos(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3, x)
```

3.471 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=211

$$-\frac{4a^3(9A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3(3A+5B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\right.}{3d}$$

```
[Out] 4/15*a^3*(21*A+20*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/5*a*A*(a+a*sec(d*x+c))
^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/15*(9*A+5*B)*(a^3+a^3*sec(d*x+c))*sin(d*
x+c)*sec(d*x+c)^(1/2)/d-4/5*a^3*(9*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d+4/3*a^3*(3*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A]
time = 0.31, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4103, 4082, 3872, 3856, 2719, 2720}

$$\frac{4a^3(21A+20B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\,(a^3\sec(c+dx)+a^3)}{15d}+\frac{4a^3(3A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(9A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-4*a^3*(9*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(21*A + 20*B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(15*d) + (2*a*A*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(5*d) + (2*(9*A + 5*B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x
])*Sin[c + d*x])/(15*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx &= \int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx \\
&= \frac{2aA\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2\sin(c-}{5d} \\
&= \frac{2aA\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2\sin(c-}{5d} \\
&= \frac{4a^3(21A+20B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d} + \\
&= \frac{4a^3(21A+20B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d} + \\
&= \frac{4a^3(21A+20B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d} + \\
&= -\frac{4a^3(9A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right|}{5d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.38, size = 268, normalized size = 1.27

$$
\frac{a^3e^{-idx}\csc(c)\sec(c)\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(2(9A+5B)e^{-i(c-dx)}(-1+e^{4ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\frac{1}{2}\sec^2(c+dx)\sin(2c)\left(-18i(9A+5B)\cos(c+dx)-54iA\cos(3(c+dx))-30iB\cos(3(c+dx))+40(3A+5B)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+66A\sin(c+dx)+45B\sin(c+dx)+30A\sin(2(c+dx))+10B\sin(2(c+dx))+54A\sin(3(c+dx))+45B\sin(3(c+dx))\right)\right)}{30d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^3*Csc[c]*Sec[c]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((2*(9*A + 5*
B)*(-1 + E^((4*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c - d*x)) + (Sec[c + d*x]^2*Sin[2*c]
*((-18*I)*(9*A + 5*B)*Cos[c + d*x] - (54*I)*A*Cos[3*(c + d*x)] - (30*I)*B*C
os[3*(c + d*x)] + 40*(3*A + 5*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2,
2] + 66*A*Sin[c + d*x] + 45*B*Sin[c + d*x] + 30*A*Sin[2*(c + d*x)] + 10*B*S
in[2*(c + d*x)] + 54*A*Sin[3*(c + d*x)] + 45*B*Sin[3*(c + d*x)]))/2))/(30*d
*E^(I*d*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(239) = 478.
time = 0.93, size = 916, normalized size = 4.34

method	result	size

default	Expression too large to display	916

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin(
1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*
d*x+1/2*c)^3*(216*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*A*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-108*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4+180*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-100*B*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-60*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d
*x+1/2*c)^4-246*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*A*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+108*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^2-190*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+100*B*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+60*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2+72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+50*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2-25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 243, normalized size = 1.15

$$-\frac{2\left(5i\sqrt{2}\,(3A+5B)a^3\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,(3A+5B)a^3\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}\,(9A+5B)a^3\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,(9A+5B)a^3\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(9(6A+5B)a^3\cos(dx+c)^2+5(3A+B)a^3\cos(dx+c)+3Aa^3\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{15\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*(3*A + 5*B)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(3*A + 5*B)*a^3*cos(d*x + c)
^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*
(9*A + 5*B)*a^3*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(9*A + 5*B)*a^3*cos(d*x
 + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c))) - (9*(6*A + 5*B)*a^3*cos(d*x + c)^2 + 5*(3*A + B)*a^3*cos(d*
x + c) + 3*A*a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}\,{(a+a\,\cos(c+d\,x))}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3, x)
```

3.472 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=199

$$-\frac{4a^3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{3d}$$

```
[Out] 2/3*a*B*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4/3*a^3*(4*A+B)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d+2/3*(A-B)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)*sec(d
*x+c)^(1/2)/d-4*a^3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+20
/3*a^3*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.30, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4102, 4103, 4082, 3872, 3856, 2719, 2720}

$$\frac{4a^3(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}\,(a^3\sec(c+dx)+a^3)}{3d}+\frac{20a^3(A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (20*a^3*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (4*a^3*(4*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
3*d) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
 + (2*(A - B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(3*
d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a}{}$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2(A-}{}$$

$$= \frac{4a^3(4A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aB}{}$$

$$= \frac{4a^3(4A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aB}{}$$

$$= \frac{4a^3(4A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aB}{}$$

$$= -\frac{4a^3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.07, size = 202, normalized size = 1.02

$$\frac{a^3e^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-12iA+12iB-12iA\cos(2(c+dx))+12iB\cos(2(c+dx))+40(A+B)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+4i(A-B)\left(1+e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+4A\sin(c+dx)+B\sin(c+dx)+18A\sin(2(c+dx))+6B\sin(2(c+dx))+B\sin(3(c+dx))\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^3*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((-12*I)*A + (12*I)*B - (12
*I)*A*Cos[2*(c + d*x)] + (12*I)*B*Cos[2*(c + d*x)] + 40*(A + B)*Cos[c + d*x
]^(3/2)*EllipticF[(c + d*x)/2, 2] + (4*I)*(A - B)*(1 + E^((2*I)*(c + d*x)))
^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 4*A*Sin[c +
 d*x] + B*Sin[c + d*x] + 18*A*Sin[2*(c + d*x)] + 6*B*Sin[2*(c + d*x)] + B*S
in[3*(c + d*x)]))/(6*d*E^(I*d*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(229) = 458.
time = 0.49, size = 654, normalized size = 3.29

method	result

default	$-\frac{4\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/3*(-4*B*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(9*A+5*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+2*B)*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
*sin(1/2*d*x+1/2*c)^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-3*B*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*a^3/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d
*x+1/2*c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 215, normalized size = 1.08

$$-\frac{2\left(5i\sqrt{2}(A+B)a^3\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(A+B)a^3\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}(A-B)a^3\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}(A-B)a^3\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(Ba^3\cos(dx+c)^2+3(3A+B)a^3\cos(dx+c)+Aa^3\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] -2/3*(5*I*sqrt(2)*(A + B)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(A + B)*a^3*cos(d*x + c)*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(A - B)*a^3*c
os(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) - 3*I*sqrt(2)*(A - B)*a^3*cos(d*x + c)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (B*a^3*c
os(d*x + c)^2 + 3*(3*A + B)*a^3*cos(d*x + c) + A*a^3)*sin(d*x + c)/sqrt(cos
(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3, x)
```

3.473 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=211

$$\frac{4a^3(5A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3(5A+3B)\sqrt{\cos(c+dx)}\,F\Big(\frac{1}{2}(c+dx)\Big|\,2}{3d}$$

```
[Out] 2/5*a*B*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/15*(5*A+9*B)*(a^
3+a^3*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4/15*a^3*(5*A-6*B)*sin(d*x+
c)*sec(d*x+c)^(1/2)/d+4/5*a^3*(5*A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+4/3*a^3*(5*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d
```

Rubi [A]

time = 0.31, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4102, 4082, 3872, 3856, 2719, 2720}

$$\frac{4a^3(5A-6B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(5A+9B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^3(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(5A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^3*(5*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(5*A - 6*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c
+ d*x]^(3/2)) + (2*(5*A + 9*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d
*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx &= \int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \\
&= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5A}{} \\
&= \frac{4a^3(5A-6B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d} + \frac{2}{} \\
&= \frac{4a^3(5A-6B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d} + \frac{2}{} \\
&= \frac{4a^3(5A-6B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d} + \frac{2}{} \\
&= \frac{4a^3(5A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.74, size = 207, normalized size = 0.98

$$
\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(120iA\cos(c+dx)+216iB\cos(c+dx)+40(5A+3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-8i(5A+9B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+60A\sin(c+dx)+3B\sin(c+dx)+10A\sin(2(c+dx))+30B\sin(2(c+dx))+3B\sin(3(c+dx))\right)}{30d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((120*I)*A*Cos[c + d*x] + (
216*I)*B*Cos[c + d*x] + 40*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2] - (8*I)*(5*A + 9*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*
Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 60*A*Sin[c + d*x]
+ 3*B*Sin[c + d*x] + 10*A*Sin[2*(c + d*x)] + 30*B*Sin[2*(c + d*x)] + 3*B*Si
n[3*(c + d*x)]))/(30*d*E^(I*d*x))
```

Maple [A]
time = 0.48, size = 337, normalized size = 1.60

method	result

default	$-\frac{4a^3\left(-12B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+42B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(s\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/15*a^3*(-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+10*A*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^4+42*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*
A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+25*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-18*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+15*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-27*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 197, normalized size = 0.93

$$-\frac{2\left(5i\sqrt{2}\,(5A+3B)a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,(5A+3B)a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}\,(5A+9B)a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\,(5A+9B)a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(3Ba^3\cos(dx+c)^2+5(A+3B)a^3\cos(dx+c)+15Aa^3\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*(5*A + 3*B)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) - 5*I*sqrt(2)*(5*A + 3*B)*a^3*weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(5*A + 9*B)*a^3*weierstrassZet
```

```
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*s
qrt(2)*(5*A + 9*B)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))) - (3*B*a^3*cos(d*x + c)^2 + 5*(A + 3*B)*a^3*c
os(d*x + c) + 15*A*a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="giac")
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\,\cos\left(c + d\,x\right))\,{\left(\frac{1}{\cos\left(c + d\,x\right)}\right)}^{3/2}\,{\left(a + a\,\cos\left(c + d\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3,x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3, x)
```

3.474 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\, d$

Optimal. Leaf size=211

$$\frac{4a^3(9A+7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3(21A+13B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\right.}{21d}$$

```
[Out] 2/7*a*B*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/35*(7*A+11*B)*(a
^3+a^3*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/105*a^3*(42*A+41*B)*sin(
d*x+c)/d/sec(d*x+c)^(1/2)+4/5*a^3*(9*A+7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+4/21*a^3*(21*A+13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d
```

Rubi [A]

time = 0.32, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4102, 4081, 3872, 3856, 2719, 2720}

$$\frac{2(7A+11B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^3(21A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(9A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a^3*(9*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(21*A + 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(42*A + 41*B)*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*
Sec[c + d*x]^(5/2)) + (2*(7*A + 11*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x]
)/(35*d*Sec[c + d*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,d$$
$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int$$
$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(7A}{}$$
$$= \frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aB(a+a\,\mathrm{sec}}{7d\,\mathrm{s}}$$
$$= \frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aB(a+a\,\mathrm{sec}}{7d\,\mathrm{s}}$$
$$= \frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aB(a+a\,\mathrm{sec}}{7d\,\mathrm{s}}$$
$$= \frac{4a^3(9A+7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.56, size = 194, normalized size = 0.92

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(40(21A+13B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-56i(9A+7B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(168i(9A+7B)+5(84A+107B)\sin(c+dx)+42(A+3B)\sin(2(c+dx))+15B\sin(3(c+dx)))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(21*A + 13*B)*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(9*A + 7*B)*E^(I*(c + d*x))*Sq
rt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))] + Cos[c + d*x]*((168*I)*(9*A + 7*B) + 5*(84*A + 107*B)*Sin[c + d*x]
 + 42*(A + 3*B)*Sin[2*(c + d*x)] + 15*B*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x
))
```

Maple [A]
time = 0.44, size = 385, normalized size = 1.82

method	result

default	$-\frac{4\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^{3}\left(120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-84A-432B)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-432*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(294*A+602*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
26*A-208*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+65*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 211, normalized size = 1.00

$$-\frac{2\left(5i\sqrt{2}\,(21A+13B)a^{3}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,(21A+13B)a^{3}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21i\sqrt{2}\,(9A+7B)a^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}\,(9A+7B)a^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(15Ba^{3}\cos(dx+c)^{3}+21(A+3B)a^{3}\cos(dx+c)^{2}+5(21A+26B)a^{3}\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/105*(5*I*sqrt(2)*(21*A + 13*B)*a^3*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(21*A + 13*B)*a^3*weierstrassPInverse(-4
```

```
, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(9*A + 7*B)*a^3*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 21*I*sqrt(2)*(9*A + 7*B)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*B*a^3*cos(d*x + c)^3 + 21*(A + 3
*B)*a^3*cos(d*x + c)^2 + 5*(21*A + 26*B)*a^3*cos(d*x + c))*sin(d*x + c)/sqr
t(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int A\sqrt{\sec{\left(c+dx \right)}}\, dx+\int 3A\cos{\left(c+dx \right)}\sqrt{\sec{\left(c+dx \right)}}\, dx+\int 3A\cos^{2}{\left(c+dx \right)}\sqrt{\sec{\left(c+dx \right)}}\, dx+\int A\cos^{3}{\left(c+dx \right)}\sqrt{\sec{\left(c+dx \right)}}\, dx+\int B\cos{\left(c+dx \right)}\sqrt{\sec{\left(c+dx \right)}}\, dx+\int 3B\cos^{2}{\left(c+dx \right)}\sqrt{\sec{\left(c+dx \right)}}\, dx+\int 3B\cos^{3}{\left(c+dx \right)}\sqrt{\sec{\left(c+dx \right)}}\, dx+\int B\cos^{4}{\left(c+dx \right)}\sqrt{\sec{\left(c+dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] a**3*(Integral(A*sqrt(sec(c + d*x)), x) + Integral(3*A*cos(c + d*x)*sqrt(se
c(c + d*x)), x) + Integral(3*A*cos(c + d*x)**2*sqrt(sec(c + d*x)), x) + Int
egral(A*cos(c + d*x)**3*sqrt(sec(c + d*x)), x) + Integral(B*cos(c + d*x)*sq
rt(sec(c + d*x)), x) + Integral(3*B*cos(c + d*x)**2*sqrt(sec(c + d*x)), x)
+ Integral(3*B*cos(c + d*x)**3*sqrt(sec(c + d*x)), x) + Integral(B*cos(c +
d*x)**4*sqrt(sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3, x)
```

3.475 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=244

$$\frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{4a^3(13A+11B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{21d}$$

```
[Out] 4/105*a^3*(24*A+23*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/9*a*B*(a+a*sec(d*x+c)
)^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2/63*(9*A+13*B)*(a^3+a^3*sec(d*x+c))*sin(
d*x+c)/d/sec(d*x+c)^(5/2)+4/21*a^3*(13*A+11*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2
)+4/15*a^3*(21*A+17*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*
a^3*(13*A+11*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.34, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4102, 4081, 3872, 3854, 3856, 2720, 2719}

$$\frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(9A+13B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(13A+11B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^3(13A+11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a^3*(21*A + 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(13*A + 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(24*A + 23*B)*Sin[c + d*x])/
(105*d*Sec[c + d*x]^(3/2)) + (4*a^3*(13*A + 11*B)*Sin[c + d*x])/(21*d*Sqrt[
Sec[c + d*x]]) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d
*x]^(7/2)) + (2*(9*A + 13*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(63*d*S
ec[c + d*x]^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$=\frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2}{9}\int \frac{(a+a\sec(c+$$

$$=\frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2(9A+13B)\,(a^3}{63d}$$

$$=\frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{2aB(a+a\sec(c+dx))^2\,\mathrm{si}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$=\frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{2aB(a+a\sec(c+dx))^2\,\mathrm{si}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$=\frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(13A+11B)\sin(c+d}{21d\sqrt{\sec(c+dx)}}$$

$$=\frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{15d}$$

$$=\frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{15d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.95, size = 196, normalized size = 0.80

$$\frac{a^3\sqrt{\sec(c+dx)}\left(240(13A+11B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-112i(21A+17B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(7056iA+5712iB+30(107A+97B)\sin(c+dx)+14(54A+73B)\sin(2(c+dx))+90A\sin(3(c+dx))+270B\sin(3(c+dx))+35B\sin(4(c+dx)))\right)}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(240*(13*A + 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2] - (112*I)*(21*A + 17*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*
x]*((7056*I)*A + (5712*I)*B + 30*(107*A + 97*B)*Sin[c + d*x] + 14*(54*A + 7
3*B)*Sin[2*(c + d*x)] + 90*A*Sin[3*(c + d*x)] + 270*B*Sin[3*(c + d*x)] + 35
*B*Sin[4*(c + d*x)])))/(1260*d)
```

Maple [A]
time = 0.40, size = 413, normalized size = 1.69

method	result

default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(-560B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(360A+2200B)\left(\sin^8\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(360*A+2200*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-1296*A-3412*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1806*A+2702*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-624*A-738*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+195*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-441*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+165*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 231, normalized size = 0.95

$$-\frac{2\left(15i\sqrt{2}\,(13A+11B)a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-15i\sqrt{2}\,(13A+11B)a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21i\sqrt{2}\,(21A+17B)a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}\,(21A+17B)a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(35Ba^3\cos(dx+c)^4+45(A+3B)a^3\cos(dx+c)^3+7(27A+34B)a^3\cos(dx+c)^2+30(13A+11B)a^3\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/315*(15*I*sqrt(2)*(13*A + 11*B)*a^3*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 15*I*sqrt(2)*(13*A + 11*B)*a^3*weierstrassPInverse(
```

```
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(21*A + 17*B)*a^3*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + 21*I*sqrt(2)*(21*A + 17*B)*a^3*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*B*a^3*cos(d*x + c)^4 + 45*
(A + 3*B)*a^3*cos(d*x + c)^3 + 7*(27*A + 34*B)*a^3*cos(d*x + c)^2 + 30*(13*
A + 11*B)*a^3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int \frac{A}{\sqrt{\sec{\left(c+dx \right)}}}\, dx + \int \frac{3A\cos{\left(c+dx \right)}}{\sqrt{\sec{\left(c+dx \right)}}}\, dx + \int \frac{3A\cos^{2}{\left(c+dx \right)}}{\sqrt{\sec{\left(c+dx \right)}}}\, dx + \int \frac{A\cos^{3}{\left(c+dx \right)}}{\sqrt{\sec{\left(c+dx \right)}}}\, dx + \int \frac{B\cos{\left(c+dx \right)}}{\sqrt{\sec{\left(c+dx \right)}}}\, dx + \int \frac{3B\cos^{2}{\left(c+dx \right)}}{\sqrt{\sec{\left(c+dx \right)}}}\, dx + \int \frac{3B\cos^{3}{\left(c+dx \right)}}{\sqrt{\sec{\left(c+dx \right)}}}\, dx + \int \frac{B\cos^{4}{\left(c+dx \right)}}{\sqrt{\sec{\left(c+dx \right)}}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a**3*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(3*A*cos(c + d*x)/sqrt(se
c(c + d*x)), x) + Integral(3*A*cos(c + d*x)**2/sqrt(sec(c + d*x)), x) + Int
egral(A*cos(c + d*x)**3/sqrt(sec(c + d*x)), x) + Integral(B*cos(c + d*x)/sq
rt(sec(c + d*x)), x) + Integral(3*B*cos(c + d*x)**2/sqrt(sec(c + d*x)), x)
+ Integral(3*B*cos(c + d*x)**3/sqrt(sec(c + d*x)), x) + Integral(B*cos(c +
d*x)**4/sqrt(sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2), x
)
```

3.476 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=193

$$\frac{3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{(5A-3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}$$

```
[Out] 1/3*(5*A-3*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-(A-B)*sec(d*x+c)^(5/2)*sin(d*
x+c)/d/(a+a*sec(d*x+c))-3*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d+3*(A-B)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+1/3*(5*A-3*B)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.19, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4104, 3872, 3853, 3856, 2719, 2720}

$$-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(5A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{(5A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) + ((5*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*a*d) - (3*(A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (
(5*A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) - ((A - B)*Sec[c + d*x
]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}a(A-B\right.}{}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{(5A-3B)\int \sec^{\frac{5}{2}}(c+dx)\,d}{2a}$$

$$= -\frac{3(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} + \frac{(5A-3B)\sec^{\frac{3}{2}}(c+dx)\,s}{3ad}$$

$$= -\frac{3(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} + \frac{(5A-3B)\sec^{\frac{3}{2}}(c+dx)\,s}{3ad}$$

$$= \frac{3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(5A-}{}$$

<u>Mathematica</u> **[C]** Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 7.39, size = 650, normalized size = 3.37

$$-\frac{Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sec\left(\frac{c}{2}\right)}{\sqrt{2}\,d(a+a\cos(c+dx))}+\frac{Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sec\left(\frac{c}{2}\right)}{\sqrt{2}\,d(a+a\cos(c+dx))}+\frac{5A\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\csc\left(\frac{c}{2}\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)\sec\left(\frac{c}{2}\right)\sqrt{\sec(c+dx)}\sin(c)}{3d(a+a\cos(c+dx))}-\frac{B\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\csc\left(\frac{c}{2}\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)\sec\left(\frac{c}{2}\right)\sqrt{\sec(c+dx)}\sin(c)}{d(a+a\cos(c+dx))}+\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(-\frac{3(A-B)\cos(dx)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)}{d}+\frac{2\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{d}+\frac{4A\sec(c)\sec(c+dx)\sin(dx)}{3d}+\frac{2(2A+5A\cos(c)-3B\cos(c))\sec(c)\tan\left(\frac{c}{2}\right)}{3d}\right)}{a+a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] -((A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E
^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])*Sec[c/2])/(Sqrt[2]*d*E^(I*d*x)*(a + a*Cos[c + d*x]))) + (B*Sqr
t[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*
Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*
d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])*Sec[c/2])/(Sqrt[2]*d*E^(I*d*x)*(a + a*Cos[c + d*x])) + (5*A*Cos[c/2 +
(d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*S
qrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])) - (B*Cos[c/2 + (d*x)/2
]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec
[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Se
c[c + d*x]]*((-3*(A - B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (2*Sec[c/2]*Sec[c/
2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x
]*Sin[d*x])/(3*d) + (2*(2*A + 5*A*Cos[c] - 3*B*Cos[c])*Sec[c]*Tan[c/2])/(3*
d)))/(a + a*Cos[c + d*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(227) = 454$.
time = 0.77, size = 466, normalized size = 2.41

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{(A-B)\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-}\right.}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((A-B)*(cos(1/
2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*(-1/6*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2)))+(-2*A+2*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 308, normalized size = 1.60

$$\frac{\left(\sqrt{2}\,(-5i\,A+3i\,B)\cos(dx+c)^2+\sqrt{2}\,(-5i\,A+3i\,B)\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\left(\sqrt{2}\,(5i\,A-3i\,B)\cos(dx+c)^2+\sqrt{2}\,(5i\,A-3i\,B)\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-9\left(\sqrt{2}\,(-i\,A+i\,B)\cos(dx+c)^2+\sqrt{2}\,(-i\,A+i\,B)\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-9\left(\sqrt{2}\,(i\,A-i\,B)\cos(dx+c)^2+\sqrt{2}\,(i\,A-i\,B)\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(9\,(A-B)\cos(dx+c)^2+2\,(2\,A-3\,B)\cos(dx+c)-2\,A\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/6*((sqrt(2)*(-5*I*A + 3*I*B)*cos(d*x + c)^2 + sqrt(2)*(-5*I*A + 3*I*B)*co
s(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sq
rt(2)*(5*I*A - 3*I*B)*cos(d*x + c)^2 + sqrt(2)*(5*I*A - 3*I*B)*cos(d*x + c)
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*(sqrt(2)*(-
I*A + I*B)*cos(d*x + c)^2 + sqrt(2)*(-I*A + I*B)*cos(d*x + c))*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*(
sqrt(2)*(I*A - I*B)*cos(d*x + c)^2 + sqrt(2)*(I*A - I*B)*cos(d*x + c))*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)) - 2*(9*(A - B)*cos(d*x + c)^2 + 2*(2*A - 3*B)*cos(d*x + c) - 2*A)*sin(d*
x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x)),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x)), x)
```

3.477 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=159

$$-\frac{(3A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{(A-B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}$$

```
[Out] -(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))+(3*A-B)*sin(d*x+c)*se
c(d*x+c)^(1/2)/a/d-(3*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
-(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.17, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4104, 3872, 3856, 2720, 3853, 2719}

$$-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(3A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(((3*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) - ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a*d) + ((3*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - ((A
- B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sqrt{\sec(c+dx)}\,\left(-\frac{1}{2}a(A\right.}{}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-B)\int \sqrt{\sec(c+dx)}}{2a}$$

$$= \frac{(3A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin}{d(a+a\sec(c+dx}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(3A}{}$$

$$= -\frac{(3A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.75, size = 400, normalized size = 2.52

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(6\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)-2\sqrt{2}Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)-12A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}+12B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}+6\sqrt{\sec(c+dx)}\left(2(3A-B)\cos(dx)\csc(c)+2(-A+B)\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{6ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))]))/E^(I*d*x) - (2*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((
2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I
)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4
, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - 12*A*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 12*B*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 6*Sqrt[Sec[c + d*x]]*(2*(3*A - B)*Cos[d*
x]*Csc[c] + 2*(-A + B)*Tan[(c + d*x)/2])))/(6*a*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.49, size = 319, normalized size = 2.01

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A-B)*sin(1/2*d*x+1/2*c)^4+(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A-B)*sin(1/2*d*x+1/2*c)^2
)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 250, normalized size = 1.57

$$\frac{\left(\sqrt{2}\,(iA-iB)\cos(dx+c)+\sqrt{2}\,(iA-iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(\sqrt{2}\,(-iA+iB)\cos(dx+c)+\sqrt{2}\,(-iA+iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+\left(\sqrt{2}\,(-3iA+iB)\cos(dx+c)+\sqrt{2}\,(-3iA+iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+\left(\sqrt{2}\,(3iA-iB)\cos(dx+c)+\sqrt{2}\,(3iA-iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2((3A-B)\cos(dx+c)+2A)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/2*((sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(-I*A + I*B)*cos(d*
x + c) + sqrt(2)*(-I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) + (sqrt(2)*(-3*I*A + I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A + I*
B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
```

```
d*x + c))) + (sqrt(2)*(3*I*A - I*B)*cos(d*x + c) + sqrt(2)*(3*I*A - I*B))*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*((3*A - B)*cos(d*x + c) + 2*A)*sin(d*x + c)/sqrt(cos(d*x + c)))/(
a*d*cos(d*x + c) + a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{a+a\,\cos\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x)),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x)), x)
```

3.478 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=123

$$\frac{(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{ad}$$

```
[Out] -(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))+(A-B)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a/d
```

Rubi [A]
time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3039, 4104, 3872, 3856, 2719, 2720}

$$-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a*d) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(a*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d
*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}\,(B+A\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{1}{2}a(A+B)\sec(c\cdots}{\sqrt{\sec(c+dx)}}}{a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{(A-B)\int \frac{1}{\sqrt{\sec(c+d\cdots}}}{2a}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\left((A-B)\sqrt{\cos(c+dx\cdots}\right.}{\cdots}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A+\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 1.20, size = 200, normalized size = 1.63

$$-\frac{e^{-\frac{1}{2}i(4c+dx)}(-1+e^{2ic})\left(3i(A+B)\left(1+e^{i(c+dx)}\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(A-B)\left(-3\left(1+e^{2i(c+dx)}\right)+e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\right)\left(\csc\left(\frac{c}{2}\right)+i\sec\left(\frac{c}{2}\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x]),x]
```

```
[Out] -1/24*((-1 + E^((2*I)*c))*((3*I)*(A + B)*(1 + E^(I*(c + d*x)))*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2] + (A - B)*(-3*(1 + E^((2*I)*(c + d*x))) +
E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))*(Csc[c/2] + I*Sec[c/2])*S
ec[(c + d*x)/2]*Sqrt[Sec[c + d*x]])/(a*d*E^((I/2)*(4*c + d*x)))
```

Maple [A]
time = 0.37, size = 243, normalized size = 1.98

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 241, normalized size = 1.96

$$-\frac{2\,(A-B)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\left(\sqrt{2}\,(-i\,A-i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(-i\,A-i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-\left(\sqrt{2}\,(i\,A+i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(i\,A+i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-\left(\sqrt{2}\,(i\,A-i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(i\,A-i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-\left(\sqrt{2}\,(-i\,A+i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(-i\,A+i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{2\,(ad\cos\left(dx+c\right)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/2*(2*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(-I*A - I*B)*cos
(d*x + c) + sqrt(2)*(-I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) - (sqrt(2)*(I*A + I*B)*cos(d*x + c) + sqrt(2)*(I*A + I*B))
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (sqrt(2)*(I*A
- I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - (sqrt(2)*(-I*A + I*B)*c
os(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\sec\left(c+dx\right)}}{\cos\left(c+dx\right)+1}\,dx+\int \frac{B\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}}{\cos\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x) + Integral(B*cos(c +
d*x)*sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{a+a\,\cos\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x)),x)
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x)), x)
```

3.479 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=125

$$-\frac{(A-3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{(A-B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}$$

```
[Out] (A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))-(A-3*B)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.16, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3039, 4105, 3872, 3856, 2719, 2720}

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -(((A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) + ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[
c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(A-3B)+\frac{1}{2}a(A-B)\sec(c+dx}{\sqrt{\sec(c+dx)}}}{a^2}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-3B)\int \frac{1}{\sqrt{\sec(c+dx)}}}{2a}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left((A-3B)\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{(A-3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 2.76, size = 422, normalized size = 3.38

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{2}\,Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)-6\sqrt{2}\,Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)+\frac{6\left((A-2B)\cos\left(\frac{1}{2}(c-dx)\right)-B\cos\left(\frac{1}{2}(3c+dx)\right)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sec(c+dx)}}+12A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}-12B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}\right)}{6ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))]))/E^(I*d*x) - (6*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((
2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I
)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4
, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + (6*((A - 2*B)*Cos[(c - d*x)/2] -
 B*Cos[(3*c + d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/Sqrt[Sec[c + d*x
]] + 12*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] -
 12*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]))/(6*
a*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.43, size = 244, normalized size = 1.95

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 237, normalized size = 1.90

$$\frac{2\,(A-B)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\left(\sqrt{2}\,(-i\,A+i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(-i\,A+i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(\sqrt{2}\,(i\,A-i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(i\,A-i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+\left(\sqrt{2}\,(-i\,A+3i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(-i\,A+3i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\left(\sqrt{2}\,(i\,A-3i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(i\,A-3i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/2*(2*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(-I*A + I*B)*cos(
d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + (sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (sqrt(2)*(-I*A
+ 3*I*B)*cos(d*x + c) + sqrt(2)*(-I*A + 3*I*B))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + (sqrt(2)*(I*A - 3*
I*B)*cos(d*x + c) + sqrt(2)*(I*A - 3*I*B))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}+\sqrt{\sec\left(c+dx\right)}}\,dx+\int \frac{B\cos\left(c+dx\right)}{\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}+\sqrt{\sec\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x) + In
tegral(B*cos(c + d*x)/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))
), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(a+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))),x)
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))), x)
```

3.480 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{(3A-5B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{3ad}$$

```
[Out] -1/3*(3*A-5*B)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)+(A-B)*sin(d*x+c)/d/(a+a*sec(
d*x+c))/sec(d*x+c)^(1/2)+3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/a/d-1/3*(3*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]

time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3039, 4105, 3872, 3854, 3856, 2720, 2719}

$$-\frac{(3A-5B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}+\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)}-\frac{(3A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) - ((3*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*a*d) - ((3*A - 5*B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
 + ((A - B)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(3A-5B)+\frac{3}{2}a(A-B)\sec(c+dx}{\sec^{\frac{3}{2}}(c+dx)}}{a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} - \frac{(3A-5B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} +$$

$$= -\frac{(3A-5B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} - \frac{(}{}$$

$$= \frac{3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(3A-5}{3ad\sqrt{}}$$

$$= \frac{3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(3A-5}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.35, size = 444, normalized size = 2.72

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-6\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)+6\sqrt{2}Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)-12A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}+20B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left((12A-13B)\cos\left(\frac{1}{2}(c-dx)\right)+(6A-5B)\cos\left(\frac{1}{2}(3c+dx)\right)-2B\sin(c)\sin\left(\frac{3}{2}(c+dx)\right)\right)}{\sqrt{\sec(c+dx)}}\right)}{6ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((-6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))]))/E^(I*d*x) + (6*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*
I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - 12*A*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 20*B*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - (Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]*((1
2*A - 13*B)*Cos[(c - d*x)/2] + (6*A - 5*B)*Cos[(3*c + d*x)/2] - 2*B*Sin[c]*
Sin[(3*(c + d*x))/2]))/Sqrt[Sec[c + d*x]]))/(6*a*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.38, size = 262, normalized size = 1.61

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(3\right.\right.}{3a\cos\left(\frac{dx}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/
2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*A*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))+8*B*sin(1/2*d*x+1/2*c)^6+(6*A-18*B)*sin(1/2*d*x+1/2*c)^4+(-3*A+
7*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 261, normalized size = 1.60

$$\frac{\left(\sqrt{2}\,(3i\,A-5i\,B)\cos(dx+c)+\sqrt{2}\,(3i\,A-5i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(\sqrt{2}\,(-3i\,A+5i\,B)\cos(dx+c)+\sqrt{2}\,(-3i\,A+5i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-9\left(\sqrt{2}\,(-i\,A+i\,B)\cos(dx+c)+\sqrt{2}\,(-i\,A+i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-9\left(\sqrt{2}\,(i\,A-i\,B)\cos(dx+c)+\sqrt{2}\,(i\,A-i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(2\,B\cos(dx+c)^2-(3\,A-5\,B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/6*((sqrt(2)*(3*I*A - 5*I*B)*cos(d*x + c) + sqrt(2)*(3*I*A - 5*I*B))*weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(-3*I*A + 5
*I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A + 5*I*B))*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 9*(sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt
```

```
(2)*(-I*A + I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - 9*(sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I
*A - I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + 2*(2*B*cos(d*x + c)^2 - (3*A - 5*B)*cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos\left(c+dx\right)\sec^{\frac{3}{2}}\left(c+dx\right)+\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int \frac{B\cos\left(c+dx\right)}{\cos\left(c+dx\right)\sec^{\frac{3}{2}}\left(c+dx\right)+\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] (Integral(A/(cos(c + d*x)*sec(c + d*x)**(3/2) + sec(c + d*x)**(3/2)), x) +
Integral(B*cos(c + d*x)/(cos(c + d*x)*sec(c + d*x)**(3/2) + sec(c + d*x)**(
3/2)), x))/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))), x)
```

3.481 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=196

$$-\frac{3(5A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad}+\frac{5(A-B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\text{s}}}{3ad}$$

```
[Out] -1/5*(5*A-7*B)*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)+(A-B)*sin(d*x+c)/d/sec(d*x+c
)^(3/2)/(a+a*sec(d*x+c))+5/3*(A-B)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-3/5*(5*A
-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+5/3*(A-B)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.19, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4105, 3872, 3854, 3856, 2719, 2720}

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{(5A-7B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}+\frac{5(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(5A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (-3*(5*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a*d) + (5*(A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a*d) - ((5*A - 7*B)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(
3/2)) + (5*(A - B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) + ((A - B)*Sin[
c + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(5A-7B)+\frac{5}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}}{a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{(5A-7B)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{2a} + $$

$$= -\frac{(5A-7B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} + \frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{(A-}{d\sec^{\frac{3}{2}}(c+dx}$$

$$= -\frac{(5A-7B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} + \frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{(A-}{d\sec^{\frac{3}{2}}(c+dx}$$

$$= -\frac{3(5A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{5(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.44, size = 518, normalized size = 2.64

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(60\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)-84\sqrt{2}Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)+200A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}-200B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}\left(3(40A-51B+(20A-33B)\cos(2c))\cos(dx)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)+40(A-B)\cos(2dx)\sin(2c)+12B\cos(3dx)\sin(3c)-120(A-B)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)-12(20A-33B)\cos(c)\sin(dx)+40(A-B)\cos(2c)\sin(2dx)+12B\cos(3c)\sin(3dx)-120(A-B)\tan\left(\frac{c}{2}\right)\right)\right)}{60ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((60*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))]))/E^(I*d*x) - (84*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^
((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2
*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3
/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + 200*A*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 200*B*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + Sqrt[Sec[c + d*x]]*(3*(40*A - 51*B +
 (20*A - 33*B)*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2] + 40*(A - B)*Cos[2*d*x]
*Sin[2*c] + 12*B*Cos[3*d*x]*Sin[3*c] - 120*(A - B)*Sec[c/2]*Sec[(c + d*x)/2
]*Sin[(d*x)/2] - 12*(20*A - 33*B)*Cos[c]*Sin[d*x] + 40*(A - B)*Cos[2*c]*Sin
[2*d*x] + 12*B*Cos[3*c]*Sin[3*d*x] - 120*(A - B)*Tan[c/2])))/(60*a*d*(1 + C
os[c + d*x]))
```

Maple [A]
time = 0.39, size = 281, normalized size = 1.43

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right.\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+
1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25*A*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*A*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2)))+48*B*sin(1/2*d*x+1/2*c)^8+(-40*A-56*B)*sin(1/2*d*x+1/2*c)
^6+(90*A-30*B)*sin(1/2*d*x+1/2*c)^4+(-35*A+23*B)*sin(1/2*d*x+1/2*c)^2)/a/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 278, normalized size = 1.42

$$-\frac{25\left(\sqrt{2}\,(i\,A-i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(i\,A-i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+25\left(\sqrt{2}\,(-i\,A+i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(-i\,A+i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+9\left(\sqrt{2}\,(5i\,A-7i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(5i\,A-7i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+9\left(\sqrt{2}\,(-5i\,A+7i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(-5i\,A+7i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-\frac{2\left(6\,B\cos(dx+c)^3+2\,(5\,A-2\,B)\cos(dx+c)^2+25\,(A-B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{30\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/30*(25*(sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*(sqrt(2)*(-I*A + I*B
```

```
)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 9*(sqrt(2)*(5*I*A - 7*I*B)*cos(d*x + c) + sqrt(2)*(
5*I*A - 7*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + 9*(sqrt(2)*(-5*I*A + 7*I*B)*cos(d*x + c) + sqrt(2)
*(-5*I*A + 7*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) - 2*(6*B*cos(d*x + c)^3 + 2*(5*A - 2*B)*cos(d*x +
 c)^2 + 25*(A - B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(
d*x + c) + a*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
giac")
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))),x)
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))), x)
```

3.482 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=208

$$-\frac{(4A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(5A-2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}$$

```
[Out] -1/3*(5*A-2*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*(A-B)*s
ec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+(4*A-B)*sin(d*x+c)*sec(d*x+
c)^(1/2)/a^2/d-(4*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d-1
/3*(5*A-2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.27, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4104, 3872, 3856, 2720, 3853, 2719}

$$-\frac{(5A-2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(4A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(5A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(4A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -(((4*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) - ((5*A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a^2*d) + ((4*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a
^2*d) - ((5*A - 2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c +
 d*x])) - ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x
])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}a(A-B)+\frac{1}{2}a(7A-}{a+a\sec(c+dx)}}{3a^2}$$

$$= -\frac{(5A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin}{3d(a+a\sec(c+dx}$$

$$= -\frac{(5A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin}{3d(a+a\sec(c+dx}$$

$$= \frac{(4A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} - \frac{(5A-2B)\sec^{\frac{3}{2}}(c+dx)\sin}{3a^2d(1+\sec(c+dx}$$

$$= -\frac{(5A-2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{(4A}{}$$

$$= -\frac{(4A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{(5A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.42, size = 303, normalized size = 1.46

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(29iA-5iB+2i(25A-7B)\cos(c+dx)+17iA\cos(2(c+dx))-5iB\cos(2(c+dx))-i(4A-B)e^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+8(5A-2B)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-i\sin\left(\frac{1}{2}(c+dx)\right)\right)-12A\sin(c+dx)-7A\sin(2(c+dx))+B\sin(2(c+dx))\right)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)}{6a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] -1/6*(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*((29*I)*A - (5*I)*B + (2*I)*(25*A
 - 7*B)*Cos[c + d*x] + (17*I)*A*Cos[2*(c + d*x)] - (5*I)*B*Cos[2*(c + d*x)]
 - (I*(4*A - B)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyper
geometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 8*(5*A
 - 2*B)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Co
s[(c + d*x)/2] - I*Sin[(c + d*x)/2]) - 12*A*Sin[c + d*x] - 7*A*Sin[2*(c + d
*x)] + B*Sin[2*(c + d*x)])*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(a^
2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(240) = 480.
time = 0.52, size = 494, normalized size = 2.38

method	result
default	$-\frac{2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(5A\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(4*A-B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A-10*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(37*A-7*B)*sin(1/2*d*x+1/2*c)^2)/
a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 367, normalized size = 1.76

$$\frac{\left(\sqrt{2}\,(5i\,A-2i\,B)\cos(dx+c)^2-2\sqrt{2}\,(-5i\,A+2i\,B)\cos(dx+c)+\sqrt{2}\,(5i\,A-2i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\left(\sqrt{2}\,(-5i\,A+2i\,B)\cos(dx+c)^2-2\sqrt{2}\,(5i\,A-2i\,B)\cos(dx+c)+\sqrt{2}\,(-5i\,A+2i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3\left(\sqrt{2}\,(4i\,A-i\,B)\cos(dx+c)^2+2\sqrt{2}\,(4i\,A-i\,B)\cos(dx+c)+\sqrt{2}\,(4i\,A-i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3\left(\sqrt{2}\,(-4i\,A+i\,B)\cos(dx+c)^2+2\sqrt{2}\,(-4i\,A+i\,B)\cos(dx+c)+\sqrt{2}\,(-4i\,A+i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+\frac{2\left(3\,(4A-B)\cos(dx+c)^2+(19A-4B)\cos(dx+c)+6A\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] 1/6*((sqrt(2)*(5*I*A - 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-5*I*A + 2*I*B)*c
os(d*x + c) + sqrt(2)*(5*I*A - 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + (sqrt(2)*(-5*I*A + 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2
)*(5*I*A - 2*I*B)*cos(d*x + c) + sqrt(2)*(-5*I*A + 2*I*B))*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(4*I*A - I*B)*cos(d*
x + c)^2 + 2*sqrt(2)*(4*I*A - I*B)*cos(d*x + c) + sqrt(2)*(4*I*A - I*B))*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 3*(sqrt(2)*(-4*I*A + I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-4*I*A + I*B)*
cos(d*x + c) + sqrt(2)*(-4*I*A + I*B))*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*(4*A - B)*cos(d*x + c)
^2 + (19*A - 4*B)*cos(d*x + c) + 6*A)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2
*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^2, x
)
```

3.483 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=161

$$\frac{A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{(2A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}$$

```
[Out] -1/3*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-A*sin(d*x+c)*se
c(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))+A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/a^2/d+1/3*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/
d
```

Rubi [A]
time = 0.25, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3039, 4104, 3872, 3856, 2719, 2720}

$$\frac{(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{A\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + ((2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*a^2*d) - (A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x
])) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2
)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sqrt{\sec(c+dx)}\,(-\frac{1}{2}a(A-}{a+a\sec(}}{3a}$$

$$= -\frac{A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3d(a+a\sec(c+dx))^2}$$

$$= \frac{A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A+B)\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.13, size = 256, normalized size = 1.59

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-iAe^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+8(2A+B)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-i\sin\left(\frac{1}{2}(c+dx)\right)\right)+2i\cos(c+dx)(7A-B+(5A+B)\cos(c+dx)+i(A-B)\sin(c+dx))\right)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)}{6a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-I)*A*(1 + E^(I*(c + d*x)))^3*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^(I*(c + d*x)) + 8*(2*A + B)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*Co
s[c + d*x]*(7*A - B + (5*A + B)*Cos[c + d*x] + I*(A - B)*Sin[c + d*x]))*(Co
s[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d
*x])^2)
```

Maple [A]
time = 0.42, size = 350, normalized size = 2.17

method	result

default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*d
*x+1/2*c)^6-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2*
d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^3-16*A*cos(1/2*d*x+1/2*c)^4-2*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1
/2*d*x+1/2*c)^2+3*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 326, normalized size = 2.02

$$\frac{\left(\sqrt{2}\left(-2iA-iB\right)\cos\left(dx+c\right)^2-2\sqrt{2}\left(2iA+iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-2iA-iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\left(\sqrt{2}\left(2iA+iB\right)\cos\left(dx+c\right)^2-2\sqrt{2}\left(-2iA-iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(2iA+iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-3\left(-i\sqrt{2}A\cos\left(dx+c\right)^2-2i\sqrt{2}A\cos\left(dx+c\right)-i\sqrt{2}A\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-3\left(i\sqrt{2}A\cos\left(dx+c\right)^2+2i\sqrt{2}A\cos\left(dx+c\right)+i\sqrt{2}A\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{2\left(3A\cos(dx+c)^2+(4A-B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{6\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] 1/6*((sqrt(2)*(-2*I*A - I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(2*I*A + I*B)*cos(d
*x + c) + sqrt(2)*(-2*I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + (sqrt(2)*(2*I*A + I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-2*I*
```

```
A - I*B)*cos(d*x + c) + sqrt(2)*(2*I*A + I*B))*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 3*(-I*sqrt(2)*A*cos(d*x + c)^2 - 2*I*sqrt(2
)*A*cos(d*x + c) - I*sqrt(2)*A)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(I*sqrt(2)*A*cos(d*x + c)^2 + 2*
I*sqrt(2)*A*cos(d*x + c) + I*sqrt(2)*A)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*A*cos(d*x + c)^2 + (4
*A - B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^
2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\sec{(c+dx)}}}{\cos^2{(c+dx)}+2\cos{(c+dx)}+1}\,dx + \int \frac{B\cos{(c+dx)}\sqrt{\sec{(c+dx)}}}{\cos^2{(c+dx)}+2\cos{(c+dx)}+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) +
 Integral(B*cos(c + d*x)*sqrt(sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*
x) + 1), x))/a**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos{(c+d\,x)})\,\sqrt{\frac{1}{\cos{(c+d\,x)}}}}{{(a+a\,\cos{(c+d\,x)})}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^2, x
)
```

3.484 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=168

$$-\frac{B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{(A+2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}$$

```
[Out] 1/3*(A+2*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))-1/3*(A-B)*sin(
d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^2-B*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/a^2/d+1/3*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a^2/d
```

Rubi [A]
time = 0.25, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4104, 4105, 3872, 3856, 2719, 2720}

$$\frac{(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -((B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d)) + ((A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*a^2*d) + ((A + 2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1
+ Sec[c + d*x])) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Se
c[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}\,(B+A\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{3}{2}a(A+B)\sec(c\cdots}{\sqrt{\sec(c+dx)}\,(a+a\sec(\cdots}}{3a^2}$$

$$= \frac{(A+2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sqrt{\sec(c+dx)}\,\text{si}\cdots}{3d(a+a\sec(c+d\cdots}$$

$$= \frac{(A+2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sqrt{\sec(c+dx)}\,\text{si}\cdots}{3d(a+a\sec(c+d\cdots}$$

$$= \frac{(A+2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sqrt{\sec(c+dx)}\,\text{si}\cdots}{3d(a+a\sec(c+d\cdots}$$

$$= -\frac{B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(A+2B)\cdots}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.57, size = 256, normalized size = 1.52

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(8(A+2B)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-i\sin\left(\frac{1}{2}(c+dx)\right)\right)+i\left(Be^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+2\cos(c+dx)(-A-5B+(A-7B)\cos(c+dx)-i(A-B)\sin(c+dx))\right)\right)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)}{6a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(8*(A + 2*B)*Cos[(c + d*x)/2]^3*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/
2]) + I*((B*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 2*Cos[c +
d*x]*(-A - 5*B + (A - 7*B)*Cos[c + d*x] - I*(A - B)*Sin[c + d*x])))*(Cos[(c
 + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])
^2)
```

Maple [A]
time = 0.44, size = 350, normalized size = 2.08

method	result

default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{Ellipti}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+12*B*cos(1/2*d*x+1/2*c)^6+4*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4-20*B*cos(1/2*d*x+1/2*c)^4-3*A*cos(
1/2*d*x+1/2*c)^2+9*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

<u>Fricas</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 324, normalized size = 1.93

$$\frac{\left(\sqrt{2}\,(-iA-2iB)\cos(dx+c)^2-2\sqrt{2}\,(iA+2iB)\cos(dx+c)+\sqrt{2}\,(-iA-2iB)\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(\sqrt{2}\,(iA+2iB)\cos(dx+c)^2-2\sqrt{2}\,(-iA-2iB)\cos(dx+c)+\sqrt{2}\,(iA+2iB)\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\left(i\sqrt{2}\,B\cos(dx+c)^2+2i\sqrt{2}\,B\cos(dx+c)+i\sqrt{2}\,B\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\left(-i\sqrt{2}\,B\cos(dx+c)^2-2i\sqrt{2}\,B\cos(dx+c)-i\sqrt{2}\,B\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3B\cos(dx+c)^2+(A+2B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/6*((sqrt(2)*(-I*A - 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(I*A + 2*I*B)*cos(d
*x + c) + sqrt(2)*(-I*A - 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + (sqrt(2)*(I*A + 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-I*A
```

```
- 2*I*B)*cos(d*x + c) + sqrt(2)*(I*A + 2*I*B))*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*B*cos(d*x + c)^2 + 2*I*sqrt(2)
*B*cos(d*x + c) + I*sqrt(2)*B)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*B*cos(d*x + c)^2 - 2*
I*sqrt(2)*B*cos(d*x + c) - I*sqrt(2)*B)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B*cos(d*x + c)^2 + (A
 + 2*B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^
2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos^2(c+dx)\sqrt{\sec(c+dx)}+2\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx+\int \frac{B\cos(c+dx)}{\cos^2(c+dx)\sqrt{\sec(c+dx)}+2\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(cos(c + d*x)**2*sqrt(sec(c + d*x)) + 2*cos(c + d*x)*sqrt(sec(c
 + d*x)) + sqrt(sec(c + d*x))), x) + Integral(B*cos(c + d*x)/(cos(c + d*x)*
*2*sqrt(sec(c + d*x)) + 2*cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*
x))), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2), x
)
```

3.485 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=176

$$-\frac{(A-4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{(2A-5B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{3a^2d}$$

```
[Out] 1/3*(2*A-5*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))+1/3*(A-B)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^2-(A-4*B)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+1/3*(2*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.26, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3039, 4105, 3872, 3856, 2719, 2720}

$$\frac{(2A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -(((A - 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) + ((2*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a^2*d) + ((2*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(3*a^2*d*(1 + Sec[c + d*x])) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d*(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(A-7B)+\frac{3}{2}a(A-B)\sec(c+dx}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx}}{3a^2}$$

$$= \frac{(2A-5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}\,\mathrm{si}}{3d(a+a\sec(c+dx}$$

$$= \frac{(2A-5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}\,\mathrm{si}}{3d(a+a\sec(c+dx}$$

$$= \frac{(2A-5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}\,\mathrm{si}}{3d(a+a\sec(c+dx}$$

$$= -\frac{(A-4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.83, size = 732, normalized size = 4.16

$$\frac{\sqrt{2}\,Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sec\left(\frac{c}{2}\right)}{3d(a+a\cos(c+dx))^2}-\frac{4\sqrt{2}\,Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sec\left(\frac{c}{2}\right)}{3d(a+a\cos(c+dx))^2}+\frac{4A\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\csc\left(\frac{c}{2}\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)\sec\left(\frac{c}{2}\right)\sqrt{\sec(c+dx)}\sin(c)}{3d(a+a\cos(c+dx))^2}-\frac{10B\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\csc\left(\frac{c}{2}\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)\sec\left(\frac{c}{2}\right)\sqrt{\sec(c+dx)}\sin(c)}{3d(a+a\cos(c+dx))^2}+\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(-\frac{2(-A+3B+B\cos(2c))\cos(dx)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)}{d}-\frac{4\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\left(4A\sin\left(\frac{dx}{2}\right)-7B\sin\left(\frac{dx}{2}\right)\right)}{3d}+\frac{2\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{3d}+\frac{8B\cos(c)\sin(dx)}{d}-\frac{4(4A-7B)\tan\left(\frac{c}{2}\right)}{3d}+\frac{2(A-B)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\tan\left(\frac{c}{2}\right)}{3d}\right)}{(a+a\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) - (4*Sq
rt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(
c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]
+ E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*
I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) + (4*A*Cos
[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec
[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2) - (10*B*Cos[c
/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c
/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2) + (Cos[c/2 + (d
*x)/2]^4*Sqrt[Sec[c + d*x]]*((-2*(-A + 3*B + B*Cos[2*c])*Cos[d*x]*Csc[c/2]*
Sec[c/2])/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(4*A*Sin[(d*x)/2] - 7*B*Sin[(d
*x)/2]))/(3*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(
d*x)/2]))/(3*d) + (8*B*Cos[c]*Sin[d*x])/d - (4*(4*A - 7*B)*Tan[c/2])/(3*d)
+ (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [A]
time = 0.49, size = 421, normalized size = 2.39

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
```

```
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*B*cos(1/2*d*x+1/2*c)^6-10*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-24*B*cos(1/2*d*x+1/2*c)^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4+38*B*cos(1/2*d*x+1/2*c)^4+9*A
*cos(1/2*d*x+1/2*c)^2-15*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)
^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 362, normalized size = 2.06

$$\frac{\left(\sqrt{2}\,(-2i\,A+5i\,B)\cos(dx+c)^2-2\sqrt{2}\,(2i\,A-5i\,B)\cos(dx+c)+\sqrt{2}\,(-2i\,A+5i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(\sqrt{2}\,(2i\,A-5i\,B)\cos(dx+c)^2-2\sqrt{2}\,(-2i\,A+5i\,B)\cos(dx+c)+\sqrt{2}\,(2i\,A-5i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\left(\sqrt{2}\,(i\,A-4i\,B)\cos(dx+c)^2+2\sqrt{2}\,(i\,A-4i\,B)\cos(dx+c)+\sqrt{2}\,(i\,A-4i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\left(\sqrt{2}\,(-i\,A+4i\,B)\cos(dx+c)^2+2\sqrt{2}\,(-i\,A+4i\,B)\cos(dx+c)+\sqrt{2}\,(-i\,A+4i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3(A-2B)\cos(dx+c)^2+(2A-5B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/6*((sqrt(2)*(-2*I*A + 5*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(2*I*A - 5*I*B)*c
os(d*x + c) + sqrt(2)*(-2*I*A + 5*I*B))*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + (sqrt(2)*(2*I*A - 5*I*B)*cos(d*x + c)^2 - 2*sqrt(2
)*(-2*I*A + 5*I*B)*cos(d*x + c) + sqrt(2)*(2*I*A - 5*I*B))*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(I*A - 4*I*B)*cos(d*
x + c)^2 + 2*sqrt(2)*(I*A - 4*I*B)*cos(d*x + c) + sqrt(2)*(I*A - 4*I*B))*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 3*(sqrt(2)*(-I*A + 4*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-I*A + 4*I*B)*
cos(d*x + c) + sqrt(2)*(-I*A + 4*I*B))*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*(A - 2*B)*cos(d*x + c)
^2 + (2*A - 5*B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(
d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2), x
)
```

3.486 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=206

$$\frac{(4A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5(A-2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{3a^2d}$$

```
[Out] -5/3*(A-2*B)*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)+1/3*(4*A-7*B)*sin(d*x+c)/a^2
/d/(1+sec(d*x+c))/sec(d*x+c)^(1/2)+1/3*(A-B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^
2/sec(d*x+c)^(1/2)+(4*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a
^2/d-5/3*(A-2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.28, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4105, 3872, 3854, 3856, 2720, 2719}

$$-\frac{5(A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(\sec(c+dx)+1)}-\frac{5(A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((4*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d) - (5*(A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*a^2*d) - (5*(A - 2*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c
+ d*x]]) + ((4*A - 7*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[
c + d*x])) + ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c +
d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2} + \frac{\int \frac{-\frac{3}{2}a(A-3B)+\frac{5}{2}a(A-B)\sec(c+}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}}{3a^2}$$

$$= \frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} + \frac{(A-B)\sin(c+d}{3d\sqrt{\sec(c+dx)}\,(a+a\sec}$$

$$= \frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} + \frac{(A-B)\sin(c+d}{3d\sqrt{\sec(c+dx)}\,(a+a\sec}$$

$$= -\frac{5(A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} + \frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} +$$

$$= \frac{(4A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5(A-}{3a^2d}$$

$$= \frac{(4A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5(A-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.96, size = 777, normalized size = 3.77

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] (-4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) + (7
*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) - (10*A
*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]
*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2) + (20*B*C
```

```
os[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*S
ec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2) + (Cos[c/2
+ (d*x)/2]^4*Sqrt[Sec[c + d*x]]*((-2*(3*A - 5*B + A*Cos[2*c] - 2*B*Cos[2*c]
)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (4*B*Cos[2*d*x]*Sin[2*c])/(3*d) + (4*Sec[
c/2]*Sec[c/2 + (d*x)/2]*(7*A*Sin[(d*x)/2] - 10*B*Sin[(d*x)/2]))/(3*d) - (2*
Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (8
*(A - 2*B)*Cos[c]*Sin[d*x])/d + (4*B*Cos[2*c]*Sin[2*d*x])/(3*d) + (4*(7*A -
 10*B)*Tan[c/2])/(3*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/
(a + a*Cos[c + d*x])^2
```

Maple [A]
time = 0.50, size = 435, normalized size = 2.11

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-16B\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*B*cos(1/2*
d*x+1/2*c)^8+24*A*cos(1/2*d*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1
/2*d*x+1/2*c)^3+24*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*co
s(1/2*d*x+1/2*c)^6-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-42*B
*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1/2*c)^4+4
8*B*cos(1/2*d*x+1/2*c)^4+15*A*cos(1/2*d*x+1/2*c)^2-21*B*cos(1/2*d*x+1/2*c)^
2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 376, normalized size = 1.83

$$-\frac{5\left(\sqrt{2}(-iA+2iB)\cos(dx+c)^2+2\sqrt{2}(-iA+2iB)\cos(dx+c)+\sqrt{2}(-iA+2iB)\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(\sqrt{2}(iA-2iB)\cos(dx+c)^2+2\sqrt{2}(iA-2iB)\cos(dx+c)+\sqrt{2}(iA-2iB)\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\left(\sqrt{2}(-4iA+7iB)\cos(dx+c)^2+2\sqrt{2}(-4iA+7iB)\cos(dx+c)+\sqrt{2}(-4iA+7iB)\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(\sqrt{2}(4iA-7iB)\cos(dx+c)^2+2\sqrt{2}(4iA-7iB)\cos(dx+c)+\sqrt{2}(4iA-7iB)\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(2B\cos(dx+c)^3-(6A-13B)\cos(dx+c)^2-5(A-2B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] -1/6*(5*(sqrt(2)*(-I*A + 2*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-I*A + 2*I*B)*cos(d*x + c) + sqrt(2)*(-I*A + 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)*(I*A - 2*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(I*A - 2*I*B)*cos(d*x + c) + sqrt(2)*(I*A - 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(-4*I*A + 7*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-4*I*A + 7*I*B)*cos(d*x + c) + sqrt(2)*(-4*I*A + 7*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(4*I*A - 7*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(4*I*A - 7*I*B)*cos(d*x + c) + sqrt(2)*(4*I*A - 7*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(2*B*cos(d*x + c)^3 - (6*A - 13*B)*cos(d*x + c)^2 - 5*(A - 2*B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2),x)
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2), x
)
```

3.487 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=261

$$-\frac{(49A-9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{6a^3d}$$

```
[Out] -1/5*(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*(8*A-3*B)*
sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-1/6*(13*A-3*B)*sec(d*x+c
)^(3/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))+1/10*(49*A-9*B)*sin(d*x+c)*sec(d*
x+c)^(1/2)/a^3/d-1/10*(49*A-9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/a^3/d-1/6*(13*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.39, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4104, 3872, 3856, 2720, 3853, 2719}

$$-\frac{(13A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(49A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(49A-9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{(8A-3B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -1/10*((49*A - 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(a^3*d) - ((13*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((49*A - 9*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(10*a^3*d) - ((A - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*
Sec[c + d*x])^3) - ((8*A - 3*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(15*a*d*(a
 + a*Sec[c + d*x])^2) - ((13*A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d
*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{5}{2}a(A-B)+\frac{1}{2}a(11A}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(8A-3B)\sec^{\frac{5}{2}}(c+dx)\sin}{15ad(a+a\sec(c+dx}$$

$$= -\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(8A-3B)\sec^{\frac{5}{2}}(c+dx)\sin}{15ad(a+a\sec(c+dx}$$

$$= -\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(8A-3B)\sec^{\frac{5}{2}}(c+dx)\sin}{15ad(a+a\sec(c+dx}$$

$$= \frac{(49A-9B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{10a^3d} - \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin}{5d(a+a\sec(c+d}$$

$$= -\frac{(13A-3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d} + \frac{(4}{}$$

$$= -\frac{(49A-9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(1}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.77, size = 358, normalized size = 1.37

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-i(49A-9B)e^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)^5\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+160(13A-3B)\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-i\sin\left(\frac{1}{2}(c+dx)\right)\right)+2i(642A-102B+(1082A-207B)\cos(c+dx)+6(87A-17B)\cos(2(c+dx))+106A\cos(3(c+dx))-21B\cos(3(c+dx))+161iA\sin(c+dx)-6iB\sin(c+dx)+148iA\sin(2(c+dx))-18iB\sin(2(c+dx))+41iA\sin(3(c+dx))-6iB\sin(3(c+dx)))\right)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)}{120a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] -1/120*(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-I)*(49*A - 9*B)*(1 + E^(I*(
c + d*x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + 160*(13*A - 3*B)*Cos[(c + d*x
)/2]^5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*S
in[(c + d*x)/2]) + (2*I)*(642*A - 102*B + (1082*A - 207*B)*Cos[c + d*x] + 6
*(87*A - 17*B)*Cos[2*(c + d*x)] + 106*A*Cos[3*(c + d*x)] - 21*B*Cos[3*(c +
d*x)] + (161*I)*A*Sin[c + d*x] - (6*I)*B*Sin[c + d*x] + (148*I)*A*Sin[2*(c
+ d*x)] - (18*I)*B*Sin[2*(c + d*x)] + (41*I)*A*Sin[3*(c + d*x)] - (6*I)*B*S
in[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(a^3*d*E^(I*
d*x)*(1 + Cos[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(285) = 570$.
time = 0.61, size = 685, normalized size = 2.62

method	result
default	$-\frac{-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(65A\,\mathrm{EllipticF}\left(c\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x
+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(49*A-9*B)*
sin(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(817*A-147*B)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(248*A-43*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(439*A-69*B)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*
d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 481, normalized size = 1.84

$$-\frac{5\left(\sqrt{2}(-13iA+3iB)\cos(dx+c)^3+3\sqrt{2}(-13iA+3iB)\cos(dx+c)^2+3\sqrt{2}(-13iA+3iB)\cos(dx+c)+\sqrt{2}(-13iA+3iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(\sqrt{2}(13iA-3iB)\cos(dx+c)^3+3\sqrt{2}(13iA-3iB)\cos(dx+c)^2+3\sqrt{2}(13iA-3iB)\cos(dx+c)+\sqrt{2}(13iA-3iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\left(\sqrt{2}(49iA-9iB)\cos(dx+c)^3+3\sqrt{2}(49iA-9iB)\cos(dx+c)^2+3\sqrt{2}(49iA-9iB)\cos(dx+c)+\sqrt{2}(49iA-9iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(\sqrt{2}(-49iA+9iB)\cos(dx+c)^3+3\sqrt{2}(-49iA+9iB)\cos(dx+c)^2+3\sqrt{2}(-49iA+9iB)\cos(dx+c)+\sqrt{2}(-49iA+9iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(3(49A-9B)\cos(dx+c)^3+2(188A-33B)\cos(dx+c)^2+5(59A-9B)\cos(dx+c)+60A\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(5*(sqrt(2)*(-13*I*A + 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-13*I*A + 3
*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-13*I*A + 3*I*B)*cos(d*x + c) + sqrt(2)*(
-13*I*A + 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
 + 5*(sqrt(2)*(13*I*A - 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(13*I*A - 3*I*B)*
cos(d*x + c)^2 + 3*sqrt(2)*(13*I*A - 3*I*B)*cos(d*x + c) + sqrt(2)*(13*I*A
- 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sq
rt(2)*(49*I*A - 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(49*I*A - 9*I*B)*cos(d*x
+ c)^2 + 3*sqrt(2)*(49*I*A - 9*I*B)*cos(d*x + c) + sqrt(2)*(49*I*A - 9*I*B)
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) + 3*(sqrt(2)*(-49*I*A + 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-49*I*A
 + 9*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-49*I*A + 9*I*B)*cos(d*x + c) + sqrt(
2)*(-49*I*A + 9*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))) - 2*(3*(49*A - 9*B)*cos(d*x + c)^3 + 2*(188*A
- 33*B)*cos(d*x + c)^2 + 5*(59*A - 9*B)*cos(d*x + c) + 60*A)*sin(d*x + c)/s
qrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d
*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x
)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{(a+a\cos(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^3,x)

[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^3, x
)

3.488 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=222

$$\frac{(9A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(3A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d}$$

```
[Out] -1/5*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*(6*A-B)*se
c(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-1/10*(9*A+B)*sin(d*x+c)*se
c(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))+1/10*(9*A+B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(3*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.37, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3039, 4104, 3872, 3856, 2719, 2720}

$$-\frac{(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{(6A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((9*A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(10*a^3*d) + ((3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(6*a^3*d) - ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(
a + a*Sec[c + d*x])^3) - ((6*A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*
d*(a + a*Sec[c + d*x])^2) - ((9*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10
*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}a(A-B)+\frac{1}{2}a(9\right.}{(a+a\sec(c+dx))}}{5a^2}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(6A-B)\sec^{\frac{3}{2}}(c+dx)\sin}{15ad(a+a\sec(c+d}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(6A-B)\sec^{\frac{3}{2}}(c+dx)\sin}{15ad(a+a\sec(c+d}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(6A-B)\sec^{\frac{3}{2}}(c+dx)\sin}{15ad(a+a\sec(c+d}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(6A-B)\sec^{\frac{3}{2}}(c+dx)\sin}{15ad(a+a\sec(c+d}$$

$$= \frac{(9A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.03, size = 793, normalized size = 3.57

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] (-3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) - (S
qrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*
(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]
 + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A*C
os[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*S
ec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) + (2*B*Cos[c/
2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/
```

```
2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 + (d*
x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(9*A + B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d)
 + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d
) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(3*
d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(3*A*Sin[(d*x)/2] + 2*B*Sin[(d*x)/2])
)/(15*d) + (4*(3*A + B)*Tan[c/2])/(3*d) + (4*(3*A + 2*B)*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a
+ a*Cos[c + d*x])^3
```

Maple [A]
time = 0.47, size = 451, normalized size = 2.03

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(108A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-30A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/2
*d*x+1/2*c)^8-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos(
1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8-10*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*B*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-138*A*cos(1/2*d*x+1/2*c)^6-22*B*cos(1/2*d*x+1/2*c)^6+
24*A*cos(1/2*d*x+1/2*c)^4+6*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*x+1/2*c)^2
+7*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 476, normalized size = 2.14

$$-\frac{5\left(\sqrt{2}\,(3iA+iB)\cos(dx+c)^3+3\sqrt{2}\,(3iA+iB)\cos(dx+c)^2+3\sqrt{2}\,(3iA+iB)\cos(dx+c)+\sqrt{2}\,(3iA+iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(\sqrt{2}\,(-3iA-iB)\cos(dx+c)^3+3\sqrt{2}\,(-3iA-iB)\cos(dx+c)^2+3\sqrt{2}\,(-3iA-iB)\cos(dx+c)+\sqrt{2}\,(-3iA-iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\left(\sqrt{2}\,(-9iA-iB)\cos(dx+c)^3+3\sqrt{2}\,(-9iA-iB)\cos(dx+c)^2+3\sqrt{2}\,(-9iA-iB)\cos(dx+c)+\sqrt{2}\,(-9iA-iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(\sqrt{2}\,(9iA+iB)\cos(dx+c)^3+3\sqrt{2}\,(9iA+iB)\cos(dx+c)^2+3\sqrt{2}\,(9iA+iB)\cos(dx+c)+\sqrt{2}\,(9iA+iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3(9A+B)\cos(dx+c)^3+2(33A+2B)\cos(dx+c)^2+5(9A-B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(5*(sqrt(2)*(3*I*A + I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(3*I*A + I*B)*co
s(d*x + c)^2 + 3*sqrt(2)*(3*I*A + I*B)*cos(d*x + c) + sqrt(2)*(3*I*A + I*B)
)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)*(-
3*I*A - I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-3*I*A - I*B)*cos(d*x + c)^2 + 3*s
qrt(2)*(-3*I*A - I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A - I*B))*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(-9*I*A - I*B)*cos
(d*x + c)^3 + 3*sqrt(2)*(-9*I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-9*I*A -
 I*B)*cos(d*x + c) + sqrt(2)*(-9*I*A - I*B))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(9*I*A + I
*B)*cos(d*x + c)^3 + 3*sqrt(2)*(9*I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(9*
I*A + I*B)*cos(d*x + c) + sqrt(2)*(9*I*A + I*B))*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*(9*A + B)*co
s(d*x + c)^3 + 2*(33*A + 2*B)*cos(d*x + c)^2 + 5*(9*A - B)*cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)
^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\sec{(c+dx)}}}{\cos^3{(c+dx)}+3\cos^2{(c+dx)}+3\cos{(c+dx)}+1}\,dx+\int \frac{B\cos{(c+dx)}\sqrt{\sec{(c+dx)}}}{\cos^3{(c+dx)}+3\cos^2{(c+dx)}+3\cos{(c+dx)}+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos
(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*sqrt(sec(c + d*x))/(cos(c + d*
x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^3, x
)
```

3.489 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=216

$$\frac{(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{6a^3d}$$

```
[Out] -1/5*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*(4*A+B)*si
n(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*(A+B)*sin(d*x+c)*sec(d
*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))+1/10*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/a^3/d+1/6*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a^3/d
```

Rubi [A]
time = 0.36, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4104, 4105, 3872, 3856, 2719, 2720}

$$\frac{(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
10*a^3*d) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(6*a^3*d) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a +
a*Sec[c + d*x])^3) - ((4*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a
 + a*Sec[c + d*x])^2) + ((A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3
 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx=\int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$=-\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\ldots$$

$$=-\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(4A+B)\sqrt{\sec(c+dx)}\ldots}{15ad(a+a\sec(c+\ldots}$$

$$=-\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(4A+B)\sqrt{\sec(c+dx)}\ldots}{15ad(a+a\sec(c+\ldots}$$

$$=-\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(4A+B)\sqrt{\sec(c+dx)}\ldots}{15ad(a+a\sec(c+\ldots}$$

$$=-\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(4A+B)\sqrt{\sec(c+dx)}\ldots}{15ad(a+a\sec(c+\ldots}$$

$$=\frac{(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.00, size = 792, normalized size = 3.67

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] -1/15*(Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c
+ d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4
, -E^((2*I)*(c + d*x))])*Sec[c/2])/(d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (
Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)
*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))
] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((
2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A*
Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*
Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (2*B*Cos
```

```
[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec
[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 +
(d*x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(A - B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d
) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(2*A*Sin[(d*x)/2] - 7*B*Sin[(d*x)/2]))
/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]
))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + B*Sin[(d*x)/2])
)/(3*d) + (4*(A + B)*Tan[c/2])/(3*d) + (4*(2*A - 7*B)*Sec[c/2 + (d*x)/2]^2*
Tan[c/2])/(15*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a
*Cos[c + d*x])^3
```

Maple [A]
time = 0.48, size = 451, normalized size = 2.09

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8-10*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*B*cos(1/2*d*x+1/2*c)^5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6+2*B*cos(1/2*d*x+1/2*c)^6+6*A*
cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/2*c)^2-17*
B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 472, normalized size = 2.19

$$-\frac{5\left(\sqrt{2}(iA+iB)\cos(dx+c)^3+3\sqrt{2}(iA+iB)\cos(dx+c)^2+3\sqrt{2}(iA+iB)\cos(dx+c)+\sqrt{2}(iA+iB)\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(\sqrt{2}(-iA-iB)\cos(dx+c)^3+3\sqrt{2}(-iA-iB)\cos(dx+c)^2+3\sqrt{2}(-iA-iB)\cos(dx+c)+\sqrt{2}(-iA-iB)\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\left(\sqrt{2}(-iA+iB)\cos(dx+c)^3+3\sqrt{2}(-iA+iB)\cos(dx+c)^2+3\sqrt{2}(-iA+iB)\cos(dx+c)+\sqrt{2}(-iA+iB)\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(\sqrt{2}(iA-iB)\cos(dx+c)^3+3\sqrt{2}(iA-iB)\cos(dx+c)^2+3\sqrt{2}(iA-iB)\cos(dx+c)+\sqrt{2}(iA-iB)\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3(A-B)\cos(dx+c)^3+2(2A-7B)\cos(dx+c)^2-5(A+B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/60*(5*(sqrt(2)*(I*A + I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A + I*B)*cos(d*
x + c)^2 + 3*sqrt(2)*(I*A + I*B)*cos(d*x + c) + sqrt(2)*(I*A + I*B))*weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)*(-I*A - I*
B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-I*A
 - I*B)*cos(d*x + c) + sqrt(2)*(-I*A - I*B))*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(-I*A + I*B)*cos(d*x + c)^3 + 3*sq
rt(2)*(-I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-I*A + I*B)*cos(d*x + c) + s
qrt(2)*(-I*A + I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(I*A - I*B)*cos(d*x + c)^3 + 3*sqr
t(2)*(I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt
(2)*(I*A - I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))) + 2*(3*(A - B)*cos(d*x + c)^3 + 2*(2*A - 7*B)*cos(d
*x + c)^2 - 5*(A + B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d
*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3), x
)
```

3.490 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=222

$$-\frac{(A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(A+3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d}$$

```
[Out] -1/5*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3+1/15*(2*A+3*B)*
sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*(A+3*B)*sin(d*x+c)*s
ec(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))-1/10*(A+9*B)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.37, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4104, 4105, 3872, 3856, 2719, 2720}

$$\frac{(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(2A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -1/10*((A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a^3*d) + ((A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(6*a^3*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*
d*(a + a*Sec[c + d*x])^3) + ((2*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
15*a*d*(a + a*Sec[c + d*x])^2) + ((A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}\,(B+A\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{5}{2}a(A+B)\sec(c+}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+}}{5a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\sqrt{\sec(c+dx)}}{15ad(a+a\sec(c+}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\sqrt{\sec(c+dx)}}{15ad(a+a\sec(c+}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\sqrt{\sec(c+dx)}}{15ad(a+a\sec(c+}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\sqrt{\sec(c+dx)}}{15ad(a+a\sec(c+}$$

$$= -\frac{(A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.13, size = 793, normalized size = 3.57

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (3*S
qrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*
(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]
 + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A*Co
s[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Se
c[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (2*B*Cos[c
```

```
/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c
/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 + (d*x
)/2]^6*Sqrt[Sec[c + d*x]]*((2*(A + 9*B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) -
 (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(7*A*Sin[(d*x)/2] - 12*B*Sin[(d*x)/2]))/(
15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - 9*B*Sin[(d*x)/2]))
/(3*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2])
)/(5*d) + (4*(A - 9*B)*Tan[c/2])/(3*d) - (4*(7*A - 12*B)*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a
+ a*Cos[c + d*x])^3
```

Maple [A]
time = 0.49, size = 451, normalized size = 2.03

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8+30*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*B*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6-198*B*cos(1/2*d*x+1/2*c)^6-
24*A*cos(1/2*d*x+1/2*c)^4+114*B*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c
)^2-27*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 474, normalized size = 2.14

$$-\frac{5 \left(\sqrt{2} (i A + 3 i B) \cos (d x + c)^3 + 3 \sqrt{2} (i A + 3 i B) \cos (d x + c)^2 + 3 \sqrt{2} (i A + 3 i B) \cos (d x + c) + \sqrt{2} (i A + 3 i B)\right) \mathrm{weierstrassPInverse}(-4, 0, \cos (d x + c) + i \sin (d x + c)) + 5 \left(\sqrt{2} (-i A - 3 i B) \cos (d x + c)^3 + 3 \sqrt{2} (-i A - 3 i B) \cos (d x + c)^2 + 3 \sqrt{2} (-i A - 3 i B) \cos (d x + c) + \sqrt{2} (-i A - 3 i B)\right) \mathrm{weierstrassPInverse}(-4, 0, \cos (d x + c) - i \sin (d x + c)) + 3 \left(\sqrt{2} (i A + 9 i B) \cos (d x + c)^3 + 3 \sqrt{2} (i A + 9 i B) \cos (d x + c)^2 + 3 \sqrt{2} (i A + 9 i B) \cos (d x + c) + \sqrt{2} (i A + 9 i B)\right) \mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos (d x + c) + i \sin (d x + c))) + 3 \left(\sqrt{2} (-i A - 9 i B) \cos (d x + c)^3 + 3 \sqrt{2} (-i A - 9 i B) \cos (d x + c)^2 + 3 \sqrt{2} (-i A - 9 i B) \cos (d x + c) + \sqrt{2} (-i A - 9 i B)\right) \mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos (d x + c) - i \sin (d x + c))) - \frac{2 \left(3 (A + 9 B) \cos (d x + c)^3 + 2 (7 A + 18 B) \cos (d x + c)^2 + 5 (A + 3 B) \cos (d x + c)\right) \sin (d x + c)}{\sqrt{\cos (d x + c)}}}{60 \left(a^3 d \cos (d x + c)^3 + 3 a^3 d \cos (d x + c)^2 + 3 a^3 d \cos (d x + c) + a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="fricas")

[Out] -1/60*(5*(sqrt(2)*(I*A + 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A + 3*I*B)*co
s(d*x + c)^2 + 3*sqrt(2)*(I*A + 3*I*B)*cos(d*x + c) + sqrt(2)*(I*A + 3*I*B)
)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)*(-
I*A - 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A - 3*I*B)*cos(d*x + c)^2 + 3*s
qrt(2)*(-I*A - 3*I*B)*cos(d*x + c) + sqrt(2)*(-I*A - 3*I*B))*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(I*A + 9*I*B)*cos(
d*x + c)^3 + 3*sqrt(2)*(I*A + 9*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*A + 9*I*
B)*cos(d*x + c) + sqrt(2)*(I*A + 9*I*B))*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(-I*A - 9*I*B)
*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A - 9*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-I*A
 - 9*I*B)*cos(d*x + c) + sqrt(2)*(-I*A - 9*I*B))*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*(A + 9*B)*co
s(d*x + c)^3 + 2*(7*A + 18*B)*cos(d*x + c)^2 + 5*(A + 3*B)*cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)
^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="giac")

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3),x)
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3), x
)
```

3.491 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=228

$$-\frac{(9A-49B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(3A-13B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{6a^3d}$$

```
[Out] 1/5*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3+1/15*(3*A-8*B)*s
in(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*(3*A-13*B)*sin(d*x+c)
*sec(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))-1/10*(9*A-49*B)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(3*A-13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.38, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3039, 4105, 3872, 3856, 2719, 2720}

$$\frac{(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-49B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A-8B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15ad(a\sec(c+dx)+a)^2}+\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

```
[Out] -1/10*((9*A - 49*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(a^3*d) + ((3*A - 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(5*d*(a + a*Sec[c + d*x])^3) + ((3*A - 8*B)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A - 13*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{-\frac{1}{2}a(A-11B)+\frac{5}{2}a(A-B)\sec(c+d}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx}}{5a^2}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sqrt{\sec(c+dx)}\,\mathrm{si}}{15ad(a+a\sec(c+dx}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sqrt{\sec(c+dx)}\,\mathrm{si}}{15ad(a+a\sec(c+dx}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sqrt{\sec(c+dx)}\,\mathrm{si}}{15ad(a+a\sec(c+dx}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sqrt{\sec(c+dx)}\,\mathrm{si}}{15ad(a+a\sec(c+dx}$$

$$= -\frac{(9A-49B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.22, size = 817, normalized size = 3.58

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] (3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) - (49
*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A
*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]
*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) - (26*B*Cos
```

```
[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec
[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 +
(d*x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(-9*A + 39*B + 10*B*Cos[2*c])*Cos[d*x]*C
sc[c/2]*Sec[c/2])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(9*A*Sin[(d*x)/2]
- 23*B*Sin[(d*x)/2]))/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(12*A*Sin[(d
*x)/2] - 17*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*S
in[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) + (16*B*Cos[c]*Sin[d*x])/d - (4*(9*A -
 23*B)*Tan[c/2])/(3*d) + (4*(12*A - 17*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(1
5*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x
])^3
```

Maple [A]
time = 0.49, size = 451, normalized size = 1.98

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(108A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+30A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{d}{2}\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348*B*cos(1/2*d*x+1/2*c)^8-130*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-294*B*cos(1/2*d*x+1/2*c)^5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6+578*B*cos(1/2*d*x+1/2
*c)^6+114*A*cos(1/2*d*x+1/2*c)^4-264*B*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*
x+1/2*c)^2+37*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 478, normalized size = 2.10

$$-\frac{5\left(\sqrt{2}(3iA-13iB)\cos(dx+c)^3+3\sqrt{2}(3iA-13iB)\cos(dx+c)^2+3\sqrt{2}(3iA-13iB)\cos(dx+c)+\sqrt{2}(3iA-13iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(\sqrt{2}(-3iA+13iB)\cos(dx+c)^3+3\sqrt{2}(-3iA+13iB)\cos(dx+c)^2+3\sqrt{2}(-3iA+13iB)\cos(dx+c)+\sqrt{2}(-3iA+13iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\left(\sqrt{2}(9iA-49iB)\cos(dx+c)^3+3\sqrt{2}(9iA-49iB)\cos(dx+c)^2+3\sqrt{2}(9iA-49iB)\cos(dx+c)+\sqrt{2}(9iA-49iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(\sqrt{2}(-9iA+49iB)\cos(dx+c)^3+3\sqrt{2}(-9iA+49iB)\cos(dx+c)^2+3\sqrt{2}(-9iA+49iB)\cos(dx+c)+\sqrt{2}(-9iA+49iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(3(9A-29B)\cos(dx+c)^3+2(18A-73B)\cos(dx+c)^2+5(3A-13B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/60*(5*(sqrt(2)*(3*I*A - 13*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(3*I*A - 13*I
*B)*cos(d*x + c)^2 + 3*sqrt(2)*(3*I*A - 13*I*B)*cos(d*x + c) + sqrt(2)*(3*I
*A - 13*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5
*(sqrt(2)*(-3*I*A + 13*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-3*I*A + 13*I*B)*co
s(d*x + c)^2 + 3*sqrt(2)*(-3*I*A + 13*I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A +
 13*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sq
rt(2)*(9*I*A - 49*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(9*I*A - 49*I*B)*cos(d*x
+ c)^2 + 3*sqrt(2)*(9*I*A - 49*I*B)*cos(d*x + c) + sqrt(2)*(9*I*A - 49*I*B)
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) + 3*(sqrt(2)*(-9*I*A + 49*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-9*I*A
+ 49*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-9*I*A + 49*I*B)*cos(d*x + c) + sqrt(
2)*(-9*I*A + 49*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))) - 2*(3*(9*A - 29*B)*cos(d*x + c)^3 + 2*(18*A -
 73*B)*cos(d*x + c)^2 + 5*(3*A - 13*B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
 + c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3), x
)
```

3.492 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=259

$$\frac{7(7A-17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-33B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{6a^3d}$$

```
[Out] -1/6*(13*A-33*B)*sin(d*x+c)/a^3/d/sec(d*x+c)^(1/2)+1/5*(A-B)*sin(d*x+c)/d/(
a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2)+1/3*(A-2*B)*sin(d*x+c)/a/d/(a+a*sec(d*x+
c))^2/sec(d*x+c)^(1/2)+7/30*(7*A-17*B)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/se
c(d*x+c)^(1/2)+7/10*(7*A-17*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^3/d-1/6*(13*A-33*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]

time = 0.40, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4105, 3872, 3854, 3856, 2720, 2719}

$$-\frac{(13A-33B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}+\frac{7(7A-17B)\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\,(a^3\sec(c+dx)+a^3)}-\frac{(13A-33B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(7A-17B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-2B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^2}+\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (7*(7*A - 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) - ((13*A - 33*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((13*A - 33*B)*Sin[c + d*x])/(6*a^3*d*S
qrt[Sec[c + d*x]]) + ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]*(a + a*
Sec[c + d*x])^3) + ((A - 2*B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]*(a +
a*Sec[c + d*x])^2) + (7*(7*A - 17*B)*Sin[c + d*x])/(30*d*Sqrt[Sec[c + d*x]]
*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} + \frac{\int \frac{-\frac{1}{2}a(3A-13B)+\frac{7}{2}a(A-B)\sec(}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}}{5a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} + \frac{(A-2B)\sin(c+}{3ad\sqrt{\sec(c+dx)}\,(a+a\,s}$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} + \frac{(A-2B)\sin(c+}{3ad\sqrt{\sec(c+dx)}\,(a+a\,s}$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} + \frac{(A-2B)\sin(c+}{3ad\sqrt{\sec(c+dx)}\,(a+a\,s}$$

$$= -\frac{(13A-33B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}} + \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}$$

$$= \frac{7(7A-17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(13}{(}$$

$$= \frac{7(7A-17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(13}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.89, size = 589, normalized size = 2.27

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),
x]
```

```
[Out] (Cos[(c + d*x)/2]^6*((-98*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))]))/E^(I*d*x) + (238*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^(
(2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - (((806*A - 1961*B)*Cos[(c -
```

```
d*x)/2] + (664*A - 1609*B)*Cos[(3*c + d*x)/2] + 470*A*Cos[(c + 3*d*x)/2] -
1165*B*Cos[(c + 3*d*x)/2] + 265*A*Cos[(5*c + 3*d*x)/2] - 620*B*Cos[(5*c + 3
*d*x)/2] + 117*A*Cos[(3*c + 5*d*x)/2] - 292*B*Cos[(3*c + 5*d*x)/2] + 30*A*C
os[(7*c + 5*d*x)/2] - 65*B*Cos[(7*c + 5*d*x)/2] - 5*B*Cos[(5*c + 7*d*x)/2]
+ 5*B*Cos[(9*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(8*Sqrt[S
ec[c + d*x]]) - 260*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]] + 660*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]]))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]
time = 0.46, size = 465, normalized size = 1.80

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-160B\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+348A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130A\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-160*B*cos(1/
2*d*x+1/2*c)^10+348*A*cos(1/2*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
cos(1/2*d*x+1/2*c)^5+294*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-4
68*B*cos(1/2*d*x+1/2*c)^8-330*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c
)^5-714*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+
1/2*c)^6+1058*B*cos(1/2*d*x+1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-474*B*cos(1
/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2+47*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B
)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 489, normalized size = 1.89

$$-\frac{5\left(\sqrt{2}(-13iA+33iB)\cos(dx+c)^3+3\sqrt{2}(-13iA+33iB)\cos(dx+c)^2+3\sqrt{2}(-13iA+33iB)\cos(dx+c)+\sqrt{2}(-13iA+33iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(\sqrt{2}(13iA-33iB)\cos(dx+c)^3+3\sqrt{2}(13iA-33iB)\cos(dx+c)^2+3\sqrt{2}(13iA-33iB)\cos(dx+c)+\sqrt{2}(13iA-33iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21\left(\sqrt{2}(-7iA+17iB)\cos(dx+c)^3+3\sqrt{2}(-7iA+17iB)\cos(dx+c)^2+3\sqrt{2}(-7iA+17iB)\cos(dx+c)+\sqrt{2}(-7iA+17iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21\left(\sqrt{2}(7iA-17iB)\cos(dx+c)^3+3\sqrt{2}(7iA-17iB)\cos(dx+c)^2+3\sqrt{2}(7iA-17iB)\cos(dx+c)+\sqrt{2}(7iA-17iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(20B\cos(dx+c)^4-3(29A-79B)\cos(dx+c)^3-2(73A-188B)\cos(dx+c)^2-5(13A-33B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] -1/60*(5*(sqrt(2)*(-13*I*A + 33*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-13*I*A +
33*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-13*I*A + 33*I*B)*cos(d*x + c) + sqrt(2
)*(-13*I*A + 33*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + 5*(sqrt(2)*(13*I*A - 33*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(13*I*A - 33
*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(13*I*A - 33*I*B)*cos(d*x + c) + sqrt(2)*(
13*I*A - 33*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 + 21*(sqrt(2)*(-7*I*A + 17*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-7*I*A + 17*I*
B)*cos(d*x + c)^2 + 3*sqrt(2)*(-7*I*A + 17*I*B)*cos(d*x + c) + sqrt(2)*(-7*
I*A + 17*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + 21*(sqrt(2)*(7*I*A - 17*I*B)*cos(d*x + c)^3 + 3*sqr
t(2)*(7*I*A - 17*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(7*I*A - 17*I*B)*cos(d*x +
 c) + sqrt(2)*(7*I*A - 17*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(20*B*cos(d*x + c)^4 - 3*(29*A -
 79*B)*cos(d*x + c)^3 - 2*(73*A - 188*B)*cos(d*x + c)^2 - 5*(13*A - 33*B)*c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3
*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3), x
)
```

3.493 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=220

$$\frac{32a(8A+9B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}+\frac{16a(8A+9B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}+\frac{4a(8A+9B)\sec^{\frac{5}{2}}(c+d\ldots}{105d\sqrt{a+a\cos(\ldots}}$$

```
[Out] 16/315*a*(8*A+9*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+4/1
05*a*(8*A+9*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/63*a*
(8*A+9*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/9*a*A*sec(
d*x+c)^(9/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+32/315*a*(8*A+9*B)*sin(d*x
+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3040, 3059, 2851, 2850}

$$\frac{2a(8A+9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{4a(8A+9B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a(8A+9B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{32a(8A+9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

```
[Out] (32*a*(8*A + 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c
+ d*x]]) + (16*a*(8*A + 9*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d*Sqrt[a
 + a*Cos[c + d*x]]) + (4*a*(8*A + 9*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(10
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(8*A + 9*B)*Sec[c + d*x]^(7/2)*Sin[c +
 d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(9/2)*Sin[c +
d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a}}{}$$

$$= \frac{2aA\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{1}{9}\Big((8A+9$$

$$= \frac{2a(8A+9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a}{}$$

$$= \frac{4a(8A+9B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a}{}$$

$$= \frac{16a(8A+9B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{4}{}$$

$$= \frac{32a(8A+9B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} +$$

Mathematica [A]

time = 0.62, size = 124, normalized size = 0.56

$$\frac{2\sqrt{a(1+\cos(c+dx))}\,(107A+81B+11(8A+9B)\cos(c+dx)+11(8A+9B)\cos(2(c+dx))+16A\cos(3(c+dx))+18B\cos(3(c+dx))+16A\cos(4(c+dx))+18B\cos(4(c+dx)))\sec^{\frac{9}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2)
,x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(107*A + 81*B + 11*(8*A + 9*B)*Cos[c + d*x] +
 11*(8*A + 9*B)*Cos[2*(c + d*x)] + 16*A*Cos[3*(c + d*x)] + 18*B*Cos[3*(c +
d*x)] + 16*A*Cos[4*(c + d*x)] + 18*B*Cos[4*(c + d*x)])*Sec[c + d*x]^(9/2)*T
an[(c + d*x)/2])/(315*d)
```

Maple [A]
time = 2.28, size = 138, normalized size = 0.63

method	result
default	$-\frac{2(-1+\cos(dx+c))(128A(\cos^4(dx+c))+144B(\cos^4(dx+c))+64A(\cos^3(dx+c))+72B(\cos^3(dx+c))+48A(\cos^2(dx+c))+54B(\cos^2(c}{315d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(128*A*cos(d*x+c)^4+144*B*cos(d*x+c)^4+64*A*cos(d*
x+c)^3+72*B*cos(d*x+c)^3+48*A*cos(d*x+c)^2+54*B*cos(d*x+c)^2+40*A*cos(d*x+c
)+45*B*cos(d*x+c)+35*A)*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))
^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(190) = 380$.
time = 0.56, size = 659, normalized size = 3.00

$$\frac{2\left(\frac{A\left(\frac{315\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{735\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1302\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1206\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{431\sqrt{2}\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{107\sqrt{2}\sqrt{a}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^5}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+1\right)}+\frac{9B\left(\frac{35\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{105\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{154\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{142\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{67\sqrt{2}\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{9\sqrt{2}\sqrt{a}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^5}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+1\right)}\right)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

```
[Out] 2/315*(A*(315*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 735*sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1302*sqrt(2)*sqrt(a)*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 1206*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 + 431*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 10
7*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin(d*x + c)^2/(c
```

```
os(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(5*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(co
s(d*x + c) + 1)^10 + 1)) + 9*B*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x +
c) + 1) - 105*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 154*sqr
t(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 142*sqrt(2)*sqrt(a)*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7 + 67*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*
x + c) + 1)^9 - 9*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(5*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin
(d*x + c)^10/(cos(d*x + c) + 1)^10 + 1)))/d
```

Fricas [A]
time = 0.36, size = 121, normalized size = 0.55

$$\frac{2\left(16\left(8A+9B\right)\cos\left(dx+c\right)^{4}+8\left(8A+9B\right)\cos\left(dx+c\right)^{3}+6\left(8A+9B\right)\cos\left(dx+c\right)^{2}+5\left(8A+9B\right)\cos\left(dx+c\right)+35A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(16*(8*A + 9*B)*cos(d*x + c)^4 + 8*(8*A + 9*B)*cos(d*x + c)^3 + 6*(8*
A + 9*B)*cos(d*x + c)^2 + 5*(8*A + 9*B)*cos(d*x + c) + 35*A)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*cos(d*x + c)^4)*sqrt(cos(d*x
 + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(11/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [B]

time = 5.60, size = 479, normalized size = 2.18

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(256\,A+288\,B)\,1i}{315\,d}-\frac{e^{c\,9i+d\,x\,9i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(256\,A+288\,B)\,1i}{315\,d}+\frac{e^{c\,2i+d\,x\,2i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(1152\,A+1296\,B)\,1i}{315\,d}-\frac{e^{c\,7i+d\,x\,7i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(1152\,A+1296\,B)\,1i}{315\,d}+\frac{e^{c\,4i+d\,x\,4i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(2016\,A+1008\,B)\,1i}{315\,d}-\frac{e^{c\,5i+d\,x\,5i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(2016\,A+1008\,B)\,1i}{315\,d}\right)}{e^{c\,1i+d\,x\,1i}+4\,e^{c\,2i+d\,x\,2i}+4\,e^{c\,3i+d\,x\,3i}+6\,e^{c\,4i+d\,x\,4i}+6\,e^{c\,5i+d\,x\,5i}+4\,e^{c\,6i+d\,x\,6i}+4\,e^{c\,7i+d\,x\,7i}+e^{c\,8i+d\,x\,8i}+e^{c\,9i+d\,x\,9i}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + a*cos(c + d*x))^(1/2)
,x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(((a + a*(exp(-
c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(256*A + 288*B)*1i)/(315*d)
 - (exp(c*9i + d*x*9i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/
2))^(1/2)*(256*A + 288*B)*1i)/(315*d) + (exp(c*2i + d*x*2i)*(a + a*(exp(- c
*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(1152*A + 1296*B)*1i)/(315*d
) - (exp(c*7i + d*x*7i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)
/2))^(1/2)*(1152*A + 1296*B)*1i)/(315*d) + (exp(c*4i + d*x*4i)*(a + a*(exp(
- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(2016*A + 1008*B)*1i)/(31
5*d) - (exp(c*5i + d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*
1i)/2))^(1/2)*(2016*A + 1008*B)*1i)/(315*d)))/(exp(c*1i + d*x*1i) + 4*exp(c
*2i + d*x*2i) + 4*exp(c*3i + d*x*3i) + 6*exp(c*4i + d*x*4i) + 6*exp(c*5i +
d*x*5i) + 4*exp(c*6i + d*x*6i) + 4*exp(c*7i + d*x*7i) + exp(c*8i + d*x*8i)
+ exp(c*9i + d*x*9i) + 1)
```

3.494 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=175

$$\frac{16a(6A+7B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{8a(6A+7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2a(6A+7B)\sec^{\frac{5}{2}}(c+d}{35d\sqrt{a+a\cos(c}$$

```
[Out] 8/105*a*(6*A+7*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/35
*a*(6*A+7*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a*A*s
ec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+16/105*a*(6*A+7*B)*sin(
d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3040, 3059, 2851, 2850}

$$\frac{2a(6A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{8a(6A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (16*a*(6*A + 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c
+ d*x]]) + (8*a*(6*A + 7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a
+ a*Cos[c + d*x]]) + (2*a*(6*A + 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*
d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*
Sqrt[a + a*Cos[c + d*x]])
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\,\mathrm{co}}}{}$$

$$= \frac{2aA\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}\Big((6A+7B)$$

$$= \frac{2a(6A+7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2aA}{7}$$

$$= \frac{8a(6A+7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a(6}{}$$

$$= \frac{16a(6A+7B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{8}{}$$

Mathematica [A]
time = 0.50, size = 102, normalized size = 0.58

$$\frac{2\sqrt{a(1+\cos(c+dx))}\,(27A+14B+9(6A+7B)\cos(c+dx)+2(6A+7B)\cos(2(c+dx))+12A\cos(3(c+dx))+14B\cos(3(c+dx)))\sec^{\frac{7}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),
x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(27*A + 14*B + 9*(6*A + 7*B)*Cos[c + d*x] + 2
*(6*A + 7*B)*Cos[2*(c + d*x)] + 12*A*Cos[3*(c + d*x)] + 14*B*Cos[3*(c + d*x
)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(105*d)
```

Maple [A]
time = 0.39, size = 116, normalized size = 0.66

method	result
default	$-\frac{2(-1+\cos(dx+c))(48A(\cos^3(dx+c))+56B(\cos^3(dx+c))+24A(\cos^2(dx+c))+28B(\cos^2(dx+c))+18A\cos(dx+c)+21B\cos(dx+c)}{105d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(48*A*cos(d*x+c)^3+56*B*cos(d*x+c)^3+24*A*cos(d*x+
c)^2+28*B*cos(d*x+c)^2+18*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*cos(d*x+c)*(1/
cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 568 vs. 2(151) = 302.
time = 0.60, size = 568, normalized size = 3.25

$$\frac{2\left(\frac{3A\left(\frac{35\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{70\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{58\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{9\sqrt{2}\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}+\frac{7B\left(\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{42\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{24\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7\sqrt{2}\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/105*(3*A*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 7*B*(15*sq
rt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sqrt(2)*sqrt(a)*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 42*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 - 24*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sqrt
(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x +
```

```
c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A]
time = 0.37, size = 104, normalized size = 0.59

$$\frac{2 \left(8 \left(6 A+7 B\right) \cos \left(dx+c\right)^{3}+4 \left(6 A+7 B\right) \cos \left(dx+c\right)^{2}+3 \left(6 A+7 B\right) \cos \left(dx+c\right)+15 A\right) \sqrt{a \cos \left(dx+c\right)+a}\, \sin \left(dx+c\right)}{105 \left(d \cos \left(dx+c\right)^{4}+d \cos \left(dx+c\right)^{3}\right) \sqrt{\cos \left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(8*(6*A + 7*B)*cos(d*x + c)^3 + 4*(6*A + 7*B)*cos(d*x + c)^2 + 3*(6*A
 + 7*B)*cos(d*x + c) + 15*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(
d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.55, size = 441, normalized size = 2.52

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(96\,A+112\,B)\,1i}{105\,d}-\frac{e^{c\,7i+d\,x\,7i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(96\,A+112\,B)\,1i}{105\,d}+\frac{e^{c\,2i+d\,x\,2i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(336\,A+392\,B)\,1i}{105\,d}-\frac{e^{c\,5i+d\,x\,5i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(336\,A+392\,B)\,1i}{105\,d}-\frac{B\,e^{c\,3i+d\,x\,3i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,8i}{3\,d}+\frac{B\,e^{c\,4i+d\,x\,4i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,8i}{3\,d}\right)}{e^{c\,1i+d\,x\,1i}+3\,e^{c\,2i+d\,x\,2i}+3\,e^{c\,3i+d\,x\,3i}+3\,e^{c\,4i+d\,x\,4i}+3\,e^{c\,5i+d\,x\,5i}+e^{c\,6i+d\,x\,6i}+e^{c\,7i+d\,x\,7i}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(1/2),
x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(((a + a*(exp(-
c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(96*A + 112*B)*1i)/(105*d)
- (exp(c*7i + d*x*7i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2
))^(1/2)*(96*A + 112*B)*1i)/(105*d) + (exp(c*2i + d*x*2i)*(a + a*(exp(- c*1
i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(336*A + 392*B)*1i)/(105*d) -
(exp(c*5i + d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))
^(1/2)*(336*A + 392*B)*1i)/(105*d) - (B*exp(c*3i + d*x*3i)*(a + a*(exp(- c*
1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*8i)/(3*d) + (B*exp(c*4i + d*x
*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*8i)/(3*d
)))/(exp(c*1i + d*x*1i) + 3*exp(c*2i + d*x*2i) + 3*exp(c*3i + d*x*3i) + 3*e
xp(c*4i + d*x*4i) + 3*exp(c*5i + d*x*5i) + exp(c*6i + d*x*6i) + exp(c*7i +
d*x*7i) + 1)
```

3.495 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=130

$$\frac{4a(4A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a(4A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+d}{5d\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 2/15*a*(4*A+5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*a
*A*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+4/15*a*(4*A+5*B)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3040, 3059, 2851, 2850}

$$\frac{2a(4A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{4a(4A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (4*a*(4*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*a*(4*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a
*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*C
os[c + d*x]])
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a}}{}$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{1}{5}\Big((4A+5B$$

$$= \frac{2a(4A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a}{}$$

$$= \frac{4a(4A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

Mathematica [A]
time = 0.31, size = 78, normalized size = 0.60

$$\frac{2\sqrt{a(1+\cos(c+dx))}\,(7A+5B+(4A+5B)\cos(c+dx)+(4A+5B)\cos(2(c+dx)))\sec^{\frac{5}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),
x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(7*A + 5*B + (4*A + 5*B)*Cos[c + d*x] + (4*A
+ 5*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

Maple [A]
time = 0.38, size = 94, normalized size = 0.72

method	result
default	$-\frac{2(-1+\cos(dx+c))\left(8A\left(\cos^2(dx+c)\right)+10B\left(\cos^2(dx+c)\right)+4A\cos(dx+c)+5B\cos(dx+c)+3A\right)\cos(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}\sqrt{a\,(1+c}}{15d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(8*A*cos(d*x+c)^2+10*B*cos(d*x+c)^2+4*A*cos(d*x+c)+
5*B*cos(d*x+c)+3*A)*cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2
)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(112) = 224$.
time = 0.55, size = 475, normalized size = 3.65

$$\frac{2\left(\frac{A\left(\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{7\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}+\frac{5B\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(A*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sq
rt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 5*B*(3*sqrt(2)*sqrt(a)*sin(d*x + c)
/(cos(d*x + c) + 1) - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
 + 5*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - sqrt(2)*sqrt(a)*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A]
time = 0.36, size = 86, normalized size = 0.66

$$\frac{2\left(2\left(4A+5B\right)\cos\left(dx+c\right)^{2}+\left(4A+5B\right)\cos\left(dx+c\right)+3A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(2*(4*A + 5*B)*cos(d*x + c)^2 + (4*A + 5*B)*cos(d*x + c) + 3*A)*sqrt(a
*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt
(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 2.69, size = 196, normalized size = 1.51

$$\frac{4\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sqrt{\frac{1}{\cos\left(c+dx\right)}}\left(14A\sin\left(c+dx\right)+10B\sin\left(c+dx\right)+8A\sin\left(2c+2dx\right)+18A\sin\left(3c+3dx\right)+4A\sin\left(4c+4dx\right)+4A\sin\left(5c+5dx\right)+10B\sin\left(2c+2dx\right)+15B\sin\left(3c+3dx\right)+5B\sin\left(4c+4dx\right)+5B\sin\left(5c+5dx\right)\right)}{15d\left(10\cos\left(c+dx\right)+8\cos\left(2c+2dx\right)+5\cos\left(3c+3dx\right)+2\cos\left(4c+4dx\right)+\cos\left(5c+5dx\right)+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(1/2),
x)
```

```
[Out] (4*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(14*A*sin(c + d*x) +
 10*B*sin(c + d*x) + 8*A*sin(2*c + 2*d*x) + 18*A*sin(3*c + 3*d*x) + 4*A*sin
(4*c + 4*d*x) + 4*A*sin(5*c + 5*d*x) + 10*B*sin(2*c + 2*d*x) + 15*B*sin(3*c
 + 3*d*x) + 5*B*sin(4*c + 4*d*x) + 5*B*sin(5*c + 5*d*x)))/(15*d*(10*cos(c +
 d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos(4*c + 4*d*x) + cos(
5*c + 5*d*x) + 6))
```

3.496 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=85

$$\frac{2a(2A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 2/3*a*A*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*a*(2*A+3*B
)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3040, 3059, 2850}

$$\frac{2a(2A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a*(2*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d
*x]]) + (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x
]])
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
```

```
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a}}{}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{3}\Big((2A+3B$$

$$= \frac{2a(2A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

Mathematica [A]
time = 0.20, size = 57, normalized size = 0.67

$$\frac{2\sqrt{a(1+\cos(c+dx))}\,(A+(2A+3B)\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),
x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(A + (2*A + 3*B)*Cos[c + d*x])*Sec[c + d*x]^(
3/2)*Tan[(c + d*x)/2])/(3*d)
```

Maple [A]
time = 0.36, size = 70, normalized size = 0.82

method	result	size
default	$-\frac{2(-1+\cos(dx+c))(2A\cos(dx+c)+3B\cos(dx+c)+A)\cos(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}}{3d\sin(dx+c)}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(2*A*cos(d*x+c)+3*B*cos(d*x+c)+A)*cos(d*x+c)*(1/cos(
d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(73) = 146.
time = 0.57, size = 380, normalized size = 4.47

$$\frac{2\left(\frac{A\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}+\frac{3B\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(A*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/
2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) +
1)^4 + 1)) + 3*B*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sqrt(
2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 1)))/d
```

Fricas [A]
time = 0.34, size = 65, normalized size = 0.76

$$\frac{2\left((2A+3B)\cos(dx+c)+A\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/3*((2*A + 3*B)*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((
d*cos(d*x + c)^2 + d*cos(d*x + c))*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 1.07, size = 114, normalized size = 1.34

$$\frac{2\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(2\,A\,\sin\left(c+d\,x\right)+3\,B\,\sin\left(c+d\,x\right)+2\,A\,\sin\left(2\,c+2\,d\,x\right)+2\,A\,\sin\left(3\,c+3\,d\,x\right)+3\,B\,\sin\left(3\,c+3\,d\,x\right)\right)}{3\,d\,\left(3\,\cos\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(1/2),
x)
```

```
[Out] (2*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(2*A*sin(c + d*x) +
3*B*sin(c + d*x) + 2*A*sin(2*c + 2*d*x) + 2*A*sin(3*c + 3*d*x) + 3*B*sin(3*
c + 3*d*x)))/(3*d*(3*cos(c + d*x) + 2*cos(2*c + 2*d*x) + cos(3*c + 3*d*x) +
 2))
```

3.497 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{2\sqrt{a}\,B\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d}+\frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d+2*a*A*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))
^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3040, 3059, 2853, 222}

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}\,B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a]*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cdots}}{\cdots}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \left(B\sqrt{\cos(\cdots}\right.$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2B\sqrt{\cos\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{a}\,B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos\cdots}}{d}$$

Mathematica [A]
time = 0.24, size = 86, normalized size = 0.90

$$\frac{\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}\,B\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2A\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),
x]
```

[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*B*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*A*Sin[(c + d*x)/2])
)/d

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(82) = 164$.
time = 0.39, size = 171, normalized size = 1.78

method	result
default	$\frac{2\left(B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\cos(dx+c)+B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right.}{d(1+\cos(dx+c))}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)

[Out] 2/d*(B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+A*sin(d*x
+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c)
)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(82) = 164$.
time = 0.71, size = 906, normalized size = 9.44

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")

[Out] 1/2*(B*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1

```
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)) + 4*A*(sqrt(2
)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*
x + c)/(cos(d*x + c) + 1) + 1)^(3/2)))/d
```

Fricas [A]
time = 0.37, size = 91, normalized size = 0.95

$$-\frac{2\left((B\cos(dx+c)+B)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\sqrt{a\cos(dx+c)+a}\,A\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -2*((B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*A*sin(d*x + c)/
sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2),
 x)
```

3.498 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{a}\,(2A+B)\mathrm{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d}+\frac{aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}}$$

```
[Out] a*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(2*A+B)*arcsin(sin
(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3040, 3060, 2853, 222}

$$\frac{\sqrt{a}\,(2A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\mathrm{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(2*A + B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*B*Sin[c + d*x])/(d*Sqrt[a + a
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a}}{}$$

$$= \frac{aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \frac{1}{2}\Big(\Big($$

$$\Big((2.$$

$$= \frac{aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} - $$

$$= \frac{\sqrt{a}\,(2A+B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$

Mathematica [A]
time = 0.24, size = 103, normalized size = 1.05

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}\,(2A+B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sqrt{\cos(c+dx)}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(Sqrt[2]*(2*A + B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*B*Sqrt[Cos[
c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

Maple [A]
time = 0.40, size = 168, normalized size = 1.71

method	result
default	$-\frac{\left(B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+2A\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+B\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\right)\sqrt{a\,(}}{d\sin(dx+c)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*A*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+B*arctan(sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c
))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 939 vs. 2(84) = 168.
time = 0.68, size = 939, normalized size = 9.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(4*A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
```

```
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

Fricas [A]
time = 0.40, size = 97, normalized size = 0.99

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}\,B\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\left(\left(2\,A+B\right)\cos\left(dx+c\right)+2\,A+B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A + B)*co
s(d*x + c) + 2*A + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)\right)\sqrt{\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sqrt(sec(c + d*x))
, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}\sqrt{a+a\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2),
 x)
```

3.499 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{a}\,(4A+3B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{4d}+\frac{aB\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c$$

```
[Out] 1/2*a*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/4*a*(4*A+3*B
)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/4*(4*A+3*B)*arcsin
(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3060, 2849, 2853, 222}

$$\frac{\sqrt{a}\,(4A+3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(4A+3B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(4*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*B*Sin[c + d*x])/(2*d*Sq
rt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*(4*A + 3*B)*Sin[c + d*x])/(
4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,\sqrt{a+$$

$$= \frac{aB\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\Big((4A+3B)\sqrt{c$$

$$= \frac{aB\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{a(4A+3I}{4d\sqrt{a+a\cos(c-}$$

$$= \frac{aB\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{a(4A+3I}{4d\sqrt{a+a\cos(c-}$$

$$= \frac{\sqrt{a}\,(4A+3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+}}{4d}$$

Mathematica [A]
time = 0.43, size = 120, normalized size = 0.79

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}\,(4A+3B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\,(4A+3B+2B\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(Sqrt[2]*(4*A + 3*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[
c + d*x]]*(4*A + 3*B + 2*B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [A]
time = 0.44, size = 238, normalized size = 1.58

method	result
default	$\frac{(-1+\cos(dx+c))^2\left(2B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)+4A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+3B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+\right.}{4d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{}{\text{co}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))^2*(2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+3*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c))+3*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(
d*x+c)))^(3/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1851 vs. 2(127) = 254.
time = 0.73, size = 1851, normalized size = 12.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
```

```
+ c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A + (2*(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2
*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x +
 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d
*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2
*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - cos(2*d*
x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqr
t(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
```

```
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

Fricas [A]
time = 0.41, size = 127, normalized size = 0.84

$$-\frac{((4A+3B)\cos(dx+c)+4A+3B)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(2B\cos(dx+c)^2+(4A+3B)\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(((4*A + 3*B)*cos(d*x + c) + 4*A + 3*B)*sqrt(a)*arctan(sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*B*cos(d*x + c)^2
+ (4*A + 3*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(
d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos(c+dx)+1\right)}\,\left(A+B\cos(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/sqrt(sec(c + d*x))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2
), x)
```

3.500 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=196

$$\frac{\sqrt{a}\,(6A+5B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{8d}+\frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c$$

```
[Out] 1/3*a*B*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+1/12*a*(6*A+5*
B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/8*a*(6*A+5*B)*sin
(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/8*(6*A+5*B)*arcsin(sin(
d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/d
```

Rubi [A]
time = 0.26, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3060, 2849, 2853, 222}

$$\frac{\sqrt{a}\,(6A+5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(6A+5B)\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a(6A+5B)\sin(c+dx)}{8d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)}{3d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*B*Sin[c + d*x])/(3*d*Sq
rt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a*(6*A + 5*B)*Sin[c + d*x])/(
12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*(6*A + 5*B)*Sin[c +
d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{co}$$

$$= \frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{6}\Big((6A+5B)\sqrt{\mathrm{co}$$

$$= \frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a(6A+5B)}{12d\sqrt{a+a\cos(c+}}$$

$$= \frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a(6A+5B)}{12d\sqrt{a+a\cos(c+}}$$

$$= \frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a(6A+5B)}{12d\sqrt{a+a\cos(c+}}$$

$$= \frac{\sqrt{a}\,(6A+5B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+d}}{8d}$$

Mathematica [A]
time = 0.77, size = 138, normalized size = 0.70

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(3\sqrt{2}\,(6A+5B)\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\,(18A+19B+2(6A+5B)\cos(c+dx)+4B\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(3*Sqrt[2]*(6*A + 5*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Co
s[c + d*x]]*(18*A + 19*B + 2*(6*A + 5*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)
])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [A]
time = 1.63, size = 308, normalized size = 1.57

method	result

default	$-\frac{(-1+\cos(dx+c))^3\left(8B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))+12A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+10B\sin(dx+c)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^3*(8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)^2+12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+
10*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+18*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)+18*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x
+c))+15*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*
cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/co
s(d*x+c))^(3/2)/sin(d*x+c)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2981 vs. $2(166) = 332$.
time = 0.84, size = 2981, normalized size = 15.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc
```

tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A + (4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x

```
 + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*...
```

Fricas [A]
time = 0.43, size = 146, normalized size = 0.74

$$-\frac{3\left((6A+5B)\cos(dx+c)+6A+5B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(8B\cos(dx+c)^3+2(6A+5B)\cos(dx+c)^2+3(6A+5B)\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{24\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/24*(3*((6*A + 5*B)*cos(d*x + c) + 6*A + 5*B)*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*B*cos(d*x + c)
^3 + 2*(6*A + 5*B)*cos(d*x + c)^2 + 3*(6*A + 5*B)*cos(d*x + c))*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos(c+dx)+1\right)}\,\left(A+B\cos(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/sec(c + d*x)**(3/2
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+dx))\sqrt{a+a\cos(c+dx)}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2
), x)
```

3.501 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=275

$$\frac{32a^2(168A+187B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}}+\frac{16a^2(168A+187B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}}+\frac{4a^2(168A+\ldots}{115\ldots}$$

```
[Out] 16/3465*a^2*(168*A+187*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1
/2)+4/1155*a^2*(168*A+187*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))
^(1/2)+2/693*a^2*(168*A+187*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c
))^(1/2)+2/99*a^2*(12*A+11*B)*sec(d*x+c)^(9/2)*sin(d*x+c)/d/(a+a*cos(d*x+c)
)^(1/2)+2/11*a*A*sec(d*x+c)^(11/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+32/3
465*a^2*(168*A+187*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3054, 3059, 2851, 2850}

$$\frac{2a^2(12A+11B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(168A+187B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(168A+187B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{1155d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2(168A+187B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3465d\sqrt{a\cos(c+dx)+a}}+\frac{32a^2(168A+187B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3465d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{11}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/2),x]
```

```
[Out] (32*a^2*(168*A + 187*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a
*Cos[c + d*x]]) + (16*a^2*(168*A + 187*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(3465*d*Sqrt[a + a*Cos[c + d*x]]) + (4*a^2*(168*A + 187*B)*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(168*A + 187*B)
*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(12*A + 11*B)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d*Sqrt[a + a*Cos[c + d*
x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(1
1*d)
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
```

```
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos}{}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{11}{2}}(c+dx)\sin(}{11d}$$

$$= \frac{2a^2(12A+11B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^2(168A+187B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx}{693d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{4a^2(168A+187B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx}{1155d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{16a^2(168A+187B)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{3465d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{32a^2(168A+187B)\sqrt{\sec(c+dx)}\,\sin(c+}{3465d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A]

time = 0.85, size = 146, normalized size = 0.53

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,(2478A+2057B+(6342A+6193B)\cos(c+dx)+13(168A+187B)\cos(2(c+dx))+2184A\cos(3(c+dx))+2431B\cos(3(c+dx))+336A\cos(4(c+dx))+374B\cos(4(c+dx))+336A\cos(5(c+dx))+374B\cos(5(c+dx)))\sec^{\frac{11}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{3465d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/
2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(2478*A + 2057*B + (6342*A + 6193*B)*Cos[c +
d*x] + 13*(168*A + 187*B)*Cos[2*(c + d*x)] + 2184*A*Cos[3*(c + d*x)] + 2431
*B*Cos[3*(c + d*x)] + 336*A*Cos[4*(c + d*x)] + 374*B*Cos[4*(c + d*x)] + 336
*A*Cos[5*(c + d*x)] + 374*B*Cos[5*(c + d*x)])*Sec[c + d*x]^(11/2)*Tan[(c +
d*x)/2])/(3465*d)
```

Maple [A]

time = 0.53, size = 161, normalized size = 0.59

method	result
default	$-\frac{2(-1+\cos(dx+c))(2688A(\cos^5(dx+c))+2992B(\cos^5(dx+c))+1344A(\cos^4(dx+c))+1496B(\cos^4(dx+c))+1008A(\cos^3(dx+c))+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/3465/d*(-1+cos(d*x+c))*(2688*A*cos(d*x+c)^5+2992*B*cos(d*x+c)^5+1344*A*c
os(d*x+c)^4+1496*B*cos(d*x+c)^4+1008*A*cos(d*x+c)^3+1122*B*cos(d*x+c)^3+840
*A*cos(d*x+c)^2+935*B*cos(d*x+c)^2+735*A*cos(d*x+c)+385*B*cos(d*x+c)+315*A)
*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(13/2)/sin(d*x+c)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(239) = 478$.
time $= 0.60$, size $= 712$, normalized size $= 2.59$

$$\frac{4\left(\frac{21\left(\frac{165\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{495\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{1056\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{1254\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{781\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{299\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{46\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}\right)A\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{5}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}\left(\frac{5\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{10\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{5\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+1\right)}+\frac{11\left(\frac{315\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{1155\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{2184\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{2586\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{1759\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{611\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{94\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}\right)B\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{5}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}\left(\frac{5\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{10\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{5\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+1\right)}\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] 4/3465*(21*(165*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 495*sqrt(
2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1056*sqrt(2)*a^(3/2)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 1254*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 781*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
299*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 46*sqrt(2)*a^(3
/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c) +
 1)^10 + 1)) + 11*(315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 11
55*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2184*sqrt(2)*a^(3/
2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2586*sqrt(2)*a^(3/2)*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 + 1759*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c)
+ 1)^9 - 611*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 94*sqr
t(2)*a^(3/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*B*(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(
d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d
*x + c) + 1)^10 + 1)))/d
```

Fricas [A]
time $= 0.36$, size $= 144$, normalized size $= 0.52$

$$\frac{2\left(16(168A+187B)a\cos(dx+c)^{5}+8(168A+187B)a\cos(dx+c)^{4}+6(168A+187B)a\cos(dx+c)^{3}+5(168A+187B)a\cos(dx+c)^{2}+35(21A+11B)a\cos(dx+c)+315Aa\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{3465\left(d\cos(dx+c)^{6}+d\cos(dx+c)^{5}\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(16*(168*A + 187*B)*a*cos(d*x + c)^5 + 8*(168*A + 187*B)*a*cos(d*x +
 c)^4 + 6*(168*A + 187*B)*a*cos(d*x + c)^3 + 5*(168*A + 187*B)*a*cos(d*x +
c)^2 + 35*(21*A + 11*B)*a*cos(d*x + c) + 315*A*a)*sqrt(a*cos(d*x + c) + a)*
sin(d*x + c)/((d*cos(d*x + c)^6 + d*cos(d*x + c)^5)*sqrt(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.14, size = 348, normalized size = 1.27

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(-\frac{32\,a\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A+3\,B\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{5\,d}+\frac{64\,a\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\left(21\,A+19\,B\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{35\,d}+\frac{32\,a\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)\,\left(168\,A+187\,B\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{315\,d}+\frac{64\,a\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)\,\left(168\,A+187\,B\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{3465\,d}\right)}{20\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+20\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+10\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)+10\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)+2\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)+2\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(13/2)*(a + a*cos(c + d*x))^(3/2)
,x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((64*a*exp((c*11
i)/2 + (d*x*11i)/2)*sin((3*c)/2 + (3*d*x)/2)*(21*A + 19*B)*(a + a*cos(c + d
*x))^(1/2))/(35*d) - (32*a*exp((c*11i)/2 + (d*x*11i)/2)*sin(c/2 + (d*x)/2)*
(2*A + 3*B)*(a + a*cos(c + d*x))^(1/2))/(5*d) + (32*a*exp((c*11i)/2 + (d*x*
11i)/2)*sin((7*c)/2 + (7*d*x)/2)*(168*A + 187*B)*(a + a*cos(c + d*x))^(1/2)
)/(315*d) + (64*a*exp((c*11i)/2 + (d*x*11i)/2)*sin((11*c)/2 + (11*d*x)/2)*(
```

```
168*A + 187*B)*(a + a*cos(c + d*x))^(1/2))/(3465*d)))/(20*exp((c*11i)/2 + (
d*x*11i)/2)*cos(c/2 + (d*x)/2) + 20*exp((c*11i)/2 + (d*x*11i)/2)*cos((3*c)/
2 + (3*d*x)/2) + 10*exp((c*11i)/2 + (d*x*11i)/2)*cos((5*c)/2 + (5*d*x)/2) +
 10*exp((c*11i)/2 + (d*x*11i)/2)*cos((7*c)/2 + (7*d*x)/2) + 2*exp((c*11i)/2
 + (d*x*11i)/2)*cos((9*c)/2 + (9*d*x)/2) + 2*exp((c*11i)/2 + (d*x*11i)/2)*c
os((11*c)/2 + (11*d*x)/2))
```

3.502 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=228

$$\frac{16a^2(34A+39B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}+\frac{8a^2(34A+39B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(34A+39B)\,s}{105d\sqrt{a\cdot}}$$

```
[Out] 8/315*a^2*(34*A+39*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+
2/105*a^2*(34*A+39*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+
2/63*a^2*(10*A+9*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/
9*a*A*sec(d*x+c)^(9/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+16/315*a^2*(34*A
+39*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3054, 3059, 2851, 2850}

$$\frac{2a^2(10A+9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(34A+39B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{8a^2(34A+39B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2(34A+39B)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

```
[Out] (16*a^2*(34*A + 39*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Co
s[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d
*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(34*A + 39*B)*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(10*A + 9*B)*Sec[c + d*x]
^(7/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Co
s[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\cos}{}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c}{9d}$$

$$= \frac{2a^2(10A+9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2a^2(34A+39B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{8a^2(34A+39B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{16a^2(34A+39B)\sqrt{\sec(c+dx)}\sin(c+dx}{315d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A]
time = 0.76, size = 124, normalized size = 0.54

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,(376A+351B+(374A+324B)\cos(c+dx)+11(34A+39B)\cos(2(c+dx))+68A\cos(3(c+dx))+78B\cos(3(c+dx))+68A\cos(4(c+dx))+78B\cos(4(c+dx)))\sec^{\frac{9}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(376*A + 351*B + (374*A + 324*B)*Cos[c + d*x]
 + 11*(34*A + 39*B)*Cos[2*(c + d*x)] + 68*A*Cos[3*(c + d*x)] + 78*B*Cos[3*(
c + d*x)] + 68*A*Cos[4*(c + d*x)] + 78*B*Cos[4*(c + d*x)])*Sec[c + d*x]^(9/
2)*Tan[(c + d*x)/2])/(315*d)
```

Maple [A]
time = 0.39, size = 139, normalized size = 0.61

method	result
default	$-\frac{2(-1+\cos(dx+c))(272A(\cos^4(dx+c))+312B(\cos^4(dx+c))+136A(\cos^3(dx+c))+156B(\cos^3(dx+c))+102A(\cos^2(dx+c))+117B}{315d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
```

[Out] -2/315/d*(-1+cos(d*x+c))*(272*A*cos(d*x+c)^4+312*B*cos(d*x+c)^4+136*A*cos(d
*x+c)^3+156*B*cos(d*x+c)^3+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+85*A*cos(d
*x+c)+45*B*cos(d*x+c)+35*A)*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+
c))^(11/2)/sin(d*x+c)*a

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(198) = 396$.
time = 0.58, size = 619, normalized size = 2.71

$$\frac{4\left(\frac{\left(\frac{315\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{840\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1344\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1242\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{517\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{94\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)A\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}+\frac{3\left(\frac{105\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{350\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{518\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{444\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{209\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{38\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)B\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="maxima")

[Out] 4/315*((315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 840*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sqrt(2)*a^(3/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 1242*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x +
c) + 1)^7 + 517*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 94*s
qrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 3*(105*sqrt(2)*a^(
3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 350*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + 518*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - 444*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 209*sqrt(2)*
a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 38*sqrt(2)*a^(3/2)*sin(d*x +
c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/
((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(11/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 + 1)))/d

Fricas [A]
time = 0.37, size = 126, normalized size = 0.55

$$\frac{2\left(8(34A+39B)a\cos(dx+c)^4+4(34A+39B)a\cos(dx+c)^3+3(34A+39B)a\cos(dx+c)^2+5(17A+9B)a\cos(dx+c)+35Aa\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{315\left(d\cos(dx+c)^5+d\cos(dx+c)^4\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="fricas")

[Out] 2/315*(8*(34*A + 39*B)*a*cos(d*x + c)^4 + 4*(34*A + 39*B)*a*cos(d*x + c)^3
+ 3*(34*A + 39*B)*a*cos(d*x + c)^2 + 5*(17*A + 9*B)*a*cos(d*x + c) + 35*A*a

```
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*cos(d*x + c)
^4)*sqrt(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.91, size = 316, normalized size = 1.39

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(\frac{96\,a\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\left(A+B\right)}{5\,d}-\frac{16\,B\,a\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{3\,d}+\frac{16\,a\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)\,\left(34\,A+39\,B\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{35\,d}+\frac{32\,a\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)\,\left(34\,A+39\,B\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{315\,d}\right)}{12\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+8\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)+2\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)+2\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + a*cos(c + d*x))^(3/2)
,x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((96*a*exp((c*9i
)/2 + (d*x*9i)/2)*sin(c/2 + (d*x)/2)*(a + a*cos(c + d*x))^(1/2)*(A + B))/(5
*d) - (16*B*a*exp((c*9i)/2 + (d*x*9i)/2)*sin((3*c)/2 + (3*d*x)/2)*(a + a*co
s(c + d*x))^(1/2))/(3*d) + (16*a*exp((c*9i)/2 + (d*x*9i)/2)*sin((5*c)/2 + (
5*d*x)/2)*(34*A + 39*B)*(a + a*cos(c + d*x))^(1/2))/(35*d) + (32*a*exp((c*9
i)/2 + (d*x*9i)/2)*sin((9*c)/2 + (9*d*x)/2)*(34*A + 39*B)*(a + a*cos(c + d*
x))^(1/2))/(315*d)))/(12*exp((c*9i)/2 + (d*x*9i)/2)*cos(c/2 + (d*x)/2) + 8*
exp((c*9i)/2 + (d*x*9i)/2)*cos((3*c)/2 + (3*d*x)/2) + 8*exp((c*9i)/2 + (d*x
*9i)/2)*cos((5*c)/2 + (5*d*x)/2) + 2*exp((c*9i)/2 + (d*x*9i)/2)*cos((7*c)/2
 + (7*d*x)/2) + 2*exp((c*9i)/2 + (d*x*9i)/2)*cos((9*c)/2 + (9*d*x)/2))
```

3.503 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=181

$$\frac{4a^2(52A+63B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(52A+63B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(8A+7B)\sec^{\frac{5}{2}}(}{35d\sqrt{a+a\,\mathrm{c}}}$$

```
[Out] 2/105*a^2*(52*A+63*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+
2/35*a^2*(8*A+7*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7
*a*A*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+4/105*a^2*(52*A+6
3*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3054, 3059, 2851, 2850}

$$\frac{2a^2(8A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(52A+63B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (4*a^2*(52*A + 63*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos
[c + d*x]]) + (2*a^2*(52*A + 63*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(8*A + 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[
c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{7}{2}}(c+dx)\sin(c+}{7d}$$

$$= \frac{2a^2(8A+7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a}{}$$

$$= \frac{2a^2(52A+63B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{4a^2(52A+63B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} +$$

Mathematica [A]
time = 0.62, size = 102, normalized size = 0.56

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,(82A+63B+3(78A+77B)\cos(c+dx)+(52A+63B)\cos(2(c+dx))+52A\cos(3(c+dx))+63B\cos(3(c+dx)))\sec^{\frac{7}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(82*A + 63*B + 3*(78*A + 77*B)*Cos[c + d*x] +
 (52*A + 63*B)*Cos[2*(c + d*x)] + 52*A*Cos[3*(c + d*x)] + 63*B*Cos[3*(c + d
*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(105*d)
```

Maple [A]
time = 0.38, size = 117, normalized size = 0.65

method	result
default	$-\frac{2(-1+\cos(dx+c))(104A(\cos^3(dx+c))+126B(\cos^3(dx+c))+52A(\cos^2(dx+c))+63B(\cos^2(dx+c))+39A\cos(dx+c)+21B\cos(dx+c}{105d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(104*A*cos(d*x+c)^3+126*B*cos(d*x+c)^3+52*A*cos(d*
x+c)^2+63*B*cos(d*x+c)^2+39*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*cos(d*x+c)*(
a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(9/2)/sin(d*x+c)*a
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(157) = 314$.

time = 0.55, size = 527, normalized size = 2.91

$$\frac{4\left(\frac{\left(\frac{105\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{245\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{273\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{171\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{38\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)A\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{3}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}+\frac{21\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{17\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{9\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)B\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{3}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 4/105*((105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1
)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 1)) + 21*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c
) + 1) - 15*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2
)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 9*sqrt(2)*a^(3/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [**A**]

time = 0.37, size = 107, normalized size = 0.59

$$\frac{2\left(2\left(52A+63B\right)a\cos\left(dx+c\right)^{3}+\left(52A+63B\right)a\cos\left(dx+c\right)^{2}+3\left(13A+7B\right)a\cos\left(dx+c\right)+15Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(2*(52*A + 63*B)*a*cos(d*x + c)^3 + (52*A + 63*B)*a*cos(d*x + c)^2 +
3*(13*A + 7*B)*a*cos(d*x + c) + 15*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.80, size = 259, normalized size = 1.43

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(-\frac{8\,a\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A+3\,B\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{3\,d}+\frac{16\,a\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\left(13\,A+12\,B\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{15\,d}+\frac{8\,a\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)\,\left(52\,A+63\,B\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{105\,d}\right)}{6\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+2\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)+2\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(3/2),
x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((16*a*exp((c*7i
)/2 + (d*x*7i)/2)*sin((3*c)/2 + (3*d*x)/2)*(13*A + 12*B)*(a + a*cos(c + d*x
))^(1/2))/(15*d) - (8*a*exp((c*7i)/2 + (d*x*7i)/2)*sin(c/2 + (d*x)/2)*(2*A
+ 3*B)*(a + a*cos(c + d*x))^(1/2))/(3*d) + (8*a*exp((c*7i)/2 + (d*x*7i)/2)*
sin((7*c)/2 + (7*d*x)/2)*(52*A + 63*B)*(a + a*cos(c + d*x))^(1/2))/(105*d))
)/(6*exp((c*7i)/2 + (d*x*7i)/2)*cos(c/2 + (d*x)/2) + 6*exp((c*7i)/2 + (d*x*
7i)/2)*cos((3*c)/2 + (3*d*x)/2) + 2*exp((c*7i)/2 + (d*x*7i)/2)*cos((5*c)/2
+ (5*d*x)/2) + 2*exp((c*7i)/2 + (d*x*7i)/2)*cos((7*c)/2 + (7*d*x)/2))
```

3.504 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=134

$$\frac{2a^2(18A+25B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(6A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a\cos(c+\ldots}}{}$$

```
[Out] 2/15*a^2*(6*A+5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5
*a*A*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+2/15*a^2*(18*A+25
*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3040, 3054, 3059, 2850}

$$\frac{2a^2(6A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(18A+25B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a^2*(18*A + 25*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[
c + d*x]]) + (2*a^2*(6*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*
Sin[c + d*x])/(5*d)
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos\cdots}{\cdots}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+\cdots}{5d}$$

$$= \frac{2a^2(6A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a\cdots}{\cdots}$$

$$= \frac{2a^2(18A+25B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \cdots$$

Mathematica [A]
time = 0.37, size = 80, normalized size = 0.60

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,(24A+25B+2(9A+5B)\cos(c+dx)+(18A+25B)\cos(2(c+dx)))\sec^{\frac{5}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(24*A + 25*B + 2*(9*A + 5*B)*Cos[c + d*x] + (
18*A + 25*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

Maple [A]
time = 0.36, size = 95, normalized size = 0.71

method	result
default	$-\frac{2(-1+\cos(dx+c))\left(18A\left(\cos^2(dx+c)\right)+25B\left(\cos^2(dx+c)\right)+9A\cos(dx+c)+5B\cos(dx+c)+3A\right)\cos(dx+c)\sqrt{a\left(1+\cos\left(dx+\right.\right.}}{15d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(18*A*cos(d*x+c)^2+25*B*cos(d*x+c)^2+9*A*cos(d*x+c)
+5*B*cos(d*x+c)+3*A)*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(7/
2)/sin(d*x+c)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(116) = 232.
time = 0.59, size = 436, normalized size = 3.25

$$\frac{4\left(\frac{3\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)A\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}+\frac{5\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{8\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)B\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 4/15*(3*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(
3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) +
5*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 8*sqrt(2)*a^(3/2)*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*B*(
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1
) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A]

time = 0.35, size = 88, normalized size = 0.66

$$\frac{2\left((18\,A+25\,B)a\cos\left(dx+c\right)^{2}+(9\,A+5\,B)a\cos\left(dx+c\right)+3\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{15\,\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*((18*A + 25*B)*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) + 3*A*a)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2
)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [B]

time = 2.44, size = 197, normalized size = 1.47

$$\frac{2\,a\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(48\,A\,\sin\left(c+d\,x\right)+50\,B\,\sin\left(c+d\,x\right)+36\,A\,\sin\left(2\,c+2\,d\,x\right)+66\,A\,\sin\left(3\,c+3\,d\,x\right)+18\,A\,\sin\left(4\,c+4\,d\,x\right)+18\,A\,\sin\left(5\,c+5\,d\,x\right)+20\,B\,\sin\left(2\,c+2\,d\,x\right)+75\,B\,\sin\left(3\,c+3\,d\,x\right)+10\,B\,\sin\left(4\,c+4\,d\,x\right)+25\,B\,\sin\left(5\,c+5\,d\,x\right)\right)}{15\,d\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(5\,c+5\,d\,x\right)+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(3/2),
x)
```

```
[Out] (2*a*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(48*A*sin(c + d*x)
 + 50*B*sin(c + d*x) + 36*A*sin(2*c + 2*d*x) + 66*A*sin(3*c + 3*d*x) + 18*A
*sin(4*c + 4*d*x) + 18*A*sin(5*c + 5*d*x) + 20*B*sin(2*c + 2*d*x) + 75*B*si
n(3*c + 3*d*x) + 10*B*sin(4*c + 4*d*x) + 25*B*sin(5*c + 5*d*x)))/(15*d*(10*
cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos(4*c + 4*d*x)
 + cos(5*c + 5*d*x) + 6))
```

3.505 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=145

$$\frac{2a^{3/2}B\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d}+\frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\,\sin(c+$$

```
[Out] 2/3*a*A*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+2*a^(3/2)*B*ar
csin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d+2/3*a^2*(4*A+3*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(
1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3054, 3059, 2853, 222}

$$\frac{2a^{3/2}B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a^(3/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(4*A + 3*B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*
x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{c}}{}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c}{3d}$$

$$= \frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2a^{3/2}B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos}}{d}$$

Mathematica [A]
time = 0.43, size = 106, normalized size = 0.73

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(3\sqrt{2}\,B\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+2(A+(5A+3B)\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(3*Sqrt[2
]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(A + (5*A + 3*B
)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(123) = 246$.
time = 0.42, size = 287, normalized size = 1.98

method	result
default	$-\frac{2\left(3B\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\left(\cos^2(dx+c)\right)+6B\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+}{1+\cos(dx}}}{\cos(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2+6*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*
cos(d*x+c)+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+5*A*sin(d*x+c)*cos(d*x+c)+3*B*sin(d*x
+c)*cos(d*x+c)+A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1462 vs. $2(123) = 246$.
time = 0.69, size = 1462, normalized size = 10.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/6*(3*(6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*a^(3/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
((2*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c)
 - a*sin(2*d*x + 2*c) - 2*(a*cos(2*d*x + 2*c) + a)*sin(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + (2*a*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + a*cos(2*d*x + 2*c) + 2*(a*cos(2*d*x + 2*c) + a)*cos(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a)*sin(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + ((a*cos(2*d*x + 2*c)^2 + a*sin(2*d*
x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a*cos(2*d*x + 2*c)^2 + a*
sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
```

```
*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - (a*cos(2*d*x + 2*c
)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + 1) + (a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2
 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
1))*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1) + 8*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)))/d
```

Fricas [**A**]
time = 0.40, size = 130, normalized size = 0.90

$$-\frac{2\left(3\left(Ba\cos\left(dx+c\right)^{2}+Ba\cos\left(dx+c\right)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{((5A+3B)a\cos(dx+c)+Aa)\sqrt{a\cos\left(dx+c\right)+a}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}\right)}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -2/3*(3*(B*a*cos(d*x + c)^2 + B*a*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - ((5*A + 3*B)*a*co
s(d*x + c) + A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2),
 x)
```

3.506 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=146

$$\frac{a^{3/2}(2A+3B)\text{ArcSin}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}}{d}-\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\ \sqrt{\sec(c+dx)}}$$

[Out] -a^2*(2*A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+a^(3/2)*(2*A+3*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*a*A*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A]
time = 0.29, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3054, 3060, 2853, 222}

$$\frac{a^{3/2}(2A+3B)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}\ \text{ArcSin}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\ \sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}\ \sqrt{a\cos(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]

[Out] (a^(3/2)*(2*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^2*(2*A - B)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,c}{}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin(}{d}$$

$$= -\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \frac{2a}{}$$

$$= -\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \frac{2a}{}$$

$$= \frac{a^{3/2}(2A+3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right.}{d}$$

Mathematica [A]
time = 0.35, size = 107, normalized size = 0.73

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}\,(2A+3B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2(2A+B\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*
(2*A + 3*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*A +
B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(128) = 256$.
time = 0.42, size = 308, normalized size = 2.11

method	result
default	$\frac{\left(2A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\cos(dx+c)+3B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c}}}{\cos(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d
*x+c)+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arct
an(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+B*sin(d*x+c)*co
s(d*x+c)+2*A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))
^(1/2)/(1+cos(d*x+c))*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1801 vs. $2(128) = 256$.
time = 0.75, size = 1801, normalized size = 12.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*B + 2*(
(a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
```

```
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*s
qrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4))/d
```

Fricas [A]
time = 0.40, size = 119, normalized size = 0.82

$$-\frac{((2A+3B)a\cos(dx+c)+(2A+3B)a)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{(Ba\cos(dx+c)+2Aa)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -((((2*A + 3*B)*a*cos(d*x + c) + (2*A + 3*B)*a)*sqrt(a)*arctan(sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (B*a*cos(d*x + c)
+ 2*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d
*x + c) + d)
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2),
 x)
```

3.507 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx}$

Optimal. Leaf size=153

$$\frac{a^{3/2}(12A+7B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{4d}+\frac{a^2(4A+5B)\sin(c+}{4d\sqrt{a+a\cos(c+dx)}\,\sqrt{\text{se}}}$$

```
[Out] 1/4*a^2*(4*A+5*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/2*
a*B*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*a^(3/2)*(12*A+
7*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.29, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3055, 3060, 2853, 222}

$$\frac{a^{3/2}(12A+7B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2(4A+5B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*(4*A + 5*B)*Sin[c +
d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a*B*Sqrt[a + a*C
os[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
```

```
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a}{}$$

$$= \frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{2}\Big($$

$$= \frac{a^2(4A+5B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} +$$

$$= \frac{a^2(4A+5B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} +$$

$$= \frac{a^{3/2}(12A+7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+d}}\right.}{4d}$$

Mathematica [A]
time = 0.49, size = 121, normalized size = 0.79

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}(12A+7B)\mathrm{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}(4A+7B+2B\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[
c + d*x]]*(Sqrt[2]*(12*A + 7*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[C
os[c + d*x]]*(4*A + 7*B + 2*B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [A]
time = 0.41, size = 233, normalized size = 1.52

method	result
default	$-\frac{\left(2B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)+4A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+7B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+12A\arctan\Big(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*(2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+4*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+7*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)+12*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c))+7*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c
)))*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1884 vs. $2(129) = 258$.
time = 0.73, size = 1884, normalized size = 12.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A +
(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c)
+ a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(2*d*x
```

```
 + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2
((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - 1))*sqrt(a))*B)/d
```

Fricas [A]
time = 0.40, size = 133, normalized size = 0.87

$$-\frac{((12\,A+7\,B)a\cos\left(dx+c\right)+(12\,A+7\,B)a)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(2\,Ba\cos(dx+c)^2+(4\,A+7\,B)a\cos(dx+c)\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(((12*A + 7*B)*a*cos(d*x + c) + (12*A + 7*B)*a)*sqrt(a)*arctan(sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*B*a*cos(d
*x + c)^2 + (4*A + 7*B)*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\,\cos(c + d\,x))\,\sqrt{\frac{1}{\cos(c + d\,x)}}\,(a + a\,\cos(c + d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2),
 x)
```

3.508 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=200

$$\frac{a^{3/2}(14A+11B)\text{ArcSin}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}}{8d}+\frac{a^2(6A+7B)\sin(c+}{12d\sqrt{a+a\cos(c+dx)}\ \text{se}}$$

```
[Out] 1/12*a^2*(6*A+7*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/3
*a*B*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+1/8*a^2*(14*A+11*
B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/8*a^(3/2)*(14*A+1
1*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.35, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3055, 3060, 2849, 2853, 222}

$$\frac{a^{3/2}(14A+11B)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}\ \text{ArcSin}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(6A+7B)\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(14A+11B)\sin(c+dx)}{8d\sqrt{\sec(c+dx)}\ \sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(6*A + 7*B)*Sin[c +
 d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*B*Sqrt[a + a
*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (a^2*(14*A + 11*B)*
Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,(a+$$

$$= \frac{aB\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\right.$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos}}{3d\sec}$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos}}{3d\sec}$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos}}{3d\sec}$$

$$= \frac{a^{3/2}(14A+11B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c}}{8d}$$

Mathematica [A]
time = 0.55, size = 141, normalized size = 0.70

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(3\sqrt{2}\,(14A+11B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+(42A+37B+2(6A+11B)\cos(c+dx)+4B\cos(2(c+dx)))\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt[2
]*(14*A + 11*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (42*A
 + 37*B + 2*(6*A + 11*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*(-Sin[(c + d*
x)/2] + Sin[(3*(c + d*x))/2])))/(48*d)
```

Maple [A]
time = 0.45, size = 309, normalized size = 1.54

method	result

default	$\frac{(-1+\cos(dx+c))^2\left(8B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))+12A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+22B\sin(dx+c)\sqrt{\frac{}{1}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/24/d*(-1+cos(d*x+c))^2*(8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
cos(d*x+c)^2+12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+2
2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+42*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+33*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)+42*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+
c))+33*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*c
os(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/cos
(d*x+c))^(1/2)/sin(d*x+c)^4*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3023 vs. $2(170) = 340$.
time = 0.82, size = 3023, normalized size = 15.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
```

*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(

```
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1))) + 1) - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/
4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*...
```

Fricas [A]
time = 0.40, size = 153, normalized size = 0.76

$$-\frac{3\left((14\,A+11\,B)a\cos\left(dx+c\right)+(14\,A+11\,B)a\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(8\,Ba\cos(dx+c)^3+2\,(6\,A+11\,B)a\cos(dx+c)^2+3\,(14\,A+11\,B)a\cos(dx+c)\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/24*(3*((14*A + 11*B)*a*cos(d*x + c) + (14*A + 11*B)*a)*sqrt(a)*arctan(sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*B*a*
cos(d*x + c)^3 + 2*(6*A + 11*B)*a*cos(d*x + c)^2 + 3*(14*A + 11*B)*a*cos(d*
x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*
x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{3/2}}{\sqrt{\frac{1}{\cos(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2
), x)
```

3.509 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=247

$$\frac{a^{3/2}(88A+75B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{64d}+\frac{a^2(8A+9B)\sin(c+}{24d\sqrt{a+a\cos(c+dx)}\,\sec}$$

```
[Out] 1/24*a^2*(8*A+9*B)*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+1/9
6*a^2*(88*A+75*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/4*
a*B*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+1/64*a^2*(88*A+75*
B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/64*a^(3/2)*(88*A+
75*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.40, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3055, 3060, 2849, 2853, 222}

$$\frac{a^{3/2}(88A+75B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(88A+75B)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(8A+9B)\sin(c+dx)}{24d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)}{64d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{4d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(8*A + 9*B)*Sin[c
+ d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a*B*Sqrt[a +
a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sec[c + d*x]^(5/2)) + (a^2*(88*A + 75*B)
*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*(8
8*A + 75*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\,\text{co}$$

$$= \frac{aB\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\right.$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c}{4d\sec}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c}{4d\sec}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c}{4d\sec}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c}{4d\sec}$$

$$= \frac{a^{3/2}(88A+75B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+}{64d}$$

Mathematica [A]

time = 0.81, size = 158, normalized size = 0.64

$$\frac{a\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(3\sqrt{2}\,(88A+75B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+(296A+285B+2(88A+93B)\cos(c+dx)+4(8A+15B)\cos(2(c+dx))+12B\cos(3(c+dx)))\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt[2
]*(88*A + 75*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (296*
A + 285*B + 2*(88*A + 93*B)*Cos[c + d*x] + 4*(8*A + 15*B)*Cos[2*(c + d*x)]
+ 12*B*Cos[3*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(384*
d)
```

Maple [A]

time = 0.37, size = 381, normalized size = 1.54

method	result

default	$-\frac{(-1+\cos(dx+c))^3\left(48B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^3(dx+c))+64A\sin(dx+c)(\cos^2(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+120B\sin(d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/192/d*(-1+cos(d*x+c))^3*(48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)^3+64*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)+120*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+176*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+150*B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+264*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)+225*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+264*A*
arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+225*B*arcta
n(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(
1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+c))^(3/2)
/sin(d*x+c)^6*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 8901 vs. $2(211) = 422$.
time = 1.13, size = 8901, normalized size = 36.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*
d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x +
```

```
3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqr
t(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/
4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(si
n(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1)
 - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*arc
tan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
```

```
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqrt(
a))*A + 3*(2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((5*a*cos(4*d*x + 4*c)^2*sin(4*
d*x + 4*c) + 5*a*sin(4*d*x + 4*c)^3 + 20*(a*sin...
```

Fricas [A]
time = 0.45, size = 171, normalized size = 0.69

$$-\frac{3\left((88A+75B)a\cos(dx+c)+(88A+75B)a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(48Ba\cos(dx+c)^4+8(8A+15B)a\cos(dx+c)^3+2(88A+75B)a\cos(dx+c)^2+3(88A+75B)a\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{192\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/192*(3*((88*A + 75*B)*a*cos(d*x + c) + (88*A + 75*B)*a)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (48*B*
a*cos(d*x + c)^4 + 8*(8*A + 15*B)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(
d*x + c)^2 + 3*(88*A + 75*B)*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{3/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2
), x)
```

3.510 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{15}{2}}(c+dx)\,dx$

Optimal. Leaf size=322

$$\frac{32a^3(4184A+4615B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{45045d\sqrt{a+a\cos(c+dx)}}+\frac{16a^3(4184A+4615B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45045d\sqrt{a+a\cos(c+dx)}}+\frac{4a^3(41$$

```
[Out] 2/13*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(13/2)*sin(d*x+c)/d+16/45045*a^3
*(4184*A+4615*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+4/150
15*a^3*(4184*A+4615*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
+2/9009*a^3*(4184*A+4615*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^
(1/2)+2/1287*a^3*(280*A+299*B)*sec(d*x+c)^(9/2)*sin(d*x+c)/d/(a+a*cos(d*x+c
))^(1/2)+2/143*a^2*(16*A+13*B)*sec(d*x+c)^(11/2)*sin(d*x+c)*(a+a*cos(d*x+c)
)^(1/2)/d+32/45045*a^3*(4184*A+4615*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*c
os(d*x+c))^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3054, 3059, 2851, 2850}

$$\frac{2a^3(280A+299B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{1287d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(4184A+4615B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{9009d\sqrt{a\cos(c+dx)+a}}+\frac{4a^3(4184A+4615B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{15015d\sqrt{a\cos(c+dx)+a}}+\frac{16a^3(4184A+4615B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{45045d\sqrt{a\cos(c+dx)+a}}+\frac{32a^3(4184A+4615B)\sin(c+dx)\sqrt{\sec(c+dx)}}{45045d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(16A+13B)\sin(c+dx)\sec^{\frac{11}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{143d}+\frac{2aA\sin(c+dx)\sec^{\frac{13}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(15/2),x]
```

```
[Out] (32*a^3*(4184*A + 4615*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(45045*d*Sqrt[a
+ a*Cos[c + d*x]]) + (16*a^3*(4184*A + 4615*B)*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(45045*d*Sqrt[a + a*Cos[c + d*x]]) + (4*a^3*(4184*A + 4615*B)*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(15015*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(4184*
A + 4615*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(9009*d*Sqrt[a + a*Cos[c + d*x
]]) + (2*a^3*(280*A + 299*B)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(1287*d*Sqrt[
a + a*Cos[c + d*x]]) + (2*a^2*(16*A + 13*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c
+ d*x]^(11/2)*Sin[c + d*x])/(143*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec
[c + d*x]^(13/2)*Sin[c + d*x])/(13*d)
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{15}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,c}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{13}{2}}(c+dx)\sin}{13d}$$

$$= \frac{2a^2(16A+13B)\sqrt{a+a\cos(c+dx)}\ \sec^{\frac{11}{2}}}{143d}$$

$$= \frac{2a^3(280A+299B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx}{1287d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^3(4184A+4615B)\sec^{\frac{7}{2}}(c+dx)\sin(c+}{9009d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{4a^3(4184A+4615B)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{15015d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{16a^3(4184A+4615B)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{45045d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{32a^3(4184A+4615B)\sqrt{\sec(c+dx)}\ \sin(c}{45045d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A]
time = 0.97, size = 171, normalized size = 0.53

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,(171806A+162955B+35(5552A+5083B)\cos(c+dx)+14(15167A+15925B)\cos(2(c+dx))+62760A\cos(3(c+dx))+69225B\cos(3(c+dx))+62760A\cos(4(c+dx))+69225B\cos(4(c+dx))+8368A\cos(5(c+dx))+9230B\cos(5(c+dx))+8368A\cos(6(c+dx))+9230B\cos(6(c+dx)))\sec^{\frac{13}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{90090d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(15/
2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(171806*A + 162955*B + 35*(5552*A + 5083*B)
*Cos[c + d*x] + 14*(15167*A + 15925*B)*Cos[2*(c + d*x)] + 62760*A*Cos[3*(c
+ d*x)] + 69225*B*Cos[3*(c + d*x)] + 62760*A*Cos[4*(c + d*x)] + 69225*B*Cos
[4*(c + d*x)] + 8368*A*Cos[5*(c + d*x)] + 9230*B*Cos[5*(c + d*x)] + 8368*A*
Cos[6*(c + d*x)] + 9230*B*Cos[6*(c + d*x)])*Sec[c + d*x]^(13/2)*Tan[(c + d*
x)/2])/(90090*d)
```

Maple [A]
time = 0.57, size = 185, normalized size = 0.57

method	result
default	$-\frac{2(-1+\cos(dx+c))\left(66944A\left(\cos^6(dx+c)\right)+73840B\left(\cos^6(dx+c)\right)+33472A\left(\cos^5(dx+c)\right)+36920B\left(\cos^5(dx+c)\right)+25104A\left(\cos^4(dx+c\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/45045/d*(-1+cos(d*x+c))*(66944*A*cos(d*x+c)^6+73840*B*cos(d*x+c)^6+33472
*A*cos(d*x+c)^5+36920*B*cos(d*x+c)^5+25104*A*cos(d*x+c)^4+27690*B*cos(d*x+c
)^4+20920*A*cos(d*x+c)^3+23075*B*cos(d*x+c)^3+18305*A*cos(d*x+c)^2+14560*B*
cos(d*x+c)^2+11970*A*cos(d*x+c)+4095*B*cos(d*x+c)+3465*A)*cos(d*x+c)*(a*(1+
cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(15/2)/sin(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(280) = 560$.
time = 0.62, size = 763, normalized size = 2.37

$$\frac{8\left(\frac{\left(\frac{45045\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{165165\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{414414\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{604890\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{522665\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{289185\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{88980\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{11864\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}\right)A\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^5}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{15}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{15}{2}}\left(\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+1\right)}+\frac{65\left(\frac{693\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{3003\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6930\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{10098\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{9053\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{4875\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{1500\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{200\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}\right)B\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^5}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{15}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{15}{2}}\left(\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+1\right)}\right)}{45045\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x, algo
rithm="maxima")
```

```
[Out] 8/45045*((45045*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 165165*sq
rt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 414414*sqrt(2)*a^(5/2)*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 604890*sqrt(2)*a^(5/2)*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 522665*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 - 289185*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 88
980*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 11864*sqrt(2)*a
^(5/2)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*A*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c
) + 1)^10 + 1)) + 65*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) -
 3003*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6930*sqrt(2)*a^
(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 10098*sqrt(2)*a^(5/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 9053*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x +
 c) + 1)^9 - 4875*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 1
500*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 200*sqrt(2)*a^(
5/2)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*B*(sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin(d*x + c)
```

```
/(cos(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c)
+ 1)^10 + 1)))/d
```

Fricas [A]
time = 0.40, size = 176, normalized size = 0.55

$$\frac{2 \left(16 \left(4184 A + 4615 B\right) a^{2} \cos \left(d x + c\right)^{6} + 8 \left(4184 A + 4615 B\right) a^{2} \cos \left(d x + c\right)^{5} + 6 \left(4184 A + 4615 B\right) a^{2} \cos \left(d x + c\right)^{4} + 5 \left(4184 A + 4615 B\right) a^{2} \cos \left(d x + c\right)^{3} + 35 \left(523 A + 416 B\right) a^{2} \cos \left(d x + c\right)^{2} + 315 \left(38 A + 13 B\right) a^{2} \cos \left(d x + c\right) + 3465 A a^{2}\right) \sqrt{a \cos \left(d x + c\right) + a} \sin \left(d x + c\right)}{45045 \left(d \cos \left(d x + c\right)^{7} + d \cos \left(d x + c\right)^{6}\right) \sqrt{\cos \left(d x + c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x, algo
rithm="fricas")
```

```
[Out] 2/45045*(16*(4184*A + 4615*B)*a^2*cos(d*x + c)^6 + 8*(4184*A + 4615*B)*a^2*
cos(d*x + c)^5 + 6*(4184*A + 4615*B)*a^2*cos(d*x + c)^4 + 5*(4184*A + 4615*
B)*a^2*cos(d*x + c)^3 + 35*(523*A + 416*B)*a^2*cos(d*x + c)^2 + 315*(38*A +
 13*B)*a^2*cos(d*x + c) + 3465*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
/((d*cos(d*x + c)^7 + d*cos(d*x + c)^6)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(15/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.19, size = 789, normalized size = 2.45

$$\frac{\sqrt{\frac{1}{\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}\,\left(\frac{a^2\,\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}\,\left(4184\,A+4615\,B\right)\,32{}\mathrm{i}}{45045\,d}-\frac{a^2\,{\mathrm{e}}^{c\,5{}\mathrm{i}+d\,x\,5{}\mathrm{i}}\,\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}\,\left(2\,A+5\,B\right)\,16{}\mathrm{i}}{5\,d}+\frac{a^2\,{\mathrm{e}}^{c\,8{}\mathrm{i}+d\,x\,8{}\mathrm{i}}\,\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}\,\left(2\,A+5\,B\right)\,16{}\mathrm{i}}{5\,d}+\frac{a^2\,{\mathrm{e}}^{c\,6{}\mathrm{i}+d\,x\,6{}\mathrm{i}}\,\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}\,\left(116\,A+115\,B\right)\,16{}\mathrm{i}}{35\,d}-\frac{a^2\,{\mathrm{e}}^{c\,7{}\mathrm{i}+d\,x\,7{}\mathrm{i}}\,\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}\,\left(116\,A+115\,B\right)\,16{}\mathrm{i}}{35\,d}+\frac{a^2\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}\,\left(1046\,A+1075\,B\right)\,16{}\mathrm{i}}{315\,d}-\frac{a^2\,{\mathrm{e}}^{c\,9{}\mathrm{i}+d\,x\,9{}\mathrm{i}}\,\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}\,\left(1046\,A+1075\,B\right)\,16{}\mathrm{i}}{315\,d}+\frac{a^2\,{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}\,\left(4184\,A+4615\,B\right)\,16{}\mathrm{i}}{3465\,d}-\frac{a^2\,{\mathrm{e}}^{c\,11{}\mathrm{i}+d\,x\,11{}\mathrm{i}}\,\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}\,\left(4184\,A+4615\,B\right)\,16{}\mathrm{i}}{3465\,d}-\frac{a^2\,{\mathrm{e}}^{c\,13{}\mathrm{i}+d\,x\,13{}\mathrm{i}}\,\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}\,\left(4184\,A+4615\,B\right)\,32{}\mathrm{i}}{45045\,d}\right)}{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}+6\,{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+6\,{\mathrm{e}}^{c\,3{}\mathrm{i}+d\,x\,3{}\mathrm{i}}+15\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}+15\,{\mathrm{e}}^{c\,5{}\mathrm{i}+d\,x\,5{}\mathrm{i}}+20\,{\mathrm{e}}^{c\,6{}\mathrm{i}+d\,x\,6{}\mathrm{i}}+20\,{\mathrm{e}}^{c\,7{}\mathrm{i}+d\,x\,7{}\mathrm{i}}+15\,{\mathrm{e}}^{c\,8{}\mathrm{i}+d\,x\,8{}\mathrm{i}}+15\,{\mathrm{e}}^{c\,9{}\mathrm{i}+d\,x\,9{}\mathrm{i}}+6\,{\mathrm{e}}^{c\,10{}\mathrm{i}+d\,x\,10{}\mathrm{i}}+6\,{\mathrm{e}}^{c\,11{}\mathrm{i}+d\,x\,11{}\mathrm{i}}+{\mathrm{e}}^{c\,12{}\mathrm{i}+d\,x\,12{}\mathrm{i}}+{\mathrm{e}}^{c\,13{}\mathrm{i}+d\,x\,13{}\mathrm{i}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(15/2)*(a + a*cos(c + d*x))^(5/2)
,x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(a + a*(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(4184*A + 4615*B)*32i)/
(45045*d) - (a^2*exp(c*5i + d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*
1i + d*x*1i)/2))^(1/2)*(2*A + 5*B)*16i)/(5*d) + (a^2*exp(c*8i + d*x*8i)*(a
+ a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(2*A + 5*B)*16i)
/(5*d) + (a^2*exp(c*6i + d*x*6i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i
+ d*x*1i)/2))^(1/2)*(116*A + 115*B)*16i)/(35*d) - (a^2*exp(c*7i + d*x*7i)*(
a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(116*A + 115*B
)*16i)/(35*d) + (a^2*exp(c*4i + d*x*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 + ex
p(c*1i + d*x*1i)/2))^(1/2)*(1046*A + 1075*B)*16i)/(315*d) - (a^2*exp(c*9i +
 d*x*9i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(104
6*A + 1075*B)*16i)/(315*d) + (a^2*exp(c*2i + d*x*2i)*(a + a*(exp(- c*1i - d
*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(4184*A + 4615*B)*16i)/(3465*d) - (
a^2*exp(c*11i + d*x*11i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i
)/2))^(1/2)*(4184*A + 4615*B)*16i)/(3465*d) - (a^2*exp(c*13i + d*x*13i)*(a
+ a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(4184*A + 4615*B
)*32i)/(45045*d)))/(exp(c*1i + d*x*1i) + 6*exp(c*2i + d*x*2i) + 6*exp(c*3i
+ d*x*3i) + 15*exp(c*4i + d*x*4i) + 15*exp(c*5i + d*x*5i) + 20*exp(c*6i + d
*x*6i) + 20*exp(c*7i + d*x*7i) + 15*exp(c*8i + d*x*8i) + 15*exp(c*9i + d*x*
9i) + 6*exp(c*10i + d*x*10i) + 6*exp(c*11i + d*x*11i) + exp(c*12i + d*x*12i
) + exp(c*13i + d*x*13i) + 1)
```

3.511 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=275

$$\frac{16a^3(710A+803B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}}+\frac{8a^3(710A+803B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}}+\frac{2a^3(710A+}{115}$$

```
[Out] 2/11*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(11/2)*sin(d*x+c)/d+8/3465*a^3*(
710*A+803*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/1155*a^
3*(710*A+803*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/693*
a^3*(194*A+209*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/99
*a^2*(14*A+11*B)*sec(d*x+c)^(9/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+16/34
65*a^3*(710*A+803*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3054, 3059, 2851, 2850}

$$\frac{2a^3(194A+209B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(710A+803B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{1155d\sqrt{a\cos(c+dx)+a}}+\frac{8a^3(710A+803B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3465d\sqrt{a\cos(c+dx)+a}}+\frac{16a^3(710A+803B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3465d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(14A+11B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{99d}+\frac{2aA\sin(c+dx)\sec^{\frac{11}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/2),x]
```

```
[Out] (16*a^3*(710*A + 803*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a
*Cos[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
3465*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(710*A + 803*B)*Sec[c + d*x]^(5/2
)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(194*A + 209*B)*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99
*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(
11*d)
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
```

```
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a + a\cos(c+dx))^{5/2}(A + B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a + a\,\mathrm{c}}{}$$

$$= \frac{2aA(a + a\cos(c+dx))^{3/2}\sec^{\frac{11}{2}}(c+dx)\sin}{11d}$$

$$= \frac{2a^2(14A + 11B)\sqrt{a + a\cos(c+dx)}\,\sec^{\frac{9}{2}}(}{99d}$$

$$= \frac{2a^3(194A + 209B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx}{693d\sqrt{a + a\cos(c+dx)}}$$

$$= \frac{2a^3(710A + 803B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx}{1155d\sqrt{a + a\cos(c+dx)}}$$

$$= \frac{8a^3(710A + 803B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{3465d\sqrt{a + a\cos(c+dx)}}$$

$$= \frac{16a^3(710A + 803B)\sqrt{\sec(c+dx)}\,\sin(c+}{3465d\sqrt{a + a\cos(c+dx)}}$$

Mathematica [A]

time = 1.35, size = 147, normalized size = 0.53

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,(9070A + 7678B + (25070A + 24827B)\cos(c+dx) + (9230A + 9284B)\cos(2(c+dx)) + 9230A\cos(3(c+dx)) + 10439B\cos(3(c+dx)) + 1420A\cos(4(c+dx)) + 1606B\cos(4(c+dx)) + 1420A\cos(5(c+dx)) + 1606B\cos(5(c+dx)))\sec^{\frac{11}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{6930d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/
2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(9070*A + 7678*B + (25070*A + 24827*B)*Cos[
c + d*x] + (9230*A + 9284*B)*Cos[2*(c + d*x)] + 9230*A*Cos[3*(c + d*x)] + 1
0439*B*Cos[3*(c + d*x)] + 1420*A*Cos[4*(c + d*x)] + 1606*B*Cos[4*(c + d*x)]
 + 1420*A*Cos[5*(c + d*x)] + 1606*B*Cos[5*(c + d*x)])*Sec[c + d*x]^(11/2)*T
an[(c + d*x)/2])/(6930*d)
```

Maple [A]

time = 0.42, size = 163, normalized size = 0.59

method	result
default	$-\frac{2(-1+\cos(dx+c))(5680A(\cos^5(dx+c))+6424B(\cos^5(dx+c))+2840A(\cos^4(dx+c))+3212B(\cos^4(dx+c))+2130A(\cos^3(dx+c))+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/3465/d*(-1+cos(d*x+c))*(5680*A*cos(d*x+c)^5+6424*B*cos(d*x+c)^5+2840*A*c
os(d*x+c)^4+3212*B*cos(d*x+c)^4+2130*A*cos(d*x+c)^3+2409*B*cos(d*x+c)^3+177
5*A*cos(d*x+c)^2+1430*B*cos(d*x+c)^2+1120*A*cos(d*x+c)+385*B*cos(d*x+c)+315
*A)*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(13/2)/sin(d*x+c)*a^
2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(239) = 478$.
time = 0.57, size = 672, normalized size = 2.44

$$\frac{8\left(\frac{5\left(\frac{693\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2310\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4620\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5478\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3575\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{1300\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{200\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}\right)A\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}+\frac{11\left(\frac{315\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{1260\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2394\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2736\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{1859\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{676\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{104\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}\right)B\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] 8/3465*(5*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 2310*sqrt(
2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4620*sqrt(2)*a^(5/2)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 5478*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 3575*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
 1300*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 200*sqrt(2)*a
^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 11*(315*sqrt(2)*a^(5/2)*si
n(d*x + c)/(cos(d*x + c) + 1) - 1260*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 2394*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
2736*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1859*sqrt(2)*a^(
5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 676*sqrt(2)*a^(5/2)*sin(d*x + c)
^11/(cos(d*x + c) + 1)^11 + 104*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x +
c) + 1)^13)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(c
os(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*
(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 + 1)))/d
```

Fricas [A]
time = 0.39, size = 156, normalized size = 0.57

$$\frac{2\left(8(710A+803B)a^2\cos(dx+c)^5+4(710A+803B)a^2\cos(dx+c)^4+3(710A+803B)a^2\cos(dx+c)^3+5(355A+286B)a^2\cos(dx+c)^2+35(32A+11B)a^2\cos(dx+c)+315Aa^2\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{3465\left(d\cos(dx+c)^6+d\cos(dx+c)^5\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(8*(710*A + 803*B)*a^2*cos(d*x + c)^5 + 4*(710*A + 803*B)*a^2*cos(d*
x + c)^4 + 3*(710*A + 803*B)*a^2*cos(d*x + c)^3 + 5*(355*A + 286*B)*a^2*cos
(d*x + c)^2 + 35*(32*A + 11*B)*a^2*cos(d*x + c) + 315*A*a^2)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^6 + d*cos(d*x + c)^5)*sqrt(cos(d*x
 + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.81, size = 751, normalized size = 2.73

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(710\,A+803\,B)\,16i}{3465\,d}-\frac{B\,a^2\,e^{c\,3i+d\,x\,3i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,8i}{3\,d}+\frac{B\,a^2\,e^{c\,8i+d\,x\,8i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,8i}{3\,d}-\frac{a^2\,e^{c\,5i+d\,x\,5i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(30\,A+41\,B)\,8i}{15\,d}+\frac{a^2\,e^{c\,6i+d\,x\,6i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(30\,A+41\,B)\,8i}{15\,d}+\frac{a^2\,e^{c\,4i+d\,x\,4i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(160\,A+157\,B)\,8i}{35\,d}-\frac{a^2\,e^{c\,7i+d\,x\,7i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(160\,A+157\,B)\,8i}{35\,d}+\frac{a^2\,e^{c\,2i+d\,x\,2i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(710\,A+803\,B)\,8i}{315\,d}-\frac{a^2\,e^{c\,9i+d\,x\,9i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(710\,A+803\,B)\,8i}{315\,d}-\frac{a^2\,e^{c\,11i+d\,x\,11i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(710\,A+803\,B)\,16i}{3465\,d}\right)}{e^{c\,1i+d\,x\,1i}+5\,e^{c\,2i+d\,x\,2i}+5\,e^{c\,3i+d\,x\,3i}+10\,e^{c\,4i+d\,x\,4i}+10\,e^{c\,5i+d\,x\,5i}+10\,e^{c\,6i+d\,x\,6i}+10\,e^{c\,7i+d\,x\,7i}+5\,e^{c\,8i+d\,x\,8i}+5\,e^{c\,9i+d\,x\,9i}+e^{c\,10i+d\,x\,10i}+e^{c\,11i+d\,x\,11i}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(13/2)*(a + a*cos(c + d*x))^(5/2)
,x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(a + a*(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(710*A + 803*B)*16i)/(3
465*d) - (B*a^2*exp(c*3i + d*x*3i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1
i + d*x*1i)/2))^(1/2)*8i)/(3*d) + (B*a^2*exp(c*8i + d*x*8i)*(a + a*(exp(- c
*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*8i)/(3*d) - (a^2*exp(c*5i +
d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(30*A
```

```
 + 41*B)*8i)/(15*d) + (a^2*exp(c*6i + d*x*6i)*(a + a*(exp(- c*1i - d*x*1i)/
2 + exp(c*1i + d*x*1i)/2))^(1/2)*(30*A + 41*B)*8i)/(15*d) + (a^2*exp(c*4i +
 d*x*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(160
*A + 157*B)*8i)/(35*d) - (a^2*exp(c*7i + d*x*7i)*(a + a*(exp(- c*1i - d*x*1
i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(160*A + 157*B)*8i)/(35*d) + (a^2*exp(c
*2i + d*x*2i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)
*(710*A + 803*B)*8i)/(315*d) - (a^2*exp(c*9i + d*x*9i)*(a + a*(exp(- c*1i -
 d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(710*A + 803*B)*8i)/(315*d) - (a^
2*exp(c*11i + d*x*11i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/
2))^(1/2)*(710*A + 803*B)*16i)/(3465*d)))/(exp(c*1i + d*x*1i) + 5*exp(c*2i
+ d*x*2i) + 5*exp(c*3i + d*x*3i) + 10*exp(c*4i + d*x*4i) + 10*exp(c*5i + d*
x*5i) + 10*exp(c*6i + d*x*6i) + 10*exp(c*7i + d*x*7i) + 5*exp(c*8i + d*x*8i
) + 5*exp(c*9i + d*x*9i) + exp(c*10i + d*x*10i) + exp(c*11i + d*x*11i) + 1)
```

3.512 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=228

$$\frac{4a^3(292A+345B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}+\frac{2a^3(292A+345B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}+\frac{2a^3(124A+1}{315d}$$

```
[Out] 2/9*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2)*sin(d*x+c)/d+2/315*a^3*(292
*A+345*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/315*a^3*(1
24*A+135*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/21*a^2*(
4*A+3*B)*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+4/315*a^3*(29
2*A+345*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3054, 3059, 2851, 2850}

$$\frac{2a^3(124A+135B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(292A+345B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{4a^3(292A+345B)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(4A+3B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{21d}+\frac{2aA\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

```
[Out] (4*a^3*(292*A + 345*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(124*A + 135*B)*Sec[c + d*x]^(5/2)*S
in[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(4*A + 3*B)*Sqrt[a +
 a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*d) + (2*a*A*(a + a*Co
s[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\cdots}{\cdots} \\
&= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\sin(\cdots}{9d} \\
&= \frac{2a^2(4A+3B)\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{7}{2}}(c\cdots}{21d} \\
&= \frac{2a^3(124A+135B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx\cdots}{315d\sqrt{a+a\cos(c+dx)}} \\
&= \frac{2a^3(292A+345B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx\cdots}{315d\sqrt{a+a\cos(c+dx)}} \\
&= \frac{4a^3(292A+345B)\sqrt{\sec(c+dx)}\,\sin(c+\cdots}{315d\sqrt{a+a\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.99, size = 126, normalized size = 0.55

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,(1454A+1395B+(1396A+1215B)\cos(c+dx)+2(803A+870B)\cos(2(c+dx))+292A\cos(3(c+dx))+345B\cos(3(c+dx))+292A\cos(4(c+dx))+345B\cos(4(c+dx)))\sec^{\frac{9}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(1454*A + 1395*B + (1396*A + 1215*B)*Cos[c
+ d*x] + 2*(803*A + 870*B)*Cos[2*(c + d*x)] + 292*A*Cos[3*(c + d*x)] + 345*
B*Cos[3*(c + d*x)] + 292*A*Cos[4*(c + d*x)] + 345*B*Cos[4*(c + d*x)])*Sec[c
 + d*x]^(9/2)*Tan[(c + d*x)/2])/(630*d)
```

Maple [A]
time = 0.40, size = 141, normalized size = 0.62

method	result
default	$-\frac{2(-1+\cos(dx+c))\left(584A\left(\cos^4(dx+c)\right)+690B\left(\cos^4(dx+c)\right)+292A\left(\cos^3(dx+c)\right)+345B\left(\cos^3(dx+c)\right)+219A\left(\cos^2(dx+c)\right)+180B\cdots\right.}{315d\sin(dx+c\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(584*A*cos(d*x+c)^4+690*B*cos(d*x+c)^4+292*A*cos(d
*x+c)^3+345*B*cos(d*x+c)^3+219*A*cos(d*x+c)^2+180*B*cos(d*x+c)^2+130*A*cos(
d*x+c)+45*B*cos(d*x+c)+35*A)*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x
+c))^(11/2)/sin(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(198) = 396$.
time = 0.58, size = 579, normalized size = 2.54

$$\frac{8\left(\frac{\left(\frac{315\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{945\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1449\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1287\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{572\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{104\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)A\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}+\frac{15\left(\frac{21\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{119\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{99\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{44\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)B\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 8/315*((315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*
sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 1)) + 15*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) -
77*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 119*sqrt(2)*a^(5/2
)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 99*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 44*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 1)))/d
```

Fricas [A]
time = 0.38, size = 135, normalized size = 0.59

$$\frac{2\,(2\,(292\,A+345\,B)a^2\cos{(dx+c)}^4+(292\,A+345\,B)a^2\cos{(dx+c)}^3+3\,(73\,A+60\,B)a^2\cos{(dx+c)}^2+5\,(26\,A+9\,B)a^2\cos{(dx+c)}+35\,Aa^2)\sqrt{a\cos{(dx+c)}+a}\,\sin{(dx+c)}}{315\,(d\cos{(dx+c)}^5+d\cos{(dx+c)}^4)\sqrt{\cos{(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(2*(292*A + 345*B)*a^2*cos(d*x + c)^4 + (292*A + 345*B)*a^2*cos(d*x +
 c)^3 + 3*(73*A + 60*B)*a^2*cos(d*x + c)^2 + 5*(26*A + 9*B)*a^2*cos(d*x + c
```

```
) + 35*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*
cos(d*x + c)^4)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="giac")
[Out] Timed out
```

Mupad [B]
time = 5.73, size = 617, normalized size = 2.71

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(292\,A+345\,B)\,4i}{315\,d}-\frac{a^2\,e^{c\,3i+d\,x\,3i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(2\,A+5\,B)\,4i}{3\,d}+\frac{a^2\,e^{c\,6i+d\,x\,6i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(2\,A+5\,B)\,4i}{3\,d}+\frac{a^2\,e^{c\,4i+d\,x\,4i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(24\,A+25\,B)\,4i}{5\,d}-\frac{a^2\,e^{c\,5i+d\,x\,5i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(24\,A+25\,B)\,4i}{5\,d}+\frac{a^2\,e^{c\,2i+d\,x\,2i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(146\,A+155\,B)\,4i}{35\,d}-\frac{a^2\,e^{c\,7i+d\,x\,7i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(146\,A+155\,B)\,4i}{35\,d}-\frac{a^2\,e^{c\,9i+d\,x\,9i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(292\,A+345\,B)\,4i}{315\,d}\right)}{e^{c\,1i+d\,x\,1i}+4\,e^{c\,2i+d\,x\,2i}+4\,e^{c\,3i+d\,x\,3i}+6\,e^{c\,4i+d\,x\,4i}+6\,e^{c\,5i+d\,x\,5i}+4\,e^{c\,6i+d\,x\,6i}+4\,e^{c\,7i+d\,x\,7i}+e^{c\,8i+d\,x\,8i}+e^{c\,9i+d\,x\,9i}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + a*cos(c + d*x))^(5/2)
,x)
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(a + a*(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(292*A + 345*B)*4i)/(31
5*d) - (a^2*exp(c*3i + d*x*3i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i +
d*x*1i)/2))^(1/2)*(2*A + 5*B)*4i)/(3*d) + (a^2*exp(c*6i + d*x*6i)*(a + a*(e
xp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(2*A + 5*B)*4i)/(3*d)
+ (a^2*exp(c*4i + d*x*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1
i)/2))^(1/2)*(24*A + 25*B)*4i)/(5*d) - (a^2*exp(c*5i + d*x*5i)*(a + a*(exp(
- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(24*A + 25*B)*4i)/(5*d) +
 (a^2*exp(c*2i + d*x*2i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i
)/2))^(1/2)*(146*A + 155*B)*4i)/(35*d) - (a^2*exp(c*7i + d*x*7i)*(a + a*(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(146*A + 155*B)*4i)/(35
*d) - (a^2*exp(c*9i + d*x*9i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d
*x*1i)/2))^(1/2)*(292*A + 345*B)*4i)/(315*d)))/(exp(c*1i + d*x*1i) + 4*exp(
c*2i + d*x*2i) + 4*exp(c*3i + d*x*3i) + 6*exp(c*4i + d*x*4i) + 6*exp(c*5i +
 d*x*5i) + 4*exp(c*6i + d*x*6i) + 4*exp(c*7i + d*x*7i) + exp(c*8i + d*x*8i)
 + exp(c*9i + d*x*9i) + 1)
```

3.513 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=181

$$\frac{2a^3(230A+301B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2a^3(10A+11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(10A+7B)\sqrt{}}{}$$

```
[Out] 2/7*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2/15*a^3*(10*A
+11*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/35*a^2*(10*A+
7*B)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+2/105*a^3*(230*A+
301*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.43, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3040, 3054, 3059, 2850}

$$\frac{2a^3(10A+11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(230A+301B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(10A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{35d}+\frac{2aA\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^3*(230*A + 301*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(10*A + 11*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d
*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(10*A + 7*B)*Sqrt[a + a*Cos[c + d*x]]*S
ec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,c}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\sin(}{7d}$$

$$= \frac{2a^2(10A+7B)\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c}{35d}$$

$$= \frac{2a^3(10A+11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2a^3(230A+301B)\sqrt{\sec(c+dx)}\,\sin(c+dx}{105d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A]
time = 0.74, size = 104, normalized size = 0.57

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,(290A+196B+(930A+987B)\cos(c+dx)+2(115A+98B)\cos(2(c+dx))+230A\cos(3(c+dx))+301B\cos(3(c+dx)))\sec^{\frac{7}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(290*A + 196*B + (930*A + 987*B)*Cos[c + d*
x] + 2*(115*A + 98*B)*Cos[2*(c + d*x)] + 230*A*Cos[3*(c + d*x)] + 301*B*Cos
[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(210*d)
```

Maple [A]
time = 0.38, size = 119, normalized size = 0.66

method	result
default	$-\frac{2(-1+\cos(dx+c))(230A(\cos^3(dx+c))+301B(\cos^3(dx+c))+115A(\cos^2(dx+c))+98B(\cos^2(dx+c))+60A\cos(dx+c)+21B\cos(dx+}{105d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(230*A*cos(d*x+c)^3+301*B*cos(d*x+c)^3+115*A*cos(d
*x+c)^2+98*B*cos(d*x+c)^2+60*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*cos(d*x+c)*
(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(9/2)/sin(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(157) = 314$.
time = 0.56, size = 488, normalized size = 2.70

$$\frac{8\left(\frac{5\left(\frac{21\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{56\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{36\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)A\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}+\frac{7\left(\frac{15\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{50\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{36\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)B\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 8/105*(5*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + 7*(15*sqrt(2)*a
^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 50*sqrt(2)*a^(5/2)*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 8*sqrt(2)*a^(
5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*B*(sin(d*x + c)^2/(cos(d*x + c) +
```

```
1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(c
os(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A]
time = 0.37, size = 114, normalized size = 0.63

$$\frac{2\left((230\,A+301\,B)a^2\cos\left(dx+c\right)^3+(115\,A+98\,B)a^2\cos\left(dx+c\right)^2+3\,(20\,A+7\,B)a^2\cos\left(dx+c\right)+15\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*((230*A + 301*B)*a^2*cos(d*x + c)^3 + (115*A + 98*B)*a^2*cos(d*x + c)
^2 + 3*(20*A + 7*B)*a^2*cos(d*x + c) + 15*A*a^2)*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.00, size = 579, normalized size = 3.20

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(230\,A+301\,B)\,2i}{105\,d}-\frac{B\,a^2\,e^{c\,1i+d\,x\,1i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,2i}{d}+\frac{B\,a^2\,e^{c\,6i+d\,x\,6i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,2i}{d}-\frac{a^2\,e^{c\,3i+d\,x\,3i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(10\,A+17\,B)\,2i}{3\,d}+\frac{a^2\,e^{c\,4i+d\,x\,4i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(10\,A+17\,B)\,2i}{3\,d}+\frac{a^2\,e^{c\,2i+d\,x\,2i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(100\,A+113\,B)\,2i}{15\,d}-\frac{a^2\,e^{c\,5i+d\,x\,5i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(100\,A+113\,B)\,2i}{15\,d}-\frac{a^2\,e^{c\,7i+d\,x\,7i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(230\,A+301\,B)\,2i}{105\,d}\right)}{e^{c\,1i+d\,x\,1i}+3\,e^{c\,2i+d\,x\,2i}+3\,e^{c\,3i+d\,x\,3i}+3\,e^{c\,4i+d\,x\,4i}+3\,e^{c\,5i+d\,x\,5i}+e^{c\,6i+d\,x\,6i}+e^{c\,7i+d\,x\,7i}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(5/2),
x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(a + a*(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(230*A + 301*B)*2i)/(10
5*d) - (B*a^2*exp(c*1i + d*x*1i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i
+ d*x*1i)/2))^(1/2)*2i)/d + (B*a^2*exp(c*6i + d*x*6i)*(a + a*(exp(- c*1i -
d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*2i)/d - (a^2*exp(c*3i + d*x*3i)*(a
 + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(10*A + 17*B)*2
i)/(3*d) + (a^2*exp(c*4i + d*x*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1
i + d*x*1i)/2))^(1/2)*(10*A + 17*B)*2i)/(3*d) + (a^2*exp(c*2i + d*x*2i)*(a
+ a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(100*A + 113*B)*
2i)/(15*d) - (a^2*exp(c*5i + d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c
*1i + d*x*1i)/2))^(1/2)*(100*A + 113*B)*2i)/(15*d) - (a^2*exp(c*7i + d*x*7i
)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(230*A + 30
1*B)*2i)/(105*d)))/(exp(c*1i + d*x*1i) + 3*exp(c*2i + d*x*2i) + 3*exp(c*3i
+ d*x*3i) + 3*exp(c*4i + d*x*4i) + 3*exp(c*5i + d*x*5i) + exp(c*6i + d*x*6i
) + exp(c*7i + d*x*7i) + 1)
```

3.514 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=192

$$\frac{2a^{5/2}B\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d}+\frac{2a^3(32A+35B)\sqrt{\sec(c+dx)}\,\sin(c}{15d\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 2/5*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/15*a^2*(8*A+
5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+2*a^(5/2)*B*arcsi
n(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d+2/15*a^3*(32*A+35*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(
1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3054, 3059, 2853, 222}

$$\frac{2a^{5/2}B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^3(32A+35B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a^(5/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(32*A + 35*B)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a
 + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*(a + a*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{c}}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\sin(}{5d}$$

$$= \frac{2a^2(8A+5B)\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+}{15d}$$

$$= \frac{2a^3(32A+35B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^3(32A+35B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^{5/2}B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos}}{d}$$

Mathematica [A]
time = 0.89, size = 130, normalized size = 0.68

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(30\sqrt{2}\,B\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{5}{2}}(c+dx)+2(49A+40B+2(14A+5B)\cos(c+dx)+(43A+40B)\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(30*Sqr
t[2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(49*A + 40*B
 + 2*(14*A + 5*B)*Cos[c + d*x] + (43*A + 40*B)*Cos[2*(c + d*x)])*Sin[(c + d
*x)/2]))/(30*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(164) = 328$.
time = 0.43, size = 389, normalized size = 2.03

method	result

default	$\frac{2\left(15B(\cos^3(dx+c))\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+45B(\cos^2(dx+c))\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{}}{\cos}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/15/d*(15*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+45*B*cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)/cos(d*x+c))+45*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+15*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))+43*A*cos(d*x+c)^2*sin(d*x+c)+40*B*sin(d*x+c)*cos(d*x+c)^2+14*A*sin(d
*x+c)*cos(d*x+c)+5*B*sin(d*x+c)*cos(d*x+c)+3*A*sin(d*x+c))*cos(d*x+c)*(1/co
s(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^4/(-1+cos(d*x+c))^2/(1+
cos(d*x+c))^3*a^2
```

<u>Maxima</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 1713 vs. $2(164) = 328$.
time = 0.68, size = 1713, normalized size = 8.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/30*(5*(10*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)*a^(5/2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c
) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2
*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
```

```
 + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x
+ 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*c
os(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt
(a) + 2*((3*a^2*sin(4*d*x + 4*c) + 7*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*sin(4*
d*x + 4*c) + 7*a^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 4*(3*a^2*cos(4*d*x + 4*c) + 7*a^2*cos(2*d*x + 2*c) + 4*a^2
)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (3*a^2*cos(4*d*x + 4*c) + 7*a^2*cos
(2*d*x + 2*c) + 4*a^2 + 4*(3*a^2*cos(4*d*x + 4*c) + 7*a^2*cos(2*d*x + 2*c)
+ 4*a^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(3*a^2*si
n(4*d*x + 4*c) + 7*a^2*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + 15*(a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x +
 2*c) + a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt
(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
5/4) + 16*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*
a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(7/2)))/d
```

Fricas [A]

time = 0.40, size = 162, normalized size = 0.84

$$-\frac{2\left(15\left(Ba^{2}\cos\left(dx+c\right)^{3}+Ba^{2}\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left((43\,A+40\,B)a^{2}\cos(dx+c)^{2}+(14\,A+5\,B)a^{2}\cos(dx+c)+3\,Aa^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

[Out] -2/15*(15*(B*a^2*cos(d*x + c)^3 + B*a^2*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt
(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - ((43*A +
40*B)*a^2*cos(d*x + c)^2 + (14*A + 5*B)*a^2*cos(d*x + c) + 3*A*a^2)*sqrt(a*
cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*co
s(d*x + c)^2)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)

[Out] Timed out

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="giac")

[Out] Timed out

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\,\cos\left(c + d\,x\right))\,\left(\frac{1}{\cos\left(c + d\,x\right)}\right)^{7/2}\,\left(a + a\,\cos\left(c + d\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2),
x)

[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2),
 x)

3.515 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=193

$$\frac{a^{5/2}(2A+5B)\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}-\frac{a^3(14A+3B)\sin(c+}{3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec}}$$

```
[Out] 2/3*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d-1/3*a^3*(14*A+
3*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+a^(5/2)*(2*A+5*B)
*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2*a^2*(2*A+B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A]

time = 0.43, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3054, 3060, 2853, 222}

$$\frac{a^{5/2}(2A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(2A+B)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}{d}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^3*(14*A + 3*B)*Sin[c + d*x]
)/(3*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*(2*A + B)*Sqrt
[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*A*(a + a*Cos
[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{c}}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sin(}{3d}$$

$$= \frac{2a^2(2A+B)\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+}}{d}$$

$$= -\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + $$

$$= -\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + $$

$$= \frac{a^{5/2}(2A+5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right.}{d}$$

Mathematica [A]
time = 0.78, size = 130, normalized size = 0.67

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(3\sqrt{2}(2A+5B)\mathrm{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+(4A+3B+4(8A+3B)\cos(c+dx)+3B\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(3*Sqrt
[2]*(2*A + 5*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (4*A
+ 3*B + 4*(8*A + 3*B)*Cos[c + d*x] + 3*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]
))/(6*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. 2(167) = 334.
time = 0.45, size = 492, normalized size = 2.55

method	result

default	$-\frac{\left(6A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)(\cos^2(dx+c))+15B\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/3/d*(6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2+15*B*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))
*cos(d*x+c)^2+12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+30*B*(cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c))*cos(d*x+c)+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+15*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*B
*sin(d*x+c)*cos(d*x+c)^2+16*A*sin(d*x+c)*cos(d*x+c)+6*B*sin(d*x+c)*cos(d*x+
c)+2*A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2780 vs. $2(167) = 334$.
time = 0.75, size = 2780, normalized size = 14.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(2*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2
*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3
*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)
 + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
```

c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x +
 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*co
s(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqr
t(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
+ 3*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*cos(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x + 2*c)
^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d*x + 2
*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*c) + 4*a
^2*cos(d*x + c) + 5*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - ((a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c)^2 + a^2*cos(d*x + c) +
(a^2*cos(d*x + c) - a^2)*sin(2*d*x + 2*c)^2 - a^2 + 2*(a^2*cos(d*x + c) - a
^2)*cos(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + 5*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos

```
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 +
 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((...
```

Fricas [A]
time = 0.47, size = 166, normalized size = 0.86

$$-\frac{3\left((2A+5B)a^2\cos\left(dx+c\right)^2+(2A+5B)a^2\cos\left(dx+c\right)\right)\sqrt{a}\ \arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\ \sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\ \sin(dx+c)}\right)-\frac{\left(3Ba^2\cos(dx+c)^2+2\,(8A+3B)a^2\cos(dx+c)+2Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\ \sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{3\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*(3*((2*A + 5*B)*a^2*cos(d*x + c)^2 + (2*A + 5*B)*a^2*cos(d*x + c))*sqr
t(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
c))) - (3*B*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 2*A*a^2)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2
 + d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{5/2}(a+a\cos(c+dx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2),
 x)
```

3.516 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=198

$$\frac{a^{5/2}(20A+19B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{4d}-\frac{a^3(4A-9B)\sin(c+}{4d\sqrt{a+a\cos(c+dx)}\,\sqrt{se}}$$

```
[Out] -1/4*a^3*(4*A-9*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-1/2
*a^2*(4*A-B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+2*a*A*(a+
a*cos(d*x+c))^(3/2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+1/4*a^(5/2)*(20*A+19*B)*a
rcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d
```

Rubi [A]
time = 0.42, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3054, 3055, 3060, 2853, 222}

$$\frac{a^{5/2}(20A+19B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}-\frac{a^2(4A-B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{2d\sqrt{\sec(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) - (a^3*(4*A - 9*B)*Sin[c +
 d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^2*(4*A - B)*S
qrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a
+ a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,\sin(}{d}$$

$$= -\frac{a^2(4A-B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx}{2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} - \frac{a^2}{}$$

$$= -\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} - \frac{a^2}{}$$

$$= \frac{a^{5/2}(20A+19B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right.}{4d}$$

Mathematica [A]
time = 0.84, size = 126, normalized size = 0.64

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}\,(20A+19B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2(8A+B+(4A+11B)\cos(c+dx)+B\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2
]*(20*A + 19*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(8*
A + B + (4*A + 11*B)*Cos[c + d*x] + B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/
(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(170) = 340.
time = 0.46, size = 344, normalized size = 1.74

method	result

default	$\frac{\left(20A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\cos(dx+c)+19B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+}{1+\cos(dx}}}{\cos(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(20*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+19*B*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*c
os(d*x+c)+2*B*sin(d*x+c)*cos(d*x+c)^2+20*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+4*A*sin(d
*x+c)*cos(d*x+c)+19*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+11*B*sin(d*x+c)*cos(d*x+c)+8*A
*sin(d*x+c))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(3/2)/(1+co
s(d*x+c))*a^2
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.40, size = 147, normalized size = 0.74

$$-\frac{((20\,A+19\,B)a^2\cos\left(dx+c\right)+(20\,A+19\,B)a^2)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(2\,Ba^2\cos(dx+c)^2+(4\,A+11\,B)a^2\cos(dx+c)+8\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(((20*A + 19*B)*a^2*cos(d*x + c) + (20*A + 19*B)*a^2)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*B*a
^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 8*A*a^2)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}(a+a\cos(c+dx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2),
x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2),
 x)
```

3.517 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx}$

Optimal. Leaf size=200

$$\frac{a^{5/2}(38A+25B)\mathrm{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d}+\frac{a^3(54A+49B)\sin(c}{24d\sqrt{a+a\cos(c+dx)}\sqrt{}}$$

```
[Out] 1/3*a*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+1/24*a^3*(54*A
+49*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/4*a^2*(2*A+3*
B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/8*a^(5/2)*(38*A+2
5*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.42, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3055, 3060, 2853, 222}

$$\frac{a^{5/2}(38A+25B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^3(54A+49B)\sin(c+dx)}{24d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^2(2A+3B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{4d\sqrt{\sec(c+dx)}}+\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^3*(54*A + 49*B)*Sin[c
 + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*(2*A + 3
*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + (a*B*
(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a}{}$$

$$= \frac{aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}$$

$$= \frac{a^2(2A+3B)\sqrt{a+a\cos(c+dx)}\,\sin(c+}{4d\sqrt{\sec(c+dx)}}$$

$$= \frac{a^3(54A+49B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} +$$

$$= \frac{a^3(54A+49B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} +$$

$$= \frac{a^{5/2}(38A+25B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+}}\right.}{8d}$$

Mathematica [A]

time = 1.04, size = 141, normalized size = 0.70

$$\frac{a^2\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(3\sqrt{2}(38A+25B)\text{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}(66A+79B+2(6A+17B)\cos(c+dx)+4B\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Se
c[c + d*x]]*(3*Sqrt[2]*(38*A + 25*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*S
qrt[Cos[c + d*x]]*(66*A + 79*B + 2*(6*A + 17*B)*Cos[c + d*x] + 4*B*Cos[2*(c
 + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [A]

time = 0.44, size = 305, normalized size = 1.52

method	result

default	$-\frac{\left(8B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))+12A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+34B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/24/d*(8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+12*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+34*B*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+66*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)+75*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+114*A*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+75*B*arctan(
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(1/cos(d*x+c))^(1
/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2
*(cos(d*x+c)^2-1)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3071 vs. $2(170) = 340$.
time = 0.87, size = 3071, normalized size = 15.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*
d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
```

```
))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (4*
(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c)
 - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*
a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqr
t(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(
1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(
sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), c
```

```
os(3*d*x + 3*c))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) +
1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), co...
```

Fricas [A]
time = 0.45, size = 163, normalized size = 0.82

$$-\frac{3\left((38\,A+25\,B)a^2\cos\left(dx+c\right)+(38\,A+25\,B)a^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(8\,Ba^2\cos(dx+c)^3+2\,(6\,A+17\,B)a^2\cos(dx+c)^2+3\,(22\,A+25\,B)a^2\cos(dx+c)\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/24*(3*((38*A + 25*B)*a^2*cos(d*x + c) + (38*A + 25*B)*a^2)*sqrt(a)*arcta
n(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*
B*a^2*cos(d*x + c)^3 + 2*(6*A + 17*B)*a^2*cos(d*x + c)^2 + 3*(22*A + 25*B)*
a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2),
 x)
```

3.518 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=247

$$\frac{a^{5/2}(200A+163B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{64d}+\frac{a^3(104A+95B)\sin(}{96d\sqrt{a+a\cos(c+dx)}\,\text{s}}$$

```
[Out] 1/4*a*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+1/96*a^3*(104*
A+95*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/24*a^2*(8*A+
11*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+1/64*a^3*(200*A+
163*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/64*a^(5/2)*(2
00*A+163*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.47, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3055, 3060, 2849, 2853, 222}

$$\frac{a^{5/2}(200A+163B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(104A+95B)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^3(200A+163B)\sin(c+dx)}{64d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{24d\sec^{\frac{3}{2}}(c+dx)}+\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(104*A + 95*B)*S
in[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*(8*A
 + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sec[c + d*x]^(3/2)) +
 (a*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/2)) + (
a^3*(200*A + 163*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,(a+a$$

$$= \frac{aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\Big(\sqrt{\cos(c+dx)}$$

$$= \frac{a^2(8A+11B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)} + \frac{aB(a+$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(8A+11B)\surd}{24}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(8A+11B)\surd}{24}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(8A+11B)\surd}{24}$$

$$= \frac{a^{5/2}(200A+163B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(}}{64d}$$

Mathematica [A]
time = 1.02, size = 159, normalized size = 0.64

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(3\sqrt{2}\,(200A+163B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+(632A+581B+(272A+362B)\cos(c+dx)+4(8A+23B)\cos(2(c+dx))+12B\cos(3(c+dx)))\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt
[2]*(200*A + 163*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (
632*A + 581*B + (272*A + 362*B)*Cos[c + d*x] + 4*(8*A + 23*B)*Cos[2*(c + d*
x)] + 12*B*Cos[3*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(
384*d)
```

Maple [A]
time = 0.39, size = 383, normalized size = 1.55

method	result

default	$\frac{(-1+\cos(dx+c))^2\left(48B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^3(dx+c))+64A\sin(dx+c)(\cos^2(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+184B\sin(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/192/d*(-1+cos(d*x+c))^2*(48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*cos(d*x+c)^3+64*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)+184*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+272*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+326*B*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+600*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)+489*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+600*A*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+489*B*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1
+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/cos(d*x+c))^(1/2)/
sin(d*x+c)^4*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 9390 vs. $2(211) = 422$.
time = 1.11, size = 9390, normalized size = 38.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(
3*d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin
```

(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
c))) + 1)^(1/4)(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(
1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
))) - 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2
*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*

```
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) - 1))*sqrt(a))*A + (2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(4
*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a^2*sin(4*d*...
```

Fricas [**A**]
time = 0.44, size = 183, normalized size = 0.74

$$-\frac{3\left((200A+163B)a^2\cos(dx+c)+(200A+163B)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(48Ba^2\cos(dx+c)^4+8(8A+23B)a^2\cos(dx+c)^3+2(136A+163B)a^2\cos(dx+c)^2+3(200A+163B)a^2\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{192\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/192*(3*((200*A + 163*B)*a^2*cos(d*x + c) + (200*A + 163*B)*a^2)*sqrt(a)*
arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
- (48*B*a^2*cos(d*x + c)^4 + 8*(8*A + 23*B)*a^2*cos(d*x + c)^3 + 2*(136*A +
 163*B)*a^2*cos(d*x + c)^2 + 3*(200*A + 163*B)*a^2*cos(d*x + c))*sqrt(a*cos
(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{5/2}}{\sqrt{\frac{1}{\cos(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2
), x)
```

3.519 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{a^{5/2}(326A+283B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{128d}+\frac{a^3(170A+157B)\,\mathrm{s}}{240d\sqrt{a+a\cos(c+dx}}$$

```
[Out] 1/5*a*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+1/240*a^3*(170
*A+157*B)*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+1/192*a^3*(3
26*A+283*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/40*a^2*(
10*A+13*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+1/128*a^3*(
326*A+283*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/128*a^(
5/2)*(326*A+283*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.54, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3055, 3060, 2849, 2853, 222}

$$\frac{a^{5/2}(326A+283B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^3(326A+283B)\sin(c+dx)}{192d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^3(170A+157B)\sin(c+dx)}{240d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^3(326A+283B)\sin(c+dx)}{128d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{a^2(10A+13B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{40d\sec^{\frac{5}{2}}(c+dx)}+\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(170*A + 157*B)
*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a^2*(
10*A + 13*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d*Sec[c + d*x]^(5/2
)) + (a*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(5/2))
 + (a^3*(326*A + 283*B)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c
 + d*x]^(3/2)) + (a^3*(326*A + 283*B)*Sin[c + d*x])/(128*d*Sqrt[a + a*Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
```

```
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{c}$$

$$= \frac{aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{5}\left(\sqrt{\cos(c+dx}\right.$$

$$= \frac{a^2(10A+13B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{40d\sec^{\frac{5}{2}}(c+dx)} + \frac{aB(}{}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(10A+13E}{}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(10A+13E}{}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(10A+13E}{}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(10A+13E}{}$$

$$= \frac{a^{5/2}(326A+283B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos}}{128d}$$

Mathematica [A]
time = 1.50, size = 181, normalized size = 0.62

$$\frac{a^2\sqrt{a(1+\cos(c+dx))}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(15\sqrt{2}\,(326A+283B)\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+(5810A+5521B+(3620A+3874B)\cos(c+dx)+4(230A+331B)\cos(2(c+dx))+120A\cos(3(c+dx))+348B\cos(3(c+dx))+48B\cos(4(c+dx)))\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{3840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(15*Sqr
t[2]*(326*A + 283*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] +
(5810*A + 5521*B + (3620*A + 3874*B)*Cos[c + d*x] + 4*(230*A + 331*B)*Cos[2
*(c + d*x)] + 120*A*Cos[3*(c + d*x)] + 348*B*Cos[3*(c + d*x)] + 48*B*Cos[4*
(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(3840*d)
```

Maple [A]
time = 0.38, size = 455, normalized size = 1.55

method	result
default	$-\frac{(-1+\cos(dx+c))^3\left(384B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^4(dx+c))+480A(\cos^3(dx+c))\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+1392B\sin(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/1920/d*(-1+cos(d*x+c))^3*(384*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*cos(d*x+c)^4+480*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)+1392*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+184
0*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2264*B*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+3260*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+2830*B*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)+4890*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)+4245*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4890*A*arctan(sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+4245*B*arctan(sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+
c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+c))^(3/2)/sin(d*x+c
)^6*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 10042 vs. $2(252) = 504$.
time = 1.15, size = 10042, normalized size = 34.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/7680*((10*(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2
/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5
*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(3/4)*((75*a^2*sin(4/5*arctan2(sin(5*d
*x + 5*c), cos(5*d*x + 5*c))) + 88*a^2*sin(3/5*arctan2(sin(5*d*x + 5*c), co
s(5*d*x + 5*c))) + 75*a^2*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c
))))*cos(3/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))),
cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - (75*a^2*cos(4/
5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 88*a^2*cos(3/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))) - 75*a^2*cos(1/5*arctan2(sin(5*d*x + 5*c)
, cos(5*d*x + 5*c))) - 88*a^2)*sin(3/2*arctan2(sin(2/5*arctan2(sin(5*d*x +
```

5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c
))) + 1)))*sqrt(a) + 6*(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))
)^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*ar
ctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(8*(a^2*cos(2/5*arcta
n2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2*sin(5*d*x + 5*c) + a^2*sin(5*d*x
+ 5*c)*sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*a^2*cos(2
/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin(5*d*x + 5*c) + a^2*sin(
5*d*x + 5*c))*cos(5/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - 5*(15
*a^2*sin(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 15*a^2*sin(3/5*
arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 60*a^2*sin(2/5*arctan2(sin(5
*d*x + 5*c), cos(5*d*x + 5*c))) - 268*a^2*sin(1/5*arctan2(sin(5*d*x + 5*c),
 cos(5*d*x + 5*c))))*cos(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(
5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))
- 8*(a^2*cos(5*d*x + 5*c) + (a^2*cos(5*d*x + 5*c) - a^2)*cos(2/5*arctan2(si
n(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + (a^2*cos(5*d*x + 5*c) - a^2)*sin(2/5
*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 - a^2 + 2*(a^2*cos(5*d*x +
5*c) - a^2)*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))*sin(5/2*a
rctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan
2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + 5*(15*a^2*cos(4/5*arctan2(si
n(5*d*x + 5*c), cos(5*d*x + 5*c))) - 15*a^2*cos(3/5*arctan2(sin(5*d*x + 5*c
), cos(5*d*x + 5*c))) - 60*a^2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
+ 5*c))) - 148*a^2*cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 2
08*a^2)*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))
), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)))*sqrt(a) + 42
45*(a^2*arctan2(-(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 +
sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(
sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/5*ar
ctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c
), cos(5*d*x + 5*c))) + 1))*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5
*c))) - cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin(1/2*arctan
2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))), (cos(2/5*arctan2(sin(5*d*x + 5*c)
, cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)
))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(c
os(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*cos(1/2*arctan2(sin(2/5
*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x +
5*c), cos(5*d*x + 5*c))) + 1)) + sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*
x + 5*c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*
c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))) + 1) - a^
2*arctan2(-(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/
5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*
d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/5*arctan2(
sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos
(5*d*x + 5*c))) + 1))*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))

```
- cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin(1/2*arctan2(sin(
2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x
 + 5*c), cos(5*d*x + 5*c))) + 1))), (cos(2/5*arctan2(sin(5*d*x + 5*c), cos(
5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 +
 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos(1/5
*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*cos(1/2*arctan2(sin(2/5*arcta
n2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c),
cos(5*d*x + 5*c))) + 1)) + sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*
c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))),
cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))) - 1) - a^2*arct
an2((cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arcta
n2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*c...
```

Fricas [A]
time = 0.44, size = 203, normalized size = 0.69

$$-\frac{15\left((326\,A+283\,B)a^2\cos\left(dx+c\right)+(326\,A+283\,B)a^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(384\,Ba^2\cos(dx+c)^5+48\,(10\,A+29\,B)a^2\cos(dx+c)^4+8\,(230\,A+283\,B)a^2\cos(dx+c)^3+10\,(326\,A+283\,B)a^2\cos(dx+c)^2+15\,(326\,A+283\,B)a^2\cos(dx+c)\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{1920\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/1920*(15*((326*A + 283*B)*a^2*cos(d*x + c) + (326*A + 283*B)*a^2)*sqrt(a
)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))
) - (384*B*a^2*cos(d*x + c)^5 + 48*(10*A + 29*B)*a^2*cos(d*x + c)^4 + 8*(23
0*A + 283*B)*a^2*cos(d*x + c)^3 + 10*(326*A + 283*B)*a^2*cos(d*x + c)^2 + 1
5*(326*A + 283*B)*a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2
), x)
```

3.520 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=295

$$-\frac{\sqrt{2}\,(A-B)\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}+\frac{2(257A-}{31}$$

```
[Out] -2/315*(29*A-93*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/1
05*(19*A-3*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-2/63*(A-
9*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/9*A*sec(d*x+c)^
(9/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-(A-B)*arctan(1/2*sin(d*x+c)*a^(1/
2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d/a^(1/2)+2/315*(257*A-129*B)*sin(d*x+c)*sec(d*x+c)^(1/2
)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3063, 12, 2861, 211}

$$-\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2(A-9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2(19A-3B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(29A-93B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2(257A-129B)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*(257*A - 129*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a +
 a*Cos[c + d*x]]) - (2*(29*A - 93*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*
d*Sqrt[a + a*Cos[c + d*x]]) + (2*(19*A - 3*B)*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 9*B)*Sec[c + d*x]^(7/2)*Sin[
c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(9/2)*Sin[c +
 d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{11}{2}}(c+dx)\sqrt{a+a\cos(c+\cdots}}$$

$$= \frac{2A\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\cdots}{9\cdots}$$

$$= -\frac{2(A-9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{9}{2}}(c+dx)\sin(c+\cdots}{9d\sqrt{a+a\cos(c+dx)}\cdots}$$

$$= \frac{2(19A-3B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-9B)\sec^{\frac{7}{2}}(c+dx)\cdots}{63d\sqrt{a+a\cos(c\cdots}}$$

$$= -\frac{2(29A-93B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2(19A-3B)\sec^{\frac{5}{2}}(c\cdots}{105d\sqrt{a+a\,\cdots}}$$

$$= \frac{2(257A-129B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A-93B)\sec^{\frac{3}{2}}\cdots}{315d\sqrt{a+\cdots}}$$

$$= \frac{2(257A-129B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A-93B)\sec^{\frac{3}{2}}\cdots}{315d\sqrt{a+\cdots}}$$

$$= \frac{2(257A-129B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A-93B)\sec^{\frac{3}{2}}\cdots}{315d\sqrt{a+\cdots}}$$

$$= -\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\cdots}{\sqrt{a}\,d}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 9.60, size = 272, normalized size = 0.92

$$\frac{2e^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\left(-315i(A-B)\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{4}(-1279A+423B+(214A-918B)\cos(c+dx)-8(157A-69B)\cos(2(c+dx))+58A\cos(3(c+dx))-186B\cos(3(c+dx))-257A\cos(4(c+dx))+129B\cos(4(c+dx)))\sec^{\frac{9}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+i\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{315d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2))/Sqrt[a + a*Cos[c + d*x
]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((-315*I)*(A - B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - ((-1279*A + 423*B + (214*A - 918*B)*
```

```
Cos[c + d*x] - 8*(157*A - 69*B)*Cos[2*(c + d*x)] + 58*A*Cos[3*(c + d*x)] -
186*B*Cos[3*(c + d*x)] - 257*A*Cos[4*(c + d*x)] + 129*B*Cos[4*(c + d*x)])*S
ec[c + d*x]^(9/2)*(Cos[(c + d*x)/2] + I*Sin[(c + d*x)/2])*Sin[(c + d*x)/2])
/4))/(315*d*E^((I/2)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(250) = 500$.
time = 0.40, size = 793, normalized size = 2.69

method	result	size
default	Expression too large to display	793

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/315/d*(315*A*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*cos(d*x+c)^5-315*B*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*cos(d*x+c)^5+1575*A*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-1575*B*(cos(d*x+c)/(1+cos(
d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4+3150*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^
3-3150*B*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))*cos(d*x+c)^3+3150*A*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*cos(d*x+c)^2-3150*B*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcs
in((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+1575*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)-1575*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+315*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-315*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+257*A*2
^(1/2)*sin(d*x+c)*cos(d*x+c)^4-129*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)^4-29*A*2
^(1/2)*sin(d*x+c)*cos(d*x+c)^3+93*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3+57*A*2^
(1/2)*sin(d*x+c)*cos(d*x+c)^2-9*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2-5*A*2^(1/
2)*sin(d*x+c)*cos(d*x+c)+45*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)+35*A*2^(1/2)*si
n(d*x+c))*cos(d*x+c)*sin(d*x+c)^8*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))^
(1/2)/(-1+cos(d*x+c))^4/(1+cos(d*x+c))^5*2^(1/2)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 0.43, size = 198, normalized size = 0.67

$$\frac{\frac{315\sqrt{2}\left((A-B)a\cos(dx+c)^5+(A-B)a\cos(dx+c)^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((257A-129B)\cos(dx+c)^4-(29A-93B)\cos(dx+c)^3+3(19A-3B)\cos(dx+c)^2-5(A-9B)\cos(dx+c)+35A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{315\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

```
[Out] 1/315*(315*sqrt(2)*((A - B)*a*cos(d*x + c)^5 + (A - B)*a*cos(d*x + c)^4)*ar
ctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
 c)))/sqrt(a) + 2*((257*A - 129*B)*cos(d*x + c)^4 - (29*A - 93*B)*cos(d*x +
 c)^3 + 3*(19*A - 3*B)*cos(d*x + c)^2 - 5*(A - 9*B)*cos(d*x + c) + 35*A)*sq
rt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^5
 + a*d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(11/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2))/(a + a*cos(c + d*x))^(1/
2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2))/(a + a*cos(c + d*x))^(1/
2), x)
```

3.521 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=250

$$\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}-\frac{2(43A-91B}{105d}$$

```
[Out] 2/105*(31*A-7*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-2/35*
(A-7*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*A*sec(d*x+
c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+(A-B)*arctan(1/2*sin(d*x+c)*a^
(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-2/105*(43*A-91*B)*sin(d*x+c)*sec(d*x+c)^(1/
2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.54, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3063, 12, 2861, 211}

$$\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2(A-7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{2(31A-7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(43A-91B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*
d) - (2*(43*A - 91*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Co
s[c + d*x]]) + (2*(31*A - 7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt
[a + a*Cos[c + d*x]]) - (2*(A - 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d
*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqr
t[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\cos(c+a}$$

$$= \frac{2A\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)}{7}$$

$$= -\frac{2(A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{7}{2}}(c+dx)\sin(c+}{7d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2(31A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-7B)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a+a\cos(c+}}$$

$$= -\frac{2(43A-91B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(31A-7B)\sec^{\frac{3}{2}}(c}{105d\sqrt{a+a}}$$

$$= -\frac{2(43A-91B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(31A-7B)\sec^{\frac{3}{2}}(c}{105d\sqrt{a+a}}$$

$$= -\frac{2(43A-91B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(31A-7B)\sec^{\frac{3}{2}}(c}{105d\sqrt{a+a}}$$

$$= \frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{}}{\sqrt{a}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.98, size = 250, normalized size = 1.00

$$\frac{2e^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\left(105i(A-B)\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{2}(-122A+14B+3(47A-119B)\cos(c+dx)+(-62A+14B)\cos(2(c+dx))+43A\cos(3(c+dx))-91B\cos(3(c+dx)))\sec^{\frac{7}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+i\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{105d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((105*I)*(A - B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt
[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - ((-122*A + 14*B + 3*(47*A - 119*B)*Co
s[c + d*x] + (-62*A + 14*B)*Cos[2*(c + d*x)] + 43*A*Cos[3*(c + d*x)] - 91*B
*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*(Cos[(c + d*x)/2] + I*Sin[(c + d*x)/2
])*Sin[(c + d*x)/2])/2))/(105*d*E^((I/2)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x
])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(211) = 422$.
time = 0.50, size = 657, normalized size = 2.63

method	result
default	$\frac{\left(105 A \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}(\cos^4(dx+c))-105 B \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}(\cos^4(dx+c))+420 A \text{ arc}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/105/d*(105*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)
))^(7/2)*cos(d*x+c)^4-105*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/
(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^4+420*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^3-420*B*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^3+630*A*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^2-6
30*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*c
os(d*x+c)^2+420*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x
+c)))^(7/2)*cos(d*x+c)-420*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)
/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)+105*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-105*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+43*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3-91*B
*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3-31*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+7*B*2
^(1/2)*sin(d*x+c)*cos(d*x+c)^2+3*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)-21*B*2^(1/
2)*sin(d*x+c)*cos(d*x+c)-15*A*2^(1/2)*sin(d*x+c))*cos(d*x+c)*sin(d*x+c)^6*(
1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))^3/(1+cos(d*x+c
))^4*2^(1/2)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 0.42, size = 181, normalized size = 0.72

$$-\frac{\frac{105 \sqrt{2}\left((A-B) a \cos(dx+c)^4+(A-B) a \cos(dx+c)^3\right) \arctan\left(\frac{\sqrt{2} \sqrt{a \cos(dx+c)+a} \sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((43 A-91 B) \cos(dx+c)^3-(31 A-7 B) \cos(dx+c)^2+3 (A-7 B) \cos(dx+c)-15 A\right) \sqrt{a \cos(dx+c)+a} \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105\left(a d \cos(dx+c)^4+a d \cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/105*(105*sqrt(2)*((A - B)*a*cos(d*x + c)^4 + (A - B)*a*cos(d*x + c)^3)*a
rctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x
+ c)))/sqrt(a) + 2*((43*A - 91*B)*cos(d*x + c)^3 - (31*A - 7*B)*cos(d*x + c
)^2 + 3*(A - 7*B)*cos(d*x + c) - 15*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c
)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(9/2)/sqrt(a*cos(d*x + c) + a),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{9/2}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a*cos(c + d*x))^(1/2
), x)
```

3.522 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=207

$$-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}+\frac{2(13A-}{15}$$

```
[Out] -2/15*(A-5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*A*se
c(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-(A-B)*arctan(1/2*sin(d*x
+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2/15*(13*A-5*B)*sin(d*x+c)*sec(d*x+c)
^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3063, 12, 2861, 211}

$$-\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2(A-5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2(13A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*(13*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*C
os[c + d*x]]) - (2*(A - 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a
+ a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*
Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= -\frac{2(A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+}{5d\sqrt{a+a\cos(c+dx}$$

$$= \frac{2(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sec^{\frac{3}{2}}(c+d}{15d\sqrt{a+a\cos(}$$

$$= \frac{2(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sec^{\frac{3}{2}}(c+d}{15d\sqrt{a+a\cos(}$$

$$= \frac{2(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sec^{\frac{3}{2}}(c+d}{15d\sqrt{a+a\cos(}$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 8.01, size = 1718, normalized size = 8.30

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Si
n[c/2 + (d*x)/2]^2]*((2*B*Sin[c/2 + (d*x)/2])/(5*(1 - 2*Sin[c/2 + (d*x)/2]^
2)^(5/2)) + (8*B*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (
2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]))/15 - ((A - B)*Csc[
c/2 + (d*x)/2]^7*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 +
210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2
+ (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*
HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 +
```

```
2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^
12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1
500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hype
rgeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricP
FQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^
2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2)))/(675*(1 - 2*Sin[c
/2 + (d*x)/2]^2)^(7/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2))))/(d*Sqrt[a*(1 + Cos[
c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(174) = 348$.

time = 0.46, size = 521, normalized size = 2.52

method	result
default	$\frac{\left(15A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}(\cos^3(dx+c))-15B\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}(\cos^3(dx+c))+45A\arcsin\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/15/d*(15*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))
^(5/2)*cos(d*x+c)^3-15*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+
```

```
cos(d*x+c)))^(5/2)*cos(d*x+c)^3+45*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(co
s(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2-45*B*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2+45*A*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)-45*B*arcs
in((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)
+15*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-
15*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+1
3*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2-5*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2-A*2
^(1/2)*sin(d*x+c)*cos(d*x+c)+5*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)+3*A*2^(1/2)*
sin(d*x+c))*cos(d*x+c)*sin(d*x+c)^4*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))
^(1/2)/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3*2^(1/2)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 0.39, size = 164, normalized size = 0.79

$$\frac{\frac{15\sqrt{2}\left((A-B)a\cos(dx+c)^3+(A-B)a\cos(dx+c)^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((13A-5B)\cos(dx+c)^2-(A-5B)\cos(dx+c)+3A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(15*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*arct
an(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c
)))/sqrt(a) + 2*((13*A - 5*B)*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) + 3*A
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x +
c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\cos(c + dx))\left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{\sqrt{a + a\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(1/2
), x)
```

3.523 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=162

$$\frac{\sqrt{2}\,(A-B)\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}-\frac{2(A-3B)}{3d\sqrt{\ }}$$

```
[Out] 2/3*A*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+(A-B)*arctan(1/2
*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-2/3*(A-3*B)*sin(d*x+c)*sec(d*
x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3063, 12, 2861, 211}

$$\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*
d) - (2*(A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
```

```
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)}{}$$

$$= -\frac{2(A-3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c}{3d\sqrt{a+a\cos(c+a}}$$

$$= -\frac{2(A-3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c}{3d\sqrt{a+a\cos(c+a}}$$

$$= -\frac{2(A-3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c}{3d\sqrt{a+a\cos(c+a}}$$

$$= \frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{}}{\sqrt{a}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.84, size = 617, normalized size = 3.81

$$\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\left(\frac{2B\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3\left(1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{3/2}}+\frac{4B\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}+\frac{(A-B)\csc^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-12\cos^4\left(\frac{1}{2}(c+dx)\right){}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)-12\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\left(4-7\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3\sin^4\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+7\sqrt{-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\left(1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\left(15-20\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)+8\sin^4\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\left(3-7\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sqrt{-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}-3\tanh^{-1}\left(\sqrt{-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\right)\left(1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\right)\right)}{63\left(1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{7/2}}\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Si
n[c/2 + (d*x)/2]^2]*((2*B*Sin[c/2 + (d*x)/2])/(3*(1 - 2*Sin[c/2 + (d*x)/2]^
2)^(3/2)) + (4*B*Sin[c/2 + (d*x)/2])/(3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) +
 ((A - B)*Csc[c/2 + (d*x)/2]^5*(-12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2
, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*
Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin[c/2 + (d*x)/
2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (
d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*
Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*(15 - 20*Sin[c/2 + (
d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7*Sin[c/2 + (d*x)/2]^2)*Sqrt[-(Si
n[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))] - 3*ArcTanh[Sqrt[-(Sin[c/
```

```
2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*(1 - 2*Sin[c/2 + (d*x)/2]^2)
)))/(63*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))))/(d*Sqrt[a*(1 + Cos[c + d*x])]
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(135) = 270$.
time = 0.44, size = 384, normalized size = 2.37

method	result
default	$\left(3A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos^2(dx+c))-3B\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos^2(dx+c))+6A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*(3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*cos(d*x+c)^2-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*cos(d*x+c)^2+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+3*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-3*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+A*2^(1/2)*sin(d*x+c)*cos(d*x+c)-
3*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)-A*2^(1/2)*sin(d*x+c))*cos(d*x+c)*sin(d*x+
c)^2*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/(1+cos(d
*x+c))^2*2^(1/2)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 0.38, size = 143, normalized size = 0.88

$$-\frac{\frac{3\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((A-3B)\cos(dx+c)-A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*(3*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*arctan(
sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
/sqrt(a) + 2*((A - 3*B)*cos(d*x + c) - A)*sqrt(a*cos(d*x + c) + a)*sin(d*x
+ c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+a\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(1/2
), x)
```

3.524 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}+\frac{2A\sqrt{\sec(c}{d\sqrt{a+}}$$

```
[Out] -(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2*A*sin(d*x+
c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3063, 12, 2861, 211}

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+ \ldots}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)} \ldots\right.}{\ldots}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \left((A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c \ldots\right.$$

$$= \frac{2A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2a(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se} \ldots\right.}{\ldots}$$

$$= -\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right) \ldots}{\sqrt{a}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.68, size = 203, normalized size = 1.71

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\sin\left(\frac{1}{2}(c+dx)\right)\left(10B-(A-B)\sec(c+dx)\left(-\frac{5}{4}(1+4\cos(c+dx)+\cos(2(c+dx)))\csc^4\left(\frac{1}{2}(c+dx)\right)\left(1-\cos(c+dx)+\tanh^{-1}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cos(c+dx)\sqrt{2-2\sec(c+dx)}\right)+\frac{1}{2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)\tan(c+dx)\right)\right)}{5d\sqrt{a(1+\cos(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*Sin[(c + d*x)/2]*(10*B - (A - B)*Sec
[c + d*x]*((-5*(1 + 4*Cos[c + d*x] + Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^4*(
1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c
+ d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/4 + (Hypergeometric2F1[2, 5/2, 7/2, -(Sec
[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/2)))/(5*d*Sqrt[a*
(1 + Cos[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(100) = 200$.
time = 0.41, size = 231, normalized size = 1.94

method	result
default	$\frac{\left(A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)-B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)+A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*cos(d*x+c)-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*cos(d*x+c)+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))+A*2^(1/2)*sin(d*x+c)-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arc
sin((-1+cos(d*x+c))/sin(d*x+c)))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(
d*x+c)))^(1/2)/(1+cos(d*x+c))*2^(1/2)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]

time = 0.42, size = 110, normalized size = 0.92

$$\frac{\frac{\sqrt{2}\,((A-B)a\cos(dx+c)+(A-B)a)\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\sqrt{a\cos(dx+c)+a}\,A\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x
 + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt(a*co
s(d*x + c) + a)*A*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(1/2
), x)
```

3.525 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{2B\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}+\frac{\sqrt{2}(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\text{si}}{\sqrt{2}\sqrt{\cos(c+dx)}}\right.}{}$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/d/a^(1/2)+(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+
c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d/a^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3061, 2861, 211, 2853, 222}

$$\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c}$$

$$= \left((A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a-}}$$

$$= -\frac{\left(2a(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{2a^2+ax^2}\,d\right.}{d}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.24, size = 102, normalized size = 0.73

$$\frac{2\left(\sqrt{2}\,B\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+(A-B)\text{ArcTan}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (2*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + (A - B)*ArcTan[Sin[(c + d*
x)/2]/Sqrt[Cos[c + d*x]]])*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A]
time = 0.40, size = 153, normalized size = 1.09

method	result
default	$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(-B\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\sqrt{2}+A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B\arcsi\right.}{d\sin(dx+c)^2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-B*arctan(sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)+A*arcsin((-1+cos(d*x+c))
/sin(d*x+c))-B*arcsin((-1+cos(d*x+c))/sin(d*x+c)))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*2^(1/2)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 1.13, size = 96, normalized size = 0.69

$$-\frac{\sqrt{2}\,(A-B)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)+2\,B\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -(sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*B*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\,\sqrt{\sec(c+dx)}}{\sqrt{a\,(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/sqrt(a*(cos(c + d*x) + 1))
, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+a\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(1/2
), x)
```

3.526 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=181

$$\frac{(2A-B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\Big(\frac{}{\sqrt{2}\,\sqrt{\cos(c}}}{}$$

```
[Out] B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(2*A-B)*arcsin(sin(d
*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a
^(1/2)-(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*co
s(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3040, 3062, 3061, 2861, 211, 2853, 222}

$$\frac{(2A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{B\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((2*A - B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A - B)*ArcTan[(Sqrt[
a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqr
t[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(d*Sqrt[
a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(}{\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(\right.}{}$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} - \left((A-B)\sqrt{\cos(c+dx\right.$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \frac{\left(2a(A-B)\sqrt{\cos(c+\right.}{}$$

$$= \frac{(2A-B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(}{\sqrt{a}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.48, size = 467, normalized size = 2.58

$$\frac{ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)\left(B-Be^{i(c+dx)}+Be^{2i(c+dx)}-Be^{3i(c+dx)}-(2A-B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{2}Be^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+2\sqrt{2}Ae^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\sqrt{2}Be^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+2Ae^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)-Be^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] ((I/4)*(1 + E^(I*(c + d*x)))*(B - B*E^(I*(c + d*x)) + B*E^((2*I)*(c + d*x))
 - B*E^((3*I)*(c + d*x)) - (2*A - B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*B*E^(I*(c + d*x))*Sqrt[1 + E^((2
*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c
 + d*x))])] + 2*Sqrt[2]*A*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Arc
Tanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - Sqrt
[2]*B*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(-1 + E^(I*(c +
 d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + 2*A*E^(I*(c + d*x))*Sqrt
[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - B*E^(I*(
c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x)
)]])*Sqrt[Sec[c + d*x]])/(d*E^((2*I)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A]
time = 0.42, size = 232, normalized size = 1.28

method	result
default	$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos(dx+c)(-1+\cos(dx+c))^{2}\left(B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+2A\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(d}{1+\cos(}}}{\cos(dx+c)}\right.\right.}{2d\sqrt{\frac{1}{\cos(dx+c)}}\,\left(\frac{\cos(dx}{1+\cos(d}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(B*2^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*A*arctan(sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)-B*arctan(sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)+2*A*arcsin((-1+cos(d*x+c))/sin(d*x
+c))-2*B*arcsin((-1+cos(d*x+c))/sin(d*x+c)))/(1/cos(d*x+c))^(1/2)/(cos(d*x+
c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4*2^(1/2)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 1.41, size = 168, normalized size = 0.93

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}\,B\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\left(\left(2A-B\right)\cos\left(dx+c\right)+2A-B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin\left(dx+c\right)}\right)+\frac{\sqrt{2}\,\left(\left(A-B\right)a\cos\left(dx+c\right)+\left(A-B\right)a\right)\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin\left(dx+c\right)}\right)}{\sqrt{a}}}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A - B)*co
s(d*x + c) + 2*A - B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) + sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a
)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}\,\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*sqrt(sec(c + d*x)
)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos(c + d\,x)}{\sqrt{\frac{1}{\cos(c + d\,x)}}\,\sqrt{a + a\,\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2)
), x)
```

3.527 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=230

$$-\frac{(4A-7B)\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{}{\sqrt{2}\sqrt{\cos}}\right.}{}$$

```
[Out] 1/2*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/4*(4*A-B)*sin(
d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-1/4*(4*A-7*B)*arcsin(sin(d
*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a
^(1/2)+(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*co
s(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$,
Rules used = {3040, 3062, 3061, 2861, 211, 2853, 222}

$$-\frac{(4A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{(4A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{B\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -1/4*((4*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*S
qrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan
[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]
])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(
2*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((4*A - B)*Sin[c + d*x])
/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
```

```
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+d}{\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{B\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= \frac{B\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{(4A-B)\sin(c+}{4d\sqrt{a+a\cos(c+dx)}\,\sqrt{s}}$$

$$= \frac{B\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{(4A-B)\sin(c+}{4d\sqrt{a+a\cos(c+dx)}\,\sqrt{s}}$$

$$= \frac{B\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{(4A-B)\sin(c+}{4d\sqrt{a+a\cos(c+dx)}\,\sqrt{s}}$$

$$= -\frac{(4A-7B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{sec(}{4\sqrt{a}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.56, size = 412, normalized size = 1.79

$$-\frac{ie^{-3i(c+dx)}\left(1+e^{i(c+dx)}\right)\left(-B-4Ae^{i(c+dx)}+2Be^{i(c+dx)}+4Ae^{2i(c+dx)}-3Be^{2i(c+dx)}-4Ae^{3i(c+dx)}+3Be^{3i(c+dx)}+4Ae^{4i(c+dx)}-2Be^{4i(c+dx)}+Be^{5i(c+dx)}-(4A-7B)e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)-8\sqrt{2}(A-B)e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+4Ae^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)-7Be^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{\sec(c+dx)}}{16d\sqrt{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] ((-1/16*I)*(1 + E^(I*(c + d*x)))*(-B - 4*A*E^(I*(c + d*x)) + 2*B*E^(I*(c +
d*x)) + 4*A*E^((2*I)*(c + d*x)) - 3*B*E^((2*I)*(c + d*x)) - 4*A*E^((3*I)*(c
 + d*x)) + 3*B*E^((3*I)*(c + d*x)) + 4*A*E^((4*I)*(c + d*x)) - 2*B*E^((4*I)
*(c + d*x)) + B*E^((5*I)*(c + d*x)) - (4*A - 7*B)*E^((2*I)*(c + d*x))*Sqrt[
1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))] - 8*Sqrt[2]*(A - B)*E^((2
*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/
(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + 4*A*E^((2*I)*(c + d*x))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 7*B*E^((2*I)
*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*
```

```
x))]])*Sqrt[Sec[c + d*x]])/(d*E^((3*I)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])
])
```

Maple [A]
time = 0.46, size = 301, normalized size = 1.31

method	result
default	$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos(dx+c)(-1+\cos(dx+c))^{3}\left(2B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\cos(dx+c)+4A\sqrt{2}\,\sqrt{\frac{c}{1+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/8/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(2*B*2^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+4*A*2^(1/2)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)-4*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c))*2^(1/2)+7*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c))*2^(1/2)-8*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))+8*B*arcsin((-1+co
s(d*x+c))/sin(d*x+c)))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/
2)/sin(d*x+c)^6*2^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

Fricas [A]
time = 2.36, size = 194, normalized size = 0.84

$$\frac{((4A-7B)\cos(dx+c)+4A-7B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{4\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{\left(2B\cos(dx+c)^{2}+(4A-B)\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\left(ad\cos(dx+c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(((4*A - 7*B)*cos(d*x + c) + 4*A - 7*B)*sqrt(a)*arctan(sqrt(a*cos(d*x +
 c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 4*sqrt(2)*((A - B)*a*
cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + (2*B*cos(d*x + c)^2 + (4*A - B)
*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}\,\sec^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**(3/
2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,\cos(c + d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\sqrt{a + a\,\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2)
), x)
```

3.528 $\int \frac{(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=192

$$\frac{(2Ab+2aB-bB)\text{ArcSin}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}}{\sqrt{a}\ d}+\frac{\sqrt{2}\ (a-b)(A-B)\text{ArcT}}{}$$

```
[Out] b*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(2*A*b+2*B*a-B*b)*
arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/d/a^(1/2)+(a-b)*(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*
x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d/a^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4306, 3124, 3061, 2861, 211, 2853, 222}

$$\frac{(2aB+2Ab-bB)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}\ \text{ArcSin}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\ d}+\frac{\sqrt{2}\ (a-b)(A-B)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}\ \text{ArcTan}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{2}\ \sqrt{\cos(c+dx)}\ \sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\ d}+\frac{bB\sin(c+dx)}{d\sqrt{\sec(c+dx)}\ \sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x
]])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((2*A*b + 2*a*B - b*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(a - b)*(
A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (b*B*
Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
```

```
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3124

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(b*d*(m +
 n + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m +
 n + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
 + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n},
 x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ
[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 4306

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(aA + (Ab + aB)\cos(c + dx) + bB\cos^2(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a + a\cos(c + dx)}}\,dx = \left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right)\int$$

$$= \frac{bB\sin(c + dx)}{d\sqrt{a + a\cos(c + dx)}\sqrt{\sec(c + d}}$$

$$= \frac{bB\sin(c + dx)}{d\sqrt{a + a\cos(c + dx)}\sqrt{\sec(c + d}}$$

$$= \frac{bB\sin(c + dx)}{d\sqrt{a + a\cos(c + dx)}\sqrt{\sec(c + d}}$$

$$= \frac{(2Ab + 2aB - bB)\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{a +}}\right.}{}$$

Mathematica [A]
time = 0.49, size = 143, normalized size = 0.74

$$\frac{\cos\left(\frac{1}{2}(c + dx)\right)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\left(\sqrt{2}\,(2Ab + 2aB - bB)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c + dx)\right)\right) + 2(a - b)(A - B)\text{ArcTan}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c + dx)}}\right) + 2bB\sqrt{\cos(c + dx)}\,\sin\left(\frac{1}{2}(c + dx)\right)\right)}{d\sqrt{a(1 + \cos(c + dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^2)*Sqrt[Sec[c
 + d*x]])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*(2*A*b + 2
*a*B - b*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*(a - b)*(A - B)*ArcTan[Sin
[(c + d*x)/2]/Sqrt[Cos[c + d*x]]] + 2*b*B*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/
2]))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A]
time = 0.51, size = 317, normalized size = 1.65

method	result

default	$-\frac{\left(B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,b\sin(dx+c)+2A\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)b+2B\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(d}{1+\cos(}}}{\cos(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d
*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b*sin(d*x+c)+2*A*2^(1/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*b+2*B*2^(
1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*a-B*2^
(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*b-2*A
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*a+2*A*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*b+2*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*a-2*B*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*b)*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*2^(1/2)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 14.32, size = 208, normalized size = 1.08

$$\frac{\sqrt{a\cos(dx+c)+a}\,Bb\sqrt{\cos(dx+c)}\,\sin(dx+c)-(2\,Ba+(2\,A-B)b+(2\,Ba+(2\,A-B)b)\cos(dx+c))\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\sqrt{2}\,((A-B)a^2-(A-B)ab+((A-B)a^2-(A-B)ab)\cos(dx+c))\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{\sqrt{a}}}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*b*sqrt(cos(d*x + c))*sin(d*x + c) - (2*B*a + (2
*A - B)*b + (2*B*a + (2*A - B)*b)*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(2)*((A - B)*
a^2 - (A - B)*a*b + ((A - B)*a^2 - (A - B)*a*b)*cos(d*x + c))*arctan(sqrt(2
```

```
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(
a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)})\,(a + b\cos{(c + dx)})\,\sqrt{\sec{(c + dx)}}}{\sqrt{a\,(\cos{(c + dx)} + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a
+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sqrt(sec(c + d*x))/sqrt(
a*(cos(c + d*x) + 1)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos{(c + d\,x)}}}\,\left(B\,b\cos{(c + d\,x)}^2 + (A\,b + B\,a)\,\cos{(c + d\,x)} + A\,a\right)}{\sqrt{a + a\,\cos{(c + d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/cos(c + d*x))^(1/2)*(A*a + cos(c + d*x)*(A*b + B*a) + B*b*cos(c + d
*x)^2))/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(((1/cos(c + d*x))^(1/2)*(A*a + cos(c + d*x)*(A*b + B*a) + B*b*cos(c + d
*x)^2))/(a + a*cos(c + d*x))^(1/2), x)
```

3.529 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=317

$$\frac{(19A-15B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d}-\frac{(1201A-1 \ldots}{210 \ldots}$$

```
[Out] -1/2*(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/210*(397*
A-273*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/70*(67*A-
63*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/14*(11*A-7*B
)*sec(d*x+c)^(7/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/4*(19*A-15*B)*ar
ctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)-1/210*(1201*A-1029*B)
*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3057, 3063, 12, 2861, 211}

$$\frac{(19A-15B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(11A-7B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{14ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{(67A-63B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{70ad\sqrt{a\cos(c+dx)+a}}+\frac{(397A-273B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{210ad\sqrt{a\cos(c+dx)+a}}-\frac{(1201A-1029B)\sin(c+dx)\sqrt{\sec(c+dx)}}{210ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((19*A - 15*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) - ((1201*A - 1029*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(210*a*d*S
qrt[a + a*Cos[c + d*x]]) + ((397*A - 273*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(210*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((67*A - 63*B)*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(70*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(7/2)
*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((11*A - 7*B)*Sec[c + d*x
]^(7/2)*Sin[c + d*x])/(14*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+a\cos(c+dx}$$

$$= -\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}\right.}{}$$

$$= -\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(11A-7B)\sec^{\frac{7}{2}}(c+dx)\sin}{14ad\sqrt{a+a\cos(c+}}$$

$$= -\frac{(67A-63B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{70ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\,\mathrm{s}}{2d(a+a\cos(c+d}$$

$$= \frac{(397A-273B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} - \frac{(67A-63B)\sec^{\frac{5}{2}}(c+}{70ad\sqrt{a+a\cos}}$$

$$= -\frac{(1201A-1029B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A-273B)\,\mathrm{s}}{210ad\sqrt{a}}$$

$$= -\frac{(1201A-1029B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A-273B)\,\mathrm{s}}{210ad\sqrt{a}}$$

$$= -\frac{(1201A-1029B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A-273B)\,\mathrm{s}}{210ad\sqrt{a}}$$

$$= \frac{(19A-15B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{}}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.12, size = 262, normalized size = 0.83

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{i(19A-15B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)}{d}-\frac{(2339A-2751B+24(213A-217B)\cos(c+dx)+60(67A-63B)\cos(2(c+dx))+1608A\cos(3(c+dx))-1512B\cos(3(c+dx))+1201A\cos(4(c+dx))-1029B\cos(4(c+dx)))\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{840d}\right)}{(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*((I*(19*A - 15*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqr
```

```
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])/(d*E^((I/2)*(c + d*x))) - ((2339*A -
2751*B + 24*(213*A - 217*B)*Cos[c + d*x] + 60*(67*A - 63*B)*Cos[2*(c + d*x)
] + 1608*A*Cos[3*(c + d*x)] - 1512*B*Cos[3*(c + d*x)] + 1201*A*Cos[4*(c + d
*x)] - 1029*B*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]*Sec[c + d*x]^(7/2)*Tan[(c
+ d*x)/2])/(840*d)))/(a*(1 + Cos[c + d*x]))^(3/2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(270) = 540$.
time = 0.52, size = 731, normalized size = 2.31

method	result
default	$\frac{\left(1995A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos^4(dx+c))\sin(dx+c)-1575B\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos^4(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/420/d*(1995*A*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*cos(d*x+c)^4*sin(d*x+c)-1575*B*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4*sin(d*x+c)+7980*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*si
n(d*x+c)-6300*B*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*cos(d*x+c)^3*sin(d*x+c)+11970*A*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-9450*B*(cos(d*x
+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*s
in(d*x+c)+7980*A*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*cos(d*x+c)*sin(d*x+c)-6300*B*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+1995*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-1575*B*
(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*
x+c)-1201*A*cos(d*x+c)^5*2^(1/2)+1029*B*cos(d*x+c)^5*2^(1/2)+397*A*cos(d*x+
c)^4*2^(1/2)-273*B*cos(d*x+c)^4*2^(1/2)+1000*A*cos(d*x+c)^3*2^(1/2)-840*B*c
os(d*x+c)^3*2^(1/2)-232*A*cos(d*x+c)^2*2^(1/2)+168*B*cos(d*x+c)^2*2^(1/2)+9
6*A*cos(d*x+c)*2^(1/2)-84*B*cos(d*x+c)*2^(1/2)-60*A*2^(1/2))*cos(d*x+c)*(1/
cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^5/(-1+cos(d*x+c))^3/(
1+cos(d*x+c))^4*2^(1/2)/a^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [A]
time = 0.40, size = 237, normalized size = 0.75

$$-\frac{105 \sqrt{2} \left((19 A-15 B) \cos (dx+c)^{5}+2 (19 A-15 B) \cos (dx+c)^{4}+(19 A-15 B) \cos (dx+c)^{3}\right) \sqrt{a} \arctan \left(\frac{\sqrt{2} \sqrt{a \cos (dx+c)+a} \sqrt{\cos (dx+c)}}{\sqrt{a} \sin(dx+c)}\right)+\frac{2 \left((1201 A-1029 B) \cos(dx+c)^{4}+12 (67 A-63 B) \cos(dx+c)^{3}-28 (7 A-3 B) \cos(dx+c)^{2}+12 (3 A-7 B) \cos(dx+c)-60 A\right) \sqrt{a \cos (dx+c)+a} \sin(dx+c)}{\sqrt{\cos (dx+c)}}}{420 \left(a^{2} d \cos (dx+c)^{5}+2 a^{2} d \cos (dx+c)^{4}+a^{2} d \cos (dx+c)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/420*(105*sqrt(2)*((19*A - 15*B)*cos(d*x + c)^5 + 2*(19*A - 15*B)*cos(d*x
 + c)^4 + (19*A - 15*B)*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((1201*A - 1029
*B)*cos(d*x + c)^4 + 12*(67*A - 63*B)*cos(d*x + c)^3 - 28*(7*A - 3*B)*cos(d
*x + c)^2 + 12*(3*A - 7*B)*cos(d*x + c) - 60*A)*sqrt(a*cos(d*x + c) + a)*si
n(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)
^4 + a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{9/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a*cos(c + d*x))^(3/2
), x)
```

3.530 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{(15A-11B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d}+\frac{(147A-9}{30}$$

```
[Out] -1/2*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-1/30*(39*A-
35*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/10*(9*A-5*B)
*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/4*(15*A-11*B)*arc
tan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+1/30*(147*A-95*B)*sin(
d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3057, 3063, 12, 2861, 211}

$$-\frac{(15A-11B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(9A-5B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{10ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{(39A-35B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{30ad\sqrt{a\cos(c+dx)+a}}+\frac{(147A-95B)\sin(c+dx)\sqrt{\sec(c+dx)}}{30ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -1/2*((15*A - 11*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[
2]*a^(3/2)*d) + ((147*A - 95*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(30*a*d*Sq
rt[a + a*Cos[c + d*x]]) - ((39*A - 35*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
30*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x]
)/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((9*A - 5*B)*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(10*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+ \right.}{}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(9A-5B)\sec^{\frac{5}{2}}(c+dx)\sin}{10ad\sqrt{a+a\cos(c+d}}$$

$$= -\frac{(39A-35B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\,s}{2d(a+a\cos(c+d}$$

$$= \frac{(147A-95B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}} - \frac{(39A-35B)\sec^{\frac{3}{2}}(c+}{30ad\sqrt{a+a\,\mathrm{co}}}$$

$$= \frac{(147A-95B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}} - \frac{(39A-35B)\sec^{\frac{3}{2}}(c+}{30ad\sqrt{a+a\,\mathrm{co}}}$$

$$= \frac{(147A-95B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}} - \frac{(39A-35B)\sec^{\frac{3}{2}}(c+}{30ad\sqrt{a+a\,\mathrm{co}}}$$

$$= -\frac{(15A-11B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.82, size = 236, normalized size = 0.87

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-60i(15A-11B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+(264A-120B+(393A-205B)\cos(c+dx)+24(9A-5B)\cos(2(c+dx))+147A\cos(3(c+dx))-95B\cos(3(c+dx)))\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{60d(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(((-60*I)*(15*A - 11*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2
*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x))
)/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (264*A -
120*B + (393*A - 205*B)*Cos[c + d*x] + 24*(9*A - 5*B)*Cos[2*(c + d*x)] + 14
7*A*Cos[3*(c + d*x)] - 95*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]*Sec[c + d*x]
^(5/2)*Tan[(c + d*x)/2]))/(60*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(229) = 458$.

time = 0.47, size = 595, normalized size = 2.20

method	result
default	$\frac{\left(225 A \sin(dx+c) \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right) \left(\cos^3(dx+c)\right) \left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}} - 165 B \sin(dx+c) \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right) \left(\cos^3(dx+c)\right) \left(\frac{\cos}{1+\cos}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/60/d*(225*A*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*(c
os(d*x+c)/(1+cos(d*x+c)))^(5/2)-165*B*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+675*A*sin(d*x+c)*ar
csin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
5/2)-495*B*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(
d*x+c)/(1+cos(d*x+c)))^(5/2)+675*A*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-495*B*sin(d*x+c)*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+22
5*A*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)-165*B*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1
+cos(d*x+c)))^(5/2)-147*A*cos(d*x+c)^4*2^(1/2)+95*B*cos(d*x+c)^4*2^(1/2)+39
*A*cos(d*x+c)^3*2^(1/2)-35*B*cos(d*x+c)^3*2^(1/2)+120*A*cos(d*x+c)^2*2^(1/2
)-80*B*cos(d*x+c)^2*2^(1/2)-24*A*cos(d*x+c)*2^(1/2)+20*B*cos(d*x+c)*2^(1/2)
+12*A*2^(1/2))*cos(d*x+c)*sin(d*x+c)^3*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c
)))^(1/2)/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3*2^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [A]
time = 0.39, size = 220, normalized size = 0.81

$$\frac{15 \sqrt{2} \left((15 A - 11 B) \cos(dx+c)^4 + 2 (15 A - 11 B) \cos(dx+c)^3 + (15 A - 11 B) \cos(dx+c)^2\right) \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a \cos(dx+c) + a} \sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)}\right) + \frac{2 \left((147 A - 95 B) \cos(dx+c)^3 + 12 (9 A - 5 B) \cos(dx+c)^2 - 4 (3 A - 5 B) \cos(dx+c) + 12 A\right) \sqrt{a \cos(dx+c) + a} \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60 \left(a^2 d \cos(dx+c)^4 + 2 a^2 d \cos(dx+c)^3 + a^2 d \cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/60*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^4 + 2*(15*A - 11*B)*cos(d*x +
c)^3 + (15*A - 11*B)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((147*A - 95*B)*co
s(d*x + c)^3 + 12*(9*A - 5*B)*cos(d*x + c)^2 - 4*(3*A - 5*B)*cos(d*x + c) +
 12*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos
(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(3/2
), x)
```

3.531 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=223

$$\frac{(11A-7B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d}-\frac{(19A-15B}{6ad}$$

```
[Out] -1/2*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/6*(7*A-3*
B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/4*(11*A-7*B)*ar
ctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)-1/6*(19*A-15*B)*sin(d
*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3057, 3063, 12, 2861, 211}

$$\frac{(11A-7B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(7A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{(19A-15B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((11*A - 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a
^(3/2)*d) - ((19*A - 15*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a +
 a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*C
os[c + d*x])^(3/2)) + ((7*A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*a*d*
Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+d$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{6ad\sqrt{a+a\cos(c+}}$$

$$= -\frac{(19A-15B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+d}{2d(a+a\cos(c}$$

$$= -\frac{(19A-15B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+d}{2d(a+a\cos(c}$$

$$= -\frac{(19A-15B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+d}{2d(a+a\cos(c}$$

$$= \frac{(11A-7B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{}}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.46, size = 215, normalized size = 0.96

$$\frac{i\cos^3\left(\frac{1}{2}(c+dx)\right)\left(6(11A-7B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+i(11A-15B+24(A-B)\cos(c+dx)+(19A-15B)\cos(2(c+dx)))\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{6d(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] ((I/6)*Cos[(c + d*x)/2]^3*((6*(11*A - 7*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*
I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))
/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + I*(11*A -
15*B + 24*(A - B)*Cos[c + d*x] + (19*A - 15*B)*Cos[2*(c + d*x)])*Sec[(c + d
*x)/2]*Sec[c + d*x]^(3/2)*Tan[(c + d*x)/2]))/(d*(a*(1 + Cos[c + d*x]))^(3/2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(188) = 376.
time = 0.44, size = 457, normalized size = 2.05

method	result
default	$\frac{\left(33A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^2(dx+c))\sin(dx+c)-21B\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^2(dx+c))\sin(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/12/d*(33*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))
^(3/2)*cos(d*x+c)^2*sin(d*x+c)-21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*sin(d*x+c)+66*A*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+c)-
42*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*c
os(d*x+c)*sin(d*x+c)+33*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)*sin(d*x+c)-21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-19*A*cos(d*x+c)^3*2^(1/2)+15*B*cos
(d*x+c)^3*2^(1/2)+7*A*cos(d*x+c)^2*2^(1/2)-3*B*cos(d*x+c)^2*2^(1/2)+16*A*co
s(d*x+c)*2^(1/2)-12*B*cos(d*x+c)*2^(1/2)-4*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x
+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/(-1+cos(d*x+c))/(1+cos(d*x+c
))^2*2^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [A]
time = 0.38, size = 197, normalized size = 0.88

$$-\frac{3\sqrt{2}\left((11A-7B)\cos(dx+c)^3+2(11A-7B)\cos(dx+c)^2+(11A-7B)\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\left((19A-15B)\cos(dx+c)^2+12(A-B)\cos(dx+c)-4A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{12\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^3 + 2*(11*A - 7*B)*cos(d*x + c)
^2 + (11*A - 7*B)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c)
```

```
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((19*A - 15*B)*cos(d*x
+ c)^2 + 12*(A - B)*cos(d*x + c) - 4*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2
*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(3/2
), x)
```

3.532 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{(7A-3B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sqrt{\mathrm{se}}}{2d(a+}$$

[Out] -1/2*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(3/2)-1/4*(7*A-3*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+1/2*(5*A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(1/2)

Rubi [A]
time = 0.32, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3057, 3063, 12, 2861, 211}

$$-\frac{(7A-3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(5A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(3/2),x]

[Out] -1/2*((7*A - 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[2]*a^(3/2)*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((5*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
```

```
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx \cdots}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+ \cdots\right.}{\cdots}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sqrt{\sec(c+dx)}\,\mathrm{s}\cdots}{2ad\sqrt{a+a\cos(c+ \cdots}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sqrt{\sec(c+dx)}\,\mathrm{s}\cdots}{2ad\sqrt{a+a\cos(c+ \cdots}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sqrt{\sec(c+dx)}\,\mathrm{s}\cdots}{2ad\sqrt{a+a\cos(c+ \cdots}}$$

$$= -\frac{(7A-3B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cdots}}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.01, size = 191, normalized size = 1.09

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-i(7A-3B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+(4A+(5A-B)\cos(c+dx))\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(((-I)*(7*A - 3*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sq
rt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (4*A + (5*A -
B)*Cos[c + d*x])*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2]))/(d*
(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(147) = 294.
time = 0.41, size = 311, normalized size = 1.77

method	result

default	$\frac{\left(7A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)-3B\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\sin(dx}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(7*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*cos(d*x+c)*sin(d*x+c)-3*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+7*A*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-5*A*cos(d*x+c)^2*2^
(1/2)-3*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)+B*cos(d*x+c)^2*2^(1/2)+A*cos(d*x+c)*2^(1/2)-B*cos(d*x+c)*2^(
1/2)+4*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/
sin(d*x+c)/(1+cos(d*x+c))*2^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [A]
time = 0.39, size = 163, normalized size = 0.93

$$\frac{\sqrt{2}\left((7A-3B)\cos(dx+c)^2+2(7A-3B)\cos(dx+c)+7A-3B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2((5A-B)\cos(dx+c)+4A)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7*A
 - 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(sqrt(a)*sin(d*x + c))) + 2*((5*A - B)*cos(d*x + c) + 4*A)*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*c
os(d*x + c) + a^2*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(3/2
), x)
```

3.533 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$\frac{(3A+B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}-\frac{(A-}{2d(a+a\cos(c}$$

```
[Out] -1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/4*(3*A+B)
*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3057, 12, 2861, 211}

$$\frac{(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((3*A + B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3
/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c
+ d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{s}\right.}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left((3A+B)\sqrt{\cos(c-\right.}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left((3A+B)\sqrt{\cos(c-\right.}{}$$

$$= \frac{(3A+B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\mathrm{co}}}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.

time = 1.67, size = 196, normalized size = 1.54

$$\frac{i\cos^3\left(\frac{1}{2}(c+dx)\right)\left((3A+B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{2}i(A-B)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{d(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (I*Cos[(c + d*x)/2]^3*(((3*A + B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]
*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) - (I/2)*(A - B)*Sec[(
c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))
)/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(104) = 208$.
time = 0.39, size = 235, normalized size = 1.85

method	result
default	$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)+B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)+3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-A*2^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+A*2^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d
*x+c)-B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)*2^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [A]
time = 0.37, size = 144, normalized size = 1.13

$$-\frac{\sqrt{2}\left((3A+B)\cos(dx+c)^2+2(3A+B)\cos(dx+c)+3A+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2\sqrt{a\cos(dx+c)+a}(A-B)\sqrt{\cos(dx+c)}\sin(dx+c)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A +
B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(A - B)*sqrt(cos(d*x + c))*
sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a*(cos(c + d*x) + 1))**(3
/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+a\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(3/2
), x)
```

3.534 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=185

$$\frac{2B\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^{3/2}d}+\frac{(A-5B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c\text{-}}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a}}\right.}{2}$$

```
[Out] 1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*B*arcsin(s
in(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^(3/2)/d+1/4*(A-5*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/
2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1
/2)
```

Rubi [A]
time = 0.34, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3040, 3056, 3061, 2861, 211, 2853, 222}

$$\frac{(A-5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A - 5*B)*ArcTan[(Sqrt[a]*Sin[c + d
*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2*d
*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
```

```
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(}{(a+a\cos(c+dx))^{3}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}}\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left((A-5B)\sqrt{\cos(c+}\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left((A-5B)\sqrt{\cos(c+}\right.}{}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.43, size = 326, normalized size = 1.76

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\,e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\left(4Bdx-4iB\sinh^{-1}\left(e^{i(c+dx)}\right)-i\sqrt{2}\,(A-5B)\log\left(1+e^{i(c+dx)}\right)+4iB\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)+i\sqrt{2}\,A\log\left(1-e^{i(c+dx)}+\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}\right)-5i\sqrt{2}\,B\log\left(1-e^{i(c+dx)}+\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}\right)\right)+(A-B)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{2d(a(1+\cos(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*((Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
)]*Sqrt[1 + E^((2*I)*(c + d*x))]*(4*B*d*x - (4*I)*B*ArcSinh[E^(I*(c + d*x))
] - I*Sqrt[2]*(A - 5*B)*Log[1 + E^(I*(c + d*x))] + (4*I)*B*Log[1 + Sqrt[1 +
 E^((2*I)*(c + d*x))]] + I*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt
[1 + E^((2*I)*(c + d*x))]] - (5*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt
[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(c + d*x)) + (A - B)*Sec[(c +
 d*x)/2]^2*Sqrt[Sec[c + d*x]]*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/
(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [A]
time = 0.41, size = 288, normalized size = 1.56

method	result

default	$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos(dx+c)(-1+\cos(dx+c))^{2}\left(A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)-B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(A*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*cos(d*x+c)-4*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(
d*x+c)-A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*B*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*sin(d*x+c)+B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/c
os(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^5*2^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

Fricas [A]
time = 3.12, size = 203, normalized size = 1.10

$$-\frac{\sqrt{2}\left((A-5B)\cos\left(dx+c\right)^{2}+2(A-5B)\cos\left(dx+c\right)+A-5B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a\cos\left(dx+c\right)+a}\,(A-B)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+8\left(B\cos\left(dx+c\right)^{2}+2B\cos\left(dx+c\right)+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\sin(dx+c)}\right)}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*
B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(A - B)*sqrt(cos(d*x + c))*
sin(d*x + c) + 8*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}\sqrt{\sec(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(sec(c + d
*x))), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos(c + d\,x)}{\sqrt{\frac{1}{\cos(c + d\,x)}}\,{(a + a\,\cos(c + d\,x))}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2)
), x)
```

3.535 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{(2A-3B)\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^{3/2}d}-\frac{(5A-9B)\text{ArcTan}\left(\frac{}{\sqrt{2}\,\sqrt{\cos(c}\right.}}{}$$

```
[Out] 1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)-1/2*(A-3*B)*
sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(2*A-3*B)*arcsin(sin
(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a
^(3/2)/d-1/4*(5*A-9*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/
2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1
/2)
```

Rubi [A]
time = 0.47, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3040, 3056, 3062, 3061, 2861, 211, 2853, 222}

$$\frac{(2A-3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(5A-9B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-\frac{(A-3B)\sin(c+dx)}{2ad\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] ((2*A - 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - ((5*A - 9*B)*ArcTan[(Sqrt[a]
*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c +
d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) - ((A - 3*B)*Sin[
c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
```

```
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
```

```
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c-}{(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{(A-3B)\sin(}{2ad\sqrt{a+a\cos(c+dx}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{(A-3B)\sin(}{2ad\sqrt{a+a\cos(c+dx}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{(A-3B)\sin(}{2ad\sqrt{a+a\cos(c+dx}}$$

$$= \frac{(2A-3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(}{a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.68, size = 836, normalized size = 3.53

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] ((-I)*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(
c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*
```

```
x))])]*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^
(3/2)) + ((3*I)*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 +
E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*
I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c
 + d*x]))^(3/2)) + ((2*I)*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]
*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] +
ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(
c + d*x))*(a*(1 + Cos[c + d*x]))^(3/2)) - ((3*I)*Sqrt[2]*B*Sqrt[E^(I*(c + d
*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(
I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^
((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d
*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(3/2)) + (Cos[c/2 +
 (d*x)/2]^3*Sqrt[Sec[c + d*x]]*((-2*A*Cos[(d*x)/2]*Sin[c/2])/d + (Sec[c/2]*
Sec[c/2 + (d*x)/2]*(A*Sin[c/2] - B*Sin[c/2]))/d + (2*B*Cos[(3*d*x)/2]*Sin[(
3*c)/2])/d - (2*A*Cos[c/2]*Sin[(d*x)/2])/d + (Sec[c/2]*Sec[c/2 + (d*x)/2]^2
*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d + (2*B*Cos[(3*c)/2]*Sin[(3*d*x)/2])/d
))/(a*(1 + Cos[c + d*x]))^(3/2)
```

Maple [A]
time = 0.42, size = 370, normalized size = 1.56

method	result
default	$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos(dx+c)(-1+\cos(dx+c))^{3}\left(-2B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^{2}(dx+c))+A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(-2*B*2^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+A*2^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)+4*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*cos(d*x+c)-6*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
cos(d*x+c))*2^(1/2)*sin(d*x+c)+5*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d
*x+c)-A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-9*B*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*sin(d*x+c)+3*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/
cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^7*2^(1/2)/a^
2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

Fricas [A]
time = 4.11, size = 246, normalized size = 1.04

$$\frac{\sqrt{2}\left((5A-9B)\cos(dx+c)^2+2(5A-9B)\cos(dx+c)+5A-9B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-4\left((2A-3B)\cos(dx+c)^2+2(2A-3B)\cos(dx+c)+2A-3B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\left(2B\cos(dx+c)^2-(A-3B)\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5*A
 - 9*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(sqrt(a)*sin(d*x + c))) - 4*((2*A - 3*B)*cos(d*x + c)^2 + 2*(2*A - 3*B)*cos
(d*x + c) + 2*A - 3*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))/(sqrt(a)*sin(d*x + c))) + 2*(2*B*cos(d*x + c)^2 - (A - 3*B)*cos(d*x
+ c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(
d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3436 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2)
), x)
```

3.536 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{(283A-163B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d}+\frac{(2671A}{}$$

```
[Out] -1/4*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(21*A-
13*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)-1/240*(787*A-4
75*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)+1/80*(157*A-
85*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)-1/32*(283*A-
163*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+1/240*(2671*
A-1495*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3057, 3063, 12, 2861, 211}

$$-\frac{(283A-163B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(157A-85B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(787A-475B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{240a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(2671A-1495B)\sin(c+dx)\sqrt{\sec(c+dx)}}{240a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(21A-13B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] -1/16*((283*A - 163*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sq
rt[2]*a^(5/2)*d) + ((2671*A - 1495*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(240
*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((787*A - 475*B)*Sec[c + d*x]^(3/2)*Sin[
c + d*x])/(240*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(5/2
)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((21*A - 13*B)*Sec[c + d
*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((157*A - 85*
B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+d}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(21A-13B)\sec^{\frac{5}{2}}(c+dx)}{16ad(a+a\cos(c+a}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(21A-13B)\sec^{\frac{5}{2}}(c+dx)}{16ad(a+a\cos(c+a}$$

$$= -\frac{(787A-475B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{5}{2}}(c+a}{4d(a+a\cos(c}$$

$$= \frac{(2671A-1495B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A-475B)\,\mathrm{s}}{240a^2d\sqrt{a}}$$

$$= \frac{(2671A-1495B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A-475B)\,\mathrm{s}}{240a^2d\sqrt{a}}$$

$$= \frac{(2671A-1495B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A-475B)\,\mathrm{s}}{240a^2d\sqrt{a}}$$

$$= -\frac{(283A-163B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right.}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.85, size = 261, normalized size = 0.82

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-240i(283A-163B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+(15053A-7685B+10(2605A-1381B)\cos(c+dx)+108(157A-85B)\cos(2(c+dx))+9110A\cos(3(c+dx))-5030B\cos(3(c+dx))+2671A\cos(4(c+dx))-1495B\cos(4(c+dx)))\sec^3\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{960d(a(1+\cos(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-240*I)*(283*A - 163*B)*Sqrt[E^(I*(c + d*x))/(1 + E^
((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*
```

```
x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (15053
*A - 7685*B + 10*(2605*A - 1381*B)*Cos[c + d*x] + 108*(157*A - 85*B)*Cos[2*
(c + d*x)] + 9110*A*Cos[3*(c + d*x)] - 5030*B*Cos[3*(c + d*x)] + 2671*A*Cos
[4*(c + d*x)] - 1495*B*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^3*Sec[c + d*x]^(5
/2)*Tan[(c + d*x)/2]))/(960*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(270) = 540$.
time = 0.49, size = 729, normalized size = 2.30

method	result
default	$-\frac{\left(4245 A \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}} \sin(dx+c)\left(\cos^4(dx+c)\right)-2445 B \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}} \sin(dx+c)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/480/d*(4245*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+
c)))^(5/2)*sin(d*x+c)*cos(d*x+c)^4-2445*B*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*cos(d*x+c)^4+16980*A*sin(d*x
+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+
c)))^(5/2)-9780*B*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^
3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+25470*A*sin(d*x+c)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-14670*B*sin(d
*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)+16980*A*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-9780*B*sin(d*x+c)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+4245*A*sin(d*x+
c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-244
5*B*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)-2671*A*cos(d*x+c)^5*2^(1/2)+1495*B*cos(d*x+c)^5*2^(1/2)-1884*A*cos
(d*x+c)^4*2^(1/2)+1020*B*cos(d*x+c)^4*2^(1/2)+2987*A*cos(d*x+c)^3*2^(1/2)-1
715*B*cos(d*x+c)^3*2^(1/2)+1728*A*cos(d*x+c)^2*2^(1/2)-960*B*cos(d*x+c)^2*2
^(1/2)-256*A*cos(d*x+c)*2^(1/2)+160*B*cos(d*x+c)*2^(1/2)+96*A*2^(1/2))*cos(
d*x+c)*sin(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x
+c))/(1+cos(d*x+c))^3*2^(1/2)/a^3
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.43, size = 266, normalized size = 0.84

$$\frac{15\sqrt{2}\left((283A-163B)\cos(dx+c)^5+3(283A-163B)\cos(dx+c)^4+3(283A-163B)\cos(dx+c)^3+(283A-163B)\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\left((2671A-1495B)\cos(dx+c)^4+5(911A-503B)\cos(dx+c)^3+32(49A-25B)\cos(dx+c)^2-160(A-B)\cos(dx+c)+96A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{480\left(a^3d\cos(dx+c)^5+3a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+a^3d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/480*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^5 + 3*(283*A - 163*B)*cos(d
*x + c)^4 + 3*(283*A - 163*B)*cos(d*x + c)^3 + (283*A - 163*B)*cos(d*x + c)
^2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqr
t(a)*sin(d*x + c))) + 2*((2671*A - 1495*B)*cos(d*x + c)^4 + 5*(911*A - 503*
B)*cos(d*x + c)^3 + 32*(49*A - 25*B)*cos(d*x + c)^2 - 160*(A - B)*cos(d*x +
 c) + 96*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*
d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*
cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{(a+a\cos(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(5/2
), x)
```

3.537 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=270

$$\frac{(163A-75B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}-\frac{(299A-}{48}$$

[Out] -1/4*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(17*A-9*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+1/48*(95*A-39*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)+1/32*(163*A-75*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)-1/48*(299*A-147*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(1/2)

Rubi [A]
time = 0.58, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3057, 3063, 12, 2861, 211}

$$\frac{(163A-75B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(95A-39B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(299A-147B)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(17A-9B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(5/2),x]

[Out] ((163*A - 75*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((299*A - 147*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((17*A - 9*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((95*A - 39*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+d}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}\right.}{}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{16ad(a+a\cos(c+d}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{16ad(a+a\cos(c+d}$$

$$= -\frac{(299A-147B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+}{4d(a+a\cos}$$

$$= -\frac{(299A-147B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+}{4d(a+a\cos}$$

$$= -\frac{(299A-147B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+}{4d(a+a\cos}$$

$$= \frac{(163A-75B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.10, size = 243, normalized size = 0.90

$$\frac{i\cos^5\left(\frac{1}{2}(c+dx)\right)\left(3(163A-75B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+\frac{1}{8}i(878A-510B+(1537A-825B)\cos(c+dx)+2(503A-255B)\cos(2(c+dx))+299A\cos(3(c+dx))-147B\cos(3(c+dx)))\sec^3\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{12d(a(1+\cos(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] ((I/12)*Cos[(c + d*x)/2]^5*((3*(163*A - 75*B)*Sqrt[E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x
)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (I/8)*(
878*A - 510*B + (1537*A - 825*B)*Cos[c + d*x] + 2*(503*A - 255*B)*Cos[2*(c
+ d*x)] + 299*A*Cos[3*(c + d*x)] - 147*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]
^3*Sec[c + d*x]^(3/2)*Tan[(c + d*x)/2]))/(d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(229) = 458$.
time = 0.48, size = 585, normalized size = 2.17

method	result
default	$-\frac{\left(489A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos^3(dx+c))\sin(dx+c)-225B\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos^3(dx+c)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/96/d*(489*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*cos(d*x+c)^3*sin(d*x+c)-225*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ar
csin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)+1467*A*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*sin(
d*x+c)-675*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))
^(3/2)*cos(d*x+c)^2*sin(d*x+c)+1467*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(c
os(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+c)-675*B*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+c)
+489*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
*sin(d*x+c)-299*A*cos(d*x+c)^4*2^(1/2)-225*B*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+147*B*cos(d*x+c)^4*2^(1/2
)-204*A*cos(d*x+c)^3*2^(1/2)+108*B*cos(d*x+c)^3*2^(1/2)+343*A*cos(d*x+c)^2*
2^(1/2)-159*B*cos(d*x+c)^2*2^(1/2)+192*A*cos(d*x+c)*2^(1/2)-96*B*cos(d*x+c)
*2^(1/2)-32*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(
1/2)/sin(d*x+c)/(1+cos(d*x+c))^2*2^(1/2)/a^3
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.42, size = 246, normalized size = 0.91

$$-\frac{3\sqrt{2}\left((163A-75B)\cos(dx+c)^4+3(163A-75B)\cos(dx+c)^3+3(163A-75B)\cos(dx+c)^2+(163A-75B)\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\left((299A-147B)\cos(dx+c)^3+(503A-255B)\cos(dx+c)^2+32(5A-3B)\cos(dx+c)-32A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{96\left(a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+a^3d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/96*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^4 + 3*(163*A - 75*B)*cos(d*x
+ c)^3 + 3*(163*A - 75*B)*cos(d*x + c)^2 + (163*A - 75*B)*cos(d*x + c))*sqr
t(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*si
n(d*x + c))) + 2*((299*A - 147*B)*cos(d*x + c)^3 + (503*A - 255*B)*cos(d*x
+ c)^2 + 32*(5*A - 3*B)*cos(d*x + c) - 32*A)*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3
+ 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{(a+a\cos(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(5/2
), x)
```

3.538 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{(75A-19B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sqrt{}}{4d(a}$$

```
[Out] -1/4*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(13*A-
5*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)-1/32*(75*A-19*B
)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+1/16*(49*A-9*B)*s
in(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3057, 3063, 12, 2861, 211}

$$-\frac{(75A-19B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(49A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(13A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] -1/16*((75*A - 19*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt
[2]*a^(5/2)*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[
c + d*x])^(5/2)) - ((13*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a*d*(
a + a*Cos[c + d*x])^(3/2)) + ((49*A - 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx
\\
&= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}
\\
&= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}}{16ad(a+a\cos(c+}
\\
&= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}}{16ad(a+a\cos(c+}
\\
&= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}}{16ad(a+a\cos(c+}
\\
&= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}}{16ad(a+a\cos(c+}
\\
&= -\frac{(75A-19B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.20, size = 219, normalized size = 0.98

$$
\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-i(75A-19B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+\frac{1}{4}(113A-9B+2(85A-13B)\cos(c+dx)+(49A-9B)\cos(2(c+dx)))\sec^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{4d(a(1+\cos(c+dx)))^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-I)*(75*A - 19*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(
Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + ((113*A - 9*
B + 2*(85*A - 13*B)*Cos[c + d*x] + (49*A - 9*B)*Cos[2*(c + d*x)])*Sec[(c +
d*x)/2]^3*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2])/4))/(4*d*(a*(1 + Cos[c + d*x
]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(188) = 376.
time = 0.43, size = 457, normalized size = 2.05

method	result
default	$-\frac{(-1+\cos(dx+c))\left(75A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos^2(dx+c))\sin(dx+c)-19B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arcsin\left(\frac{-1+\cos}{\sin(d}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/32/d*(-1+cos(d*x+c))*(75*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-19*B*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+150*
A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(
d*x+c)*sin(d*x+c)-49*A*cos(d*x+c)^3*2^(1/2)-38*B*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+9*B*cos(d*
x+c)^3*2^(1/2)+75*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)-36*A*cos(d*x+c)^2*2^(1/2)-19*B*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4*B*cos(d*x+c)
^2*2^(1/2)+53*A*cos(d*x+c)*2^(1/2)-13*B*cos(d*x+c)*2^(1/2)+32*A*2^(1/2))*co
s(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/(1+cos(
d*x+c))*2^(1/2)/a^3
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.42, size = 210, normalized size = 0.94

$$\frac{\sqrt{2}\left((75\,A-19\,B)\cos\left(dx+c\right)^{3}+3\,(75\,A-19\,B)\cos\left(dx+c\right)^{2}+3\,(75\,A-19\,B)\cos\left(dx+c\right)+75\,A-19\,B\right)\sqrt{a}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{a\cos\left(dx+c\right)+a}\ \sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\ \sin(dx+c)}\right)+\frac{2\left((49\,A-9\,B)\cos(dx+c)^{2}+(85\,A-13\,B)\cos(dx+c)+32\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\ \sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(d*x + c)^
2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(a)*arctan(sqrt(2)*sqrt
```

```
(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((49*A
- 9*B)*cos(d*x + c)^2 + (85*A - 13*B)*cos(d*x + c) + 32*A)*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(5/2
), x)
```

3.539 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=176

$$\frac{(19A+5B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d}-\frac{(A}{4d(a+a\cos}$$

```
[Out] -1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-1/16*(9*A-B
)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/32*(19*A+5*B)*ar
ctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3057, 12, 2861, 211}

$$\frac{(19A+5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(9A-B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((19*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*
a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Se
c[c + d*x]]) - ((9*A - B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*
Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
```

```
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx}{\sqrt{\cos(c+dx)}\,(a+a\cos(c}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-B}{16ad(a+a\cos(c+}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-B}{16ad(a+a\cos(c+}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-B}{16ad(a+a\cos(c+}$$

$$= \frac{(19A+5B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 1.81, size = 216, normalized size = 1.23

$$\frac{i\cos^5\left(\frac{1}{2}(c+dx)\right)\left((19A+5B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{4}i(13A-5B+(9A-B)\cos(c+dx))\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{4d(a(1+\cos(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] ((I/4)*Cos[(c + d*x)/2]^5*(((19*A + 5*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(
Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) - (I/4)*(13*A
- 5*B + (9*A - B)*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(Sin[
(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/(d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(147) = 294$.
time = 0.40, size = 375, normalized size = 2.13

method	result

default	$-\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{a\,(1+\cos\,(dx+c))}\,\cos(dx+c)(-1+\cos(dx+c))^2\left(-9A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))+B\sqrt{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/32/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*
x+c))^2*(-9*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+B*2^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+19*A*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-4*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*cos(d*x+c)+5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d
*x+c)+4*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+19*A*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+13*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)+5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-5*B*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^5/(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [A]
time = 0.41, size = 207, normalized size = 1.18

$$-\frac{\sqrt{2}\,\left((19\,A+5\,B)\cos\,(dx+c)^3+3\,(19\,A+5\,B)\cos\,(dx+c)^2+3\,(19\,A+5\,B)\cos\,(dx+c)+19\,A+5\,B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos\,(dx+c)+a}\,\sqrt{\cos\,(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)+\frac{2\left((9\,A-B)\cos(dx+c)^2+(13\,A-5\,B)\cos(dx+c)\right)\sqrt{a\cos\,(dx+c)+a}\,\sin(dx+c)}{\sqrt{\cos\,(dx+c)}}}{32\,\left(a^3d\cos\,(dx+c)^3+3\,a^3d\cos\,(dx+c)^2+3\,a^3d\cos\,(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2
 + 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((9*A - B)
```

```
*cos(d*x + c)^2 + (13*A - 5*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
+ 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(5/2
),x)
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(5/2
), x)
```

3.540 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=174

$$\frac{(5A+3B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d}+\frac{(A-}{4d(a+a\cos(c}$$

```
[Out] 1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+1/16*(A+7*B)
*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/32*(5*A+3*B)*arct
an(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$,
Rules used = {3040, 3056, 3057, 12, 2861, 211}

$$\frac{(5A+3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(A+7B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((5*A + 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a
^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec
[c + d*x]]) + ((A + 7*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*S
qrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
```

```
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(\ldots}{(a+a\cos(c+dx))^{5\ldots}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}\ldots}\right.}{\ldots}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{(A+7B)\,\mathrm{s}\ldots}{16ad(a+a\cos(c+a\ldots}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{(A+7B)\,\mathrm{s}\ldots}{16ad(a+a\cos(c+a\ldots}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{(A+7B)\,\mathrm{s}\ldots}{16ad(a+a\cos(c+a\ldots}$$

$$= \frac{(5A+3B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\ldots}}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.86, size = 213, normalized size = 1.22

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(i(5A+3B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+\frac{1}{4}(5A+3B+(A+7B)\cos(c+dx))\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{4d(a(1+\cos(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((I*(5*A + 3*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[
2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + ((5*A + 3*B + (A
+ 7*B)*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(-Sin[(c + d*x)/
2] + Sin[(3*(c + d*x))/2]))/4))/(4*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(145) = 290$.
time = 0.41, size = 375, normalized size = 2.16

method	result

default	$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos(dx+c)(-1+\cos(dx+c))^{3}\left(A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^{2}(dx+c))+7B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(A*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+7*B*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)^2+4*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)+5*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-4*B*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+3*B*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-5*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)+5*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-3*B*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d
*x+c))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^7*
2^(1/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

Fricas [A]
time = 0.40, size = 205, normalized size = 1.18

$$-\frac{\sqrt{2}\,\left((5\,A+3\,B)\cos\left(dx+c\right)^{3}+3\,(5\,A+3\,B)\cos\left(dx+c\right)^{2}+3\,(5\,A+3\,B)\cos\left(dx+c\right)+5\,A+3\,B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{2\left((A+7\,B)\cos(dx+c)^{2}+(5\,A+3\,B)\cos(dx+c)\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{32\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 +
 3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((A + 7*B)*cos
```

```
(d*x + c)^2 + (5*A + 3*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a
^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2)
), x)
```

3.541 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{2B\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^{5/2}d}+\frac{(3A-43B)\text{ArcTan}\left(\frac{\sqrt{a}\,\text{s}}{\sqrt{2}\,\sqrt{\cos(c+dx)}}\right.}{}$$

```
[Out] 1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2)+1/16*(3*A-11
*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*B*arcsin(sin(d
*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(
5/2)/d+1/32*(3*A-43*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/
2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1
/2)
```

Rubi [A]

time = 0.46, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3040, 3056, 3061, 2861, 211, 2853, 222}

$$\frac{(3A-43B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}+\frac{(3A-11B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A - 43*B)*ArcTan[(Sqrt[a]*Sin[c
+ d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/
(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) + ((3*A - 11*B)*Sin[c +
 d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
```

```
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c-}{(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(}{}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(3A-11B)\,\mathrm{s}}{16ad(a+a\cos(c+dx}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(3A-11B)\,\mathrm{s}}{16ad(a+a\cos(c+dx}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(3A-11B)\,\mathrm{s}}{16ad(a+a\cos(c+dx}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.58, size = 347, normalized size = 1.48

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\,e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\left(32Bdx-32iB\sinh^{-1}\left(e^{i(c+dx)}\right)-i\sqrt{2}\,(3A-43B)\log\left(1+e^{i(c+dx)}\right)+32iB\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)+3i\sqrt{2}\,A\log\left(1-e^{i(c+dx)}+\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}\right)-43i\sqrt{2}\,B\log\left(1-e^{i(c+dx)}+\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}\right)\right)+\frac{1}{2}(3A-11B+(7A-15B)\cos(c+dx))\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{8d(a(1+\cos(c+dx)))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
)]*Sqrt[1 + E^((2*I)*(c + d*x))]*(32*B*d*x - (32*I)*B*ArcSinh[E^(I*(c + d*x
))] - I*Sqrt[2]*(3*A - 43*B)*Log[1 + E^(I*(c + d*x))] + (32*I)*B*Log[1 + Sq
rt[1 + E^((2*I)*(c + d*x))]] + (3*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sq
rt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]] - (43*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d
*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(c + d*x)) + ((3*A
 - 11*B + (7*A - 15*B)*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*
(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/2))/(8*d*(a*(1 + Cos[c + d*x]))
^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(195) = 390$.
time = 0.41, size = 476, normalized size = 2.03

method	result
default	$-\frac{\sqrt{a(1+\cos(dx+c))}\,(-1+\cos(dx+c))^4\cos(dx+c)\left(7A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))-15B\sqrt{2}\,\sqrt{\frac{\cos(dx+c}{1+\cos(dx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*cos(d*x+c)*(7*A*2^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-15*B*2^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)^2-32*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+3*A*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-4*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)-43*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*
sin(d*x+c)+4*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-32*B*ar
ctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d
*x+c)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-3*A*2^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)-43*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+
c)+11*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(3/2)/(co
s(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^9*2^(1/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

Fricas [A]
time = 6.50, size = 277, normalized size = 1.18

$$-\frac{\sqrt{2}\left((3A-43B)\cos(dx+c)^3+3(3A-43B)\cos(dx+c)^2+3(3A-43B)\cos(dx+c)+3A-43B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+64\left(B\cos(dx+c)^3+3B\cos(dx+c)^2+3B\cos(dx+c)+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{2\left((7A-15B)\cos(dx+c)^2+(3A-11B)\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^2
 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 64*(B*cos(d*
x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((7*A - 1
5*B)*cos(d*x + c)^2 + (3*A - 11*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c
)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2)
), x)
```

3.542 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=286

$$\frac{(2A-5B)\text{ArcSin}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}}{a^{5/2}d}-\frac{(43A-115B)\text{ArcTan}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{2}\ \sqrt{\cos(c+dx)}\ldots}\right.}{\ldots}$$

```
[Out] 1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2)+1/16*(7*A-15
*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)-1/16*(11*A-35*B)
*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(2*A-5*B)*arcsin(
sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a^(5/2)/d-1/32*(43*A-115*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x
+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)
/d*2^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3040, 3056, 3062, 3061, 2861, 211, 2853, 222}

$$\frac{(2A-5B)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}\ \text{ArcSin}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(43A-115B)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}\ \text{ArcTan}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{2}\ \sqrt{\cos(c+dx)}\ \sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\ a^{5/2}d}-\frac{(11A-35B)\sin(c+dx)}{16a^2d\sqrt{\sec(c+dx)}\ \sqrt{a\cos(c+dx)+a}}+\frac{(7A-15B)\sin(c+dx)}{16ad\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((2*A - 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - ((43*A - 115*B)*ArcTan[(Sqrt
[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sq
rt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[
c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((7*A - 15*
B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) - (
(11*A - 35*B)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+}{(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(7A-15B)\sin(}{16ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(7A-15B)\sin(}{16ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(7A-15B)\sin(}{16ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(7A-15B)\sin(}{16ad(a+a\cos(c+dx))}$$

$$= \frac{(2A-5B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}{a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.02, size = 929, normalized size = 3.25

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/
2)),x]
```

```
[Out] (((-11*I)/4)*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*
(c + d*x))])]*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c +
d*x]))^(5/2)) + (((35*I)/4)*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
)]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d*x))*(
a*(1 + Cos[c + d*x]))^(5/2)) + ((4*I)*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x)
)] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c +
 d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^5)/(
d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2)) - ((10*I)*Sqrt[2]*B*Sqr
t[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*
(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])
*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2))
 + (Cos[c/2 + (d*x)/2]^5*Sqrt[Sec[c + d*x]]*((15*(-A + B)*Cos[(d*x)/2]*Sin[
c/2])/(2*d) + (4*B*Cos[(3*d*x)/2]*Sin[(3*c)/2])/d - (15*(A - B)*Cos[c/2]*Si
n[(d*x)/2])/(2*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^2*(19*A*Sin[(d*x)/2] - 27*
B*Sin[(d*x)/2]))/(4*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^4*(-(A*Sin[(d*x)/2])
+ B*Sin[(d*x)/2]))/(2*d) + (4*B*Cos[(3*c)/2]*Sin[(3*d*x)/2])/d + ((19*A - 2
7*B)*Sec[c/2 + (d*x)/2]*Tan[c/2])/(4*d) - ((A - B)*Sec[c/2 + (d*x)/2]^3*Tan
[c/2])/(2*d)))/(a*(1 + Cos[c + d*x]))^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(241) = 482$.

time = 0.45, size = 609, normalized size = 2.13

method	result
default	$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos(dx+c)\right)^{5}\cos(dx+c)\left(-16B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{3}(dx+c)\right)+15A\sqrt{2}\,\sqrt{\frac{\cos(\cdots}{1+\cos\cdots}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*cos(d*x+c)*(-16*B*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+15*A*2^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+32*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*cos(d*x+c)*sin(d*x+c)-39*B*2^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-80*B*cos(d*x+c)*arctan(sin(d*x+c
```

```
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+43*A*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-4*A*2^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+32*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-115*B*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+20*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*cos(d*x+c)-80*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+43*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*si
n(d*x+c)-11*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-115*B*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*sin(d*x+c)+35*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2))/(1/cos(d*x+c))^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^11*
2^(1/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

Fricas [A]
time = 8.45, size = 313, normalized size = 1.09

$$\frac{\sqrt{2}\left((43\,A-115\,B)\cos\left(dx+c\right)^{3}+3\,(43\,A-115\,B)\cos\left(dx+c\right)^{2}+3\,(43\,A-115\,B)\cos\left(dx+c\right)+43\,A-115\,B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)-32\left((2\,A-5\,B)\cos\left(dx+c\right)^{3}+3\,(2\,A-5\,B)\cos\left(dx+c\right)^{2}+3\,(2\,A-5\,B)\cos\left(dx+c\right)+2\,A-5\,B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\,\sin(dx+c)}\right)+\frac{2\left(16\,B\cos(dx+c)^{3}-5\,(3\,A-11\,B)\cos(dx+c)^{2}-(11\,A-35\,B)\cos(dx+c)\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^3 + 3*(43*A - 115*B)*cos(d*x + c
)^2 + 3*(43*A - 115*B)*cos(d*x + c) + 43*A - 115*B)*sqrt(a)*arctan(sqrt(2)*
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 32*((
2*A - 5*B)*cos(d*x + c)^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + 3*(2*A - 5*B)*co
s(d*x + c) + 2*A - 5*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*
x + c))/(sqrt(a)*sin(d*x + c))) + 2*(16*B*cos(d*x + c)^3 - 5*(3*A - 11*B)*c
os(d*x + c)^2 - (11*A - 35*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*
x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2)
), x)
```

3.543 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=317

$$\frac{(1015A-363B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{64\sqrt{2}\,a^{7/2}d}-\frac{(1887A\ \cdots}{1\cdots}$$

```
[Out] -1/6*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)-1/48*(23*A-
11*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)-1/64*(109*A-41
*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)+1/192*(579*A-1
99*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^(1/2)+1/128*(1015*
A-363*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*
x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)-1/192*(188
7*A-691*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.72, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3057, 3063, 12, 2861, 211}

$$\frac{(1015A-363B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{(579A-199B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{192a^3d\sqrt{a\cos(c+dx)+a}}-\frac{(1887A-691B)\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^3d\sqrt{a\cos(c+dx)+a}}-\frac{(109A-41B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{(23A-11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48ad(a\cos(c+dx)+a)^{5/2}}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((1015*A - 363*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt
[2]*a^(7/2)*d) - ((1887*A - 691*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^
3*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) - ((23*A - 11*B)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) - ((109*A - 41*B)*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2)) + ((579*A - 199*B)
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(192*a^3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}\right.}{}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{48ad(a+a\cos(c+dx}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{48ad(a+a\cos(c+dx}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{48ad(a+a\cos(c+dx}$$

$$= -\frac{(1887A-691B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+}{6d(a+a\cos(}$$

$$= -\frac{(1887A-691B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+}{6d(a+a\cos(}$$

$$= -\frac{(1887A-691B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+}{6d(a+a\cos(}$$

$$= \frac{(1015A-363B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right.}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.03, size = 267, normalized size = 0.84

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{i(1015A-363B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)}{d} - \frac{(21641A-8469B+4(9415A-3579B)\cos(c+dx)+8(3069A-1145B)\cos(2(c+dx))+10164A\cos(3(c+dx))-3748B\cos(3(c+dx))+1887A\cos(4(c+dx))-691B\cos(4(c+dx)))\sec^5\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{96d}\right)}{8(a(1+\cos(c+dx)))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*((I*(1015*A - 363*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(
```

```
Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])/(d*E^((I/2)*(c + d*x))) - ((21641*
A - 8469*B + 4*(9415*A - 3579*B)*Cos[c + d*x] + 8*(3069*A - 1145*B)*Cos[2*(
c + d*x)] + 10164*A*Cos[3*(c + d*x)] - 3748*B*Cos[3*(c + d*x)] + 1887*A*Cos
[4*(c + d*x)] - 691*B*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^5*Sec[c + d*x]^(3/
2)*Tan[(c + d*x)/2])/(96*d)))/(8*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(270) = 540$.
time = 0.51, size = 729, normalized size = 2.30

method	result
default	$\frac{(-1+\cos(dx+c))\left(3045A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^4(dx+c))\sin(dx+c)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-1089B\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^4(dx+c))\sin\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/384/d*(-1+cos(d*x+c))*(3045*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c
)^4*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-1089*B*(cos(d*x+c)/(1+cos
(d*x+c)))^(3/2)*cos(d*x+c)^4*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+
12180*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*cos(d*x+c)^3*sin(d*x+c)-4356*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)+18270*A*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*sin(d*x+c
)-6534*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/
2)*cos(d*x+c)^2*sin(d*x+c)+12180*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(
d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+c)-1887*A*cos(d*x+c)^5*2^(1
/2)-4356*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)*cos(d*x+c)*sin(d*x+c)+691*B*cos(d*x+c)^5*2^(1/2)+3045*A*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-3195*A*co
s(d*x+c)^4*2^(1/2)-1089*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)*sin(d*x+c)+1183*B*cos(d*x+c)^4*2^(1/2)+831*A*cos(d*x+c)
^3*2^(1/2)-275*B*cos(d*x+c)^3*2^(1/2)+3355*A*cos(d*x+c)^2*2^(1/2)-1215*B*co
s(d*x+c)^2*2^(1/2)+1024*A*cos(d*x+c)*2^(1/2)-384*B*cos(d*x+c)*2^(1/2)-128*A
*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+
c)^3/(1+cos(d*x+c))^2*2^(1/2)/a^4
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.45, size = 295, normalized size = 0.93

$$-\frac{3\sqrt{2}\left((1015A-363B)\cos(dx+c)^5+4(1015A-363B)\cos(dx+c)^4+6(1015A-363B)\cos(dx+c)^3+4(1015A-363B)\cos(dx+c)^2+(1015A-363B)\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\left((1887A-691B)\cos(dx+c)^4+2(2541A-937B)\cos(dx+c)^3+39(109A-41B)\cos(dx+c)^2+128(7A-3B)\cos(dx+c)-128A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{384\left(a^4d\cos(dx+c)^5+4a^4d\cos(dx+c)^4+6a^4d\cos(dx+c)^3+4a^4d\cos(dx+c)^2+a^4d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((1015*A - 363*B)*cos(d*x + c)^5 + 4*(1015*A - 363*B)*cos
(d*x + c)^4 + 6*(1015*A - 363*B)*cos(d*x + c)^3 + 4*(1015*A - 363*B)*cos(d*
x + c)^2 + (1015*A - 363*B)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((1887*A - 69
1*B)*cos(d*x + c)^4 + 2*(2541*A - 937*B)*cos(d*x + c)^3 + 39*(109*A - 41*B)
*cos(d*x + c)^2 + 128*(7*A - 3*B)*cos(d*x + c) - 128*A)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(
d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*
x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{(a+a\cos(c+dx))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(7/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(7/2
), x)
```

3.544 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{3(121A-21B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{64\sqrt{2}\,a^{7/2}d}-\frac{(A-B}{6d}$$

[Out] -1/6*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(7/2)-1/48*(19*A-7*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)-1/192*(199*A-43*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)-3/128*(121*A-21*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)+1/192*(691*A-103*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d/(a+a*cos(d*x+c))^(1/2)

Rubi [A]
time = 0.59, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3057, 3063, 12, 2861, 211}

$$-\frac{3(121A-21B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{(691A-103B)\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^3d\sqrt{a\cos(c+dx)+a}}-\frac{(199A-43B)\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{(19A-7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{48ad(a\cos(c+dx)+a)^{5/2}}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(7/2),x]

[Out] (-3*(121*A - 21*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) - ((19*A - 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) - ((199*A - 43*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)) + ((691*A - 103*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^3*d*Sqrt[a + a*Cos[c + d*x]])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}}{48ad(a+a\cos(c+}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}}{48ad(a+a\cos(c+}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}}{48ad(a+a\cos(c+}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}}{48ad(a+a\cos(c+}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}}{48ad(a+a\cos(c+}$$

$$= -\frac{3(121A-21B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right.}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.20, size = 242, normalized size = 0.90

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(-9i(121A-21B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+\frac{1}{16}(5284A-532B+9(941A-121B)\cos(c+dx)+4(937A-133B)\cos(2(c+dx))+691A\cos(3(c+dx))-103B\cos(3(c+dx)))\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d(a(1+\cos(c+dx)))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((-9*I)*(121*A - 21*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2
*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x))
)/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + ((5284*A
- 532*B + 9*(941*A - 121*B)*Cos[c + d*x] + 4*(937*A - 133*B)*Cos[2*(c + d*x
)] + 691*A*Cos[3*(c + d*x)] - 103*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^5*Sq
rt[Sec[c + d*x]]*Tan[(c + d*x)/2])/16))/(24*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(229) = 458$.
time = 0.46, size = 595, normalized size = 2.20

method	result
default	$\frac{(-1+\cos(dx+c))^2\left(1089A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)(\cos^3(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-189B\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sin(dx+c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/384/d*(-1+cos(d*x+c))^2*(1089*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*
x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-189*B*arcsin((-1+cos(d*
x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
+3267*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*cos(d*x+c)^2*sin(d*x+c)-691*A*cos(d*x+c)^4*2^(1/2)-567*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c
)+103*B*cos(d*x+c)^4*2^(1/2)+3267*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)-1183*A*cos(d*x+c)^3*2^(
1/2)-567*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*cos(d*x+c)*sin(d*x+c)+163*B*cos(d*x+c)^3*2^(1/2)+1089*A*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+275*A*cos
(d*x+c)^2*2^(1/2)-189*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)-71*B*cos(d*x+c)^2*2^(1/2)+1215*A*cos(d*x+c)*2^
(1/2)-195*B*cos(d*x+c)*2^(1/2)+384*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/
2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/(1+cos(d*x+c))*2^(1/2)/a^4
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.40, size = 260, normalized size = 0.96

$$\frac{9\sqrt{2}\left((121A-21B)\cos(dx+c)^4+4(121A-21B)\cos(dx+c)^3+6(121A-21B)\cos(dx+c)^2+4(121A-21B)\cos(dx+c)+121A-21B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\left((691A-103B)\cos(dx+c)^3+2(937A-133B)\cos(dx+c)^2+39(41A-5B)\cos(dx+c)+384A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(9*sqrt(2)*((121*A - 21*B)*cos(d*x + c)^4 + 4*(121*A - 21*B)*cos(d*x
+ c)^3 + 6*(121*A - 21*B)*cos(d*x + c)^2 + 4*(121*A - 21*B)*cos(d*x + c) +
121*A - 21*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) + 2*((691*A - 103*B)*cos(d*x + c)^3 + 2*(937*
A - 133*B)*cos(d*x + c)^2 + 39*(41*A - 5*B)*cos(d*x + c) + 384*A)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\,\cos(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(7/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(7/2
), x)
```

3.545 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=223

$$\frac{(63A+13B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{64\sqrt{2}\,a^{7/2}d}-\frac{(}{6d(a+a\,\mathrm{c}}$$

```
[Out] -1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)-1/16*(5*A-B
)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-1/192*(103*A+5*B)*
sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/128*(63*A+13*B)*
arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3057, 12, 2861, 211}

$$\frac{(63A+13B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{(103A+5B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}-\frac{(5A-B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{5/2}}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((63*A + 13*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]
*a^(7/2)*d) - ((A - B)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[S
ec[c + d*x]]) - ((5*A - B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)
*Sqrt[Sec[c + d*x]]) - ((103*A + 5*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c
 + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx}{\sqrt{\cos(c+dx)}\,(a+a\cos(c}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{(5A-B}{16ad(a+a\cos(c+}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{(5A-B}{16ad(a+a\cos(c+}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{(5A-B}{16ad(a+a\cos(c+}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{(5A-B}{16ad(a+a\cos(c+}$$

$$= \frac{(63A+13B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.19, size = 228, normalized size = 1.02

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(48i(63A+13B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+(493A-73B+(532A-4B)\cos(c+dx)+(103A+5B)\cos(2(c+dx)))\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{384d(a(1+\cos(c+dx)))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((48*I)*(63*A + 13*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*
I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))
/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (493*A - 7
3*B + (532*A - 4*B)*Cos[c + d*x] + (103*A + 5*B)*Cos[2*(c + d*x)])*Sec[(c +
 d*x)/2]^6*Sqrt[Sec[c + d*x]]*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/(
384*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(188) = 376.
time = 0.42, size = 512, normalized size = 2.30

method	result
default	$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos(dx+c)(-1+\cos(dx+c))^3\left(-103A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^3(dx+c))-5B\sqrt{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/384/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*
x+c))^3*(-103*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-5*B*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+189*A*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-163*A*2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*cos(d*x+c)^2+39*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(
d*x+c)^2*sin(d*x+c)+7*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c
)^2+378*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+71*A*2^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+78*B*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+37*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*cos(d*x+c)+189*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+195
*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+39*B*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*sin(d*x+c)-39*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*
x+c)^7/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(7/2
), x)
```

Fricas [A]
time = 0.43, size = 257, normalized size = 1.15

$$-\frac{3\sqrt{2}\left((63A+13B)\cos\left(dx+c\right)^4+4(63A+13B)\cos\left(dx+c\right)^3+6(63A+13B)\cos\left(dx+c\right)^2+4(63A+13B)\cos\left(dx+c\right)+63A+13B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\left((103A+5B)\cos(dx+c)^3+2(133A-B)\cos(dx+c)^2+39(5A-B)\cos(dx+c)\right)\sqrt{a\cos\left(dx+c\right)+a}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{384\left(a^4d\cos\left(dx+c\right)^4+4a^4d\cos\left(dx+c\right)^3+6a^4d\cos\left(dx+c\right)^2+4a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((63*A + 13*B)*cos(d*x + c)^4 + 4*(63*A + 13*B)*cos(d*x +
 c)^3 + 6*(63*A + 13*B)*cos(d*x + c)^2 + 4*(63*A + 13*B)*cos(d*x + c) + 63*
A + 13*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)/(sqrt(a)*sin(d*x + c))) + 2*((103*A + 5*B)*cos(d*x + c)^3 + 2*(133*A - B)
*cos(d*x + c)^2 + 39*(5*A - B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3
+ 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+a\cos(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(7/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(7/2
), x)
```

3.546 $\quad \int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=221

$$\frac{(13A+7B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{64\sqrt{2}\,a^{7/2}d}+\frac{(A\ \cdots}{6d(a+a\cos(\cdots}$$

```
[Out] 1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)+1/16*(A+3*B)
*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-1/192*(5*A-17*B)*si
n(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/128*(13*A+7*B)*arc
tan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$,
Rules used = {3040, 3056, 3057, 12, 2861, 211}

$$\frac{(13A+7B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{(5A-17B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}+\frac{(A+3B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{5/2}}+\frac{(A-B)\sin(c+dx)}{6d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((13*A + 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*
a^(7/2)*d) + ((A - B)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Se
c[c + d*x]]) + ((A + 3*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*
Sqrt[Sec[c + d*x]]) - ((5*A - 17*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c +
 d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(}{(a+a\cos(c+dx))^{7/}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}}\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{(A+3B)\,\mathrm{s}}{16ad(a+a\cos(c+a}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{(A+3B)\,\mathrm{s}}{16ad(a+a\cos(c+a}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{(A+3B)\,\mathrm{s}}{16ad(a+a\cos(c+a}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{(A+3B)\,\mathrm{s}}{16ad(a+a\cos(c+a}$$

$$= \frac{(13A+7B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.02, size = 233, normalized size = 1.05

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{i(13A+7B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)}{d}-\frac{(73A+59B+4(A+35B)\cos(c+dx)+(-5A+17B)\cos(2(c+dx)))\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)}{48d}\right)}{8(a(1+\cos(c+dx)))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*((I*(13*A + 7*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt
[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/(d*E^((I/2)*(c + d*x))) - ((73*A + 59*
B + 4*(A + 35*B)*Cos[c + d*x] + (-5*A + 17*B)*Cos[2*(c + d*x)])*Sec[(c + d*
x)/2]^6*Sqrt[Sec[c + d*x]]*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/(48*d
)))/(8*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(186) = 372$.
time = 0.42, size = 512, normalized size = 2.32

method	result
default	$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos(dx+c)(-1+\cos(dx+c))^{4}\left(-5A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^{3}(dx+c))+17B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^4*(-5*A*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+17*B*2^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+39*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*c
os(d*x+c)^2*sin(d*x+c)+7*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c)^2+21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+53*B
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+78*A*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+37*A*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)+42*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c
)*sin(d*x+c)-49*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+39*A
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-39*A*2^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)+21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-21*B*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^9*2^(1/2)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sqrt(sec(d*x + c
))), x)
```

Fricas [A]
time = 0.45, size = 255, normalized size = 1.15

$$-\frac{3\sqrt{2}\left((13A+7B)\cos\left(dx+c\right)^{4}+4\left(13A+7B\right)\cos\left(dx+c\right)^{3}+6\left(13A+7B\right)\cos\left(dx+c\right)^{2}+4\left(13A+7B\right)\cos\left(dx+c\right)+13A+7B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\left((5A-17B)\cos(dx+c)^{3}-2(A+35B)\cos(dx+c)^{2}-3(13A+7B)\cos(dx+c)\right)\sqrt{a\cos\left(dx+c\right)+a}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{384\left(a^{4}d\cos\left(dx+c\right)^{4}+4a^{4}d\cos\left(dx+c\right)^{3}+6a^{4}d\cos\left(dx+c\right)^{2}+4a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((13*A + 7*B)*cos(d*x + c)^4 + 4*(13*A + 7*B)*cos(d*x + c
)^3 + 6*(13*A + 7*B)*cos(d*x + c)^2 + 4*(13*A + 7*B)*cos(d*x + c) + 13*A +
7*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sq
rt(a)*sin(d*x + c))) + 2*((5*A - 17*B)*cos(d*x + c)^3 - 2*(A + 35*B)*cos(d*
x + c)^2 - 3*(13*A + 7*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a
^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(7/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(7/2)
), x)
```

3.547 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=221

$$\frac{(7A+5B)\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d}+\frac{(A-}{6d(a+a\cos(}$$

```
[Out] 1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2)+1/48*(A-13*B
)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+1/192*(17*A+67*B)*
sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/128*(7*A+5*B)*ar
ctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3056, 3057, 12, 2861, 211}

$$\frac{(7A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{(17A+67B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)}{6d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}+\frac{(A-13B)\sin(c+dx)}{48ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] ((7*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a
^(7/2)*d) + ((A - B)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c +
d*x]^(3/2)) + ((A - 13*B)*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)*
Sqrt[Sec[c + d*x]]) + ((17*A + 67*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c
+ d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c-}{(a+a\cos(c+dx))^{7/2}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-13B)\,\mathrm{si}}{48ad(a+a\cos(c+dx}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-13B)\,\mathrm{si}}{48ad(a+a\cos(c+dx}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-13B)\,\mathrm{si}}{48ad(a+a\cos(c+dx}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-13B)\,\mathrm{si}}{48ad(a+a\cos(c+dx}$$

$$= \frac{(7A+5B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\,\sqrt{}}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.09, size = 488, normalized size = 2.21

$$\frac{i(7A+5B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d(a(1+\cos(c+dx)))^{7/2}} + \frac{\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{(17A+67B)\cos\left(\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)}{12d}+\frac{(17A+67B)\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{12d}+\frac{\sec\left(\frac{c}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(19A\sin\left(\frac{dx}{2}\right)-151B\sin\left(\frac{dx}{2}\right)\right)}{24d}+\frac{\sec\left(\frac{c}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{3d}+\frac{\sec\left(\frac{c}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-17A\sin\left(\frac{dx}{2}\right)+29B\sin\left(\frac{dx}{2}\right)\right)}{12d}+\frac{(19A-151B)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\tan\left(\frac{c}{2}\right)}{24d}-\frac{(17A-29B)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\tan\left(\frac{c}{2}\right)}{12d}+\frac{(A-B)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\tan\left(\frac{c}{2}\right)}{3d}\right)}{(a(1+\cos(c+dx)))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] ((I/8)*(7*A + 5*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 +
 E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2
*I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[
c + d*x]))^(7/2)) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Sec[c + d*x]]*(((17*A + 67*B
)*Cos[(d*x)/2]*Sin[c/2])/(12*d) + ((17*A + 67*B)*Cos[c/2]*Sin[(d*x)/2])/(12
*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^2*(19*A*Sin[(d*x)/2] - 151*B*Sin[(d*x)/2
]))/(24*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^6*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2
]))/(3*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^4*(-17*A*Sin[(d*x)/2] + 29*B*Sin[(
d*x)/2]))/(12*d) + ((19*A - 151*B)*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) - ((
```

```
17*A - 29*B)*Sec[c/2 + (d*x)/2]^3*Tan[c/2])/(12*d) + ((A - B)*Sec[c/2 + (d*
x)/2]^5*Tan[c/2])/(3*d)))/(a*(1 + Cos[c + d*x]))^(7/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(186) = 372$.
time = 0.42, size = 512, normalized size = 2.32

method	result
default	$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos(dx+c)\right)^{5}\cos(dx+c)\left(17A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{3}(dx+c)\right)+67B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*cos(d*x+c)*(17*A*2^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+67*B*2^(1/2)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+53*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*cos(d*x+c)^2+21*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(
d*x+c)-17*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+15*B*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-49*A*2^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+42*A*arcsin((-1+cos(d*x+c))/sin(d*x+
c))*cos(d*x+c)*sin(d*x+c)-35*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)+30*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-21*A
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+21*A*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*sin(d*x+c)-15*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*B*arcs
in((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)
/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^11*2^(1/2)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(3/
2)), x)
```

Fricas [A]
time = 0.42, size = 257, normalized size = 1.16

$$-\frac{3\sqrt{2}\left((7A+5B)\cos\left(dx+c\right)^{4}+4(7A+5B)\cos\left(dx+c\right)^{3}+6(7A+5B)\cos\left(dx+c\right)^{2}+4(7A+5B)\cos\left(dx+c\right)+7A+5B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{2\left((17A+67B)\cos(dx+c)^{3}+10(7A+5B)\cos(dx+c)^{2}+3(7A+5B)\cos(dx+c)\right)\sqrt{a\cos\left(dx+c\right)+a}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{384\left(a^{4}d\cos\left(dx+c\right)^{4}+4a^{4}d\cos\left(dx+c\right)^{3}+6a^{4}d\cos\left(dx+c\right)^{2}+4a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((7*A + 5*B)*cos(d*x + c)^4 + 4*(7*A + 5*B)*cos(d*x + c)^
3 + 6*(7*A + 5*B)*cos(d*x + c)^2 + 4*(7*A + 5*B)*cos(d*x + c) + 7*A + 5*B)*
sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)
*sin(d*x + c))) - 2*((17*A + 67*B)*cos(d*x + c)^3 + 10*(7*A + 5*B)*cos(d*x
+ c)^2 + 3*(7*A + 5*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*
d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(7/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(7/2)
), x)
```

3.548 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=281

$$\frac{2B\text{ArcSin}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}}{a^{7/2}d}+\frac{(5A-177B)\text{ArcTan}\left(\frac{\sqrt{a}\ \text{s}}{\sqrt{2}\ \sqrt{\cos(c+dx)}}\right.}{}$$

```
[Out] 1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2)+1/48*(5*A-17
*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2)+1/64*(5*A-49*B)*
sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*B*arcsin(sin(d*x
+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/
2)/d+1/128*(5*A-177*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/
2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1
/2)
```

Rubi [A]

time = 0.58, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3040, 3056, 3061, 2861, 211, 2853, 222}

$$\frac{(5A-177B)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}\ \text{ArcTan}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{2}\ \sqrt{\cos(c+dx)}\ \sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\ a^{7/2}d}+\frac{2B\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}\ \text{ArcSin}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}+\frac{(5A-49B)\sin(c+dx)}{64a^2d\sqrt{\sec(c+dx)}\ (a\cos(c+dx)+a)^{3/2}}+\frac{(5A-17B)\sin(c+dx)}{48ad\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}+\frac{(A-B)\sin(c+dx)}{6d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) + ((5*A - 177*B)*ArcTan[(Sqrt[a]*Sin[c
 + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Sin[c + d*x])
/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)) + ((5*A - 17*B)*Sin[c
+ d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) + ((5*A - 49
*B)*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+}{(a+a\cos(c+dx))^{7/2}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A-17B)\sin(}{48ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A-17B)\sin(}{48ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A-17B)\sin(}{48ad(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A-17B)\sin(}{48ad(a+a\cos(c+dx))}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^{7/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.17, size = 365, normalized size = 1.30

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(128Bdx-128iB\sinh^{-1}\left(e^{i(c+dx)}\right)-i\sqrt{2}(5A-177B)\log\left(1+e^{i(c+dx)}\right)+128iB\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)+5i\sqrt{2}A\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)-177i\sqrt{2}B\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)+\frac{1}{8}(97A-541B+4(25A-181B)\cos(c+dx)+(67A-247B)\cos(2(c+dx)))\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{48d(a(1+\cos(c+dx)))^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*((3*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x
)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(128*B*d*x - (128*I)*B*ArcSinh[E^(I*(c +
 d*x))] - I*Sqrt[2]*(5*A - 177*B)*Log[1 + E^(I*(c + d*x))] + (128*I)*B*Log[
1 + Sqrt[1 + E^((2*I)*(c + d*x))]] + (5*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)
) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]] - (177*I)*Sqrt[2]*B*Log[1 - E^(I
*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(c + d*x))
+ ((97*A - 541*B + 4*(25*A - 181*B)*Cos[c + d*x] + (67*A - 247*B)*Cos[2*(c
```

```
+ d*x)])*Sec[(c + d*x)/2]^6*Sqrt[Sec[c + d*x]]*(-Sin[(c + d*x)/2] + Sin[(3*
(c + d*x))/2]))/8))/(48*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(236) = 472$.
time = 0.44, size = 667, normalized size = 2.37

method	result
default	$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos(dx+c)\right)^{6}\cos(dx+c)\left(67A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{3}(dx+c)\right)-247B\sqrt{2}\,\sqrt{\frac{\cos(d}{1+\cos(}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^6*cos(d*x+c)*(67*A*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-247*B*2^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-384*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+15*A*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-17*A*2^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-531*B*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*cos(d*x+c)^2*sin(d*x+c)-115*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)^2-768*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+30*A*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*cos(d*x+c)*sin(d*x+c)-35*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(
d*x+c)-1062*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+215*
B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-384*B*arctan(sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+15*A*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-15*A*2^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)-531*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+147*B*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(5/2)/(cos(d*x+c)
/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^13*2^(1/2)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(5/
2)), x)
```

Fricas [A]
time = 8.51, size = 338, normalized size = 1.20

$$-\frac{3\sqrt{2}\left((5A-177B)\cos(dx+c)^4+4(5A-177B)\cos(dx+c)^3+6(5A-177B)\cos(dx+c)^2+4(5A-177B)\cos(dx+c)+5A-177B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+768\left(B\cos(dx+c)^4+4B\cos(dx+c)^3+6B\cos(dx+c)^2+4B\cos(dx+c)+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{2\left((67A-247B)\cos(dx+c)^3+2(25A-181B)\cos(dx+c)^2+3(5A-49B)\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((5*A - 177*B)*cos(d*x + c)^4 + 4*(5*A - 177*B)*cos(d*x +
 c)^3 + 6*(5*A - 177*B)*cos(d*x + c)^2 + 4*(5*A - 177*B)*cos(d*x + c) + 5*A
 - 177*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)/(sqrt(a)*sin(d*x + c))) + 768*(B*cos(d*x + c)^4 + 4*B*cos(d*x + c)^3 + 6*
B*cos(d*x + c)^2 + 4*B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
 + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((67*A - 247*B)*cos(d*
x + c)^3 + 2*(25*A - 181*B)*cos(d*x + c)^2 + 3*(5*A - 49*B)*cos(d*x + c))*s
qrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c
)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c
) + a^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(7/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(7/2)
), x)
```

3.549 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=333

$$\frac{(2A-7B)\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d}-\frac{(177A-637B)\text{ArcTan}\left(\frac{}{\sqrt{2}\sqrt{\text{co}}}\right.}{}$$

```
[Out] 1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2)+1/16*(3*A-7*
B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2)+1/192*(79*A-259*B
)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)-7/64*(7*A-27*B)*
sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(2*A-7*B)*arcsin(s
in(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^(7/2)/d-1/128*(177*A-637*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*
x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2
)/d*2^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3040, 3056, 3062, 3061, 2861, 211, 2853, 222}

$$\frac{(2A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{ArcSin}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{(177A-637B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{7(7A-27B)\sin(c+dx)}{64a^3d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{(79A-259B)\sin(c+dx)}{192a^2d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}+\frac{(3A-7B)\sin(c+dx)}{16ad\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}+\frac{(A-B)\sin(c+dx)}{6d\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)),x]
```

```
[Out] ((2*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) - ((177*A - 637*B)*ArcTan[(Sqr
t[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*S
qrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Sin
[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)) + ((3*A - 7*
B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + (
(79*A - 259*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c +
d*x]^(3/2)) - (7*(7*A - 27*B)*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
```

```
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c-}{(a+a\cos(c+dx))^{7/2}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin}{16ad(a+a\cos(c+dx}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin}{16ad(a+a\cos(c+dx}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin}{16ad(a+a\cos(c+dx}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin}{16ad(a+a\cos(c+dx}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin}{16ad(a+a\cos(c+dx}$$

$$= \frac{(2A-7B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(}{a^{7/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.37, size = 1017, normalized size = 3.05

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/
2)),x]
```

```
[Out] (((-49*I)/8)*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*
(c + d*x))])]*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c +
```

```
d*x]))^(7/2)) + (((189*I)/8)*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*
(a*(1 + Cos[c + d*x]))^(7/2)) + ((8*I)*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x
))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c
+ d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^7)/
(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(7/2)) - ((28*I)*Sqrt[2]*B*Sq
rt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]
*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[
2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]
)*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(7/2)
) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Sec[c + d*x]]*(((-247*A + 427*B)*Cos[(d*x)/2
]*Sin[c/2])/(12*d) + (8*B*Cos[(3*d*x)/2]*Sin[(3*c)/2])/d - ((247*A - 427*B)
*Cos[c/2]*Sin[(d*x)/2])/(12*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^2*(379*A*Sin[
(d*x)/2] - 703*B*Sin[(d*x)/2]))/(24*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^6*(A*
Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^4*(-41
*A*Sin[(d*x)/2] + 53*B*Sin[(d*x)/2]))/(12*d) + (8*B*Cos[(3*c)/2]*Sin[(3*d*x
)/2])/d + ((379*A - 703*B)*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) - ((41*A - 5
3*B)*Sec[c/2 + (d*x)/2]^3*Tan[c/2])/(12*d) + ((A - B)*Sec[c/2 + (d*x)/2]^5*
Tan[c/2])/(3*d)))/(a*(1 + Cos[c + d*x]))^(7/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 854 vs. $2(282) = 564$.
time = 0.49, size = 855, normalized size = 2.57

method	result	size
default	Expression too large to display	855

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^7*cos(d*x+c)*(-192*B*cos(
d*x+c)^4*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+247*A*2^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+384*A*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*
arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-907*B*2^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-1344*B*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+
c)+531*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+115*A*2
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+768*A*arctan(sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*cos(d*x+c)*sin(d*x
+c)-1911*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-343*B
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-2688*B*cos(d*x+c)*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(
```

```
d*x+c)+1062*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-215*
A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+384*A*arctan(sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-3822*B
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+875*B*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-1344*B*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+531*A*arcsin((-1+c
os(d*x+c))/sin(d*x+c))*sin(d*x+c)-147*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)-1911*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+567*B*2^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(7/2)/(cos(d*x+c)/(1+cos(d
*x+c)))^(9/2)/sin(d*x+c)^15*2^(1/2)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(7/
2)), x)
```

Fricas [A]
time = 14.49, size = 379, normalized size = 1.14

$$\frac{3\sqrt{2}\left((177A-637B)\cos(dx+c)^4+4(177A-637B)\cos(dx+c)^3+6(177A-637B)\cos(dx+c)^2+4(177A-637B)\cos(dx+c)+177A-637B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-384\left((2A-7B)\cos(dx+c)^4+4(2A-7B)\cos(dx+c)^3+6(2A-7B)\cos(dx+c)^2+4(2A-7B)\cos(dx+c)+2A-7B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\left(192B\cos(dx+c)^4-(247A-1099B)\cos(dx+c)^3-2(181A-721B)\cos(dx+c)^2-21(7A-27B)\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((177*A - 637*B)*cos(d*x + c)^4 + 4*(177*A - 637*B)*cos(d*
x + c)^3 + 6*(177*A - 637*B)*cos(d*x + c)^2 + 4*(177*A - 637*B)*cos(d*x + c
) + 177*A - 637*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) - 384*((2*A - 7*B)*cos(d*x + c)^4 + 4*(2
*A - 7*B)*cos(d*x + c)^3 + 6*(2*A - 7*B)*cos(d*x + c)^2 + 4*(2*A - 7*B)*cos
(d*x + c) + 2*A - 7*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))/(sqrt(a)*sin(d*x + c))) + 2*(192*B*cos(d*x + c)^4 - (247*A - 1099*B)
*cos(d*x + c)^3 - 2*(181*A - 721*B)*cos(d*x + c)^2 - 21*(7*A - 27*B)*cos(d*
x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*co
s(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*co
s(d*x + c) + a^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(7/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(7/2)
), x)
```

3.550 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=180

$$-\frac{2(3aA+5bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] 2/3*(A*b+B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*A*sec(d*x+c)^(5/2)*sin(d*
x+c)/d+2/5*(3*A*a+5*B*b)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*(3*A*a+5*B*b)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(A*b+B*a)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3039, 4082, 3872, 3853, 3856, 2719, 2720}

$$\frac{2(aB+Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(3aA+5bB)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(3aA+5bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(3*a*A + 5*b*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(5*d) + (2*(A*b + a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*
a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))(B+A\sec(c$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sec^{\frac{3}{2}}(c+$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + (Ab+aB)\int \mathrm{s}$$

$$= \frac{2(3aA+5bB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(}{}$$

$$= \frac{2(3aA+5bB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(}{}$$

$$= -\frac{2(3aA+5bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right.}{5d}$$

Mathematica [A]
time = 2.02, size = 132, normalized size = 0.73

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(-12(3aA+5bB)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(Ab+aB)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(15(aA+bB)+10(Ab+aB)\cos(c+dx)+3(3aA+5bB)\cos(2(c+dx)))\sin(c+dx)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
[Out] (Sec[c + d*x]^(5/2)*(-12*(3*a*A + 5*b*B)*Cos[c + d*x]^(5/2)*EllipticE[(c +
d*x)/2, 2] + 20*(A*b + a*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
2*(15*(a*A + b*B) + 10*(A*b + a*B)*Cos[c + d*x] + 3*(3*a*A + 5*b*B)*Cos[2*(
c + d*x)])*Sin[c + d*x]))/(30*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(208) = 416$.
time = 0.88, size = 636, normalized size = 3.53

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2(Ab+aB)\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin}}{6\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b+B*a)*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*B*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/
2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*a*A/(8
*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin
(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 235, normalized size = 1.31

$$-\frac{5\sqrt{2}\,(i\,Ba+i\,Ab)\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\,(-i\,Ba-i\,Ab)\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+3\sqrt{2}\,(3i\,Aa+5i\,Bb)\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+3\sqrt{2}\,(-3i\,Aa-5i\,Bb)\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(3\,(3\,Aa+5\,Bb)\cos(dx+c)^2+3\,Aa+5\,(Ba+Ab)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(I*B*a + I*A*b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a - I*A*b)*cos(d*x + c)^2*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(3*I*
A*a + 5*I*B*b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4
```

```
, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-3*I*A*a - 5*I*B*b)*cos(d
*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(3*(3*A*a + 5*B*b)*cos(d*x + c)^2 + 3*A*a + 5*(B*a + A
*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x)),x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x)), x)
```

3.551 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=143

$$-\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(aA+3bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{3d}$$

```
[Out] 2/3*a*A*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*(A*b+B*a)*sin(d*x+c)*sec(d*x+c)^(1/
2)/d-2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(A*a+3*
B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3039, 4082, 3872, 3856, 2720, 3853, 2719}

$$\frac{2(aB+Ab)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2(aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*(a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*d) + (2*(A*b + a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
+ (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))(B+A\sec($$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2}{3}\int \sqrt{\sec(c+}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + (Ab+aB)\int \mathrm{se}$$

$$= \frac{2(Ab+aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2aA\,\mathrm{se}}{}$$

$$= \frac{2(aA+3bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\mathrm{s}}}{3d}$$

$$= -\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{}}{d}$$

Mathematica [A]
time = 0.92, size = 104, normalized size = 0.73

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\left(-3(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\frac{(aA+3(Ab+aB)\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(A*b + a*B)*EllipticE[(c + d*x
)/2, 2] + (a*A + 3*b*B)*EllipticF[(c + d*x)/2, 2] + ((a*A + 3*(A*b + a*B)*C
os[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(179) = 358$.
time = 0.64, size = 401, normalized size = 2.80

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2Bb\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{El}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a*A
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b+B*a)/sin(1/2*d*x+1/2*c)^2
/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 205, normalized size = 1.43

$$\frac{\sqrt{2}\left(-i\,Aa-3i\,Bb\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(i\,Aa+3i\,Bb\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\,\sqrt{2}\left(i\,Ba+i\,Ab\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\,\sqrt{2}\left(-i\,Ba-i\,Ab\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+\frac{2\left(Aa+3\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A*a - 3*I*B*b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a + 3*I*B*b)*cos(d*x + c)*weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*B*a + I*
A*b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*B*a - I*A*b)*cos(d*x + c)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) +
 2*(A*a + 3*(B*a + A*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*c
os(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,(a+b\,\cos\left(c+d\,x\right))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x)),x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x)), x)
```

3.552 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=111

$$-\frac{2(aA-bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{d}$$

```
[Out] 2*a*A*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*(A*a-B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A]
time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3039, 4082, 3872, 3856, 2719, 2720}

$$\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(aA-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + 2\int \frac{\frac{1}{2}(-aA+$$
$$= \frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + (Ab+aB)\int$$
$$= \frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \Big((Ab+aB)\sqrt{}$$
$$= -\frac{2(aA-bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{d}$$

Mathematica [A]
time = 0.30, size = 85, normalized size = 0.77

$$\frac{2\sqrt{\sec(c+dx)}\left(-\left((aA-bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+(Ab+aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+aA\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-((a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]) + (A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*A*Si
n[c + d*x]))/d
```

Maple [A]
time = 0.39, size = 246, normalized size = 2.22

method	result
default	$\frac{4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,b-2A\,\mathrm{E}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a-A*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*b-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a-B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a+B*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 153, normalized size = 1.38

$$\frac{\frac{2\,Aa\sin(dx+c)}{\sqrt{\cos(dx+c)}}+\sqrt{2}\,(-i\,Ba-i\,Ab)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(i\,Ba+i\,Ab)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+\sqrt{2}\,(-i\,Aa+i\,Bb)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+\sqrt{2}\,(i\,Aa-i\,Bb)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] (2*A*a*sin(d*x + c)/sqrt(cos(d*x + c)) + sqrt(2)*(-I*B*a - I*A*b)*weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*B*a + I*A*b)*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(2)*(-I*A*a
 + I*B*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c))) + sqrt(2)*(I*A*a - I*B*b)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}(a+b\cos(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x)),x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x)), x)
```

3.553 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\, d$

Optimal. Leaf size=115

$$\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(3aA+bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{3d}$$

```
[Out] 2/3*b*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+2/3*(3*A*a+B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d
```

Rubi [A]

time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3039, 4081, 3872, 3856, 2719, 2720}

$$\frac{2(3aA+bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*(3*a*A + b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*b*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}(Ab+aB)-\frac{1}{2}(3a$$
$$= \frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - (-Ab-aB)\int \frac{1}{\sqrt{\sec(c+}}$$
$$= \frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \left((-Ab-aB)\sqrt{\cos(c+d}\right.$$
$$= \frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{d}$$

Mathematica [A]
time = 0.27, size = 90, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(6(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3aA+bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+bB\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2] + 2*(3*a*A + b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*B*S
in[2*(c + d*x)]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(153) = 306$.
time = 0.37, size = 326, normalized size = 2.83

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+3aA\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\,(}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b+3*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*b-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+B*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 156, normalized size = 1.36

$$\frac{2\,Bb\sqrt{\cos(dx+c)}\,\sin(dx+c)+\sqrt{2}\,(-3i\,Aa-i\,Bb){\rm weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(3i\,Aa+i\,Bb){\rm weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3\,\sqrt{2}\,(-i\,Ba-i\,Ab){\rm weierstrassZeta}(-4,0,{\rm weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3\,\sqrt{2}\,(i\,Ba+i\,Ab){\rm weierstrassZeta}(-4,0,{\rm weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*B*b*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-3*I*A*a - I*B*b)*wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*A*a +
 I*B*b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(
2)*(-I*B*a - I*A*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*B*a + I*A*b)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))\,(a + b\cos(c + dx))\,\sqrt{\sec(c + dx)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\,\cos(c + d\,x))\,\sqrt{\frac{1}{\cos(c + d\,x)}}\,(a + b\,\cos(c + d\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x)),x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x)), x)
```

3.554 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=148

$$\frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] 2/5*b*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*(A*b+B*a)*sin(d*x+c)/d/sec(d*x+c)
^(1/2)+2/5*(5*A*a+3*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3
*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3039, 4081, 3872, 3854, 3856, 2720, 2719}

$$\frac{2(aB+Ab)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(5*a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}(Ab+aB) - \frac{1}{2}(5aA+3bB)\sec$$
$$= \frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - (-Ab-aB)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx - \frac{1}{5}(-$$
$$= \frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}(-Ab-aB$$
$$= \frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{5d}$$
$$= \frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{5d}$$

Mathematica [A]
time = 0.61, size = 108, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)}\left(6(5aA+3bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(Ab+aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(5Ab+5aB+3bB\cos(c+dx))\sin(2(c+dx))\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(5*a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 10*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5
*A*b + 5*a*B + 3*b*B*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(180) = 360.
time = 0.37, size = 371, normalized size = 2.51

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(20Ab+20aB+24Bb)\left(\sin^4\left(\frac{d}{}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b+(20*A*b+20*B*a+24*B*b)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b-10*B*a-6*B*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b-15*A*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a+
5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a-9*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 184, normalized size = 1.24

$$-\frac{5\sqrt{2}\,(iBa+iAb)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}\,(-iBa-iAb)\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\sqrt{2}\,(-5iAa-3iBb)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\sqrt{2}\,(5iAa+3iBb)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(3Bb\cos(dx+c)^2+5(Ba+Ab)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(I*B*a + I*A*b)*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a - I*A*b)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(-5*I*A*a - 3*I*B*b)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqr
t(2)*(5*I*A*a + 3*I*B*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c))) - 2*(3*B*b*cos(d*x + c)^2 + 5*(B*a + A*b)*c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\,(a + b\cos(c + dx))}{\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))/sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/(1/cos(c + d*x))^(1/2), x)
```

3.555 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{6(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(7aA+5bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{21d}$$

```
[Out] 2/7*b*B*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*(A*b+B*a)*sin(d*x+c)/d/sec(d*x+c)
^(3/2)+2/21*(7*A*a+5*B*b)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*(A*b+B*a)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(7*A*a+5*B*b)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3039, 4081, 3872, 3854, 3856, 2719, 2720}

$$\frac{2(aB+Ab)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(7aA+5bB)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(7aA+5bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*d) + (2*(7*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)
) + (2*(A*b + a*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(7*a*A + 5*b
*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}(Ab+aB)-\frac{1}{2}(7aA+5bB)\sec(c}{\sec^{\frac{5}{2}}(c+dx)}$$
$$= \frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - (-Ab-aB)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx - \frac{1}{7}(-7a$$
$$= \frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(7aA+5bB)}{21d\sqrt{\sec(}}$$
$$= \frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(7aA+5bB)}{21d\sqrt{\sec(}}$$
$$= \frac{6(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} +$$

Mathematica [A]

time = 1.08, size = 125, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(252(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(7aA+5bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(70aA+65bB+42(Ab+aB)\cos(c+dx)+15bB\cos(2(c+dx)))\sin(2(c+dx))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(252*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2] + 20*(7*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
(70*a*A + 65*b*B + 42*(A*b + a*B)*Cos[c + d*x] + 15*b*B*Cos[2*(c + d*x)])*S
in[2*(c + d*x)]))/(210*d)
```

Maple [A]

time = 0.35, size = 413, normalized size = 2.29

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(-168Ab-168aB-360Bb)\left(\sin^6\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b+(-168*A*b-168*B*a-360*B*b)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*a+168*A*b+168*B*a+280*B*b)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*a-42*A*b-42*B*a-80*B*b)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+35*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*b+25*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 203, normalized size = 1.13

$$-\frac{5\sqrt{2}\,(7i\,Aa+5i\,Bb)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\,(-7i\,Aa-5i\,Bb)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+63\sqrt{2}\,(-i\,Ba-i\,Ab)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+63\sqrt{2}\,(i\,Ba+i\,Ab)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(15\,Bb\cos(dx+c)^3+21\,(Ba+Ab)\cos(dx+c)^2+5\,(7\,Aa+5\,Bb)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(7*I*A*a + 5*I*B*b)*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*A*a - 5*I*B*b)*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*sqrt(2)*(-I*B*a - I*A*b)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 63*sqrt(2)*(I*B*a + I*A*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*B*b*cos(d*x + c)^3 + 21*(B*a + A
*b)*cos(d*x + c)^2 + 5*(7*A*a + 5*B*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\,(a + b\cos(c + dx))}{\sec^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))/sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,\left(a+b\,\cos\left(c+d\,x\right)\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/(1/cos(c + d*x))^(3/2), x)
```

3.556 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=221

$$-\frac{2(3a^2A+5b(Ab+2aB))\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(2aAb+a^2B+3b^2B)\sqrt{\cos(c}$$

```
[Out] 2/15*a*(7*A*b+5*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*A*sec(d*x+c)^(3/2)
*(b+a*sec(d*x+c))*sin(d*x+c)/d+2/5*(3*a^2*A+5*b*(A*b+2*B*a))*sin(d*x+c)*sec
(d*x+c)^(1/2)/d-2/5*(3*a^2*A+5*b*(A*b+2*B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+2/3*(2*A*a*b+B*a^2+3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.25, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4111, 4132, 3853, 3856, 2719, 4131, 2720}

$$\frac{2(a^2B+2aAb+3b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(3a^2A+5b(2aB+Ab))\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2(3a^2A+5b(2aB+Ab))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5aB+7Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a^2*A + 5*b*(A*b + 2*a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(2*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(3*a^2*A +
5*b*(A*b + 2*a*B))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*(7*A*b + 5
*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*Sec[c + d*x]^(3/2)*(
b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2(B+A\,\text{s}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+d}{5d}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+d}{5d}$$

$$= \frac{2(3a^2A+5b(Ab+2aB))\sqrt{\sec(c+dx)}\,\sin(c}{5d}$$

$$= \frac{2(3a^2A+5b(Ab+2aB))\sqrt{\sec(c+dx)}\,\sin(c}{5d}$$

$$= -\frac{2(3a^2A+5b(Ab+2aB))\sqrt{\cos(c+dx)}\,E(}{5d}$$

Mathematica [A]
time = 2.63, size = 171, normalized size = 0.77

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(-12(3a^2A+5Ab^2+10abB)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(2aAb+a^2B+3b^2B)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(15(a^2A+Ab^2+2abB)+10a(2Ab+aB)\cos(c+dx)+3(3a^2A+5Ab^2+10abB)\cos(2(c+dx)))\sin(c+dx)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Cos[c + d*x]^(5/2)*
EllipticE[(c + d*x)/2, 2] + 20*(2*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x]^(5/
2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(a^2*A + A*b^2 + 2*a*b*B) + 10*a*(2*A*
b + a*B)*Cos[c + d*x] + 3*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Cos[2*(c + d*x)])*
Sin[c + d*x]))/(30*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(249) = 498.
time = 0.99, size = 723, normalized size = 3.27

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2Bb^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{d}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a
*(2*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(A*b+2*B*a)/sin(
1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))+2/5*a^2*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6
*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 286, normalized size = 1.29

$$-\frac{5 \sqrt{2} \left(i B a^2 + 2i A a b + 3i B b^2\right) \cos\left(dx+c\right)^2 \mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(dx+c\right) + i \sin\left(dx+c\right)\right) + 5 \sqrt{2} \left(-i B a^2 - 2i A a b - 3i B b^2\right) \cos\left(dx+c\right)^2 \mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(dx+c\right) - i \sin\left(dx+c\right)\right) + 3 \sqrt{2} \left(3i A a^2 + 10i B a b + 5i A b^2\right) \cos\left(dx+c\right)^2 \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(dx+c\right) + i \sin\left(dx+c\right)\right)\right) + 3 \sqrt{2} \left(-3i A a^2 - 10i B a b - 5i A b^2\right) \cos\left(dx+c\right)^2 \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(dx+c\right) - i \sin\left(dx+c\right)\right)\right) - \frac{2 \left(3 A a^2 + 3 \left(3 A a^2 + 10 B a b + 5 A b^2\right) \cos\left(dx+c\right)^2 + 5 \left(B a^2 + 2 A a b\right) \cos\left(dx+c\right)\right) \sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{15 \, d \cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(I*B*a^2 + 2*I*A*a*b + 3*I*B*b^2)*cos(d*x + c)^2*weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a^2 - 2*
```

```
I*A*a*b - 3*I*B*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 3*sqrt(2)*(3*I*A*a^2 + 10*I*B*a*b + 5*I*A*b^2)*cos(d*x
 + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) + 3*sqrt(2)*(-3*I*A*a^2 - 10*I*B*a*b - 5*I*A*b^2)*cos(d*x +
c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c))) - 2*(3*A*a^2 + 3*(3*A*a^2 + 10*B*a*b + 5*A*b^2)*cos(d*x + c)^2
+ 5*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos
(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2, x)
```

3.557 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=177

$$-\frac{2(2aAb+a^2B-b^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(a^2A+3Ab^2+6abB)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 2/3*a*(5*A*b+3*B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/3*a*A*(b+a*sec(d*x+c))*
sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*(2*A*a*b+B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(A*a^2+3*A*b^2+6*B*a*b)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.22, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4111, 4132, 3856, 2720, 4131, 2719}

$$\frac{2(a^2A+6abB+3Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(a^2B+2aAb-b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(3aB+5Ab)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+b)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*(2*a*A*b + a^2*B - b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(5*A*b + 3*a*B)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*Sqrt[Sec[c + d*x]]*(b + a*Se
c[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))\sin(c+d}{3d}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))\sin(c+d}{3d}$$

$$= \frac{2a(5Ab+3aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2a}{}$$

$$= \frac{2(a^2A+3Ab^2+6abB)\,\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{3d}$$

$$= -\frac{2(2aAb+a^2B-b^2B)\,\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c-\right.}{d}$$

Mathematica [A]
time = 1.24, size = 125, normalized size = 0.71

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\left(-3(2aAb+a^2B-b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+(a^2A+3Ab^2+6abB)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\frac{a(aA+3(2Ab+aB)\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(2*a*A*b + a^2*B - b^2*B)*Elli
pticE[(c + d*x)/2, 2] + (a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2,
2] + (a*(a*A + 3*(2*A*b + a*B)*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/
2)))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(211) = 422.
time = 0.68, size = 650, normalized size = 3.67

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2Bb^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,(}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*B*a*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*b^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
a^2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a*(2*A*b+B*a)/sin(1/2*d*x
+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 247, normalized size = 1.40

$$\frac{\sqrt{2}\left(-i\,Aa^2-6i\,Bab-3i\,Ab^2\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(i\,Aa^2+6i\,Bab+3i\,Ab^2\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\sqrt{2}\left(i\,Ba^2+2i\,Aab-i\,Bb^2\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\sqrt{2}\left(-i\,Ba^2-2i\,Aab+i\,Bb^2\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+\frac{2\left(Aa^2+3\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A*a^2 - 6*I*B*a*b - 3*I*A*b^2)*cos(d*x + c)*weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a^2 + 6*I*B*a*b
+ 3*I*A*b^2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
```

```
*x + c)) - 3*sqrt(2)*(I*B*a^2 + 2*I*A*a*b - I*B*b^2)*cos(d*x + c)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
3*sqrt(2)*(-I*B*a^2 - 2*I*A*a*b + I*B*b^2)*cos(d*x + c)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(A*a^2 +
 3*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(
d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+b\,\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2, x)
```

3.558 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=161

$$-\frac{2(a^2A-b(Ab+2aB))\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(6aAb+3a^2B+b^2B)\sqrt{\cos(c+}}{}$$

```
[Out] 2/3*b^2*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a^2*A*sin(d*x+c)*sec(d*x+c)^(1/2)
/d-2*(a^2*A-b*(A*b+2*B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2
/3*(6*A*a*b+3*B*a^2+B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.21, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4109, 4132, 3856, 2720, 4131, 2719}

$$\frac{2(3a^2B+6aAb+b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(a^2A-b(2aB+Ab))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a^2*A - b*(A*b + 2*a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*(6*a*A*b + 3*a^2*B + b^2*B)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*B*Sin[c + d*x])
/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4109

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}b(Ab+2aB)+\big(}{}$$

$$= \frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}b(Ab+2aB)-\frac{3}{2}}{\sqrt{\sec(c+}}$$

$$= \frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2a^2A\sqrt{\sec(c+dx)}\sin(c}{d}$$

$$= \frac{2(6aAb+3a^2B+b^2B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{3d}$$

$$= -\frac{2(a^2A-b(Ab+2aB))\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(\right.}{d}$$

Mathematica [A]
time = 0.82, size = 124, normalized size = 0.77

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left((-6a^2A+6Ab^2+12abB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(6aAb+3a^2B+b^2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{2(3a^2A+b^2B\cos(c+dx))\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-6*a^2*A + 6*A*b^2 + 12*a*b*B)*Ell
ipticE[(c + d*x)/2, 2] + 2*(6*a*A*b + 3*a^2*B + b^2*B)*EllipticF[(c + d*x)/
2, 2] + (2*(3*a^2*A + b^2*B*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c + d*x]])
)/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(197) = 394$.
time = 0.46, size = 405, normalized size = 2.52

method	result
default	$-\frac{\frac{8B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{3}+4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2-4Aab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/3*(-4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^2+6*A*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2*a^2-6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+2*B*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2*b^2-3*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*b^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 208, normalized size = 1.29

$$\frac{\sqrt{2}\,(-3i\,Ba^2-6i\,Aab-i\,Bb^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(3i\,Ba^2+6i\,Aab+i\,Bb^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3\sqrt{2}\,(i\,Aa^2-2i\,Bab-i\,Ab^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3\sqrt{2}\,(-i\,Aa^2+2i\,Bab+i\,Ab^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+\frac{2\,(Bb^2\cos(dx+c)+3\,Aa^2)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*B*a^2 - 6*I*A*a*b - I*B*b^2)*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*B*a^2 + 6*I*A*a*b + I*B*b^2)*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*A*
a^2 - 2*I*B*a*b - I*A*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*A*a^2 + 2*I*B*a*b + I*A*b
^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(B*b^2*cos(d*x + c) + 3*A*a^2)*sin(d*x + c)/sqrt(cos(d*x + c
)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="giac")
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+b\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2,x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2, x)
```

3.559 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\, d$

Optimal. Leaf size=171

$$\frac{2(10aAb+5a^2B+3b^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(3a^2A+Ab^2+2abB)\sqrt{\cos(c+}}{}$$

```
[Out] 2/5*b^2*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*b*(A*b+2*B*a)*sin(d*x+c)/d/sec(
d*x+c)^(1/2)+2/5*(10*A*a*b+5*B*a^2+3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+2/3*(3*A*a^2+A*b^2+2*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4109, 4132, 3856, 2719, 4130, 2720}

$$\frac{2(3a^2A+2abB+Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5a^2B+10aAb+3b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(2aB+Ab)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A + A*b^2 + 2*a*b*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*B*Sin[c
 + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(3*d*S
qrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4109

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx &= \int \frac{(b+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}b(Ab+2aB)+\Big(}{} \\
&= \frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}b(Ab+2aB)-\frac{5}{2}}{\sec^{\frac{3}{2}}(c+} \\
&= \frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} \\
&= \frac{2(10aAb+5a^2B+3b^2B)\sqrt{\cos(c+dx)}\,E\big(\frac{1}{2}}{5d} \\
&= \frac{2(10aAb+5a^2B+3b^2B)\sqrt{\cos(c+dx)}\,E\big(\frac{1}{2}}{5d}
\end{aligned}
$$

Mathematica [A]
time = 1.00, size = 128, normalized size = 0.75

$$
\frac{\sqrt{\sec(c+dx)}\left(6(10aAb+5a^2B+3b^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(3a^2A+Ab^2+2abB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+b(5Ab+10aB+3bB\cos(c+dx))\sin(2(c+dx))\right)}{15d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2] + 10*(3*a^2*A + A*b^2 + 2*a*b*B)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2] + b*(5*A*b + 10*a*B + 3*b*B*Cos[c + d*x])*Sin[2*
(c + d*x)]))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(203) = 406$.
time = 0.39, size = 487, normalized size = 2.85

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+\left(20A\,b^2+40Bab+24B\,b^2\right)\left(\sin\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^2+(20*A*b^2+40*B*a*b+24*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b^2-20*B*a*b-6*B*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+10*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 226, normalized size = 1.32

$$-\frac{5\sqrt{2}\left(3i\,Aa^2+2i\,Bab+i\,Ab^2\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-3i\,Aa^2-2i\,Bab-i\,Ab^2\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+3\sqrt{2}\left(-5i\,Ba^2-10i\,Aab-3i\,Bb^2\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(5i\,Ba^2+10i\,Aab+3i\,Bb^2\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(3\,Bb^2\cos\left(dx+c\right)^2+5\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] -1/15*(5*sqrt(2)*(3*I*A*a^2 + 2*I*B*a*b + I*A*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-3*I*A*a^2 - 2*I*B*a*b - I*A*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(-5*I*B*a^2 - 10*I*A*a*b - 3*I*B*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(5*I*B*a^2 + 10*I*A*a*b + 3*I*B*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*B*b^2*cos(d*x + c)^2 + 5*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))(a + b\cos(c + dx))^2 \sqrt{\sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sqrt(sec(c + d*x)), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\cos(c + d\,x))\sqrt{\frac{1}{\cos(c + d\,x)}}\,(a + b\cos(c + d\,x))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2, x)
```

3.560 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=213

$$\frac{2(5a^2A+3Ab^2+6abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(14aAb+7a^2B+5b^2B)\sqrt{\cos(c}{}$$

[Out] 2/7*b^2*B*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*b*(A*b+2*B*a)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/21*(14*A*a*b+7*B*a^2+5*B*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(14*A*a*b+7*B*a^2+5*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A]
time = 0.23, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4109, 4132, 3854, 3856, 2720, 4130, 2719}

$$\frac{2(7a^2B+14aAb+5b^2B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(7a^2B+14aAb+5b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(5a^2A+6abB+3Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(2aB+Ab)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]

[Out] (2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b^2*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4109

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}b(Ab+2aB)+\left(-7aAb+\left(-\right.\right.}{}$$

$$= \frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}b(Ab+2aB)-\frac{7}{2}a^2A\sec^2(c+}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(14aAb}{}$$

$$= \frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(14aAb}{}$$

$$= \frac{2(5a^2A+3Ab^2+6abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{5d}$$

Mathematica [A]
time = 1.47, size = 161, normalized size = 0.76

$$\frac{\sqrt{\sec(c+dx)}\left(84(5a^2A+3Ab^2+6abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(14aAb+7a^2B+5b^2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(42b(Ab+2aB)\cos(c+dx)+5(28aAb+14a^2B+13b^2B+3b^2B\cos(2(c+dx))))\sin(2(c+dx))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2] + 20*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + (42*b*(A*b + 2*a*B)*Cos[c + d*x] + 5*(28*a*
A*b + 14*a^2*B + 13*b^2*B + 3*b^2*B*Cos[2*(c + d*x)]))*Sin[2*(c + d*x)]))/(
210*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(241) = 482$.
time = 0.43, size = 548, normalized size = 2.57

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+\left(-168A\,b^2-336Bab-360B\,b\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)

[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^2+(-168*A*b^2-336*B*a*b-360*B*b^2)*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a*b+168*A*b^2+140*B*a^2+336*B*a
*b+280*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-140*A*a*b-42*A*b^2-
70*B*a^2-84*B*a*b-80*B*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*A*a*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-63*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*b^2+35*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*b^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-126*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 254, normalized size = 1.19

$$-\frac{5\sqrt{2}\left(7i\,Ba^2+14i\,Aab+5i\,Bb^2\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\left(-7i\,Ba^2-14i\,Aab-5i\,Bb^2\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+21\sqrt{2}\left(-5i\,Aa^2-6i\,Bab-3i\,Ab^2\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+21\sqrt{2}\left(5i\,Aa^2+6i\,Bab+3i\,Ab^2\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(15\,Bb^2\cos(dx+c)^3+21\,(2\,Bab+Ab^2)\cos(dx+c)^2+5\,(7\,Ba^2+14\,Aab+5\,Bb^2)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")

[Out] -1/105*(5*sqrt(2)*(7*I*B*a^2 + 14*I*A*a*b + 5*I*B*b^2)*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*B*a^2 - 14*I*A*a*b
- 5*I*B*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21
sqrt(2)(-5*I*A*a^2 - 6*I*B*a*b - 3*I*A*b^2)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(5*I*A*a^

```
2 + 6*I*B*a*b + 3*I*A*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*B*b^2*cos(d*x + c)^3 + 21*(2*B*a*
b + A*b^2)*cos(d*x + c)^2 + 5*(7*B*a^2 + 14*A*a*b + 5*B*b^2)*cos(d*x + c))*
sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))(a + b\cos(c + dx))^2}{\sqrt{\sec(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2/sqrt(sec(c + d*x)), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\cos(c + dx))\,(a + b\cos(c + dx))^2}{\sqrt{\frac{1}{\cos(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2), x
)
```

3.561 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=295

$$-\frac{2(9a^2Ab+5Ab^3+3a^3B+15ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(5a^3A+21aAb^2+2\cdots}{\cdots}$$

```
[Out] 2/21*a*(5*A*a^2+18*A*b^2+21*B*a*b)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/35*a^2*(
11*A*b+7*B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*a*A*sec(d*x+c)^(3/2)*(b+a*s
ec(d*x+c))^2*sin(d*x+c)/d+2/5*(9*A*a^2*b+5*A*b^3+3*B*a^3+15*B*a*b^2)*sin(d*
x+c)*sec(d*x+c)^(1/2)/d-2/5*(9*A*a^2*b+5*A*b^3+3*B*a^3+15*B*a*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(5*A*a^3+21*A*a*b^2+21*B*a^2*b+
21*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.38, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3039, 4111, 4161, 4132, 3853, 3856, 2719, 4131, 2720}

$$\frac{2a(5a^2A+21abB+18Ab^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a^2(7aB+11Ab)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d}+\frac{2(3a^3B+9a^2Ab+15ab^2B+5Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2(5a^3A+21a^2bB+21aAb^2+21b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2(3a^3B+9a^2Ab+15ab^2B+5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (-2*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*a^3*A + 21*a*A*b^2 + 2
1*a^2*b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(21*d) + (2*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*(5*a^2*A + 18*A*b^2 + 21*a*b*B)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a^2*(11*A*b + 7*a*B)*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(35*d) + (2*a*A*Sec[c + d*x]^(3/2)*(b + a*Sec[c + d*x]
)^2*Sin[c + d*x])/(7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^3(B+A\sec$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^2\sin(c+dx}{7d}$$

$$= \frac{2a^2(11Ab+7aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2}{}$$

$$= \frac{2a^2(11Ab+7aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2}{}$$

$$= \frac{2(9a^2Ab+5Ab^3+3a^3B+15ab^2B)\sqrt{\sec(c+d}}{5d}$$

$$= \frac{2(9a^2Ab+5Ab^3+3a^3B+15ab^2B)\sqrt{\sec(c+d}}{5d}$$

$$= -\frac{2(9a^2Ab+5Ab^3+3a^3B+15ab^2B)\sqrt{\cos(c+}}{5d}$$

Mathematica [A]
time = 3.90, size = 225, normalized size = 0.76

$$\frac{2\sqrt{\sec(c+dx)}\left(-21(9a^2Ab+5Ab^3+3a^3B+15ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(5a^3A+21aAb^2+21a^2bB+21b^3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+21(9a^2Ab+5Ab^3+3a^3B+15ab^2B)\sin(c+dx)+5a(5a^2A+21Ab^2+21abB)\tan(c+dx)+21a^2(3Ab+aB)\sec(c+dx)\tan(c+dx)+15a^3A\sec^2(c+dx)\tan(c+dx)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-21*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*a^3*A + 21*a*A*b^2 + 21*a^
2*b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 21*(9*a^2*
A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sin[c + d*x] + 5*a*(5*a^2*A + 21*A*b^
```

```
2 + 21*a*b*B)*Tan[c + d*x] + 21*a^2*(3*A*b + a*B)*Sec[c + d*x]*Tan[c + d*x]
 + 15*a^3*A*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(319) = 638$.
time = 1.33, size = 917, normalized size = 3.11

method	result	size
default	Expression too large to display	917

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*a
*b*(A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*(A*b+3*B*a)/si
n(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2)))+2*A*a^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2
*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2)))+2/5*a^2*(3*A*b+B*a)/(8*sin(1/2*d*x+1/2*c)^6-12*s
in(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x
+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 364, normalized size = 1.23

$$-\frac{5\sqrt{2}\,(5i\,Aa^3+21i\,Ba^2b+21i\,Aab^2+21i\,Bb^3)\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}\,(-5i\,Aa^3-21i\,Ba^2b-21i\,Aab^2-21i\,Bb^3)\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21\sqrt{2}\,(3i\,Ba^3+9i\,Aa^2b+15i\,Bab^2+5i\,Ab^3)\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21\sqrt{2}\,(-3i\,Ba^3-9i\,Aa^2b-15i\,Bab^2-5i\,Ab^3)\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(15\,Aa^3+21\,(3\,Ba^3+9\,Aa^2b+15\,Bab^2+5\,Ab^3)\cos(dx+c)^3+5\,(5\,Aa^3+21\,Ba^2b+21\,Aab^2)\cos(dx+c)^2+21\,(Ba^3+3\,Aa^2b)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*A*a^3 + 21*I*B*a^2*b + 21*I*A*a*b^2 + 21*I*B*b^3)*co
s(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*
sqrt(2)*(-5*I*A*a^3 - 21*I*B*a^2*b - 21*I*A*a*b^2 - 21*I*B*b^3)*cos(d*x + c
)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*
(3*I*B*a^3 + 9*I*A*a^2*b + 15*I*B*a*b^2 + 5*I*A*b^3)*cos(d*x + c)^3*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 21*sqrt(2)*(-3*I*B*a^3 - 9*I*A*a^2*b - 15*I*B*a*b^2 - 5*I*A*b^3)*cos(d*x
+ c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) - 2*(15*A*a^3 + 21*(3*B*a^3 + 9*A*a^2*b + 15*B*a*b^2 + 5*A*b^
3)*cos(d*x + c)^3 + 5*(5*A*a^3 + 21*B*a^2*b + 21*A*a*b^2)*cos(d*x + c)^2 +
21*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*co
s(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}(a+b\cos(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^3,x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^3, x)
```

3.562 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=244

$$-\frac{2(3a^3A+15aAb^2+15a^2bB-5b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(3a^2Ab+3Ab^3+a^3}{}$$

```
[Out] 2/15*a^2*(9*A*b+5*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*(3*A*a^2+14*A*b^
2+15*B*a*b)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/5*a*A*(b+a*sec(d*x+c))^2*sin(d*
x+c)*sec(d*x+c)^(1/2)/d-2/5*(3*A*a^3+15*A*a*b^2+15*B*a^2*b-5*B*b^3)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(3*A*a^2*b+3*A*b^3+B*a^3+9*B*a*
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.37, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4111, 4161, 4132, 3856, 2720, 4131, 2719}

$$\frac{2a(3a^2A+15abB+14Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a^2(5aB+9Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{2(a^3B+3a^2Ab+9ab^2B+3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(3a^3A+15a^2bB+15aAb^2-5b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A*b + 3*A*b^3 + a
^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*a*(3*a^2*A + 14*A*b^2 + 15*a*b*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(5*d) + (2*a^2*(9*A*b + 5*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(15*d) + (2*a*A*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*
d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
```

```
& !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2\sin(c+}{5d}$$

$$= \frac{2a^2(9Ab+5aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2a}{}$$

$$= \frac{2a^2(9Ab+5aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2a}{}$$

$$= \frac{2a(3a^2A+14Ab^2+15abB)\sqrt{\sec(c+dx)}\,\sin(}{5d}$$

$$= \frac{2(3a^2Ab+3Ab^3+a^3B+9ab^2B)\sqrt{\cos(c+dx}}{3d}$$

$$= -\frac{2(3a^3A+15aAb^2+15a^2bB-5b^3B)\sqrt{\cos(c}}{5d}$$

Mathematica [A]
time = 1.73, size = 192, normalized size = 0.79

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-3(3a^3A+15aAb^2+15a^2bB-5b^3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(3a^2Ab+3Ab^3+a^3B+9ab^2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{a(15(a^2A+3Ab^2+3abB)+10a(3Ab+aB)\cos(c+dx)+9(a^2A+5Ab^2+5abB)\cos(2(c+dx)))\sin(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)}\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(3*a^3*A + 15*a*A*b^2 + 15*a^2
*b*B - 5*b^3*B)*EllipticE[(c + d*x)/2, 2] + 5*(3*a^2*A*b + 3*A*b^3 + a^3*B
+ 9*a*b^2*B)*EllipticF[(c + d*x)/2, 2] + (a*(15*(a^2*A + 3*A*b^2 + 3*a*b*B)
 + 10*a*(3*A*b + a*B)*Cos[c + d*x] + 9*(a^2*A + 5*A*b^2 + 5*a*b*B)*Cos[2*(c
 + d*x)])*Sin[c + d*x])/(2*Cos[c + d*x]^(5/2))))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(272) = 544$.
time = 1.00, size = 970, normalized size = 3.98

method	result	size

default	Expression too large to display	970

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*a*b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^3*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
2*a^2*(3*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*a*b*(A*b+B*a)
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))+2/5*A*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c
)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 326, normalized size = 1.34

$$-\frac{5\sqrt{2}\left(i\,Ba^3+3i\,Aa^2b+9i\,Bab^2+3i\,Ab^3\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-i\,Ba^3-3i\,Aa^2b-9i\,Bab^2-3i\,Ab^3\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+3\sqrt{2}\left(3i\,Aa^3+15i\,Ba^2b+15i\,Aab^2-5i\,Bb^3\right)\cos\left(dx+c\right)^2\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(-3i\,Aa^3-15i\,Ba^2b-15i\,Aab^2+5i\,Bb^3\right)\cos\left(dx+c\right)^2\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(3\,Aa^3+9\left(Aa^3+5\,Ba^2b+5\,Aab^2\right)\cos(dx+c)^2+5\left(Ba^3+3\,Aa^2b\right)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{15\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(I*B*a^3 + 3*I*A*a^2*b + 9*I*B*a*b^2 + 3*I*A*b^3)*cos(d*x
+ c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2
)*(-I*B*a^3 - 3*I*A*a^2*b - 9*I*B*a*b^2 - 3*I*A*b^3)*cos(d*x + c)^2*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(3*I*A*a^3 +
 15*I*B*a^2*b + 15*I*A*a*b^2 - 5*I*B*b^3)*cos(d*x + c)^2*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)
*(-3*I*A*a^3 - 15*I*B*a^2*b - 15*I*A*a*b^2 + 5*I*B*b^3)*cos(d*x + c)^2*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)) - 2*(3*A*a^3 + 9*(A*a^3 + 5*B*a^2*b + 5*A*a*b^2)*cos(d*x + c)^2 + 5*(B*a
^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
 c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3,x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3, x)
```

3.563 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=239

$$-\frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(a^3A+9aAb^2+9a^2bB -$$

```
[Out] 2/3*a^2*(A*a-B*b)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*b*B*(b+a*sec(d*x+c))^2*
sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/3*a*(9*A*a*b+3*B*a^2-2*B*b^2)*sin(d*x+c)*se
c(d*x+c)^(1/2)/d-2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+2/3*(A*a^3+9*A*a*b^2+9*B*a^2*b+B*b^3)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.35, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4110, 4161, 4132, 3856, 2720, 4131, 2719}

$$\frac{2a(3a^2B+9aAb-2b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2a^2(aA-bB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(a^3A+9a^2bB+9aAb^2+b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(a^3B+3a^2Ab-3ab^2B-Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b
^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d
) + (2*a*(9*a*A*b + 3*a^2*B - 2*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*
d) + (2*a^2*(a*A - b*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*b*B*(b
+ a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
```

```
&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{(b\cdots}{\cdots}$$

$$= \frac{2a^2(aA-bB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2bB(\cdots}{\cdots}$$

$$= \frac{2a^2(aA-bB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2bB(\cdots}{\cdots}$$

$$= \frac{2a(9aAb+3a^2B-2b^2B)\sqrt{\sec(c+dx)}\,\sin(c\cdots}{3d}$$

$$= \frac{2(a^3A+9aAb^2+9a^2bB+b^3B)\sqrt{\cos(c+dx)}\cdots}{3d}$$

$$= -\frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B)\sqrt{\cos(c+dx\cdots}}{d}$$

Mathematica [A]
time = 2.07, size = 166, normalized size = 0.69

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-6(3a^2Ab-Ab^3+a^3B-3ab^2B)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(a^3A+9aAb^2+9a^2bB+b^3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{(2a^3A+b^3B+6a^2(3Ab+aB)\cos(c+dx)+b^3B\cos(2(c+dx)))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-6*(3*a^2*A*b - A*b^3 + a^3*B - 3*a
*b^2*B)*EllipticE[(c + d*x)/2, 2] + 2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*
B)*EllipticF[(c + d*x)/2, 2] + ((2*a^3*A + b^3*B + 6*a^2*(3*A*b + a*B)*Cos[
c + d*x] + b^3*B*Cos[2*(c + d*x)])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1209 vs. 2(269) = 538.
time = 0.82, size = 1210, normalized size = 5.06

method	result	size

default	Expression too large to display	1210

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(-8*B*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+36*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^4*a^2*b-2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*a^3-18*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*a*b^2-18*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*sin(1/2*d*x+1/2*c)^2*a^2*b+6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)
^2*b^3+12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^3+8*B*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4*b^3-18*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*
c)^2*a^2*b-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^3-6*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*a^3+18*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*s
in(1/2*d*x+1/2*c)^2*a*b^2-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3-1
8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b+A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*a^3+9*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b
-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2*a^3-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+9*a^2*b*B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
a^3-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x
)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 298, normalized size = 1.25

$$\frac{\sqrt{2}\left(-i\,Aa^3-9i\,Ba^2b-9i\,Aab^2-i\,Bb^3\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(i\,Aa^3+9i\,Ba^2b+9i\,Aab^2+i\,Bb^3\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\sqrt{2}\left(i\,Ba^3+3i\,Aa^2b-3i\,Bab^2-i\,Ab^3\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\sqrt{2}\left(-i\,Ba^3-3i\,Aa^2b+3i\,Bab^2+i\,Ab^3\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+\frac{2\left(Bb^3\cos\left(dx+c\right)^2+Aa^3+3\left(Ba^3+3\,Aa^2b\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A*a^3 - 9*I*B*a^2*b - 9*I*A*a*b^2 - I*B*b^3)*cos(d*x + c)*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a^
3 + 9*I*B*a^2*b + 9*I*A*a*b^2 + I*B*b^3)*cos(d*x + c)*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*B*a^3 + 3*I*A*a^2*b - 3
*I*B*a*b^2 - I*A*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*B*a^3 - 3*I*A*a^2
*b + 3*I*B*a*b^2 + I*A*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(B*b^3*cos(d*x + c)^2 +
 A*a^3 + 3*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{5/2}(a+b\cos(c+dx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3,x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3, x)
```

3.564 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=237

$$-\frac{2(5a^3A-15aAb^2-15a^2bB-3b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(9a^2Ab+Ab^3+3a^3$$

```
[Out] 2/5*b*B*(b+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/15*b^2*(5*A*b+9*
B*a)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/5*a^2*(5*A*a-B*b)*sin(d*x+c)*sec(d*x+c
)^(1/2)/d-2/5*(5*A*a^3-15*A*a*b^2-15*B*a^2*b-3*B*b^3)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(9*A*a^2*b+A*b^3+3*B*a^3+3*B*a*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.35, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4110, 4159, 4132, 3856, 2720, 4131, 2719}

$$\frac{2a^2(5aA-bB)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2(3a^3B+9a^2Ab+3ab^2B+Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(5a^3A-15a^2bB-15aAb^2-3b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b^2(9aB+5Ab)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(9*a^2*A*b + A*b^3 + 3*a
^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*b^2*(5*A*b + 9*a*B)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x
]]) + (2*a^2*(5*a*A - b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*B*
(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx &= \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{(b-
\\
&= \frac{2b^2(5Ab+9aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2bB(b+a\sec(c}{5d\sec
\\
&= \frac{2b^2(5Ab+9aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2bB(b+a\sec(c}{5d\sec}
\\
&= \frac{2b^2(5Ab+9aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a^2(5aA-bB)}{}
\\
&= \frac{2(9a^2Ab+Ab^3+3a^3B+3ab^2B)\sqrt{\cos(c+dx}}{3d}
\\
&= -\frac{2(5a^3A-15aAb^2-15a^2bB-3b^3B)\sqrt{\cos(c}}{5d}
\end{aligned}$$

Mathematica [A]
time = 1.54, size = 172, normalized size = 0.73

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(12(-5a^3A+15aAb^2+15a^2bB+3b^3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(9a^2Ab+Ab^3+3a^3B+3ab^2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{2(10b^2(Ab+3aB)\cos(c+dx)+3(10a^3A+b^3B+b^3B\cos(2(c+dx))))\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(12*(-5*a^3*A + 15*a*A*b^2 + 15*a^2*
b*B + 3*b^3*B)*EllipticE[(c + d*x)/2, 2] + 20*(9*a^2*A*b + A*b^3 + 3*a^3*B
+ 3*a*b^2*B)*EllipticF[(c + d*x)/2, 2] + (2*(10*b^2*(A*b + 3*a*B)*Cos[c + d
*x] + 3*(10*a^3*A + b^3*B + b^3*B*Cos[2*(c + d*x)]))*Sin[c + d*x])/Sqrt[Cos
[c + d*x]]))/(30*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(265) = 530$.
time = 0.50, size = 641, normalized size = 2.70

method	result

default	$-\frac{2\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+20A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+60B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2+24B\cos\left(\frac{dx}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15*(-24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+20*A*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^4*b^3+60*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4
*a*b^2+24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^3-30*A*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2*a^3-10*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*
b^3+45*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15
*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*a^3-45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-30*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b^2-6*B*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2*b^3+15*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*a*b^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-45*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-9*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*b^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 270, normalized size = 1.14

$$-\frac{5\sqrt{2}\,(3i\,Ba^3+9i\,Aa^2b+3i\,Bab^2+i\,Ab^3)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\,(-3i\,Ba^3-9i\,Aa^2b-3i\,Bab^2-i\,Ab^3)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+3\sqrt{2}\,(5i\,Aa^3-15i\,Ba^2b-15i\,Aab^2-3i\,Bb^3)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+3\sqrt{2}\,(-5i\,Aa^3+15i\,Ba^2b+15i\,Aab^2+3i\,Bb^3)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(3\,Bb^3\cos(dx+c)^2+15\,Aa^3+5\,(3\,Bab^2+Ab^3)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(3*I*B*a^3 + 9*I*A*a^2*b + 3*I*B*a*b^2 + I*A*b^3)*weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-3*I*B*a^3 -
 9*I*A*a^2*b - 3*I*B*a*b^2 - I*A*b^3)*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) + 3*sqrt(2)*(5*I*A*a^3 - 15*I*B*a^2*b - 15*I*A*a*b^2 -
 3*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + 3*sqrt(2)*(-5*I*A*a^3 + 15*I*B*a^2*b + 15*I*A*a*b^2 +
3*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) - 2*(3*B*b^3*cos(d*x + c)^2 + 15*A*a^3 + 5*(3*B*a*b^2 + A
*b^3)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3, x)
```

3.565 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}$

Optimal. Leaf size=245

$$\frac{2(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(21a^3A+21aAb^2+2$$

```
[Out] 2/35*b^2*(7*A*b+11*B*a)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*b*B*(b+a*sec(d*x+
c))^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/21*b*(21*A*a*b+18*B*a^2+5*B*b^2)*sin(
d*x+c)/d/sec(d*x+c)^(1/2)+2/5*(15*A*a^2*b+3*A*b^3+5*B*a^3+9*B*a*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(21*A*a^3+21*A*a*b^2+21*B*a^2
*b+5*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.34, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}=0.242$, Rules used = {3039, 4110, 4159, 4132, 3856, 2719, 4130, 2720}

$$\frac{2b(18a^2B+21aAb+5b^2B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(21a^3A+21a^2bB+21aAb^2+5b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(5a^3B+15a^2Ab+9ab^2B+3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b^2(11aB+7Ab)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^3*A + 21*a*A*b^2 + 2
1*a^2*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*b^2*(7*A*b + 11*a*B)*Sin[c + d*x])/(35*d*Sec[c + d*x
]^(3/2)) + (2*b*(21*a*A*b + 18*a^2*B + 5*b^2*B)*Sin[c + d*x])/(21*d*Sqrt[Se
c[c + d*x]]) + (2*b*B*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c + d*x
]^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,d$$

$$= \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int$$

$$= \frac{2b^2(7Ab+11aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2bB(b+a\,\mathrm{s}}{7}$$

$$= \frac{2b^2(7Ab+11aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2bB(b+a\,\mathrm{s}}{7}$$

$$= \frac{2b^2(7Ab+11aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b(21aAb+}{2}$$

$$= \frac{2(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\sqrt{\cos(c}}{5d}$$

$$= \frac{2(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\sqrt{\cos(c}}{5d}$$

Mathematica [A]
time = 1.40, size = 180, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)}\left(84(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(21a^3A+21aAb^2+21a^2bB+5b^3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+b(42b(Ab+3aB)\cos(c+dx)+5(42aAb+42a^2B+13b^2B+3b^2B\cos(2(c+dx))))\sin(2(c+dx))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(21*a^3*A + 21*a*A*b^2 + 21*a^2
*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*(42*b*(A*b
 + 3*a*B)*Cos[c + d*x] + 5*(42*a*A*b + 42*a^2*B + 13*b^2*B + 3*b^2*B*Cos[2*
(c + d*x)]))*Sin[2*(c + d*x)]))/(210*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(273) = 546$.
time = 0.44, size = 664, normalized size = 2.71

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+\left(-168A\,b^3-504Ba\,b^2-360b^3\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^3+(-168*A*b^3-504*B*a*b^2-360*B*b^3)*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a*b^2+168*A*b^3+420*B*a^2*b+5
04*B*a*b^2+280*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*A*a*b^2
-42*A*b^3-210*B*a^2*b-126*B*a*b^2-80*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)+105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3+105*A*a*b^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^
3+105*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10
5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3-189*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 295, normalized size = 1.20

$$-\frac{5\sqrt{2}\left(21i\,Aa^3+21i\,Ba^2b+21i\,Aab^2+5i\,Bb^3\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}\left(-21i\,Aa^3-21i\,Ba^2b-21i\,Aab^2-5i\,Bb^3\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21\sqrt{2}\left(-5i\,Ba^3-15i\,Aa^2b-9i\,Bab^2-3i\,Ab^3\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21\sqrt{2}\left(5i\,Ba^3+15i\,Aa^2b+9i\,Bab^2+3i\,Ab^3\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(15\,Bb^3\cos(dx+c)^3+21\left(3\,Bab^2+Ab^3\right)\cos(dx+c)^2+5\left(21\,Ba^2b+21\,Aab^2+5\,Bb^3\right)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(21*I*A*a^3 + 21*I*B*a^2*b + 21*I*A*a*b^2 + 5*I*B*b^3)*we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*I*
A*a^3 - 21*I*B*a^2*b - 21*I*A*a*b^2 - 5*I*B*b^3)*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-5*I*B*a^3 - 15*I*A*a^2*b - 9
*I*B*a*b^2 - 3*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(5*I*B*a^3 + 15*I*A*a^2*b + 9*I
*B*a*b^2 + 3*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))) - 2*(15*B*b^3*cos(d*x + c)^3 + 21*(3*B*a*b^2 +
 A*b^3)*cos(d*x + c)^2 + 5*(21*B*a^2*b + 21*A*a*b^2 + 5*B*b^3)*cos(d*x + c)
)*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))(a + b\cos(c + dx))^3 \sqrt{\sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3*sqrt(sec(c + d*x)), x
)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\,\cos(c + d\,x))\,\sqrt{\frac{1}{\cos(c + d\,x)}}\,(a + b\,\cos(c + d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3, x)
```

3.566 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=295

$$\frac{2(15a^3A+27aAb^2+27a^2bB+7b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{2(21a^2Ab+5Ab^3+7a$$

```
[Out] 2/63*b^2*(9*A*b+13*B*a)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/45*b*(27*A*a*b+22*B
*a^2+7*B*b^2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/9*b*B*(b+a*sec(d*x+c))^2*sin(
d*x+c)/d/sec(d*x+c)^(7/2)+2/21*(21*A*a^2*b+5*A*b^3+7*B*a^3+15*B*a*b^2)*sin(
d*x+c)/d/sec(d*x+c)^(1/2)+2/15*(15*A*a^3+27*A*a*b^2+27*B*a^2*b+7*B*b^3)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(21*A*a^2*b+5*A*b^3+7*B*a^
3+15*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.38, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3039, 4110, 4159, 4132, 3854, 3856, 2720, 4130, 2719}

$$\frac{2b(22a^2B+27aAb+7b^2B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(7a^3B+21a^2Ab+15ab^2B+5Ab^3)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(7a^3B+21a^2Ab+15ab^2B+5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(15a^3A+27a^2bB+27aAb^2+7b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b^2(13aB+9Ab)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(21*a^2*A*b + 5*A*b^3 +
 7*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(21*d) + (2*b^2*(9*A*b + 13*a*B)*Sin[c + d*x])/(63*d*Sec[c + d
*x]^(5/2)) + (2*b*(27*a*A*b + 22*a^2*B + 7*b^2*B)*Sin[c + d*x])/(45*d*Sec[c
 + d*x]^(3/2)) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*Sin[c + d
*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*b*B*(b + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(9*d*Sec[c + d*x]^(7/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx &= \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx \\
&= \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{(b+a\sec(c+d \\
&= \frac{2b^2(9Ab+13aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2bB(b+a\sec(c+dx))^2\sin}{9d\sec^{\frac{7}{2}}(c+dx)} \\
&= \frac{2b^2(9Ab+13aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2bB(b+a\sec(c+dx))^2\sin}{9d\sec^{\frac{7}{2}}(c+dx)} \\
&= \frac{2b^2(9Ab+13aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b(27aAb+22a^2B+7b^2B}{45d\sec^{\frac{3}{2}}(c+dx} \\
&= \frac{2b^2(9Ab+13aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b(27aAb+22a^2B+7b^2B}{45d\sec^{\frac{3}{2}}(c+dx} \\
&= \frac{2(15a^3A+27aAb^2+27a^2bB+7b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{15d}
\end{aligned}$$

Mathematica [A]
time = 1.98, size = 219, normalized size = 0.74

$$\frac{\sqrt{\sec(c+dx)}\left(168(15a^3A+27aAb^2+27a^2bB+7b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+120(21a^2Ab+5Ab^3+7a^3B+15ab^2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(7b(108aAb+108a^2B+43b^2B)\cos(c+dx)+5(252a^2Ab+78Ab^3+84a^3B+234ab^2B+18b^2(Ab+3aB)\cos(2(c+dx))+7b^3B\cos(3(c+dx))))\sin(2(c+dx))\right)}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 120*(21*a^2*A*b + 5*A*b^3 + 7*a
^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*b*(108
*a*A*b + 108*a^2*B + 43*b^2*B)*Cos[c + d*x] + 5*(252*a^2*A*b + 78*A*b^3 + 8
```

```
4*a^3*B + 234*a*b^2*B + 18*b^2*(A*b + 3*a*B)*Cos[2*(c + d*x)] + 7*b^3*B*Cos
[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(1260*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(319) = 638$.
time = 0.43, size = 745, normalized size = 2.53

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+(720A\,b^3+2160Ba\,b^2+22\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^3+(720*A*b^3+2160*B*a*b^2+2240*B*b^
3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*A*a*b^2-1080*A*b^3-1512*B
*a^2*b-3240*B*a*b^2-2072*B*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(12
60*A*a^2*b+1512*A*a*b^2+840*A*b^3+420*B*a^3+1512*B*a^2*b+2520*B*a*b^2+952*B
*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-630*A*a^2*b-378*A*a*b^2-240
*A*b^3-210*B*a^3-378*B*a^2*b-720*B*a*b^2-168*B*b^3)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+315*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+75*A*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b^2+105*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*B*a*b^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-567*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-147*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 332, normalized size = 1.13

$$-\frac{15\sqrt{2}\left(7i\,Ba^3+21i\,Aa^2b+15i\,Bab^2+5i\,Ab^3\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+15\sqrt{2}\left(-7i\,Ba^3-21i\,Aa^2b-15i\,Bab^2-5i\,Ab^3\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+21\sqrt{2}\left(-15i\,Aa^3-27i\,Ba^2b-27i\,Aab^2-7i\,Bb^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+21\sqrt{2}\left(15i\,Aa^3+27i\,Ba^2b+27i\,Aab^2+7i\,Bb^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(35\,Bb^3\cos\left(dx+c\right)^4+45\left(3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)^3+7\left(27\,Ba^2b+27\,Aab^2+7\,Bb^3\right)\cos\left(dx+c\right)^2+15\left(7\,Ba^3+21\,Aa^2b+15\,Bab^2+5\,Ab^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/315*(15*sqrt(2)*(7*I*B*a^3 + 21*I*A*a^2*b + 15*I*B*a*b^2 + 5*I*A*b^3)*we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(-7*I*
B*a^3 - 21*I*A*a^2*b - 15*I*B*a*b^2 - 5*I*A*b^3)*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-15*I*A*a^3 - 27*I*B*a^2*b -
27*I*A*a*b^2 - 7*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(15*I*A*a^3 + 27*I*B*a^2*b +
27*I*A*a*b^2 + 7*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - 2*(35*B*b^3*cos(d*x + c)^4 + 45*(3*B*a*b
^2 + A*b^3)*cos(d*x + c)^3 + 7*(27*B*a^2*b + 27*A*a*b^2 + 7*B*b^3)*cos(d*x
+ c)^2 + 15*(7*B*a^3 + 21*A*a^2*b + 15*B*a*b^2 + 5*A*b^3)*cos(d*x + c))*sin
(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\cos\left(c+dx\right)\right)\left(a+b\cos\left(c+dx\right)\right)^3}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3/sqrt(sec(c + d*x)), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^3}{\sqrt{\frac{1}{\cos(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2), x
)
```

3.567 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=210

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3ad}$$

```
[Out] 2/3*A*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-2*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/
2)/a^2/d+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+2/3
*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+2*b*(A*b-B*a)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a
+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a+b)/d
```

Rubi [A]
time = 0.51, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3039, 4118, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*d) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(3*a*d) + (2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d) - (2*(A*b - a*B)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (2*A*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(3*a*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{b+a\sec(c+dx)}\,dx$$

$$= \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{Ab}{2}+\frac{1}{2}aA\sec(c+dx)-\right.}{b+a\sec(c+dx)}}{3a}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c-}{3ad}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c-}{3ad}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c-}{3ad}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d} -$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2A\sqrt{}}{}$$

Mathematica [A]
time = 33.68, size = 225, normalized size = 1.07

$$-\frac{\cot(c+dx)\left(-a^2A\sec^{\frac{5}{2}}(c+dx)+a^2A\cos(2(c+dx))\sec^{\frac{5}{2}}(c+dx)-6a(-Ab+aB)E\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-2(3Ab^2+a^2(A-3B)+3ab(A-B))F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+6Ab^2\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-6abB\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right)}{3a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),x]

[Out] -1/3*(Cot[c + d*x]*(-(a^2*A*Sec[c + d*x]^(5/2)) + a^2*A*Cos[2*(c + d*x)]*Sec[c + d*x]^(5/2) - 6*a*(-(A*b) + a*B)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*(3*A*b^2 + a^2*(A - 3*B) + 3*a*b*(A - B))*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*A*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*a*b*B*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^3*d)

Maple [A]
time = 0.80, size = 441, normalized size = 2.10

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4(Ab-aB)b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{a^2\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(A*b-B*a)*b^2/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(-A*b+B*a)/a^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}{a+b\,\cos\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x)),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x)), x)
```

3.568 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=126

$$-\frac{2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{a(a+b)d}$$

```
[Out] 2*A*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a/d-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/a/(a+b)/d
```

Rubi [A]
time = 0.28, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4118, 4191, 3934, 2884, 12, 3856, 2719}

$$-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d
) - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{b+a\sec(c+dx)}\,dx$$

$$=\frac{2A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad}+\frac{2\int \frac{-\frac{Ab}{2}-\frac{1}{2}aA\sec(c+dx)-\frac{1}{2}(Ab-aB)\sec}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+}}{a}$$

$$=\frac{2A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad}+\frac{2\int -\frac{Ab^2}{2\sqrt{\sec(c+dx)}}\,dx}{ab^2}+\frac{(-A}{}$$

$$=\frac{2A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad}-\frac{A\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a}+\frac{((-A}{}$$

$$=-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a+b)d}$$

$$=-\frac{2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{2(Ab-a}{}$$

Mathematica [A]
time = 31.37, size = 125, normalized size = 0.99

$$-\frac{2\cos(2(c+dx))\csc(c+dx)\left(aAE\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-(aA+Ab-aB)F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+(Ab-aB)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)\sec(c+dx)\sqrt{-\tan^2(c+dx)}}{a^2d\left(-2+\sec^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*Cos[2*(c + d*x)]*Csc[c + d*x]*(a*A*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]]
, -1] - (a*A + A*b - a*B)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (A*b
- a*B)*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sec[c + d*x]*Sqr
t[-Tan[c + d*x]^2])/(a^2*d*(-2 + Sec[c + d*x]^2))
```

Maple [A]
time = 0.46, size = 300, normalized size = 2.38

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4(-Ab+aB)b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{a\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(-A*b+B*a)/a
/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x
+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x)),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x)), x)
```

3.569 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b(a+b)d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d+2*(A*b-B*a)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+
b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a+b)/d
```

Rubi [A]
time = 0.17, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3039, 4123, 3856, 2720, 3934, 2884}

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
 + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4123

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
 NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}\,(B+A\sec(c+dx))}{b+a\sec(c+dx)}\,dx$$

$$= \frac{B\int\sqrt{\sec(c+dx)}\,dx}{b} - \frac{(-Ab+aB)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx}{b}$$

$$= \frac{\left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b} - \frac{\Big((-$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} + \frac{2(Ab-a}{}$$

Mathematica [A]
time = 20.61, size = 76, normalized size = 0.75

$$\frac{2\cot(c+dx)\left(AbF\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+(-Ab+aB)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)\sqrt{-\tan^2(c+dx)}}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(A*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (-(A*b) +
a*B)*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]
^2])/(a*b*d)
```

Maple [A]
time = 0.34, size = 217, normalized size = 2.15

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{EllipticP}\ldots\right.}{b(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))*b+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))*a)/b/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos{(c+dx)})\sqrt{\sec{(c+dx)}}}{a+b\cos{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos{(c+d\,x)})\,\sqrt{\frac{1}{\cos{(c+d\,x)}}}}{a+b\,\cos{(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x)),x)
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x)), x)
```

3.570 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=149

$$\frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{b^2d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d+2*(A*b-B*a)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/d-2*a*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a+b)/d
```

Rubi [A]
time = 0.22, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4123, 3856, 2719, 3933, 2882, 2720, 2884}

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
 + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(b^2*d) - (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2882

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
```

```
b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3933

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[Sqrt[d*Sin[e + f*x]]*(Sqrt[d*Csc[e + f*x]]/d), Int[S
qrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
x] && NeQ[a^2 - b^2, 0]
```

Rule 4123

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
 NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))}\,dx$$

$$= \frac{B\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b} - \frac{(-Ab+aB)\int \frac{\sqrt{\sec(c+dx)}}{b+a\sec(c+dx)}\,dx}{b}$$

$$= \frac{\left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b} - \frac{\left((-Ab\right.}{}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{\left((-Ab+aE\right.}{}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} + \frac{2(Ab-aB)}{}$$

Mathematica [A]
time = 36.91, size = 220, normalized size = 1.48

$$\frac{\cot(c+dx)\left(-bB\sec^{\frac{3}{2}}(c+dx) - bB\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx) + bB\sec^{\frac{7}{2}}(c+dx) + bB\cos(2(c+dx))\sec^{\frac{7}{2}}(c+dx) - 2bBE\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)} + 2bBF\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)} + 2Ab\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)} - 2aB\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Cot[c + d*x]*(-(b*B*Sec[c + d*x]^(3/2)) - b*B*Cos[2*(c + d*x)]*Sec[c + d*x
]^(3/2) + b*B*Sec[c + d*x]^(7/2) + b*B*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2)
- 2*b*B*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2
*b*B*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*A*
b*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
- 2*a*B*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*
x]^2]))/(b^2*d)
```

Maple [A]
time = 0.37, size = 295, normalized size = 1.98

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\text{EllipticF}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-A*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a*b-B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+B*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2)/b^2/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{(c + dx)}}{(a + b\cos{(c + dx)})\sqrt{\sec{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(1/2),x)

[Out] Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))), x)
```

3.571 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d}-\frac{2(3aAb-3a^2B-b^2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{3b^3d}$$

```
[Out] 2/3*B*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/b^2/d-2/3*(3*A*a*b-3*B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/b^3/d+2*a^2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/b^3/(a+b)/d
```

Rubi [A]
time = 0.35, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3039, 4119, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{2a^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2(-3a^2B+3aAb-b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(b^2*d) - (2*(3*a*A*b - 3*a^2*B - b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^3*d) + (2*a^2*(A*b - a*B)*Sqrt[Cos
[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b
^3*(a + b)*d) + (2*B*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
```

```
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))}\,dx$$

$$=\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{2\int \frac{-\frac{3}{2}(Ab-aB)-\frac{1}{2}bB\sec(c+dx)-\frac{1}{2}aB\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))}\,dx}{3b}$$

$$=\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{2\int \frac{-\frac{3}{2}b(Ab-aB)-\left(\frac{b^2B}{2}-\frac{3}{2}a(Ab-aB)\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3b^3}+$$

$$=\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}+\frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b^2}-\frac{(3aAb-3}{}$$

$$=\frac{2a^2(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3(a+b)d}$$

$$=\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d}-\frac{2(3a}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 542 vs. 2(197) = 394.
time = 36.82, size = 542, normalized size = 2.75

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -1/6*((2*(-3*A*b + a*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]
], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c +
 d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 -
Cos[c + d*x]^2)) - (4*B*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c
 + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/
((a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-3*A*b + 3*a*B)*Cos[2*(c +
d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE
```

```
[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*
x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi
[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
+ d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*S
qrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(b*d) + (B*Sqrt[Sec[c + d*x]]*Sin[
2*(c + d*x)])/(3*b*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 821 vs. $2(257) = 514$.
time = 0.46, size = 822, normalized size = 4.17

method	result	size
default	Expression too large to display	822

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2+4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^4*b^3+3*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*a*b^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-3*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2*b+2*B*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2*a*b^2-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3-3*a^3*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+3*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*a*b^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a^2*b+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b
),2^(1/2))*a^3)/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{\left(c + dx \right)}}{\left(a + b\cos{\left(c + dx \right)}\right)\sec^{\frac{3}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))*sec(c + d*x)**(3/2)), x
)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cos(c+dx)}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}(a+b\cos(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))),x)
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))), x)
```

3.572 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=405

$$\frac{(4a^2Ab-5Ab^3-2a^3B+3ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a^2-b^2)d}+\frac{(2a^2A-5Ab^2+3abB)\ \sqrt{}}{}$$

```
[Out] 1/3*(2*A*a^2-5*A*b^2+3*B*a*b)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/(a^2-b^2)/d+b
*(A*b-B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))-(4*A*
a^2*b-5*A*b^3-2*B*a^3+3*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/
d+(4*A*a^2*b-5*A*b^3-2*B*a^3+3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a^3/(a^2-b^2)/d+1/3*(2*A*a^2-5*A*b^2+3*B*a*b)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d+b*(7*A*a^2*b-5*A*b^3-5*B*a^3+3*B*
a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d
*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)/(a
+b)^2/d
```

Rubi [A]

time = 0.83, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3039, 4114, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}+\frac{(2a^2A+3abB-5Ab^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(a^2-b^2)}+\frac{(2a^2A+3abB-5Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(a^2-b^2)}-\frac{(-2a^3B+4a^2Ab+3ab^2B-5Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^3d(a^2-b^2)}+\frac{(-2a^3B+4a^2Ab+3ab^2B-5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a^2-b^2)}+\frac{b(-5a^3B+7a^2Ab+3ab^2B-5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d) + ((2*a^2*A - 5*A*b^
2 + 3*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(3*a^2*(a^2 - b^2)*d) + (b*(7*a^2*A*b - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^3*(a - b)*(a + b)^2*d) - ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*
B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((2*a^2*A - 5*A*b
^2 + 3*a*b*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b*(
A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c +
 d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
```

```
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)\,d(b+a\sec(c+dx))} - \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}b(Ab-aB)+a(A \ldots\right.}{\ldots}$$

$$= \frac{(2a^2A-5Ab^2+3abB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2(a^2-b^2)\,d} + \frac{b(Ab-aB)\sec \ldots}{a(a^2-b^2)\,d \ldots}$$

$$= -\frac{(4a^2Ab-5Ab^3-2a^3B+3ab^2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^3(a^2-b^2)\,d} + \ldots$$

$$= -\frac{(4a^2Ab-5Ab^3-2a^3B+3ab^2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^3(a^2-b^2)\,d} + \ldots$$

$$= -\frac{(4a^2Ab-5Ab^3-2a^3B+3ab^2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^3(a^2-b^2)\,d} + \ldots$$

$$= \frac{b(7a^2Ab-5Ab^3-5a^3B+3ab^2B)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx \ldots\right.}{a^3(a-b)(a+b)^2d}$$

$$= \frac{(4a^2Ab-5Ab^3-2a^3B+3ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right) \ldots}{a^3(a^2-b^2)\,d}$$

<u>Mathematica</u> [A]
time = 37.16, size = 735, normalized size = 1.81

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^2,
x]

[Out] ((2*(-4*a^4*A - 44*a^2*A*b^2 + 45*A*b^4 + 30*a^3*b*B - 27*a*b^3*B)*Cos[c +
d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcS
in[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*
Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-28*a^3*A
*b + 40*a*A*b^3 + 12*a^4*B - 24*a^2*b^2*B)*Cos[c + d*x]^2*EllipticPi[-(a/b)
, ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x
]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-12*a^
2*A*b^2 + 15*A*b^4 + 6*a^3*b*B - 9*a*b^3*B)*Cos[2*(c + d*x)]*(b + a*Sec[c +
```

```
d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*El
lipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Se
c[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt
[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d
*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2
- Sec[c + d*x]^2)))/(12*a^3*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(((-
4*a^2*A*b + 5*A*b^3 + 2*a^3*B - 3*a*b^2*B)*Sin[c + d*x])/(a^3*(a^2 - b^2))
+ (-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x])/(a^2*(a^2 - b^2)*(a + b*Co
s[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^2)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. $2(461) = 922$.

time = 1.68, size = 1004, normalized size = 2.48

method	result	size
default	Expression too large to display	1004

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*(2*A*b-B
*a)/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-2*A*b+B*a)/a^3/sin(1/2*d*x+1/2*c
)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))+2*(A*b-B*a)*b/a^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b
)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2
*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2
*A/a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
```

```
^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^2, x
)
```

3.573 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=316

$$-\frac{(2a^2A-3Ab^2+abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a^2-b^2)\,d}+\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right.}{a(a^2-b^2)\,d}$$

```
[Out] b*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))+(2*A
*a^2-3*A*b^2+B*a*b)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-(2*A*a^2-3*
A*b^2+B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d+(
A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d-(5*A*a^
2*b-3*A*b^3-3*B*a^3+B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a^2/(a-b)/(a+b)^2/d
```

Rubi [A]

time = 0.60, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3039, 4114, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}+\frac{(2a^2A+abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(a^2-b^2)}+\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad(a^2-b^2)}-\frac{(2a^2A+abB-3Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a^2-b^2)}-\frac{(-3a^3B+5a^2Ab+ab^2B-3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -(((2*a^2*A - 3*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d)) + ((A*b - a*B)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*d) - ((5*a^2*
A*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)*(a + b)^2*d) + ((2*a^
2*A - 3*A*b^2 + a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*(a^2 - b^2)*d)
 + (b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*
Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\,(a^2-b^2)\,d(b+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\,\left(-\frac{1}{2}b(Ab-a\right.}{}$$

$$= \frac{(2a^2A-3Ab^2+abB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2\,(a^2-b^2)\,d} + \frac{b(Ab-aB)\,\mathrm{se}}{a\,(a^2-b^2)}$$

$$= \frac{(2a^2A-3Ab^2+abB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2\,(a^2-b^2)\,d} + \frac{b(Ab-aB)\,\mathrm{se}}{a\,(a^2-b^2)}$$

$$= \frac{(2a^2A-3Ab^2+abB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2\,(a^2-b^2)\,d} + \frac{b(Ab-aB)\,\mathrm{se}}{a\,(a^2-b^2)}$$

$$= -\frac{(5a^2Ab-3Ab^3-3a^3B+ab^2B)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\right.}{a^2(a-b)(a+b)^2d}$$

$$= -\frac{(2a^2A-3Ab^2+abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{a^2\,(a^2-b^2)\,d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 681 vs. 2(316) = 632.
time = 36.98, size = 681, normalized size = 2.16

$$-\frac{\frac{2(10a^2Ab-9Ab^3-4a^3B+3ab^2B)\cos^2(c+dx)\left(F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}+\frac{2(4a^3A-8aAb^2+4a^2bB)\cos^2(c+dx)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)}{b(a+b\cos(c+dx))(1-\cos^2(c+dx))}+\frac{(2a^2Ab-3Ab^3+ab^2B)\cos(2(c+dx))(b+a\sec(c+dx))\left(-4ab+4ab\sec^2(c+dx)-4abE\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}+2(2a-b)bF\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}-4a^2\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}+2b^2\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}\right)\sin(c+dx)}{ab^2(a+b\cos(c+dx))(1-\cos^2(c+dx))\sqrt{\sec(c+dx)}(2-\sec^2(c+dx))}}{4a^2(a-b)(a+b)d}+\frac{\sqrt{\sec(c+dx)}\left(\frac{(2a^2A-3Ab^2+abB)\sin(c+dx)}{a^2(a^2-b^2)}+\frac{Ab^2\sin(c+dx)-abB\sin(c+dx)}{a(a^2-b^2)(a+b\cos(c+dx))}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] -1/4*((2*(10*a^2*A*b - 9*A*b^3 - 4*a^3*B + 3*a*b^2*B)*Cos[c + d*x]^2*(Ellip
ticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c
 + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])
/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(4*a^3*A - 8*a*A*b^2 +
4*a^2*b*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1
]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos
[c + d*x])*(1 - Cos[c + d*x]^2)) + ((2*a^2*A*b - 3*A*b^3 + a*b^2*B)*Cos[2*(
c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*Ellip
ticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*
x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*Ellipt
icPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Se
c[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^
2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(a^2*(a - b)*(a + b)*d) + (Sqr
t[Sec[c + d*x]]*(((2*a^2*A - 3*A*b^2 + a*b*B)*Sin[c + d*x])/(a^2*(a^2 - b^2
)) + (A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x])/(a*(a^2 - b^2)*(a + b*Cos[c
+ d*x]))))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 855 vs. 2(378) = 756.
time = 0.91, size = 856, normalized size = 2.71

method	result	size
default	Expression too large to display	856

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^2/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b+B*a)/a*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
```

```
2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3
*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*
b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2)))+2*A/a^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^2, x
)
```

3.574 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=260

$$-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a^2-b^2)\,d}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b(a^2-b^2)\,d}$$

```
[Out] b*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))-(A*b
-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d-(A*b-B*a)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d+(3*A*a^2*b-A*b^3
-B*a^3-B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(
a-b)/b/(a+b)^2/d
```

Rubi [A]
time = 0.39, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3039, 4114, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd(a^2-b^2)}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad(a^2-b^2)}+\frac{\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -(((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a*(a^2 - b^2)*d)) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) + ((3*a^2*A*b - A*b^3 - a^3*
B - a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a*(a - b)*b*(a + b)^2*d) + (b*(A*b - a*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}b(Ab-aB)+a(Ab-aB)\sec}{\sqrt{\sec(c}}}{}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}b^2(Ab-aB)+\frac{1}{2}ab(Ab-aB}{\sqrt{\sec(c+dx}}}{ab^2\left(a^2-b^2\right.}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c+}}}{2a\left(a^2-b^2\right)}$$

$$= \frac{\left(3a^2Ab-Ab^3-a^3B-ab^2B\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\right|2}{a(a-b)b(a+b)^2d}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d} - \frac{(A}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 639 vs. 2(260) = 520.
time = 36.89, size = 639, normalized size = 2.46

$$\frac{2(-4a^2A+3Ab^2+abB)\cos^2(c+dx)\left(F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))} + \frac{2(4aAb-4a^2B)\cos^2(c+dx)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)}{b(a+b\cos(c+dx))(1-\cos^2(c+dx))} + [illegible]$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] ((2*(-4*a^2*A + 3*A*b^2 + a*b*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[
c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b +
a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*
x])*(1 - Cos[c + d*x]^2)) + (2*(4*a*A*b - 4*a^2*B)*Cos[c + d*x]^2*EllipticP
```

```
i[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec
[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) +
((A*b^2 - a*b*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[
c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi
[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
+ d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c +
d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*a*(
-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-(((-(A*b) + a*B)*Sin[c + d*x])/(
a*(a^2 - b^2))) + (-(A*b*Sin[c + d*x]) + a*B*Sin[c + d*x])/((a^2 - b^2)*(a
+ b*Cos[c + d*x]))))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(324) = 648$.

time = 0.72, size = 721, normalized size = 2.77

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/(-2*a*b+2*
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+2*(A*b-B*a)/b*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b
+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-
b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
```

```
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2, x
)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{(a + b\cos(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**2, x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+b\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^2, x
)
```

3.575 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=258

$$\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b\left(a^2-b^2\right)d}+\frac{\left(aAb+a^2B-2b^2B\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right.}{b^2\left(a^2-b^2\right)d}$$

```
[Out] -(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))+(A*b-B*
a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d+(A*a*b+B*a^2-
2*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-(A*a^
2*b+A*b^3+B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/(a-b)/b^2/(a+b)^2/d
```

Rubi [A]
time = 0.38, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3039, 4112, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(a^2B+aAb-2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(b*(a^2 - b^2)*d) + ((a*A*b + a^2*B - 2*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) - ((a^2*A*b + A
*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^2*(a + b)^2*d) - ((A*b - a*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4112

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-d)*(A
*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) +
 d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{\sqrt{\sec(c+dx)}\,(B+A\sec(c+dx))}{(b+a\sec(c+dx))^2}\,dx$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(b+a\sec(c+dx))}+\frac{\int \frac{\frac{1}{2}(Ab-aB)+(aA-bB)\sec(c}{\sqrt{\sec(c+dx}}}{a^2}$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(b+a\sec(c+dx))}+\frac{\int \frac{\frac{1}{2}b(Ab-aB)-(\frac{1}{2}a(Ab-aB)}{\sqrt{\sec(c}}}{b^2\,(a^2}$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(b+a\sec(c+dx))}+\frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c}}}{2b\,(a^2-b^2)}$$

$$=-\frac{(a^2Ab+Ab^3+a^3B-3ab^2B)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\right|}{(a-b)b^2(a+b)^2d}$$

$$=\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b\,(a^2-b^2)\,d}+\frac{(aA}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 626 vs. 2(258) = 516.
time = 36.87, size = 626, normalized size = 2.43

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] ((2*(-(A*b) + a*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1
] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x]
)*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c
 + d*x]^2)) + (2*(4*a*A - 4*b*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[S
```

```
qrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c
 + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((A*b - a*B)*Cos[2
*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*Ell
ipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[
c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*Elli
pticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x
]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*(a + b)*d) + (Sqr
t[Sec[c + d*x]]*(((A*b - a*B)*Sin[c + d*x])/(b*(-a^2 + b^2)) + (-(a*A*b*Sin
[c + d*x]) + a^2*B*Sin[c + d*x])/(b*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(322) = 644$.

time = 0.75, size = 808, normalized size = 3.13

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{E}}{b^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{d}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a*(A*b-B*a)/b^2*(-b^2/a/(a^2-
b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*
b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
```

```
x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b
^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{(c + dx)}}{(a + b\cos{(c + dx)})^2 \sqrt{\sec{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))**2*sqrt(sec(c + d*x))),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2), x
)
```

3.576 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=284

$$-\frac{(aAb-3a^2B+2b^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a^2-b^2)\,d}+\frac{(a^2Ab-2Ab^3-3a^3B+4ab^2B)\sqrt{c}}{b^3}$$

```
[Out] a*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b+a*sec(d*x+c))-(A*a
*b-3*B*a^2+2*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b
^2)/d+(A*a^2*b-2*A*b^3-3*B*a^3+4*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/b^3/(a^2-b^2)/d-a*(A*a^2*b-3*A*b^3-3*B*a^3+5*B*a*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a
+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^3/(a+b)^2/d
```

Rubi [A]
time = 0.42, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3039, 4115, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{(-3a^2B+aAb+2b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a^2-b^2)}+\frac{(-3a^3B+a^2Ab+4ab^2B-2Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a^2-b^2)}-\frac{a(-3a^3B+a^2Ab+5ab^2B-3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -(((a*A*b - 3*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d)) + ((a^2*A*b - 2*A*b^3 - 3*a^3*B +
 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(b^3*(a^2 - b^2)*d) - (a*(a^2*A*b - 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*Sqrt[C
os[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
((a - b)*b^3*(a + b)^2*d) + (a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(b*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(-aAb+3a^2B-2b^2B\right)-b(Ab \ldots}{\sqrt{\sec(c+ \ldots}}}{b \ldots}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}b\left(-aAb+3a^2B-2b^2B\right)-\left(b^2\ldots\right.}{\sqrt{\ldots}}}{\ldots}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\left(aAb-3a^2B+2b^2B\right)\int \ldots}{2b^2\left(a^2-\ldots\right.}$$

$$= -\frac{a\left(a^2Ab-3Ab^3-3a^3B+5ab^2B\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\ldots\right.}{(a-b)b^3(a+b)^2d}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+\ldots}}{b^2\left(a^2-b^2\right)d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 655 vs. 2(284) = 568.
time = 36.89, size = 655, normalized size = 2.31

$$\frac{\frac{2\left(-aAb-a^2B+2b^2B\right)\cos^2(c+dx)\left(F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)}{a(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)}+\frac{2\left(4Ab^2-4abB\right)\cos^2(c+dx)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)}{b(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)}+\ldots[\text{illegible}]}{4b(-a+b)(a+b)d}+\ldots[\text{illegible}]$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] ((2*(-(a*A*b) - a^2*B + 2*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[
c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b +
a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*
x])*(1 - Cos[c + d*x]^2)) + (2*(4*A*b^2 - 4*a*b*B)*Cos[c + d*x]^2*EllipticP
```

```
i[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec
[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) +
((a*A*b - 3*a^2*B + 2*b^2*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b
+ 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
rt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^
2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a
+ b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]
^2)))/(4*b*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-((a*(-(A*b) + a*B)*S
in[c + d*x])/(b^2*(a^2 - b^2))) + (a^2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x
])/(b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(348) = 696$.
time = 0.89, size = 849, normalized size = 2.99

method	result	size
default	Expression too large to display	849

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2
*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*b*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))+4*a/b^2*(2*A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(A*b-
B*a)/b^3*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*
b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
```

```
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2),x)
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2), x
)
```

3.577 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=363

$$\frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3(a^2-b^2)d}-\frac{(9a^3Ab-12aAb^3-15a^4B$$

```
[Out] -1/3*(3*A*a*b-5*B*a^2+2*B*b^2)*sin(d*x+c)/b^2/(a^2-b^2)/d/sec(d*x+c)^(1/2)+
a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(b+a*sec(d*x+c))/sec(d*x+c)^(1/2)+(3*A
*a^2*b-2*A*b^3-5*B*a^3+4*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/b^3/(a^2-b^2)/d-1/3*(9*A*a^3*b-12*A*a*b^3-15*B*a^4+16*B*a^2*b^2+2*B*b^4
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^4/(a^2-b^2)/d+a^2*(3*A*a^2
*b-5*A*b^3-5*B*a^3+7*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/(a-b)/b^4/(a+b)^2/d
```

Rubi [A]
time = 0.60, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3039, 4115, 4189, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+b)}-\frac{(-5a^2B+3aAb+2b^2B)\sin(c+dx)}{3b^2d(a^2-b^2)\sqrt{\sec(c+dx)}}+\frac{(-5a^3B+3a^2Ab+4ab^2B-2Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a^2-b^2)}+\frac{a^2(-5a^3B+3a^2Ab+7ab^2B-5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^4d(a-b)(a+b)^2}-\frac{(-15a^4B+9a^3Ab+16a^2b^2B-12aAb^3+2b^4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d) - ((9*a^3*A*b - 12*a
*A*b^3 - 15*a^4*B + 16*a^2*b^2*B + 2*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^4*(a^2 - b^2)*d) + (a^2*(3*a^2*A*b -
 5*A*b^3 - 5*a^3*B + 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^4*(a + b)^2*d) - ((3*a*A*b
 - 5*a^2*B + 2*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]
) + (a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(b + a
*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
```

```
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))} + \frac{\int \frac{1}{2}(-3aAb+5a^2B- \cdots}{\cdots}$$

$$= -\frac{(3aAb-5a^2B+2b^2B)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{a(Ab-aB)\,\mathrm{si}\cdots}{b\left(a^2-b^2\right)d\sqrt{\sec(c+dx}\cdots}$$

$$= -\frac{(3aAb-5a^2B+2b^2B)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{a(Ab-aB)\,\mathrm{si}\cdots}{b\left(a^2-b^2\right)d\sqrt{\sec(c+dx}\cdots}$$

$$= -\frac{(3aAb-5a^2B+2b^2B)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{a(Ab-aB)\,\mathrm{si}\cdots}{b\left(a^2-b^2\right)d\sqrt{\sec(c+dx}\cdots}$$

$$= \frac{a^2(3a^2Ab-5Ab^3-5a^3B+7ab^2B)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+\cdots\right.}{(a-b)b^4(a+b)^2d}$$

$$= \frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\cdots}{b^3\left(a^2-b^2\right)d}$$

Mathematica [A]
time = 37.11, size = 701, normalized size = 1.93

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] -1/12*((2*(-3*a^2*A*b + 6*A*b^3 + 5*a^3*B - 8*a*b^2*B)*Cos[c + d*x]^2*(Elli
pticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[
c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]
)/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-12*a*A*b^2 + 8*a^2*b
*B + 4*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*
Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-9*a^2*A*b + 6*A*b^3 + 15*a^3*B - 1
2*a*b^2*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*
x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*S
qrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]
```

```
], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b
), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]
^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
 d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*
(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/((a - b)*b^2
*(a + b)*d) + (Sqrt[Sec[c + d*x]]*((a^2*(-(A*b) + a*B)*Sin[c + d*x])/(b^3*(
a^2 - b^2)) - (a^3*A*b*Sin[c + d*x] - a^4*B*Sin[c + d*x])/(b^3*(-a^2 + b^2)
*(a + b*Cos[c + d*x])) + (B*Sin[2*(c + d*x)])/(3*b^2)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. $2(421) = 842$.
time = 1.03, size = 1066, normalized size = 2.94

method	result	size
default	Expression too large to display	1066

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^4/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^4*b^2+6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^2+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2-9*B*a^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b)-4*a^2/b^3*(3*A*b-4*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b-
B*a)/b^4*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*
b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
```

```
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
 x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+b\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2), x
)
```

3.578 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=480

$$-\frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3\left(a^2-b^2\right)^2d}+\frac{(11a^2Ab-$$

```
[Out] 1/2*b*(A*b-B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^
2+1/4*b*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^
2/(a^2-b^2)^2/d/(b+a*sec(d*x+c))+1/4*(8*A*a^4-29*A*a^2*b^2+15*A*b^4+9*B*a^3
*b-3*B*a*b^3)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)^2/d-1/4*(8*A*a^4-29
*A*a^2*b^2+15*A*b^4+9*B*a^3*b-3*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/a^3/(a^2-b^2)^2/d+1/4*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a*b^2)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d-1/4*(35*A*a^4*b-3
8*A*a^2*b^3+15*A*b^5-15*B*a^5+6*B*a^3*b^2-3*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)^2/(a+b)^3/d
```

Rubi [A]
time = 0.92, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3039, 4114, 4183, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b(-7a^3B+11a^2Ab+ab^2B-5Ab^3)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{(-7a^3B+11a^2Ab+ab^2B-5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^3d\left(a^2-b^2\right)^2}-\frac{(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{(-15a^5B+35a^4Ab+6a^3b^2B-38a^2Ab^3-3ab^4B+15Ab^5)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -1/4*((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sqrt[Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)^2*
d) + ((11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) - ((35*a^4*A
*b - 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos
[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4
*a^3*(a - b)^2*(a + b)^3*d) + ((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b
*B - 3*a*b^3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2*d) +
(b*(A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*S
ec[c + d*x])^2) + (b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
```

```
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}b(Ab-aB)+2a(\ldots\right.}{\ldots}$$

$$= \frac{b(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b(11a^2Ab-5Ab^3-7a^3B \ldots}{4a^2\left(a^2-b^2\right)^2 \ldots}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\sec(c+dx)}\,\sin(c\ldots}{4a^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\sec(c+dx)}\,\sin(c\ldots}{4a^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\sec(c+dx)}\,\sin(c\ldots}{4a^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(35a^4Ab-38a^2Ab^3+15Ab^5-15a^5B+6a^3b^2B-3ab^4B\right)\sqrt{\cos\ldots}}{4a^3(a-b)^2(a+b)^3\ldots}$$

$$= -\frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\cos(c+dx)}\,E(\ldots}{4a^3\left(a^2-b^2\right)^2 d}$$

Mathematica [A]
time = 37.36, size = 844, normalized size = 1.76

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] -1/16*((2*(56*a^4*A*b - 95*a^2*A*b^3 + 45*A*b^5 - 16*a^5*B + 19*a^3*b^2*B -
 9*a*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - Ell
ipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[
1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]
^2)) + (2*(16*a^5*A - 80*a^3*A*b^2 + 40*a*A*b^4 + 32*a^4*b*B - 8*a^2*b^3*B)
*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*S
ec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])
*(1 - Cos[c + d*x]^2)) + ((8*a^4*A*b - 29*a^2*A*b^3 + 15*A*b^5 + 9*a^3*b^2*
```

```
B - 3*a*b^4*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c
+ d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x
]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-
(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*
x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(a^3*(a
- b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((8*a^4*A - 29*a^2*A*b^2 + 15*A*
b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2) + (A*b^2*S
in[c + d*x] - a*b*B*Sin[c + d*x])/(2*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2)
+ (11*a^2*A*b^2*Sin[c + d*x] - 5*A*b^4*Sin[c + d*x] - 7*a^3*b*B*Sin[c + d*x
] + a*b^3*B*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1974 vs. $2(528) = 1056$.
time = 1.84, size = 1975, normalized size = 4.11

method	result	size
default	Expression too large to display	1975

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^3/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))-2*A*b/a^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^
2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a
^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/
(a-b),2^(1/2)))+2*A/a^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
```

```
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(-A*b+B*a)/a*(-1/2*b^2/a/(a^2
-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^
2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^
2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^3, x
)
```

3.579 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=405

$$-\frac{(9a^2Ab-3Ab^3-5a^3B-ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2(a^2-b^2)^2d}-\frac{(7a^2Ab-Ab^3-3a^3B-3}{}$$

```
[Out] 1/2*b*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^
2+1/4*b*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2
/(a^2-b^2)^2/d/(b+a*sec(d*x+c))-1/4*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d-1/4*(7*A*a^2*b-
A*b^3-3*B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/b/(
a^2-b^2)^2/d+1/4*(15*A*a^4*b-6*A*a^2*b^3+3*A*b^5-3*B*a^5-10*B*a^3*b^2+B*a*b
^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a-b)^2/b/(
a+b)^3/d
```

Rubi [A]

time = 0.67, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3039, 4114, 4183, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad(a^2-b^2)(a\sec(c+dx)+b)^2}+\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d(a^2-b^2)^2(a\sec(c+dx)+b)}-\frac{(-3a^3B+7a^2Ab-3ab^2B-Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd(a^2-b^2)^2}-\frac{(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d(a^2-b^2)^2}+\frac{(-3a^5B+15a^4Ab-10a^3b^2B-6a^2Ab^3+ab^4B+3Ab^5)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2bd(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -1/4*((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)^2*d) - ((7*a^2*A*b -
 A*b^3 - 3*a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(4*a*b*(a^2 - b^2)^2*d) + ((15*a^4*A*b - 6*a^2*A*b^3 +
3*A*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(
2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*b*(a + b
)^3*d) + (b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d
*(b + a*Sec[c + d*x])^2) + (b*(9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
```

```
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}b(A\right.}{}$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b(9a^2Ab-3Ab^3-5a^3B}{4a^2\left(a^2-\right.}$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b(9a^2Ab-3Ab^3-5a^3B}{4a^2\left(a^2-\right.}$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b(9a^2Ab-3Ab^3-5a^3B}{4a^2\left(a^2-\right.}$$

$$= \frac{(15a^4Ab-6a^2Ab^3+3Ab^5-3a^5B-10a^3b^2B+ab^4B)\sqrt{\cos(c}}{4a^2(a-b)^2b(a+b)^3}$$

$$= -\frac{(9a^2Ab-3Ab^3-5a^3B-ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right|}{4a^2\left(a^2-b^2\right)^2d}$$

Mathematica [A]
time = 37.08, size = 797, normalized size = 1.97

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] ((2*(16*a^4*A - 19*a^2*A*b^2 + 9*A*b^4 - 9*a^3*b*B + 3*a*b^3*B)*Cos[c + d*x
]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[
Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin
[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-32*a^3*A*b
+ 8*a*A*b^3 + 16*a^4*B + 8*a^2*b^2*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*
Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-9*a^2*A*b^
2 + 3*A*b^4 + 5*a^3*b*B + a*b^3*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-
4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
```

```
- 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x
]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b
^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c
+ d*x]^2)))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(-1/4*((-9
*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Sin[c + d*x])/(a^2*(a^2 - b^2)^2) +
 (-(A*b*Sin[c + d*x]) + a*B*Sin[c + d*x])/(2*(a^2 - b^2)*(a + b*Cos[c + d*x
])^2) + (-7*a^2*A*b*Sin[c + d*x] + A*b^3*Sin[c + d*x] + 3*a^3*B*Sin[c + d*x
] + 3*a*b^2*B*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1743 vs. $2(457) = 914$.
time = 1.51, size = 1744, normalized size = 4.31

method	result	size
default	Expression too large to display	1744

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b*(-b^2/a/(
a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a
^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*(A*b-B*a)/b*(-1/2*b^2/a/(a^2-
b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b
+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
```

```
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2
)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2
)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{(a + b\cos(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**3, x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^3, x
)
```

3.580 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=402

$$\frac{(5a^2Ab+Ab^3-a^3B-5ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4ab\,(a^2-b^2)^2\,d}+\frac{(3a^2Ab+3Ab^3+a^3B-7a$$

```
[Out] 1/2*b*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^
2-1/4*(7*A*a^2*b-A*b^3-3*B*a^3-3*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^
2-b^2)^2/d/(b+a*sec(d*x+c))+1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/b/(a^2-b^2)^2/d+1/4*(3*A*a^2*b+3*A*b^
3+B*a^3-7*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^
2)^2/d-1/4*(3*A*a^4*b+10*A*a^2*b^3-A*b^5+B*a^5-10*B*a^3*b^2-3*B*a*b^4)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2
*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a-b)^2/b^2/(a+b)^3/d
```

Rubi [A]
time = 0.67, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3039, 4114, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\,(a^2-b^2)\,(a\sec(c+dx)+b)^2}-\frac{(-3a^3B+7a^2Ab-3ab^2B-Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad\,(a^2-b^2)^2\,(a\sec(c+dx)+b)}+\frac{(a^3B+3a^2Ab-7ab^2B+3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\,(a^2-b^2)^2}+\frac{(a^3(-B)+5a^2Ab-5ab^2B+Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\,(a^2-b^2)^2}-\frac{(a^5B+3a^4Ab-10a^3b^2B+10a^2Ab^3-3ab^4B-Ab^5)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4ab^2d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b*(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*A*
b^3 + a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) - ((3*a^4*A*b + 10*a^2*A*b^3 - A*b^5
 + a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^2*b^2*(a + b)^3*d)
+ (b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a
*Sec[c + d*x])^2) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
```

```
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(b+a\sec(c+dx))^2} - \frac{\int \frac{\frac{1}{2}b(Ab-aB)+2a(Ab-aB)\sec}{\sqrt{\sec(c}}}{}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(b+a\sec(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B}{4a\,(a^2-b}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(b+a\sec(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B}{4a\,(a^2-b}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(b+a\sec(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B}{4a\,(a^2-b}$$

$$= -\frac{(3a^4Ab+10a^2Ab^3-Ab^5+a^5B-10a^3b^2B-3ab^4B)\sqrt{\cos(c}}{4a(a-b)^2b^2(a+b)^3a}$$

$$= \frac{(5a^2Ab+Ab^3-a^3B-5ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{4ab\,(a^2-b^2)^2\,d}$$

Mathematica [A]
time = 37.03, size = 784, normalized size = 1.95

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] ((2*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Cos[c + d*x]^2*(EllipticF[Ar
cSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]
]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a
+ b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16*a^3*A + 8*a*A*b^2 - 24*a^2
*b*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b
+ a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c +
d*x])*(1 - Cos[c + d*x]^2)) + ((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*Cos[
2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*El
lipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
 d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
```

```
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[
Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*Ell
ipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
 Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*
x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(16*a*(a - b)^2*(a + b)^2*d
) + (Sqrt[Sec[c + d*x]]*(((-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Sin[c +
d*x])/(4*a*b*(a^2 - b^2)^2) - (a*A*b*Sin[c + d*x] - a^2*B*Sin[c + d*x])/(2*
b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (3*a^2*A*b*Sin[c + d*x] + 3*A*b^3*
Sin[c + d*x] + a^3*B*Sin[c + d*x] - 7*a*b^2*B*Sin[c + d*x])/(4*b*(-a^2 + b^
2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1849 vs. $2(454) = 908$.
time = 1.52, size = 1850, normalized size = 4.60

method	result	size
default	Expression too large to display	1850

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/b/(-2*a*b+
2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))+2*(A*b-2*B*a)/b^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a
/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2
*b/(a-b),2^(1/2)))-2*a*(A*b-B*a)/b^2*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-
b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
```

```
os(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/
a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^
2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b
/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/
(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3), x
)
```

3.581 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=400

$$-\frac{(a^2Ab+5Ab^3+3a^3B-9ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2(a^2-b^2)^2d}+\frac{(a^3Ab-7aAb^3+3a^4B-$$

```
[Out] -1/2*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+1
/4*(3*A*a^2*b+3*A*b^3+B*a^3-7*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b
^2)^2/d/(b+a*sec(d*x+c))-1/4*(A*a^2*b+5*A*b^3+3*B*a^3-9*B*a*b^2)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d+1/4*(A*a^3*b-7*A*a*b^3
+3*B*a^4-5*B*a^2*b^2+8*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
b^3/(a^2-b^2)^2/d-1/4*(A*a^4*b-10*A*a^2*b^3-3*A*b^5+3*B*a^5-6*B*a^3*b^2+15*
B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2
*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^
3/(a+b)^3/d
```

Rubi [A]
time = 0.62, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3039, 4112, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a\sec(c+dx)+b)^2}+\frac{(a^3B+3a^2Ab-7ab^2B+3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd(a^2-b^2)^2(a\sec(c+dx)+b)}-\frac{(3a^3B+a^2Ab-9ab^2B+5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d(a^2-b^2)^2}+\frac{(3a^4B+a^3Ab-5a^2b^2B-7aAb^3+8b^4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d(a^2-b^2)^2}-\frac{(3a^5B+a^4Ab-6a^3b^2B-10a^2Ab^3+15ab^4B-3Ab^5)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -1/4*((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)^2*d) + ((a^3*A*b - 7
*a*A*b^3 + 3*a^4*B - 5*a^2*b^2*B + 8*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d) - ((a^4*A*b - 10*
a^2*A*b^3 - 3*A*b^5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^
2*b^3*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2
- b^2)*d*(b + a*Sec[c + d*x])^2) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*
B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x
]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4112

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-d)*(A
*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) +
```

```
 d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{\sec(c+dx)}\,(B+A\sec(c+dx))}{(b+a\sec(c+dx))^3}\,dx$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}+\frac{\int \frac{\frac{1}{2}(Ab-aB)+2(aA-bB)\sec(}{\sqrt{\sec(c+dx}}}{2\,(a}$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}+\frac{(3a^2Ab+3Ab^3+a^3B}{4b\left(a^2-b\right.}$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}+\frac{(3a^2Ab+3Ab^3+a^3B}{4b\left(a^2-b\right.}$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}+\frac{(3a^2Ab+3Ab^3+a^3B}{4b\left(a^2-b\right.}$$

$$=-\frac{(a^4Ab-10a^2Ab^3-3Ab^5+3a^5B-6a^3b^2B+15ab^4B)\sqrt{\cos(c}}{4(a-b)^2b^3(a+b)^3d}$$

$$=-\frac{(a^2Ab+5Ab^3+3a^3B-9ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right|2)}{4b^2\left(a^2-b^2\right)^2d}$$

Mathematica [A]
time = 37.03, size = 786, normalized size = 1.96

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] -1/16*((2*(-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[c + d*x]^2*(Elliptic
F[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c +
d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a
*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(24*a*A*b^2 - 8*a^2*b*B -
16*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]
*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[
c + d*x])*(1 - Cos[c + d*x]^2)) + ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B
)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*
a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - S
ec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
```

```
rt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b
^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sq
rt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[
c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/((a - b)^2*b*(a + b)^
2*d) + (Sqrt[Sec[c + d*x]]*(((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sin[
c + d*x])/(4*b^2*(a^2 - b^2)^2) - (-(a^2*A*b*Sin[c + d*x]) + a^3*B*Sin[c +
d*x])/(2*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (a^3*A*b*Sin[c + d*x] -
 7*a*A*b^3*Sin[c + d*x] - 5*a^4*B*Sin[c + d*x] + 11*a^2*b^2*B*Sin[c + d*x])
/(4*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1936 vs. $2(452) = 904$.
time = 1.55, size = 1937, normalized size = 4.84

method	result	size
default	Expression too large to display	1937

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
^2*(A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a/b^3*(2*A*b-3*B*a)*(-b^2/a
/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/
(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*a^2*(A*b-B*a)/b^3*(-1/2*b^2
/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2
)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
```

```
/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/
(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)
*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3), x
)
```

3.582 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=427

$$-\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^3\left(a^2-b^2\right)^2d}+\frac{(3a^4Ab-5}{}$$

```
[Out] 1/2*a*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b+a*sec(d*x+c))^
2+1/4*a*(A*a^2*b-7*A*b^3-5*B*a^3+11*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^
2/(a^2-b^2)^2/d/(b+a*sec(d*x+c))-1/4*(3*A*a^3*b-9*A*a*b^3-15*B*a^4+29*B*a^2
*b^2-8*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/
d+1/4*(3*A*a^4*b-5*A*a^2*b^3+8*A*b^5-15*B*a^5+33*B*a^3*b^2-24*B*a*b^4)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^4/(a^2-b^2)^2/d-1/4*a*(3*A*a^4*b
-6*A*a^2*b^3+15*A*b^5-15*B*a^5+38*B*a^3*b^2-35*B*a*b^4)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^4/(a+b)^3/d
```

Rubi [A]

time = 0.67, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3039, 4115, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{a(-5a^3B+a^2Ab+11ab^2B-7Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}+\frac{(-15a^5B+3a^4Ab+33a^3b^2B-5a^2Ab^3-24ab^4B+8Ab^5)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{a(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

```
[Out] -1/4*((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*Sqrt[Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)^2*
d) + ((3*a^4*A*b - 5*a^2*A*b^3 + 8*A*b^5 - 15*a^5*B + 33*a^3*b^2*B - 24*a*b
^4*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b
^4*(a^2 - b^2)^2*d) - (a*(3*a^4*A*b - 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 3
8*a^3*b^2*B - 35*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^4*(a + b)^3*d) + (a*(A*b - a
*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(b + a*Sec[c + d*x]
)^2) + (a*(a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
```

```
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(b+a\sec(c+dx))^2} + \frac{\int \frac{\frac{1}{2}(-aAb+5a^2B-4b^2B)-2b(A}{\sqrt{\sec(c+}}}{2}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(b+a\sec(c+dx))^2} + \frac{a(a^2Ab-7Ab^3-5a^3B-}{4b^2\,(a^2-b}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(b+a\sec(c+dx))^2} + \frac{a(a^2Ab-7Ab^3-5a^3B-}{4b^2\,(a^2-b}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(b+a\sec(c+dx))^2} + \frac{a(a^2Ab-7Ab^3-5a^3B-}{4b^2\,(a^2-b}$$

$$= -\frac{a(3a^4Ab-6a^2Ab^3+15Ab^5-15a^5B+38a^3b^2B-35ab^4B)\sqrt{\text{co}}}{4(a-b)^2b^4(a+b)^3}$$

$$= -\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sqrt{\cos(c+dx)}\,E(}{4b^3\,(a^2-b^2)^2\,d}$$

Mathematica [A]
time = 37.16, size = 820, normalized size = 1.92

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] ((2*(-(a^3*A*b) - 5*a*A*b^3 + 5*a^4*B - 7*a^2*b^2*B + 8*b^4*B)*Cos[c + d*x]
^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[S
qrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[
c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(8*a^2*A*b^2 +
 16*A*b^4 + 8*a^3*b*B - 32*a*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSi
n[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Si
n[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-3*a^3*A*b +
9*a*A*b^3 + 15*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*Cos[2*(c + d*x)]*(b + a*Sec[
c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b
*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[
```

```
c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt
[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c
+ d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]
*(2 - Sec[c + d*x]^2)))/(16*(a - b)^2*b^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]
]*(-1/4*(a*(-3*a^2*A*b + 9*A*b^3 + 7*a^3*B - 13*a*b^2*B)*Sin[c + d*x])/(b^3
*(a^2 - b^2)^2) - (a^3*A*b*Sin[c + d*x] - a^4*B*Sin[c + d*x])/(2*b^3*(-a^2
+ b^2)*(a + b*Cos[c + d*x])^2) + (-5*a^4*A*b*Sin[c + d*x] + 11*a^2*A*b^3*Si
n[c + d*x] + 9*a^5*B*Sin[c + d*x] - 15*a^3*b^2*B*Sin[c + d*x])/(4*b^3*(-a^2
 + b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1976 vs. $2(479) = 958$.
time = 1.66, size = 1977, normalized size = 4.63

method	result	size
default	Expression too large to display	1977

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^4/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3
*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*b*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))+12/b^3*a*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2/b^4*(3
*A*b-4*B*a)*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/
(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2*a^3*(A*b
-B*a)/b^4*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^
```

```
2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^
2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*
b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)),
 x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3), x
)
```

3.583 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=521

$$\frac{\left(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^4\left(a^2-b^2\right)^2d}-$$

```
[Out] -1/12*(15*A*a^3*b-33*A*a*b^3-35*B*a^4+61*B*a^2*b^2-8*B*b^4)*sin(d*x+c)/b^3/
(a^2-b^2)^2/d/sec(d*x+c)^(1/2)+1/2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(b+
a*sec(d*x+c))^2/sec(d*x+c)^(1/2)+1/4*a*(3*A*a^2*b-9*A*b^3-7*B*a^3+13*B*a*b^
2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(b+a*sec(d*x+c))/sec(d*x+c)^(1/2)+1/4*(15*A
*a^4*b-29*A*a^2*b^3+8*A*b^5-35*B*a^5+65*B*a^3*b^2-24*B*a*b^4)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^4/(a^2-b^2)^2/d-1/12*(45*A*a^5*b-99*A*a^3
*b^3+72*A*a*b^5-105*B*a^6+223*B*a^4*b^2-128*B*a^2*b^4-8*B*b^6)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^5/(a^2-b^2)^2/d+1/4*a^2*(15*A*a^4*b-38*A
*a^2*b^3+35*A*b^5-35*B*a^5+86*B*a^3*b^2-63*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^5/(a+b)^3/d
```

Rubi [A]
time = 0.96, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3039, 4115, 4185, 4189, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+b)^2}+\frac{a\left(-7a^3B+3a^2Ab+13ab^2B-9Ab^3\right)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+b)}-\frac{\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{12b^3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}+\frac{\left(-35a^5B+15a^4Ab+65a^3b^2B-29a^2Ab^3-24ab^4B+8Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}+\frac{a^2\left(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^5d(a-b)^2(a+b)^3}-\frac{\left(-105a^6B+45a^5Ab+223a^4b^2B-99a^3Ab^3-128a^2b^4B+72aAb^5-8b^6B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12b^5d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

```
[Out] ((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4*
B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^4*
(a^2 - b^2)^2*d) - ((45*a^5*A*b - 99*a^3*A*b^3 + 72*a*A*b^5 - 105*a^6*B + 2
23*a^4*b^2*B - 128*a^2*b^4*B - 8*b^6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*b^5*(a^2 - b^2)^2*d) + (a^2*(15*a^4*A*b -
 38*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B + 86*a^3*b^2*B - 63*a*b^4*B)*Sqrt[Cos[c
 + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(
a - b)^2*b^5*(a + b)^3*d) - ((15*a^3*A*b - 33*a*A*b^3 - 35*a^4*B + 61*a^2*b
^2*B - 8*b^4*B)*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]) +
 (a*(A*b - a*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(b + a*
Sec[c + d*x])^2) + (a*(3*a^2*A*b - 9*A*b^3 - 7*a^3*B + 13*a*b^2*B)*Sin[c +
d*x])/(4*b^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^3}\,dx$$

$$=\frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2}+\frac{\int \frac{\frac{1}{2}(-3aAb+7a^2 \cdots}{\cdots}}{\cdots}$$

$$=\frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2}+\frac{a(3a^2Ab-9 \cdots}{4b^2\left(a^2-b^2\right)^2 \cdots}$$

$$=-\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}}+\cdots$$

$$=-\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}}+\cdots$$

$$=-\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}}+\cdots$$

$$=\frac{a^2\left(15a^4Ab-38a^2Ab^3+35Ab^5-35a^5B+86a^3b^2B-63ab^4B\right)\sqrt{\cdots}}{4(a-b)^2b^5(a+b)\cdots}$$

$$=\frac{\left(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B\right)\sqrt{\cos\cdots}}{4b^4\left(a^2-b^2\right)^2d}\cdots$$

Mathematica [A]
time = 37.39, size = 865, normalized size = 1.66

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),
x]

[Out] -1/48*((2*(-15*a^4*A*b + 21*a^2*A*b^3 - 24*A*b^5 + 35*a^5*B - 73*a^3*b^2*B
+ 56*a*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - E
llipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqr
t[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*
x]^2)) + (2*(-24*a^3*A*b^2 + 96*a*A*b^4 + 56*a^4*b*B - 112*a^2*b^3*B - 16*b
^5*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b
+ a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c +
```

```
d*x])*(1 - Cos[c + d*x]^2)) + ((-45*a^4*A*b + 87*a^2*A*b^3 - 24*A*b^5 + 105
*a^5*B - 195*a^3*b^2*B + 72*a*b^4*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*
(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF
[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a
*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[
c + d*x]^2)))/((a - b)^2*b^3*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*((a^2*(-7*a
^2*A*b + 13*A*b^3 + 11*a^3*B - 17*a*b^2*B)*Sin[c + d*x])/(4*b^4*(a^2 - b^2)
^2) - (-(a^4*A*b*Sin[c + d*x]) + a^5*B*Sin[c + d*x])/(2*b^4*(-a^2 + b^2)*(a
 + b*Cos[c + d*x])^2) + (9*a^5*A*b*Sin[c + d*x] - 15*a^3*A*b^3*Sin[c + d*x]
 - 13*a^6*B*Sin[c + d*x] + 19*a^4*b^2*B*Sin[c + d*x])/(4*b^4*(-a^2 + b^2)^2
*(a + b*Cos[c + d*x])) + (B*Sin[2*(c + d*x)])/(3*b^3)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2194 vs. $2(569) = 1138$.
time = 1.91, size = 2195, normalized size = 4.21

method	result	size
default	Expression too large to display	2195

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^5/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^4*b^2+9*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^2+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2-18*B*a^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*a*b)-8*a^2/b^4*(3*A*b-5*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3/b^5*
(4*A*b-5*B*a)*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
```

```
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*
b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*a^4*(A
*b-B*a)/b^5*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*
a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^
2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2
-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-
b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2
)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)),
 x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+b\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3), x
)
```

3.584 $\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

```
[Out] 2/3*B*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 3853, 3856, 2720}

$$\frac{2B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = B\int \sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}B\int \sqrt{\sec(c+dx)}\,dx$$
$$= \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\Big(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2B\sec^{\frac{3}{2}}(c}{}$$

Mathematica [A]
time = 0.08, size = 47, normalized size = 0.73

$$\frac{2B\sec^{\frac{3}{2}}(c+dx)\left(\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*Sec[c + d*x]^(3/2)*(Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + Sin
[c + d*x]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(80) = 160.
time = 0.29, size = 214, normalized size = 3.34

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\mathrm{co}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*B*((2*cos(1/2*d*x+1/2*c)^2-
1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 91, normalized size = 1.42

$$\frac{-i\sqrt{2}\,B\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,B\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+\frac{2B\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*B*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + I*sqrt(2)*B*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 2*B*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
 c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(B\,a+B\,b\,\cos\left(c+d\,x\right))}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/cos(c + d*x))^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),
x)
```

```
[Out] int(((1/cos(c + d*x))^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),
 x)
```

3.585 $\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}$$

```
[Out] 2*B*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 3853, 3856, 2719}

$$\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(aB + bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx &= B\int \sec^{\frac{3}{2}}(c+dx)\,dx \\ &= \frac{2B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} - B\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx \\ &= \frac{2B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} - \left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right. \\ &= -\frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2B\sqrt{\sec}}{}\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 46, normalized size = 0.77

$$\frac{2B\sqrt{\sec(c+dx)}\left(-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*Sqrt[Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
Sin[c + d*x]))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(80) = 160$.
time = 0.30, size = 183, normalized size = 3.05

method	result
default	$-\frac{2B\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -2*B*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 76, normalized size = 1.27

$$\frac{-i\sqrt{2}\,B\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\,B\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2B\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] (-I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c))) + I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*B*sin(d*x + c)/sqrt(cos(d*x +
 c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int\sec^{\frac{3}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] B*Integral(sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/cos(c + d*x))^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),
x)
```

```
[Out] int(((1/cos(c + d*x))^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),
 x)
```

3.586 $\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {21, 3856, 2720}

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = B\int \sqrt{\sec(c+dx)}\,dx$$
$$= \left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A]
time = 0.04, size = 37, normalized size = 1.00

$$\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(59) = 118$.
time = 0.29, size = 134, normalized size = 3.62

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticF}(}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 53, normalized size = 1.43

$$\frac{-i\sqrt{2}\,B\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,B\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] (-I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I
*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] B*Integral(sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\sqrt{\frac{1}{\cos(c+d\,x)}}\,(B\,a+B\,b\,\cos(c+d\,x))}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/cos(c + d*x))^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),
x)
```

```
[Out] int(((1/cos(c + d*x))^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),
 x)
```

3.587 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=37

$$\frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {21, 3856, 2719}

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = B\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A]
time = 0.04, size = 37, normalized size = 1.00

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(59) = 118.
time = 0.24, size = 134, normalized size = 3.62

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticE}\left(\mathrm{co}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$
risch	$-\frac{i\sqrt{2}\,B}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}+1\right)}{\sqrt{\left(e^{2i(dx+c)}+1\right)e^{i(dx+c)}}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 59, normalized size = 1.59

$$\frac{i\sqrt{2}\,B\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\,B\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] (I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c))) - I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int\frac{1}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] B*Integral(1/sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))),
x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))),
 x)
```

3.588 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 3854, 3856, 2720}

$$\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = B\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}B\int \sqrt{\sec(c+dx)}\,dx$$
$$= \frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c}}$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2B\sin(c+d}{3d\sqrt{\sec(c+d}}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 0.78

$$\frac{B\sqrt{\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] (B*Sqrt[Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin
[2*(c + d*x)]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(80) = 160.
time = 0.30, size = 180, normalized size = 2.81

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 71, normalized size = 1.11

$$\frac{2\,B\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-i\,\sqrt{2}\,B\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+i\,\sqrt{2}\,B\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] 1/3*(2*B*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*B*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int\frac{1}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] B*Integral(sec(c + d*x)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))),
x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))),
 x)
```

3.589 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{6B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/5*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)+6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d
```

Rubi [A]

time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 3854, 3856, 2719}

$$\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = B\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3B)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(3B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c$$
$$= \frac{6B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2B\sin(c+dx}{5d\sec^{\frac{3}{2}}(c+dx}$$

Mathematica [A]
time = 0.09, size = 56, normalized size = 0.88

$$\frac{B\sqrt{\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)+\sin(3(c+dx))\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)
),x]
```

```
[Out] (B*Sqrt[Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(80) = 160$.
time = 0.34, size = 203, normalized size = 3.17

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right.}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(-8*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 77, normalized size = 1.20

$$\frac{2\,B\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+3i\,\sqrt{2}\,B\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-3i\,\sqrt{2}\,B\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] 1/5*(2*B*cos(d*x + c)^(3/2)*sin(d*x + c) + 3*I*sqrt(2)*B*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(
2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))),
x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))),
 x)
```

3.590 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=473

$$\frac{2(a-b)\sqrt{a+b}\,(19a^2Ab+8Ab^3+63a^3B-14ab^2B)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\,\sqrt{\cos(}}\right.\right.}{105a^4d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/105*(25*A*a^2-4*A*b^2+7*B*a*b)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c
))^(1/2)/a^2/d+2/35*(A*b+7*B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c)
)^(1/2)/a/d+2/7*A*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/10
5*(a-b)*(19*A*a^2*b+8*A*b^3+63*B*a^3-14*B*a*b^2)*csc(d*x+c)*EllipticE((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^
(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-
b))^(1/2)/a^4/d/sec(d*x+c)^(1/2)+2/105*(a-b)*(8*A*b^2+a^2*(25*A-63*B)+2*a*b
*(3*A-7*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x
+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3078, 3134, 3077, 2895, 3073}

$$\frac{2(25a^2A+7abB-4Ab^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d}+\frac{2(a-b)\sqrt{a+b}\,(a^2(25A-63B)+2ab(3A-7B)+8Ab^2)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{105a^3d\sqrt{\sec(c+dx)}}+\frac{2(a-b)\sqrt{a+b}\,(63a^3B+19a^2Ab-14ab^2B+8Ab^3)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{105a^4d\sqrt{\sec(c+dx)}}+\frac{2(7aB+Ab)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{35ad}+\frac{2A\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d*Sqrt[Sec[c + d*
x]]) + (2*(a - b)*Sqrt[a + b]*(8*A*b^2 + a^2*(25*A - 63*B) + 2*a*b*(3*A - 7
*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[
Sec[c + d*x]]) + (2*(25*a^2*A - 4*A*b^2 + 7*a*b*B)*Sqrt[a + b*Cos[c + d*x]]
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a^2*d) + (2*(A*b + 7*a*B)*Sqrt[a + b
*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*a*d) + (2*A*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
```

```
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos}}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{7}{2}}(c+dx)\sin(c+}{7d}$$

$$= \frac{2(Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+d}{35ad}$$

$$= \frac{2(25a^2A-4Ab^2+7abB)\sqrt{a+b\cos(c+dx}}{105a^2d}$$

$$= \frac{2(25a^2A-4Ab^2+7abB)\sqrt{a+b\cos(c+dx}}{105a^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(19a^2Ab+8Ab^3+63a^3B-}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3321 vs. $2(473) = 946$.
time = 24.03, size = 3321, normalized size = 7.02

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2), x]

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sin[c + d*x])/(105*a^3) + (2*Sec[c + d*x]^2*(A*b*Sin[c + d*x] + 7*a*B*Sin[c + d*x]))/(35*a) + (2*Sec[c + d*x]*(25*a^2*A*Sin[c + d*x] - 4*A*b^2*Sin[c + d*x] + 7*a*b*B*Sin[c + d*x]))/(105*a^2) + (2*A*Sec[c + d*x]^2*Tan[c + d*x])/7))/d + (2*((-19*A*b)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^3)/(105*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*a*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b^2*B)/(15*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a*A*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (17*A*b^2*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^4*Sqrt[Sec[c + d*x]])/(105*a^3*Sqrt[a + b*Cos[c + d*x]]) - (2*b*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*B*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a + b*Cos[c + d*x]]) - (19*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a^3*Sqrt[a + b*Cos[c + d*x]]) - (3*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^3*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (19*a^

```
2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1/2*((1
9*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*
x])*Sec[(c + d*x)/2]^4) - ((a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*
b^2*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 +
 Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])] + (a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*
B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Co
s[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])] - ((a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Co
s[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(8*A*b^2 - 2*a*b*(3*A
+ 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos
[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b
*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/
2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*
a*b^2*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)
/2] - (19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(8*A*b^2 - 2
*a*b*(3*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3435 vs. $2(427) = 854$.
time = 2.18, size = 3436, normalized size = 7.26

method	result	size
default	Expression too large to display	3436

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/105/d*(-8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+63*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-14*B*cos(d*x+c)^4*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
```

))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b^2-14*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^3-49*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+14*B*cos(d*x+c)^4*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b^2+19*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^3*b+19*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*sin(d*x+c)*cos
(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b^3-19*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3*b-2*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*A*cos(d*x+c)^3*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
3+63*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3*b-14*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-14*B*cos(d*x+c)^3*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3
-49*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^3*b-19*A*cos(d*x+c)^5*a^2*b^2+28*B*cos(d*x+c)^2*a^3*b-
25*A*cos(d*x+c)^5*a^3*b+14*B*cos(d*x+c)^5*a*b^3-19*A*cos(d*x+c)^4*a^3*b+20*
A*cos(d*x+c)^4*a^2*b^2-8*A*cos(d*x+c)^4*a*b^3+35*B*cos(d*x+c)^4*a^3*b+14*B*
cos(d*x+c)^4*a^2*b^2-14*B*cos(d*x+c)^4*a*b^3+15*A*a^4+21*B*cos(d*x+c)*a^4+8
*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*b^4-25*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+63*B*sin(d*x+c)*cos(d*x+c)^4*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*sin(d*
x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^4+8*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((

```
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^4-25*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+63*B*sin(d*x+c)*cos(d
*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4
-63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^4+18*A*cos(d*x+c)*a^3*b+14*B*cos(d*x+c)^3*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+19*A
*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^3*b+19*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2),
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{9/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(1/2),
 x)
```

3.591 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=390

$$\frac{2(a-b)\sqrt{a+b}\,(9a^2A-2Ab^2+5abB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\right.}{15a^3d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/15*(A*b+5*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d+2/5
*A*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/15*(a-b)*(9*A*a^2
-2*A*b^2+5*B*a*b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2
)-2/15*(a-b)*(9*A*a+2*A*b-5*B*a)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/
sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.61, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3078, 3134, 3077, 2895, 3073}

$$-\frac{2(a-b)\sqrt{a+b}\,(9aA-5aB+2Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^2d\sqrt{\sec(c+dx)}}+\frac{2(a-b)\sqrt{a+b}\,(9a^2A+5abB-2Ab^2)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d\sqrt{\sec(c+dx)}}+\frac{2(5aB+Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{15ad}+\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sqrt[Cos[c + d*x]]*Csc
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*
Sqrt[a + b]*(9*a*A + 2*A*b - 5*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(A*b + 5*a*B)*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d) + (2*A*Sqrt[a + b*C
os[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
```

```
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\,c}}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(c+}{5d}$$

$$= \frac{2(Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+d}{15ad}$$

$$= \frac{2(Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+d}{15ad}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(9a^2A-2Ab^2+5abB)\,\sqrt{co}}{}$$

Mathematica [A]
time = 17.67, size = 423, normalized size = 1.08

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2(a+b)\left(9a^2A-2Ab^2+5abB\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+2a(a+b)(9aA-2Ab+5aB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)-\left(9a^2A-2Ab^2+5abB\right)\cos(c+dx)(a+b\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{15a^2d\sqrt{a+b\cos(c+dx)}\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}}+\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(9a^2A-2Ab^2+5abB)\sin(c+dx)}{15a^2}+\frac{2\sec(c+dx)(Ab\sin(c+dx)+5aB\sin(c+dx))}{15a}+\frac{2}{5}A\sec(c+dx)\tan(c+dx)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),
x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(9*a^2*A - 2*A*b^2 + 5
*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
```

```
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)] + 2*a*(a + b)*(9*a*A - 2*A*b + 5*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2*A - 2*A*b^2 + 5*a*b*B)*C
os[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*
a^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sin[c + d*x]
)/(15*a^2) + (2*Sec[c + d*x]*(A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/(15*a
) + (2*A*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2486 vs. $2(350) = 700$.
time = 0.55, size = 2487, normalized size = 6.38

method	result	size
default	Expression too large to display	2487

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*(9*A*cos(d*x+c)^4*a^2*b+A*cos(d*x+c)^4*a*b^2+5*B*cos(d*x+c)^4*a^2*b
+5*B*cos(d*x+c)^4*a*b^2-5*A*cos(d*x+c)^3*a^2*b-2*A*cos(d*x+c)^3*a*b^2+5*B*c
os(d*x+c)^3*a^2*b-5*B*cos(d*x+c)^3*a*b^2-3*A*a^3+5*B*cos(d*x+c)^3*a^3-2*A*c
os(d*x+c)^4*b^3+9*A*cos(d*x+c)^3*a^3+2*A*cos(d*x+c)^3*b^3-6*A*cos(d*x+c)^2*
a^3+7*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)^3*a^2*b-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-9*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+2*
A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b^2+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-5*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-5*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^3*a*b^2+7*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
```

```
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2-9*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)
^2*a^2*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)^2*a*b^2+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b-5*B*sin(d*x+c)*cos(d
*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b-5*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^2-5*B*cos(d*x+c)*a^3+A*cos(d*x+c)^2*a*b^2-10*B*cos(
d*x+c)^2*a^2*b+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3-4*A*cos(d*x+c)*a^2*b-9*A*sin(d*x+c)*cos
(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)^3*b^3+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3-9*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)^2*a^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*cos(d*x+c)^2*b^3+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3)*cos(d*x+c)*(1/cos(d*
x+c))^(7/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2),
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(1/2),
 x)
```

3.592 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=324

$$\frac{2(a-b)\sqrt{a+b}\,(Ab+3aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{3a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*A*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*(a-b)*(A*b+3
*B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a
+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/3*(a-b)*
(A-3*B)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3078, 3077, 2895, 3073}

$$\frac{2(a-b)\sqrt{a+b}\,(3aB+Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{2(a-b)\sqrt{a+b}\,(A-3B)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad\sqrt{\sec(c+dx)}}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(A -
3*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[
c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(3*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

```
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\,c}}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3d}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3d}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(Ab+3aB)\sqrt{\cos(c+dx)}\ \mathrm{c}}{}$$

Mathematica [A]
time = 14.69, size = 346, normalized size = 1.07

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2(a+b)(Ab+3aB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+2a(a+b)(A+3B)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)-(Ab+3aB)\cos(c+dx)(a+b\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3ad\sqrt{a+b\cos(c+dx)}\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}}+\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(Ab+3aB)\sin(c+dx)}{3a}+\frac{2}{3}A\tan(c+dx)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),
x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(A*b + 3*a*B)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a +
 b)*(A + 3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] - (A*b + 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2]))/(3*a*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)
/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(A*b + 3*a*B)*Si
n[c + d*x])/(3*a) + (2*A*Tan[c + d*x])/3))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1734 vs. $2(290) = 580$.
time = 0.42, size = 1735, normalized size = 5.35

method	result	size
default	Expression too large to display	1735

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(-a^2*A+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2+A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*
x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-3*B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^2*a^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2+3*B*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2-3*B*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
cos(d*x+c)^2*a^2+A*cos(d*x+c)^2*a^2+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+3*B*cos(d*x+c)^2*a^2
-3*B*cos(d*x+c)*a^2+A*cos(d*x+c)^3*b^2-A*cos(d*x+c)^2*b^2-A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)
*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b-3*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2
*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*b^2-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c
)^2*a*b+A*cos(d*x+c)^2*a*b-2*A*cos(d*x+c)*a*b+A*cos(d*x+c)^3*a*b)*cos(d*x+c
)*(1/cos(d*x+c))^(5/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2),
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2),
 x)
```

3.593 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=411

$$\frac{2A(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2*A*(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+2*(A*b-a
*(A-B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-2*B*csc(d*
x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b
,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b)
)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3070, 2888, 3077, 2895, 3073}

$$\frac{2\sqrt{a+b}\,(Ab-a(A-B))\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}+\frac{2A(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}-\frac{2B\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(A*b - a*(A - B))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (2*Sq
rt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(d*Sqrt[Sec[c + d*x]])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
```

```
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3070

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)]])/((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[B*(d
/b^2), Int[Sqrt[b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Int[(A*c
 + (B*c + A*d)*Sin[e + f*x])/((b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\,c}}{}$$

$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{aA+(}{\cos^{\frac{3}{2}}(c+}$$

$$= -\frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\Big(\frac{a}{}}{}$$

$$= \frac{2A(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)}{}$$

Mathematica [A]
time = 17.25, size = 635, normalized size = 1.55

$$\frac{2A\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{2\left(aA\tan\left(\frac{1}{2}(c+dx)\right)+Ab\tan\left(\frac{1}{2}(c+dx)\right)-2Ab\tan^3\left(\frac{1}{2}(c+dx)\right)-aA\tan^5\left(\frac{1}{2}(c+dx)\right)+Ab\tan^5\left(\frac{1}{2}(c+dx)\right)-2bB\Pi\left(-1;\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\frac{a+b+a\tan^2\left(\frac{1}{2}(c+dx)\right)-b\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-2bB\Pi\left(-1;\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\frac{a+b+a\tan^2\left(\frac{1}{2}(c+dx)\right)-b\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+A(a+b)E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\frac{a+b+a\tan^2\left(\frac{1}{2}(c+dx)\right)-b\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-(b(A-B)+a(A+B))F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\frac{a+b+a\tan^2\left(\frac{1}{2}(c+dx)\right)-b\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}\right)}{d\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(-1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^{3/2}\sqrt{\frac{a+b+a\tan^2\left(\frac{1}{2}(c+dx)\right)-b\tan^2\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),
x]
```

```
[Out] (2*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*(a*A*
Tan[(c + d*x)/2] + A*b*Tan[(c + d*x)/2] - 2*A*b*Tan[(c + d*x)/2]^3 - a*A*Ta
n[(c + d*x)/2]^5 + A*b*Tan[(c + d*x)/2]^5 - 2*b*B*EllipticPi[-1, ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b*B*EllipticPi[
-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)] + A*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (b*(A - B) + a*(A + B
))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*
```

```
Tan[(c + d*x)/2]^2)/(a + b)]))/(d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 +
 Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c
+ d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. 2(375) = 750.
time = 0.46, size = 1361, normalized size = 3.31

method	result	size
default	Expression too large to display	1361

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/d*(A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*
x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*B*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*a-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+A*sin(d*x+c)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b-A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+2*B*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+B*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)
```

```
+A*cos(d*x+c)^2*b+A*cos(d*x+c)*a-A*cos(d*x+c)*b-a*A)*cos(d*x+c)*(1/cos(d*x+
c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2),
x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}\sqrt{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2),
 x)
```

3.594 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sqrt{\sec(c+d$

Optimal. Leaf size=445

$$-\frac{(a-b)\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{ad\sqrt{\sec(c+dx)}}$$

```
[Out] B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-(a-b)*B*csc(d*x+c)*E
llipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+(2*A+B)*csc(d*x+c)*EllipticF((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-(2*A*b+B*a)*csc(d*x+c)*EllipticPi((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(
a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3040, 3082, 3132, 2888, 3077, 2895, 3073}

$$\frac{\sqrt{a+b}\,(2A+B)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}-\frac{\sqrt{a+b}\,(aB+2Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}-\frac{B(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*(2*A + B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*
A*b + a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(b*d*Sqrt[Sec[c + d*x]]) + (B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/d
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
```

```
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3082

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a +
 b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)
*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(
a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+\ldots}}{\ldots}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin(c\ldots}{d}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin(c\ldots}{d}$$

$$= -\frac{\sqrt{a+b}\,(2Ab+aB)\sqrt{\cos(c+dx)}\,\csc(c\ldots}{\ldots}$$

$$= -\frac{(a-b)\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+\ldots}{\ldots}$$

Mathematica [A]
time = 17.29, size = 787, normalized size = 1.77

[illegible]

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]], x]

[Out] (-(a*B*Tan[(c + d*x)/2]) - b*B*Tan[(c + d*x)/2] + 2*b*B*Tan[(c + d*x)/2]^3 + a*B*Tan[(c + d*x)/2]^5 - b*B*Tan[(c + d*x)/2]^5 - 4*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*B*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*(A*b + a*(-A + B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)])/(d*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-1 + Tan[(c + d*x)/2]^4))

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1368 vs. $2(405) = 810$.
time = 0.59, size = 1369, normalized size = 3.08

method	result	size
default	Expression too large to display	1369

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/d*(4*A*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(

```
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+4*A*sin(d*x+c)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b+2*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-2*A*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b
+2*B*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a+B*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a+B*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a+B*cos(d*x+c)^3*b+B*cos(d*x+c)^2*a-b*B*cos(d*x+c)^2
-B*cos(d*x+c)*a)*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)),
x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos{(c+dx)})\sqrt{a+b\cos{(c+dx)}}\sqrt{\sec{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x)),
x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2),
 x)
```

3.595 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=533

$$-\frac{(a-b)\sqrt{a+b}\,(4Ab+aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{4abd\sqrt{\sec(c+dx)}}$$

```
[Out] 1/2*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*(4*A*b+B*a)*
sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b/d-1/4*(a-b)*(4*A*b+B*a
)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/sec(d*x+c)^(1/2)+1/4*(4*A*b+(a+
2*b)*B)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)-1/4*(4*A*a
*b-B*a^2+4*B*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)
*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*
x+c)^(1/2)
```

Rubi [A]

time = 0.79, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3040, 3082, 3140, 3132, 2888, 3077, 2895, 3073}

$$-\frac{\sqrt{a+b}\,(a^2(-B)+4aAb+4b^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d\sqrt{\sec(c+dx)}}+\frac{\sqrt{a+b}\,(B(a+2b)+4Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd\sqrt{\sec(c+dx)}}-\frac{(a-b)\sqrt{a+b}\,(aB+4Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4abd\sqrt{\sec(c+dx)}}+\frac{(aB+4Ab)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{4bd}+\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] -1/4*((a - b)*Sqrt[a + b]*(4*A*b + a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(4*A*b + (a +
2*b)*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[S
ec[c + d*x]]) - (Sqrt[a + b]*(4*a*A*b - a^2*B + 4*b^2*B)*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*Sqrt[Sec[c + d*x]]
) + (B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + ((
```

4*A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b *d)

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3082

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a +
 b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)
*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(
a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,\sqrt{a+b}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{4}\Big(\sqrt{\cos(c+dx)}\,\sqrt{s}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4Ab+aB)\sqrt{a+b\,c}}{}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4Ab+aB)\sqrt{a+b\,c}}{}$$

$$= -\frac{\sqrt{a+b}\,(4aAb-a^2B+4b^2B)\,\sqrt{\cos(c+dx)}\,\csc(c+dx}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(4Ab+aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\Big(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1121 vs. 2(533) = 1066.
time = 18.06, size = 1121, normalized size = 2.10

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (S
qrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(4*a*A*b*Tan[(c + d*x)/2] + 4*A*b^2*Tan[
(c + d*x)/2] + a^2*B*Tan[(c + d*x)/2] + a*b*B*Tan[(c + d*x)/2] - 8*A*b^2*Ta
n[(c + d*x)/2]^3 - 2*a*b*B*Tan[(c + d*x)/2]^3 - 4*a*A*b*Tan[(c + d*x)/2]^5
+ 4*A*b^2*Tan[(c + d*x)/2]^5 - a^2*B*Tan[(c + d*x)/2]^5 + a*b*B*Tan[(c + d*
x)/2]^5 + 8*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(a + b)] - 2*a^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x
)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*B*EllipticPi[-1, ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
```

```
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*a*A*b*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] - 2*a^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*B*EllipticPi[-1,
 ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2
)/(a + b)] + (a + b)*(4*A*b + a*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b*(4*a*A -
 a*B + 2*b*B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x
)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*b*d*(1 + Tan[(c + d*x)/2]^2)^(
3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c
 + d*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2053 vs. $2(479) = 958$.
time = 0.51, size = 2054, normalized size = 3.85

method	result	size
default	Expression too large to display	2054

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*(4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*b^2-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*c
os(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
```

```
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*cos(d*x+c)^4*b
^2-2*B*cos(d*x+c)^2*b^2+B*cos(d*x+c)^2*a^2-B*cos(d*x+c)*a^2+4*A*cos(d*x+c)^
3*b^2-4*A*cos(d*x+c)^2*b^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8*A*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+B
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^2+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-2*B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+3*B*cos(d*x+c)^3*a*b-B*cos(d*x+c)^2*
a*b-2*B*cos(d*x+c)*a*b+4*A*cos(d*x+c)^2*a*b-4*A*cos(d*x+c)*a*b)*(1/cos(d*x+
c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)),
x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{a + b\cos(c + dx)}}{\sqrt{\sec(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/sqrt(sec(c + d*x)),
x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\,\cos(c + d\,x))\,\sqrt{a + b\,\cos(c + d\,x)}}{\sqrt{\frac{1}{\cos(c + d\,x)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2
), x)
```

3.596 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=620

$$-\frac{(a-b)\sqrt{a+b}\,(6aAb-3a^2B+16b^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{24ab^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 1/3*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1/4*(2*A*b-B*a
)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/24*(6*A*a*b-3*B*
a^2+16*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b^2/d-1/24
*(a-b)*(6*A*a*b-3*B*a^2+16*B*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c
)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2
/d/sec(d*x+c)^(1/2)+1/24*(a+2*b)*(6*A*b-3*B*a+8*B*b)*csc(d*x+c)*EllipticF((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)+1/8*(2*A*a^2*b-8*A*b^3-B*a^3-4*B*a*b^2
)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c
))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 1.07, antiderivative size = 620, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3040, 3069, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$-\frac{(a-b)\sqrt{a+b}\,(-3a^2B+6aAb+16b^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{24ab^2d\sqrt{\sec(c+dx)}}+\frac{(-3a^2B+6aAb+16b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24b^2d}+\frac{\sqrt{a+b}\,(a^3(-B)+2a^2Ab-4ab^2B-8Ab^3)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{8b^3d\sqrt{\sec(c+dx)}}+\frac{\sqrt{a+b}\,(a+2b)(-3aB+6Ab+8bB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{24b^2d\sqrt{\sec(c+dx)}}+\frac{(2Ab-aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4bd\sqrt{\sec(c+dx)}}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] -1/24*((a - b)*Sqrt[a + b]*(6*a*A*b - 3*a^2*B + 16*b^2*B)*Sqrt[Cos[c + d*x]
]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a
 + b]*(a + 2*b)*(6*A*b - 3*a*B + 8*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(24*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(2*a^2*A*b
- 8*A*b^3 - a^3*B - 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(
a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
```

```
[c + d*x]))/(a - b)])/(8*b^3*d*Sqrt[Sec[c + d*x]]) + ((2*A*b - a*B)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Sec[c + d*x]]) + (B*(a + b*Cos[
c + d*x])^(3/2)*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]]) + ((6*a*A*b - 3*a^
2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
24*b^2*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
```

```
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\text{c}}$$

$$= \frac{B(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.}}{}$$

$$= \frac{(2Ab-aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{B(a+b\,\text{c}}{3}$$

$$= \frac{(2Ab-aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{B(a+b\,\text{c}}{3}$$

$$= \frac{(2Ab-aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{B(a+b\,\text{c}}{3}$$

$$= \frac{\sqrt{a+b}\,(2a^2Ab-8Ab^3-a^3B-4ab^2B)\,\sqrt{\cos(c+dx)}\;\text{c}}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(6aAb-3a^2B+16b^2B)\,\sqrt{\cos(c+dx)}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1533 vs. $2(620) = 1240$.
time = 14.61, size = 1533, normalized size = 2.47

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((B*Sin[c + d*x])/12 + ((6*A*b
 + a*B)*Sin[2*(c + d*x)])/(24*b) + (B*Sin[3*(c + d*x)])/12))/d + (Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]
*(6*a^2*A*b*Tan[(c + d*x)/2] + 6*a*A*b^2*Tan[(c + d*x)/2] - 3*a^3*B*Tan[(c
+ d*x)/2] - 3*a^2*b*B*Tan[(c + d*x)/2] + 16*a*b^2*B*Tan[(c + d*x)/2] + 16*b
^3*B*Tan[(c + d*x)/2] - 12*a*A*b^2*Tan[(c + d*x)/2]^3 + 6*a^2*b*B*Tan[(c +
d*x)/2]^3 - 32*b^3*B*Tan[(c + d*x)/2]^3 - 6*a^2*A*b*Tan[(c + d*x)/2]^5 + 6*
a*A*b^2*Tan[(c + d*x)/2]^5 + 3*a^3*B*Tan[(c + d*x)/2]^5 - 3*a^2*b*B*Tan[(c
+ d*x)/2]^5 - 16*a*b^2*B*Tan[(c + d*x)/2]^5 + 16*b^3*B*Tan[(c + d*x)/2]^5 -
 12*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(a + b)] + 48*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^
2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*B*EllipticPi[-1, ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a*b^2*B*EllipticPi[
-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*
a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c +
 d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*A*b^3*EllipticPi[-1, ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^
3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x
)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*
Tan[(c + d*x)/2]^2)/(a + b)] + 24*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b
)*(-6*a*A*b + 3*a^2*B - 16*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b*(-12*A*b^
2 + 2*a*b*(3*A - 7*B) + a^2*B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*b^2*d*(-1 +
Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]
*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2955 vs. $2(560) = 1120$.
time = 0.53, size = 2956, normalized size = 4.77

method	result	size
default	Expression too large to display	2956

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/24/d*(-10*B*cos(d*x+c)^4*a*b^2-18*A*cos(d*x+c)^3*a*b^2+B*cos(d*x+c)^3*a^2*b-12*A*cos(d*x+c)^4*b^3-6*A*cos(d*x+c)^2*a^2*b-6*B*cos(d*x+c)^2*a*b^2+12*A*cos(d*x+c)^2*b^3-8*B*cos(d*x+c)^5*b^3-8*B*cos(d*x+c)^3*b^3+3*B*cos(d*x+c)^2*a^3+16*B*cos(d*x+c)^2*b^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-16*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-48*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^3+24*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^3-6*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^3+3*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^3-16*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^3+12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-24*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+28*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-16*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/

```
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-48*A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-3*B*cos(d*x+
c)*a^3+6*A*cos(d*x+c)^2*a*b^2-3*B*cos(d*x+c)^2*a^2*b+12*A*cos(d*x+c)*sin(d*
x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^
2*b-12*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*a*b^2-6*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b-6*A*cos(d*x+c)*sin(d*x+c)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2-24*B*co
s(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*a*b^2-2*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b+28*B*cos(d*x+c)*sin(d*x+c)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2+3*B*cos(d*x+c
)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*a^2*b-16*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*a*b^2+2*B*cos(d*x+c)*a^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2),
x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{a + b\cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/sec(c + d*x)**(3/2),
 x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\,\cos(c + d\,x))\,\sqrt{a + b\,\cos(c + d\,x)}}{{\left(\frac{1}{\cos(c + d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2
), x)
```

3.597 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=562

$$\frac{2(a-b)\sqrt{a+b}\,(147a^4A+33a^2Ab^2+8Ab^4+246a^3bB-18ab^3B)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\Bigg(\text{ArcSin}\bigg(\frac{\ }{\sqrt{\ }}}{315a^4d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/315*(88*A*a^2*b-4*A*b^3+75*B*a^3+9*B*a*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(
a+b*cos(d*x+c))^(1/2)/a^2/d+2/315*(49*A*a^2+3*A*b^2+72*B*a*b)*sec(d*x+c)^(5
/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d+2/63*(10*A*b+9*B*a)*sec(d*x+c)^(7
/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/9*a*A*sec(d*x+c)^(9/2)*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/d+2/315*(a-b)*(147*A*a^4+33*A*a^2*b^2+8*A*b^4+246*B
*a^3*b-18*B*a*b^3)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/sec(d*x+c)^(1/
2)+2/315*(a-b)*(8*A*b^3-a^3*(147*A-75*B)+3*a^2*b*(13*A-57*B)+6*a*b^2*(A-3*B
))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b)
)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 1.30, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3068, 3134, 3077, 2895, 3073}

$$\frac{2(49a^2A+72abB+3Ab^2)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad}+\frac{2(a-b)\sqrt{a+b}\,(-(a^3(147A-75B))+3a^2b(13A-57B)+6ab^2(A-3B)+8Ab^3)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{315a^3d\sqrt{\sec(c+dx)}}+\frac{2(75a^3B+88a^2Ab+9ab^2B-4Ab^3)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315a^2d}+\frac{2(a-b)\sqrt{a+b}\,(147a^4A+246a^3bB+33a^2Ab^2-18ab^3B+8Ab^4)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{315a^4d\sqrt{\sec(c+dx)}}+\frac{2(9aB+10Ab)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{63d}+\frac{2aA\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B -
18*a*b^3*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*
d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(8*A*b^3 - a^3*(147*A - 75*B
) + 3*a^2*b*(13*A - 57*B) + 6*a*b^2*(A - 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(315*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(88*a^2*A*b -
 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(315*a^2*d) + (2*(49*a^2*A + 3*A*b^2 + 72*a*b*B)*Sqrt[a + b
*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*a*d) + (2*(10*A*b + 9*
```

```
a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*
a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{c}}{}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{9}{2}}(c+dx)\sin(c}{9d}$$

$$= \frac{2(10Ab+9aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{7}{2}}(c}{63d}$$

$$= \frac{2(49a^2A+3Ab^2+72abB)\sqrt{a+b\cos(c+}}{315ad}$$

$$= \frac{2(88a^2Ab-4Ab^3+75a^3B+9ab^2B)\sqrt{a+}}{315a^2}$$

$$= \frac{2(88a^2Ab-4Ab^3+75a^3B+9ab^2B)\sqrt{a+}}{315a^2}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(147a^4A+33a^2Ab^2+8Ab}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3739 vs. $2(562) = 1124$.
time = 25.76, size = 3739, normalized size = 6.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4*A + 33*a^2*A*b^2
+ 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sin[c + d*x])/(315*a^3) + (2*Sec[c +
d*x]^3*(10*A*b*Sin[c + d*x] + 9*a*B*Sin[c + d*x]))/63 + (2*Sec[c + d*x]^2*(
49*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 72*a*b*B*Sin[c + d*x]))/(315
*a) + (2*Sec[c + d*x]*(88*a^2*A*b*Sin[c + d*x] - 4*A*b^3*Sin[c + d*x] + 75*
a^3*B*Sin[c + d*x] + 9*a*b^2*B*Sin[c + d*x]))/(315*a^2) + (2*a*A*Sec[c + d*
x]^3*Tan[c + d*x])/9))/d + (2*((-7*a^2*A)/(15*Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) - (11*A*b^2)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (8*A*b^4)/(315*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (82
*a*b*B)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b^3*B)/(35*a
```

*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (13*a*A*b*Sqrt[Sec[c + d*x]
])/(105*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^3*Sqrt[Sec[c + d*x]])/(315*a*Sq
rt[a + b*Cos[c + d*x]]) - (8*A*b^5*Sqrt[Sec[c + d*x]])/(315*a^3*Sqrt[a + b*
Cos[c + d*x]]) + (5*a^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]])
 - (31*b^2*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) + (2*b^4*B*
Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos[c + d*x]]) - (7*a*A*b*Cos[2*(c +
 d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (11*A*b^3*Cos[2*
(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^5*
Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*a^3*Sqrt[a + b*Cos[c + d*x]]) - (
82*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]
) + (2*b^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos[c
+ d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(147*a^4*A + 33
*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 - 6*a
b^2(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (147*a^4*A +
33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[
c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^3*d*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*Sin[c + d*x]*(-2*(a + b)*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B
 - 18*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*
B) + 3*a^2*b*(13*A + 57*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)] - (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b
*B - 18*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]))/(315*a^3*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2])
 - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(147
*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*
A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B))*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (1
47*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Cos[c + d*x]*
(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^3*Sqrt[a
+ b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Se
c[c + d*x]]*(-1/2*((147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a
*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4) - ((a + b)*(1
47*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - S

```
in[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a
*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A
 + 57*B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(
1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])] - ((a + b)*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3
*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]
)^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)
*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B)
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Ellipti...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4398 vs. $2(510) = 1020$.
time = 0.77, size = 4399, normalized size = 7.83

method	result	size
default	Expression too large to display	4399

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/315/d*(8*A*cos(d*x+c)^6*b^5-45*B*cos(d*x+c)*a^5-98*A*cos(d*x+c)^4*a^5-14
*A*cos(d*x+c)^2*a^5+147*A*cos(d*x+c)^5*a^5-8*A*cos(d*x+c)^5*b^5+75*B*cos(d*
x+c)^5*a^5-30*B*cos(d*x+c)^3*a^5-18*B*cos(d*x+c)^6*a*b^4-18*B*cos(d*x+c)^5*
a^2*b^3+8*A*cos(d*x+c)^5*a*b^4+A*cos(d*x+c)^3*a^2*b^3-53*A*cos(d*x+c)^2*a^3
*b^2-204*B*cos(d*x+c)^4*a^4*b+9*B*cos(d*x+c)^4*a^2*b^3-52*A*cos(d*x+c)^3*a^
4*b+75*B*cos(d*x+c)^6*a^4*b+246*B*cos(d*x+c)^6*a^3*b^2+9*B*cos(d*x+c)^6*a^2
*b^3-10*A*cos(d*x+c)^5*a^4*b+33*A*cos(d*x+c)^5*a^3*b^2-34*A*cos(d*x+c)^5*a^
2*b^3+246*B*cos(d*x+c)^5*a^4*b-165*B*cos(d*x+c)^5*a^3*b^2+18*B*cos(d*x+c)^5
*a*b^4-68*A*cos(d*x+c)^4*a^3*b^2-4*A*cos(d*x+c)^4*a*b^4+147*A*cos(d*x+c)^6*
a^4*b+88*A*cos(d*x+c)^6*a^3*b^2+33*A*cos(d*x+c)^6*a^2*b^3-4*A*cos(d*x+c)^6*
a*b^4-35*A*a^5-81*B*cos(d*x+c)^3*a^3*b^2-117*B*cos(d*x+c)^2*a^4*b-85*A*cos(
d*x+c)*a^4*b+147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a^5-8*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)
*b^5+75*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^5+147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
```

s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^5-147*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*b^5+75*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^5+8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-147*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-33*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^2*b^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^4+246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^4*b+153*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b^2-18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^2*b^3-246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^4*b-246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b^2+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^2*b^3+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^4+186*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+33*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a^2*b^3+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x

```
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(11/
2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(11/
2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{11/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(3/2)
,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(3/2)
, x)
```

3.598 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=473

$$\frac{2(a-b)\sqrt{a+b}\,(82a^2Ab-6Ab^3+63a^3B+21ab^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\,\sqrt{\cos(}}\right.\right.}{105a^3d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/105*(25*A*a^2+3*A*b^2+42*B*a*b)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+
c))^(1/2)/a/d+2/35*(8*A*b+7*B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c
))^(1/2)/d+2/7*a*A*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/1
05*(a-b)*(82*A*a^2*b-6*A*b^3+63*B*a^3+21*B*a*b^2)*csc(d*x+c)*EllipticE((a+b
*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)
^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a
-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)-2/105*(a-b)*(6*A*b^2-a^2*(25*A-63*B)+3*a*
b*(19*A-7*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d
*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3068, 3134, 3077, 2895, 3073}

$$-\frac{2(a-b)\sqrt{a+b}\,\left(-\left(a^2(25A-63B)\right)+3ab(19A-7B)+6Ab^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{105a^2d\sqrt{\sec(c+dx)}}+\frac{2(25a^2A+42abB+3Ab^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad}+\frac{2(a-b)\sqrt{a+b}\,(63a^3B+82a^2Ab+21ab^2B-6Ab^3)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{105a^3d\sqrt{\sec(c+dx)}}+\frac{2(7aB+8Ab)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{35d}+\frac{2aA\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[Sec[c + d*
x]]) - (2*(a - b)*Sqrt[a + b]*(6*A*b^2 - a^2*(25*A - 63*B) + 3*a*b*(19*A -
7*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d*Sqrt
[Sec[c + d*x]]) + (2*(25*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b*Cos[c + d*x
]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a*d) + (2*(8*A*b + 7*a*B)*Sqrt[a +
 b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a*A*Sqrt[a +
b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
```

```
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{co}}{}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{7}{2}}(c+dx)\sin(c}{7d}$$

$$= \frac{2(8Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+}{35d}$$

$$= \frac{2(25a^2A+3Ab^2+42abB)\,\sqrt{a+b\cos(c+d}}{105ad}$$

$$= \frac{2(25a^2A+3Ab^2+42abB)\,\sqrt{a+b\cos(c+d}}{105ad}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(82a^2Ab-6Ab^3+63a^3B+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3318 vs. $2(473) = 946$.

time = 24.14, size = 3318, normalized size = 7.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(-82*a^2*A*b + 6*A*b^3 -
63*a^3*B - 21*a*b^2*B)*Sin[c + d*x])/(105*a^2) + (2*Sec[c + d*x]^2*(8*A*b*S
in[c + d*x] + 7*a*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(25*a^2*A*Sin[c + d
*x] + 3*A*b^2*Sin[c + d*x] + 42*a*b*B*Sin[c + d*x]))/(105*a) + (2*a*A*Sec[c
 + d*x]^2*Tan[c + d*x])/7))/d + (2*((-82*a*A*b)/(105*Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]) + (2*A*b^3)/(35*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) - (3*a^2*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b
^2*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a^2*A*Sqrt[Sec[c
 + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^2*Sqrt[Sec[c + d*x]])/(10
5*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a +
 b*Cos[c + d*x]]) + (a*b*B*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]])
 - (b^3*B*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) - (82*A*b^2*Co
s[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^
4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos[c + d*x]]) -
(3*a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]])
- (b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]
))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(82*a^2*A*b - 6*A*b^3
+ 63*a^3*B + 21*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(
25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] - (82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(
a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^2*d*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec
[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a
*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)] + 2*a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(25*A + 63*B))*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (82
*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x
])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^2*(a + b*Cos[c + d*x])^(3/2
)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c
 + d*x)/2]*(-2*(a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(
```

```
a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (82*a^2*A*b - 6*
A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c +
 d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1/2*((82*a^2*A*b
- 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c
 + d*x)/2]^4) - ((a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d
*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])] + (a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(25*A + 63*B))*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*
x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])] - ((a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*
x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B)
 + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d
*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(82*a
^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*S
in[c + d*x]*Tan[(c + d*x)/2] + (82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*
B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] -
(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c +
d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(-6*A*b^2 + 3*a*b*
(19*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + ...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3420 vs. $2(427) = 854$.

time = 0.58, size = 3421, normalized size = 7.23

method	result	size
default	Expression too large to display	3421

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*(-6*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-63*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/
```

(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-21*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-21*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+84*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+21*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-82*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-82*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+82*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+51*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-6*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-63*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-21*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-21*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+84*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+82*A*cos(d*x+c)^5*a^2*b^2-63*B*cos(d*x+c)^2*a^3*b+25*A*cos(d*x+c)^5*a^3*b+21*B*cos(d*x+c)^5*a*b^3+82*A*cos(d*x+c)^4*a^3*b-55*A*cos(d*x+c)^4*a^2*b^2-6*A*cos(d*x+c)^4*a*b^3+21*B*cos(d*x+c)^4*a^2*b^2-21*B*cos(d*x+c)^4*a*b^3-15*A*a^4-21*B*cos(d*x+c)*a^4+6*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+63*B*sin(d*x+c)*cos(d*x+c)^4*(cos

```
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+6*A*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^4+25*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+63*B*sin(d*x+c)*cos(d
*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4
-39*A*cos(d*x+c)*a^3*b+21*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-82*A*cos(d*x+c)^4*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-82
*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b^2+6*A*cos(d*x+c)^4*sin(d*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{9/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(3/2),
 x)
```

3.599 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=393

$$\frac{2(a-b)\sqrt{a+b}\,(9a^2A+3Ab^2+20abB)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\right.}{15a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/15*(6*A*b+5*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/5
*a*A*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/15*(a-b)*(9*A*a
^2+3*A*b^2+20*B*a*b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2
)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(
1/2)-2/15*(a-b)*(9*A*a-3*A*b-5*B*a+15*B*b)*csc(d*x+c)*EllipticF((a+b*cos(d*
x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*
cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.69, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3068, 3134, 3077, 2895, 3073}

$$\frac{2(a-b)\sqrt{a+b}\,(9a^2A+20abB+3Ab^2)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^2d\sqrt{\sec(c+dx)}}-\frac{2(a-b)\sqrt{a+b}\,(9aA-5aB-3Ab+15bB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15ad\sqrt{\sec(c+dx)}}+\frac{2(5aB+6Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{15d}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)
*Sqrt[a + b]*(9*a*A - 3*A*b - 5*a*B + 15*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) + (2*(6*A*b + 5*a*B)*S
qrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*Sq
rt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
```

```
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{co}}{}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(c}{5d}$$

$$= \frac{2(6Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+}{15d}$$

$$= \frac{2(6Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+}{15d}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(9a^2A+3Ab^2+20abB)\,\sqrt{}}{}$$

Mathematica [A]
time = 18.12, size = 427, normalized size = 1.09

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2(a+b)\left(9a^2A+3Ab^2+20abB\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+2a(a+b)(3b(A+5B)+a(9A+5B))\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)-\left(9a^2A+3Ab^2+20abB\right)\cos(c+dx)(a+b\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{15ad\sqrt{a+b\cos(c+dx)}\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}}+\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\left(9a^2A+3Ab^2+20abB\right)\sin(c+dx)}{15a}+\frac{2}{15}\sec(c+dx)(6Ab\sin(c+dx)+5aB\sin(c+dx))+\frac{2}{5}aA\sec(c+dx)\tan(c+dx)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2
),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(9*a^2*A + 3*A*b^2 + 2
0*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + 2*a*(a + b)*(3*b*(A + 5*B) + a*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2*A + 3*A*b^2 + 2
0*a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]))/(15*a*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a
+ b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sin
[c + d*x])/(15*a) + (2*Sec[c + d*x]*(6*A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x
]))/15 + (2*a*A*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2673 vs. $2(353) = 706$.
time = 0.48, size = 2674, normalized size = 6.80

method	result	size
default	Expression too large to display	2674

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*(9*A*cos(d*x+c)^4*a^2*b+6*A*cos(d*x+c)^4*a*b^2+5*B*cos(d*x+c)^4*a^2
*b+20*B*cos(d*x+c)^4*a*b^2+3*A*cos(d*x+c)^3*a*b^2+20*B*cos(d*x+c)^3*a^2*b-2
0*B*cos(d*x+c)^3*a*b^2-3*A*a^3+5*B*cos(d*x+c)^3*a^3+3*A*cos(d*x+c)^4*b^3+9*
A*cos(d*x+c)^3*a^3-3*A*cos(d*x+c)^3*b^3-6*A*cos(d*x+c)^2*a^3+12*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^3*a^2*b+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-3*A*sin(d*x+c)*cos(
d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^2+20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*cos(d*x+c)^3*a^2*b-20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-20*B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2
```

```
+12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)^2*a^2*b+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2-9*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b-3*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
cos(d*x+c)^2*a*b^2+20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b-20*B*sin(d*x+c)*cos(d*x+c)^2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-20*B*sin
(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^2+15*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+15*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-5*B*cos(d*x
+c)*a^3-9*A*cos(d*x+c)^2*a*b^2-25*B*cos(d*x+c)^2*a^2*b+9*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3
-9*A*cos(d*x+c)*a^2*b-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^3+5*B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)^3*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^2*a^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3-3*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^3+5*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*a^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(7/2)/
sin(d*x+c)/a
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{7/2}(a+b\cos(c+dx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(3/2),
 x)
```

3.600 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=479

$$\frac{2(a-b)\sqrt{a+b}\,(4Ab+3aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{3ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*a*A*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*(a-b)*(4*A
*b+3*B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+2/3*(3*A
*b^2+a^2*(A-3*B)-a*(4*A*b-6*B*b))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c
)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/s
ec(d*x+c)^(1/2)-2*b*B*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(
1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*
x+c)^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3040, 3068, 3132, 2888, 3077, 2895, 3073}

$$\frac{2\sqrt{a+b}\,(a^2(A-3B)-a(4Ab-6bB)+3Ab^2)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad\sqrt{\sec(c+dx)}}+\frac{2(a-b)\sqrt{a+b}\,(3aB+4Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad\sqrt{\sec(c+dx)}}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}-\frac{2bB\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(4*A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(3*A*b^2 + a^
2*(A - 3*B) - a*(4*A*b - 6*b*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[a + b]*B*Sqrt[Cos[c + d*
x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) +
 (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
```

```
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3d}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3d}$$

$$= -\frac{2b\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\Big(}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(4Ab+3aB)\sqrt{\cos(c+dx)}\,c}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5981 vs. $2(479) = 958$.
time = 24.45, size = 5981, normalized size = 12.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2
),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2328 vs. $2(433) = 866$.
time = 0.42, size = 2329, normalized size = 4.86

method	result	size
default	Expression too large to display	2329

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3/d*(a^2*A-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-4*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*
x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+3*B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^2*a^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2-3*B*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2+3*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*a^2-A*cos(d*x+c)^2*a^2-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-3*A*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
```

```
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2
-3*B*cos(d*x+c)^2*a^2+3*B*cos(d*x+c)*a^2-4*A*cos(d*x+c)^3*b^2+4*A*cos(d*x+c
)^2*b^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b-6*B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)^2*a*b+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+3*B*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+3
*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*b^2-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^2-6*B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin
(d*x+c)*b^2+3*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-3*B*cos(d*x+c)^3*a*b+3*B*cos(d*x+c)^2*a*b-
4*A*cos(d*x+c)^2*a*b+5*A*cos(d*x+c)*a*b-A*cos(d*x+c)^3*a*b)*cos(d*x+c)/(a+b
*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2
), x)
```

Fricas [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}(a+b\cos(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2),
 x)
```

3.601 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=509

$$\frac{(a-b)\sqrt{a+b}\,(2aA-bB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2*a*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-(2*A*a-B*b)*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d+(a-b)*(2*A*a-B*b)*csc(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-(2*a*(A-B)-b*(4*A+B))*csc(
d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-(2*A*b+3*B*a)*csc(d*x+c)
*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.86, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3040, 3068, 3140, 3132, 2888, 3077, 2895, 3073}

$$-\frac{\sqrt{a+b}\,(2a(A-B)-b(4A+B))\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}+\frac{(a-b)\sqrt{a+b}\,(2aA-bB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}-\frac{\sqrt{a+b}\,(3aB+2Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}-\frac{(2aA-bB)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{d}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(2*a*A - b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*a*(A - B) - b*(4*A
 + B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c
+ d*x]]) - (Sqrt[a + b]*(2*A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + b*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d - ((2*a*A - b*B)*Sqrt[a + b
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
```

```
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{co}}{}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin(}{d}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin(}{d}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin(}{d}$$

$$= -\frac{\sqrt{a+b}\,(2Ab+3aB)\sqrt{\cos(c+dx)}\,\csc(c}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,(2aA-bB)\sqrt{\cos(c+dx)}\,\mathrm{cs}}{}$$

Mathematica [A]
time = 16.23, size = 927, normalized size = 1.82

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (Sqrt[
(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-2*a^2*A*Tan[(c + d*x)/2] - 2*a*A*b*Tan[(c
+ d*x)/2] + a*b*B*Tan[(c + d*x)/2] + b^2*B*Tan[(c + d*x)/2] + 4*a*A*b*Tan[(
c + d*x)/2]^3 - 2*b^2*B*Tan[(c + d*x)/2]^3 + 2*a^2*A*Tan[(c + d*x)/2]^5 - 2
*a*A*b*Tan[(c + d*x)/2]^5 - a*b*B*Tan[(c + d*x)/2]^5 + b^2*B*Tan[(c + d*x)/
2]^5 + 4*A*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*S
qrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
 d*x)/2]^2)/(a + b)] + 6*a*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2
]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*A*b^2*EllipticPi[-1, ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6
*a*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c +
```

```
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(2*a*A - b*B)*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
 + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)] + 2*(-(A*b^2) + 2*a*b*(A - B) + a^2*(A + B))*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d
*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b
)]))/(d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2192 vs. $2(465) = 930$.
time = 0.43, size = 2193, normalized size = 4.31

method	result	size
default	Expression too large to display	2193

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*cos(d*x+c)*a^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2+B*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2+6*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a
*b-2*a^2*A-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*b^2+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+2*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2+
4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
```

```
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*cos(d*x+c)^3*b^2+2*A*cos(d*x+c)*a^2-4
*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b-2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+c)^2*b^2+6*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*
a*b-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-4*B*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*A*sin(d*x+c)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+B*cos(d*x+c)^2
*a*b-B*cos(d*x+c)*a*b+2*A*cos(d*x+c)^2*a*b-2*A*cos(d*x+c)*a*b+2*A*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+2*B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2),
 x)
```

3.602 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}$

Optimal. Leaf size=532

$$-\frac{(a-b)\sqrt{a+b}\,(4Ab+5aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{4ad\sqrt{\sec(c+dx)}}$$

```
[Out] 1/2*b*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*(4*A*b+5*B
*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-1/4*(a-b)*(4*A*b+5
*B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a
+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+1/4*(8*A*a+4
*A*b+5*B*a+2*B*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-1/
4*(12*A*a*b+3*B*a^2+4*B*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(
a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d
*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b
/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.87, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3040, 3069, 3140, 3132, 2888, 3077, 2895, 3073}

$$-\frac{\sqrt{a+b}\,(3a^2B+12aAb+4b^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd\sqrt{\sec(c+dx)}}+\frac{\sqrt{a+b}\,(8aA+5aB+4Ab+2bB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4d\sqrt{\sec(c+dx)}}-\frac{(a-b)\sqrt{a+b}\,(5aB+4Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4ad\sqrt{\sec(c+dx)}}+\frac{(5aB+4Ab)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{4d}+\frac{bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] -1/4*((a - b)*Sqrt[a + b]*(4*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*a*A + 4*A*b
 + 5*a*B + 2*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*
d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(12*a*A*b + 3*a^2*B + 4*b^2*B)*Sqrt[Co
s[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec[
c + d*x]]) + (b*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c +
d*x]]) + ((4*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(4*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
```

```
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{c}}{}$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{2}\Big(\surd$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4Ab}{}$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4Ab}{}$$

$$= -\frac{\sqrt{a+b}\,(12aAb+3a^2B+4b^2B)\sqrt{\cos(c}}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(4Ab+5aB)\sqrt{\cos(c+dx}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1134 vs. 2(532) = 1064.
time = 18.47, size = 1134, normalized size = 2.13

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (b*B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) +
(Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(4*a*A*b*Tan[(c + d*x)/2] + 4*A*b^2*Ta
n[(c + d*x)/2] + 5*a^2*B*Tan[(c + d*x)/2] + 5*a*b*B*Tan[(c + d*x)/2] - 8*A*
b^2*Tan[(c + d*x)/2]^3 - 10*a*b*B*Tan[(c + d*x)/2]^3 - 4*a*A*b*Tan[(c + d*x
)/2]^5 + 4*A*b^2*Tan[(c + d*x)/2]^5 - 5*a^2*B*Tan[(c + d*x)/2]^5 + 5*a*b*B*
Tan[(c + d*x)/2]^5 + 24*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*B*EllipticPi[-1, ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*B*EllipticPi[-1
, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a*
```

```
A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x
)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*
Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*B*E
llipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^
2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(a + b)] + (a + b)*(4*A*b + 5*a*B)*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x
)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
 + 2*(4*a^2*(A - B) - 2*b^2*B + a*b*(-8*A + B))*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/
2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))
/(4*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2431 vs. $2(478) = 956$.
time = 0.44, size = 2432, normalized size = 4.57

method	result	size
default	Expression too large to display	2432

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*(8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2-8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2+4*A*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+5*B*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*a^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*b^2-16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
```

```
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+5*B*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b+2*B*cos(d*x+c)^4*b^2-2*B*cos(d*x+c)^2*b^2+5*B*cos
(d*x+c)^2*a^2-5*B*cos(d*x+c)*a^2+4*A*cos(d*x+c)^3*b^2-4*A*cos(d*x+c)^2*b^2+
4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b-16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+5*B*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*s
in(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*b^2+5*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*b^2+7*B*cos(d*x+c)^3*a*b-5*B*cos(d*x+c)^2*a*b-2*B*cos(d*x+c)*
a*b+4*A*cos(d*x+c)^2*a*b-4*A*cos(d*x+c)*a*b+8*A*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-8*B*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2)/(a+b*cos(d*x+c
))^(1/2)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2),
 x)
```

3.603 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=626

$$-\frac{(a-b)\sqrt{a+b}\,(30aAb+3a^2B+16b^2B)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|\right.}{24abd\sqrt{\sec(c+dx)}}$$

```
[Out] 1/3*b*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+1/12*(6*A*b+7*
B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/24*(30*A*a*b+3*
B*a^2+16*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b/d-1/24
*(a-b)*(30*A*a*b+3*B*a^2+16*B*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+
c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/
d/sec(d*x+c)^(1/2)+1/24*(30*A*a*b+12*A*b^2+3*B*a^2+14*B*a*b+16*B*b^2)*csc(d
*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)
/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)-1/8*(6*A*a^2*b+8*A*b^3-
B*a^3+12*B*a*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)
*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*
x+c)^(1/2)
```

Rubi [A]

time = 1.25, antiderivative size = 626, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3040, 3069, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$\frac{\sqrt{a+b}\,(3a^2B+30aAb+14abB+12Ab^2+16b^2B)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{24bd\sqrt{\sec(c+dx)}}-\frac{(a-b)\sqrt{a+b}\,(3a^2B+30aAb+16b^2B)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{24abd\sqrt{\sec(c+dx)}}+\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{24bd}-\frac{\sqrt{a+b}\,(a^3(-B)+6a^2Ab+12ab^2B+8Ab^3)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{8b^2d\sqrt{\sec(c+dx)}}+\frac{(7aB+6Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{12d\sqrt{\sec(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] -1/24*((a - b)*Sqrt[a + b]*(30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[Cos[c + d*x
]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a
+ b]*(30*a*A*b + 12*A*b^2 + 3*a^2*B + 14*a*b*B + 16*b^2*B)*Sqrt[Cos[c + d*x
]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a
 + b]*(6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
```

```
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^2*d*Sqrt[Sec[c + d*x]]) + (b*B
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + ((6*A*b
+ 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[Sec[c + d*x]]) +
 ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(24*b*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,(a+b$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\right.$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a}}{12d}$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a}}{12d}$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a}}{12d}$$

$$= -\frac{\sqrt{a+b}\,(6a^2Ab+8Ab^3-a^3B+12ab^2B)\,\sqrt{\cos(c+dx}}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(30aAb+3a^2B+16b^2B)\,\sqrt{\cos(c+dx}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1489 vs. $2(626) = 1252$.
time = 19.11, size = 1489, normalized size = 2.38

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*B*Sin[c + d*x])/12 + ((6*A
*b + 7*a*B)*Sin[2*(c + d*x)])/24 + (b*B*Sin[3*(c + d*x)])/12))/d + (Sqrt[(1
 - Tan[(c + d*x)/2]^2)^(-1)]*(30*a^2*A*b*Tan[(c + d*x)/2] + 30*a*A*b^2*Tan[
(c + d*x)/2] + 3*a^3*B*Tan[(c + d*x)/2] + 3*a^2*b*B*Tan[(c + d*x)/2] + 16*a
*b^2*B*Tan[(c + d*x)/2] + 16*b^3*B*Tan[(c + d*x)/2] - 60*a*A*b^2*Tan[(c + d
*x)/2]^3 - 6*a^2*b*B*Tan[(c + d*x)/2]^3 - 32*b^3*B*Tan[(c + d*x)/2]^3 - 30*
a^2*A*b*Tan[(c + d*x)/2]^5 + 30*a*A*b^2*Tan[(c + d*x)/2]^5 - 3*a^3*B*Tan[(c
 + d*x)/2]^5 + 3*a^2*b*B*Tan[(c + d*x)/2]^5 - 16*a*b^2*B*Tan[(c + d*x)/2]^5
 + 16*b^3*B*Tan[(c + d*x)/2]^5 + 36*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Ta
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*A*b^3*EllipticPi[-1,
 ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^3*
B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(
a + b)] + 72*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Ta
n[(c + d*x)/2]^2)/(a + b)] + 36*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*A*b^3
*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2
]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan
[(c + d*x)/2]^2)/(a + b)] - 6*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 72*a*b^2*B*E
llipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^
2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(a + b)] + (a + b)*(30*a*A*b + 3*a^2*B + 16*b^2*B)*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(
1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x
)/2]^2)/(a + b)] - 2*b*(12*A*b^2 + a^2*(24*A - 7*B) + a*(-6*A*b + 26*b*B))*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan
[(c + d*x)/2]^2)/(a + b)]))/(24*b*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)
])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3140 vs. $2(566) = 1132$.

time = 0.54, size = 3141, normalized size = 5.02

method	result	size
default	Expression too large to display	3141

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/24/d*(22*B*cos(d*x+c)^4*a*b^2+42*A*cos(d*x+c)^3*a*b^2+17*B*cos(d*x+c)^3*a^2*b+12*A*cos(d*x+c)^4*b^3+30*A*cos(d*x+c)^2*a^2*b-6*B*cos(d*x+c)^2*a*b^2-12*A*cos(d*x+c)^2*b^3+8*B*cos(d*x+c)^5*b^3+8*B*cos(d*x+c)^3*b^3+3*B*cos(d*x+c)^2*a^3-16*B*cos(d*x+c)^2*b^3-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+16*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+48*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^3-24*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^3-6*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^3+3*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^3+16*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^3+36*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+12*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+30*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+30*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+14*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-52*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+16*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d

```
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+48*A*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-3*B*c
os(d*x+c)*a^3-30*A*cos(d*x+c)^2*a*b^2-3*B*cos(d*x+c)^2*a^2*b+36*A*cos(d*x+c
)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*a^2*b+12*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2+30*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b+30*A*cos(d*x+c)*sin
(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b
^2+72*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*a*b^2+14*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b-52*B*cos(d*x+c)*sin(d*x+
c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2+3*
B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*a^2*b+16*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2-14*B*c...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2
), x)
```

3.604 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=730

$$-\frac{(a-b)\sqrt{a+b}\,(24a^2Ab+128Ab^3-9a^3B+156ab^2B)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\,\sqrt{\text{co}}}\right.\right.}{192ab^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 1/24*(8*A*b-3*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1
/4*B*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1/32*(8*A*a*b-3
*B*a^2+12*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/1
92*(24*A*a^2*b+128*A*b^3-9*B*a^3+156*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(
1/2)*sec(d*x+c)^(1/2)/b^2/d-1/192*(a-b)*(24*A*a^2*b+128*A*b^3-9*B*a^3+156*B
*a*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d/sec(d*x+c)^(1/2)-1/192*
(9*a^3*B-6*a^2*b*(4*A+B)-8*b^3*(16*A+9*B)-4*a*b^2*(28*A+39*B))*csc(d*x+c)*E
llipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)+1/64*(8*A*a^3*b-96*A*a*b^3-3
*B*a^4-24*B*a^2*b^2-48*B*b^4)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(
d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/
b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 1.52, antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3040, 3069, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$\frac{(-3a^2B+8aAb+12b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{32bd\sqrt{\sec(c+dx)}}-\frac{\sqrt{a+b}\,(9a^3B-6a^2b(4A+B)-4ab^2(28A+39B)-8b^3(16A+9B))\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{192b^2d\sqrt{\sec(c+dx)}}-\frac{(a-b)\sqrt{a+b}\,(-9a^3B+24a^2Ab+156ab^2B+128Ab^3)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{192ab^2d\sqrt{\sec(c+dx)}}+\frac{(-9a^3B+24a^2Ab+156ab^2B+128Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{192b^2d}+\frac{\sqrt{a+b}\,(-3a^4B+8a^3Ab-24a^2b^2B-96aAb^3-48b^4B)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{64b^3d\sqrt{\sec(c+dx)}}+\frac{(8Ab-3aB)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{24bd\sqrt{\sec(c+dx)}}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{4bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] -1/192*((a - b)*Sqrt[a + b]*(24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B
)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*d*Sqrt[Sec[c
 + d*x]]) - (Sqrt[a + b]*(9*a^3*B - 6*a^2*b*(4*A + B) - 8*b^3*(16*A + 9*B)
- 4*a*b^2*(28*A + 39*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
```

```
)])/(192*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*a^3*A*b - 96*a*A*b^3 -
 3*a^4*B - 24*a^2*b^2*B - 48*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(64*b^3*d*Sqrt[Sec[c + d*x]]) + ((8*a*A*b - 3*a^
2*B + 12*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*b*d*Sqrt[Sec[c +
 d*x]]) + ((8*A*b - 3*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*b*d
*Sqrt[Sec[c + d*x]]) + (B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*b*d*S
qrt[Sec[c + d*x]]) + ((24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Sqrt
[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b^2*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
```

```
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\,\mathrm{c}$$

$$= \frac{B(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{(8Ab-3aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{24bd\sqrt{\sec(c+dx)}} + \frac{B(a+}{}$$

$$= \frac{(8aAb-3a^2B+12b^2B)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx}{32bd\sqrt{\sec(c+dx)}}$$

$$= \frac{(8aAb-3a^2B+12b^2B)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx}{32bd\sqrt{\sec(c+dx)}}$$

$$= \frac{(8aAb-3a^2B+12b^2B)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx}{32bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{a+b}\,(8a^3Ab-96aAb^3-3a^4B-24a^2b^2B-48b^4B)}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(24a^2Ab+128Ab^3-9a^3B+156ab^2B)}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1888 vs. 2(730) = 1460.
time = 21.02, size = 1888, normalized size = 2.59

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((8*A*b + 9*a*B)*Sin[c + d*x]
)/96 + ((56*a*A*b + 3*a^2*B + 48*b^2*B)*Sin[2*(c + d*x)])/(192*b) + ((8*A*b
 + 9*a*B)*Sin[3*(c + d*x)])/96 + (b*B*Sin[4*(c + d*x)])/32))/d - (Sqrt[(1 -
 Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(24*a^3*A*b*Tan[(c + d*x)/2] + 24*a^2*
A*b^2*Tan[(c + d*x)/2] + 128*a*A*b^3*Tan[(c + d*x)/2] + 128*A*b^4*Tan[(c +
d*x)/2] - 9*a^4*B*Tan[(c + d*x)/2] - 9*a^3*b*B*Tan[(c + d*x)/2] + 156*a^2*b
^2*B*Tan[(c + d*x)/2] + 156*a*b^3*B*Tan[(c + d*x)/2] - 48*a^2*A*b^2*Tan[(c
+ d*x)/2]^3 - 256*A*b^4*Tan[(c + d*x)/2]^3 + 18*a^3*b*B*Tan[(c + d*x)/2]^3
- 312*a*b^3*B*Tan[(c + d*x)/2]^3 - 24*a^3*A*b*Tan[(c + d*x)/2]^5 + 24*a^2*A
*b^2*Tan[(c + d*x)/2]^5 - 128*a*A*b^3*Tan[(c + d*x)/2]^5 + 128*A*b^4*Tan[(c
 + d*x)/2]^5 + 9*a^4*B*Tan[(c + d*x)/2]^5 - 9*a^3*b*B*Tan[(c + d*x)/2]^5 -
156*a^2*b^2*B*Tan[(c + d*x)/2]^5 + 156*a*b^3*B*Tan[(c + d*x)/2]^5 - 48*a^3*
A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)] + 576*a*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(a + b)] + 18*a^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(
c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 144*a^2*b^2*B*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 288*
b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^
2)/(a + b)] - 48*a^3*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Ta
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 576*a*A*b^3*EllipticPi[
-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)] + 18*a^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Ta
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 144*a^2*b^2*B*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] + 288*b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a +
```

```
b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(-24*a^2*A*b
 - 128*A*b^3 + 9*a^3*B - 156*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b*(2*a^
2*b*(28*A - 57*B) - 4*a*b^2*(52*A - 9*B) + 3*a^3*B - 72*b^3*B)*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 +
 Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)]))/(192*b^2*d*Sqrt[1 + Tan[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d
*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4055 vs. $2(664) = 1328$.
time = 0.68, size = 4056, normalized size = 5.56

method	result	size
default	Expression too large to display	4056

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/192/d*(-112*A*cos(d*x+c)*a^2*b^2-128*A*cos(d*x+c)*a*b^3-6*B*cos(d*x+c)*a
^3*b-156*B*cos(d*x+c)*a^2*b^2-72*B*cos(d*x+c)*a*b^3+9*B*cos(d*x+c)^2*a^3*b+
120*B*cos(d*x+c)^5*a*b^3+176*A*cos(d*x+c)^4*a*b^3+78*B*cos(d*x+c)^4*a^2*b^2
+128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*b^4-144*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^4-9*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4+18*B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(
d*x+c)*a^4+288*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^4+112*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-416*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+24*A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+24
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*
```

sin(d*x+c)+128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b^3*sin(d*x+c)-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+112*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2-416*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*a*b^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+128*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a
*b^3-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)*sin(d*x+c)*a^3*b+576*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+6*B*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^
3*b-228*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)*sin(d*x+c)*a^2*b^2+72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3-9*B*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+156
B(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)*sin(d*x+c)*a^2*b^2+156*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+144*B*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+6
4*A*cos(d*x+c)^3*b^4-128*A*cos(d*x+c)^2*b^4-9*B*cos(d*x+c)^2*a^4-72*B*cos(d
*x+c)^2*b^4+48*B*cos(d*x+c)^6*b^4+24*B*cos(d*x+c)^4*b^4+9*B*cos(d*x+c)*a^4+
128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4*s
in(d*x+c)-24*A*cos(d*x+c)*a^3*b+64*A*cos(d*x+c)^5*b^4-24*A*cos(d*x+c)^2*a^2
*b^2+78*B*cos(d*x+c)^2*a^2*b^2-156*B*cos(d*x+c)^2*a*b^3+136*A*cos(d*x+c)^3*
a^2*b^2-3*B*cos(d*x+c)^3*a^3*b+108*B*cos(d*x+c)...

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\ (a+b\cos(c+dx))^{3/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2
),x)
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2
), x)
```

3.605 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=662

$$\frac{2(a-b)\sqrt{a+b}\,(3705a^4Ab+255a^2Ab^3+40Ab^5+1617a^5B+3069a^3b^2B-110ab^4B)\,\sqrt{\cos(c+dx)}\,\csc(c}{3465a^4d\sqrt{\sec(c}$$

```
[Out] 2/11*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(11/2)*sin(d*x+c)/d+2/3465*(675*
A*a^4+1025*A*a^2*b^2-20*A*b^4+1793*B*a^3*b+55*B*a*b^3)*sec(d*x+c)^(3/2)*sin
(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/d+2/3465*(1145*A*a^2*b+15*A*b^3+539*B*a^
3+825*B*a*b^2)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d+2/693
*(81*A*a^2+113*A*b^2+209*B*a*b)*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c)
)^(1/2)/d+2/99*a*(14*A*b+11*B*a)*sec(d*x+c)^(9/2)*sin(d*x+c)*(a+b*cos(d*x+c
))^(1/2)/d+2/3465*(a-b)*(3705*A*a^4*b+255*A*a^2*b^3+40*A*b^5+1617*B*a^5+306
9*B*a^3*b^2-110*B*a*b^4)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/sec(d*x+
c)^(1/2)+2/3465*(a-b)*(40*A*b^4+3*a^4*(225*A-539*B)-6*a^3*b*(505*A-209*B)+1
5*a^2*b^2*(19*A-121*B)+10*a*b^3*(3*A-11*B))*csc(d*x+c)*EllipticF((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)
*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/a^3/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 1.85, antiderivative size = 662, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3040, 3068, 3126, 3134, 3077, 2895, 3073}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*
B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(3465*a^4*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(40*A
*b^4 + 3*a^4*(225*A - 539*B) - 6*a^3*b*(505*A - 209*B) + 15*a^2*b^2*(19*A -
 121*B) + 10*a*b^3*(3*A - 11*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
```

```
)/(a - b)])/(3465*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(675*a^4*A + 1025*a^2*A*b^
2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(3465*a^2*d) + (2*(1145*a^2*A*b + 15*A*b^3 + 539*a
^3*B + 825*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(3465*a*d) + (2*(81*a^2*A + 113*A*b^2 + 209*a*b*B)*Sqrt[a + b*Cos[c + d*
x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d) + (2*a*(14*A*b + 11*a*B)*Sqrt[
a + b*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*a*A*(a + b
*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
```

```
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{co}}{}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{11}{2}}(c+dx)\sin(c}{11d}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{9}{2}}(c}{99d}$$

$$= \frac{2(81a^2A+113Ab^2+209abB)\sqrt{a+b\cos(c}{693d}$$

$$= \frac{2(1145a^2Ab+15Ab^3+539a^3B+825ab^2B)}{346}$$

$$= \frac{2(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB}{}$$

$$= \frac{2(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB}{}$$

$$= \frac{2(a-b)\sqrt{a+b}(3705a^4Ab+255a^2Ab^3+40}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4198 vs. $2(662) = 1324$.
time = 27.05, size = 4198, normalized size = 6.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/
2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3705*a^4*A*b + 255*a^2*A*
b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sin[c + d*x])/(
3465*a^3) + (2*Sec[c + d*x]^4*(23*a*A*b*Sin[c + d*x] + 11*a^2*B*Sin[c + d*x
]))/99 + (2*Sec[c + d*x]^3*(81*a^2*A*Sin[c + d*x] + 113*A*b^2*Sin[c + d*x]
+ 209*a*b*B*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2*(1145*a^2*A*b*Sin[c + d*
x] + 15*A*b^3*Sin[c + d*x] + 539*a^3*B*Sin[c + d*x] + 825*a*b^2*B*Sin[c + d
*x]))/(3465*a) + (2*Sec[c + d*x]*(675*a^4*A*Sin[c + d*x] + 1025*a^2*A*b^2*S
in[c + d*x] - 20*A*b^4*Sin[c + d*x] + 1793*a^3*b*B*Sin[c + d*x] + 55*a*b^3*
```

B*Sin[c + d*x]))/(3465*a^2) + (2*a^2*A*Sec[c + d*x]^4*Tan[c + d*x])/11))/d
+ (2*((-247*a^2*A*b)/(231*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (1
7*A*b^3)/(231*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^5)/(693
*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (7*a^3*B)/(15*Sqrt[a +
b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (31*a*b^2*B)/(35*Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]) + (2*b^4*B)/(63*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]) + (15*a^3*A*Sqrt[Sec[c + d*x]])/(77*Sqrt[a + b*Cos[c + d*x]])
- (26*a*A*b^2*Sqrt[Sec[c + d*x]])/(231*Sqrt[a + b*Cos[c + d*x]]) - (7*A*b^4
*Sqrt[Sec[c + d*x]])/(99*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^6*Sqrt[Sec[c
+ d*x]])/(693*a^3*Sqrt[a + b*Cos[c + d*x]]) + (38*a^2*b*B*Sqrt[Sec[c + d*x]
])/(105*Sqrt[a + b*Cos[c + d*x]]) - (124*b^3*B*Sqrt[Sec[c + d*x]])/(315*Sqr
t[a + b*Cos[c + d*x]]) + (2*b^5*B*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Co
s[c + d*x]]) - (247*a*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*Sqrt[
a + b*Cos[c + d*x]]) - (17*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*
a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^6*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])
/(693*a^3*Sqrt[a + b*Cos[c + d*x]]) - (7*a^2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[
c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (31*b^3*B*Cos[2*(c + d*x)]*Sqrt[
Sec[c + d*x]])/(35*Sqrt[a + b*Cos[c + d*x]]) + (2*b^5*B*Cos[2*(c + d*x)]*Sq
rt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*(-2*(a + b)*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617
*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*
A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3*a^4*(22
5*A + 539*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] - (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*
a^3*b^2*B - 110*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(3465*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*
x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)
*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 1
10*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] + 2*a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*
A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3*a^4*(225*A + 539*B))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3705*a^4*A*b
 + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Co
s[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3465
a^3(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(3705*a^4*A*b + 255*a^
2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)
*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(5

```
05*A + 209*B) + 3*a^4*(225*A + 539*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A
*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3465*a^3*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*(-1/2*((3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^
2*B - 110*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4) -
((a + b)*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b
^2*B - 110*a*b^4*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c
 + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B)
 + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(505*A +...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5380 vs. $2(604) = 1208$.
time = 1.00, size = 5381, normalized size = 8.13

method	result	size
default	Expression too large to display	5381

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x,method=_RET
URNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(13/
2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(13/
2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(13/
2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{13/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(13/2)*(a + b*cos(c + d*x))^(5/2)
,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(13/2)*(a + b*cos(c + d*x))^(5/2)
, x)
```

3.606 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=562

$$\frac{2(a-b)\sqrt{a+b}\,(147a^4A+279a^2Ab^2-10Ab^4+435a^3bB+45ab^3B)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\Bigg(\text{ArcSin}\Bigg(}{315a^3d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/9*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2)*sin(d*x+c)/d+2/315*(163*A*a
^2*b+5*A*b^3+75*B*a^3+135*B*a*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x
+c))^(1/2)/a/d+2/315*(49*A*a^2+75*A*b^2+135*B*a*b)*sec(d*x+c)^(5/2)*sin(d*x
+c)*(a+b*cos(d*x+c))^(1/2)/d+2/21*a*(4*A*b+3*B*a)*sec(d*x+c)^(7/2)*sin(d*x+
c)*(a+b*cos(d*x+c))^(1/2)/d+2/315*(a-b)*(147*A*a^4+279*A*a^2*b^2-10*A*b^4+4
35*B*a^3*b+45*B*a*b^3)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)
^(1/2)-2/315*(a-b)*(10*A*b^3-6*a^2*b*(19*A-60*B)+3*a^3*(49*A-25*B)+15*a*b^2
*(11*A-3*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x
+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 1.29, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3040, 3068, 3126, 3134, 3077, 2895, 3073}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B
+ 45*a*b^3*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^
3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(10*A*b^3 - 6*a^2*b*(19*A
- 60*B) + 3*a^3*(49*A - 25*B) + 15*a*b^2*(11*A - 3*B))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(163*
a^2*A*b + 5*A*b^3 + 75*a^3*B + 135*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(315*a*d) + (2*(49*a^2*A + 75*A*b^2 + 135*a*b*B)
*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*a*(
```

```
4*A*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(2
1*d) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(
9*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\text{c}}{}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\sin(}{9d}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{7}{2}}(c}{21d}$$

$$= \frac{2(49a^2A+75Ab^2+135abB)\sqrt{a+b\cos(c}}{315d}$$

$$= \frac{2(163a^2Ab+5Ab^3+75a^3B+135ab^2B)\sqrt{}}{315}$$

$$= \frac{2(163a^2Ab+5Ab^3+75a^3B+135ab^2B)\sqrt{}}{315}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(147a^4A+279a^2Ab^2-10}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3755 vs. 2(562) = 1124.
time = 25.90, size = 3755, normalized size = 6.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4*A + 279*a^2*A*b^2
 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sin[c + d*x])/(315*a^2) + (2*Sec[c
+ d*x]^3*(19*a*A*b*Sin[c + d*x] + 9*a^2*B*Sin[c + d*x]))/63 + (2*Sec[c + d*
x]^2*(49*a^2*A*Sin[c + d*x] + 75*A*b^2*Sin[c + d*x] + 135*a*b*B*Sin[c + d*x
]))/315 + (2*Sec[c + d*x]*(163*a^2*A*b*Sin[c + d*x] + 5*A*b^3*Sin[c + d*x]
+ 75*a^3*B*Sin[c + d*x] + 135*a*b^2*B*Sin[c + d*x]))/(315*a) + (2*a^2*A*Sec
[c + d*x]^3*Tan[c + d*x])/9))/d + (2*((-7*a^3*A)/(15*Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]) - (31*a*A*b^2)/(35*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) + (2*A*b^4)/(63*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
- (29*a^2*b*B)/(21*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b^3*B)/(
7*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (38*a^2*A*b*Sqrt[Sec[c + d
```

*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) - (124*A*b^3*Sqrt[Sec[c + d*x]])/(315*
Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^5*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b
*Cos[c + d*x]]) + (5*a^3*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]
) - (2*a*b^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (b^4*B*S
qrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Cos[c + d*x]]) - (7*a^2*A*b*Cos[2*(c + d
*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^3*Cos[2*(c
 + d*x)]*Sqrt[Sec[c + d*x]])/(35*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^5*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Cos[c + d*x]]) - (29*a*b
^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (
b^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Cos[c + d*x]]))*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(147*a^4*A + 279*a^2*A*b^
2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-10*A*b^3 + 15*a*b^2*
(11*A + 3*B) + 3*a^3*(49*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (147*a^4*A + 27
9*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[
c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^2*d*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*Sin[c + d*x]*(-2*(a + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b
*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)] + 2*a*(a + b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3*(49
*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)] - (147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 +
 435*a^3*b*B + 45*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(315*a^2*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d
*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a
 + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a
(a + b)(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3*(49*A + 25*B) + 6*a^2*b
*(19*A + 60*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d
x])/((a + b)(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)] - (147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*
b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
]))/(315*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*(-1/2*((147*a^4*A + 279*a^2*A*b^2 - 10*A*b^
4 + 435*a^3*b*B + 45*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*
x)/2]^4) - ((a + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 4
5*a*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
E[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/
(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/

```
(1 + Cos[c + d*x])] + (a*(a + b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3
*(49*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c
 + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(147*a^4*A + 27
9*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*S
in[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c +
d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))] + (a*(a + b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3
*(49*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[Co...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4399 vs. $2(510) = 1020$.
time = 0.73, size = 4400, normalized size = 7.83

method	result	size
default	Expression too large to display	4400

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/315/d*(-10*A*cos(d*x+c)^6*b^5-45*B*cos(d*x+c)*a^5-98*A*cos(d*x+c)^4*a^5-
14*A*cos(d*x+c)^2*a^5+147*A*cos(d*x+c)^5*a^5+10*A*cos(d*x+c)^5*b^5+75*B*cos
(d*x+c)^5*a^5-30*B*cos(d*x+c)^3*a^5+45*B*cos(d*x+c)^6*a*b^4+45*B*cos(d*x+c)
^5*a^2*b^3-10*A*cos(d*x+c)^5*a*b^4-80*A*cos(d*x+c)^3*a^2*b^3-170*A*cos(d*x+
c)^2*a^3*b^2-330*B*cos(d*x+c)^4*a^4*b-180*B*cos(d*x+c)^4*a^2*b^3-82*A*cos(d
*x+c)^3*a^4*b+75*B*cos(d*x+c)^6*a^4*b+435*B*cos(d*x+c)^6*a^3*b^2+135*B*cos(
d*x+c)^6*a^2*b^3+65*A*cos(d*x+c)^5*a^4*b+279*A*cos(d*x+c)^5*a^3*b^2-199*A*c
os(d*x+c)^5*a^2*b^3+435*B*cos(d*x+c)^5*a^4*b-165*B*cos(d*x+c)^5*a^3*b^2-45*
B*cos(d*x+c)^5*a*b^4-272*A*cos(d*x+c)^4*a^3*b^2+5*A*cos(d*x+c)^4*a*b^4+147*
A*cos(d*x+c)^6*a^4*b+163*A*cos(d*x+c)^6*a^3*b^2+279*A*cos(d*x+c)^6*a^2*b^3+
5*A*cos(d*x+c)^6*a*b^4-35*A*a^5-270*B*cos(d*x+c)^3*a^3*b^2-180*B*cos(d*x+c)
^2*a^4*b-130*A*cos(d*x+c)*a^4*b+147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a
^5+10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)^5*sin(d*x+c)*b^5+75*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+147*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
```

```
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^5-147*A
*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^5+10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*b^5+75*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^5-10*A*cos(d*x+
c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b^4-147*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-279*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-279*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(
d*x+c)*a^2*b^3+10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^4+435*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^4*b+405*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin
(d*x+c)*a^3*b^2+45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^2*b^3-435*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^4*b-435*B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*
sin(d*x+c)*a^3*b^2-45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^2*b^3-45*B*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^4+261*A*
cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^4*b+279*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+155*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a^2*b^3-10
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(11/
2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(11/
2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(11/
2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos (c+d\,x))\,{\left(\frac{1}{\cos (c+d\,x)}\right)}^{11/2}\,{(a+b\cos (c+d\,x))}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(5/2)
,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(5/2)
, x)
```

3.607 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=474

$$\frac{2(a-b)\sqrt{a+b}\,(145a^2Ab+15Ab^3+63a^3B+161ab^2B)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\,\sqrt{c}}\right.\right.}{105a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/7*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2/105*(25*A*a^
2+45*A*b^2+77*B*a*b)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2
/35*a*(10*A*b+7*B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2
/105*(a-b)*(145*A*a^2*b+15*A*b^3+63*B*a^3+161*B*a*b^2)*csc(d*x+c)*EllipticE
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/105*(a-b)*(a^2*(25*A-63*B)+15*b^2*
(A-7*B)-8*a*b*(15*A-7*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c
)^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3040, 3068, 3126, 3134, 3077, 2895, 3073}

$$\frac{2(a-b)\sqrt{a+b}\,(a^2(25A-63B)-8ab(15A-7B)+15b^2(A-7B))\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{105ad\sqrt{\sec(c+dx)}}+\frac{2(25a^2A+77abB+45Ab^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}+\frac{2(a-b)\sqrt{a+b}\,(63a^3B+145a^2Ab+161ab^2B+15Ab^3)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{105a^2d\sqrt{\sec(c+dx)}}+\frac{2a(7aB+10Ab)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{35d}+\frac{2aA\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d*Sqrt[Sec[c +
 d*x]]) + (2*(a - b)*Sqrt[a + b]*(a^2*(25*A - 63*B) + 15*b^2*(A - 7*B) - 8*
a*b*(15*A - 7*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(10
5*a*d*Sqrt[Sec[c + d*x]]) + (2*(25*a^2*A + 45*A*b^2 + 77*a*b*B)*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*a*(10*A*b + 7*a
*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a
*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\sin(c-}{7d}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c-}{35d}$$

$$= \frac{2(25a^2A+45Ab^2+77abB)\,\sqrt{a+b\cos(c+d}}{105d}$$

$$= \frac{2(25a^2A+45Ab^2+77abB)\,\sqrt{a+b\cos(c+d}}{105d}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(145a^2Ab+15Ab^3+63a^3B-}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3348 vs. $2(474) = 948$.
time = 24.59, size = 3348, normalized size = 7.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(145*a^2*A*b + 15*A*b^3 +
63*a^3*B + 161*a*b^2*B)*Sin[c + d*x])/(105*a) + (2*Sec[c + d*x]^2*(15*a*A*b
*Sin[c + d*x] + 7*a^2*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(25*a^2*A*Sin[c
 + d*x] + 45*A*b^2*Sin[c + d*x] + 77*a*b*B*Sin[c + d*x]))/105 + (2*a^2*A*Se
c[c + d*x]^2*Tan[c + d*x])/7))/d + (2*((-29*a^2*A*b)/(21*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) - (A*b^3)/(7*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]]) - (3*a^3*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (23
*a*b^2*B)/(15*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a^3*A*Sqrt[
Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (2*a*A*b^2*Sqrt[Sec[c + d*x]
])/(21*Sqrt[a + b*Cos[c + d*x]]) - (A*b^4*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a +
 b*Cos[c + d*x]]) + (8*a^2*b*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d
*x]]) - (8*b^3*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (29*a*
A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) -
(A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Cos[c + d*x]])
```

- (3*a^2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x
]]) - (23*b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c +
 d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(145*a^2*A*b + 1
5*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15
*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)] - (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b
^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
))/(105*a*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(145*a^2*A*b + 15*A*b
^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15*A +
7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)] - (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1
05*a*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(145*a^2*A*b + 15*A*b^
3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15*A + 7
B) + a^2(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] - (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*
Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(10
5*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(-1/2*((145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^
2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4) - ((a + b)*(145*
a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 +
Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(15*b^2*
(A + 7*B) + 8*a*b*(15*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x
]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(14
5*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x]
)/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))] + (a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15*A + 7*B) + a^2*(
25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c

```
 + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2))
)/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(145*a^2*A*b
+ 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c
+ d*x]*Tan[(c + d*x)/2] + (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)
*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (1
45*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + ...
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3635 vs. $2(428) = 856$.
time = 0.58, size = 3636, normalized size = 7.67

method	result	size
default	Expression too large to display	3636

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*(15*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-63*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-161*B*cos(d*x+c)^
4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b^2-161*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+119*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+161*B*cos(d*x+c
)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b^2-145*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-145*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-15*A*sin(d*
x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^3+145*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+135*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
```

```
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+15*A*cos(d
*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^3-63*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-161*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-161*B*cos(
d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b^3+119*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+145*A*cos(d*x+c)^5*a^2*b^2-98*B*cos
(d*x+c)^2*a^3*b+25*A*cos(d*x+c)^5*a^3*b+161*B*cos(d*x+c)^5*a*b^3+145*A*cos(
d*x+c)^4*a^3*b-55*A*cos(d*x+c)^4*a^2*b^2+15*A*cos(d*x+c)^4*a*b^3+35*B*cos(d
*x+c)^4*a^3*b+161*B*cos(d*x+c)^4*a^2*b^2-161*B*cos(d*x+c)^4*a*b^3-15*A*a^4-
21*B*cos(d*x+c)*a^4-15*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*cos(d*x+c)^4*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*sin(d*
x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^4+63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-15*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*sin(d*x+c)*cos(
d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
4-63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^4+63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-60*A*cos(d*x+c)*a^3*b+161*B*c
os(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*b^2-145*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-145*A*cos(d*x+c)^4*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2
), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{9/2}(a+b\cos(c+dx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(5/2),
 x)
```

3.608 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=553

$$\frac{2(a-b)\sqrt{a+b}\,(9a^2A+23Ab^2+35abB)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\right.}{15ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2/5*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/15*a*(8*A*b+
5*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/15*(a-b)*(9*A
*a^2+23*A*b^2+35*B*a*b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a
*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^
(1/2)+2/15*(15*A*b^3-a*b^2*(23*A-45*B)+a^2*b*(17*A-35*B)-a^3*(9*A-5*B))*csc
(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-2*b^2*B*csc(d*x+c)*El
lipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.91, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3040, 3068, 3126, 3132, 2888, 3077, 2895, 3073}

$$\frac{2(a-b)\sqrt{a+b}\,(9a^2A+35abB+23Ab^2)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15ad\sqrt{\sec(c+dx)}}+\frac{2\sqrt{a+b}\,(-(a^3(9A-5B))+a^2b(17A-35B)-ab^2(23A-45B)+15Ab^3)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15ad\sqrt{\sec(c+dx)}}+\frac{2a(5aB+8Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{15d}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}-\frac{2b^2B\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a +
 b]*(15*A*b^3 - a*b^2*(23*A - 45*B) + a^2*b*(17*A - 35*B) - a^3*(9*A - 5*B)
)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c
+ d*x]]) - (2*b^2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[
(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a*(8*A*b + 5*a*B)*Sqrt[a
```

```
 + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*(a + b*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
```

```
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\sin(c}{5d}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c-}{15d}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c-}{15d}$$

$$= -\frac{2b^2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(9a^2A+23Ab^2+35abB)\,\surd}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7032 vs. 2(553) = 1106.
time = 25.52, size = 7032, normalized size = 12.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2
),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3281 vs. 2(501) = 1002.
time = 0.50, size = 3282, normalized size = 5.93

method	result	size
default	Expression too large to display	3282

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

[Out] -2/15/d*(9*A*cos(d*x+c)^4*a^2*b+11*A*cos(d*x+c)^4*a*b^2+5*B*cos(d*x+c)^4*a^2*b+35*B*cos(d*x+c)^4*a*b^2+5*A*cos(d*x+c)^3*a^2*b+23*A*cos(d*x+c)^3*a*b^2+35*B*cos(d*x+c)^3*a^2*b-35*B*cos(d*x+c)^3*a*b^2-3*A*a^3+5*B*cos(d*x+c)^3*a^3+23*A*cos(d*x+c)^4*b^3+9*A*cos(d*x+c)^3*a^3-23*A*cos(d*x+c)^3*b^3-6*A*cos(d*x+c)^2*a^3+17*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+23*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-23*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+35*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-35*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-35*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2+17*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b+23*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b-23*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2+35*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b-35*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-35*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+45*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+45*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-5*B*cos(d*x+c)*a^3-34*A*cos(d*x+c)^2*a*b^2-40*B*cos(d*x+c)^2*a^2*b+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3-14*A*cos(d*x+c)*a^2*b-9*A*sin(d*x+c)*cos(d

```
*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
-23*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)^3*b^3+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3-9*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^2*a^3-23*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*cos(d*x+c)^2*b^3+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3-15*B*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3
+30*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*b^3+15*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2
), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(5/2),
 x)
```

3.609 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=596

$$\frac{(a-b)\sqrt{a+b}\,(14aAb+6a^2B-3b^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{3ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*(2*A*b+B*a
)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-1/3*(14*A*a*b+6*B*a^
2-3*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d+1/3*(a-b)*(
14*A*a*b+6*B*a^2-3*B*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c
)^(1/2)-1/3*(2*a*b*(7*A-9*B)-2*a^2*(A-3*B)-3*b^2*(6*A+B))*csc(d*x+c)*Ellipt
icF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2
))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*
x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-b*(2*A*b+5*B*a)*csc(d*x+c)*EllipticPi
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 1.19, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3040, 3068, 3126, 3140, 3132, 2888, 3077, 2895, 3073}

$$-\frac{\sqrt{a+b}\left(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6A+B)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3d\sqrt{\sec(c+dx)}}+\frac{(a-b)\sqrt{a+b}\left(6a^2B+14aAb-3b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad\sqrt{\sec(c+dx)}}-\frac{\left(6a^2B+14aAb-3b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{3d}-\frac{b\sqrt{a+b}(5aB+2Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}+\frac{2a(aB+2Ab)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2
*a*b*(7*A - 9*B) - 2*a^2*(A - 3*B) - 3*b^2*(6*A + B))*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d*Sqrt[Sec[c + d*x]]) - (b*Sqrt[a + b]*
(2*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
```

```
- b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a*(2*A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d - ((14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + b*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

<u>Rule 3077</u>

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

<u>Rule 3126</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

<u>Rule 3132</u>

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

<u>Rule 3140</u>

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sin(c}{3d}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+}}{d}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+}}{d}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+}}{d}$$

$$= -\frac{b\sqrt{a+b}(2Ab+5aB)\sqrt{\cos(c+dx)}\csc(c}{}$$

$$= \frac{(a-b)\sqrt{a+b}(14aAb+6a^2B-3b^2B)\sqrt{\cos}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7700 vs. $2(596) = 1192$.
time = 25.76, size = 7700, normalized size = 12.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2
),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3211 vs. $2(540) = 1080$.

time = 0.48, size = 3212, normalized size = 5.39

method	result	size
default	Expression too large to display	3212

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/3/d*(2*A*cos(d*x+c)^3*a^2*b+14*A*cos(d*x+c)^3*a*b^2+6*B*cos(d*x+c)^3*a^2
*b+3*B*cos(d*x+c)^3*a*b^2-2*A*a^3+2*A*cos(d*x+c)^2*a^3+3*B*cos(d*x+c)^4*b^3
+14*A*cos(d*x+c)^2*a^2*b-3*B*cos(d*x+c)^2*a*b^2-3*B*cos(d*x+c)^3*b^3+6*B*co
s(d*x+c)^2*a^3+12*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^3-6*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^3-6*B*cos(d*x+c)*si
n(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^
3+3*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*b^3+14*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b+18*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2-14*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)^2*a^2*b-14*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b-6*B*sin(d
*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-18*B*sin(d
*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
```

```
^(1/2))*a*b^2-6*B*cos(d*x+c)*a^3-14*A*cos(d*x+c)^2*a*b^2-6*B*cos(d*x+c)^2*a
^2*b+18*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*a*b^2-14*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b-14*A*cos(d*x+c)*sin(d*x+c)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2+30*B
*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*a*b^2+18*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b-18*B*cos(d*x+c)*sin(d*x+c)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2-6*B*cos(d
*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*a^2*b+3*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2-16*A*cos(d*x+c)*a^2*b+14*A*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*
b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)^2*a^3+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3+12*A*cos(d*x+c)^2*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-6*B*c
os(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^3+3*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+2*A*cos(d*...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2
), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2),
 x)
```

3.610 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=607

$$\frac{(a-b)\sqrt{a+b}\,(8a^2A-4Ab^2-9abB)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\cdots}{4ad\sqrt{\sec(c+dx)}}$$

```
[Out] -1/2*b*(4*A*a-B*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+2*a
*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-1/4*(8*A*a^2-4*A*b^
2-9*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d+1/4*(a-b)*(
8*A*a^2-4*A*b^2-9*B*a*b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)
^(1/2)-1/4*(8*a^2*(A-B)-2*b^2*(2*A+B)-3*a*b*(8*A+3*B))*csc(d*x+c)*EllipticF
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-1/4*(20*A*a*b+15*B*a^2+4*B*b^2)*csc(d*x+
c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(
(-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 1.19, antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3040, 3068, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$-\frac{\sqrt{a+b}\,(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+B))\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4d\sqrt{\sec(c+dx)}}+\frac{(a-b)\sqrt{a+b}\,(8a^2A-9abB-4Ab^2)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4ad\sqrt{\sec(c+dx)}}-\frac{\sqrt{a+b}\,(15a^2B+20aAb+4b^2B)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4d\sqrt{\sec(c+dx)}}-\frac{(8a^2A-9abB-4Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{4d}-\frac{b(4aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{2d\sqrt{\sec(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(8*a^2*A - 4*A*b^2 - 9*a*b*B)*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(4*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*
a^2*(A - B) - 2*b^2*(2*A + B) - 3*a*b*(8*A + 3*B))*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(20*a
*A*b + 15*a^2*B + 4*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a +
b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
```

```
 d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (b*(4*a*A - b*B)*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) - ((8*a^2*A - 4*A*b^2 -
9*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d) +
(2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
```

```
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{co}}{}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,\sin}{d}$$

$$= -\frac{b(4aA-bB)\sqrt{a+b\cos(c+dx)}\,\sin(c+}{2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{b(4aA-bB)\sqrt{a+b\cos(c+dx)}\,\sin(c+}{2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{b(4aA-bB)\sqrt{a+b\cos(c+dx)}\,\sin(c+}{2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\,(20aAb+15a^2B+4b^2B)\,\sqrt{\cos(c}}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,(8a^2A-4Ab^2-9abB)\,\sqrt{\cos}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1278 vs. 2(607) = 1214.
time = 19.08, size = 1278, normalized size = 2.11

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(2*a^2*A*Sin[c + d*x] + (b^2*B
*Sin[2*(c + d*x)])/4))/d + (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-8*a^3*A*T
```

```
an[(c + d*x)/2] - 8*a^2*A*b*Tan[(c + d*x)/2] + 4*a*A*b^2*Tan[(c + d*x)/2] +
 4*A*b^3*Tan[(c + d*x)/2] + 9*a^2*b*B*Tan[(c + d*x)/2] + 9*a*b^2*B*Tan[(c +
 d*x)/2] + 16*a^2*A*b*Tan[(c + d*x)/2]^3 - 8*A*b^3*Tan[(c + d*x)/2]^3 - 18*
a*b^2*B*Tan[(c + d*x)/2]^3 + 8*a^3*A*Tan[(c + d*x)/2]^5 - 8*a^2*A*b*Tan[(c
+ d*x)/2]^5 - 4*a*A*b^2*Tan[(c + d*x)/2]^5 + 4*A*b^3*Tan[(c + d*x)/2]^5 - 9
*a^2*b*B*Tan[(c + d*x)/2]^5 + 9*a*b^2*B*Tan[(c + d*x)/2]^5 + 40*a*A*b^2*Ell
ipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b
)] + 30*a^2*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)] + 8*b^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 40*a*A*b^2*EllipticPi[-1, ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
 + 30*a^2*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)
/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^3*B*EllipticPi[-1, ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] -
 (a + b)*(8*a^2*A - 4*A*b^2 - 9*a*b*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*(12*a^
2*b*(A - B) - 2*b^3*B + a*b^2*(-12*A + B) + 4*a^3*(A + B))*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)]))/(4*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*
x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3277 vs. $2(549) = 1098$.

time = 0.51, size = 3278, normalized size = 5.40

method	result	size
default	Expression too large to display	3278

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(-11*B*cos(d*x+c)^3*a*b^2+8*A*a^3-30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+8*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-4
```

```
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)*sin(d*x+c)*b^3-8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3-40*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-
4*A*cos(d*x+c)^3*b^3-2*B*cos(d*x+c)^4*b^3-8*A*cos(d*x+c)^2*a^2*b+9*B*cos(d*
x+c)^2*a*b^2+4*A*cos(d*x+c)^2*b^3+2*B*cos(d*x+c)^2*b^3+24*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+8*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)
-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*
sin(d*x+c)+24*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b*sin(d*x+c)-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b^2*sin(d*x+c)-9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-8*A*cos(d*x+c)*a^3-40*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*a*b^2-30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b-4*A*cos(d*x+c)^2*a*b^2-9*B*
cos(d*x+c)^2*a^2*b+24*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2+8*A*cos(d*x+c)*sin(d*x+c)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b-4*A*cos(d*x+c
)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*a*b^2+24*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*a^2*b-2*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2-9*B*cos(d*x+c)*sin(d*x+c
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b-9*B
```

```
*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*a*b^2+9*B*cos(d*x+c)*a^2*b+2*B*cos(d*x+c)*a*b^2+8*A*cos(d*x+c)*
a^2*b+4*A*cos(d*x+c)*a*b^2-24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+8*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-4*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-8*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-8*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3*s
in(d*x+c)-8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^3*sin(d*x+c)+4*B*(cos(d*x+c)/(1+cos(d*x+c)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2
), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2),
 x)
```

3.611 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}$

Optimal. Leaf size=624

$$-\frac{(a-b)\sqrt{a+b}\,(54aAb+33a^2B+16b^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\right.}{24ad\sqrt{\sec(c+dx)}}$$

```
[Out] 1/3*b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+1/4*b*(2*A*b+3
*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/24*(54*A*a*b+3
3*B*a^2+16*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-1/24
*(a-b)*(54*A*a*b+33*B*a^2+16*B*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x
+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
/sec(d*x+c)^(1/2)+1/24*(4*b^2*(3*A+4*B)+a^2*(48*A+33*B)+a*(54*A*b+26*B*b))*
csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((
-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-1/8*(30*A*a^2*b+8*A*
b^3+5*B*a^3+20*B*a*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/se
c(d*x+c)^(1/2)
```

Rubi [A]
time = 1.22, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3040, 3069, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$\frac{\sqrt{a+b}\,(a^2(48A+33B)+a(54Ab+26bB)+4b^2(3A+4B))\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{24d\sqrt{\sec(c+dx)}} - \frac{(a-b)\sqrt{a+b}\,(33a^2B+54aAb+16b^2B)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{24ad\sqrt{\sec(c+dx)}} + \frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24d} - \frac{\sqrt{a+b}\,(5a^3B+30a^2Ab+20ab^2B+8Ab^3)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{8bd\sqrt{\sec(c+dx)}} + \frac{b(3aB+2Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4d\sqrt{\sec(c+dx)}} + \frac{bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] -1/24*((a - b)*Sqrt[a + b]*(54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[Cos[c + d*
x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a +
 b]*(4*b^2*(3*A + 4*B) + a^2*(48*A + 33*B) + a*(54*A*b + 26*b*B))*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d*Sqrt[Sec[c + d*x]]) - (S
qrt[a + b]*(30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
```

```
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b*d*Sqrt[Sec[c + d*x]])
+ (b*(2*A*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c
 + d*x]]) + (b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
 d*x]]) + ((54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(24*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
```

```
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b}{}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\Big($$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+}{4d\sqrt{\sec(c+dx)}}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+}{4d\sqrt{\sec(c+dx)}}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+}{4d\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\,(30a^2Ab+8Ab^3+5a^3B+20ab}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(54aAb+33a^2B+16b^2B}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1504 vs. $2(624) = 1248$.

time = 19.62, size = 1504, normalized size = 2.41

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*B*Sin[c + d*x])/12 + (b*
(6*A*b + 13*a*B)*Sin[2*(c + d*x)])/24 + (b^2*B*Sin[3*(c + d*x)])/12))/d + (
```

```
Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(54*a^2*A*b*Tan[(c + d*x)/2] + 54*a*A*b
^2*Tan[(c + d*x)/2] + 33*a^3*B*Tan[(c + d*x)/2] + 33*a^2*b*B*Tan[(c + d*x)/
2] + 16*a*b^2*B*Tan[(c + d*x)/2] + 16*b^3*B*Tan[(c + d*x)/2] - 108*a*A*b^2*
Tan[(c + d*x)/2]^3 - 66*a^2*b*B*Tan[(c + d*x)/2]^3 - 32*b^3*B*Tan[(c + d*x)
/2]^3 - 54*a^2*A*b*Tan[(c + d*x)/2]^5 + 54*a*A*b^2*Tan[(c + d*x)/2]^5 - 33*
a^3*B*Tan[(c + d*x)/2]^5 + 33*a^2*b*B*Tan[(c + d*x)/2]^5 - 16*a*b^2*B*Tan[(
c + d*x)/2]^5 + 16*b^3*B*Tan[(c + d*x)/2]^5 + 180*a^2*A*b*EllipticPi[-1, Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*A*b^3*
EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
+ b)] + 30*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
 + d*x)/2]^2)/(a + b)] + 120*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 180*a^2*A*b*EllipticPi[-1, Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)] + 48*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^3*B*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
b)] + 120*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(54*a*A*b + 33*a^2*B +
 16*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2
]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*(-12*A*b^3 + 2*a*b^2*(3*A - 19*B)
+ 24*a^3*(A - B) + a^2*(-72*A*b + 13*b*B))*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*
d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan
[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3513 vs. $2(564) = 1128$.

time = 0.54, size = 3514, normalized size = 5.63

method	result	size
default	Expression too large to display	3514

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

[Out] -1/24/d*(34*B*cos(d*x+c)^4*a*b^2+66*A*cos(d*x+c)^3*a*b^2+59*B*cos(d*x+c)^3*
a^2*b+12*A*cos(d*x+c)^4*b^3+54*A*cos(d*x+c)^2*a^2*b-18*B*cos(d*x+c)^2*a*b^2
-12*A*cos(d*x+c)^2*b^3+8*B*cos(d*x+c)^5*b^3+8*B*cos(d*x+c)^3*b^3+33*B*cos(d
*x+c)^2*a^3-16*B*cos(d*x+c)^2*b^3-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+30*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+33*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+16*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+4
8*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*b^3-24*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^3+30*B*cos(d*x+c)*sin(d*x+c)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^3+33*B*cos
(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*a^3+16*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*b^3+180*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+12*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+54*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+54*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d
*x+c)+120*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*a*b^2*sin(d*x+c)+26*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-76*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+33*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+16*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+48*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x

```
+c)-33*B*cos(d*x+c)*a^3-54*A*cos(d*x+c)^2*a*b^2-33*B*cos(d*x+c)^2*a^2*b+180
*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*a^2*b+12*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2+54*A*cos(d*x+c)*sin(d*x+c)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b+54*A*co
s(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*a*b^2+120*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2+26*B*cos(d*x+c)*sin(d*x+c)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b-76*B*cos(d*
x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*a*b^2+33*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b+16*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)
), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2),
 x)
```

3.612 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=724

$$-\frac{(a-b)\sqrt{a+b}\,(264a^2Ab+128Ab^3+15a^3B+284ab^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\,c}}{\sqrt{a+b}\,\sqrt{}}\right.\right.\cdots}{192abd\sqrt{\sec(c+dx)}}$$

```
[Out] 1/4*b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+1/24*(8*A*b+11
*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+1/32*(24*A*a*b+5
*B*a^2+12*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/192
*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(
1/2)*sec(d*x+c)^(1/2)/b/d-1/192*(a-b)*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B
*a*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/sec(d*x+c)^(1/2)+1/192*(1
5*a^3*B+8*b^3*(16*A+9*B)+2*a^2*b*(132*A+59*B)+4*a*b^2*(52*A+71*B))*csc(d*x+
c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)-1/64*(40*A*a^3*b+160*A*a*b
^3-5*B*a^4+120*B*a^2*b^2+48*B*b^4)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)
*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/b^2/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 1.58, antiderivative size = 724, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3040, 3069, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$\frac{(5a^2B+24aAb+12b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{32d\sqrt{\sec(c+dx)}}+\frac{\sqrt{a+b}\,(15a^3B+2a^2b(132A+59B)+4ab^2(52A+71B)+8b^3(16A+9B))\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{192bd\sqrt{\sec(c+dx)}}-\frac{(a-b)\sqrt{a+b}\,(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{192abd\sqrt{\sec(c+dx)}}+\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{192bd}-\frac{\sqrt{a+b}\,(-5a^4B+40a^3Ab+120a^2b^2B+160aAb^3+48b^4B)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{64b^2d\sqrt{\sec(c+dx)}}+\frac{(11aB+8Ab)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{24d\sqrt{\sec(c+dx)}}+\frac{bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{4d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] -1/192*((a - b)*Sqrt[a + b]*(264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2
*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d*Sqrt[Sec[c
 + d*x]]) + (Sqrt[a + b]*(15*a^3*B + 8*b^3*(16*A + 9*B) + 2*a^2*b*(132*A +
59*B) + 4*a*b^2*(52*A + 71*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
```

(a - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(40*a^3*A*b + 160*a*A*b^3 - 5*a^4*B + 120*a^2*b^2*B + 48*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d*Sqrt[Sec[c + d*x]]) + (b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/2)) + ((24*a*A*b + 5*a^2*B + 12*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*d*Sqrt[Sec[c + d*x]]) + ((8*A*b + 11*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*d*Sqrt[Sec[c + d*x]]) + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b*d)

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
```

```
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,(a+b$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\right.$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{(8Ab+11aB)(a}{24}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{(24aAb+5a^2B}{}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{(24aAb+5a^2B}{}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{(24aAb+5a^2B}{}$$

$$= -\frac{\sqrt{a+b}\,(40a^3Ab+160aAb^3-5a^4B+120a^2b^2B+48}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(264a^2Ab+128Ab^3+15a^3B+284ab^2}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1857 vs.

2(724) = 1448.
time = 20.23, size = 1857, normalized size = 2.56

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*(8*A*b + 17*a*B)*Sin[c + d
*x])/96 + ((104*a*A*b + 59*a^2*B + 48*b^2*B)*Sin[2*(c + d*x)])/192 + (b*(8*
A*b + 17*a*B)*Sin[3*(c + d*x)])/96 + (b^2*B*Sin[4*(c + d*x)])/32))/d + (Sqr
t[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(264*a^3*A*b*Tan[(c + d*x)/2] + 264*a^2*A*
b^2*Tan[(c + d*x)/2] + 128*a*A*b^3*Tan[(c + d*x)/2] + 128*A*b^4*Tan[(c + d*
x)/2] + 15*a^4*B*Tan[(c + d*x)/2] + 15*a^3*b*B*Tan[(c + d*x)/2] + 284*a^2*b
^2*B*Tan[(c + d*x)/2] + 284*a*b^3*B*Tan[(c + d*x)/2] - 528*a^2*A*b^2*Tan[(c
 + d*x)/2]^3 - 256*A*b^4*Tan[(c + d*x)/2]^3 - 30*a^3*b*B*Tan[(c + d*x)/2]^3
 - 568*a*b^3*B*Tan[(c + d*x)/2]^3 - 264*a^3*A*b*Tan[(c + d*x)/2]^5 + 264*a^
2*A*b^2*Tan[(c + d*x)/2]^5 - 128*a*A*b^3*Tan[(c + d*x)/2]^5 + 128*A*b^4*Tan
[(c + d*x)/2]^5 - 15*a^4*B*Tan[(c + d*x)/2]^5 + 15*a^3*b*B*Tan[(c + d*x)/2]
^5 - 284*a^2*b^2*B*Tan[(c + d*x)/2]^5 + 284*a*b^3*B*Tan[(c + d*x)/2]^5 + 24
0*a^3*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] + 960*a*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^4*B*EllipticPi[-1, ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 720*a^2*b^2*B*Ellipt
icPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
+ 288*b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(a + b)] + 240*a^3*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 960*a*A*b^3*Elli
pticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*S
qrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
 d*x)/2]^2)/(a + b)] - 30*a^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 720*a^2*b^2*B*El
lipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
 + d*x)/2]^2)/(a + b)] + 288*b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(264*
```

```
a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]
^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2
*b*(a^3*(192*A - 59*B) + 4*a*b^2*(76*A - 9*B) + 72*b^3*B + a^2*(-104*A*b +
322*b*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(192*b*d*(1 + Tan[(c + d*x)/2]^2)^(3/
2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
 d*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4239 vs. $2(658) = 1316$.
time = 0.70, size = 4240, normalized size = 5.86

method	result	size
default	Expression too large to display	4240

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/192/d*(-208*A*cos(d*x+c)*a^2*b^2-128*A*cos(d*x+c)*a*b^3-118*B*cos(d*x+c)
*a^3*b-284*B*cos(d*x+c)*a^2*b^2-72*B*cos(d*x+c)*a*b^3-15*B*cos(d*x+c)^2*a^3
*b+184*B*cos(d*x+c)^5*a*b^3+272*A*cos(d*x+c)^4*a*b^3+254*B*cos(d*x+c)^4*a^2
*b^2+128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)*sin(d*x+c)*b^4-144*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^4+15*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4-30*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
*sin(d*x+c)*a^4+288*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^4+208*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-608*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+264*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x
+c)+264*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b^2*sin(d*x+c)+128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
```

```
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^3*sin(d*x+c)+240*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+208*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2-608*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*a*b^3+264*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+264*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+128*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a*b^3+240*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+960*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+118*B*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
sin(d*x+c)*a^3*b-644*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+72*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+15*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*a^3*b+284*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+284*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+720*B*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*a^2*b^2+64*A*cos(d*x+c)^3*b^4-128*A*cos(d*x+c)^2*b^4+15*B*cos(d*x+c)^2
*a^4-72*B*cos(d*x+c)^2*b^4+48*B*cos(d*x+c)^6*b^4+24*B*cos(d*x+c)^4*b^4-15*B
*cos(d*x+c)*a^4-384*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b+128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4*sin(d*x+c...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)
), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{5/2}}{\sqrt{\frac{1}{\cos(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2
),x)
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2
), x)
```

3.613 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=839

$$-\frac{(a-b)\sqrt{a+b}\,(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2B+1024b^4B)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\Big(\mathrm{A}}{1920ab^2d\sqrt{\sec(c+dx}}$$

[Out] 1/240*(50*A*a*b-15*B*a^2+64*B*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1/40*(10*A*b-3*B*a)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1/5*B*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1/320*(50*A*a^2*b+120*A*b^3-15*B*a^3+172*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/1920*(150*A*a^3*b+2840*A*a*b^3-45*B*a^4+1692*B*a^2*b^2+1024*B*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b^2/d-1/1920*(a-b)*(150*A*a^3*b+2840*A*a*b^3-45*B*a^4+1692*B*a^2*b^2+1024*B*b^4)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d/sec(d*x+c)^(1/2)-1/1920*(45*a^4*B-30*a^3*b*(5*A+B)-16*b^4*(45*A+64*B)-8*a*b^3*(355*A+193*B)-4*a^2*b^2*(295*A+423*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)+1/128*(10*A*a^4*b-240*A*a^2*b^3-96*A*b^5-3*B*a^5-40*B*a^3*b^2-240*B*a*b^4)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)

Rubi [A]
time = 2.24, antiderivative size = 839, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3040, 3069, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

Antiderivative was successfully verified.

[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]

[Out] -1/1920*((a - b)*Sqrt[a + b]*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2*B + 1024*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(45*a^4*B - 30*a^3*b*(5*A + B) - 16*b^4*(45*A + 64*B) - 8*a*b^3*(355*A + 193*B) - 4*a^2*b^2*(295*A + 42

```
3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*b^2*d*Sqr
t[Sec[c + d*x]]) + (Sqrt[a + b]*(10*a^4*A*b - 240*a^2*A*b^3 - 96*A*b^5 - 3*
a^5*B - 40*a^3*b^2*B - 240*a*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(128*b^3*d*Sqrt[Sec[c + d*x]]) + ((50*a^2*A*b +
120*A*b^3 - 15*a^3*B + 172*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(320*b*d*Sqrt[Sec[c + d*x]]) + ((50*a*A*b - 15*a^2*B + 64*b^2*B)*(a + b*Cos
[c + d*x])^(3/2)*Sin[c + d*x])/(240*b*d*Sqrt[Sec[c + d*x]]) + ((10*A*b - 3*
a*B)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(40*b*d*Sqrt[Sec[c + d*x]]) +
 (B*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(5*b*d*Sqrt[Sec[c + d*x]]) + (
(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2*B + 1024*b^4*B)*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1920*b^2*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
```

```
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
```

```
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\,\mathrm{c}$$

$$= \frac{B(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{5bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{(10Ab-3aB)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{40bd\sqrt{\sec(c+dx)}} + \frac{B(a}{}$$

$$= \frac{\left(50aAb-15a^2B+64b^2B\right)(a+b\cos(c+dx))^{3/2}\sin(c+}{240bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(50a^2Ab+120Ab^3-15a^3B+172ab^2B\right)\sqrt{a+b\cos(c+}}{320bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(50a^2Ab+120Ab^3-15a^3B+172ab^2B\right)\sqrt{a+b\cos(c+}}{320bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(50a^2Ab+120Ab^3-15a^3B+172ab^2B\right)\sqrt{a+b\cos(c+}}{320bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{a+b}\,\left(10a^4Ab-240a^2Ab^3-96Ab^5-3a^5B-40a^3b^2\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\left(150a^3Ab+2840aAb^3-45a^4B+1692\right.}{}$$

Mathematica [A]
time = 16.37, size = 703, normalized size = 0.84

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((170*a*A*b + 93*a^2*B + 88*b
^2*B)*Sin[c + d*x])/960 + ((590*a^2*A*b + 480*A*b^3 + 15*a^3*B + 1024*a*b^2
```

```
*B)*Sin[2*(c + d*x)])/(1920*b) + ((170*a*A*b + 93*a^2*B + 100*b^2*B)*Sin[3*
(c + d*x)])/960 + (b*(10*A*b + 21*a*B)*Sin[4*(c + d*x)])/320 + (b^2*B*Sin[5
*(c + d*x)])/80))/d - (-(b*(a + b)*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B +
 1692*a^2*b^2*B + 1024*b^4*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)]) + a*(a + b)*(45*a^4*B - 30*a^3*b*(5*A + 3*B) + 60*a^2*b^2*(5*A +
11*B) + 16*b^4*(45*A + 64*B) + 8*a*b^3*(265*A + 129*B))*EllipticF[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c +
d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 15*(10*a^4*A*b - 240*a^2*A*b^3 - 96*A*
b^5 - 3*a^5*B - 40*a^3*b^2*B - 240*a*b^4*B)*((a - b)*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] + 2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)] - b*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^
2*b^2*B + 1024*b^4*B)*(a + b*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2
)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2])/(1920*b^3*d*Sqrt[a + b*Cos[c + d*x]]
*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5171 vs. $2(767) = 1534$.
time = 0.92, size = 5172, normalized size = 6.16

method	result	size
default	Expression too large to display	5172

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2
), x)
```

3.614 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=403

$$\frac{2(a-b)\sqrt{a+b}\,(9a^2A+8Ab^2-10abB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\right.}{15a^4d\sqrt{\sec(c+dx)}}$$

```
[Out] -2/15*(4*A*b-5*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/
d+2/5*A*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d+2/15*(a-b)*(
9*A*a^2+8*A*b^2-10*B*a*b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/sec(d*x
+c)^(1/2)-2/15*(8*A*b^2+a^2*(9*A-5*B)-2*a*b*(A+5*B))*csc(d*x+c)*EllipticF((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.63, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3079, 3134, 3077, 2895, 3073}

$$-\frac{2(4Ab-5aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{15a^2d}+\frac{2(a-b)\sqrt{a+b}\,(9a^2A-10abB+8Ab^2)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{15a^4d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{a+b}\,(a^2(9A-5B)-2ab(A+5B)+8Ab^2)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{15a^3d\sqrt{\sec(c+dx)}}+\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^4*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a
+ b]*(8*A*b^2 + a^2*(9*A - 5*B) - 2*a*b*(A + 5*B))*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(4*A*b - 5
*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d)
+ (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
```

```
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
```

)

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+d}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{\left(2\sqrt{\cos(c+d}\right.}{}$$

$$= -\frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} + \frac{2}{}$$

$$= -\frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} + \frac{2}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(9a^2A+8Ab^2-10abB)\,\sqrt{\cos(c+dx)}\,\csc(c+d}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2987 vs. $2(403) = 806$.
time = 22.29, size = 2987, normalized size = 7.41

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + b*Cos[c + d*x]
],x]

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A + 8*A*b^2 - 10*a*
b*B)*Sin[c + d*x])/(15*a^3) + (2*Sec[c + d*x]*(-4*A*b*Sin[c + d*x] + 5*a*B*
Sin[c + d*x]))/(15*a^2) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(5*a)))/d + (2*((
-3*A)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^2)/(15*a^2*S
qrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b*B)/(3*a*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]]) - (7*A*b*Sqrt[Sec[c + d*x]])/(15*a*Sqrt[a + b*
Cos[c + d*x]]) - (8*A*b^3*Sqrt[Sec[c + d*x]])/(15*a^3*Sqrt[a + b*Cos[c + d*
x]]) + (B*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*B*Sqrt[
Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Cos[c + d*x]]) - (3*A*b*Cos[2*(c + d*x)]*S
qrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^3*Cos[2*(c + d*x
)]*Sqrt[Sec[c + d*x]])/(15*a^3*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*B*Cos[2*(
c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(
8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2*A + 8*A*b^2 - 10*a*b*B)
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1
5*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(9*a^2*A + 8*A*b^2 - 10*a
*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)] + 2*a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2*A + 8*A*b^2
- 10*a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2]))/(15*a^3*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(9*a^2*A +
 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)] + 2*a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a
^2*A + 8*A*b^2 - 10*a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(15*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)
/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1/2*((9*a^2*A + 8*A*b^
2 - 10*a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4) - ((a +
 b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos
[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*
x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(8*A*b^2 + 2*a*b*(A - 5*B)

```
 + a^2*(9*A + 5*B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c
 + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Co
s[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(8*A*b^2 + 2*a*b*(A - 5*B) + a
^2*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2
)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(9*a^2*A +
8*A*b^2 - 10*a*b*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d
*x)/2] + (9*a^2*A + 8*A*b^2 - 10*a*b*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (9*a^2*A + 8*A*b^2 - 10*a*b*B)*Cos[c +
 d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(8*A*
b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2
]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(
a + b)]) - ((a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(
c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan
[(c + d*x)/2]^2]))/(15*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2
]) + ((-2*(a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2487 vs. $2(363) = 726$.
time = 0.48, size = 2488, normalized size = 6.17

method	result	size
default	Expression too large to display	2488

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*(9*A*cos(d*x+c)^4*a^2*b-4*A*cos(d*x+c)^4*a*b^2+5*B*cos(d*x+c)^4*a^2
*b-10*B*cos(d*x+c)^4*a*b^2-10*A*cos(d*x+c)^3*a^2*b+8*A*cos(d*x+c)^3*a*b^2-1
0*B*cos(d*x+c)^3*a^2*b+10*B*cos(d*x+c)^3*a*b^2-3*A*a^3+5*B*cos(d*x+c)^3*a^3
+8*A*cos(d*x+c)^4*b^3+9*A*cos(d*x+c)^3*a^3-8*A*cos(d*x+c)^3*b^3-6*A*cos(d*x
+c)^2*a^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
```

+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2-10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b+10*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+10*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-5*B*cos(d*x+c)*a^3-4*A*cos(d*x+c)^2*a*b^2+5*B*cos(d*x+c)^2*a^2*b+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3+A*cos(d*x+c)*a^2*b-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^3+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b

```
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^3+5*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3)*cos(d*x+c
)*(1/cos(d*x+c))^(7/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b*cos(c + d*x))^(1/2
), x)
```

3.615 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=330

$$-\frac{2(a-b)\sqrt{a+b}\,(2Ab-3aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{3a^3d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*A*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d-2/3*(a-b)*(2*A
*b-3*B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)+2/3*(2
*A*b+a*(A-3*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos
(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3079, 3077, 2895, 3073}

$$-\frac{2(a-b)\sqrt{a+b}\,(2Ab-3aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{a+b}\,(a(A-3B)+2Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(3*a^3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(2*A*b + a
*(A - 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d*
Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[
c + d*x])/(3*a*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

```
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+d}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c+d}}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\left((2Ab+a(A-\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(2Ab-3aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-}\right.}{3}$$

Mathematica [A]
time = 15.61, size = 355, normalized size = 1.08

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2(a+b)(-2Ab+3aB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+2a(-2Ab+a(A+3B))\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+(2Ab-3aB)\cos(c+dx)(a+b\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^2d\sqrt{a+b\cos(c+dx)}\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}}+\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(-2Ab+3aB)\sin(c+dx)}{3a^2}+\frac{2A\tan(c+dx)}{3a}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-2*A*b + 3*a*B)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(
-2*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)] + (2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-2*
A*b + 3*a*B)*Sin[c + d*x])/(3*a^2) + (2*A*Tan[c + d*x])/(3*a)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1543 vs. $2(296) = 592$.
time = 0.42, size = 1544, normalized size = 4.68

method	result	size
default	Expression too large to display	1544

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(-a^2*A+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-2*A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(
d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-3*B*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*cos(d*x+c)^2*a^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2+3*B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2-
3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)^2*a^2+A*cos(d*x+c)^2*a^2+3*B*cos(d*x+c)^2*a^2-3*B*cos(d*x+c)
*a^2-2*A*cos(d*x+c)^3*b^2+2*A*cos(d*x+c)^2*b^2+2*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+
c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)^2*a*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*cos(d*x+c)^2*a*b-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+2*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*c
os(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-3*
B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2+3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*b-2*A*cos(d*x+c)^2
*a*b+A*cos(d*x+c)*a*b+A*cos(d*x+c)^3*a*b)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(
a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a),
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(1/2
), x)
```

3.616 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=270

$$\frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+a}{a+b}}}{a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 2*A*(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)-2*(A-B
)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3040, 3077, 2895, 3073}

$$\frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{a+b}(A-B)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(A - B)*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+}}\\ &= \left(A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c}}\\ &= \frac{2A(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right.\right.}{a^2d\sqrt{s}}\end{aligned}$$

Mathematica [A]

time = 14.16, size = 279, normalized size = 1.03

$$\frac{2\left(A(a+b\cos(c+dx))\sqrt{\sec(c+dx)}\,\sin(c+dx)-\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2A(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}-2a(A+B)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}+A\cos(c+dx)(a+b\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*(A*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*A*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] - 2*a*(A + B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] + A*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)/2]^2]))/(a*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(246) = 492$.
time = 0.46, size = 812, normalized size = 3.01

method	result
default	$-\frac{2\left(A\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)a-A\sin(dx+c)\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/d*(A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+A*cos(d*x+c)^2*b+A*cos(d*x+c)*a-A*cos(d*x+c)*b-a*A)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(1/2
), x)
```

3.617 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=268

$$\frac{2A\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2*A*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-2*B*csc(d*x+c)
*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3040, 3085, 2888, 2895}

$$\frac{2A\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}-\frac{2B\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*A*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d
*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3085

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[B/d, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Dist
[(B*c - A*d)/d, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c}}$$

$$= \left(A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(}}$$

$$= \frac{2A\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\,\sqrt{\cos(}}\right.\right.}{ad\sqrt{\sec(}}$$

Mathematica [A]
time = 2.63, size = 157, normalized size = 0.59

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left((A-B)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)+2B\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\right)\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\,\sqrt{1+\sec(c+dx)}}{d\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((A - B)*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*B*EllipticPi[-1, ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]
*Sqrt[1 + Sec[c + d*x]])/(d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^
2])
```

Maple [A]
time = 0.44, size = 199, normalized size = 0.74

method	result
default	$\frac{2\sqrt{\frac{1}{\cos(dx+c)}}\left(A\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)-B\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)+2B\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+}{\sin(dx+c)}\right.\right.}{d\sqrt{a+b\cos(dx+c)}\,(-1+\cos(dx+c))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(A*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))+2*B*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(
a+b))^(1/2)))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a + b\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/sqrt(a + b*cos(c + d*x)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\,\cos(c + d\,x))\,\sqrt{\frac{1}{\cos(c + d\,x)}}}{\sqrt{a + b\,\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(1/2
), x)
```

3.618 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=487

$$-\frac{(a-b)\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{abd\sqrt{\sec(c+dx)}}$$

```
[Out] B*sin(d*x+c)/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+a*B*sin(d*x+c)*sec(d
*x+c)^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)-(a-b)*B*csc(d*x+c)*EllipticE((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1
/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b)
)^(1/2)/a/b/d/sec(d*x+c)^(1/2)+B*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/se
c(d*x+c)^(1/2)-(2*A*b-B*a)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x
+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2
/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.79, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3040, 3082, 3130, 2888, 3072, 3077, 2895, 3073}

$$-\frac{\sqrt{a+b}\,(2Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}+\frac{B\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}-\frac{B(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd\sqrt{\sec(c+dx)}}+\frac{aB\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\cos(c+dx)}}+\frac{B\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*A*b
- a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b
^2*d*Sqrt[Sec[c + d*x]]) + (B*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) + (a*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[a + b*Co
s[c + d*x]])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3082

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a +
 b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)
*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(
a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3130

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{1}{2}\Big(\sqrt{\cos(c+dx)}\sqrt{\sec(}$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c}\right.}{}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\right.\right.}{b}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\right.\right.}{b}$$

$$= -\frac{(a-b)\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+b}}\right.\right.}{abd\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.06, size = 1091, normalized size = 2.24

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] (Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d
*x)/2]^2)]*(a*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2] + b*Sqrt[(a - b)/(a
+ b)]*B*Tan[(c + d*x)/2] - 2*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2]^3 -
 a*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + b*Sqrt[(a - b)/(a + b)]*B*T
an[(c + d*x)/2]^5 - (4*I)*A*b*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a
 - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
 + (2*I)*a*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Ta
n[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*A*b*Elli
pticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]],
-((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
```

```
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*a*B*El
lipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]]
, -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*
B*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a
 - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b +
 a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(A*b - a*B)*
EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a -
 b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b*Sqrt[(a - b)/(a +
 b)]*d*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c
+ d*x)/2]^2)]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. $2(443) = 886$.
time = 0.55, size = 1007, normalized size = 2.07

method	result	size
default	Expression too large to display	1007

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*cos(d*x+c)*b-4*A*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a-B*sin(d*x+c
)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b+2*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*b-4*A*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b+2*B*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a-B*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-B*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*Elliptic
```

```
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-B*cos(d*x+c)^3*b-B*cos
(d*x+c)^2*a+b*B*cos(d*x+c)^2+B*cos(d*x+c)*a)*(1/cos(d*x+c))^(1/2)/sin(d*x+c
)/(a+b*cos(d*x+c))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)))
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{(c + dx)}}{\sqrt{a + b\cos{(c + dx)}}\,\sqrt{\sec{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x)))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2)
), x)
```

3.619 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=539

$$-\frac{(a-b)\sqrt{a+b}\,(4Ab-3aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{4ab^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 1/2*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/4*(4*A*b-3*B
*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b^2/d-1/4*(a-b)*(4*A
*b-3*B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d/sec(d*x+c)^(1/2)+1/4*
(4*A*b-3*B*a+2*B*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-
sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1
/2)+1/4*(4*A*a*b-3*B*a^2-4*B*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*
cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.78, antiderivative size = 539, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3040, 3069, 3140, 3132, 2888, 3077, 2895, 3073}

$$\frac{\sqrt{a+b}\,(-3a^2B+4aAb-4b^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^3d\sqrt{\sec(c+dx)}}+\frac{\sqrt{a+b}\,(-3aB+4Ab+2bB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d\sqrt{\sec(c+dx)}}-\frac{(a-b)\sqrt{a+b}\,(4Ab-3aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4ab^2d\sqrt{\sec(c+dx)}}+\frac{(4Ab-3aB)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{4b^2d}+\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{2bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -1/4*((a - b)*Sqrt[a + b]*(4*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(a*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(4*A*b - 3
*a*B + 2*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*
d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(4*a*A*b - 3*a^2*B - 4*b^2*B)*Sqrt[Cos
[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d*Sqrt[Sec
[c + d*x]]) + (B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d*Sqrt[Sec[c +
```

```
d*x]]) + ((4*A*b - 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(4*b^2*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
```

```
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{(4Ab-3aB)\sqrt{a+b\cos(c}{}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{(4Ab-3aB)\sqrt{a+b\cos(c}{}$$

$$= \frac{\sqrt{a+b}\,(4aAb-3a^2B-4b^2B)\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+}{b}\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(4Ab-3aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-}\right.}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1157 vs. $2(539) = 1078$.
time = 19.09, size = 1157, normalized size = 2.15

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] (B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*b*d) +
(Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-4*a*A*b*Tan[(c + d*x)/2] -
 4*A*b^2*Tan[(c + d*x)/2] + 3*a^2*B*Tan[(c + d*x)/2] + 3*a*b*B*Tan[(c + d*x
)/2] + 8*A*b^2*Tan[(c + d*x)/2]^3 - 6*a*b*B*Tan[(c + d*x)/2]^3 + 4*a*A*b*Ta
n[(c + d*x)/2]^5 - 4*A*b^2*Tan[(c + d*x)/2]^5 - 3*a^2*B*Tan[(c + d*x)/2]^5
+ 3*a*b*B*Tan[(c + d*x)/2]^5 + 8*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c
+ d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^2*B*EllipticPi[-1, ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
```

```
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*b^2*B*Elli
pticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)
] + 8*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[
(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2
]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^2*B*EllipticPi[-1, ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8
*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(-4*A*b + 3*a*B)*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)] - 2*(a - 2*b)*b*B*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*b^2*d*Sqrt[1 + T
an[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2
)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1878 vs. $2(485) = 970$.
time = 0.44, size = 1879, normalized size = 3.49

method	result	size
default	Expression too large to display	1879

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*(4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)
*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
a*b-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
```

```
a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*cos(d*x+c)^4*b^2-2*B*cos(d*x+
c)^2*b^2-3*B*cos(d*x+c)^2*a^2+3*B*cos(d*x+c)*a^2+4*A*cos(d*x+c)^3*b^2-4*A*c
os(d*x+c)^2*b^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*A*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b-3*B*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*b^2-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2
+8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+c)^3*a*b+3*B*cos(d*x+c)^2*a*b-2*B*cos(d*x
+c)*a*b+4*A*cos(d*x+c)^2*a*b-4*A*cos(d*x+c)*a*b)*cos(d*x+c)*(1/cos(d*x+c))^
(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/b^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2))
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{\left(c + dx \right)}}{\sqrt{a + b\cos{\left(c + dx \right)}}\sec^{\frac{3}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**(3/2)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2)
), x)
```

3.620 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=433

$$-\frac{2(5a^2Ab-8Ab^3-3a^3B+6ab^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{3a^4\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

```
[Out] 2*b*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1
/2)+2/3*(A*a^2-4*A*b^2+3*B*a*b)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c)
)^(1/2)/a^2/(a^2-b^2)/d-2/3*(5*A*a^2*b-8*A*b^3-3*B*a^3+6*B*a*b^2)*csc(d*x+c
)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-
b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/a^4/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+2/3*(a+2*b)*(4*A*b+a*(A-3
*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/
2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.73, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3079, 3134, 3077, 2895, 3073}

$$\frac{2(a+2b)(a(A-3B)+4Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d\sqrt{a+b}\sqrt{\sec(c+dx)}}+\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2(-3a^3B+5a^2Ab+6ab^2B-8Ab^3)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^4d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(a
 + 2*b)*(4*A*b + a*(A - 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(3*a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*
(a^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
```

```
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
```

)

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+d \ldots}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec \ldots}\right.}{\ldots}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2(a^2A-4Ab^2+3abB) \ldots}{\ldots}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2(a^2A-4Ab^2+3abB) \ldots}{\ldots}$$

$$= -\frac{2(5a^2Ab-8Ab^3-3a^3B+6ab^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)\ldots}{\ldots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3433 vs. $2(433) = 866$.

time = 24.33, size = 3433, normalized size = 7.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-5*a^2*A*b + 8*A*b^3 + 3*
a^3*B - 6*a*b^2*B)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)) + (2*(-(A*b^3*Sin[c +
d*x]) + a*b^2*B*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*
A*Tan[c + d*x])/(3*a^2)))/d + (2*((5*A*b)/(3*(a^2 - b^2)*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^3)/(3*a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) - (a*B)/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) + (2*b^2*B)/(a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) + (a*A*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]) + (7*A*b^2*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[a + b*Cos[c +
 d*x]]) - (8*A*b^4*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]) - (2*b*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]
) + (2*b^3*B*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]])
 + (5*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[a +
b*Cos[c + d*x]]) - (8*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^
2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x
]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*B*Cos[2*(c + d*x)]*Sqrt
[Sec[c + d*x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^
2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] + 2*a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (-5*a^2*A*b + 8*A*b^3 +
 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(
c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(
a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a^2 - a*b - 2*b^2)
*(-4*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] - (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Cos[c
+ d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a
^2 - b^2)*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(-5*a^2*A*b + 8*A
*b^3 + 3*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)] + 2*a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] -
```

```
(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos
[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d
*x]]*(-1/2*((-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^4) - ((a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^
3*B - 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c +
d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])] + (a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*S
qrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c +
 d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])] - ((a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d
*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A +
3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) +
((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(-5*a^2*A*b + 8*A*b^3
 + 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c
 + d*x)/2] + (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*(a + b*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-5*a^2*A*b + 8*A*b^
3 + 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]^2 + (a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Co...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3342 vs. $2(395) = 790$.
time = 0.46, size = 3343, normalized size = 7.72

method	result	size
default	Expression too large to display	3343

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(A*a^2*b^2-4*A*cos(d*x+c)*a*b^3+3*B*cos(d*x+c)*a^2*b^2+5*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x
+c)*a^3*b+5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^2-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
```

```
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^3-3*B*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*
a^3*b-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)^2*sin(d*x+c)*a^2*b^2-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b+6*B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*
b^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)^2*sin(d*x+c)*a*b^3-3*B*cos(d*x+c)^2*a^3*b-8*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^4-3*B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a^4-A*a^4-5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+8*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*a*b^3+5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a^3*b+5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2-8*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3
-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*a^3*b-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2-3*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+6*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d
*x+c)*a^2*b^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+8*A*cos(d*x+c)^3*b^4-8*A*cos(d*x+c)^2*b^
4+3*B*cos(d*x+c)^2*a^4-3*B*cos(d*x+c)*a^4-5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
```

```
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b+4*A*cos(d*x+
c)*a^3*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)^2*sin(d*x+c)*a^2*b^2+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^3+A*cos(d*x+c)^2*a^4
+4*A*cos(d*x+c)^2*a^2*b^2+3*B*cos(d*x+c)^3*a^2*b^2+A*cos(d*x+c)^3*a^3*b-4*A
*cos(d*x+c)^3*a*b^3-6*B*cos(d*x+c)^2*a^2*b^2+6*B*cos(d*x+c)^2*a*b^3-5*A*cos
(d*x+c)^3*a^2*b^2+3*B*cos(d*x+c)^3*a^3*b-6*B*cos(d*x+c)^3*a*b^3-5*A*cos(d*x
+c)^2*a^3*b+8*A*cos(d*x+c)^2*a*b^3+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4-8*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^4+
3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(3/2
), x)
```

3.621 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=345

$$\frac{2(a^2A-2Ab^2+abB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a+}}}{a^3\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

```
[Out] 2*b*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1
/2)+2*(A*a^2-2*A*b^2+B*a*b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)/sec(d
*x+c)^(1/2)-2*(2*A*b+a*(A-B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(
a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)/sec
(d*x+c)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3079, 3077, 2895, 3073}

$$-\frac{2(a(A-B)+2Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{a+b}\,\sqrt{\sec(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\,(a^2-b^2)\,\sqrt{a+b\cos(c+dx)}}+\frac{2(a^2A+abB-2Ab^2)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^3d\sqrt{a+b}\,\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2*A - 2*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(2*A*b + a*(A - B))*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d*Sqrt[
Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2
- b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

```
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+d$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}\right.}{}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left((a-b)(2Ab+a(A-\right.}{}$$

$$= \frac{2\left(a^2A-2Ab^2+abB\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.\right.}{a^3\sqrt{a+}}$$

Mathematica [**A**]
time = 18.18, size = 433, normalized size = 1.26

$$\frac{\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}\left(\frac{2\left(a^2A-2Ab^2+abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)}-\frac{2\left(-Ab^2\sin(c+dx)+abB\sin(c+dx)\right)}{a\left(a^2-b^2\right)(a+b\cos(c+dx))}\right)}{d}+\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2(a+b)\left(a^2A-2Ab^2+abB\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+2a(a+b)(-2Ab+a(A+B))\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)-\left(a^2A-2Ab^2+abB\right)\cos(c+dx)(a+b\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a^2*A - 2*A*b^2 + a*b*B)*
Sin[c + d*x])/(a^2*(a^2 - b^2)) - (2*(-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c +
 d*x]))/(a*(a^2 - b^2)*(a + b*Cos[c + d*x]))))/d + (2*Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*(-2*(a + b)*(a^2*A - 2*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b
+ a*(A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)] - (a^2*A - 2*A*b^2 + a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2290 vs. $2(317) = 634$.
time = 0.51, size = 2291, normalized size = 6.64

method	result	size

default	Expression too large to display	2291

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/d*(A*a*b^2-A*a^3-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+A*cos(d*x+c)^2*a^2*
b+B*cos(d*x+c)^2*a*b^2-2*A*cos(d*x+c)^2*b^3-2*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*
x+c)-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b*sin(d*x+c)-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^2*sin(d*x+c)+A*cos(d*x+c)*a^3+A*cos(d*x+c)^2*a*b^2-B*cos(d*x+c)^2*a
^2*b+2*A*cos(d*x+c)*b^3-2*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2-A*cos(d*x+c)*sin(d*x+c)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b+2*A*cos(d*x
+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*a*b^2+B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*a^2*b-B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b-B*cos(d*x+c)*sin(d*x+c)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2+B*cos(d*
x+c)*a^2*b-B*cos(d*x+c)*a*b^2-A*cos(d*x+c)*a^2*b-2*A*cos(d*x+c)*a*b^2-A*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*a^2*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
```

```
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3*sin(d*x+c)+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*b^3*sin(d*x+c)+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+A*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+
c)*a^3)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a
^2/(a-b)/(a+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(3/2
), x)
```

3.622 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=324

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{a^2\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

```
[Out] -2*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
+2*(A*b-B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+2*(
A+B)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/
2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3040, 3072, 3077, 2895, 3073}

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{a+b}\,\sqrt{\sec(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(A+B)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}\,\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*S
qrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A + B)*Sqrt[Cos[c + d*x]]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(A*b - a
*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]
])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx}{\sqrt{\cos(c+dx)}\,(a+b\cos(c}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{\left((a-b)(A+B)\sqrt{\right.}{}$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\,\sqrt{\mathrm{co}}}\right.\right.}{a^2\sqrt{a+b}\,d}$$

Mathematica [**A**]
time = 13.61, size = 305, normalized size = 0.94

$$\frac{2\left(\frac{b(Ab-aB)\sin(c+dx)}{\sqrt{\sec(c+dx)}} + \frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(a+b)(-Ab+aB)E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{b+a\sec(c+dx)}{(a+b)(1+\sec(c+dx))}} + 2a(a+b)(A-B)F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{b+a\sec(c+dx)}{(a+b)(1+\sec(c+dx))}} - (Ab-aB)\cos(c+dx)(a+b\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}}\right)}{(a^3-ab^2)\,d\sqrt{a+b\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (2*((b*(A*b - a*B)*Sin[c + d*x])/Sqrt[Sec[c + d*x]] + (Sqrt[Cos[(c + d*x)/2
]^2*Sec[c + d*x]]*(2*(a + b)*(-(A*b) + a*B)*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*
x])/((a + b)*(1 + Sec[c + d*x]))] + 2*a*(a + b)*(A - B)*EllipticF[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b +
a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (A*b - a*B)*Cos[c + d*x]*(a
 + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)
/2]^2]))/((a^3 - a*b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1635 vs. $2(296) = 592$.
time = 0.50, size = 1636, normalized size = 5.05

method	result	size
default	Expression too large to display	1636

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/d*(A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*cos(d*x+c)^2*a*b+A*cos(d*x+c)^2*b^2+B*cos(d*x+c)^2*a^2-B*cos(d*x+c)^2*a*b+A*cos(d*x+c)*a*b-A*cos(d*x+c)*b^2-B*cos(d*x+c)*a^2+B*cos(d*x+c)*a*b)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/a/(a-b)/(a+b)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**(3/2
), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(3/2
), x)
```

3.623 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=476

$$-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{ab\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

```
[Out] 2*a*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1
/2)-2*(A*b-B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos
(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+
2*(A*b-B*a)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x
+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)-2*B*
csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))
/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3040, 3071, 2888, 2873, 2874, 2895, 3073}

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd\sqrt{a+b}\sqrt{\sec(c+dx)}}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd\sqrt{a+b}\sqrt{\sec(c+dx)}}-\frac{2B\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*
Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*S
qrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(b^2*d*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2873

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[-2*a*d*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2874

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3071

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> D
```

```
ist[B/b, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Di
st[(A*b - a*B)/b, Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos}{(a+b\cos(c+dx))}$$

$$= \frac{\left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{b^2 c}$$

$$= -\frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{b^2 c}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,c}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{ab\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.90, size = 1403, normalized size = 2.95

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(A*b - a*B)*Sin[c + d*x])/
(b*(-a^2 + b^2)) - (2*(a*A*b*Sin[c + d*x] - a^2*B*Sin[c + d*x]))/(b*(-a^2 +
 b^2)*(a + b*Cos[c + d*x]))))/d + (2*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a*A*b*Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2] + A*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] - a^2*Sqrt
[(a - b)/(a + b)]*B*Tan[(c + d*x)/2] - a*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c +
 d*x)/2] - 2*A*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 + 2*a*b*Sqrt[(a
 - b)/(a + b)]*B*Tan[(c + d*x)/2]^3 - a*A*b*Sqrt[(a - b)/(a + b)]*Tan[(c +
d*x)/2]^5 + A*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 + a^2*Sqrt[(a -
b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - a*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x
)/2]^5 - (2*I)*a^2*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a
+ b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*
b^2*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c +
d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*B*EllipticP
i[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a
+ b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*b^2*B*Ellip
ticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -
((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*(-(
A*b) + a*B)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -(
(a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b
)*(-(A*b) + (2*a + b)*B)*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c +
 d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d
*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b
)]))/(b*Sqrt[(a - b)/(a + b)]*(a^2 - b^2)*d*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[
(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b*(-1 + Tan[(c + d*x)/2
]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2015 vs. $2(436) = 872$.

time = 0.49, size = 2016, normalized size = 4.24

method	result	size
default	Expression too large to display	2016

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/d*(A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
```

^(1/2))*b^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*b^2+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-B*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+c)^2*a^2+B*cos(d*x+c)*a^
2-A*cos(d*x+c)^2*b^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*
x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
b^2-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2-2*B*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*b^2+A*cos(d*x+c)*b^2-A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*b^2+B*cos(d*x+c)^2*a*b-B*cos(d*x+c)*a*b+A*cos(d
*x+c)^2*a*b-A*cos(d*x+c)*a*b)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+
c))^(1/2)/b/(a-b)/(a+b)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{(c + dx)}}{(a + b\cos{(c + dx)})^{\frac{3}{2}}\sqrt{\sec{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))**(3/2)*sqrt(sec(c + d*x
))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2)
),x)
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2)
), x)
```

3.624 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=560

$$\frac{(2aAb-3a^2B+b^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c}{a+}}{ab^2\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

```
[Out] 2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1
/2)-(2*A*a*b-3*B*a^2+B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1
/2)/b^2/(a^2-b^2)/d+(2*A*a*b-3*B*a^2+B*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^
(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d
/(a+b)^(1/2)/sec(d*x+c)^(1/2)-(2*A*b-(3*a+b)*B)*csc(d*x+c)*EllipticF((a+b*c
os(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x
+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2
/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)-(2*A*b-3*B*a)*csc(d*x+c)*EllipticPi((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.94, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3040, 3068, 3140, 3132, 2888, 3077, 2895, 3073}

$$\frac{(-3a^2B+2aAb+b^2B)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ab^2d\sqrt{a+b}\sqrt{\sec(c+dx)}}-\frac{(-3a^2B+2aAb+b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{b^2d(a^2-b^2)}+\frac{2a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\sqrt{a+b}(2Ab-3aB)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^3d\sqrt{\sec(c+dx)}}-\frac{(2Ab-B(3a+b))\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(a*b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - ((2*A*b - (3*a + b)*B)*S
qrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*Sqrt[a + b]*d*Sqr
t[Sec[c + d*x]]) - (Sqrt[a + b]*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) + (2*a*(A
*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
```

```
c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
```

```
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+}{(a+b\cos(c+dx))^{3/2}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}} - \frac{(2aAb-3a^2}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}} - \frac{(2aAb-3a^2}{}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-3aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\right.}{}$$

$$= \frac{(2aAb-3a^2B+b^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.\right.}{ab^2\sqrt{a+}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1551 vs. $2(560) = 1120$.

time = 19.39, size = 1551, normalized size = 2.77

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a*(-(A*b) + a*B)*Sin[c +
d*x])/(b^2*(a^2 - b^2)) + (2*(a^2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x]))/(
b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d - (Sqrt[(1 - Tan[(c + d*x)/2]^2)
^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[
(c + d*x)/2]^2)]*(2*a^2*A*b*Tan[(c + d*x)/2] + 2*a*A*b^2*Tan[(c + d*x)/2] -
 3*a^3*B*Tan[(c + d*x)/2] - 3*a^2*b*B*Tan[(c + d*x)/2] + a*b^2*B*Tan[(c + d
*x)/2] + b^3*B*Tan[(c + d*x)/2] - 4*a*A*b^2*Tan[(c + d*x)/2]^3 + 6*a^2*b*B*
Tan[(c + d*x)/2]^3 - 2*b^3*B*Tan[(c + d*x)/2]^3 - 2*a^2*A*b*Tan[(c + d*x)/2
]^5 + 2*a*A*b^2*Tan[(c + d*x)/2]^5 + 3*a^3*B*Tan[(c + d*x)/2]^5 - 3*a^2*b*B
*Tan[(c + d*x)/2]^5 - a*b^2*B*Tan[(c + d*x)/2]^5 + b^3*B*Tan[(c + d*x)/2]^5
```

```
 - 4*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d
*x)/2]^2)/(a + b)] + 4*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^
2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*B*EllipticPi[-1, ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a*b^2*B*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a^
2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d
*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*B
*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2
]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan
[(c + d*x)/2]^2)/(a + b)] - 6*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(-
2*a*A*b + 3*a^2*B - b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b*(a + b)*(-(A*b)
+ a*B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b^2*(-a^2 + b^2)*d*Sqrt[1 + Tan[(c + d*
x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2889 vs. $2(516) = 1032$.
time = 0.44, size = 2890, normalized size = 5.16

method	result	size
default	Expression too large to display	2890

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(-B*cos(d*x+c)^3*a^2*b+2*A*cos(d*x+c)^2*a^2*b+B*cos(d*x+c)^2*a*b^2+B*co
s(d*x+c)^3*b^3-3*B*cos(d*x+c)^2*a^3-B*cos(d*x+c)^2*b^3-2*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+6*B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-
3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
```

))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin
(d*x+c)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^3*sin(d*x+c)+4*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^3-2*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^3+6*B*cos(d*x+c)*sin(
d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
a^3-3*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*a^3+B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^3-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-2*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2
A(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*si
n(d*x+c)-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*a*b^2*sin(d*x+c)+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+B*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+4*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+3*
B*cos(d*x+c)*a^3-2*A*cos(d*x+c)^2*a*b^2+3*B*cos(d*x+c)^2*a^2*b-4*A*cos(d*x+
c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*a^2*b-2*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2+2*A*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b+2*A*cos(d*x+c)*sin(d
x+c)((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c

```
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2
-6*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*a*b^2+2*B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b+2*B*cos(d*x+c)*sin(d*x+c)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2-3*B*cos
(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*a^2*b+B*cos(d*x+c)*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2-2*B*cos(d*x+c)*a^2*b-B*cos(d*x+c)*a*b^2-2*
A*cos(d*x+c)*a^2*b+2*A*cos(d*x+c)*a*b^2)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/si
n(d*x+c)/(a+b*cos(d*x+c))^(1/2)/b^2/(a-b)/(a+b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^(3/2)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3436 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2)
), x)
```

3.625 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=607

$$-\frac{2(8a^4Ab-28a^2Ab^3+16Ab^5-3a^5B+15a^3b^2B-8ab^4B)\sqrt{\cos(c+dx)}\csc(c+dx)E\Bigg(\mathrm{ArcSin}\Bigg(\frac{\sqrt{a+b}}{\sqrt{a+b}}\cdots}{3a^5(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*b*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^
(3/2)+2/3*b*(10*A*a^2*b-6*A*b^3-7*B*a^3+3*B*a*b^2)*sec(d*x+c)^(3/2)*sin(d*x
+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(A*a^4-13*A*a^2*b^2+8*A*b^
4+8*B*a^3*b-4*B*a*b^3)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a
^3/(a^2-b^2)^2/d-2/3*(8*A*a^4*b-28*A*a^2*b^3+16*A*b^5-3*B*a^5+15*B*a^3*b^2-
8*B*a*b^4)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^5/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)
-2/3*(16*A*b^4-a^4*(A-3*B)+4*a*b^3*(3*A-2*B)-9*a^3*b*(A-B)-2*a^2*b^2*(8*A+3
*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/
2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a^2-b^2)/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 1.41, antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3079, 3134, 3077, 2895, 3073}

$$\frac{2b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}}+\frac{2b(-7a^3B+10a^2Ab+3ab^2B-6Ab^3)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(-(a^4(A-3B))-9a^3b(A-B)-2a^2b^2(8A+3B)+4ab^3(3A-2B)+16Ab^4)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^4d\sqrt{a+b}(a^2-b^2)\sqrt{\sec(c+dx)}}+\frac{2(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d(a^2-b^2)^2}-\frac{2(-3a^5B+8a^4Ab+15a^3b^2B-28a^2Ab^3-8ab^4B+16Ab^5)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^5d(a-b)(a+b)^{3/2}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(8*a^4*A*b - 28*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4
*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5*(a - b)*(a
 + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(16*A*b^4 - a^4*(A - 3*B) + 4*a*b^3*
(3*A - 2*B) - 9*a^3*b*(A - B) - 2*a^2*b^2*(8*A + 3*B))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec
[c + d*x]]) + (2*b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 -
 b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B
+ 3*a*b^2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a
```

+ b*Cos[c + d*x]]) + (2*(a^4*A - 13*a^2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d)

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
```

```
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+d$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec\right.}{}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b(10a^2Ab-6Ab^3-7a}{3a^2\left(a^2-b\right.}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b(10a^2Ab-6Ab^3-7a}{3a^2\left(a^2-b\right.}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b(10a^2Ab-6Ab^3-7a}{3a^2\left(a^2-b\right.}$$

$$= -\frac{2(8a^4Ab-28a^2Ab^3+16Ab^5-3a^5B+15a^3b^2B-8ab^4B)\sqrt{c}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4316 vs. 2(607) = 1214.
time = 27.04, size = 4316, normalized size = 7.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-8*a^4*A*b + 28*a^2*A*b^3
 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sin[c + d*x])/(3*a^4*(a^2
 - b^2)^2) + (2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x]))/(3*a^2*(a^2
 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-11*a^2*A*b^3*Sin[c + d*x] + 7*A*b^5*
Sin[c + d*x] + 8*a^3*b^2*B*Sin[c + d*x] - 4*a*b^4*B*Sin[c + d*x]))/(3*a^3*(
a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^3)))/d + (2*((
8*a*A*b)/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2
8*A*b^3)/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) +
(16*A*b^5)/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
) - (a^2*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (
5*b^2*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*b
^4*B)/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (
```

a^2*A*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (5*A *b^2*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (32*A*b ^4*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (16 *A*b^6*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (3*a*b*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (1 7*b^3*B*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (8*b^5*B*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (8*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (28*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a ^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (16*A*b^6*Cos[2*(c + d*x)]*Sqrt[Sec [c + d*x]])/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (5 *b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)^2*Sqrt[a + b*Cos [c + d*x]]) - (8*b^5*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b ^2)^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2* (a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8* a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] *Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[T an[(c + d*x)/2]], (-a + b)/(a + b)] - (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Se c[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]] *Sin[c + d*x]*(-2*(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]* EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (-8*a^4*A*b + 28*a^ 2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2* (a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/ 2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos [c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti cE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-16*A*b^4 + 2 *a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b) *(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4 *B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])) /(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) +

```
(2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1/2*((-8*a^4*A*b + 28*a^2*A*b^3
- 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c
+ d*x])*Sec[(c + d*x)/2]^4) - ((a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^
5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c
 + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(-16*A*b^4 +
2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*
B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c ...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8100 vs. $2(561) = 1122$.
time = 0.66, size = 8101, normalized size = 13.35

method	result	size
default	Expression too large to display	8101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(5/2
), x)
```

3.626 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=496

$$\frac{2(3a^4A-15a^2Ab^2+8Ab^4+6a^3bB-2ab^3B)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{3a^4(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^
(3/2)+2/3*b*(8*A*a^2*b-4*A*b^3-5*B*a^3+B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)
/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(3*A*a^4-15*A*a^2*b^2+8*A*b^4
+6*B*a^3*b-2*B*a*b^3)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/
2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d/sec(d
*x+c)^(1/2)+2/3*(8*A*b^3-3*a^3*(A-B)+2*a*b^2*(3*A-B)-3*a^2*b*(3*A+B))*csc(d
*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)
/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*
x+c))/(a-b))^(1/2)/a^3/(a^2-b^2)/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.89, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3079, 3134, 3077, 2895, 3073}

$$\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}}+\frac{2(-3a^3(A-B)-3a^2b(3A+B)+2ab^2(3A-B)+8Ab^3)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d\sqrt{a+b}(a^2-b^2)\sqrt{\sec(c+dx)}}+\frac{2b(-5a^3B+8a^2Ab+ab^2B-4Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}}+\frac{2(3a^4A+6a^3bB-15a^2Ab^2-2ab^3B+8Ab^4)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^4d(a-b)(a+b)^{3/2}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d*Sqr
t[Sec[c + d*x]]) + (2*(8*A*b^3 - 3*a^3*(A - B) + 2*a*b^2*(3*A - B) - 3*a^2*
b*(3*A + B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*S
qrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b
*(8*a^2*A*b - 4*A*b^3 - 5*a^3*B + a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+d$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{se\right.}{}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4Ab^3-5a}{3a^2\left(a^2-\right.}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4Ab^3-5a}{3a^2\left(a^2-\right.}$$

$$= \frac{2(3a^4A-15a^2Ab^2+8Ab^4+6a^3bB-2ab^3B)\sqrt{\cos(c+dx)}\,\csc}{3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3891 vs. $2(496) = 992$.

time = 25.85, size = 3891, normalized size = 7.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^4*A - 15*a^2*A*b^2 +
8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2) - (2*(
-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c
+ d*x])^2) - (2*(-8*a^2*A*b^2*Sin[c + d*x] + 4*A*b^4*Sin[c + d*x] + 5*a^3*b
*B*Sin[c + d*x] - a*b^3*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c
+ d*x]))))/d + (2*(-((a^2*A)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]])) + (5*A*b^2)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) - (8*A*b^4)/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[
Sec[c + d*x]]) - (2*a*b*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) + (2*b^3*B)/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]) - (3*a*A*b*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c
 + d*x]]) + (17*A*b^3*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos
[c + d*x]]) - (8*A*b^5*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*
Cos[c + d*x]]) + (a^2*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c
 + d*x]]) - (5*b^2*B*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c
+ d*x]]) + (2*b^4*B*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos
[c + d*x]]) - (a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sq
rt[a + b*Cos[c + d*x]]) + (5*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a*
(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^5*Cos[2*(c + d*x)]*Sqrt[Se
c[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (2*b^2*B*Cos[
2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) +
 (2*b^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a
+ b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(3*a^
4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^3
+ 3*a^2*b*(-3*A + B) + 3*a^3*(A + B) - 2*a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^4*A - 15*a^2
*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Co
s[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*Sin[c + d*x]*(-2*(a + b)*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B
- 2*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)] + 2*a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*a^3*(A + B) - 2*
a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
```

```
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)] - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3
*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(3*a^4*A
 - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 + 3
*a^2*b*(-3*A + B) + 3*a^3*(A + B) - 2*a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^4*A - 15*a^2*A*
b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*(-1/2*((3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cos[c +
d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4) - ((a + b)*(3*a^4*A - 15*a^2*
A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[
c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(8*A*b^3 + 3
*a^2*b*(-3*A + B) + 3*a^3*(A + B) - 2*a*b^2*(3*A + B))*Sqrt[(a + b*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c +
d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*
(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*
Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*
a^3*(A + B) - 2*a*b^2*(3*A + B))*Sqrt[Cos[c + d...
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6505 vs. $2(456) = 912$.
time = 0.55, size = 6506, normalized size = 13.12

method	result	size
default	Expression too large to display	6506

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\,\cos\left(c + d\,x\right))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a + b\,\cos\left(c + d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(5/2
), x)
```

3.627 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=469

$$\frac{2(6a^2Ab-2Ab^3-3a^3B-ab^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{3a^3(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^
(1/2)-2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a
/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^
2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-2/3*(2*
A*b^2-3*a^2*(A+B)+a*b*(3*A+B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a^2-b^2)/d/(a+b
)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.76, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3079, 3072, 3077, 2895, 3073}

$$-\frac{2(-3a^2(A+B)+ab(3A+B)+2Ab^2)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{a+b}\,(a^2-b^2)\sqrt{\sec(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\,(a^2-b^2)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}+\frac{2(-3a^3B+6a^2Ab-ab^2B-2Ab^3)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d(a-b)(a+b)^{3/2}\sqrt{\sec(c+dx)}}-\frac{2(-3a^3B+6a^2Ab-ab^2B-2Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\,(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]])
- (2*(2*A*b^2 - 3*a^2*(A + B) + a*b*(3*A + B))*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x
]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^(3/2)*Sqrt[Sec[c + d*x]]) - (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*
x]])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c\right.}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{2(6a^2Ab-}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{2(6a^2Ab-}{}$$

$$= \frac{2(6a^2Ab-2Ab^3-3a^3B-ab^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\Big(}{3a^3(a-}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 3493 vs. $2(469) = 938$.
time = 24.31, size = 3493, normalized size = 7.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(-6*a^2*A*b + 2*A*b^3 + 3
*a^3*B + a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2) + (2*(-(A*b*Sin[c + d
*x]) + a*B*Sin[c + d*x]))/(3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-5*a
^2*A*b*Sin[c + d*x] + A*b^3*Sin[c + d*x] + 2*a^3*B*Sin[c + d*x] + 2*a*b^2*B
*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + (2*((-2*a*A*
b)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*b^3)/
(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*B)/(
(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (b^2*B)/(3*(a^
2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*A*Sqrt[Sec[c
 + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (5*A*b^2*Sqrt[Sec[c +
d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*Sqrt[Sec[c + d
*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (a*b*B*Sqrt[Sec[c +
d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (b^3*B*Sqrt[Sec[c + d*x
]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (2*A*b^2*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*C
os[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c +
 d*x]]) + (a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a
 + b*Cos[c + d*x]]) + (b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2
 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*
(2*(a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b^2
 + 3*a^2*(A - B) + a*b*(3*A - B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] + (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B
)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(
3*(a^3 - a*b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-6*a^2*A*b +
2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3
*A - B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x]*(a + b*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^2*(a + b*
Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Se
c[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a
*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)] + 2*a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3*A - B))*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (-6*a^2*A*b
+ 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-6*a^2
```

```
*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(
c + d*x)/2]^4)/2 + ((a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*
x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])] + (a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3*A - B))*Sqrt[(a + b*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Si
n[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a
 + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*
x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))] + (a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3*A - B))*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b
*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - b*(-6*a^2*A*b + 2*A*b^3 + 3*a^
3*B + a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2
] - (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*(a + b*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B
 + a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*S...
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5204 vs. $2(429) = 858$.
time = 0.46, size = 5205, normalized size = 11.10

method	result	size
default	Expression too large to display	5205

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}}{(a+b\cos(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(5/2
), x)
```

3.628 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=431

$$-\frac{2(3a^2A+Ab^2-4abB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\text{se}}{a}}}{3a^2(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

```
[Out] -2/3*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/
2)+2/3*(3*A*a^2+A*b^2-4*B*a*b)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a
+b*cos(d*x+c))^(1/2)-2/3*(3*A*a^2+A*b^2-4*B*a*b)*csc(d*x+c)*EllipticE((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*
x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^
2/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)+2/3*(a*(3*A+B)-b*(A+3*B))*csc(d*x+c)
*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b
))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.68, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3040, 3078, 3072, 3077, 2895, 3073}

$$-\frac{2(3a^2A-4abB+Ab^2)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d(a-b)(a+b)^{3/2}\sqrt{\sec(c+dx)}}+\frac{2(3a^2A-4abB+Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}+\frac{2(a(3A+B)-b(A+3B))\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad(a-b)(a+b)^{3/2}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*(3*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(3*a^2*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(a*(3*A
+ B) - b*(A + 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
3*a*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(A*b - a*B)*Sin[c + d*
x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) + (2*(3
*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)^2
*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
```

```
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
```

```
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(\ldots}{(a+b\cos(c+dx))^{5/\ldots}}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(\ldots}}{\ldots}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{2(3a^2A+\ldots}{3\ldots}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{2(3a^2A+\ldots}{3\ldots}$$

$$= -\frac{2(3a^2A+Ab^2-4abB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\ldots\right.\right.}{3a^2(a-b\ldots}$$

Mathematica [A]
time = 19.10, size = 528, normalized size = 1.23

$$\frac{\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}\left(-\frac{2(3a^2A+Ab^2-4abB)\sin(c+dx)}{3a(a^2-b^2)^2}+\frac{2(-aAb\sin(c+dx)+a^2B\sin(c+dx))}{3b(-a^2+b^2)(a+b\cos(c+dx))^2}+\frac{2(2a^2Ab\sin(c+dx)+2Ab^3\sin(c+dx)+a^3B\sin(c+dx)-5ab^2B\sin(c+dx))}{3b(-a^2+b^2)^2(a+b\cos(c+dx))}\right)}{d}-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2(a+b)(3a^2A+Ab^2-4abB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)+2a(a+b)(3aA+Ab-aB-3bB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)-(3a^2A+Ab^2-4abB)\cos(c+dx)(a+b\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a\left(a^2-b^2\right)^2d\sqrt{a+b\cos(c+dx)}\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(3*a^2*A + A*b^2 - 4*a*b*
B)*Sin[c + d*x])/(3*a*(a^2 - b^2)^2) + (2*(-(a*A*b*Sin[c + d*x]) + a^2*B*Si
n[c + d*x]))/(3*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(2*a^2*A*b*Sin[
c + d*x] + 2*A*b^3*Sin[c + d*x] + a^3*B*Sin[c + d*x] - 5*a*b^2*B*Sin[c + d*
x]))/(3*b*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d - (2*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*(-2*(a + b)*(3*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Cos[c + d*x
```

```
]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(3*a
*A + A*b - a*B - 3*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] - (3*a^2*A + A*b^2 - 4*a*b*B)*Cos[c + d*x]*(a + b*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a*(a^2 - b^2)^2*d*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4242 vs. $2(391) = 782$.
time = 0.50, size = 4243, normalized size = 9.84

method	result	size
default	Expression too large to display	4243

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3/d*(A*cos(d*x+c)*a^2*b^2-4*B*cos(d*x+c)*a^3*b+3*B*cos(d*x+c)*a^2*b^2-3*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
^2*sin(d*x+c)*a^3*b-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^3-B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d
*x+c)*a^3*b-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^2+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^2+4*B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x
+c)*a*b^3-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^2*a*b^3+4*B*cos(d*x+c)^2*a^3*b-A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^4+A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*sin
(d*x+c)-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
```

```
*a^3*b*sin(d*x+c)-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^2*b^2*sin(d*x+c)-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+3*A*cos(d*x+c)*a^4+3*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3
*b+5*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*a^2*b^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3-6*A*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b-4*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d
*x+c)*a^2*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b-7*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*
b^2-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*a*b^3+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+8*B*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+4*B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a*b^3+A*cos(d*x+c)^3*b^4-A*cos(d*x+c)^2*b^4+B*cos(d*x+c)*a^4+7*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)*a^3*b+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^4*sin(d*x+c)-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4-4*A*cos(d*x+c)*a^3*b+4*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(1...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3
 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sqrt(sec(d*x + c)))
, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2)
), x)
```

3.629 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=602

$$\frac{2(4Ab^3+3a^3B-7ab^2B)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{3a(a-b)b^2(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^
(1/2)-2/3*a*(4*A*b^3+3*B*a^3-7*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^
2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(4*A*b^3+3*B*a^3-7*B*a*b^2)*csc(d*x+c
)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-
b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/a/(a-b)/b^2/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-2/3*(3*A*b^3+3*a^
3*B+a^2*b*B-a*b^2*(A+6*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b
)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d
*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/b^2/(a+b)^(3/2)/
d/sec(d*x+c)^(1/2)-2*B*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^
(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/s
ec(d*x+c)^(1/2)
```

Rubi [A]
time = 1.05, antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3040, 3068, 3130, 2888, 3072, 3077, 2895, 3073}

$$\frac{2(3a^3B-7ab^2B+4Ab^3)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ab^2d(a-b)(a+b)^{3/2}\sqrt{\sec(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}-\frac{2(3a^3B+a^2bB-ab^2(A+6B)+3Ab^3)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ab^2d(a-b)(a+b)^{3/2}\sqrt{\sec(c+dx)}}-\frac{2a(3a^3B-7ab^2B+4Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2B\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\mathrm{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(3*
A*b^3 + 3*a^3*B + a^2*b*B - a*b^2*(A + 6*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]
) - (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b,
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*
```

```
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) - (2*a*(4*A*
b^3 + 3*a^3*B - 7*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b
^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3130

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+}{(a+b\cos(c+dx))^{5/2}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+\right.}{}$$

$$= -\frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}\ \sqrt{}}\right.\right.}{b^3d\sqrt{}}$$

$$= -\frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}\ \sqrt{}}\right.\right.}{b^3d\sqrt{}}$$

$$= \frac{2(4Ab^3+3a^3B-7ab^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\right.\right.}{3a(a-b)b^2(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.07, size = 1994, normalized size = 3.31

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(4*A*b^3 + 3*a^3*B - 7*a*b
^2*B)*Sin[c + d*x])/(3*b^2*(-a^2 + b^2)^2) - (2*(-(a^2*A*b*Sin[c + d*x]) +
a^3*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) - (2*(-(a^
3*A*b*Sin[c + d*x]) + 5*a*A*b^3*Sin[c + d*x] + 4*a^4*B*Sin[c + d*x] - 8*a^2
*b^2*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d - (2*
Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*
x)/2]^2)]*(4*a*A*b^3*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] + 4*A*b^4*Sqrt[
(a - b)/(a + b)]*Tan[(c + d*x)/2] + 3*a^4*Sqrt[(a - b)/(a + b)]*B*Tan[(c +
d*x)/2] + 3*a^3*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2] - 7*a^2*b^2*Sqrt
[(a - b)/(a + b)]*B*Tan[(c + d*x)/2] - 7*a*b^3*Sqrt[(a - b)/(a + b)]*B*Tan[
(c + d*x)/2] - 8*A*b^4*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 - 6*a^3*b*S
```

```
qrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + 14*a*b^3*Sqrt[(a - b)/(a + b)]*
B*Tan[(c + d*x)/2]^3 - 4*a*A*b^3*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 +
 4*A*b^4*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 - 3*a^4*Sqrt[(a - b)/(a +
 b)]*B*Tan[(c + d*x)/2]^5 + 3*a^3*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2
]^5 + 7*a^2*b^2*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - 7*a*b^3*Sqrt[(
a - b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + (6*I)*a^4*B*EllipticPi[(a + b)/(a -
b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)] - (12*I)*a^2*b^2*B*EllipticPi[(a + b)/(a - b), I*ArcS
inh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^
2)/(a + b)] + (6*I)*b^4*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b
)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (
6*I)*a^4*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[
(c + d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
 - (12*I)*a^2*b^2*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a +
 b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)] + (6*I)*b^4*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/
(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)] + I*(a - b)*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*EllipticE[I*ArcSi
nh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*(3*b^3*(A - B) + 6*a^3*B +
4*a^2*b*B - a*b^2*(A + 9*B))*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[
(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
 + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)]))/(3*b^2*Sqrt[(a - b)/(a + b)]*(a^2 - b^2)^2*d*(-1 + Tan[(c + d*x)/2
]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b*(-1 + Tan[(
c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5758 vs. $2(550) = 1100$.
time = 0.48, size = 5759, normalized size = 9.57

method	result	size
default	Expression too large to display	5759

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3
 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(3/2))
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \cos(c + dx)}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2} (a + b \cos(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2)
), x)
```

3.630 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=733

$$\frac{(6a^3Ab-14aAb^3-15a^4B+26a^2b^2B-3b^4B)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{3a(a-b)b^3(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^
(3/2)+2/3*a*(2*A*a^2*b-6*A*b^3-5*B*a^3+9*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^
2/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-1/3*(6*A*a^3*b-14*A*a*b^3-15*B*
a^4+26*B*a^2*b^2-3*B*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2
)/b^3/(a^2-b^2)^2/d+1/3*(6*A*a^3*b-14*A*a*b^3-15*B*a^4+26*B*a^2*b^2-3*B*b^4
)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1
+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/b^3/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)+1/3*(3
*b^3*(4*A-B)+15*a^3*B-a*b^2*(2*A+21*B)-a^2*(6*A*b-5*B*b))*csc(d*x+c)*Ellipt
icF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2
))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/(a-b)/b^3/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-(2*A*b-5*B*a)*csc(d*x+c)*El
lipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 1.61, antiderivative size = 733, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3040, 3068, 3126, 3140, 3132, 2888, 3077, 2895, 3073}

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}+\frac{(15a^3B-a^2(6Ab-5bB)-ab^2(2A+21B)+3b^3(4A-B))\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3b^3d(a-b)(a+b)^{3/2}\sqrt{\sec(c+dx)}}+\frac{2a(-5a^3B+2a^2Ab+9ab^2B-6Ab^3)\sin(c+dx)}{3b^2d(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{(-15a^4B+6a^3Ab+26a^2b^2B-14aAb^3-3b^4B)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ab^3d(a-b)(a+b)^{3/2}\sqrt{\sec(c+dx)}}-\frac{(-15a^4B+6a^3Ab+26a^2b^2B-14aAb^3-3b^4B)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{3b^3d(a^2-b^2)^2}-\frac{\sqrt{a+b}(2Ab-5aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\operatorname{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^4d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d*S
qrt[Sec[c + d*x]]) + ((3*b^3*(4*A - B) + 15*a^3*B - a*b^2*(2*A + 21*B) - a^
2*(6*A*b - 5*b*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*(a - b)*b^3*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*A*b - 5*
a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
```

```
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^4
*d*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3
- 5*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]) - ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^
2*b^2*B - 3*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(3*b^3*(a^2 - b^2)^2*d)
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3040

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Sin[e + f*x])^m*((c + d
*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+}{(a+b\cos(c+dx))^{5/2}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(2\sqrt{\cos(c+}\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(2a^2Ab}{3b^2\left(a^2-b^2\right.}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(2a^2Ab}{3b^2\left(a^2-b^2\right.}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(2a^2Ab}{3b^2\left(a^2-b^2\right.}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-5aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\right.}{}$$

$$= \frac{\left(6a^3Ab-14aAb^3-15a^4B+26a^2b^2B-3b^4B\right)\sqrt{\cos(c+dx)}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2318 vs. 2(733) = 1466.

time = 22.28, size = 2318, normalized size = 3.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/
2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a*(-3*a^2*A*b + 7*A*b^3 +
 6*a^3*B - 10*a*b^2*B)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2) + (2*(-(a^3*A*b*
Sin[c + d*x]) + a^4*B*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)*(a + b*Cos[c + d*x
])^2) + (2*(-4*a^4*A*b*Sin[c + d*x] + 8*a^2*A*b^3*Sin[c + d*x] + 7*a^5*B*Si
n[c + d*x] - 11*a^3*b^2*B*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)^2*(a + b*Cos[c
 + d*x]))))/d + (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c
 + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(6*a^4*A*b*T
an[(c + d*x)/2] + 6*a^3*A*b^2*Tan[(c + d*x)/2] - 14*a^2*A*b^3*Tan[(c + d*x)
/2] - 14*a*A*b^4*Tan[(c + d*x)/2] - 15*a^5*B*Tan[(c + d*x)/2] - 15*a^4*b*B*
Tan[(c + d*x)/2] + 26*a^3*b^2*B*Tan[(c + d*x)/2] + 26*a^2*b^3*B*Tan[(c + d*
x)/2] - 3*a*b^4*B*Tan[(c + d*x)/2] - 3*b^5*B*Tan[(c + d*x)/2] - 12*a^3*A*b^
2*Tan[(c + d*x)/2]^3 + 28*a*A*b^4*Tan[(c + d*x)/2]^3 + 30*a^4*b*B*Tan[(c +
d*x)/2]^3 - 52*a^2*b^3*B*Tan[(c + d*x)/2]^3 + 6*b^5*B*Tan[(c + d*x)/2]^3 -
6*a^4*A*b*Tan[(c + d*x)/2]^5 + 6*a^3*A*b^2*Tan[(c + d*x)/2]^5 + 14*a^2*A*b^
3*Tan[(c + d*x)/2]^5 - 14*a*A*b^4*Tan[(c + d*x)/2]^5 + 15*a^5*B*Tan[(c + d*
x)/2]^5 - 15*a^4*b*B*Tan[(c + d*x)/2]^5 - 26*a^3*b^2*B*Tan[(c + d*x)/2]^5 +
 26*a^2*b^3*B*Tan[(c + d*x)/2]^5 + 3*a*b^4*B*Tan[(c + d*x)/2]^5 - 3*b^5*B*T
an[(c + d*x)/2]^5 - 12*a^4*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2
]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a^2*A*b^3*EllipticPi[-1, ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*A*b^5*Ellipt
icPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
+ 30*a^5*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x
)/2]^2)/(a + b)] - 60*a^3*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a*b^4*B*EllipticPi[-1, ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^4*A*b*Ellip
ticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
d*x)/2]^2)/(a + b)] + 24*a^2*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*A*b^5*Elli
pticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*S
```

```
qrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
 d*x)/2]^2)/(a + b)] + 30*a^5*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 60*a^3*b^2*B*Ell
ipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)] + 30*a*b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(-6*a
^3*A*b + 14*a*A*b^3 + 15*a^4*B - 26*a^2*b^2*B + 3*b^4*B)*EllipticE[ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(
c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(
a + b)] + 2*b*(a + b)*(3*A*b^3 + 3*a*b^2*(A - 2*B) + 5*a^3*B - a^2*b*(2*A +
 3*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*b^3*(a^2 - b^2)^2*d*Sqrt[1 + Tan[(c +
 d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8620 vs. $2(675) = 1350$.
time = 0.62, size = 8621, normalized size = 11.76

method	result	size
default	Expression too large to display	8621

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2)
), x)
```

3.631 $\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx$

Optimal. Leaf size=266

$$\frac{2(a-b)\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 2*(a-b)*B*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)-2*B*cs
c(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/
2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {21, 4307, 2880, 2895, 3073}

$$\frac{2B(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2B\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2880

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
```

```
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c}}$$

$$= -\left(\left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b}}\right.$$

$$= \frac{2(a-b)\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+b}}\right.\right.}{a^2d\sqrt{}}$$

Mathematica [A]
time = 6.12, size = 298, normalized size = 1.12

$$B\left(\frac{2\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{ad}-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)-2a\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+\cos(c+dx)(a+b\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad\sqrt{a+b\cos(c+dx)}\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]
)^(3/2),x]
```

```
[Out] B*((2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (2*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + Cos[c + d*x]*(a + b*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a*d*Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[(c + d*x)/2]^2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(242) = 484$.
time = 0.38, size = 621, normalized size = 2.33

method	result
default	$-\frac{2B\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos(dx+c)\sin(dx+c)a-\sqrt{\frac{\cos(dx+c}{1+\cos(dx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -2*B/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+cos(d*x+c)^2*b+a*cos(d
```

```
*x+c)-b*cos(d*x+c)-a)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2
)/sin(d*x+c)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/cos(c + d*x))^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^
(3/2),x)
```

```
[Out] int(((1/cos(c + d*x))^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^
(3/2), x)
```

3.632 $\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\cdots}}{ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2*B*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {21, 4307, 2895}

$$\frac{2B\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d
*Sqrt[Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b}$$

$$= \frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\,\mathrm{co}}}{\sqrt{a+b}\,\sqrt{c}}\right.\right.}{ad\sqrt{}}$$

Mathematica [A]
time = 0.16, size = 104, normalized size = 0.80

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)}{d\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\,\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]
)^(3/2),x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.40, size = 126, normalized size = 0.97

method	result	size
default	$\frac{2B\sqrt{\frac{1}{\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\sin^2(dx+c))}{d\sqrt{a+b\cos(dx+c)}\,(-1+\cos(dx+c))}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 2*B/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^
(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int\frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] B*Integral(sqrt(sec(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^
(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/cos(c + d*x))^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^
(3/2),x)
```

```
[Out] int(((1/cos(c + d*x))^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^
(3/2), x)
```

3.633 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=137

$$-\frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+d}{a+b}}}{bd\sqrt{\sec(c+dx)}}$$

```
[Out] -2*B*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$,
Rules used = {21, 4307, 2888}

$$-\frac{2B\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] (-2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(b*d*Sqrt[Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = B\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$

$$= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a-}}{\sqrt{a+}}\right.\right.}{bd}$$

Mathematica [A]
time = 0.20, size = 147, normalized size = 1.07

$$-\frac{2B\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left(F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)-2\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\right)\sqrt{1+\sec(c+dx)}}{d\sqrt{\frac{1}{1+\cos(c+dx)}}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c +
 d*x]]),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sqrt[1
 + Sec[c + d*x]])/(d*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Cos[c + d*x]]
)
```

Maple [A]
time = 0.37, size = 144, normalized size = 1.05

method	result	size
default	$\frac{2B\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)-2\,\text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\sqrt{-\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}}{d\sqrt{a+b\cos\left(dx+c\right)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{\cos(dx+c)}}}$	144

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 2*B/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-2*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))/(a+b*cos(d*x+c))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/(1/cos(d*x+c))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x
 + c))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{1}{\sqrt{a+b\cos{(c+dx)}}\,\sqrt{\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] B*Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x
 + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(
3/2)),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(
3/2)), x)
```

3.634 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=479

$$-\frac{(a-b)\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{abd\sqrt{\sec(c+dx)}}$$

```
[Out] B*sin(d*x+c)/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+a*B*sin(d*x+c)*sec(d
*x+c)^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)-(a-b)*B*csc(d*x+c)*EllipticE((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1
/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b)
)^(1/2)/a/b/d/sec(d*x+c)^(1/2)+B*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/se
c(d*x+c)^(1/2)+a*B*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2
)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {21, 4307, 2899, 2888, 3082, 3072, 12, 2880, 2895, 3073}

$$\frac{aB\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}+\frac{B\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}-\frac{B(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd\sqrt{\sec(c+dx)}}+\frac{aB\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\cos(c+dx)}}+\frac{B\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b]*B*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d*Sqrt[
Sec[c + d*x]]) + (B*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c +
d*x]]) + (a*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[a + b*Cos[c + d*x]
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2880

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 2899

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> Dist[(-a)*(d/(2*b)), Int[Sqrt[d*Sin[e + f*x]]/Sqrt
[a + b*Sin[e + f*x]], x], x] + Dist[d/(2*b), Int[Sqrt[d*Sin[e + f*x]]*((a +
 2*b*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3082

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a +
 b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)
*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(
a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx &= B\int \frac{1}{\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}\,dx \\
&= \left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx \\
&= \frac{\left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(a+2b\cos(c+dx}{\sqrt{a+b\cos(c+dx)}}}{2b} \\
&= \frac{a\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\,}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{b^2d\sqrt{s}} \\
&= \frac{a\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\,}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{b^2d\sqrt{s}} \\
&= \frac{a\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\,}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{b^2d\sqrt{s}} \\
&= \frac{a\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\,}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{b^2d\sqrt{s}} \\
&= -\frac{(a-b)\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a-}}{\sqrt{a+}}\right.\right.}{abd}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.15, size = 508, normalized size = 1.06

$$\frac{B\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{1+\sec(c+dx)}\left(2i(a-b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(i\sinh^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\frac{a+b}{a-b}\right)-4ia\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(i\sinh^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\frac{a+b}{a-b}\right)+4ia\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\Pi\left(\frac{a+b}{a-b};i\sinh^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\frac{a+b}{a-b}\right)+b\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{3}{2}(c+dx)\right)+2a\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\tan\left(\frac{1}{2}(c+dx)\right)-b\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{4b\sqrt{\frac{a-b}{a+b}}\,d\left(\frac{1}{1+\cos(c+dx)}\right)^{3/2}\sqrt{a+b\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]
^(3/2)),x]
```

```
[Out] (B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^2*Sqrt[1 + Sec[c
+ d*x]]*((2*I)*(a - b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/
(a - b))] - (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a
```

```
- b))] + (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]]
, -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)
]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(
a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(4*b*Sqrt[
(a - b)/(a + b)]*d*((1 + Cos[c + d*x])^(-1))^(3/2)*Sqrt[a + b*Cos[c + d*x]]
)
```

Maple [A]
time = 0.45, size = 631, normalized size = 1.32

method	result
default	$-\frac{B\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\ \mathrm{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos(dx+c)\sin(dx+c)a+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -B/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*b-2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a+(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+cos(d*x+c)^3
*b+cos(d*x+c)^2*a-cos(d*x+c)^2*b-a*cos(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3
/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)
^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3436 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)
^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(
3/2)),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(
3/2)), x)
```

3.635 $\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=59

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Int}\left((c\cos(e+fx))^{-m}(a+b\cos(e+fx))^n(A+B\cos(e+fx)),x\right)$$

```
[Out] (c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Unintegrable((a+b*cos(f*x+e))^n*(A+B*cos(
f*x+e))/((c*cos(f*x+e))^m),x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Defer[Int][((a + b*Cos[e + f*x])^n*(A
 + B*Cos[e + f*x]))/(c*Cos[e + f*x])^m, x]
```

Rubi steps

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = ((c\cos(e+fx))^m(c\sec(e+fx))^m)\int (c\cos$$

Mathematica [A]
time = 8.54, size = 0, normalized size = 0.00

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m, x
]
```

Maple [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int (a+b\cos(fx+e))^n\,(A+B\cos(fx+e))\,(c\sec(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*sec(f*x + e))^m, x
)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*sec(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**n*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*sec(f*x + e))^m, x
)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{c}{\cos\left(e+f\,x\right)}\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^n,x)
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^n, x)
```

3.636 $\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=644

$$-\frac{c^6(4a^3Ab(15-8m+m^2)+a^4B(15-8m+m^2)+4aAb^3(10-7m+m^2)+6a^2b^2B(10-7m+m^2)+b^4}{f(2-m)(4-m)(6-m)\sqrt{\sin^2(}}$$

```
[Out] -c^6*(4*a^3*A*b*(m^2-8*m+15)+a^4*B*(m^2-8*m+15)+4*a*A*b^3*(m^2-7*m+10)+6*a^
2*b^2*B*(m^2-7*m+10)+b^4*B*(m^2-6*m+8))*hypergeom([1/2, 3-1/2*m],[4-1/2*m],
cos(f*x+e)^2)*(c*sec(f*x+e))^(-6+m)*sin(f*x+e)/f/(-m^3+12*m^2-44*m+48)/(sin
(f*x+e)^2)^(1/2)-c^5*(a^4*A*(m^2-6*m+8)+6*a^2*A*b^2*(m^2-5*m+4)+4*a^3*b*B*(
m^2-5*m+4)+A*b^4*(m^2-4*m+3)+4*a*b^3*B*(m^2-4*m+3))*hypergeom([1/2, 5/2-1/2
*m],[7/2-1/2*m],cos(f*x+e)^2)*(c*sec(f*x+e))^(-5+m)*sin(f*x+e)/f/(1-m)/(m^2
-8*m+15)/(sin(f*x+e)^2)^(1/2)-a*c^5*(4*a^2*A*b*(m^2-4*m+3)+a^3*B*(m^2-4*m+3
)+2*A*b^3*(m^2-2*m+4)+a*b^2*B*(5*m^2-13*m+8))*(c*sec(f*x+e))^(-5+m)*tan(f*x
+e)/f/(1-m)/(m^2-6*m+8)-a^2*c^5*(2*a*b*B*(1-m)^2+a^2*A*(2-m)^2+A*b^2*(m^2-m
+6))*sec(f*x+e)*(c*sec(f*x+e))^(-5+m)*tan(f*x+e)/f/(-m^3+6*m^2-11*m+6)-a*c^
5*(a*B*(1-m)-A*b*(2+m))*(c*sec(f*x+e))^(-5+m)*(b+a*sec(f*x+e))^2*tan(f*x+e)
/f/(m^2-3*m+2)-a*A*c^5*(c*sec(f*x+e))^(-5+m)*(b+a*sec(f*x+e))^3*tan(f*x+e)/
f/(1-m)
```

Rubi [A]
time = 1.32, antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4111, 4181, 4161, 4132, 3857, 2722, 4131}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] -((c^6*(4*a^3*A*b*(15 - 8*m + m^2) + a^4*B*(15 - 8*m + m^2) + 4*a*A*b^3*(10
 - 7*m + m^2) + 6*a^2*b^2*B*(10 - 7*m + m^2) + b^4*B*(8 - 6*m + m^2))*Hyper
geometric2F1[1/2, (6 - m)/2, (8 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-
6 + m)*Sin[e + f*x])/(f*(2 - m)*(4 - m)*(6 - m)*Sqrt[Sin[e + f*x]^2])) - (c
^5*(a^4*A*(8 - 6*m + m^2) + 6*a^2*A*b^2*(4 - 5*m + m^2) + 4*a^3*b*B*(4 - 5*
m + m^2) + A*b^4*(3 - 4*m + m^2) + 4*a*b^3*B*(3 - 4*m + m^2))*Hypergeometri
c2F1[1/2, (5 - m)/2, (7 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-5 + m)*S
in[e + f*x])/(f*(1 - m)*(3 - m)*(5 - m)*Sqrt[Sin[e + f*x]^2]) - (a*c^5*(4*a
^2*A*b*(3 - 4*m + m^2) + a^3*B*(3 - 4*m + m^2) + 2*A*b^3*(4 - 2*m + m^2) +
a*b^2*B*(8 - 13*m + 5*m^2))*(c*Sec[e + f*x])^(-5 + m)*Tan[e + f*x])/(f*(1 -
 m)*(2 - m)*(4 - m)) - (a^2*c^5*(2*a*b*B*(1 - m)^2 + a^2*A*(2 - m)^2 + A*b^
2*(6 - m + m^2))*Sec[e + f*x]*(c*Sec[e + f*x])^(-5 + m)*Tan[e + f*x])/(f*(1
```

```
 - m)*(2 - m)*(3 - m)) - (a*c^5*(a*B*(1 - m) - A*b*(2 + m))*(c*Sec[e + f*x]
)^(-5 + m)*(b + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(1 - m)*(2 - m)) - (a*A*
c^5*(c*Sec[e + f*x])^(-5 + m)*(b + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(1 -
m))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
```

```
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rule 4181

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e +
 f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*
B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2
- b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^5\int (c\sec(e+fx))^{-5+m}(b+a\sec(e+fx)$$

$$= -\frac{aAc^5(c\sec(e+fx))^{-5+m}(b+a\sec(e+f}{f(1-m)}$$

$$= -\frac{ac^5(aB(1-m)-Ab(2+m))(c\sec(e+f}{f(1-m}$$

$$= -\frac{a^2c^5(2abB(1-m)^2+a^2A(2-m)^2+Ab}{f(3}$$

$$= -\frac{a^2c^5(2abB(1-m)^2+a^2A(2-m)^2+Ab}{f(3}$$

$$= -\frac{ac^5(4a^2Ab(3-4m+m^2)+a^3B(3-4m}{}$$

$$= -\frac{(a^4A(8-6m+m^2)+6a^2Ab^2(4-5m+}{}$$

$$= -\frac{(a^4A(8-6m+m^2)+6a^2Ab^2(4-5m+}{}$$

Mathematica [A]
time = 4.41, size = 317, normalized size = 0.49

$$\frac{\cot(e+fx)\left(\frac{b^4B\cos^5(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);\sec^2(e+fx)\right)}{-5+m}+\frac{b^3(Ab+4aB)\cos^4(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-4+m);\frac{1}{2}(-2+m);\sec^2(e+fx)\right)}{-4+m}+a\left(\frac{2b^2(2Ab+3aB)\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sec^2(e+fx)\right)}{-3+m}+a\left(\frac{2b(3Ab+2aB)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+m);\frac{m}{2};\sec^2(e+fx)\right)}{-2+m}+a\left(\frac{(4Ab+aB)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sec^2(e+fx)\right)}{-1+m}+\frac{aA\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};\sec^2(e+fx)\right)}{m}\right)\right)\right)\right)(c\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[e + f*x]*((b^4*B*Cos[e + f*x]^5*Hypergeometric2F1[1/2, (-5 + m)/2, (-3
 + m)/2, Sec[e + f*x]^2])/(-5 + m) + (b^3*(A*b + 4*a*B)*Cos[e + f*x]^4*Hype
rgeometric2F1[1/2, (-4 + m)/2, (-2 + m)/2, Sec[e + f*x]^2])/(-4 + m) + a*((
2*b^2*(2*A*b + 3*a*B)*Cos[e + f*x]^3*Hypergeometric2F1[1/2, (-3 + m)/2, (-1
 + m)/2, Sec[e + f*x]^2])/(-3 + m) + a*((2*b*(3*A*b + 2*a*B)*Cos[e + f*x]^2
*Hypergeometric2F1[1/2, (-2 + m)/2, m/2, Sec[e + f*x]^2])/(-2 + m) + a*(((4
*A*b + a*B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[
e + f*x]^2])/(-1 + m) + (a*A*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[e +
 f*x]^2])/m))))*(c*Sec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/f
```

Maple [F]
time = 0.87, size = 0, normalized size = 0.00

$$\int \left(a+b\cos\left(fx+e\right)\right)^4\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*sec(f*x + e))^m, x
)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((B*b^4*cos(f*x + e)^5 + A*a^4 + (4*B*a*b^3 + A*b^4)*cos(f*x + e)^4
 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*cos(f*x + e)^3 + 2*(2*B*a^3*b + 3*A*a^2*b^2)
*cos(f*x + e)^2 + (B*a^4 + 4*A*a^3*b)*cos(f*x + e))*(c*sec(f*x + e))^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**4*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*sec(f*x + e))^m, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{c}{\cos\left(e+f\,x\right)}\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^4,x)
```

```
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^4, x)
```

3.637 $\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=455

$$-\frac{c^5(a^3A(8-6m+m^2)+3aAb^2(4-5m+m^2)+3a^2bB(4-5m+m^2)+b^3B(3-4m+m^2))\,{}_2F_1\left(\frac{1}{2},\frac{5-m}{2}\right.}{f(1-m)(3-m)(5-m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -c^5*(a^3*A*(m^2-6*m+8)+3*a*A*b^2*(m^2-5*m+4)+3*a^2*b*B*(m^2-5*m+4)+b^3*B*(
m^2-4*m+3))*hypergeom([1/2, 5/2-1/2*m],[7/2-1/2*m],cos(f*x+e)^2)*(c*sec(f*x
+e))^(-5+m)*sin(f*x+e)/f/(1-m)/(m^2-8*m+15)/(sin(f*x+e)^2)^(1/2)-c^4*(A*b^3
*(2-m)+3*a*b^2*B*(2-m)+3*a^2*A*b*(3-m)+a^3*B*(3-m))*hypergeom([1/2, 2-1/2*m
],[3-1/2*m],cos(f*x+e)^2)*(c*sec(f*x+e))^(-4+m)*sin(f*x+e)/f/(m^2-6*m+8)/(s
in(f*x+e)^2)^(1/2)-a*c^4*(3*a*b*B*(1-m)+a^2*A*(2-m)-2*A*b^2*m)*(c*sec(f*x+e
))^(-4+m)*tan(f*x+e)/f/(m^2-4*m+3)-a^2*c^4*(a*B*(1-m)-A*b*(1+m))*sec(f*x+e)
*(c*sec(f*x+e))^(-4+m)*tan(f*x+e)/f/(m^2-3*m+2)-a*A*c^4*(c*sec(f*x+e))^(-4+
m)*(b+a*sec(f*x+e))^2*tan(f*x+e)/f/(1-m)
```

Rubi [A]

time = 0.74, antiderivative size = 455, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3039, 4111, 4161, 4132, 3857, 2722, 4131}

$$-\frac{ac^4\tan(e+fx)\left(a^2A(2-m)+3abB(1-m)-2Ab^2m\right)(c\sec(e+fx))^{m-4}}{f(1-m)(3-m)}-\frac{a^2c^4\tan(e+fx)\sec(e+fx)(aB(1-m)-Ab(m+1))(c\sec(e+fx))^{m-4}}{f(1-m)(2-m)}-\frac{c^5\sin(e+fx)\left(a^3A(m^2-6m+8)+3a^2bB(m^2-5m+4)+3aAb^2(m^2-5m+4)+b^3B(m^2-4m+3)\right)(c\sec(e+fx))^{m-5}\,{}_2F_1\left(\frac{1}{2},\frac{5-m}{2};\frac{7-m}{2};\cos^2(e+fx)\right)}{f(1-m)(3-m)(5-m)\sqrt{\sin^2(e+fx)}}-\frac{c^4\sin(e+fx)\left(a^3B(3-m)+3a^2Ab(3-m)+3ab^2B(2-m)+Ab^3(2-m)\right)(c\sec(e+fx))^{m-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-m}{2};\frac{6-m}{2};\cos^2(e+fx)\right)}{f(2-m)(4-m)\sqrt{\sin^2(e+fx)}}-\frac{aAc^4\tan(e+fx)(a\sec(e+fx)+b)^2(c\sec(e+fx))^{m-4}}{f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] -((c^5*(a^3*A*(8 - 6*m + m^2) + 3*a*A*b^2*(4 - 5*m + m^2) + 3*a^2*b*B*(4 -
5*m + m^2) + b^3*B*(3 - 4*m + m^2))*Hypergeometric2F1[1/2, (5 - m)/2, (7 -
m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-5 + m)*Sin[e + f*x])/(f*(1 - m)*(3
 - m)*(5 - m)*Sqrt[Sin[e + f*x]^2])) - (c^4*(A*b^3*(2 - m) + 3*a*b^2*B*(2 -
 m) + 3*a^2*A*b*(3 - m) + a^3*B*(3 - m))*Hypergeometric2F1[1/2, (4 - m)/2,
(6 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-4 + m)*Sin[e + f*x])/(f*(2 -
m)*(4 - m)*Sqrt[Sin[e + f*x]^2]) - (a*c^4*(3*a*b*B*(1 - m) + a^2*A*(2 - m)
- 2*A*b^2*m)*(c*Sec[e + f*x])^(-4 + m)*Tan[e + f*x])/(f*(1 - m)*(3 - m)) -
(a^2*c^4*(a*B*(1 - m) - A*b*(1 + m))*Sec[e + f*x]*(c*Sec[e + f*x])^(-4 + m)
*Tan[e + f*x])/(f*(1 - m)*(2 - m)) - (a*A*c^4*(c*Sec[e + f*x])^(-4 + m)*(b
+ a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(1 - m))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
```

```
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^4\int (c\sec(e+fx))^{-4+m}(b+a\sec(e+fx$$

$$= -\frac{aAc^4(c\sec(e+fx))^{-4+m}(b+a\sec(e+}{f(1-m)}$$

$$= -\frac{a^2c^4(aB(1-m)-Ab(1+m))\sec(e+}{f(1-m)(}$$

$$= -\frac{a^2c^4(aB(1-m)-Ab(1+m))\sec(e+}{f(1-m)(}$$

$$= -\frac{ac^4(3abB(1-m)+a^2A(2-m)-2Ab}{f(1-m)(}$$

$$= -\frac{(Ab^3(2-m)+3ab^2B(2-m)+3a^2Ab(}{}$$

$$= -\frac{(Ab^3(2-m)+3ab^2B(2-m)+3a^2Ab(}{}$$

Mathematica [A]
time = 2.11, size = 259, normalized size = 0.57

$$\frac{\cot(e+fx)\left(\frac{b^3B\cos^4(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-4+m);\frac{1}{2}(-2+m);\sec^2(e+fx)\right)}{-4+m}+\frac{b^2(Ab+3aB)\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sec^2(e+fx)\right)}{-3+m}+a\left(\frac{3b(Ab+aB)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+m);\frac{m}{2};\sec^2(e+fx)\right)}{-2+m}+a\left(\frac{(3Ab+aB)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sec^2(e+fx)\right)}{-1+m}+\frac{aA\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};\sec^2(e+fx)\right)}{m}\right)\right)\right)(c\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[e + f*x]*((b^3*B*Cos[e + f*x]^4*Hypergeometric2F1[1/2, (-4 + m)/2, (-2
 + m)/2, Sec[e + f*x]^2])/(-4 + m) + (b^2*(A*b + 3*a*B)*Cos[e + f*x]^3*Hype
rgeometric2F1[1/2, (-3 + m)/2, (-1 + m)/2, Sec[e + f*x]^2])/(-3 + m) + a*((
3*b*(A*b + a*B)*Cos[e + f*x]^2*Hypergeometric2F1[1/2, (-2 + m)/2, m/2, Sec[
e + f*x]^2])/(-2 + m) + a*(((3*A*b + a*B)*Cos[e + f*x]*Hypergeometric2F1[1/
2, (-1 + m)/2, (1 + m)/2, Sec[e + f*x]^2])/(-1 + m) + (a*A*Hypergeometric2F
1[1/2, m/2, (2 + m)/2, Sec[e + f*x]^2])/m)))*(c*Sec[e + f*x])^m*Sqrt[-Tan[e
 + f*x]^2])/f
```

Maple [F]
time = 0.71, size = 0, normalized size = 0.00

$$\int (a+b\cos(fx+e))^3 (A+B\cos(fx+e)) (c\sec(fx+e))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*sec(f*x + e))^m, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(f*x + e)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(f*x + e)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(f*x + e)^2 + (B*a^3 + 3*A*a^2*b)*cos(f*x + e))
*(c*sec(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sec(e+fx))^m (A+B\cos(e+fx)) (a+b\cos(e+fx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**3*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))**3,
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*sec(f*x + e))^m, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{c}{\cos\left(e+f\,x\right)}\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^3,x)
```

```
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^3, x)
```

3.638 $\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=327

$$-\frac{c^4(b^2B(2-m)+2aAb(3-m)+a^2B(3-m))\ {}_2F_1\left(\frac{1}{2},\frac{4-m}{2};\frac{6-m}{2};\cos^2(e+fx)\right)(c\sec(e+fx))^{-4+m}\sin(e+}{f(2-m)(4-m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -c^4*(b^2*B*(2-m)+2*a*A*b*(3-m)+a^2*B*(3-m))*hypergeom([1/2, 2-1/2*m],[3-1/
2*m],cos(f*x+e)^2)*(c*sec(f*x+e))^(-4+m)*sin(f*x+e)/f/(m^2-6*m+8)/(sin(f*x+
e)^2)^(1/2)-c^3*(A*b^2*(1-m)+2*a*b*B*(1-m)+a^2*A*(2-m))*hypergeom([1/2, 3/2
-1/2*m],[5/2-1/2*m],cos(f*x+e)^2)*(c*sec(f*x+e))^(-3+m)*sin(f*x+e)/f/(m^2-4
*m+3)/(sin(f*x+e)^2)^(1/2)-a*c^3*(a*B*(1-m)-A*b*m)*(c*sec(f*x+e))^(-3+m)*ta
n(f*x+e)/f/(m^2-3*m+2)-a*A*c^3*(c*sec(f*x+e))^(-3+m)*(b+a*sec(f*x+e))*tan(f
*x+e)/f/(1-m)
```

Rubi [A]
time = 0.41, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3039, 4111, 4132, 3857, 2722, 4131}

$$-\frac{c^4\sin(e+fx)\left(a^2B(3-m)+2aAb(3-m)+b^2B(2-m)\right)(c\sec(e+fx))^{m-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-m}{2};\frac{6-m}{2};\cos^2(e+fx)\right)}{f(2-m)(4-m)\sqrt{\sin^2(e+fx)}}-\frac{c^3\sin(e+fx)\left(a^2A(2-m)+2abB(1-m)+Ab^2(1-m)\right)(c\sec(e+fx))^{m-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-m}{2};\frac{5-m}{2};\cos^2(e+fx)\right)}{f(1-m)(3-m)\sqrt{\sin^2(e+fx)}}-\frac{ac^3\tan(e+fx)(aB(1-m)-Abm)(c\sec(e+fx))^{m-3}}{f(1-m)(2-m)}-\frac{aAc^3\tan(e+fx)(a\sec(e+fx)+b)(c\sec(e+fx))^{m-3}}{f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] -((c^4*(b^2*B*(2 - m) + 2*a*A*b*(3 - m) + a^2*B*(3 - m))*Hypergeometric2F1[
1/2, (4 - m)/2, (6 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-4 + m)*Sin[e
+ f*x])/(f*(2 - m)*(4 - m)*Sqrt[Sin[e + f*x]^2])) - (c^3*(A*b^2*(1 - m) + 2
*a*b*B*(1 - m) + a^2*A*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m)/2
, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-3 + m)*Sin[e + f*x])/(f*(1 - m)*(3 - m
)*Sqrt[Sin[e + f*x]^2]) - (a*c^3*(a*B*(1 - m) - A*b*m)*(c*Sec[e + f*x])^(-3
 + m)*Tan[e + f*x])/(f*(1 - m)*(2 - m)) - (a*A*c^3*(c*Sec[e + f*x])^(-3 + m
)*(b + a*Sec[e + f*x])*Tan[e + f*x])/(f*(1 - m))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^3\int (c\sec(e+fx))^{-3+m}(b+a\sec(e+fx)$$
$$= -\frac{aAc^3(c\sec(e+fx))^{-3+m}(b+a\sec(e+f}{f(1-m)}$$
$$= -\frac{aAc^3(c\sec(e+fx))^{-3+m}(b+a\sec(e+f}{f(1-m)}$$
$$= -\frac{ac^3(aB(1-m)-Abm)(c\sec(e+fx))^{-3}}{f(1-m)(2-m)}$$
$$= -\frac{(Ab^2(1-m)+2abB(1-m)+a^2A(2-}{}$$
$$= -\frac{(Ab^2(1-m)+2abB(1-m)+a^2A(2-}{}$$

Mathematica [A]
time = 1.00, size = 205, normalized size = 0.63

$$\frac{\cot(e+fx)\left(\frac{b^2B\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sec^2(e+fx)\right)}{-3+m}+\frac{b(Ab+2aB)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+m);\frac{m}{2};\sec^2(e+fx)\right)}{-2+m}+a\left(\frac{(2Ab+aB)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sec^2(e+fx)\right)}{-1+m}+\frac{aA\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};\sec^2(e+fx)\right)}{m}\right)\right)(c\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[e + f*x]*((b^2*B*Cos[e + f*x]^3*Hypergeometric2F1[1/2, (-3 + m)/2, (-1
 + m)/2, Sec[e + f*x]^2])/(-3 + m) + (b*(A*b + 2*a*B)*Cos[e + f*x]^2*Hyperg
eometric2F1[1/2, (-2 + m)/2, m/2, Sec[e + f*x]^2])/(-2 + m) + a*(((2*A*b +
a*B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[e + f*x
]^2])/(-1 + m) + (a*A*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[e + f*x]^2
])/m))*(c*Sec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/f
```

Maple [F]
time = 0.59, size = 0, normalized size = 0.00

$$\int (a+b\cos(fx+e))^2\,(A+B\cos(fx+e))\,(c\sec(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*sec(f*x + e))^m, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(f*x + e)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(f*x + e)^2 +
 (B*a^2 + 2*A*a*b)*cos(f*x + e))*(c*sec(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c \sec (e + fx))^{m} (A + B \cos (e + fx)) (a + b \cos (e + fx))^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**2*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))**2,
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*sec(f*x + e))^m, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{c}{\cos\left(e+f\,x\right)}\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^2,x)
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^2, x)
```

3.639 $\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=217

$$-\frac{c^3(bB(1-m)+aA(2-m))\,{}_2F_1\left(\frac{1}{2},\frac{3-m}{2};\frac{5-m}{2};\cos^2(e+fx)\right)(c\sec(e+fx))^{-3+m}\sin(e+fx)}{f(1-m)(3-m)\sqrt{\sin^2(e+fx)}}-\frac{(Ab+aB)}{}$$

```
[Out] -c^3*(b*B*(1-m)+a*A*(2-m))*hypergeom([1/2, 3/2-1/2*m],[5/2-1/2*m],cos(f*x+e
)^2)*(c*sec(f*x+e))^(-3+m)*sin(f*x+e)/f/(m^2-4*m+3)/(sin(f*x+e)^2)^(1/2)-(A
*b+B*a)*c^2*hypergeom([1/2, 1-1/2*m],[2-1/2*m],cos(f*x+e)^2)*(c*sec(f*x+e))
^(-2+m)*sin(f*x+e)/f/(2-m)/(sin(f*x+e)^2)^(1/2)-a*A*c^2*(c*sec(f*x+e))^(-2+
m)*tan(f*x+e)/f/(1-m)
```

Rubi [A]
time = 0.24, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3039, 4082, 3872, 3857, 2722}

$$-\frac{c^3\sin(e+fx)(aA(2-m)+bB(1-m))(c\sec(e+fx))^{m-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-m}{2};\frac{5-m}{2};\cos^2(e+fx)\right)}{f(1-m)(3-m)\sqrt{\sin^2(e+fx)}}-\frac{c^2(aB+Ab)\sin(e+fx)(c\sec(e+fx))^{m-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{2};\frac{4-m}{2};\cos^2(e+fx)\right)}{f(2-m)\sqrt{\sin^2(e+fx)}}-\frac{aAc^2\tan(e+fx)(c\sec(e+fx))^{m-2}}{f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] -((c^3*(b*B*(1 - m) + a*A*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m
)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-3 + m)*Sin[e + f*x])/(f*(1 - m)*(3
- m)*Sqrt[Sin[e + f*x]^2])) - ((A*b + a*B)*c^2*Hypergeometric2F1[1/2, (2 -
m)/2, (4 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-2 + m)*Sin[e + f*x])/(f
*(2 - m)*Sqrt[Sin[e + f*x]^2]) - (a*A*c^2*(c*Sec[e + f*x])^(-2 + m)*Tan[e +
 f*x])/(f*(1 - m))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^2\int (c\sec(e+fx))^{-2+m}(b+a\sec(e+fx))($$

$$= -\frac{aAc^2(c\sec(e+fx))^{-2+m}\tan(e+fx)}{f(1-m)} - \frac{c}{}$$

$$= -\frac{aAc^2(c\sec(e+fx))^{-2+m}\tan(e+fx)}{f(1-m)} + (($$

$$= -\frac{aAc^2(c\sec(e+fx))^{-2+m}\tan(e+fx)}{f(1-m)} + \Big($$

$$= -\frac{(Ab+aB)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{2};\frac{4-m}{2};\right.}{f(2-m)\sqrt{s}}$$

Mathematica [A]
time = 0.40, size = 163, normalized size = 0.75

$$\frac{\cot(e+fx)\left(bB(-1+m)m\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+m);\frac{m}{2};\sec^2(e+fx)\right)+(-2+m)\left((Ab+aB)m\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sec^2(e+fx)\right)+aA(-1+m)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};\sec^2(e+fx)\right)\right)\right)(c\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{f(-2+m)(-1+m)m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[e + f*x]*(b*B*(-1 + m)*m*Cos[e + f*x]^2*Hypergeometric2F1[1/2, (-2 + m
)/2, m/2, Sec[e + f*x]^2] + (-2 + m)*((A*b + a*B)*m*Cos[e + f*x]*Hypergeome
tric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[e + f*x]^2] + a*A*(-1 + m)*Hypergeo
metric2F1[1/2, m/2, (2 + m)/2, Sec[e + f*x]^2]))*(c*Sec[e + f*x])^m*Sqrt[-T
an[e + f*x]^2])/(f*(-2 + m)*(-1 + m)*m)
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int (a+b\cos (fx+e)) (A+B\cos (fx+e)) (c\sec (fx+e))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*(c*sec(f*x +
 e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sec (e+fx))^m (A+B\cos (e+fx)) (a+b\cos (e+fx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{c}{\cos\left(e+f\,x\right)}\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)),x)
```

```
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)), x)
```

3.640 $\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx$

Optimal. Leaf size=299

$$-\frac{(Ab-aB)F_1\left(\frac{1}{2};\frac{m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{m/2}(c\sec(e+fx))^{1+m}\sin(e}{(a^2-b^2)\,cf}$$

```
[Out] -(A*b-B*a)*AppellF1(1/2,1/2*m,1,3/2,sin(f*x+e)^2,-b^2*sin(f*x+e)^2/(a^2-b^2
))*cos(f*x+e)*(cos(f*x+e)^2)^(1/2*m)*(c*sec(f*x+e))^(1+m)*sin(f*x+e)/(a^2-b
^2)/c/f+a*(A*b-B*a)*AppellF1(1/2,1/2+1/2*m,1,3/2,sin(f*x+e)^2,-b^2*sin(f*x+
e)^2/(a^2-b^2))*(cos(f*x+e)^2)^(1/2+1/2*m)*(c*sec(f*x+e))^(1+m)*sin(f*x+e)/
b/(a^2-b^2)/c/f-B*c*hypergeom([1/2, 1/2-1/2*m],[3/2-1/2*m],cos(f*x+e)^2)*(c
*sec(f*x+e))^(-1+m)*sin(f*x+e)/b/f/(1-m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3039, 4123, 3857, 2722, 3954, 2902, 3268, 440}

$$-\frac{(Ab-aB)\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{m/2}(c\sec(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)}{cf\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(e+fx)\cos^2(e+fx)^{\frac{m+1}{2}}(c\sec(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{m+1}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)}{bcf\left(a^2-b^2\right)}-\frac{Bc\sin(e+fx)(c\sec(e+fx))^{m-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)}{bf(1-m)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x]),x]
```

```
[Out] -(((A*b - a*B)*AppellF1[1/2, m/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*
x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(Cos[e + f*x]^2)^(m/2)*(c*Sec[e + f*x])^(1
 + m)*Sin[e + f*x])/((a^2 - b^2)*c*f)) + (a*(A*b - a*B)*AppellF1[1/2, (1 +
m)/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*x]^2)/(a^2 - b^2))]*(Cos[e +
 f*x]^2)^((1 + m)/2)*(c*Sec[e + f*x])^(1 + m)*Sin[e + f*x])/(b*(a^2 - b^2)*
c*f) - (B*c*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[e + f*x]^2]*(c
*Sec[e + f*x])^(-1 + m)*Sin[e + f*x])/(b*f*(1 - m)*Sqrt[Sin[e + f*x]^2])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2902

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3039

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] && !IntegerQ[m]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 4123

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
 NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx = \int \frac{(c\sec(e+fx))^m(B+A\sec(e+fx))}{b+a\sec(e+fx)}\,dx$$
$$= \frac{B\int (c\sec(e+fx))^m\,dx}{b} + \frac{(Ab-aB)\int \frac{(c\sec(e+fx))^{1+m}}{b+a\sec(e+fx)}\,dx}{bc}$$
$$= \frac{\left(B\left(\frac{\cos(e+fx)}{c}\right)^m (c\sec(e+fx))^m\right)\int \left(\frac{\cos(e+fx)}{c}\right)^{-m}\,dx}{b} + \frac{((A$$
$$= -\frac{B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)(c\sec(e+fx$$
$$bf(1-m)\sqrt{\sin^2(e+fx)}$$

$$= -\frac{B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)(c\sec(e+fx$$
$$bf(1-m)\sqrt{\sin^2(e+fx)}$$
$$= -\frac{(Ab-aB)F_1\left(\frac{1}{2};\frac{m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e}{(a^2-b^2)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 10630 vs. 2(299) = 598.
time = 26.34, size = 10630, normalized size = 35.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x]),x]
[Out] Result too large to show
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(fx+e))(c\sec(fx+e))^m}{a+b\cos(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x)
[Out] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c \sec (e + fx))^m (A + B \cos (e + fx))}{a + b \cos (e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))**m/(a+b*cos(f*x+e)),x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))/(a + b*cos(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[0,1,0,0]%%%} / %%%{1,[0,0,1,0]%%%}+%%%{-1,[0,0,0,1]%%%} E
rror:
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{c}{\cos(e+f\,x)}\right)^m\,(A+B\,\cos\left(e+f\,x\right))}{a+b\,\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x)),x)
[Out] int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x)), x)
```

3.641 $\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=210

$$\frac{2bB\cos(e+fx)\sqrt{a+b\cos(e+fx)}\,(c\sec(e+fx))^m\sin(e+fx)}{f(5-2m)}+\frac{2(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Int}\Big(\frac{(c}{}$$

```
[Out] 2*b*B*cos(f*x+e)*(c*sec(f*x+e))^m*sin(f*x+e)*(a+b*cos(f*x+e))^(1/2)/f/(5-2*
m)+2*(c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Unintegrable((1/2*a*c*(2*b*B*(1-m)+2
*a*A*(5/2-m))+1/2*c*(b^2*B*(3-2*m)+a*(2*A*b+B*a)*(5-2*m))*cos(f*x+e)+1/2*b*
c*(A*b*(5-2*m)+2*a*B*(3-m))*cos(f*x+e)^2)/((c*cos(f*x+e))^m)/(a+b*cos(f*x+e
))^(1/2),x)/c/(5-2*m)
```

Rubi [A]
time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (2*b*B*Cos[e + f*x]*Sqrt[a + b*Cos[e + f*x]]*(c*Sec[e + f*x])^m*Sin[e + f*x
])/(f*(5 - 2*m)) + (2*(c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Defer[Int][((a*
c*(2*b*B*(1 - m) + 2*a*A*(5/2 - m)))/2 + (c*(b^2*B*(3 - 2*m) + a*(2*A*b + a
*B)*(5 - 2*m))*Cos[e + f*x])/2 + (b*c*(A*b*(5 - 2*m) + 2*a*B*(3 - m))*Cos[e
 + f*x]^2)/2)/((c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]), x])/(c*(5 - 2*
m))
```

Rubi steps

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx=((c\cos(e+fx))^m(c\sec(e+fx))^m)\int (c\cos$$

$$=\frac{2bB\cos(e+fx)\sqrt{a+b\cos(e+fx)}\,(c\sec}{f(5-2m)}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^
m,x]
```

```
[Out] $Aborted
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \left(a+b\cos \left(fx+e\right)\right)^{\frac{3}{2}} \left(A+B\cos \left(fx+e\right)\right) \left(c\sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*sec(f*x + e))^
m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*sqrt(b*cos(f
*x + e) + a)*(c*sec(f*x + e))^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="giac")
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*sec(f*x + e))^
m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{c}{\cos\left(e+f\,x\right)}\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(3/2),x)
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(3/2), x)
```

3.642 $\int \sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=61

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Int}\Big((c\cos(e+fx))^{-m}\sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx)),x\Big)$$

```
[Out] (c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Unintegrable((A+B*cos(f*x+e))*(a+b*cos(f*
x+e))^(1/2)/((c*cos(f*x+e))^m),x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Defer[Int][(Sqrt[a + b*Cos[e + f*x]]*
(A + B*Cos[e + f*x]))/(c*Cos[e + f*x])^m, x]
```

Rubi steps

$$\int \sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = ((c\cos(e+fx))^m(c\sec(e+fx))^m)\int (c\,c$$

Mathematica [A]
time = 116.80, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,
x]
```

```
[Out] Integrate[Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,
 x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos(fx+e)}\,(A+B\cos(fx+e))\,(c\sec(fx+e))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
[Out] int((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="maxima")
[Out] integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m,
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="fricas")
[Out] integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m,
x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sec(e+fx))^{m}\,(A+B\cos(e+fx))\,\sqrt{a+b\cos(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*sqrt(a + b*cos(e + f*x)),
 x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m,
 x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{c}{\cos\left(e+f\,x\right)}\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\sqrt{a+b\,\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(1/2),x)
```

```
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(1/2), x)
```

3.643 $\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx$

Optimal. Leaf size=61

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\mathrm{Int}\left(\frac{(c\cos(e+fx))^{-m}(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}},x\right)$$

```
[Out] (c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Unintegrable((A+B*cos(f*x+e))/((c*cos(f*x
+e))^m)/(a+b*cos(f*x+e))^(1/2),x)
```

Rubi [A]
time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/Sqrt[a + b*Cos[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Defer[Int][(A + B*Cos[e + f*x])/((c*C
os[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]), x]
```

Rubi steps

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx = ((c\cos(e+fx))^m(c\sec(e+fx))^m)\int \frac{(c\cos(e+fx))^{-m}(A+}{\sqrt{a+b\cos(e+}}$$

Mathematica [A]
time = 13.88, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/Sqrt[a + b*Cos[e + f*x]
],x]
```

```
[Out] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/Sqrt[a + b*Cos[e + f*x]
], x]
```

Maple [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(fx+e))(c\sec(fx+e))^m}{\sqrt{a+b\cos(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x)
[Out] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="maxima")
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="fricas")
[Out] integral((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sec(e+fx))^m (A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))**m/(a+b*cos(f*x+e))**(1/2),x)
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))/sqrt(a + b*cos(e + f*x)),
 x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{c}{\cos(e+f\,x)}\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)}{\sqrt{a+b\,\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(1/2),x)
```

```
[Out] int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(1/2), x
)
```

3.644 $\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=213

$$\frac{2b(Ab-aB)\cos(e+fx)(c\sec(e+fx))^m\sin(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\cos(e+fx)}}+\frac{2(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Int}\Bigg(\frac{(c\cos(e+fx))^{-m}}{}$$

```
[Out] 2*b*(A*b-B*a)*cos(f*x+e)*(c*sec(f*x+e))^m*sin(f*x+e)/a/(a^2-b^2)/f/(a+b*cos
(f*x+e))^(1/2)+2*(c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Unintegrable((1/2*c*(a^2
*A+A*b^2*(1-2*m)-2*a*b*B*(1-m))-1/2*a*(A*b-B*a)*c*cos(f*x+e)-1/2*b*(A*b-B*a
)*c*(3-2*m)*cos(f*x+e)^2)/((c*cos(f*x+e))^m)/(a+b*cos(f*x+e))^(1/2),x)/a/(a
^2-b^2)/c
```

Rubi [A]
time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x])^(3/2),x]
```

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x]*(c*Sec[e + f*x])^m*Sin[e + f*x])/(a*(a^2 - b^
2)*f*Sqrt[a + b*Cos[e + f*x]]) + (2*(c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*D
efer[Int][((c*(a^2*A + A*b^2*(1 - 2*m) - 2*a*b*B*(1 - m)))/2 - (a*(A*b - a*
B)*c*Cos[e + f*x])/2 - (b*(A*b - a*B)*c*(3 - 2*m)*Cos[e + f*x]^2)/2)/((c*Co
s[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]), x])/(a*(a^2 - b^2)*c)
```

Rubi steps

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx = \left((c\cos(e+fx))^m(c\sec(e+fx))^m\right)\int \frac{(c\cos(e+fx))^{-m}(A+}{(a+b\cos(e+}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)(c\sec(e+fx))^m\sin(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\cos(e+fx)}}+\frac{(2(c}{}$$

Mathematica [A]
time = 14.27, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x])^(3
/2),x]
```

```
[Out] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x])^(3
/2), x]
```

Maple [A]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(fx+e))\,(c\sec(fx+e))^{m}}{(a+b\cos(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x)
```

```
[Out] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a)^(3/2
), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m/(
b^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sec(e+fx))^{m}\,(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))**m/(a+b*cos(f*x+e))**(3/2),x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))/(a + b*cos(e + f*x))**(3/
2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a)^(3/2
), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{c}{\cos(e+f\,x)}\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)}{{\left(a+b\,\cos\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(3/2),x)
```

```
[Out] int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(3/2), x
)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```